Tackling the Axion Isocurvature Problem with Modulus Field Kination

Chandrika Chandrashekar

with Matthew Reece

Harvard University

2026 Phenomenology Symposium

Motivation

Construct a Cosmological Scenario for the QCD axion* where:

* QCD axion = axion that solves the strong CP problem

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1. QCD axion is high quality

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2. QCD axion comprises *all* the dark matter

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Construct a Cosmological Scenario for the QCD axion* where:

1. QCD axion is high quality
2. QCD axion comprises *all* the dark matter
3. Supports high inflationary energy scale

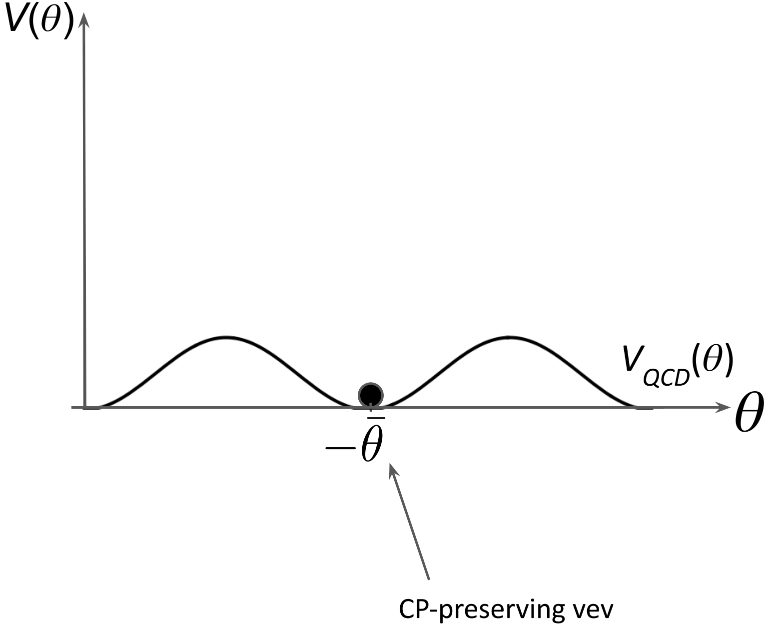
* QCD axion = axion that solves the strong CP problem

Motivation (*i.e., our Hopes and Dreams*)

QCD Axion of High Quality

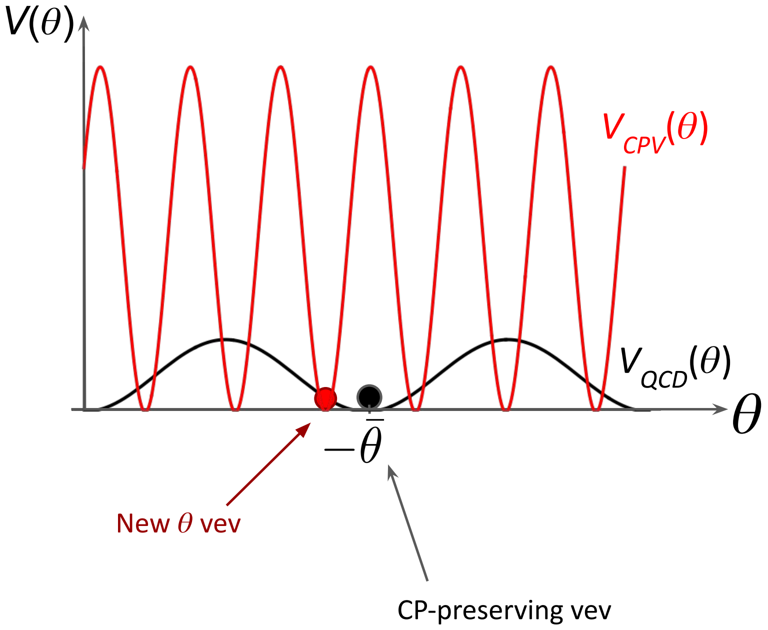
Quality Problem Lightning Review

'V_{QCD}' preserves CP symmetry



Quality Problem Lightning Review

Quality Problem: Higher order operators generically break CP symmetry[1]



Alleviating the Quality Problem with Extra Dimensional Axions

QCD Axion of High Quality

⇒ Consider: θ comes from higher dimensional gauge field *A*[1]

Gauge Symmetries of *A* will *Restrict* Axion Operators in the 4D EFT

$$V_{extra} \sim V_{QCD} \sim e^{-\#R}$$

Overall size of the extra dimensions

Motivation *Continued*

QCD Axion of High Quality

⇩

Axion from extra dimensions

[1] David Marsh. arXiv: arXiv:1510.07633v2 (2016)
[2] Ciaran A. J. O'Hare. arXiv:2403.17697v2 (2024)

Motivation *Continued*

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Axion present during inflation, seeds isocurvature[1,2] with amplitude $(H_{inf}/f)^2$

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QCD Axion Comprises *all* the Dark Matter

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Axion decay constant $f \approx 10^{12}$ GeV [1,2]

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[3] Clough, Katy, et al. "Robustness of inflation to inhomogeneous initial conditions." JCAP 2017.09 (2017): 025.

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Low H_{inf}, less robust inflation[3]

"**Axion Isocurvature Problem**"

Roadmap

Allowing a Larger H_{inf} with *Time Varying* Axion Decay Constant

f need *not* be constant *throughout* cosmic history.

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⇒ Need a mechanism that allows f to *decrease* between inflation and reheating

Decreasing Axion Decay Constant with Bulk Modulus Evolution

Consider: a (4+n)-dimensional spacetime manifold $M = X_{4D} \times Y_{nD}$ with gauge field A

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$$f^2 \propto \frac{1}{\mathcal{V}_Y}$$

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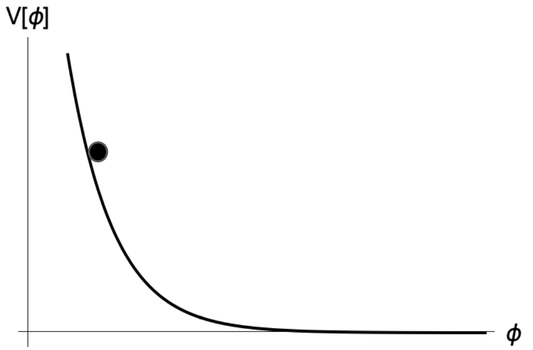

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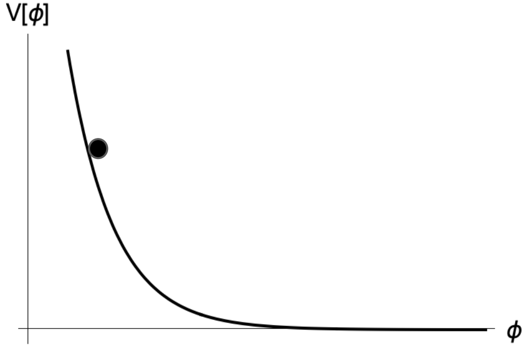

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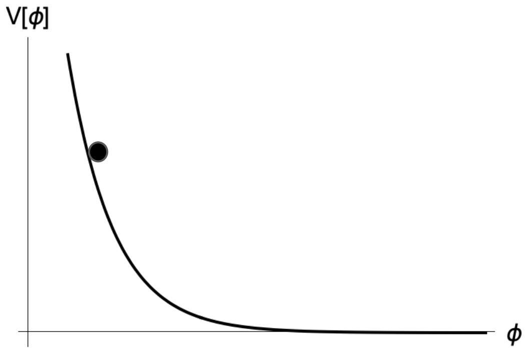
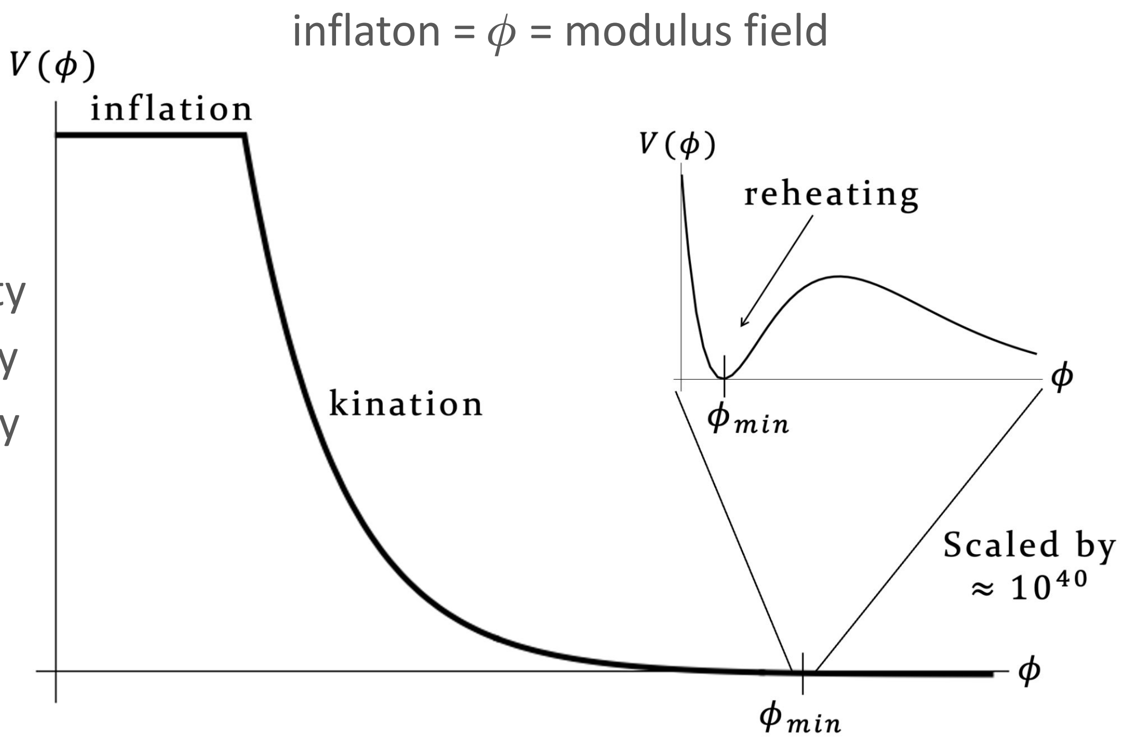


