Stochasticity in Resonant Reheating

Dr. Leia Price — lyprice@mit.edu

2507.08075; Phys Rev D 112, 083505

With co-authors Fred Adams, Anthony Bloch, Scott Watson

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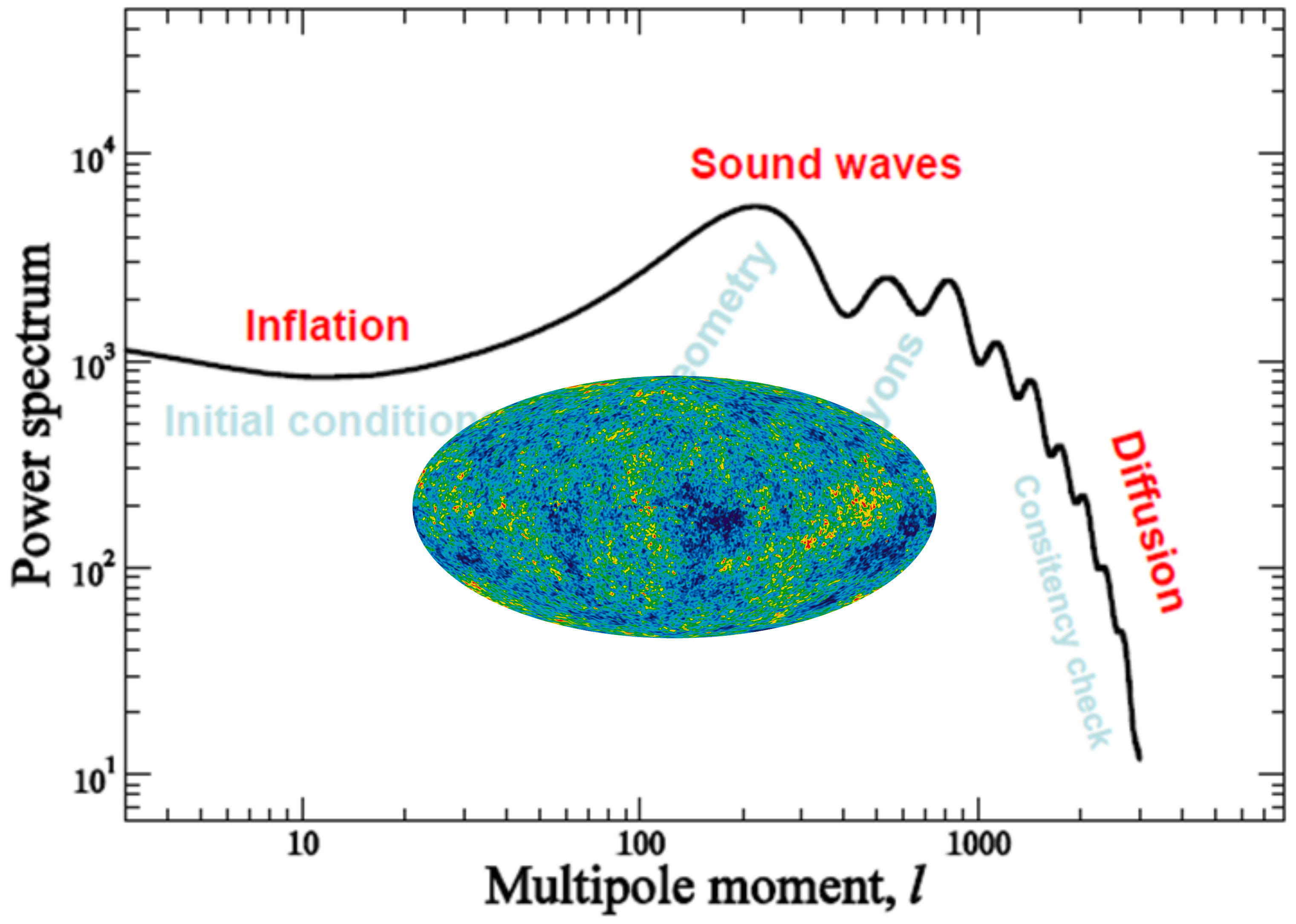
Inflation needs a beginning and an end

I'm focused on the end



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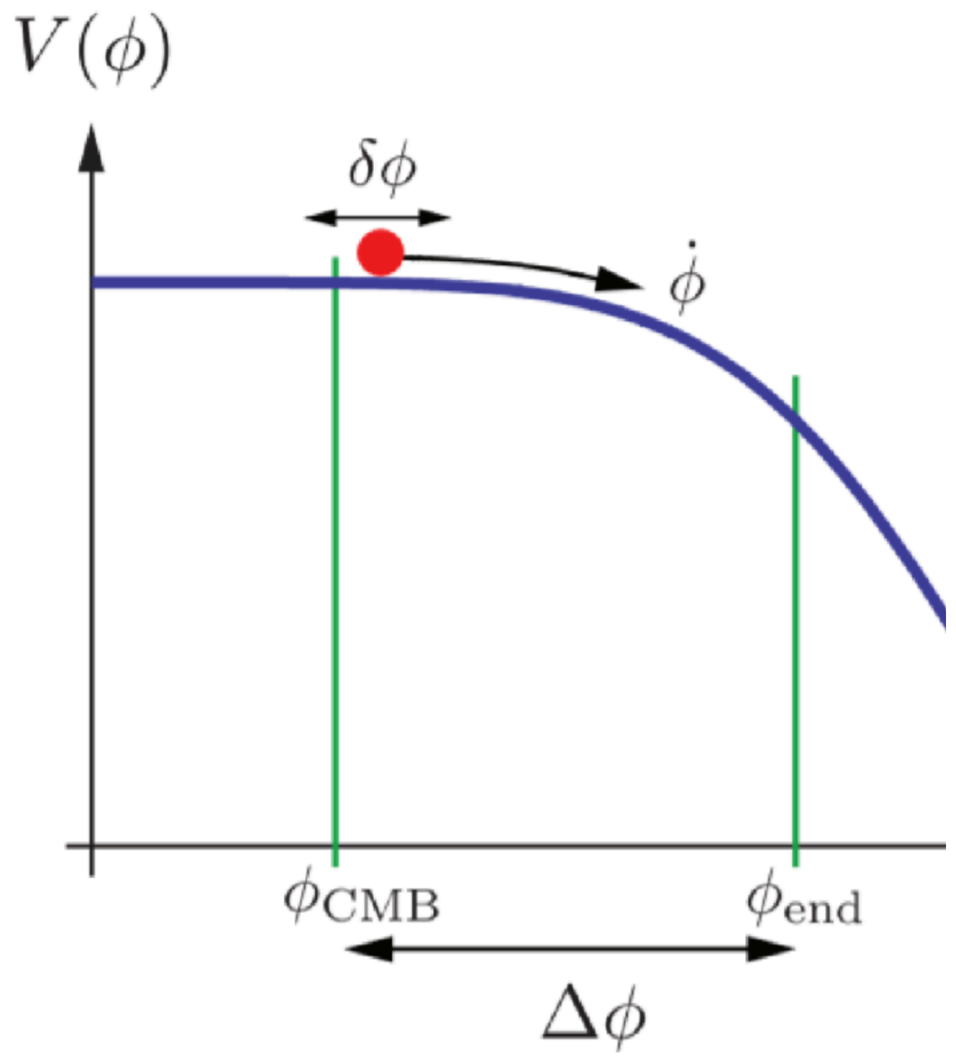

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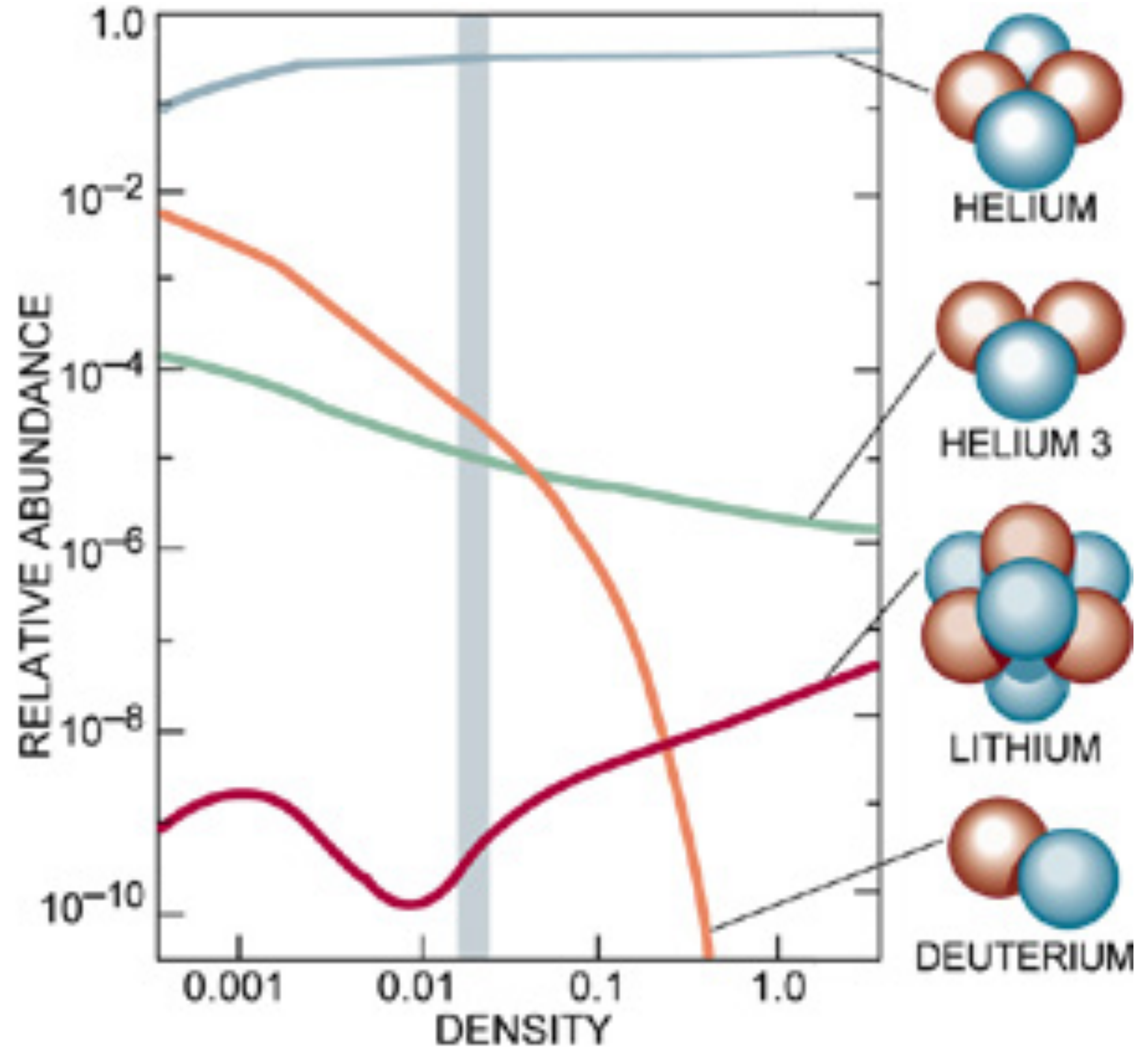

CMB mysteries → Slow-roll inflation → BBN

Perturbative description is incomplete

$$V_{\text{intrc}} = \sigma\phi\chi^2 + g^2\phi^2\chi^2 + h_\phi\phi\bar{\psi}\psi + h_\chi\chi\bar{\psi}\psi$$

ϕ : inflaton, χ and ψ : scalar and fermion we want to produce

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Tree-level decay rates $\Gamma_{\phi\to\chi\chi} = \dfrac{\sigma^2}{8\pi m_\phi}$, $\Gamma_{b\to\psi\bar{\psi}} = \dfrac{h_b^2\, m_b}{8\pi}$

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Bose enhancement: $n_\chi \sim \exp\left(\dfrac{\pi\sigma|\phi|}{2m_\phi}t\right)$, Instant reheating: $\Gamma_{\chi\to\psi\bar{\psi}} = \dfrac{h_\chi^2 g|\phi|}{8\pi}$

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I will focus on populating χ through the σ coupling

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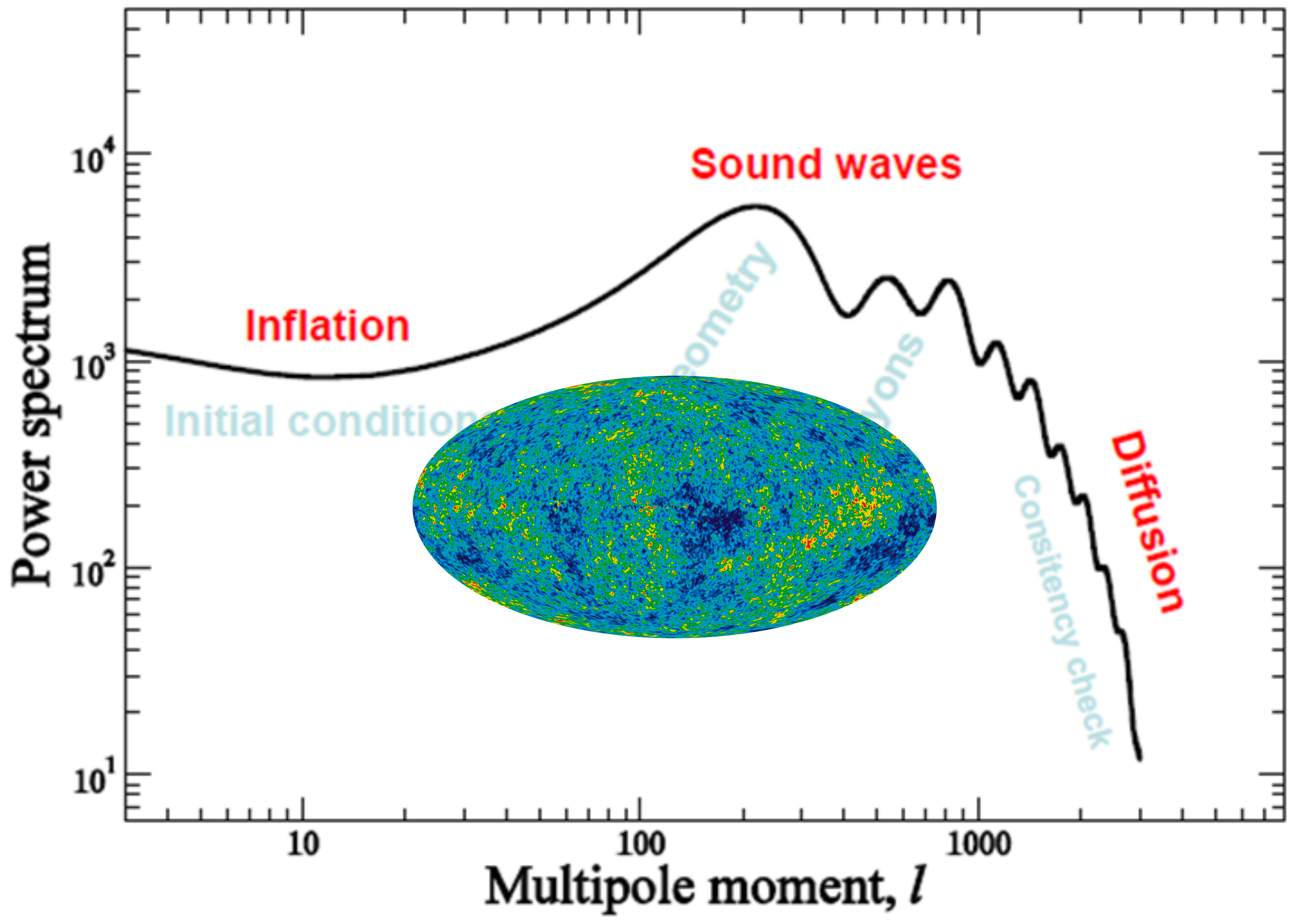

