String Cosmology and the Hagedorn Transition

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May 11, 2025

What Sets Up Inflation?

- Big Bang + rapid expansion: the current paradigm
- Inflation predicts CMB temperature fluctuations
- **Initial conditions problem:** inflaton potential, e-folds, singularity
- If string theory is the UV completion of gravity, the universe must have passed through a stringy regime

What does the pre-inflationary universe look like in string theory?

Stringy Origins: The Brandenberger–Vafa Picture

Classical cosmology:

- Run the universe backwards → temperature rises without bound → singularity

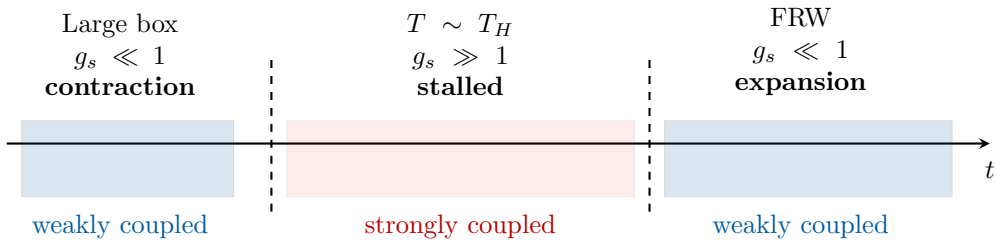
String theory:

- All string theories exhibit a maximum temperature
- Density of states grows exponentially: energy → new states, not higher T
- ⇒ **Hagedorn temperature** T_H

At T_H:

- Universe stalls at finite size; strings act like rubber bands around dimensions
- Wound string + anti-wound string → string loop → dimension grows
- ⇒ 3 spatial dimensions expand, rest stay compact

The Cosmological Narrative



- Start: large box of weakly coupled strings contracts adiabatically
- Middle: temperature saturates at T_H; string coupling grows; universe **stalls**
- End: winding modes annihilate → three dimensions expand → FRW cosmology

Can we make this narrative precise using an effective field theory?

How to Model a Stringy Cosmology

Low-energy effective string action:

$$S = \frac{1}{2\kappa_N^2} \int d^N x \, \sqrt{-g} \, e^{-\phi} \left(-R - (\partial\phi)^2 + \mathcal{L}_m \right)$$

- ϕ = dilaton, controls string coupling $g_s = e^{\phi}$
- $\mathcal{L}_m$ = matter content (previously modeled thermodynamically)

Homogeneous cosmological background:

$$ds^2 = n^2(t)\, dt^2 - e^{2\lambda(t)}\, d\vec{x}^{\,2}$$

- $\lambda = \ln a$ (log of scale factor)
- Extra dimensions assumed compact; study $3 + 1$ dimensions

The Thermal Scalar

Matter content: complex scalar field describing ± 1 winding modes

$$\mathcal{L}_m = |\partial\chi|^2 - \mu^2(\beta)|\chi|^2 \qquad \mu^2(\beta) = \frac{\beta^2 - \beta_H^2}{(2\pi\alpha')^2}$$

$$\begin{array}{lll} T < T_H: & \mu^2 > 0 & \text{(massive)} \\ T = T_H: & \mu^2 = 0 & \text{(massless)} \\ T > T_H: & \mu^2 < 0 & \text{(tachyonic)} \end{array}$$

Atick–Witten (1988) [Nucl. Phys. B310, 291]; Horowitz–Polchinski (1998) [hep-th/9707170]

The gravi-dilaton action becomes:

$$S = \frac{1}{2\kappa_N^2} \int d^N x \, \sqrt{-g}\, e^{-\phi} \left(-R - (\partial\phi)^2 + |\partial\chi|^2 - \mu^2(\beta)|\chi|^2 \right)$$

Equations of Motion

Shifted dilaton: $\varphi = \phi - 3\lambda$ (T-duality invariant)

Hamiltonian constraint (must hold for all t):

$$\dot{\varphi}^2 - d\dot{\lambda}^2 = \dot{\chi}^2 + \mu^2\chi^2, \qquad \dot{\varphi} = \pm\sqrt{3H_s^2 + \rho_\chi}$$