$\Rightarrow f^2 \sim \mathrm{Exp}[-\#\phi/M_P]$

ϕ increases $\Rightarrow$ Volume $\mathcal{V}$ increases $\Rightarrow$ *f decreases*

Cosmic History with Bulk Modulus Field

Kination = energy density dominated by kinetic energy of scalar field[1,2]



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Cosmic History with Bulk Modulus Field

$f \gg 10^{12}$ GeV

inflaton = ϕ = modulus field

$f \approx 10^{12}$ GeV

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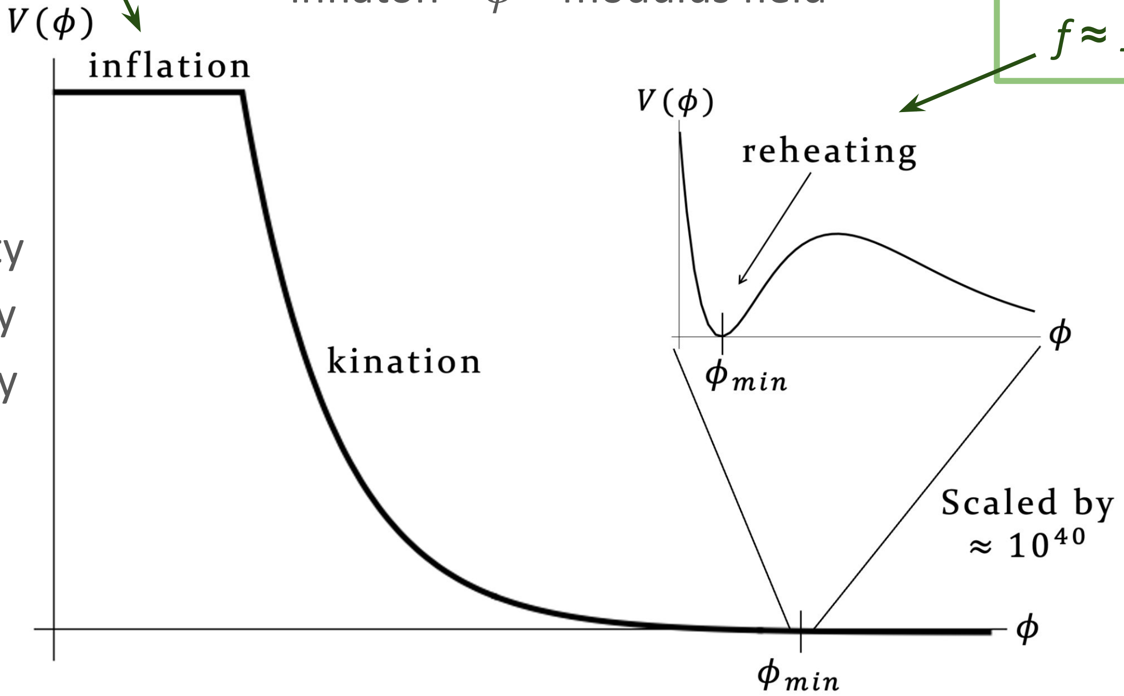

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Ending Kination with Radiation

Need to end kination to avoid decompactification

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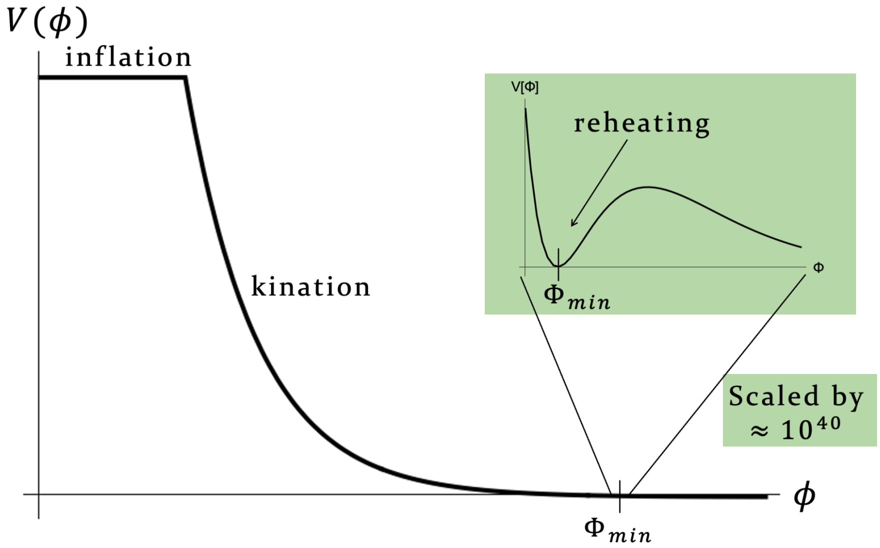
$\Rightarrow$ modulus overshoots the minimum[1]



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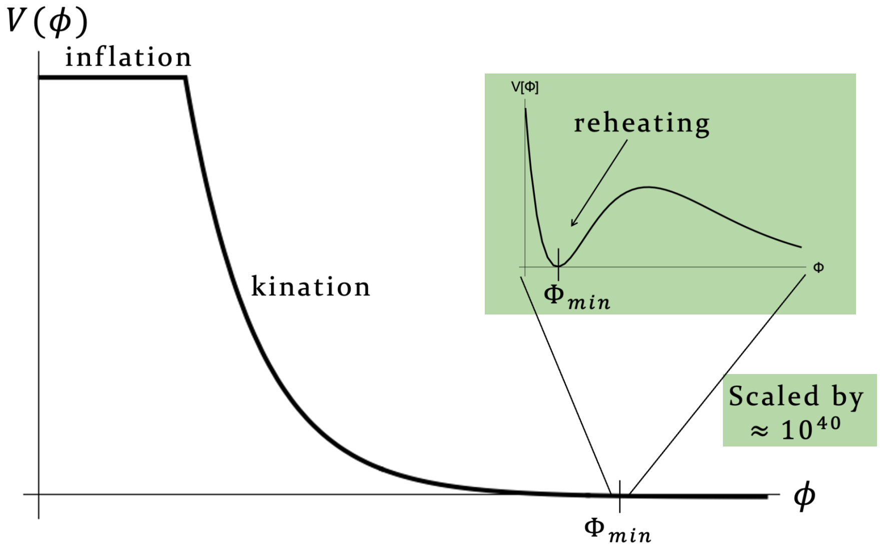