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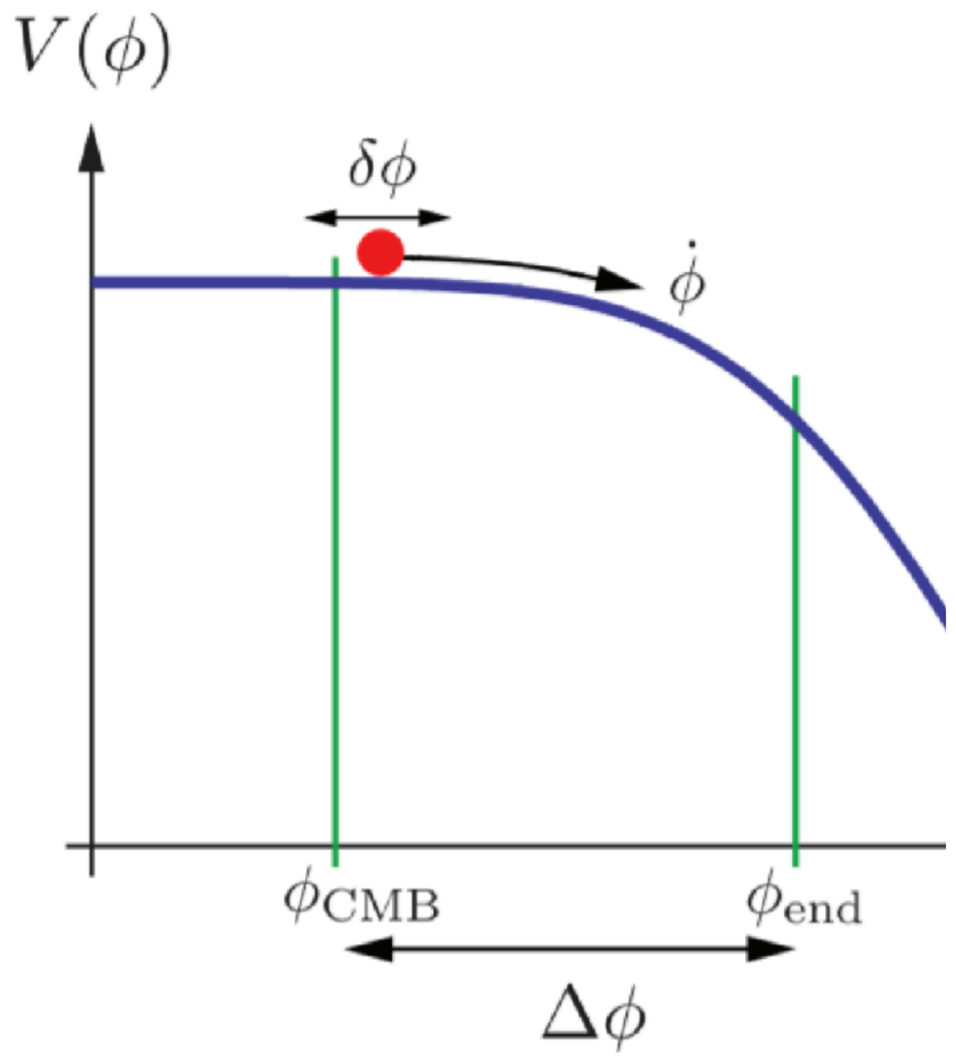

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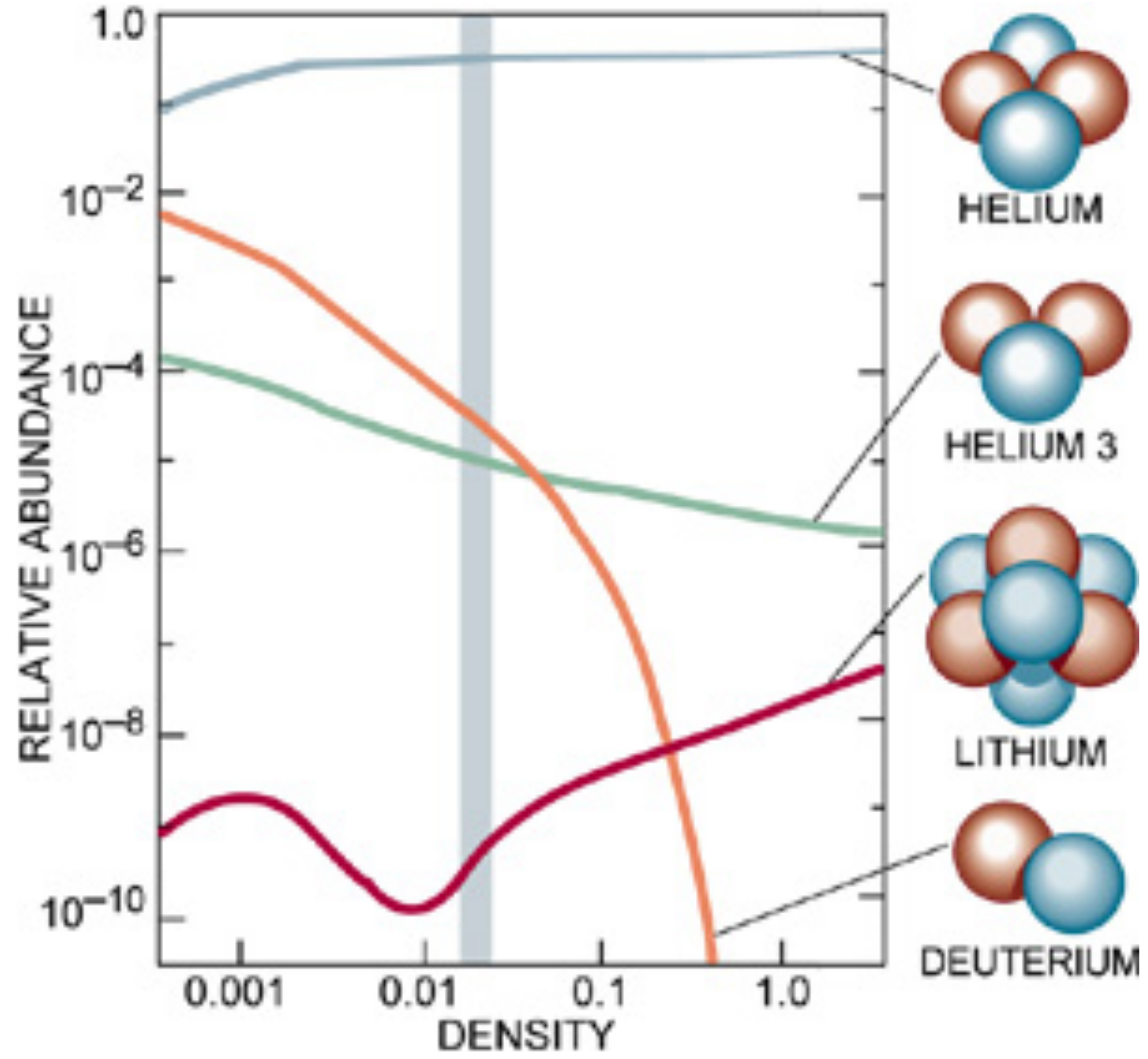

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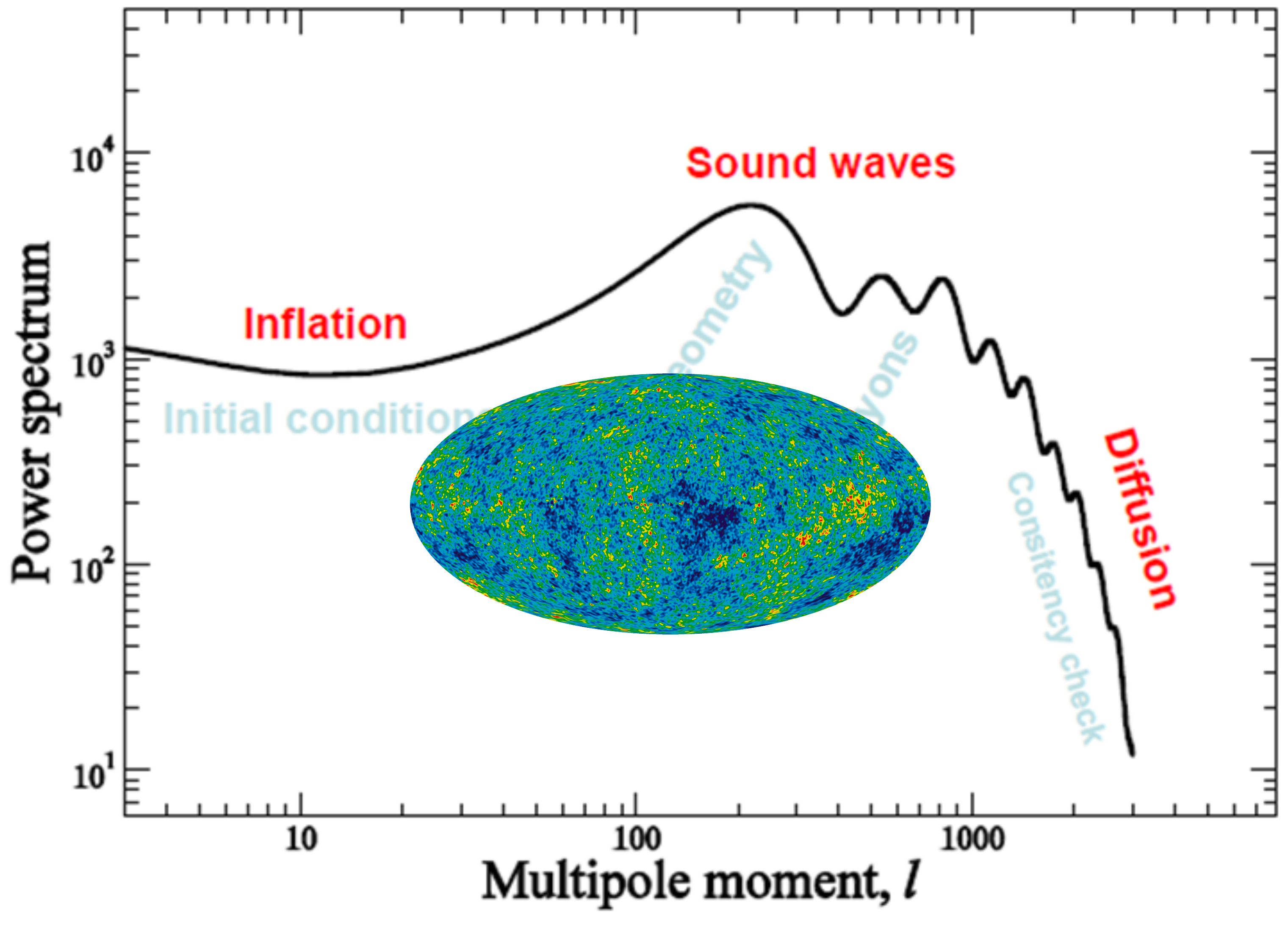

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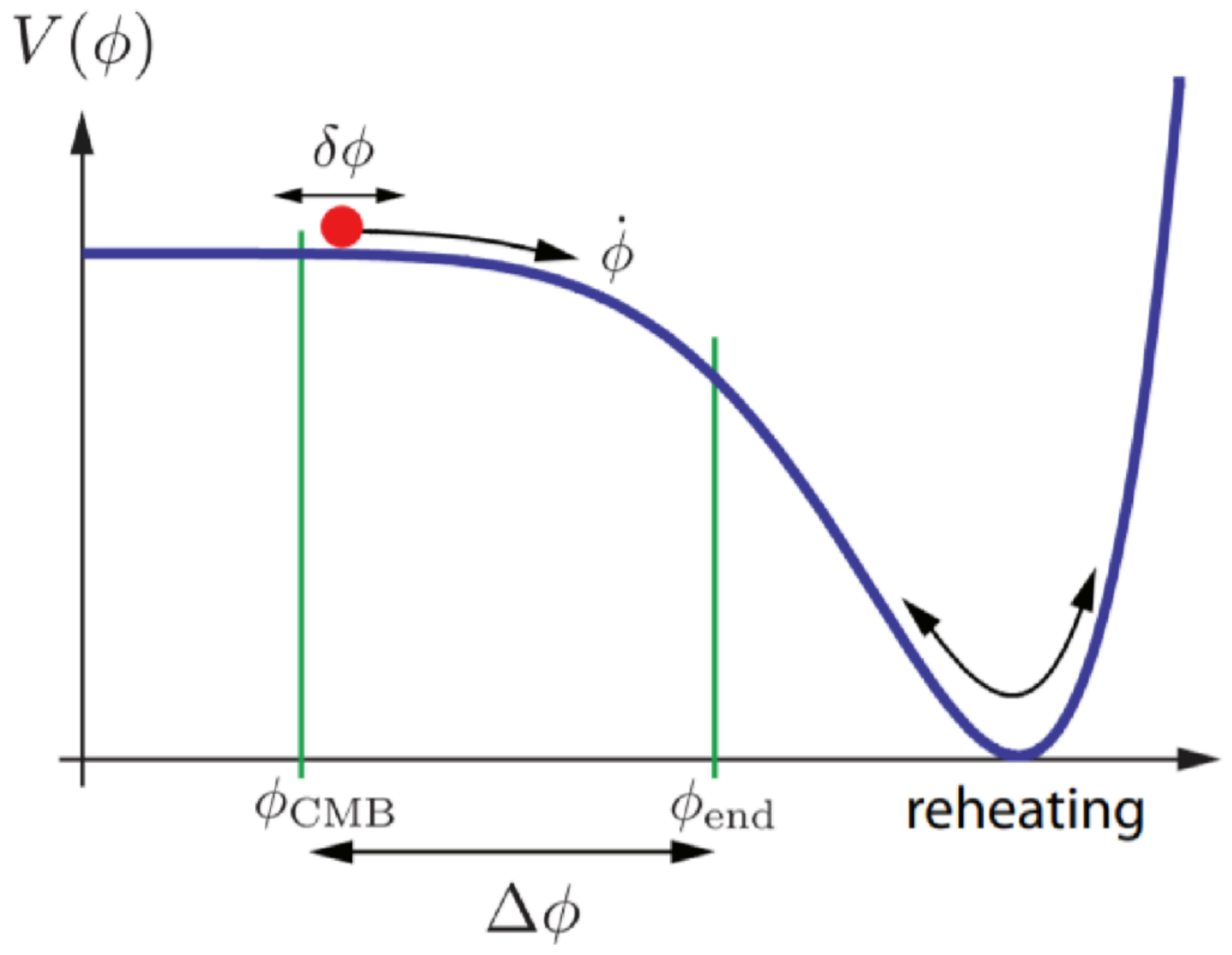
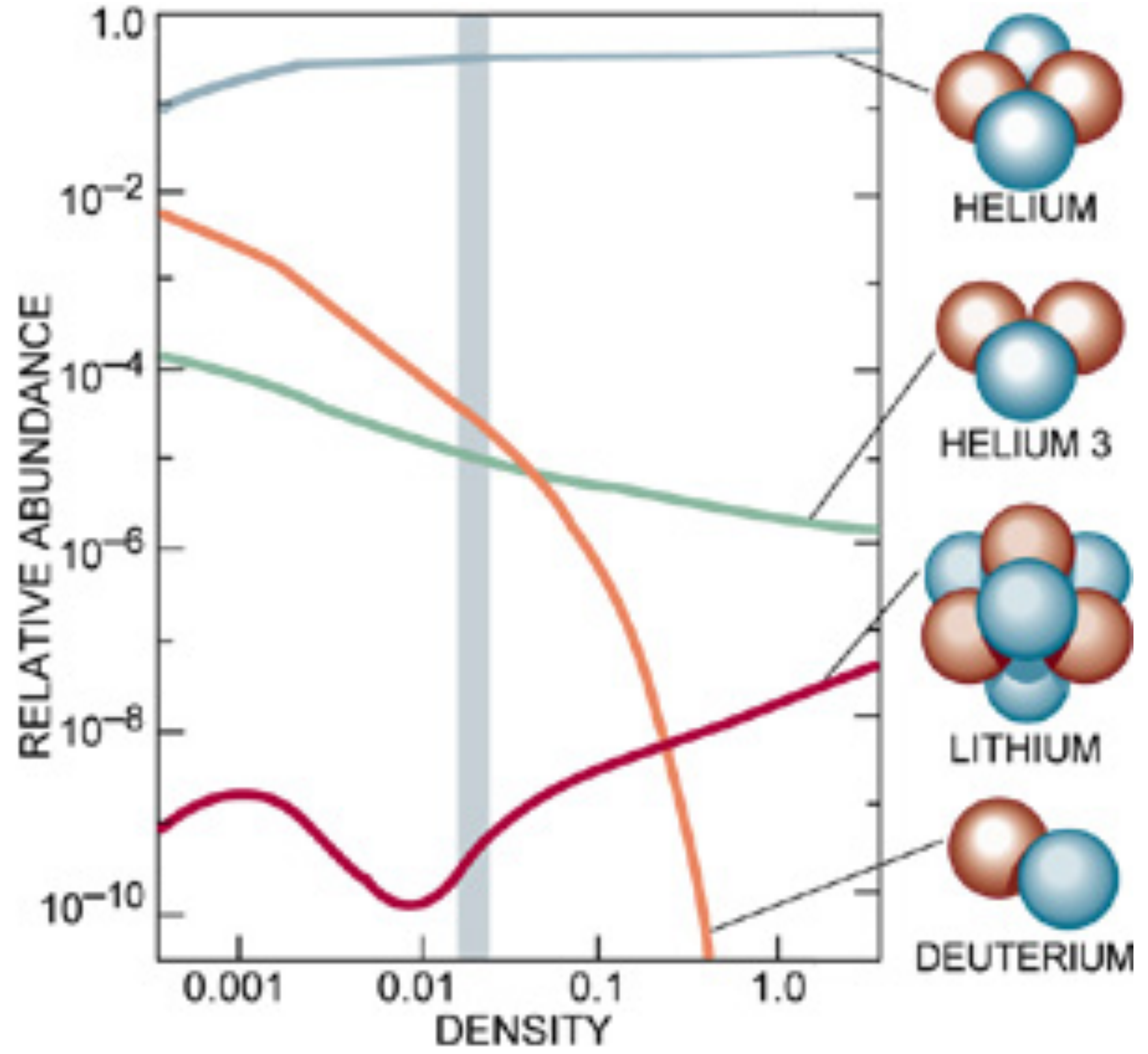




