

EFFECTS OF A NEW FORCE ON SCALAR DARK MATTER SOLITONS

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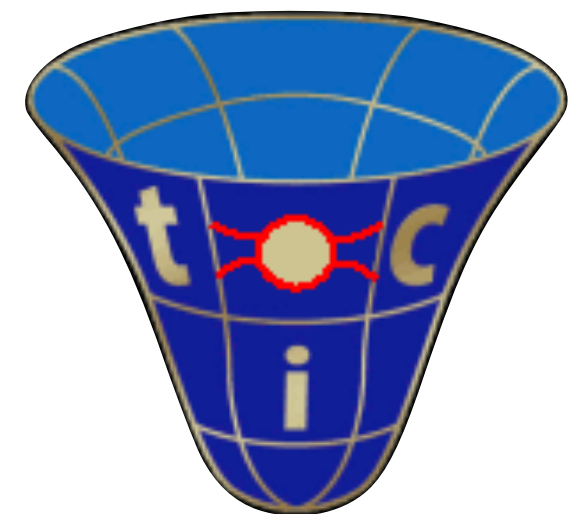
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Based on: **arXiv: 2511.15916v2**

w/ **Mark P. Hertzberg, Michiru Uwabo-Niibo**



OUTLINE

1. Introduction

1. Background

2. Motivation

2. The theory

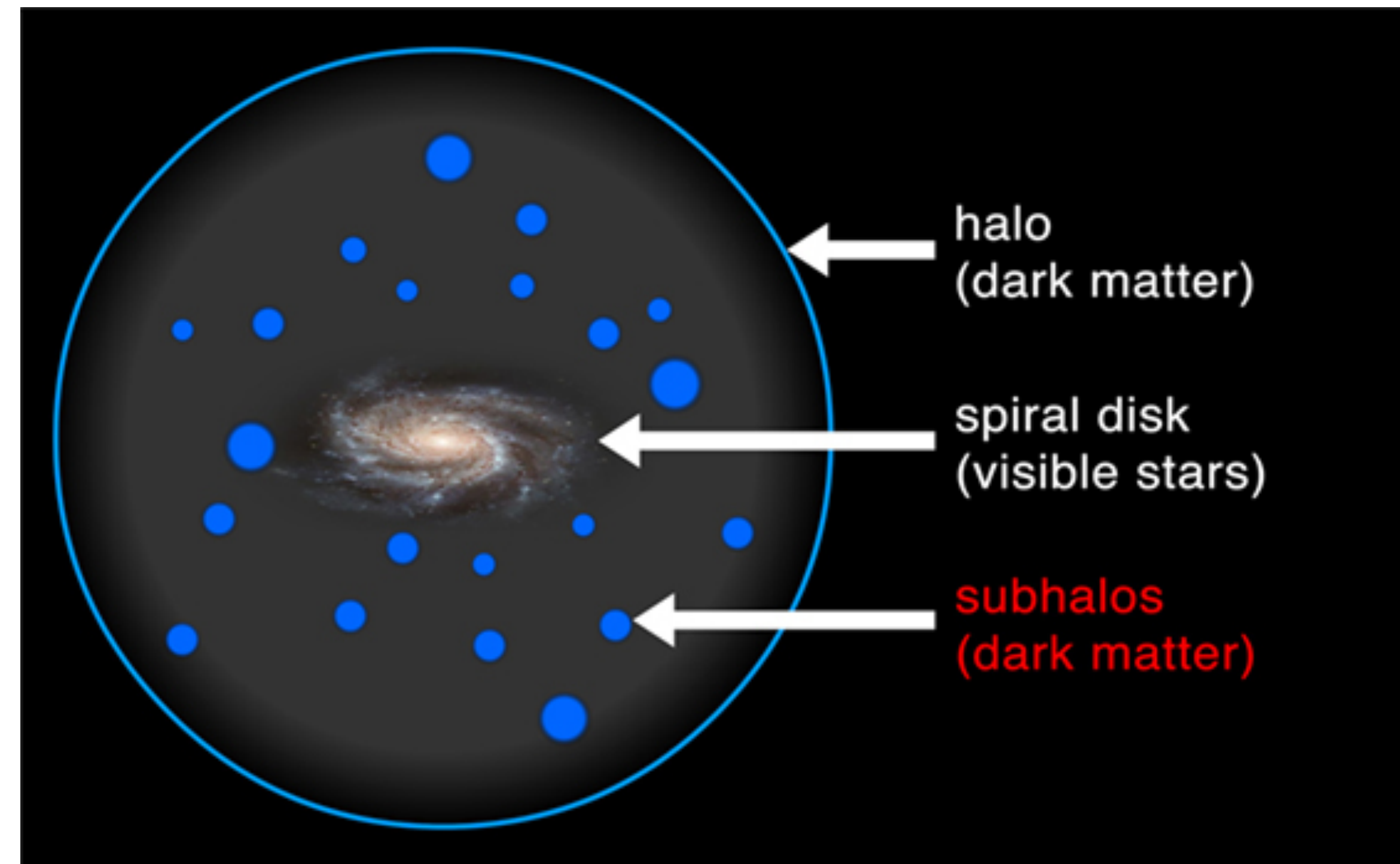
1. Single mediator case

2. Multiple mediator case

3. Numerical methods

4. Results

5. Takeaways



Cartoon of a dark matter halo surrounding a galaxy.

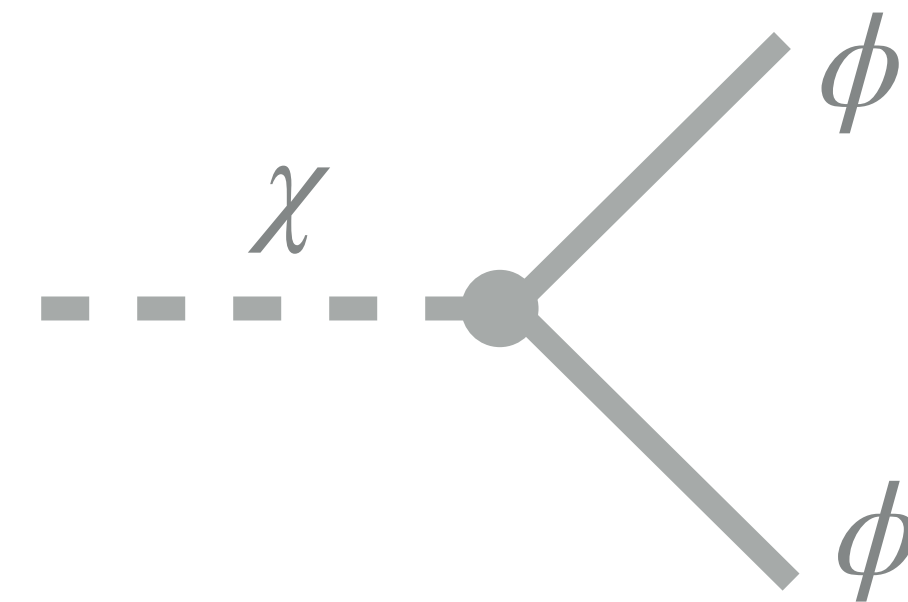
THE SCALAR DM AND THE SCALAR DM CLUMP

- ▶ Light ($m_\phi \ll 1\text{eV}$) scalar dark matter (DM)
- ▶ Astronomical consequences when $m_\phi \sim 10^{-22} - 10^{-19}\text{eV}$
 - ▶ Can form condensates: "Bose stars" , "clumps"
- ▶ Cosine potential expanded around $\phi = 0$

$$V(\phi) = m_\phi^2 f_{PQ}^2 \left(1 - \cos \left(\frac{\phi}{f_{PQ}} \right) \right) \rightarrow V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \dots$$

THE NEW FORCE

- ▶ Non-gravitational force
 - ▶ Yukawa-like
- ▶ Scalar, light mediator
 - ▶ 3-point interaction with DM
 - ▶ Coupling to the trace of energy-momentum tensor of the DM



