Leggett-Garg Inequality Violation in Muon g-2 Experiments

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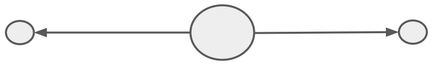
Outline

- What are Leggett-Garg Inequalities
- Muon g-2 experiment ⟶ muon polarization
- LGI violation
- Conclusion

Introduction: Bell’s Inequality

Similarities between Bell’s inequality and Leggett-Garg inequalities

Motivation: EPR thought experiment

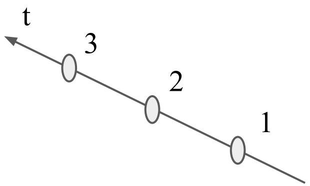


1.) Realism: observable has well defined value regardless of measurement
2.) Locality: information cannot travel faster than light speed. Measurements by different observers far apart aren’t affected by one another

$$1 + P(\vec{b}, \vec{c}) \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})|$$

[J.S. Bell 1964]

Leggett-Garg Inequality



Motivation: macroscopic coherence [A.J.Leggett, A. Garg 1985]

Leggett-Garg: correlations for same operator at different times $Q = \pm 1$

1.) Realism: Observable has well defined value at all times
2.) Non-Invasive Measurability: performing a measurement on the system does not affect future measurements

[C. Emary, N. Lambert, F. Nori 2014]

$$C_{ij} = \sum_{Q_i, Q_j = \pm 1} Q_i Q_j P_{ij}(Q_i, Q_j) \qquad P_{ij}(Q_i, Q_j) = \sum_{Q_k; k \neq i,j} P_{ij}(Q_3, Q_2, Q_1)$$

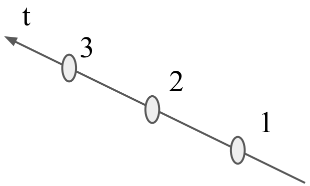
Then the correlation functions can be written as combinations of probabilities

$$K_3 = C_{21} + C_{32} - C_{31} = 1 - 4\left[P(+,-,+) + P(-,+,-)\right] \longrightarrow \boxed{1 \geq K_3}$$

Leggett-Garg Inequality Violation

Leggett-Garg: correlations for same operator at different times

[A.J.Leggett, A. Garg 1985]



1.) Realism: Observable has well defined value at all times
2.) Non-Invasive Measurability: performing a measurement on the system does not affect future measurements

[C. Emary, N. Lambert, F. Nori 2014]

QM: violates assumptions

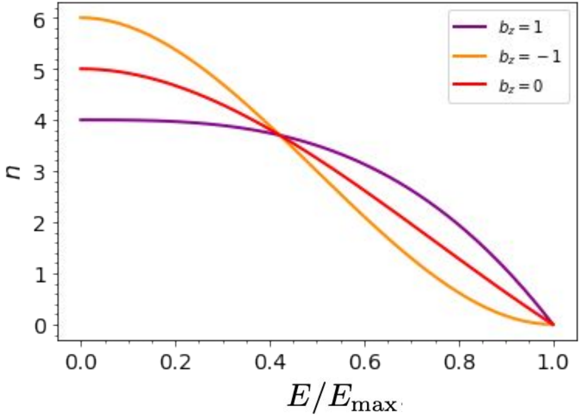
A single qubit: $\hat{Q} = \overrightarrow{b} \cdot \hat{\sigma}$

The correlator: $C_{ij} = \frac{1}{2}\left\langle\left\{\hat{Q}_i, \hat{Q}_j\right\}\right\rangle$ [T. Fritz 2014]

$$\mathcal{C}_{ij} = \frac{1}{2}\langle\phi|\{\hat{Q}(t_i), \hat{Q}(t_j)\}|\phi\rangle = \vec{b}_i \cdot \vec{b}_j.$$

$$K_3 = \mathcal{C}_{21} + \mathcal{C}_{32} - \mathcal{C}_{31} = \vec{b}_2 \cdot \vec{b}_1 + \vec{b}_3 \cdot \vec{b}_2 - \vec{b}_3 \cdot \vec{b}_1 \leq \frac{3}{2}$$

Muon g-2

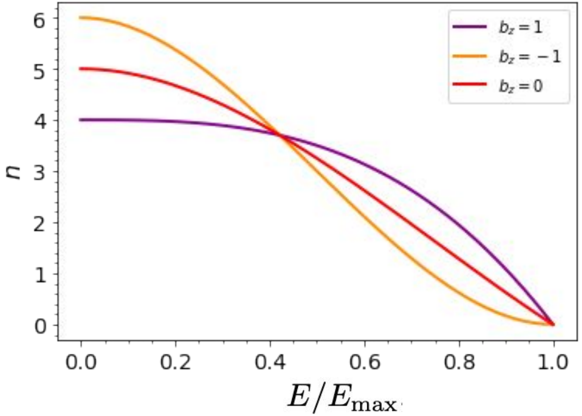


-Muon g-2 experiment provides opportunity to measure K_3 . First LG test with muons

-Muon spin precession around ring. The longitudinal polarization of the muon will oscillate in time.