CMB mysteries → Slow-roll inflation → BBN

Relation to Baryogenesis

Resonance processes are non-thermal and ubiquitous

Discontinuous tunneling transition ruled out

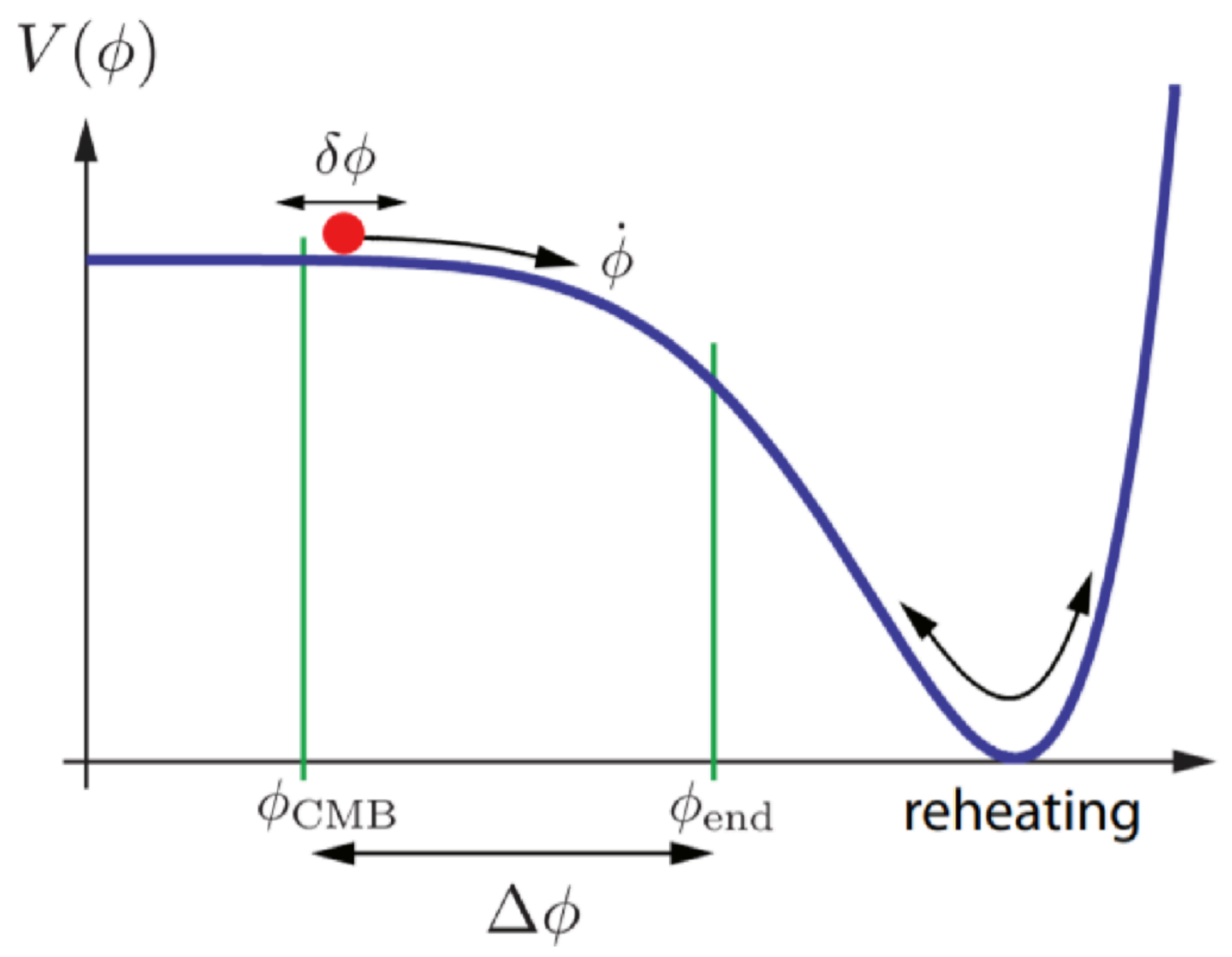
so reheating is necessarily non-perturbative anyway

ϕ can coherently (classically) oscillate around the minimum

$\phi(t) \approx \Phi(t)\cos(m_\phi t)$ around quadratic minimum

Clearly Feynman diagrams alone won't work

EOM of χ-modes $\ddot{\chi}_k + \left(k^2 + m_\chi^2 + 2\sigma\phi(t)\right)\chi_k = 0$



Exponentially forced oscillator solution

AKA “Hill’s equation”

EOM: $\ddot{\chi}_k + \left(k^2 + m_\chi^2 + 2\sigma\phi(t)\right)\chi_k = 0 \quad \leftrightarrow \quad \chi_k'' + \left(A + 2qP_T(\tau)\right)\chi_k = 0$

Redefinitions: $A = \dfrac{4\left(k^2 + m_\chi^2\right)}{m_\phi^2}$, $q = \dfrac{4\sigma\Phi}{m_\phi^2}$, $\tau = \dfrac{m_\phi t}{2}$, $T = \pi$

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Floquet's theorem: $\chi_k(\tau) = c_1\, e^{\mu_k \tau}\, u_{T1}(\tau) + c_2\, e^{-\mu_k \tau}\, u_{T2}(\tau)$

periodic basis solutions u with exponentially changing coefficients

Growth rate $\gamma_k \equiv \Re(\mu_k)$ defines solution

Exponentially forced oscillator solution

Floquet's theorem reminder: $\chi_k(t) = c_1\, e^{\mu_k t}\, u_{T1}(t) + c_2\, e^{-\mu_k t}\, u_{T2}(t)$

Solution for each period n: $x_n(t) = \alpha_n x_{T1}(t) + \beta_n x_{T2}(t)$

Impose continuity: $\begin{pmatrix} x_{T1}(T) & x_{T2}(T) \\ \dot{x}_{T1}(T) & \dot{x}_{T2}(T) \end{pmatrix} \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} \equiv \mathbb{M} \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} = \begin{pmatrix} \alpha_{n+1} \\ \beta_{n+1} \end{pmatrix}$

Transfer matrix $\mathbb{M}$ has eigenvalues λ, $1/\lambda$

$u_{Ti}(t)$ are the diagonal solutions

$$x_n(T) = c_1\, \lambda^n\, u_{T1}(0) + c_2\, \lambda^{-n}\, u_{T2}(0) = c_1\, e^{n\mu T}\, u_{T1}(T) + c_2\, e^{-n\mu T}\, u_{T2}(T) \quad \Rightarrow \quad \mu = \frac{\ln \lambda}{T}$$

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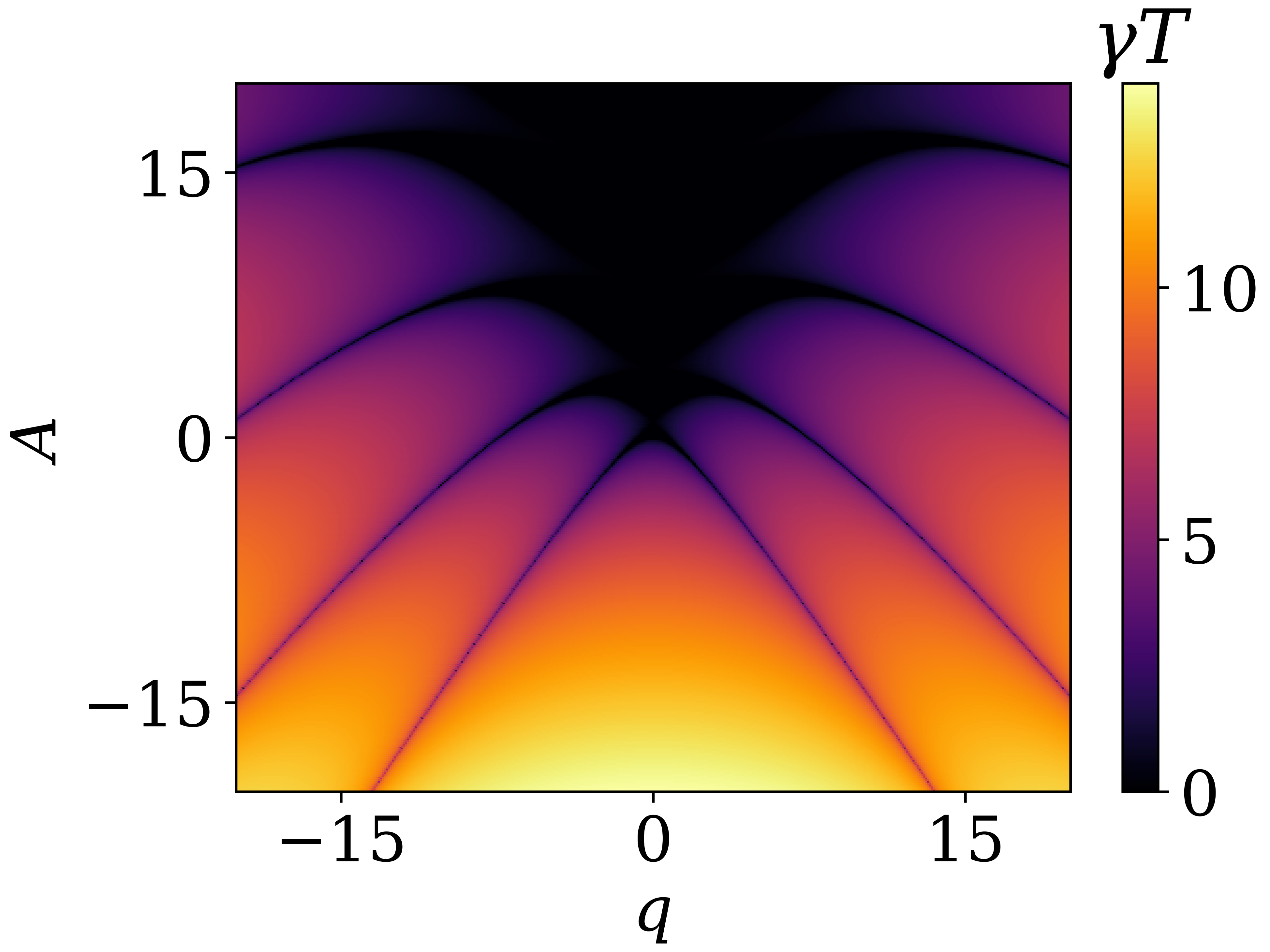
Exponentially forced oscillator solution

Map of growth rate for Mathieu's equation

"Mathieu's equation" is a special case of "Hill's" with $P_T(\tau) = \cos(2\tau)$



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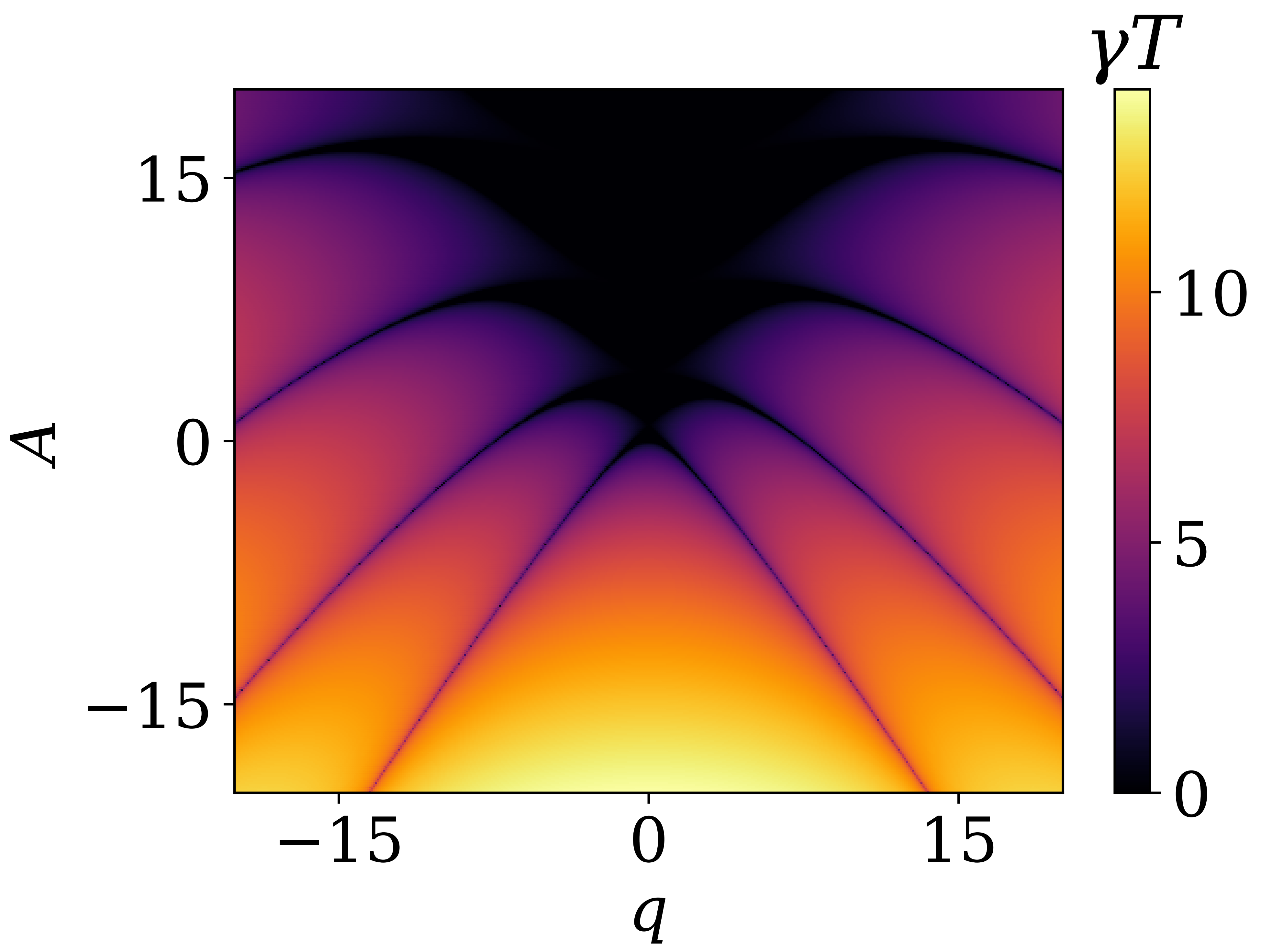