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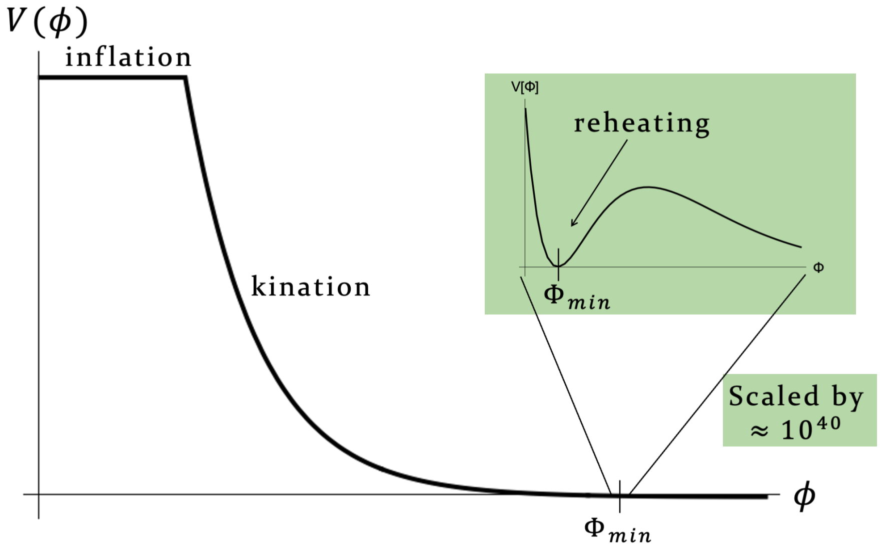
Radiation Dilutes More Slowly that Modulus Kinetic Energy
⇒ Can Eventually Dominate and End Kination

⇒Modulus trapped if kination ends before ϕ_{min}[1,2]



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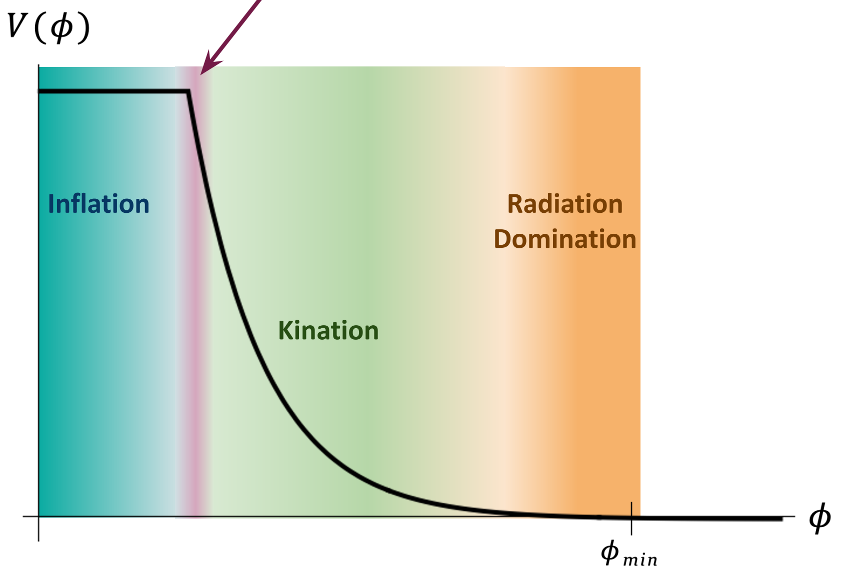

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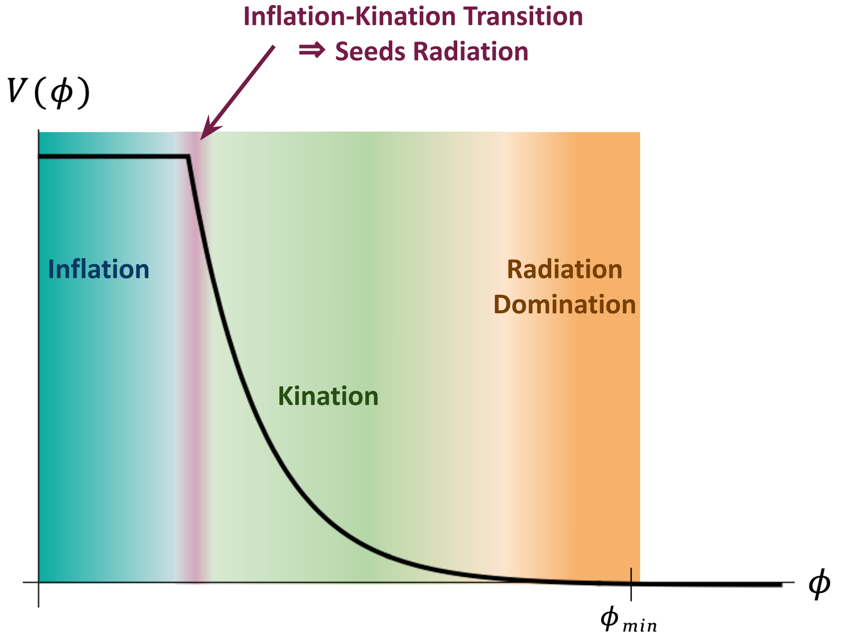

Remainder of the Talk: **How to Source Radiation during the Inflation-Kination Transition to End Kination**

Trapping the Bulk Modulus with Generic Sources of Radiation

Extra Dimensional Gauge Fields as Generic Sources of Radiation

Extra-Dimensional Theory → compactifying → **4D EFT**

higher dimensional gauge field → modulus field, axion

Extra Dimensional Gauge Fields as Generic Sources of Radiation

Extra-Dimensional Theory —compactifying→ **4D EFT**

(i) p-form gauge field $C^{(p)}$ → modulus field

(ii) n independent p-cycles → n light compact scalars[1]

(iii) m independent (p-1)-cycles → m massless gauge fields

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⇒ **4D scalars and gauge fields couple to modulus** via kinetic prefactors $f^2(\phi)$ that depend exponentially on ϕ

[1] Matthew Reece ArXiv: 2406.08543v2 (2024)

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4D EFT:

$$\mathcal{L} \supset \underbrace{\frac{1}{2}\partial_\mu\Phi\,\partial^\mu\Phi}_{\text{modulus kinetic term}} + \underbrace{\frac{1}{2}f_s^2(\Phi)\,\partial_\mu\theta\,\partial^\mu\theta}_{\text{compact scalar kinetic term}} - \underbrace{\frac{1}{4}f_g^2(\Phi)\,F_{\mu\nu}F^{\mu\nu}}_{\text{4D gauge field kinetic term}} - \underbrace{V_0\exp(-\lambda\Phi/M_P)}_{\text{modulus potential}}$$

where $f_s^2 \sim \exp(-\alpha_s\Phi/M_P)$ with $\alpha_s > 0$ and $f_g^2 \sim \exp(-\alpha_g\Phi/M_P)$ with $\alpha_g < 0$

α's depend on which cycle the scalar/gauge field corresponds to

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Coupling to the modulus can enhance particle production of 4D scalar and gauge fields and source the radiation needed to end kination

Scalar Field Particle Production

EOM for scalar field mode functions θ_k

$$\theta_k'' + \left(\frac{2a'}{a} - \frac{\alpha_s\, \phi'}{M_P}\right)\theta_k' + k^2\theta_k = 0$$

Scalar Field Particle Production

EOM for scalar field mode functions θ_k can be rewritten as a **time-dependent simple harmonic oscillator (SHO)**

$$\theta_k'' + \underbrace{\left(\frac{2a'}{a} - \frac{\alpha_s \phi'}{M_P}\right)}_{\equiv b(\eta)} \theta_k' + k^2 \theta_k = 0$$

rescale:

$$\chi_k(\eta) \equiv \exp\left(\frac{1}{2}\int^{\eta} b(\eta')\mathrm{d}\eta'\right) \theta_k(\eta)$$

Time-dependent SHO

$$\chi_k'' + \underbrace{\left(k^2 - U_{eff}\right)}_{\equiv \omega_k^2(\eta)} \chi_k = 0$$

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Initial condition: $n_k = 0$ at the start of inflation.

Time varying frequency $\omega_k \Rightarrow$ particle production[1]