CLUMP BEHAVIOR

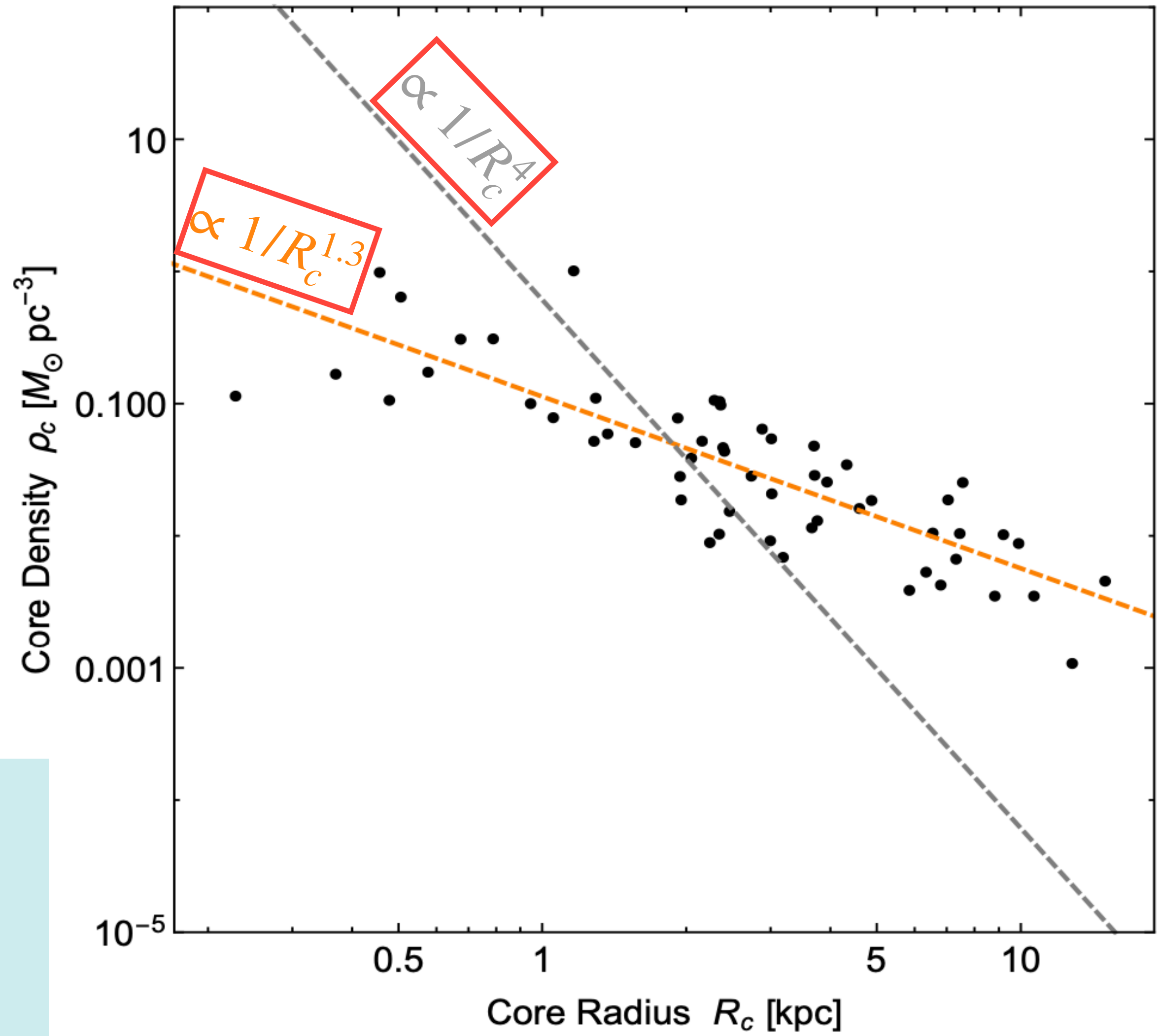
Some observations point towards $\rho_c \propto 1/R_c^q$ with $q \approx 1.3$

MNRAS
3823-3838
1701.02698

- ▶ $q = 4$ if the ultralight DM that interacts via gravity

2310.19762

**GOOD QUESTION TO ASK:
WHAT HAPPENS WHEN WE INTRODUCE A NEW FORCE?**



Core density versus core radius relation of 56 mock galaxies
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THE ACTION