-polarization can be determined from the energy distribution of decay products

Muon g-2

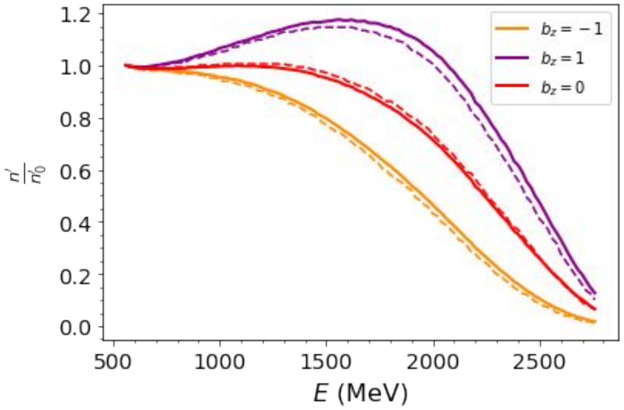


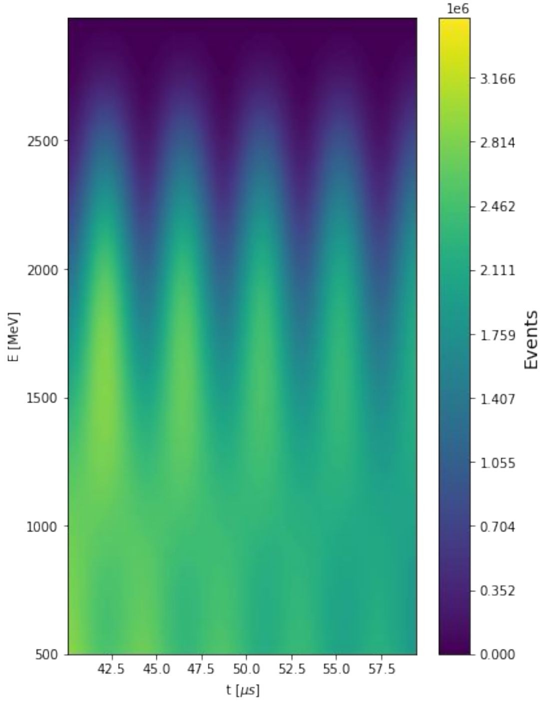
$$n_{\mathrm{th}}(E, b_z) = N(E)\Big(1 + b_z A(E)\Big)$$

Muon g-2 + LGI Construction

Model the detector efficiency:

$$\epsilon(E, b_z) = \epsilon_0(E) + b_z \epsilon_1(E)$$





[J. LaBounty 2024]

LGI Construction

$$P_{\pm} = \frac{1 \pm b_z}{2}$$

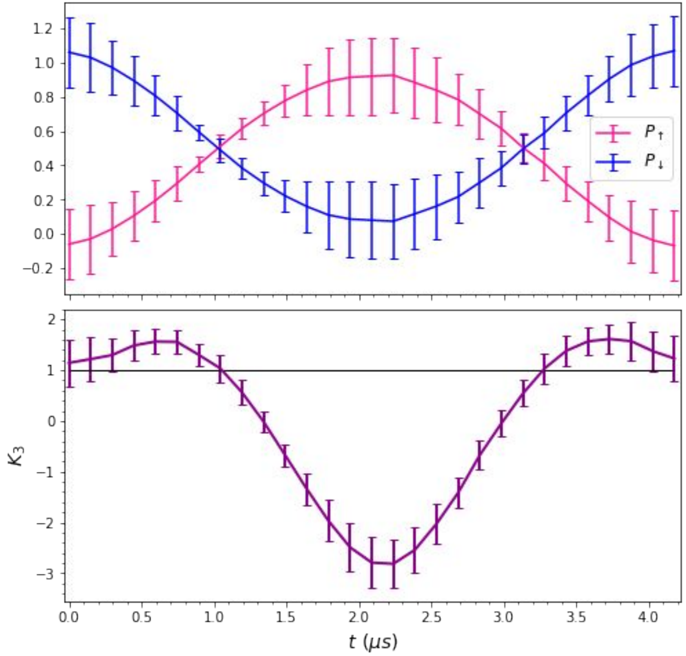
Assume known initial state

Stationarity [C. Emary, N. Lambert, F. Nori 2014]