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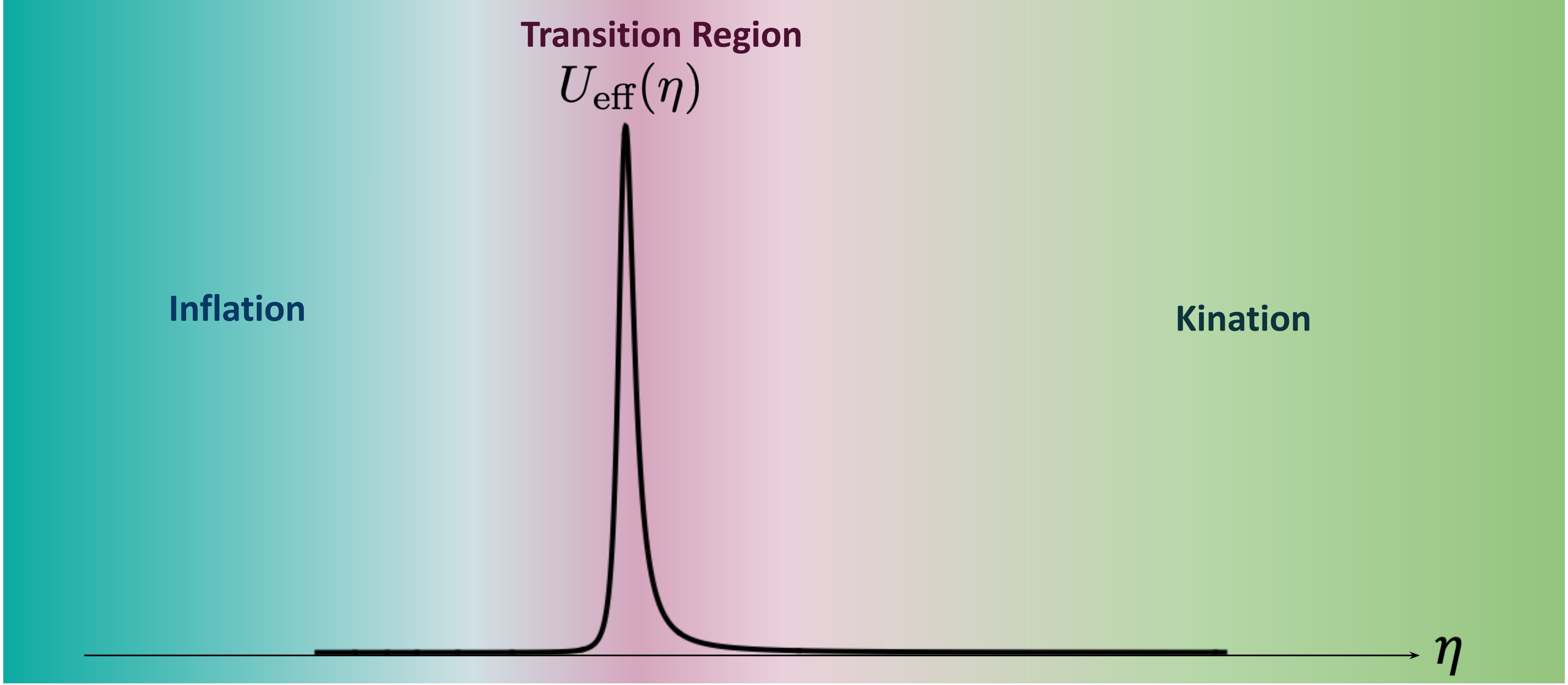
⇒ Most particle production when U_{eff} is changing rapidly and/or when $k^2 = U_{eff}$ (when modes re-enter the horizon)

Plotting the Effective Potential



U_{eff} changes rapidly during inflation-kination transition
⇒ Potentially large particle production for modes re-entering the horizon during this time

Calculating Particle Number

Bogoliubov Coefficients

Ansatz for SHO mode functions χ_k:

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k}} e^{-i\int \omega_k \mathrm{d}\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k}} e^{i\int \omega_k \mathrm{d}\eta}$$

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$$\chi_k'' + (k^2 - U_{eff})\chi_k = 0 \iff \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{2m}{\hbar^2}(E - V(x))\,\psi = 0$$

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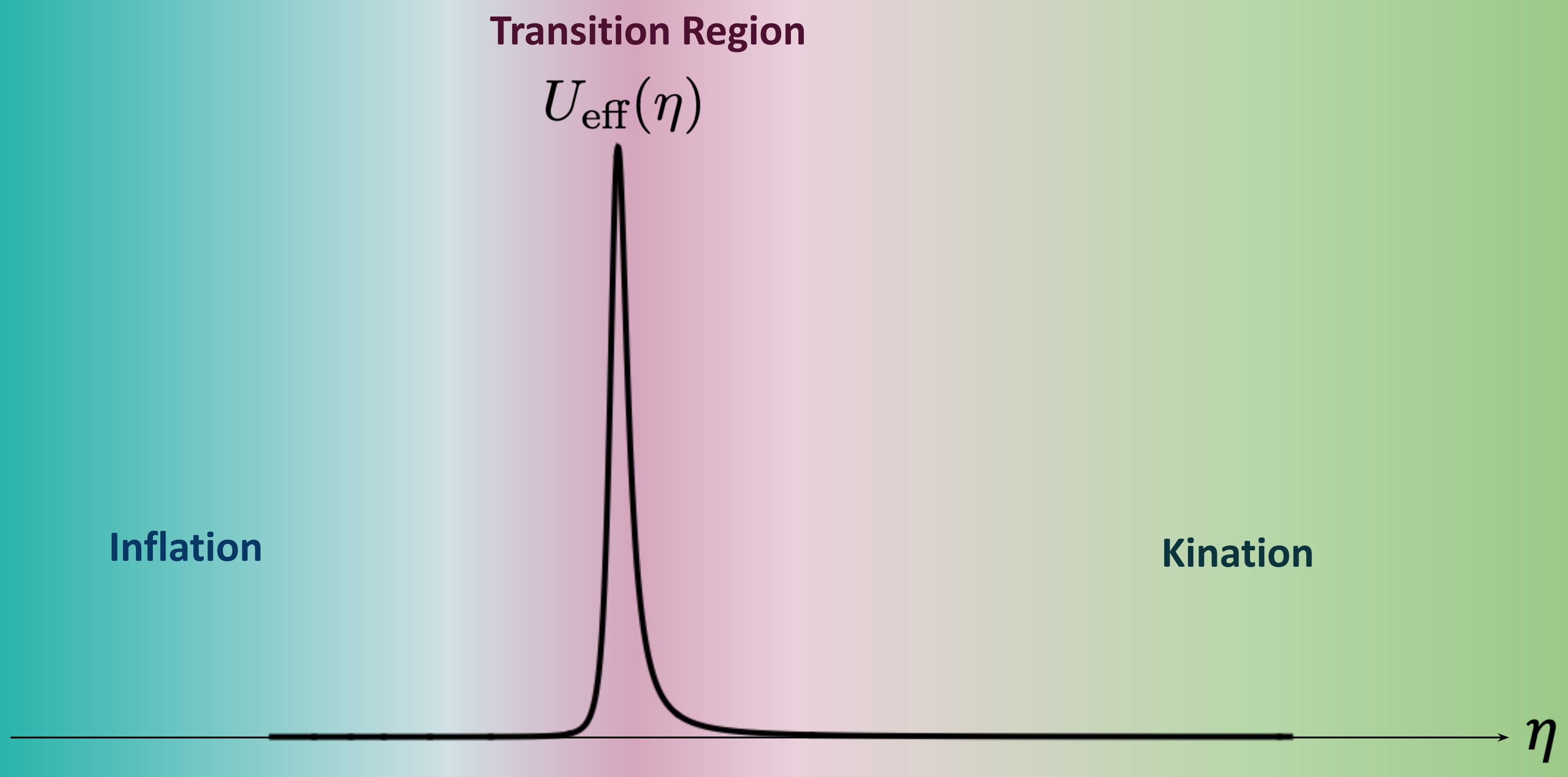
⇒ this calculation is analogous to a QM scattering problem!