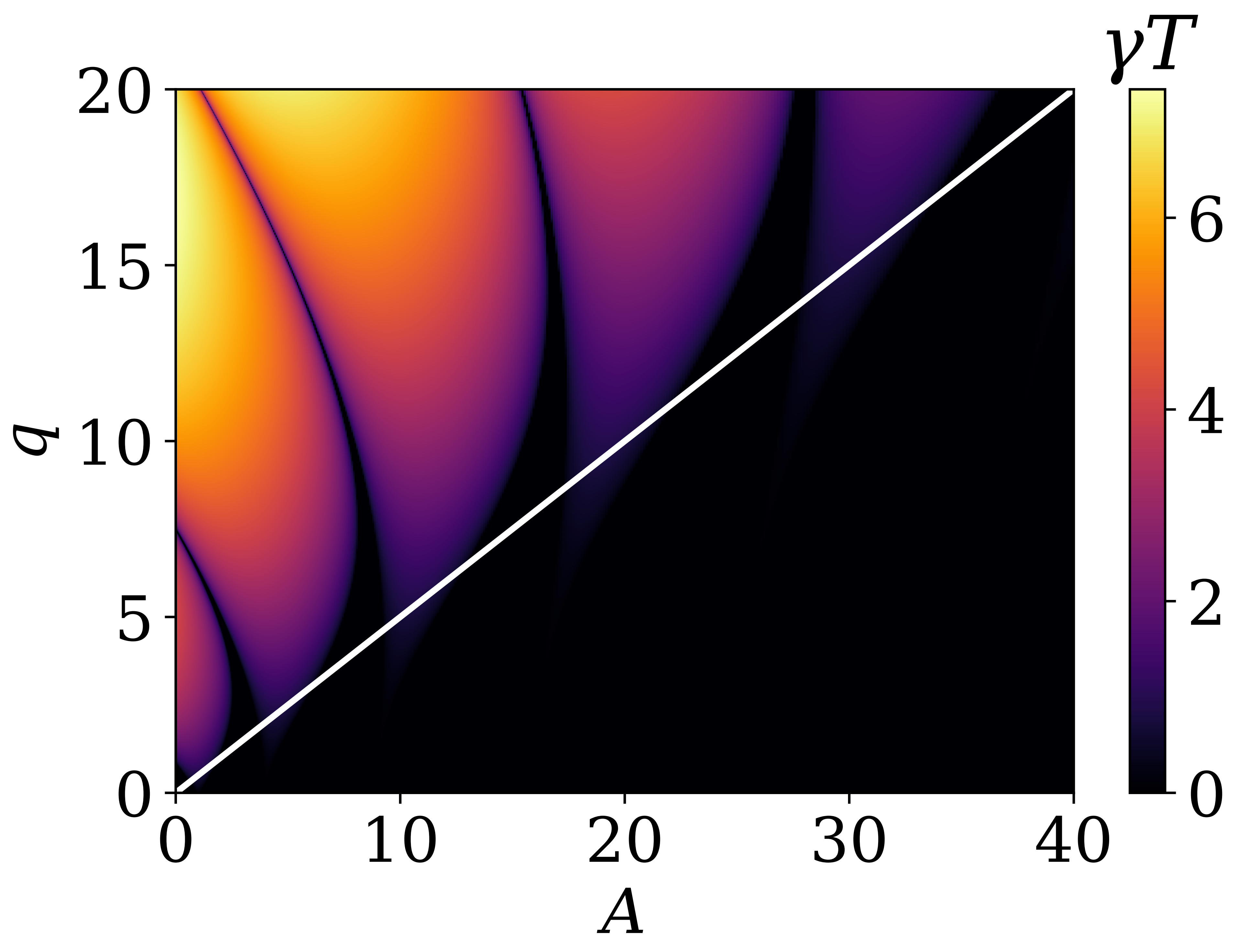

“Stochastic” preheating

Multiplying random matrices

$$\left(\text{Eigenvalues of } \prod_n \mathbb{M}_n\right) \neq \prod_n \left(\text{Eigenvalues of } \mathbb{M}_n\right)$$

Growth rate has two parts, $\gamma_N = \gamma_R + \frac{1}{N}\sum_n^N \gamma_n$

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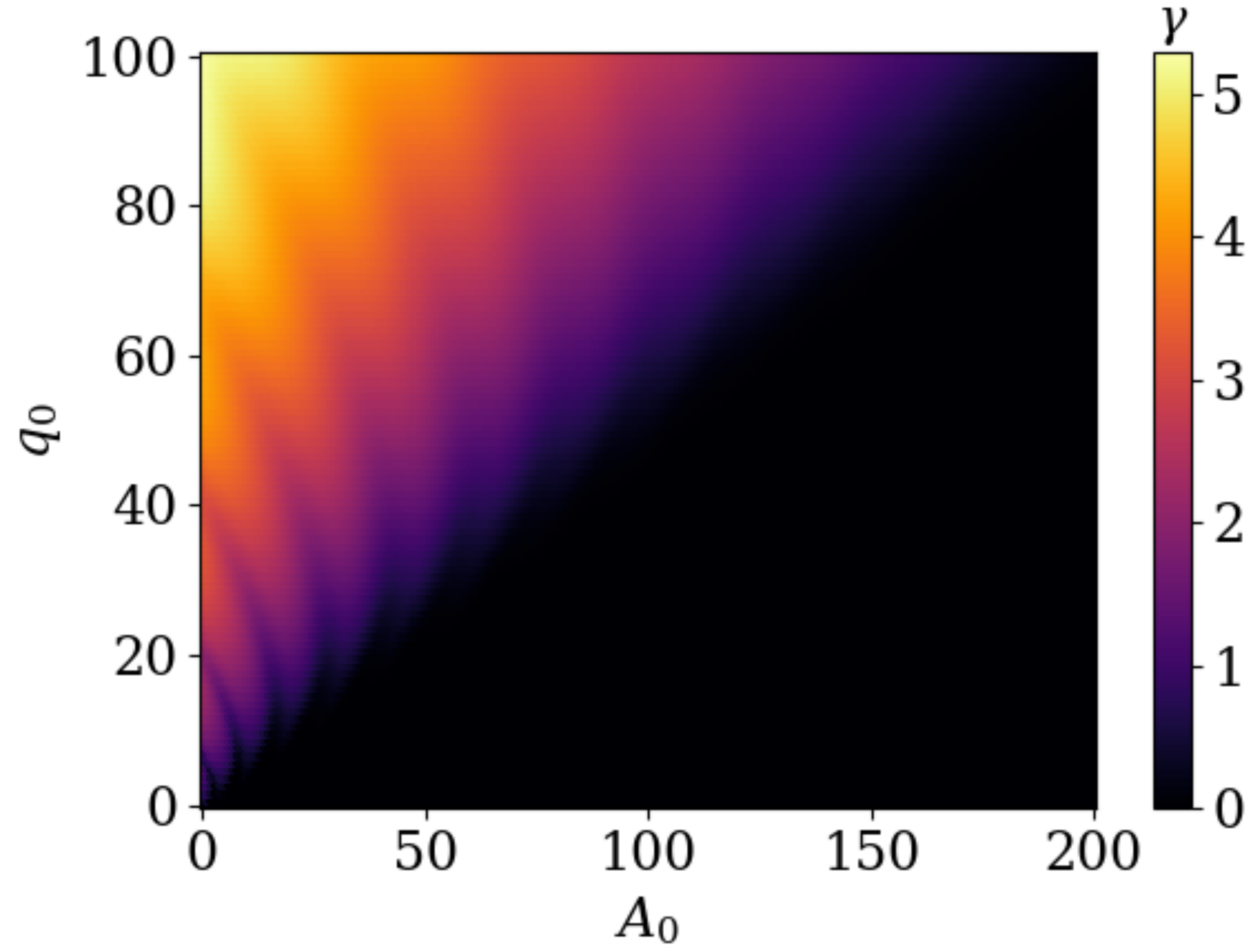
Second term (local average) dominates for high q_0:

Top line is a good approximation



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Growth rate randomly walks away from zero

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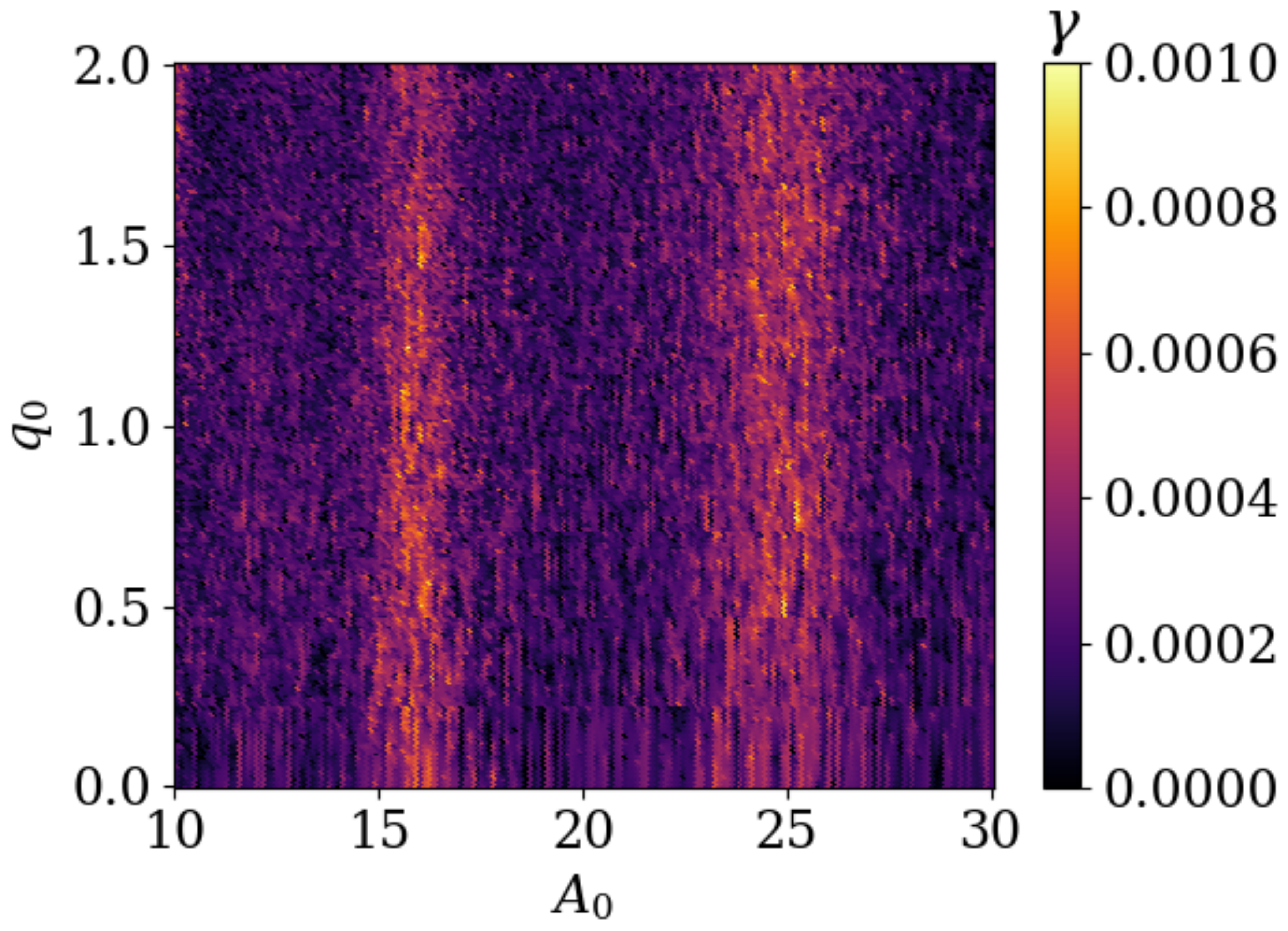
Growth rate has two parts, $\gamma_N = \gamma_R + \frac{1}{N}\sum_n^N \gamma_n$

First term dominates for regions with $\gamma_{\text{noiseless}} = 0$

R for "random walk"



“Stochastic” preheating

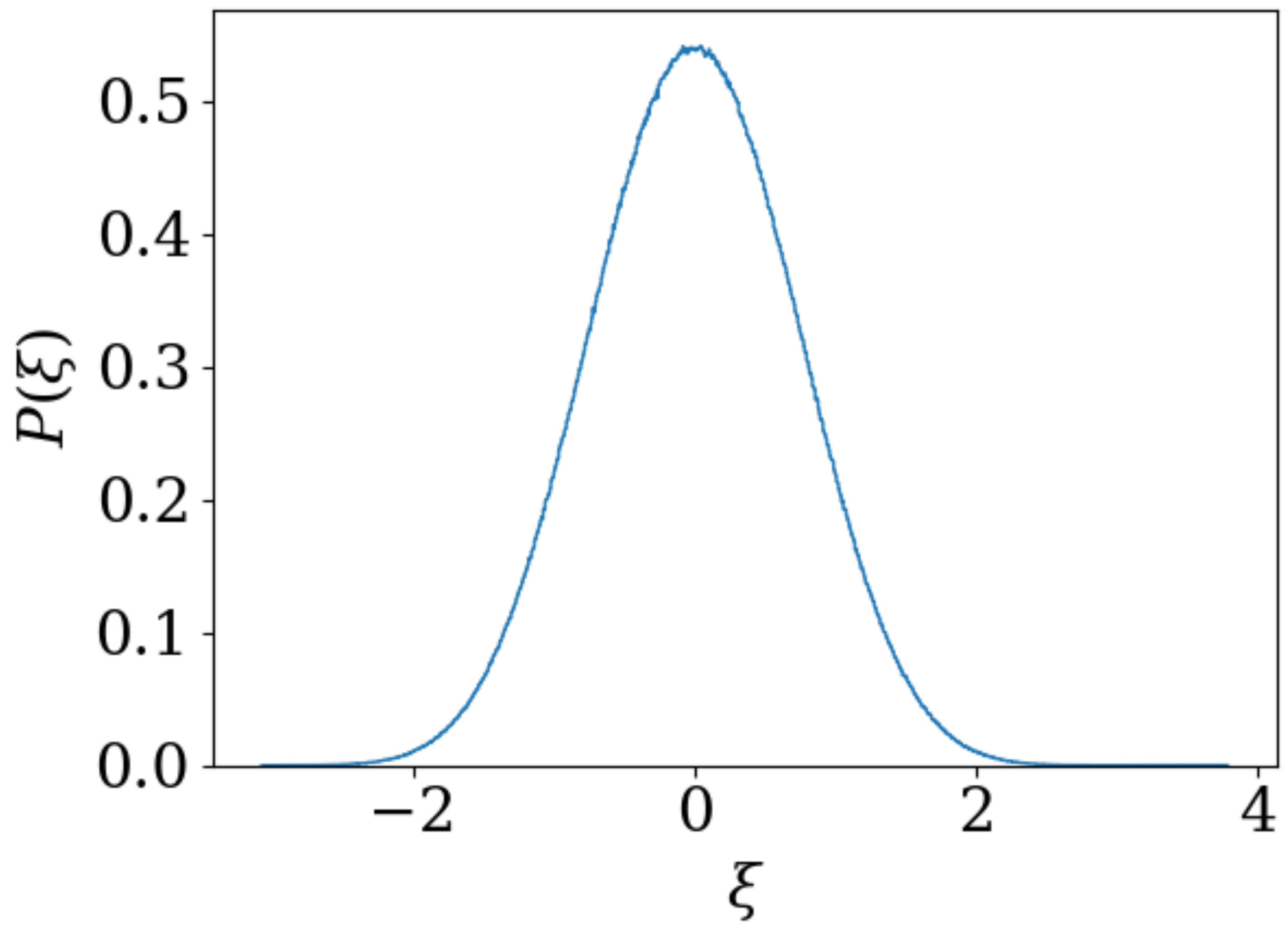
Fluctuations through adding scalars

Add a bunch of little scalars ψ_i with $V_{\text{intrc}} = \dfrac{\Lambda}{2} \sum_i^L g_i \phi^2 \psi_i$

Inflaton EOM becomes

$$\ddot{\phi} + m_\phi^2 (1 + \xi)\,\phi = 0 \quad \text{where} \quad \xi = \frac{\Lambda}{m_\phi} \sum_i^L g_i \frac{\psi_i}{m_\phi}$$

ξ turns out to be normally distributed when sampled at each period, $\langle \xi \rangle = 0$.