- Time difference: $\Delta t \equiv t_j - t_i$

$$K_3 = 2C(\Delta t) - C(2\Delta t)$$

Small statistical uncertainty



Conclusions

- What are LGIs: family of inequalities for a system adhering to realism and non-invasive measurement.
- QM can violate these types inequalities
- Muon g-2 experiments can be used as a "Leggett-Garg" test (+ assumptions)
    - First test of LG with muons
- LGI violation consistent with QM predictions

extra

LGI

The correlation functions:

$$
\begin{aligned}
C_{21} = {} & P(+,+,+) - P(+,+,-) - P(-,-,+) + P(-,-,-) \\
& - P(+,-,+) + P(+,-,-) + P(-,+,+) - P(-,+,-); \\
C_{32} = {} & P(+,+,+) + P(+,+,-) + P(-,-,+) + P(-,-,-) \\
& - P(+,-,+) - P(+,-,-) - P(-,+,+) - P(-,+,-); \\
C_{31} = {} & P(+,+,+) - P(+,+,-) - P(-,-,+) + P(-,-,-) \\
& + P(+,-,+) - P(+,-,-) - P(-,+,+) + P(-,+,-),
\end{aligned}
$$

LGI

For a hidden variable theory,

$$\left\langle \hat{A}(t) \right\rangle = \int d\lambda A(\lambda, t)\rho(\lambda)$$

$$\left|\left\langle \hat{Q}(t_2)\hat{Q}(t_1)\right\rangle - \left\langle \hat{Q}(t_4)\hat{Q}(t_1)\right\rangle\right| \leq 2 \pm \left[\left\langle \hat{Q}(t_3)\hat{Q}(t_2)\right\rangle + \left\langle \hat{Q}(t_4)\hat{Q}(t_3)\right\rangle\right]$$

Leggett-Garg Inequality

Leggett-Garg: correlations for same operator at different times [A.J.Leggett, A. Garg 1985]

1.) Realism: state has well defined value at all times
2.) Non-Invasive Measurability: performing a measurement on the system does not affect future measurements

[C. Emary, N. Lambert, F. Nori 2014]

Most general: $K_n = C_{21} + C_{32} + C_{43} + \ldots + C_{n(n-1)} - C_{n1}$

$$-n \leq K_n \leq n-2 \qquad n \geq 3,\ \text{odd}$$
$$-(n-2) \leq K_n \leq n-2 \qquad n \geq 4,\ \text{even}$$

LGI Construction

[J. LaBounty 2024]

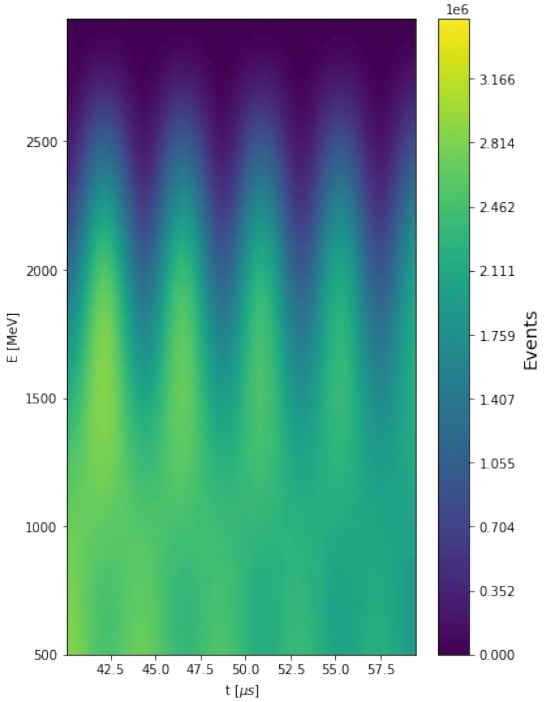
Get polarization at each time

$$\chi^2 = \sum_i \left(\frac{n_{\mathrm{exp}}(E_i, t) - n'_{\mathrm{th}}(E_i, b_z)}{\Delta n_i} \right)^2 \qquad P_\pm = \frac{1 \pm b_z}{2}$$

Assume stationarity:

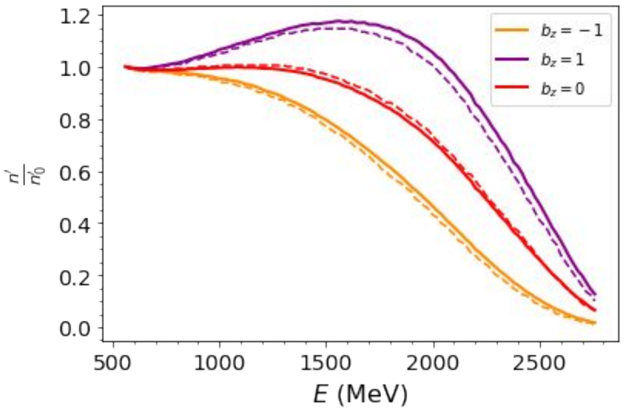
- known initial state
- Time difference: $\Delta t \equiv t_j - t_i$

$$K_3 = 2C(\Delta t) - C(2\Delta t)$$



Muon g-2 + LGI Construction

Model the detector efficiency: $\epsilon(E, b_z) = \epsilon_0(E) + b_z \epsilon_1(E)$



$$n'_{\text{th}}(E, b_z) = n_{\text{th}}(E, b_z) \times \epsilon(E, b_z)$$