Particle Production Recast as a Scattering Problem



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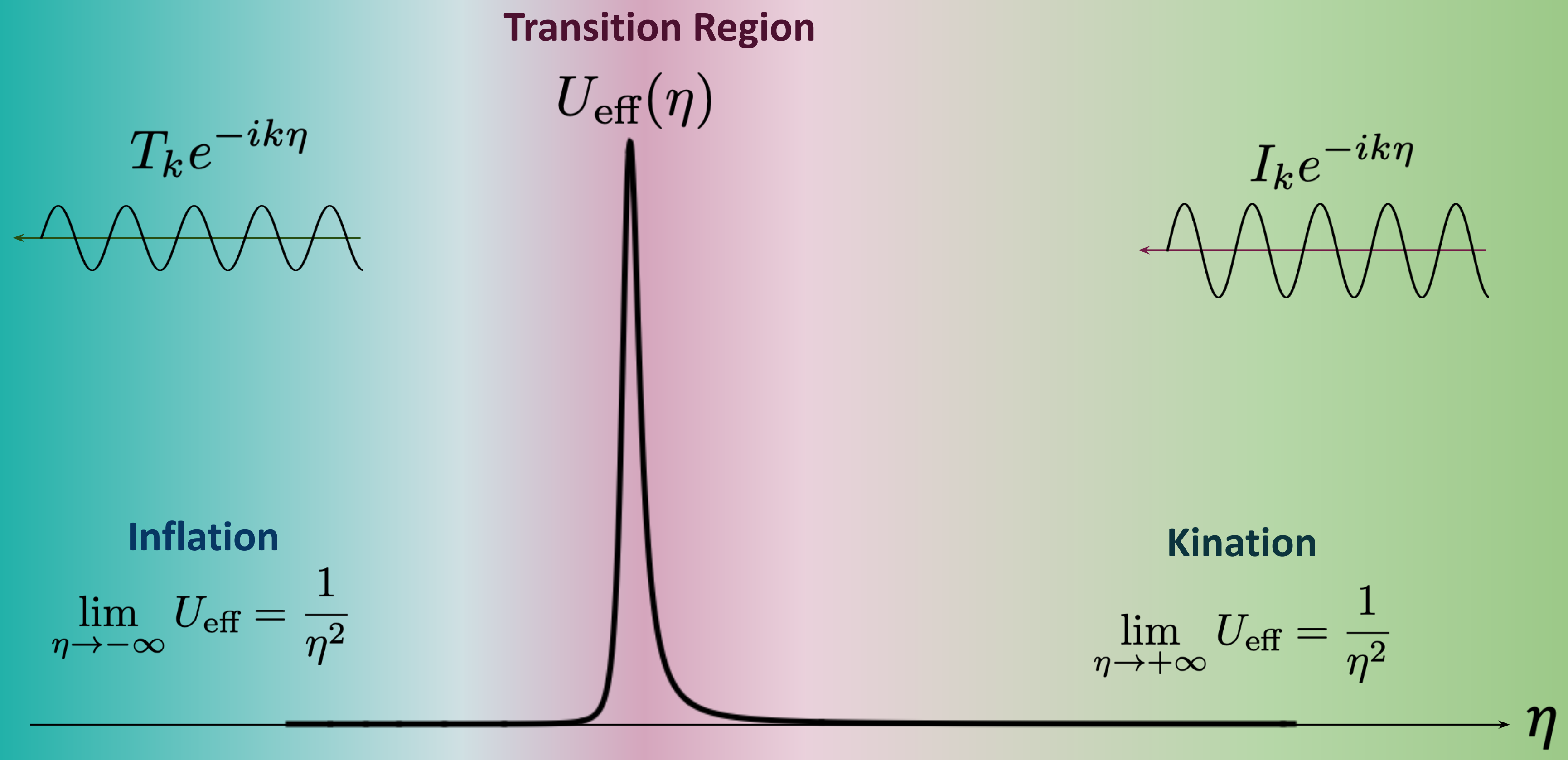

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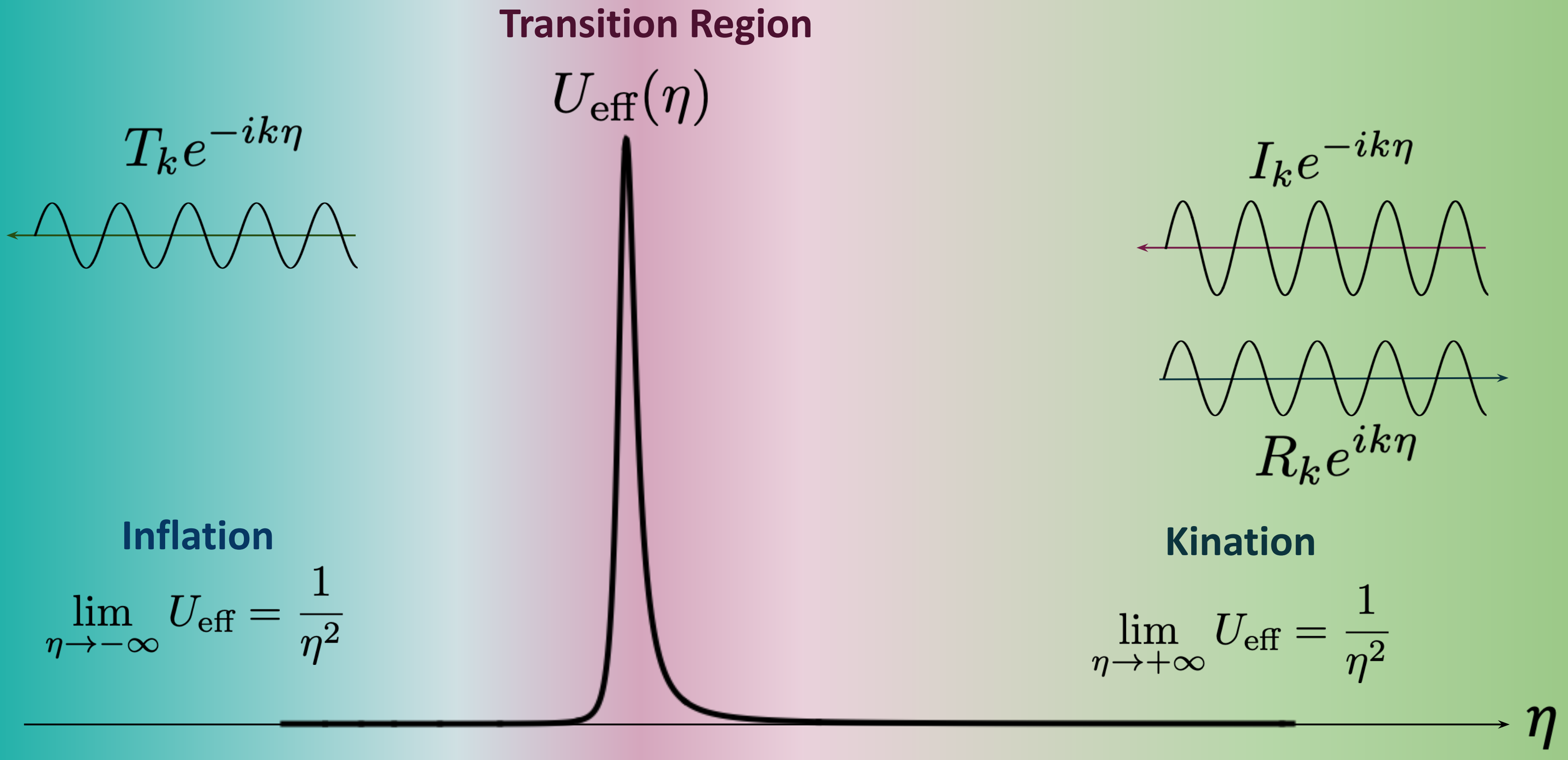
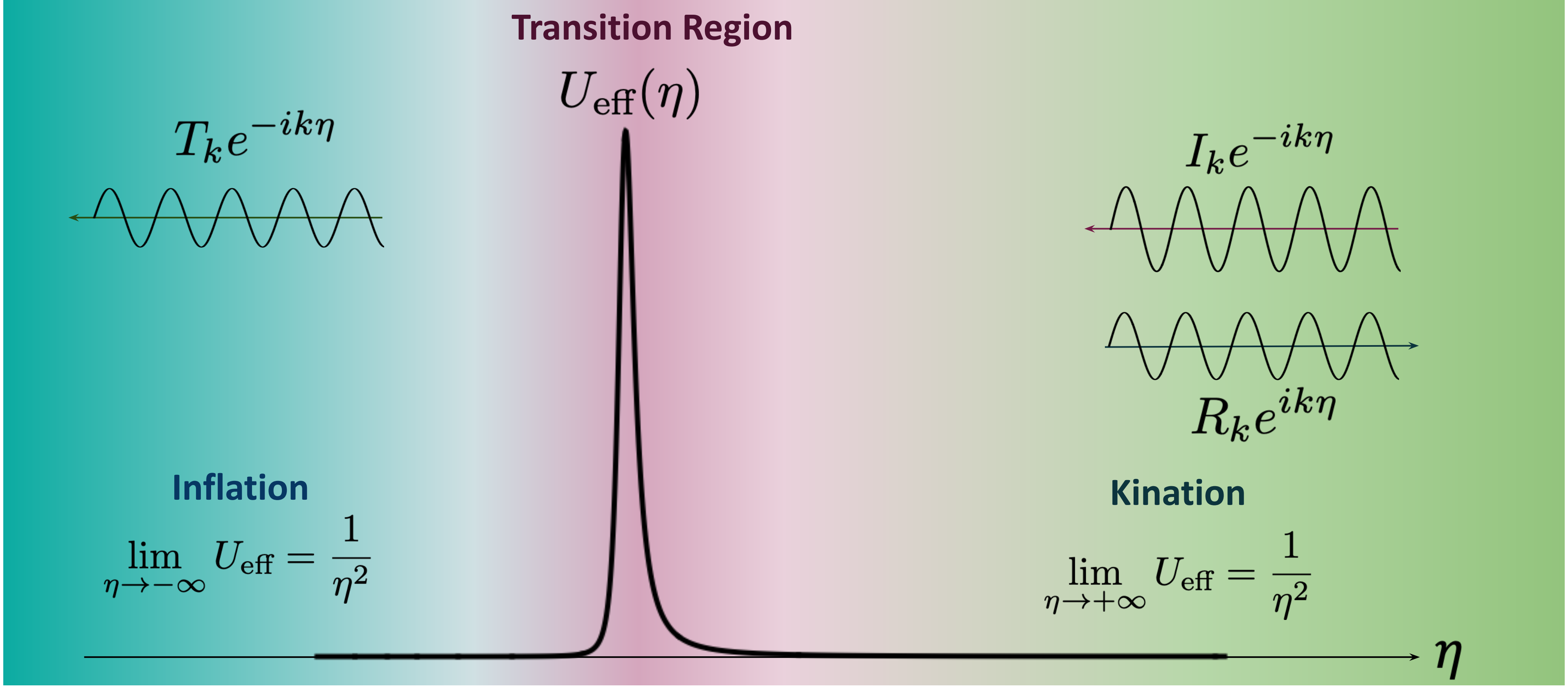