The action in (+, -, -, -) signature

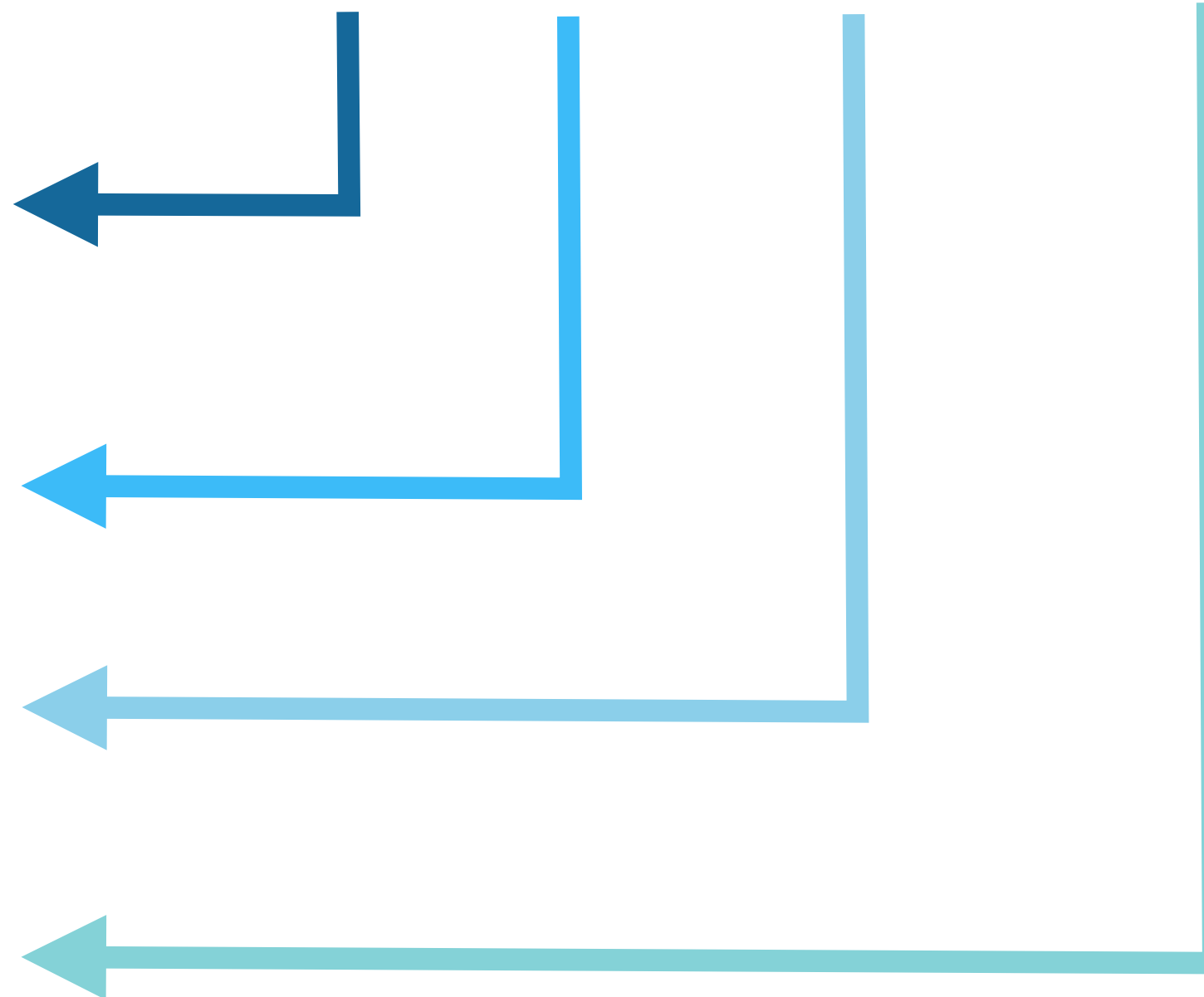
$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_\phi + \mathcal{L}_\chi + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{grav}} \right]$$

▶ $\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_\phi$

▶ $\mathcal{L}_\chi = \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2$

▶ $\mathcal{L}_{\text{int}} = -\chi \left[c_1 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + c_2 \tilde{V}_\phi \right]$

▶ $\mathcal{L}_{\text{grav}} = \frac{-\mathcal{R}}{16\pi G}$



c_1, c_2 : kinetic, potential coupling constants

THE \tilde{V}_ϕ POTENTIAL

The interaction term goes as $\sim \chi T^\phi$

The trace of energy-momentum tensor:

$$T^\phi = -2 \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V_\phi \right)$$