"Stochastic" preheating

Fluctuations through adding scalars

Inflaton effective mass becomes $m_{\text{eff}}^2 = (1+\xi) m_\phi^2$

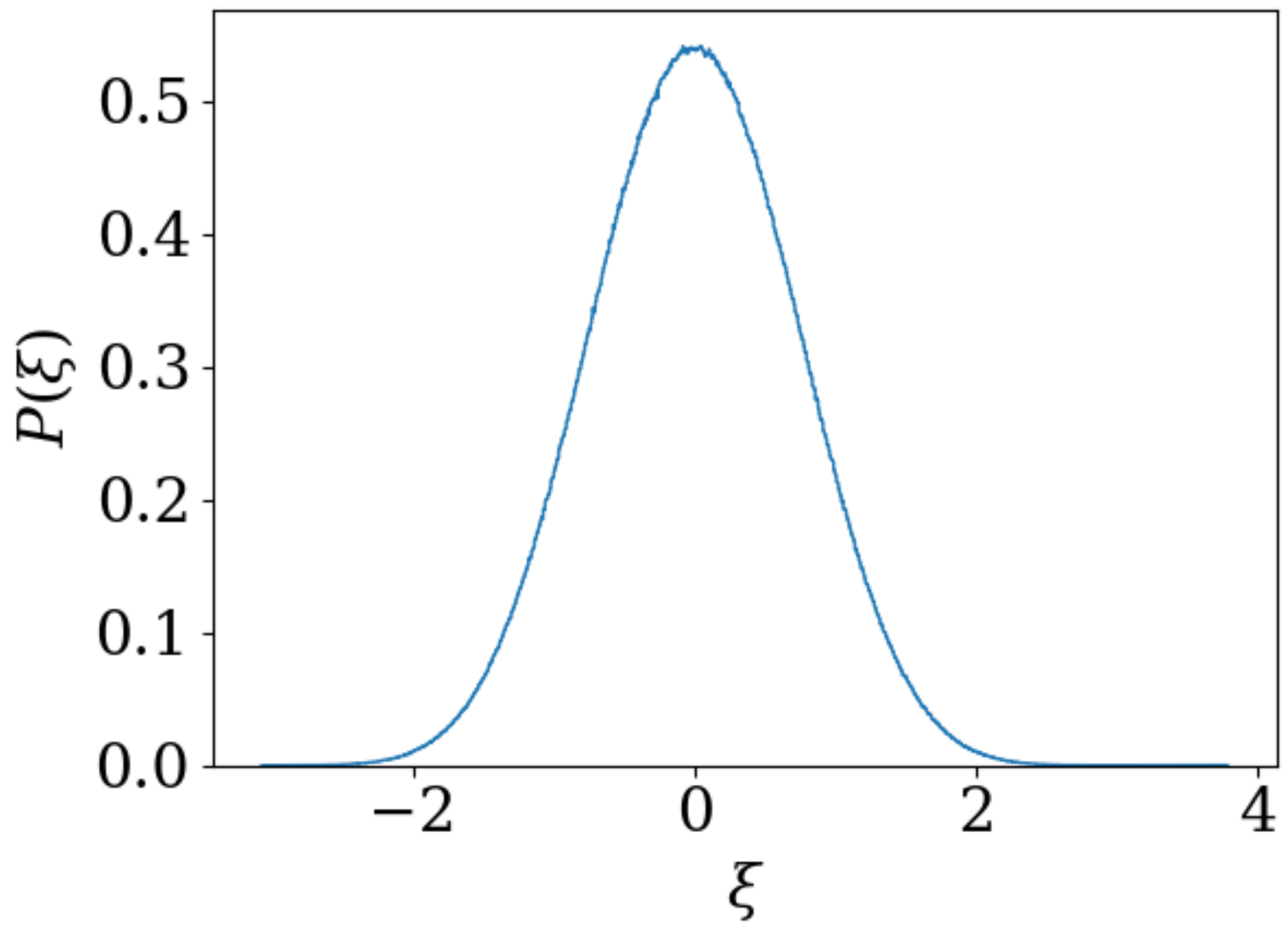
So oscillation frequency becomes random

χ-mode EOM:

$$\ddot{\chi}_k + \left(k^2 + m_\chi^2 + 2\sigma\Phi_0 \cos\left(\sqrt{1+\xi}\, m_\phi t\right) \right) \chi_k = 0$$

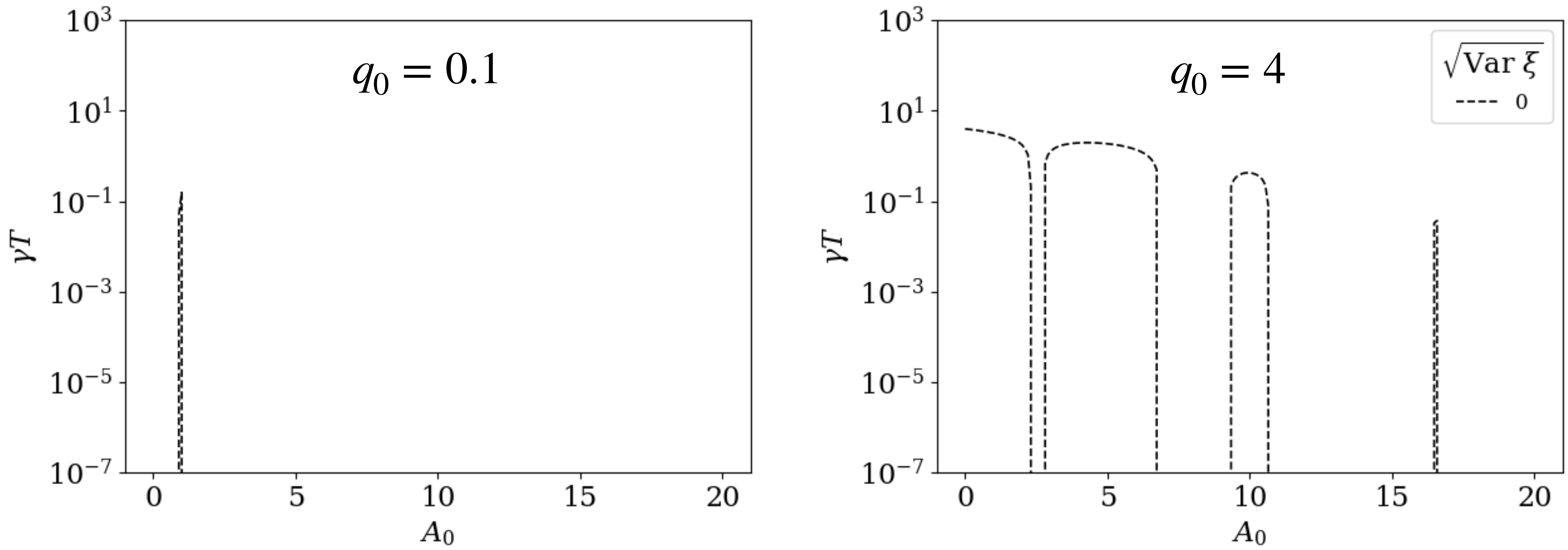
Equivalently, change time variable to $\tau = \sqrt{1+\xi}\, t$

$$\chi_k'' + \left(\frac{k^2 + m_\chi^2}{1+\xi} + \frac{2\sigma\Phi_0}{1+\xi} \cos(m_\phi \tau) \right) \chi_k = 0$$



“Stochastic” preheating

Slices through stability maps



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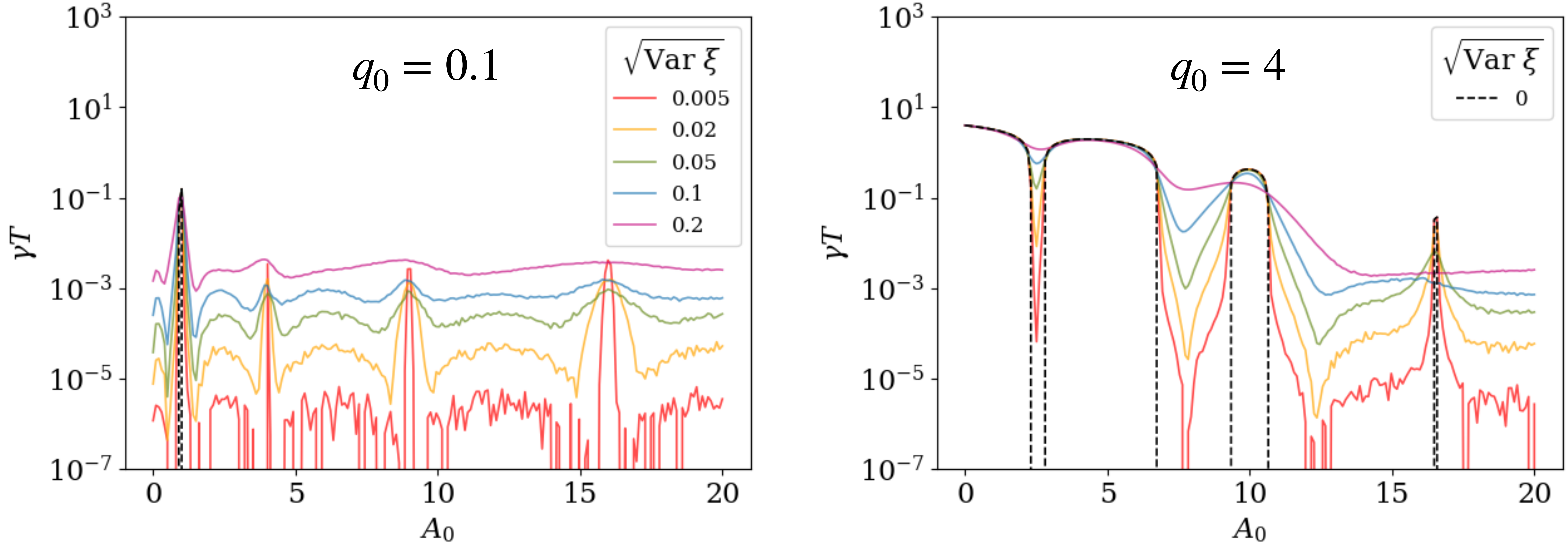
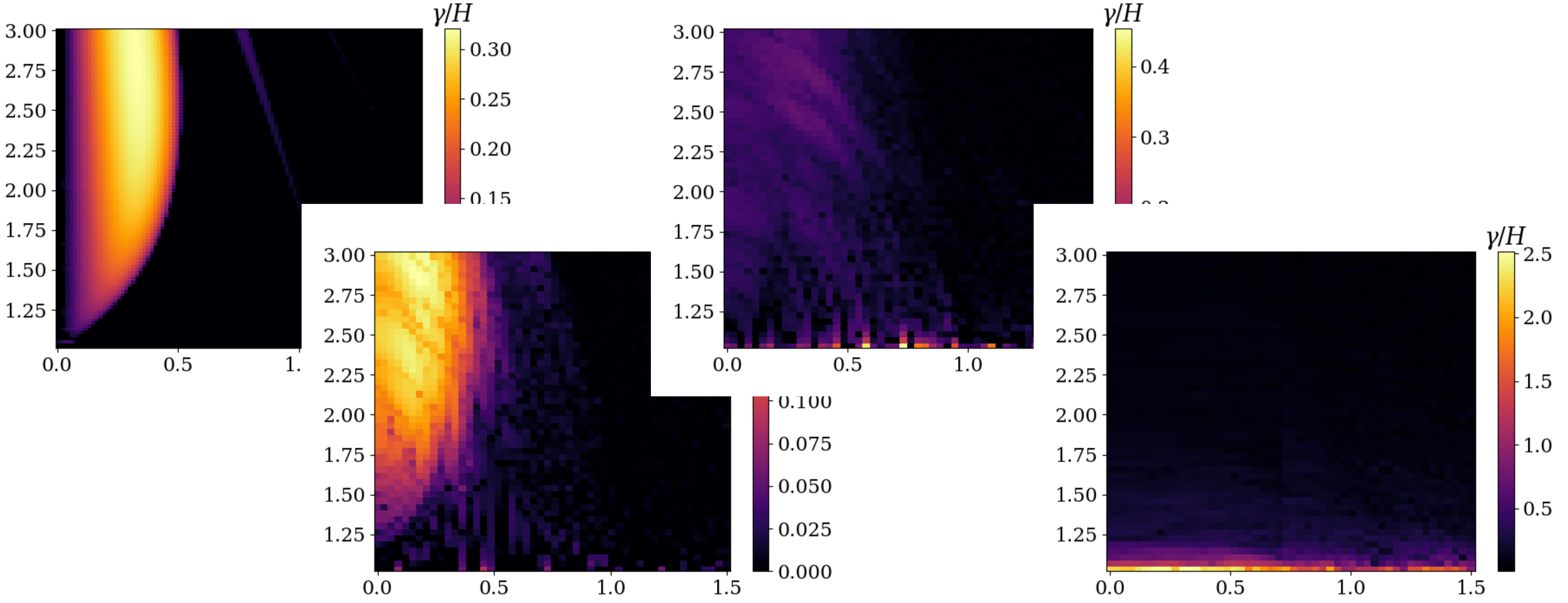


Coming soon 26xx.xxxxx (Sinha, LP)

Application to cosmological moduli and swampland conjectures



Summary

Reheating must have occurred, likely through resonant particle production

Stochasticity is helpful to increase efficiency from zero

This is particularly applicable when considering UV completions of inflation

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Stochasticity is helpful to increase efficiency from zero

This is particularly applicable when considering UV completions of inflation

Thank you

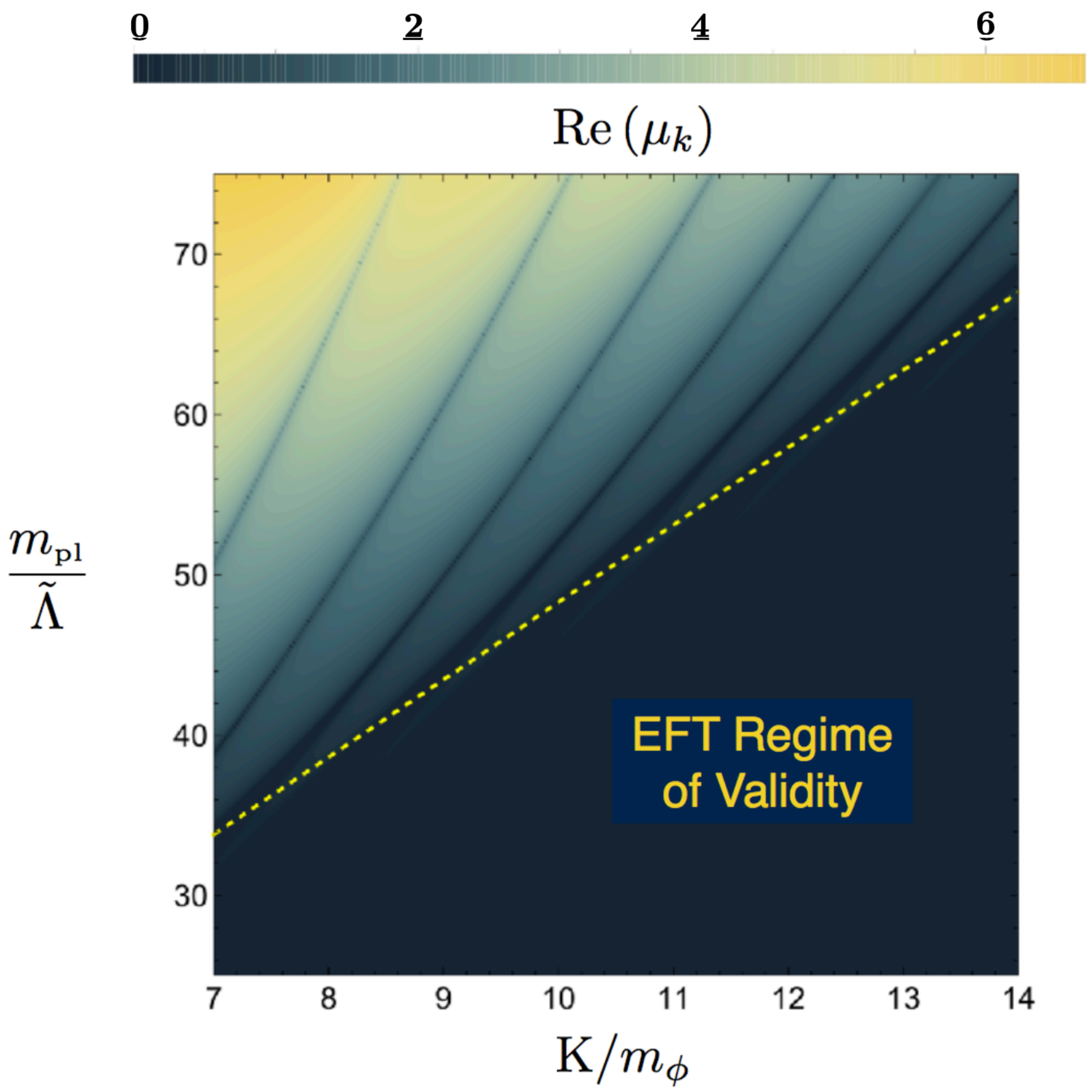
Extra slides

Reheating is *phenomenologically* successful, but:

Typically happens instantly then thermalizes

Embedding reheating in a higher-energy theory presents issues: EFTs, QG models, cosmological moduli