Particle Production Recast as a Scattering Problem



occupation number: $n_k(\eta) = |\beta_k(\eta)|^2 = \frac{|R|^2}{|T|^2}$

Particle Production at Large and Small *k*

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$k^2 > U_{\text{eff}}$ and $kL \gg 1$	
$|\beta_k|^2 \sim \exp(-kL)$ [1] ↑ Width of potential	

[1] *Michael Berry: Journal of Physics A: Mathematical and General* 15.12 (1982)

Particle Production at Large and Small k

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Implications for Energy Density

$$\text{energy density: } \rho(\eta) \sim \int_{k_{min}}^{k_{max}} \frac{\mathrm{d}k}{a^4(\eta)} \underbrace{k^3\, |\beta_k(\eta)|^2}$$

$$\lim_{k\to 0} k^3 |\beta_k|^2 \sim \frac{1}{k^{2(\nu_\iota+\nu_\kappa)-3}} \sim \frac{1}{k^n}$$

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$$\implies \text{ for } \alpha_s \geq \sqrt{\frac{2}{3}}, \text{ we find IR divergences in the energy}$$

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Common in string theories

$$\implies \text{for } \alpha_s \geq \sqrt{\frac{2}{3}}, \text{ we find IR divergences in the energy}$$

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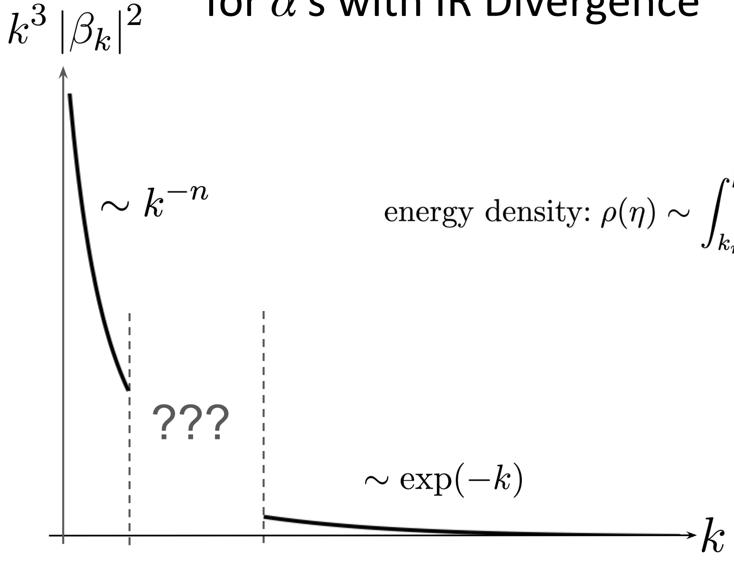
Next Steps: (1) Numerically Solve for β_k at intermediate k to find total energy stored in radiation

for α's with IR Divergence



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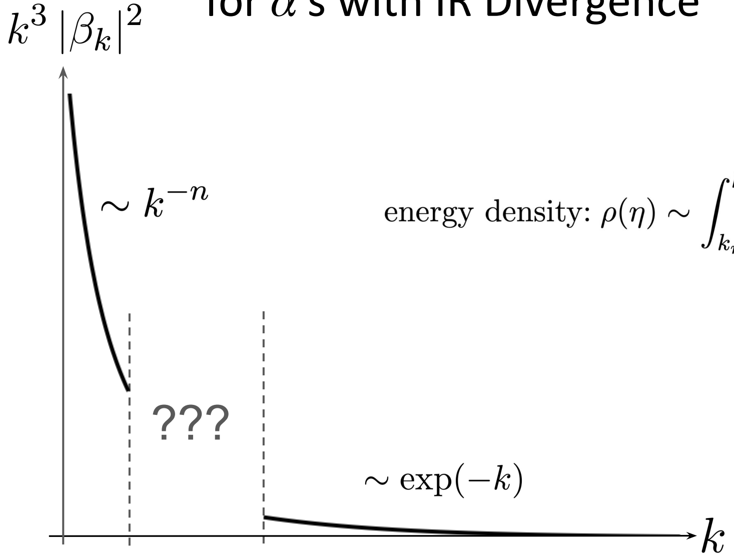

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(2) Determine the number of scalar and gauge fields needed to end kination ⇒ address the axion isocurvature problem!

Next Steps: (1) Numerically Solve for β_k at intermediate *k* to find total energy stored in radiation

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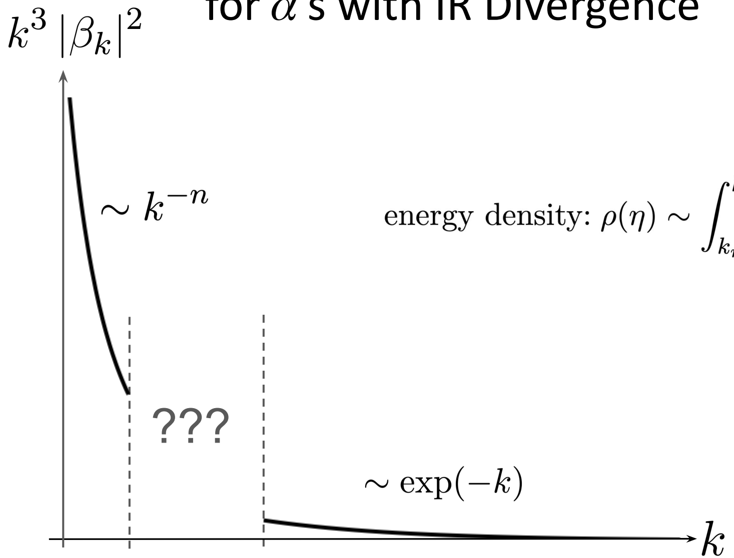


$$\text{energy density: } \rho(\eta) \sim \int_{k_{min}}^{k_{max}} \frac{\mathrm{d}k}{a^4(\eta)}\, k^3\,|\beta_k(\eta)|^2$$

Thank You!

(2) Determine the number of scalar and gauge fields needed to end kination ⇒ address the axion isocurvature problem!

Thank You!

Backup Slides

QCD Axion: A Solution to the Strong CP Problem

[1] Hook, Anson. arXiv:1812.02669 (2018).
[2] Reece, Matthew. arXiv:2304.08512 (2023).
[3] Abel, Christopher, et al. Physical Review Letters 124.8 (2020): 081803.

The QCD Lagrangian has a CP violating term[1,2]:

$$\mathcal{L}_{QCD} \supset \bar{\theta}\left(\frac{g_s^2}{32\pi^2}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad \text{where} \quad \tilde{G}^{a\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}$$

Extent of CP violation parametrized by $\bar{\theta}$

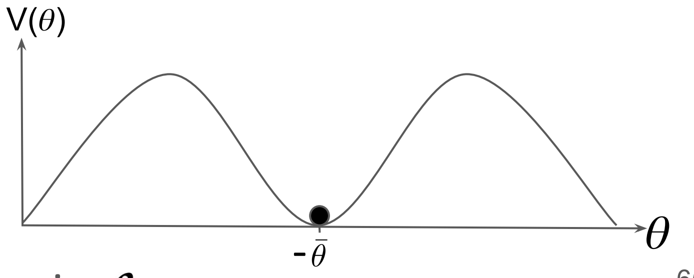
From neutron EDM measurements[3]: $\bar{\theta} \leq 10^{-10}$

Introducing the Axion field, θ, with gluon coupling[1,2]: $\mathcal{L} \supset \mathcal{L}_{QCD} + \frac{f^2}{2}\partial_\mu\theta\,\partial^\mu\theta + \theta\left(\frac{g_s^2}{32\pi^2}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$

Generates periodic axion potential[1]

$$V(\theta) = -F_\pi^2 m_\pi^2\sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2(\bar{\theta}+\theta(x))}$$

minimized at $\langle\theta\rangle = -\bar{\theta}$



⇒ Axion dynamically relaxes and **cancels CP violating term** in $\mathcal{L}_{QCD}$