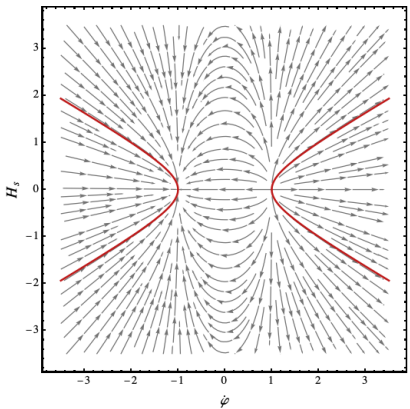
Shifted dilaton: $\ddot{\varphi} = \dot{\varphi}^2 - \mu^2\chi^2$

Thermal scalar: $\ddot{\chi} - \dot{\varphi}\,\dot{\chi} + \mu^2\chi = 0$

Scale factor: $\ddot{\lambda} - \dot{\varphi}\,\dot{\lambda} = -\dfrac{1}{2d}\dfrac{d\mu^2}{d\lambda}\,\chi^2$

Branch Structure and the Hagedorn Transition

Branches defined by sign of shifted dilaton velocity: $\dot{\varphi} = \pm\sqrt{3H_s^2 + \rho_\chi}$



Phase portrait at $\mu^2 > 0$ ($T < T_H$)

Fixed points (momentarily stalled):
$(\dot{\varphi}, H_s) = (\pm\mu\chi,\, 0)$

- $(+)$ branch (right): evolves toward future singularity
- $(-)$ branch (left): FRW cosmology
- **Branches are disconnected**; $(+) \to (-)$ classically forbidden

Approaching T_H ($\mu^2 \to 0$):

- Fixed points collapse to origin; gap **closes**
- $\dot{\varphi}$ can sit at zero $\Rightarrow$ system stalls indefinitely

Multiple Hagedorn Temperatures and Wilson Lines

Everything so far assumes a **single** Hagedorn temperature.

Heterotic string: 10D superstring (right-movers) + 26D bosonic (left-movers). The 16D mismatch → compactified on a self-dual lattice → **gauge charges**

Wilson line shifts momentum and gauge charges:

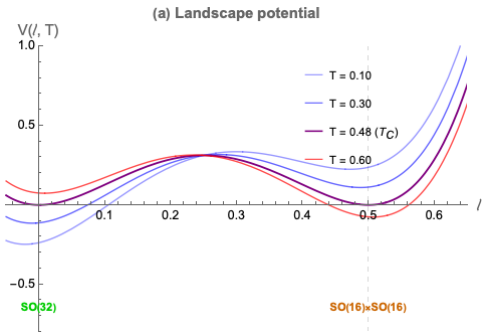
$$m \to m + \vec{\lambda}\cdot\vec{\ell} - \frac{w\,\ell^2}{2}\,, \qquad \vec{\lambda} \to \vec{\lambda} - w\vec{\ell}, \qquad w \to w$$

Changes which state becomes massless first ⇒ changes T_H (Dienes et al.):

$$T_H(\vec{\ell}) = \frac{\sqrt{2}\,\mathcal{M}}{\sqrt{3 - \vec{\ell}\cdot\vec{\ell} + \sqrt{8 - 4\,\vec{\ell}\cdot\vec{\ell}}}}$$

$SO(32)_A$:	$\vec{\ell}\cdot\vec{\ell} = 0$	$\Rightarrow\ T_H = (2-\sqrt{2})\,\mathcal{M}$
$SO(32)_B$, $SO(8)\times SO(24)$, $U(16)$:	$\vec{\ell}\cdot\vec{\ell} = 1$	$\Rightarrow\ T_H = \mathcal{M}/\sqrt{2}$
$SO(16)\times SO(16)$:	$\vec{\ell}\cdot\vec{\ell} = 2$	$\Rightarrow\ T_H = \sqrt{2}\,\mathcal{M}$

Free Energy and Cosmological Implications



With Wilson lines, T_H depends on the **full string spectrum** — not just a single mode

Wilson line enters as a dynamical parameter in the free energy; different values → different gauge sectors

As T changes, the preferred minimum shifts ⇒ Wilson line is **dynamical**

New questions:

- Which T_H did the universe settle at?
- Cascade of phase transitions?
- How does the universe navigate this landscape?

— *ongoing work*

Summary

- The thermal scalar provides a concrete **effective field theory** for the Hagedorn phase, replacing thermodynamic approximations with dynamical fields
- The Hamiltonian constraint organizes cosmologies into **branch sectors** — the pre-inflationary phase and FRW live on different branches, separated by a classical barrier
- Wilson lines reveal a **landscape of Hagedorn temperatures** — the cosmology of this landscape is new and largely unexplored

Thank You

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