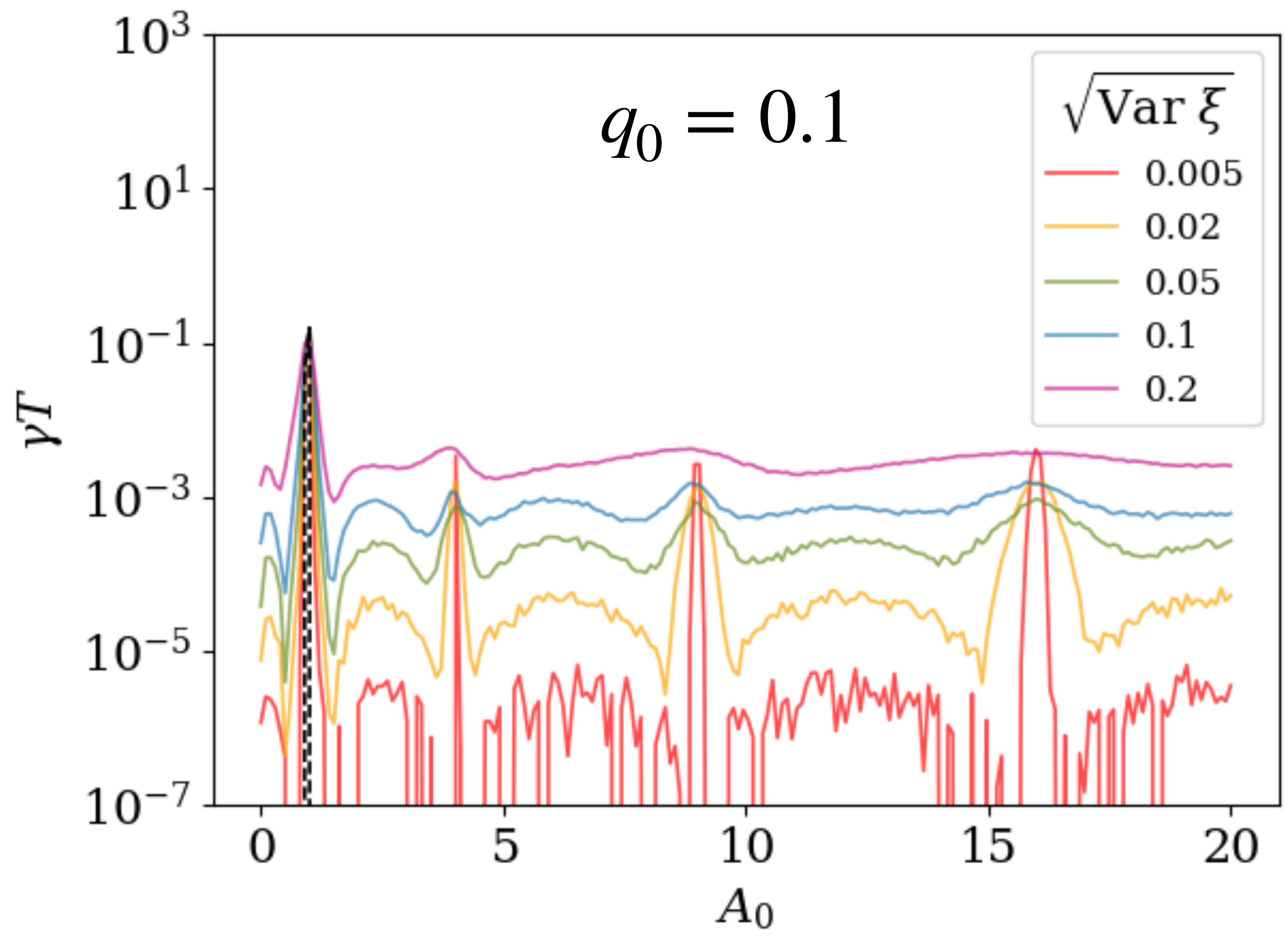
It's worth trying to make **reheating more efficient**

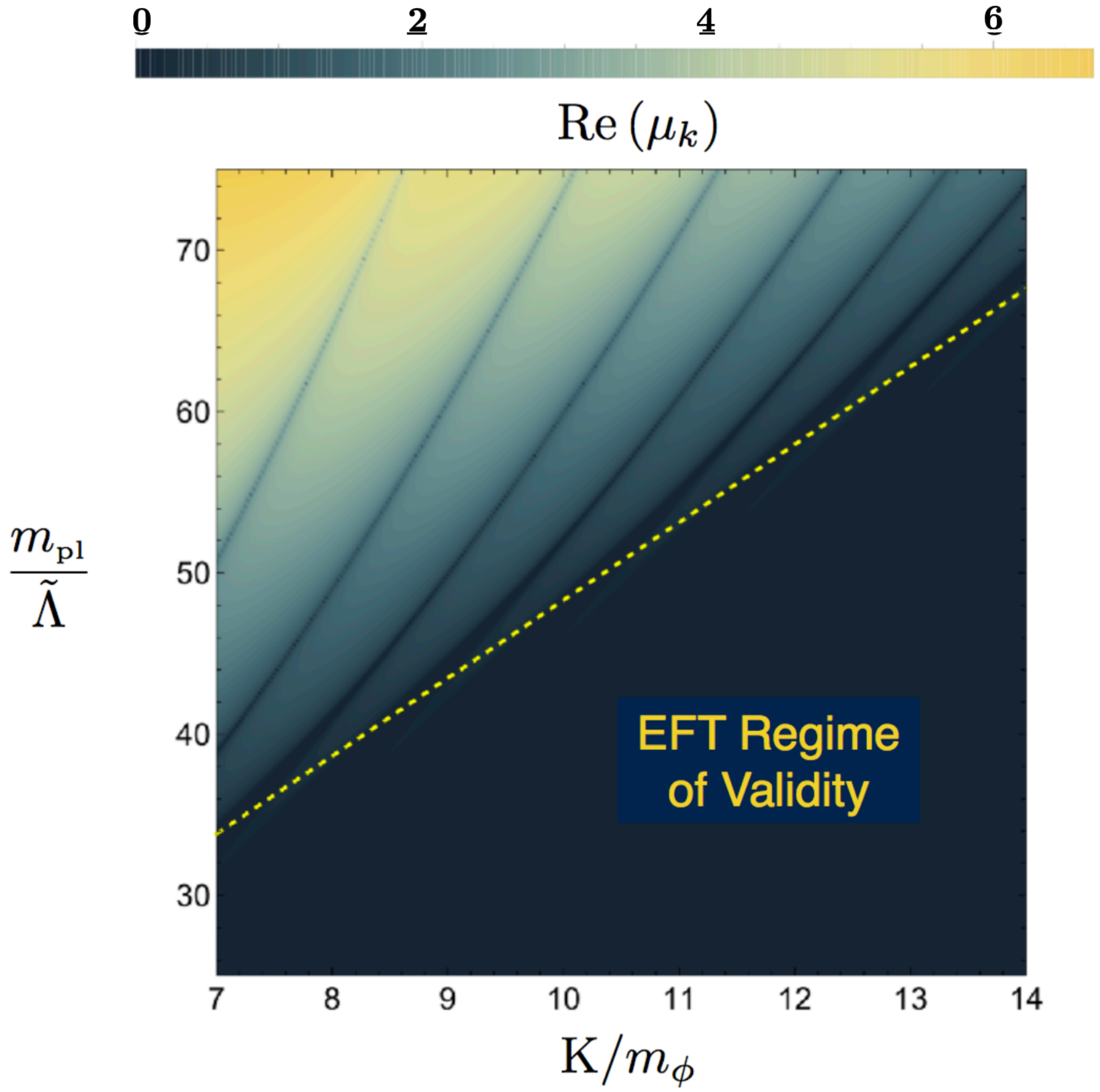


PhysRevD.96.123524

“Stochastic” preheating

Stable regions now have hope





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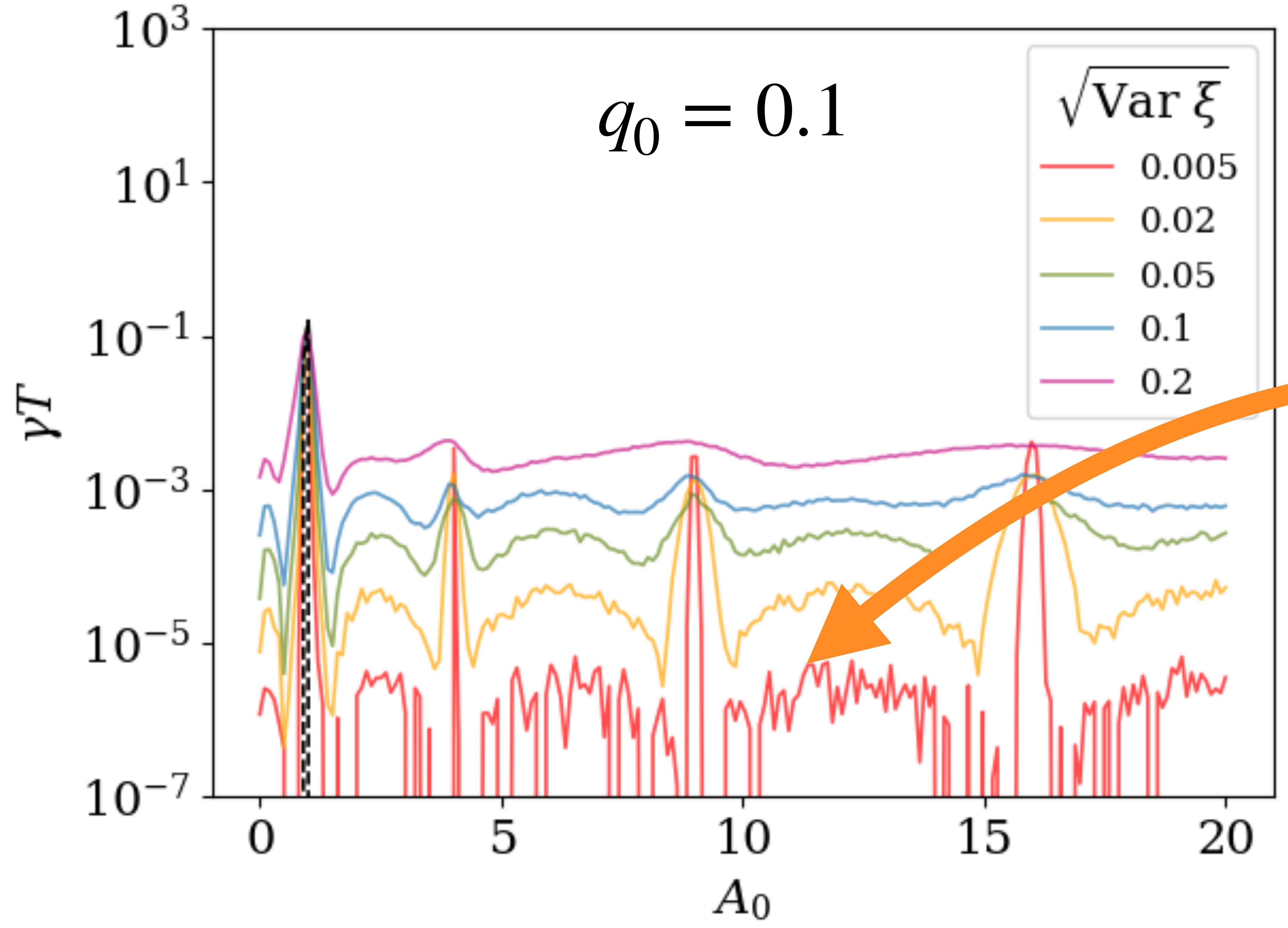

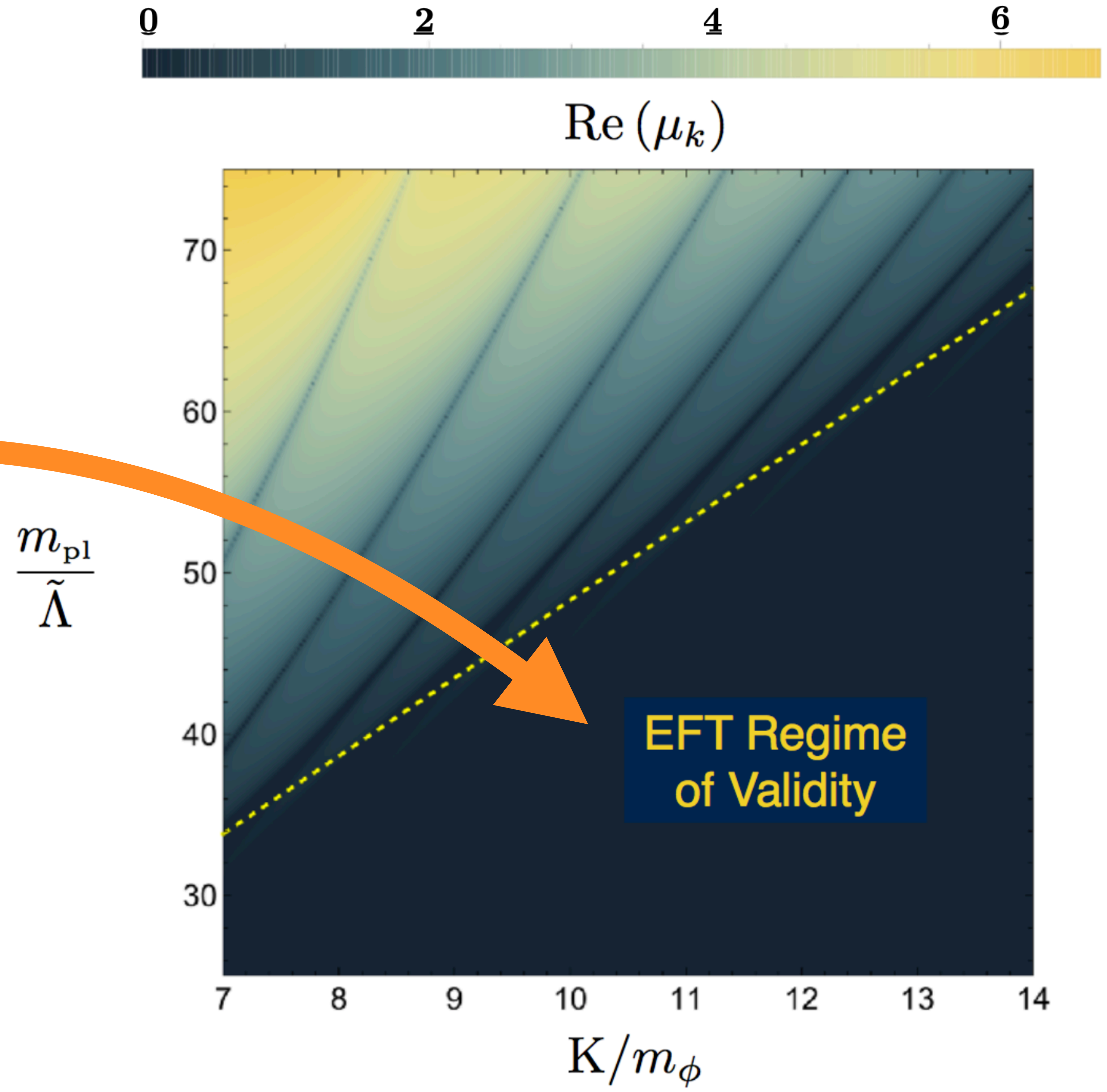

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Expansion leads to changing A and q

$$A = \frac{4\left(k_0^2/a^2 + m_\chi^2\right)}{m_\phi^2} \quad , \quad q = \frac{4\sigma}{m_\phi^2}\frac{\Phi_0}{a^{3/2}}$$

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Expansion naturally ends preheating

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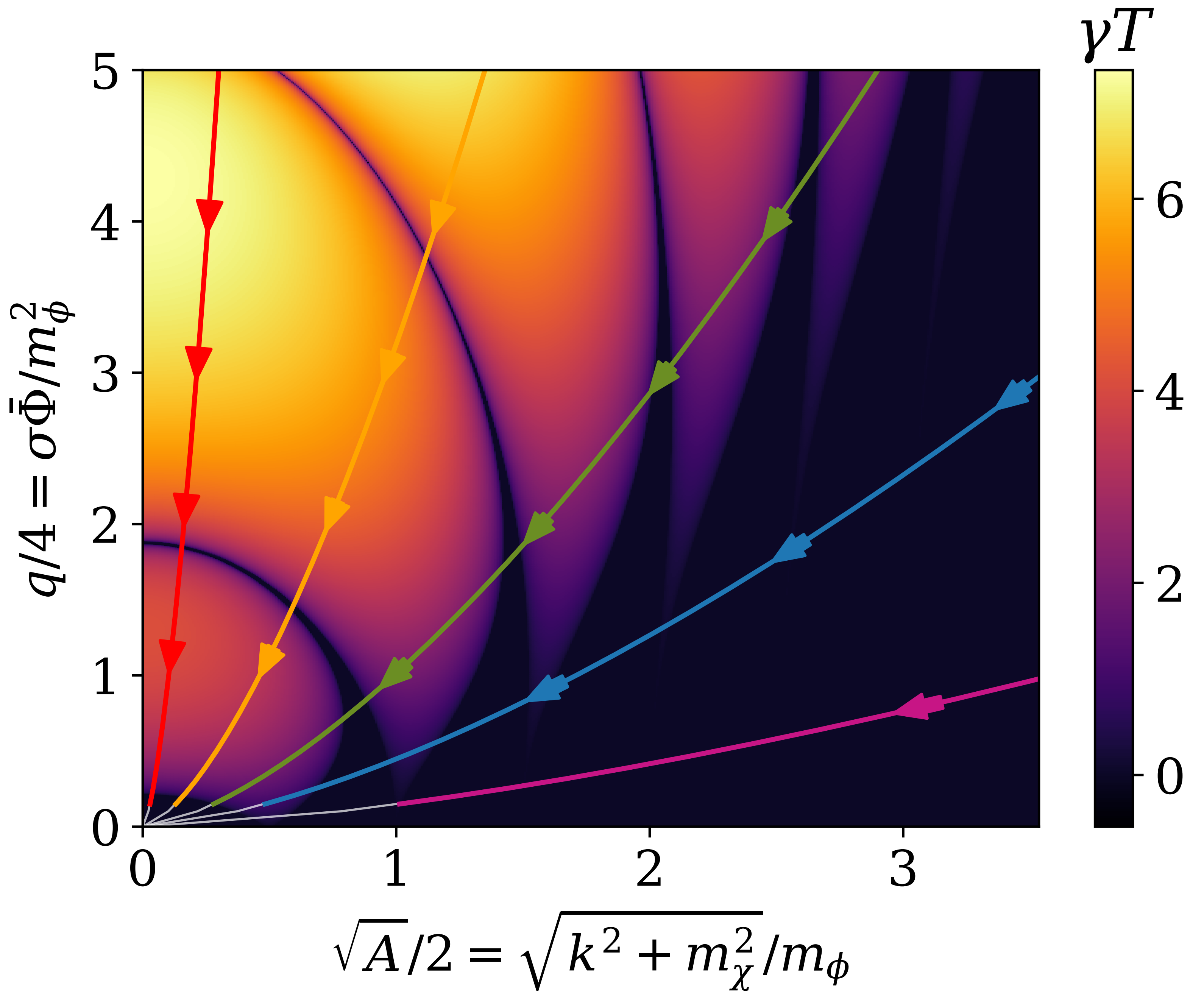