True Axion Solutions to CP-Problem Avoid *Quality Problem*

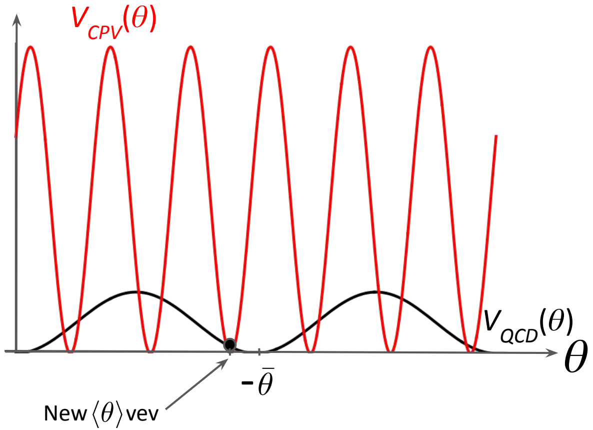
But, we can always consider additional operators in $\mathscr{L}$[1]:

$$\mathcal{L} \supset \frac{|c|e^{i\varphi}}{M_P^{n-4}}\phi^n + h.c. \quad \text{where} \quad \phi(x) = \frac{1}{\sqrt{2}}(f + s(x))\exp(i\theta(x))$$

CP-Violating (CPV) Potential: $V_{CPV}(\theta) = 2|c|M_P^4\left(\frac{f}{\sqrt{2}M_p}\right)^n \cos(n\theta+\varphi)$

e.g., for n = 8
and $f = 10^{12}$ *GeV*[1]: $V_{CPV}(\theta) \approx (250\,\mathrm{TeV})^4\cos(8\theta+\varphi)$

$$V_{QCD}(\theta) \approx (110\,\mathrm{MeV})^4\cos(\theta)$$



⇒ "**The Quality Problem**": higher dimensional operators generically break CP symmetry.

Can be alleviated with axions coming from *gauge fields* living *in extra dimensions*[1]

Alleviating the Quality Problem with Extra Dimensional Axions

Set up: θ comes from higher dimensional gauge field *A*[1]

Coupling of *A* to *G* generates V_{QCD} in 4D

To spoil CP, we need non-derivative axion terms

⇒ non-derivative gauge terms in higher dimensions

⇒ **gauge symmetries** permit two kinds of terms

(1) *A* couples to other gauge fields

(2) *A* couples to charged fields

Covariant derivative generates axion potential in 4D

$$V_{extra} \sim V_{QCD} \sim e^{-\#R}$$

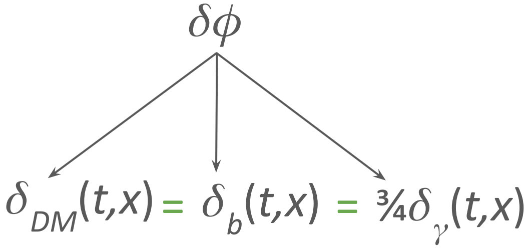
Where R is the size of the extra dimensions

⇒ We will be working with **extra dimensional axions**

Extra Dimensional Axions Seed Isocurvature Fluctuations

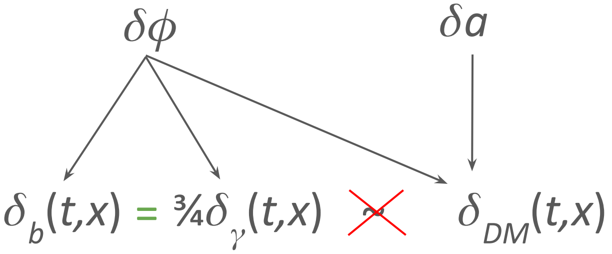
Extra-dimensional axions ⟶ around during inflation ⟶ seed isocurvature[1,2]

Inflation with Inflaton Only



⇒ initial fluctuations are *adiabatic*[3]

Inflation with Inflaton and Spectator Field



⇒ initial fluctuations are *non-adiabatic*

Known as "isocurvature"

[1] David Marsh. arXiv: arXiv:1510.07633v2 (2016)
[2] Ciaran A. J. O'Hare. arXiv:2403.17697v2 (2024)
[3] Baumann, Daniel. Cosmology. Cambridge University Press, 2022.

Isocurvature Constraints from the CMB

$$\mathcal{P}_{a,\,iso} = \mathcal{A}_{a,\,iso}\left(\frac{k}{k_*}\right)^{n_{iso}}$$

$$\mathcal{A}_{a,\,iso} = \left(\frac{\delta\rho_{a,\,iso}}{\rho_a}\right)^2 = \left(\frac{H_{inf}}{\pi f \theta_i}\right)^2 \lesssim 8\cdot 10^{-11}$$ [1]

$$\longrightarrow \quad \boxed{H_{inf} \lesssim \theta_i\left(\frac{f}{10^{12}\,GeV}\right)10^7\,GeV}$$

⇒ Low inflationary Hubble scale

Problem: inflation with low H_{inf} tends to end prematurely[2]

[1] Ciaran A. J. O'Hare. arXiv:2403.17697v2 (2024)

[2] Clough, Katy, et al. "Robustness of inflation to inhomogeneous initial conditions." Journal of Cosmology and Astroparticle Physics 2017.09 (2017): 025.

Extra Dimensional Axions are Pre-Inflationary and Seed Isocurvature Fluctuations

Extra-dimensional axions $\longrightarrow$ around during inflation

$\longrightarrow$ seed isocurvature (fluctuations independent from the inflation's) [1,2]

CMB constraints: $$\left(\frac{H_{inf}}{\pi f \theta_i}\right)^2 \lesssim 8 \cdot 10^{-11}$$ [1]

$$\longrightarrow \quad \boxed{H_{inf} \lesssim \theta_i \left(\frac{f}{10^{12}\, GeV}\right) 10^7\, GeV}$$

For QCD axion to comprise **all the dark matter**, we need $f \approx 10^{12}$ GeV [2,3]

$\Rightarrow$ Low inflationary Hubble scale

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[1] David Marsh. arXiv: arXiv:1510.07633v2 (2016)
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[3] Baumann, Daniel. Cosmology. Cambridge University Press, 2022.

Low Inflationary Energy Scale ⇒ Small Field Displacement⇒not enough inflationary e-folds

Tensor power spectrum amplitude

Scalar power spectrum amplitude

$$r \equiv \frac{A_t}{A_s} \qquad A_t = \frac{2H_*^2}{\pi^2 M_P^2} \qquad A_s = \left(\frac{H_*^2}{2\pi\dot{\phi}_*}\right)^2 \quad \Rightarrow \quad r = \frac{8}{M_P^2}\left(\frac{d\phi}{dN}\right)^2$$

⇘ ⇙ ⇓

$$\frac{H_*}{M_P} = \frac{\pi}{\sqrt{2}}\sqrt{A_s r} \quad \Rightarrow \quad \frac{\Delta\phi}{H_*} = \frac{N}{2\pi}\sqrt{\frac{1}{A_s}} \quad \Leftarrow \quad \frac{\Delta\phi}{M_P} = N\sqrt{\frac{r}{8}}$$

⇘

$$N \sim \ln T_R \quad \& \quad T_R \sim \sqrt{M_P H_{inf}}$$

$$\boxed{\Delta\phi \sim H_{inf} \ln\left(\sqrt{H_{inf}}\right)}$$

p-cycles and $C^{(p)}$ compactifications

“p-cycles” are p-dimensional submanifolds which:

- Have no boundary
- Are not a boundary themselves

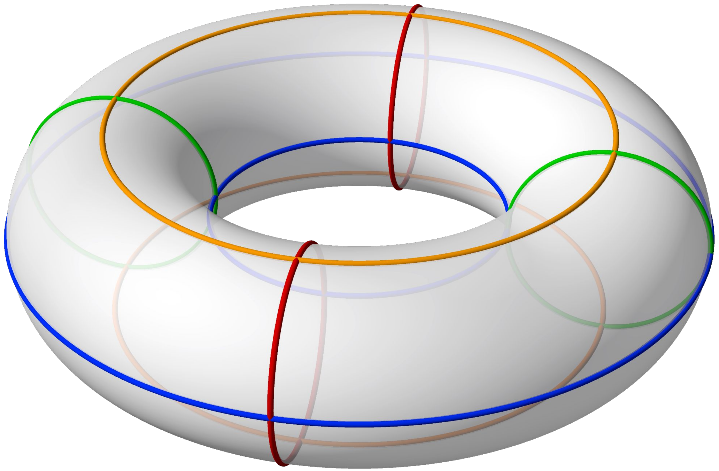

Image credit: Wikipedia

Axions from Extra Dimensional Gauge Fields

Consider: 5 dimensional manifold with 5th dimension a compact circle of radius R,

$M = X_{4d} \times S^1$

with 1-form gauge field *A* living in 5D $\vec{A} = A_\mu x^\mu + A_5 x^5$

How does the 5D field *A* manifest in 4D?