Coupling choice:

$$\tilde{V}_\phi = V_\phi \text{ and } c_2 = -4c_1$$

$$\tilde{V}_\phi = V_\phi \text{ and } c_1 = \frac{1}{2}c_2$$

THE METRIC AND THE WEAK FIELD REGIME

The metric:
$$ds^2 = (1 + 2\phi_N) dt^2 - (1 - 2\psi) d\vec{x} \cdot d\vec{x}$$

The full Lagrangian density:

$$\begin{aligned} \mathcal{L}_{\text{full}} = & \frac{1}{2}(\partial_\mu \phi)^2 - V_\phi + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}m_\chi^2 \chi^2 - \phi_N \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla} \phi)^2 + V_\phi \right] - \psi \left[\frac{3}{2}\dot{\phi}^2 - \frac{1}{2}(\vec{\nabla} \phi)^2 - 3V_\phi \right] \\ & - \chi \left[c_1(\dot{\phi}^2 - (\vec{\nabla} \phi)^2) + c_2 \tilde{V}_\phi \right] + \frac{1}{8\pi G} [(\vec{\nabla} \psi)^2 - 3\dot{\psi}^2 - 2\vec{\nabla} \phi_N \cdot \vec{\nabla} \psi] \end{aligned}$$

- ▶ Weak field regime: $\phi, \psi, c_i \chi \ll 1$ for $i = 1, 2$

THE NONRELATIVISTIC REGIME

The field decomposition into **slowly** and **rapidly** varying parts:

$$\phi(x) = \frac{1}{\sqrt{2m_\phi}} \left[F(x)e^{-im_\phi t} + F^*(x)e^{+im_\phi t} \right]$$

- ▶ In this limit $\rho_\phi = m_\phi |F|^2$

We find

$$\frac{\delta S}{\delta \phi_N} \implies \nabla^2 \psi = 4\pi G \rho_\phi, \quad \frac{\delta S}{\delta \psi} \implies \phi_N = \psi$$

THE NONRELATIVISTIC REGIME

Assuming

- ▶ χ does not oscillate rapidly: $e^{-2im_\phi t} \rightarrow 0$
- ▶ χ is slowly varying: $\dot{\chi} \rightarrow 0$
- ▶ the small field regime: $V_\phi \approx \frac{1}{2}m_\phi^2\phi^2$

$$\mathcal{L}_{\text{time avg}} = \frac{i}{2} (\dot{F}F^* - F\dot{F}^*) - \frac{1}{2m_\phi} \left| \vec{\nabla} F \right|^2 - \phi_N \rho_\phi - \frac{1}{8\pi G} (\phi_N)^2 - \frac{1}{2} (\vec{\nabla} \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 - C \chi \rho_\phi$$

THE EQUATIONS OF MOTION

- ▶ EoM of F :

$$i\dot{F} = -\frac{1}{2m_\phi} \nabla^2 F + m_\phi (\phi_N + C\chi)$$

- ▶ The constraint equations for ϕ_N, χ :

$$\nabla^2 \phi_N = 4\pi G \rho_\phi$$

$$\nabla^2 \chi - m_\chi^2 \chi = C \rho_\phi$$

THE CONDENSATE SOLUTIONS

- ▶ Solutions of the form $F(x) = f(\vec{x}) e^{-i\mu t}$ with $\rho_\phi = m_\phi |f|^2$
- ▶ Ground state configuration assumed to be spherically symmetric with $f = f(r)$

$$\mu f(r) = -\frac{1}{2m_\phi} \left(f'' + \frac{2}{r} f' \right) + m_\phi (\phi_N + C\chi) f$$

$$\phi_N(r) = -\frac{4\pi G m_\phi}{r} \int_0^\infty dr' r' r_{<} |f(r')|^2 + \text{const}$$

$$\chi(r) = \frac{C}{4\pi G} \phi_N + \text{const} \quad (\text{massless}) \quad , \quad \chi(r) = -\frac{C m_\phi}{r} \int_0^\infty dr' \frac{r' |f(r')|^2}{2m_\chi} e^{-m_\chi(r+r')} [e^{2m_\chi r_{<}} - 1] \quad (\text{massive})$$

DIMENSIONLESS QUANTITIES

Choosing $\alpha = f(0) \implies \tilde{f}(0) = 1$,

$$\beta = (Gm_\phi^3 \alpha^2)^{-1/4}, \quad \gamma = \sqrt{\frac{G\alpha^2}{m_\phi}}, \quad \varepsilon = m_\phi \gamma$$

And we eventually plot

$$\rho_c = \frac{1}{Gm_\phi^2 \beta^4}, \quad \tilde{f} = \sqrt{\frac{\rho}{\rho_c}}, \quad \tilde{r} = \left(Gm_\phi^2 \rho_c\right)^{1/4}$$