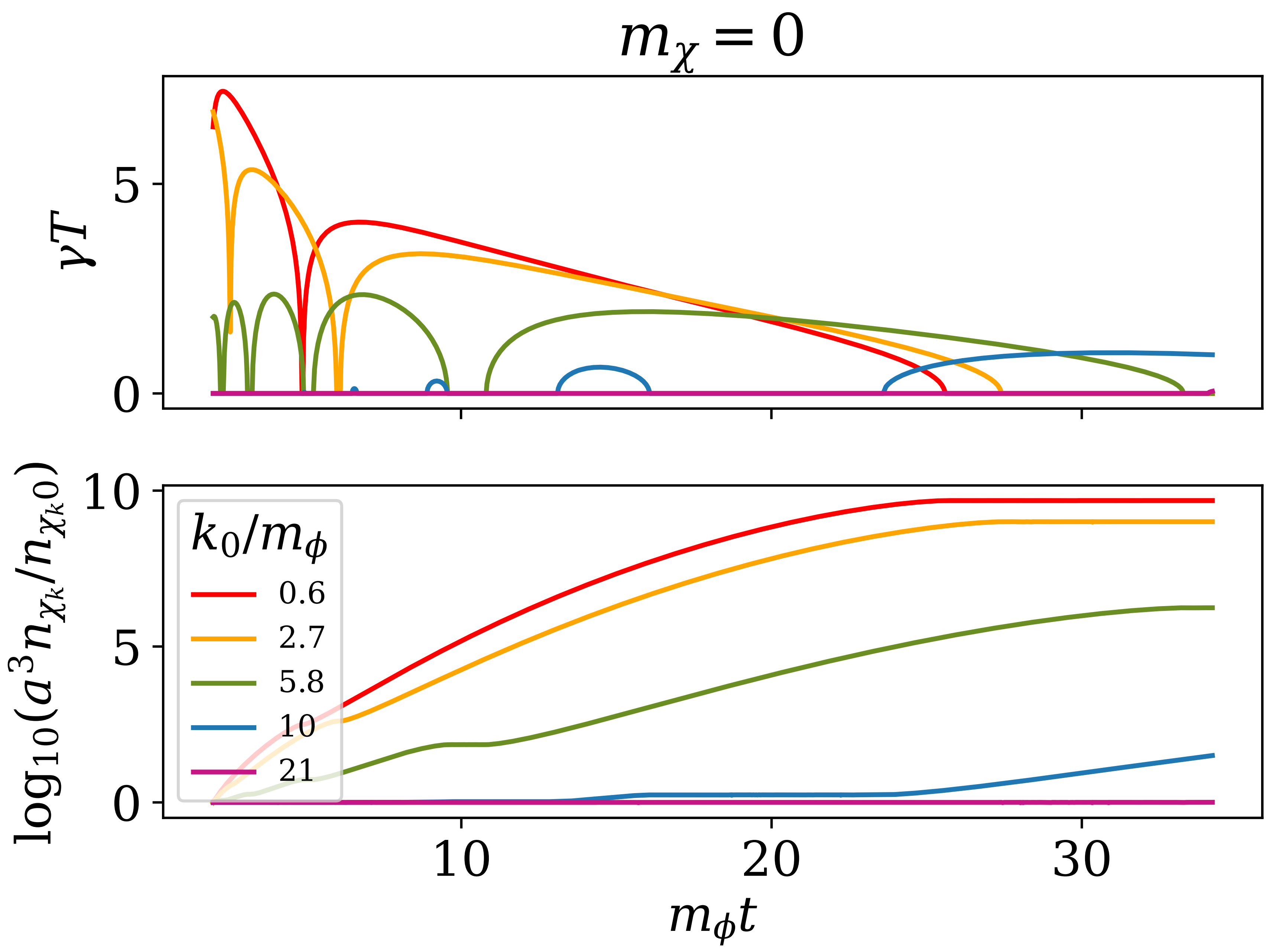

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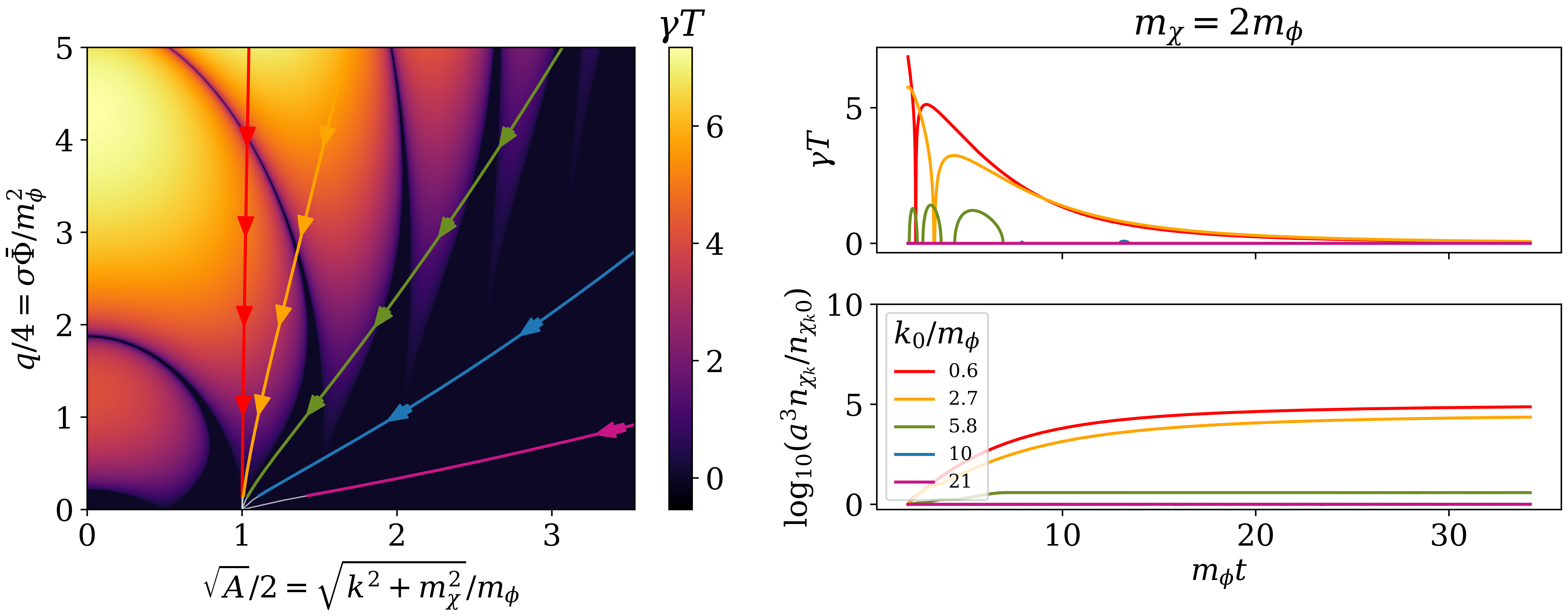


Quadratic + quartic oscillation solution

Split into amplitude and periodic $\phi = \Phi\tilde{\phi}$ and solve for $\tilde{\phi}$

$$\text{Energy conservation} \quad \frac{1}{2}\dot{\tilde{\phi}}^2 = \frac{1}{2}m_\phi^2 + \frac{1}{4}\lambda\Phi^2 - \frac{1}{2}m_\phi^2\tilde{\phi}^2 - \frac{1}{4}\lambda\Phi^2\tilde{\phi}^4$$

$$t = \frac{\sqrt{2}k}{\sqrt{\lambda}\Phi}\int_0^\theta \frac{d\theta'}{\sqrt{1 - k^2\sin^2\theta'}} \quad , \quad \text{where} \quad k^2 = \frac{1}{2}\left(1 + \frac{m_\phi^2}{\lambda\Phi^2}\right)^{-1} \quad , \quad \theta = \cos^{-1}\tilde{\phi}$$

$$\text{Incomplete elliptical integral of the first kind, } t = \frac{\sqrt{2}k}{\sqrt{\lambda}\Phi}F\left(\theta, k^2\right),$$

$$\text{with exact solution elliptic cosine, } \tilde{\phi} = \cos\theta = \text{cn}\left(\frac{\sqrt{\lambda}\Phi t}{\sqrt{2}k}, k^2\right)$$

$$\tilde{\phi} \approx \cos(\omega t)\left[1 - \epsilon\sin^2(\omega t)\right] \quad , \quad \text{where} \quad \omega = \frac{(m^2 + \lambda\Phi^2)^{3/2}}{m^2 + \frac{9}{8}\lambda\Phi^2} \quad , \quad \epsilon = \frac{1}{8}\frac{\lambda\Phi^2}{m^2 + \frac{9}{8}\lambda\Phi^2}$$

Roles of other conformal interaction terms

and excluding most of them

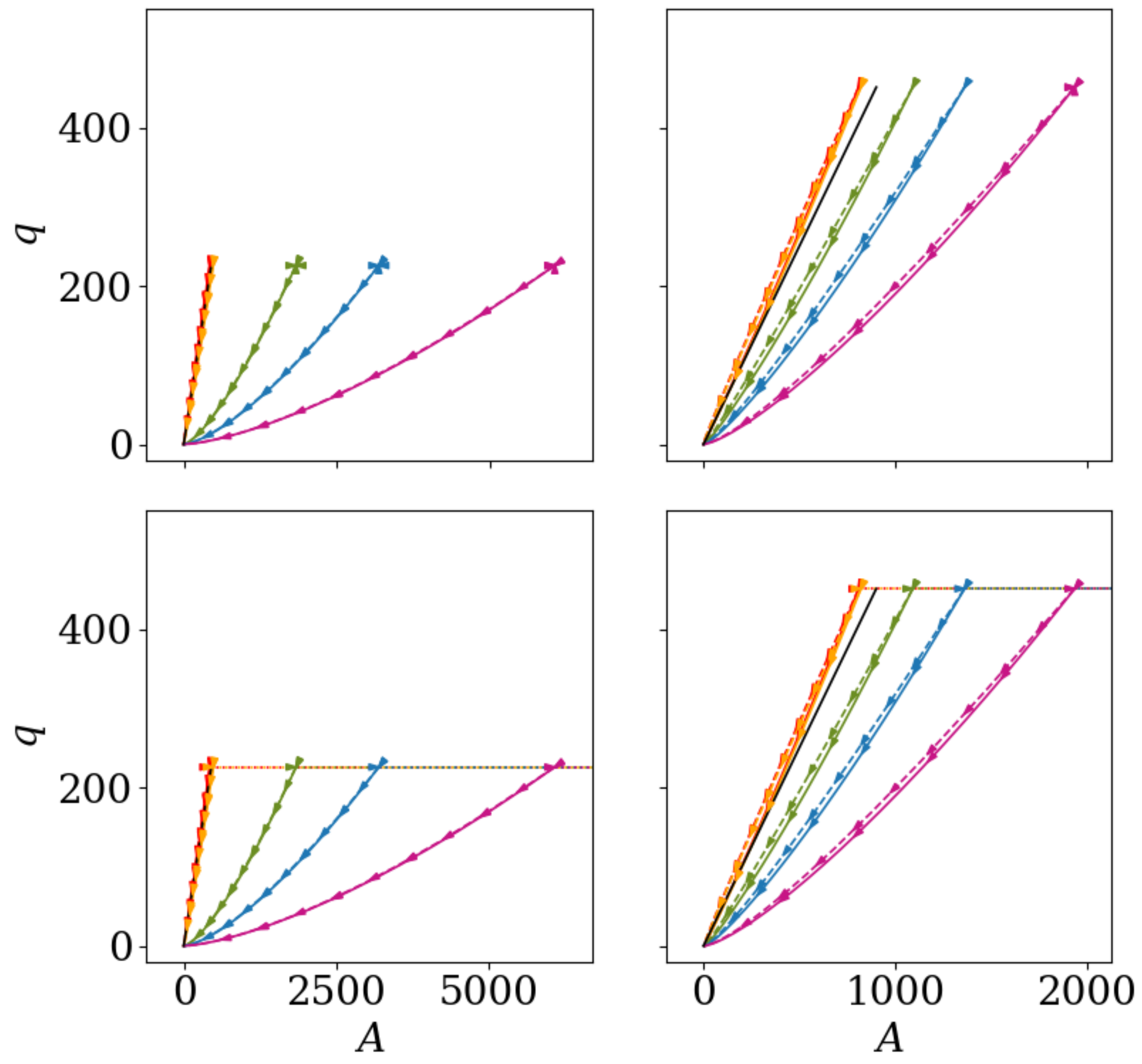
$$V(\phi,\chi) = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_\chi}{4}\chi^4 + g_1\phi^3\chi + g^2\phi^2\chi^2 + g_3\phi\chi^3$$

$\lambda_\chi > 2\sigma^2/m_\phi^2$: Bounds potential from below; if too large can act as a strong restoring force

g_1: Ordinary forcing term that results in *linear* growth in n_χ

g^2: Only interesting one, turns out to result in $\lesssim$ growth compared to $V \supset \sigma\phi\chi^2$ (see right)