5D kinetic term: $S = \int\limits_{X_{4d}\times S^1} \sqrt{\det[g_{MN}]}\, \mathrm{d}^5x \left(-\frac{1}{4e_5^2} F_{MN}F^{MN}\right)$

where $F_{MN} = \partial_M A_N - \partial_N A_M \sim (\partial_\mu A_\nu - \partial_\nu A_\mu) + (\partial_\mu A_5 - \partial_5 A_\mu)$

$$A_\mu(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \tilde{A}_\mu^{(n)}(x^\mu) \qquad A_5(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \phi^{(n)}(x^\mu)$$

$$\tilde{A}^{(-n)} = \left(\tilde{A}^{(n)}\right)^*$$

$$\phi^{(-n)} = \left(\phi^{(n)}\right)^*$$

Define **axion** as zero mode of A_5[1]: $\theta(x_\mu) \equiv \int_0^{2\pi R} A_5(x_\mu, x_5)\, \mathrm{d}x_5 = 2\pi R\, \phi^{(0)}(x^\mu)$

[1] Matthew Reece. *arXiv:2304.08512* (2023).

Extra Dimensional Axions have Decay Constants that Depend on Extra Dimensional Geometry

$A_5 \to A_5 + \partial_5 \alpha$ where $\alpha = x_5/R$

$\theta(x_\mu) \equiv \int_0^{2\pi R} A_5(x_\mu, x_5)\, \mathrm{d}x_5 \to \theta(x_\mu) + 2\pi$ $\Rightarrow$ θ is 2π periodic ✓

Recall: $A_5(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \phi^{(n)}(x^\mu)$

$$\mathcal{L}_{4d} \supset -\frac{1}{4e_5^2} \int_{S^1} (\partial_\mu A_5)^2 = \frac{2\pi R}{4e_5^2} \sum_n \left(\partial_\mu \phi^{(n)}\right)\left(\partial^\mu \phi^{(n)}\right)$$

⇓ Zero mode, using $\theta(x_\mu) = 2\pi R\, \phi^{(0)}(x^\mu)$

$$\boxed{\mathcal{L}_{4d} \supset \frac{1}{2} f^2 \partial_\mu \theta\, \partial^\mu \theta}$$ where $$\boxed{f^2 = \frac{1}{2\pi R e_5^2}}$$

More *generally*, for a higher dimensional manifold[1]

$M = X_{4D} \times Y_{nD}$,

$$f^2 \propto \frac{1}{\mathcal{V}_Y}$$

where $\mathcal{V}_Y$ = volume of Y_{nd}

[1] Matthew Reece. arXiv: 2406.08543 (2024).

Compactifying Extra Dimensions Generically Leads to a Scalar Field with Exponential Potential in 4D

Again start with higher dimensional manifold $M = X_{4D} \times Y_{nD}$

$$ds^2 = g_{MN}(X)\, dX^M dX^N = g_{\mu\nu}(x)\, dx^\mu dx^\nu + \boxed{\lambda^2(x)}\, g_{mn}(y)\, dy^m dy^n$$

Overall volume of Y depends on position in 4D

$$S_{EH} = \int_M \boxed{\sqrt{\det[g_{MN}]}}\, \boxed{\mathcal{R}_{(4+n)D}}\, \boxed{M^{2+n}_{(4+n)}}\, d^4x\, d^n y$$

Higher dimensional Planck constant

$$\boxed{\det[g_{MN}]} = \lambda^{2n}(x) \det[g_{\mu\nu}(x)] \det[g_{mn}(y)]$$

$$\boxed{\mathcal{R}_{(4+n)D}} = \mathcal{R}_{4D} + \underbrace{\frac{\mathcal{R}_{nD}(y)}{\lambda^2(x)}}_{\text{Potential term}} + \underbrace{\#\frac{1}{\lambda^2(x)}\, g^{\mu\nu}(x)\, \partial_\mu \lambda(x)\, \partial_\nu \lambda(x)}_{\text{Kinetic term}}$$

Kinetic term $\rightarrow\ \sim \partial_\mu \Phi\, \partial^\mu \Phi$

Potential term $\rightarrow$

$$\boxed{V(\Phi) \sim \left(e^{-\Phi}\right)^{n+2}}$$

bulk modulus field: $\phi = M_p \Phi$

Weyl rescaling: $\tilde{g}_{\mu\nu} = \lambda^n(x) g_{\mu\nu}$

Canonical field: $\lambda(x) = e^{\Phi(x)}$

Particle Number in Curved Spacetimes

Bulk axion quantum field in terms of rescaled mode functions:

$$\hat{\theta}_b(x,t) = \int d^3k\, a^{1/2} \left[\hat{a}_k X_k(t) e^{-ikx} + \hat{a}_k^\dagger X_k^*(t) e^{ikx} \right]$$

$$\ddot{X}_k + \omega_k^2 X_k = 0 \xrightarrow{\text{ansatz}} X_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega}} e^{i\int \omega dt} + \frac{\beta_k(t)}{\sqrt{2\omega}} e^{-i\int \omega dt}$$

$$|\alpha|^2 - |\beta|^2 = 1$$

Rewriting mode expansion in terms of α_k and β_k:

$$\hat{\theta}_b(x,t) = \int \frac{d^3k\, a^{1/2}}{\sqrt{2\omega}} \left[\left(\alpha_k(t)\, \hat{a}_k + \beta^*_{-k}(t) \hat{a}^\dagger_{-k} \right) e^{-i\int \omega dt} e^{-ikx} + \left(\beta_{-k}(t) \hat{a}_{-k} + \alpha_k^*(t) a_k^\dagger \right) e^{i\int \omega dt} e^{ikx} \right]$$

$$\hat{b}_k(t) \equiv \alpha_k(t) \hat{a}_k + \beta^*_{-k}(t) \hat{a}^\dagger_{-k}$$

$$\hat{b}_k^\dagger(t) \equiv \alpha_k^*(t) \hat{a}_k^\dagger + \beta_{-k}(t) \hat{a}_{-k}$$

$$\left[\hat{b}_k, \hat{b}_k^\dagger \right] = 1$$

"Bogoliubov Transformations"

Particle Number in Curved Spacetimes Cont'd

$$\hat{a}_k|\Omega\rangle = 0$$

$$n_k(t) \equiv \langle\Omega\,|\,\hat{N}_k(t)\,|\,\Omega\rangle = \langle\Omega\,|\,\hat{b}_k^\dagger(t)\hat{b}_k(t)\,|\,\Omega\rangle = |\beta_k(t)|^2$$

$$= \frac{\omega_k}{2}\left(\frac{|\dot{X}_k|^2}{\omega_k^2} + |X_k|^2\right) - \frac{1}{2}$$

⇒ solve for mode functions $X_k(t)$

⇒ determine comoving $n_k(t)$

⇒ physical number density given by $n_k(t)/a^3(t)$

Scenario with Largest Possible H_{inf}

$f \lesssim M_P$ → $H_{inf} \lesssim 5 \cdot 10^{13}\,\mathrm{GeV}$

isocurvature constraint

→ $\Delta\phi \lesssim 24$

taking $f_{RH} \approx 10^{12}\, GeV$

⇒ e-folds of kination ~ 7.7

Implications for Energy Density

Recall, $\text{energy density: } \rho(\eta) \sim \int_{k_{min}}^{k_{max}} \frac{\mathrm{d}k}{a^4(\eta)} \underbrace{k^3 \, |\beta_k(\eta)|^2}$

$$\lim_{k \to 0} k^3 |\beta_k|^2 \sim \frac{1}{k^{2(\nu_\iota + \nu_\kappa) - 3}} \sim \frac{1}{k^n}$$

For n ≥ 1, the integral will have an IR divergence as $k_{min} \to 0$.

Scalars	Gauge Fields
$\nu_\iota = 3/2$	$\nu_\iota = 1/2$
$n \geq 1 \implies \nu_\kappa \geq 1/2$ $\quad \nu_\kappa = \frac{\alpha}{2}\sqrt{\frac{3}{2}}$	$n \geq 1 \implies \nu_\kappa \geq 3/2$
$\implies \alpha \geq \sqrt{2/3}$	$\nu_\kappa = \frac{1}{4}\sqrt{6\alpha^2 + 4\sqrt{6}\alpha + 4}$
$\alpha_{QCD} = \sqrt{3/2}$ $\quad \alpha_{bulk} = 2\sqrt{2/3}$	$\implies \alpha \leq -4\sqrt{2/3}$
⇒ IR divergences for common values of scalar α's	smaller than common values of α