g_3: Double exponential growth, not numerically tractable



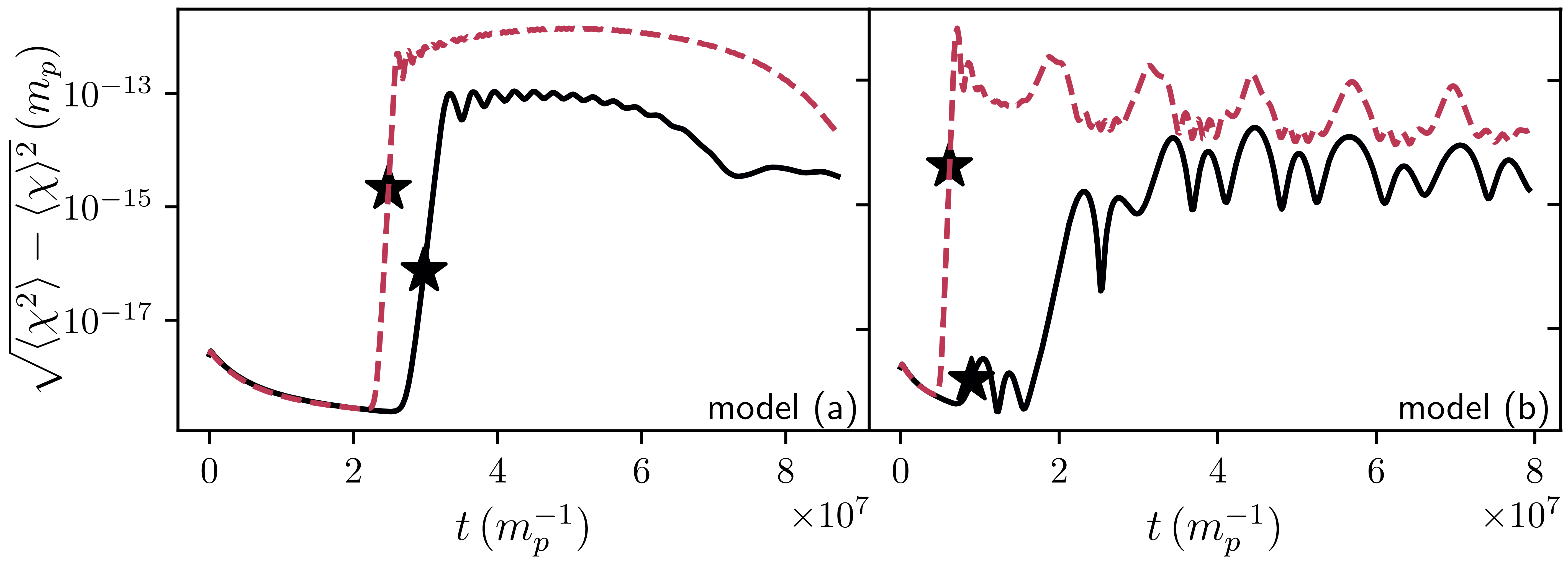
g_2 trajectories

Lattice results for quartic potential

Field variances show growth of nonzero modes

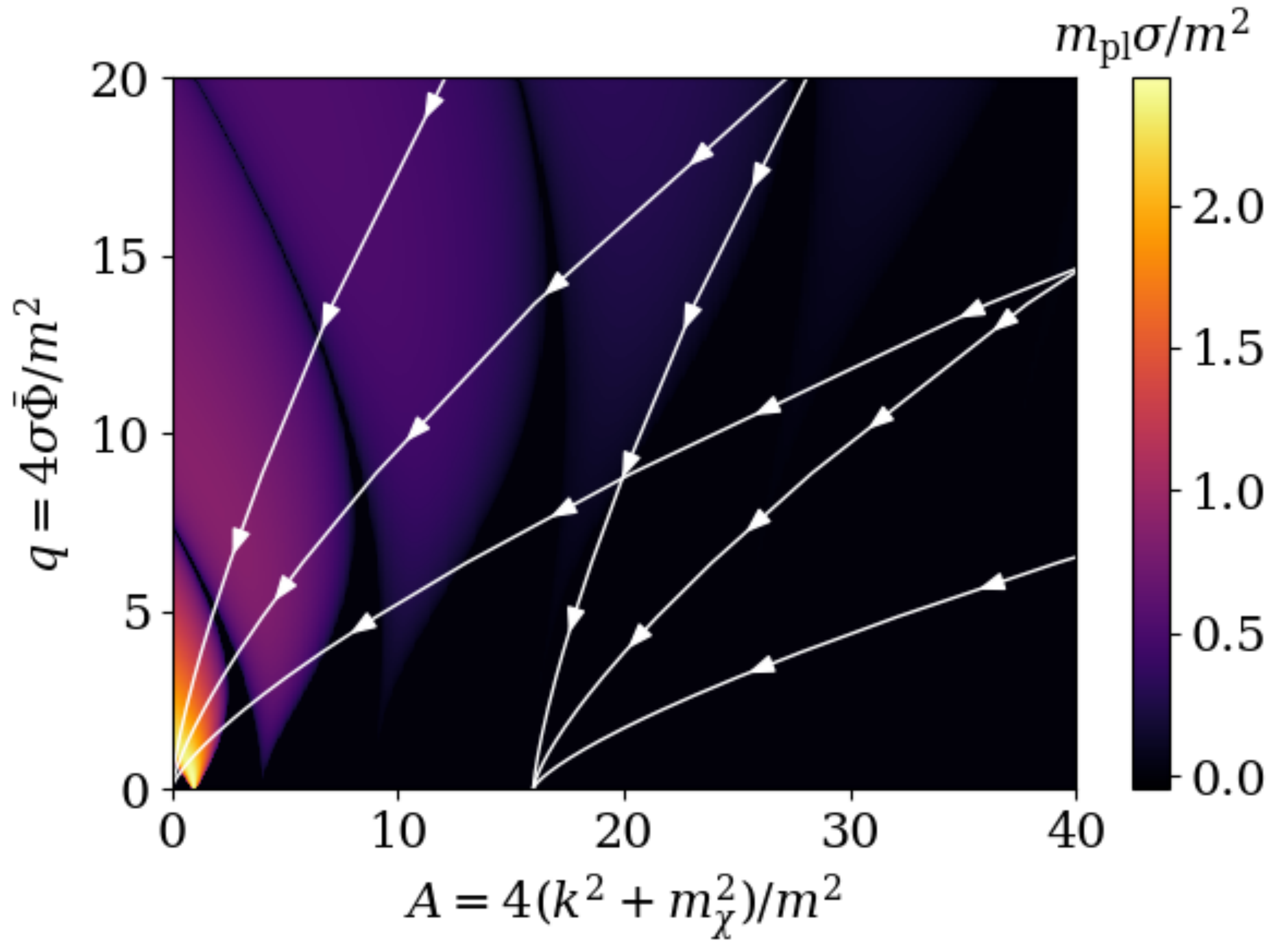
Model A: $\lambda = 10^{-14}$, Model B: $m_\phi = 5 \times 10^{-7} m_{\rm pl}$

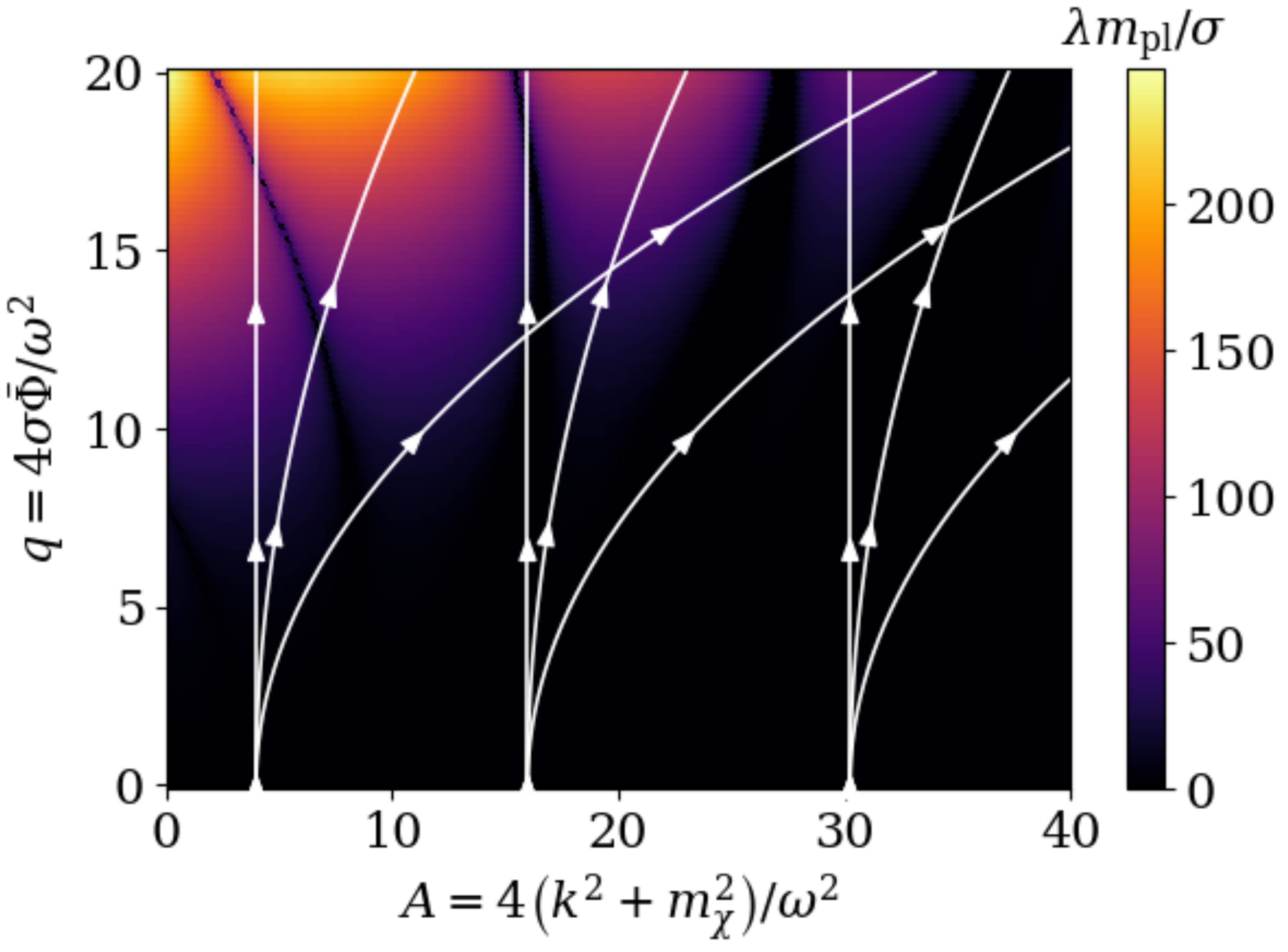
Red: $\sigma = 5 \times 10^{13}$, Black: $\sigma = 5 \times 10^{12}$



Comparing growth and expansion rates

For real efficiency we need $\gamma/H \gtrsim 1$





Stochastic DEs versus cycle-to-cycle variation

Avoiding actually evaluating stochastic integrals

Correspondence between the two types of randomness

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \big(A_0 + 2q_0 P_T(t)\big)y(\tau) = \epsilon \quad \leftrightarrow \quad \frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \big((A_0 + \ell_n) + 2(q_0 + p_n)P_T(t)\big)y(\tau) = 0$$

through these expressions containing stochastic integrals:

$$\ell_n = \frac{J_2\Xi_1 - J_1\Xi_2}{I_1J_2 - I_2J_1} \quad , \quad p_n = \frac{I_1\Xi_2 - I_2\Xi_1}{I_1J_2 - I_2J_1}$$

$$\text{where} \quad I_i = \int_0^T u_i^2(t)\mathrm{d}t \quad , \quad J_i = \int_0^T P(t)u_i^2(t)\mathrm{d}t \quad , \quad \Xi_i = \int_0^T \epsilon u_i(t)\mathrm{d}t$$

Cosmological moduli problem

Similar setup to reheating, slightly different issue

Light scalar fields ϕ ("moduli") are a generic prediction of higher dimensions

Tree level decay rate is tiny, e.g. $\Gamma_{\phi\to aa} = \frac{1}{48\pi}\frac{m_\phi^3}{m_{\rm pl}^2}$ for $\mathscr{L} \supset \frac{\phi}{m_{\rm pl}}\partial^\mu a \partial_\mu a$

Early matter domination threatens BBN success

Moduli also generically oscillate in a potential well: hope?

Attractor-tracker solution to overshoot

Sets initial energy conditions for cosmological moduli

For a potential $V(\phi) \approx V_0\, e^{-\lambda\phi/M_{\rm pl}} + (\text{small barrier})$, there are two generic phases:

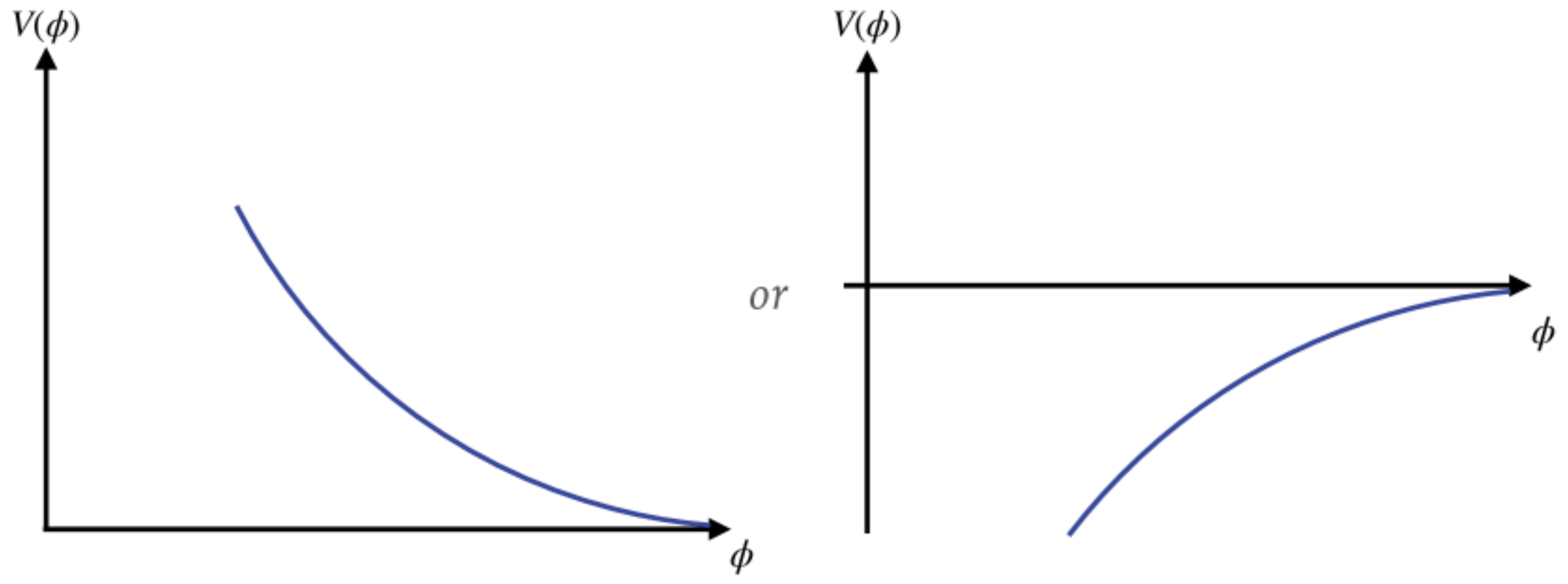
1) Kination: opposite of potential energy domination, $w_{\rm kin} = +1$, so energy could possibly dilute fast enough to become less than height of barrier

2) If there's any background radiation with w, an attractor solution might be reached:

$$\Omega_{\rm KE} = \frac{3}{2}\frac{(1+w)^2}{\lambda^2}\ , \quad \Omega_{\rm PE} = \frac{3}{2}\frac{1-w^2}{\lambda^2}\ , \quad \Omega_w = 1 - 3\frac{1+w}{\lambda^2}$$

Dine-Seiberg problem

"When corrections can be computed, they are not important, and when they are important they cannot be computed."



From presentation by Severin Lüst

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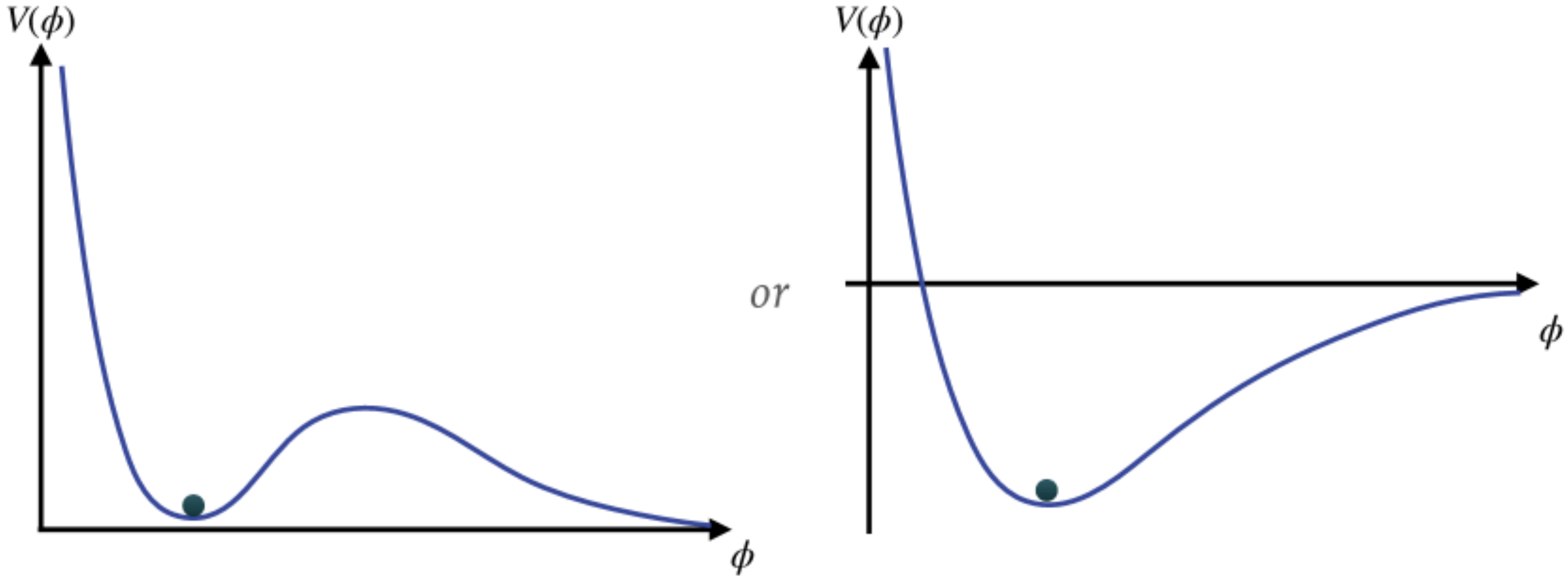

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KKLT and LVS

Two ways around Dine-Seiberg problem

Last equation of Wess and Bagger: $V = e^K K^{ij} D_i W \overline{D_j W}$

Scalar potential from superpotential W and Kahler potential $K = -3\ln(T + \overline{T}) + \ldots$

T is modulus whose potential we want to change

1) Classical flux additions

2) Quantum corrections

LVS: perturbative corrections to K

KKLT: nonperturbative corrections to W

3) Uplift, usually with anti-brane, to get a de Sitter vacuum for our Λ (still finely-tuned)