Our definitions:

$$r \equiv \beta \tilde{r}$$

$$\mu \equiv \varepsilon \tilde{\mu}$$

$$f(r) \equiv \alpha \tilde{f}(r)$$

$$\phi_N(r) \equiv \gamma \tilde{\phi}_N(r)$$

$$\chi(r) \equiv \frac{\gamma}{C} \tilde{\chi}(r)$$

DIMENSIONLESS EQUATIONS OF THE MOTION

$$\tilde{\mu} \tilde{f} = -\frac{\tilde{\nabla}^2}{2} \tilde{f} + (\phi_N + \chi) \tilde{f}$$

$$\tilde{\nabla}^2 \tilde{\phi}_N = 4\pi \tilde{f}$$

$$\tilde{\nabla}^2 \tilde{\chi} - (\beta')^2 = 4\pi g_\chi \tilde{f}$$

where the measures for **radial size** and **coupling** are:

$$\beta' \equiv m_\chi \beta \quad g_\chi \equiv \frac{C^2}{4\pi G}$$

EFFECTIVE GRAVITATIONAL COUPLING

Two limits to consider:

▶ $\beta' \ll 1$:

$$G_{\text{eff}} = \left(1 + g_\chi\right) G$$

▶ $\beta' \gg 1$:

$$G_{\text{eff}} = G$$

New force effects when $\beta' \sim 1$

Core density:

$$\rho_c = \frac{\zeta}{G_{\text{eff}} m_\phi^2 R_c^4}$$

EVERYTHING AGAIN, WITH AN ADJUSTMENT

Multiple bosons:

$$\chi \rightarrow \chi_1 + \chi_2$$

$$\mathcal{L}_\chi = \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}m_\chi^2\chi^2 \rightarrow \mathcal{L}_\chi = \frac{1}{2}g^{\mu\nu}\partial_\mu\chi_1\partial_\nu\chi_1 - \frac{1}{2}m_{\chi_1}^2\chi_1^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\chi_2\partial_\nu\chi_2 - \frac{1}{2}m_{\chi_2}^2\chi_2^2$$

$$\mathcal{L}_{\text{int}} = -\chi \left[c_1 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + c_2 \tilde{V}_\phi \right] \rightarrow \mathcal{L}_{\text{int}} = -\chi_1 \left[c_1 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + c_2 \tilde{V}_\phi \right] - \chi_2 \left[\hat{c}_1 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \hat{c}_2 \tilde{V}_\phi \right]$$

EVERYTHING AGAIN, WITH AN ADJUSTMENT

Constraint equations:

$$\nabla^2 \chi_1 - m_{\chi_1}^2 \chi_1 = C_1 \rho_\phi, \quad \nabla^2 \chi_2 - m_{\chi_2}^2 \chi_2 = C_2 \rho_\phi$$

EoM for F:

$$i\dot{F} = -\frac{1}{2m_\phi} \nabla^2 F + m_\phi (\phi_N + C_1 \chi_1 + C_2 \chi_2)$$

$$\text{with } C_1 = c_1 + \frac{c_2}{2} = c_2, \quad C_2 = \hat{c}_1 + \frac{\hat{c}_2}{2} = \hat{c}_2$$

EFFECTIVE GRAVITATIONAL COUPLING NOW

Three limits to consider when $m_{\chi_2} \gg m_{\chi_1}$:

▸ $R_c \ll m_{\chi_2}^{-1}$

$$G_{\text{eff}} = \left(1 + g_{\chi_1} + g_{\chi_2}\right) G$$

▸ $m_{\chi_2}^{-1} \ll R_c \ll m_{\chi_1}^{-1}$

$$G_{\text{eff}} = \left(1 + g_{\chi_1}\right) G$$

▸ $R_c \gg m_{\chi_1}^{-1}$

$$G_{\text{eff}} = G$$

METHOD

Numerically solved the system of equations!

- ▶ Started with $\tilde{\chi} = 0$
 - ▶ Boundary conditions: $\tilde{f} = 0, \tilde{f}' = 0, \tilde{\phi}_N(0) = 0, \tilde{\phi}'_N(0)$
- ▶ Guessed $\tilde{\mu}$ and altered to find solitons
 - ▶ Convergence via bisecting the search interval
- ▶ Used the solution to feed into the $\tilde{\chi}$ integral
- ▶ Rinsed and repeated until convergence reached

$g_\chi \gg 1 \implies$ SLOW CONVERGENCE

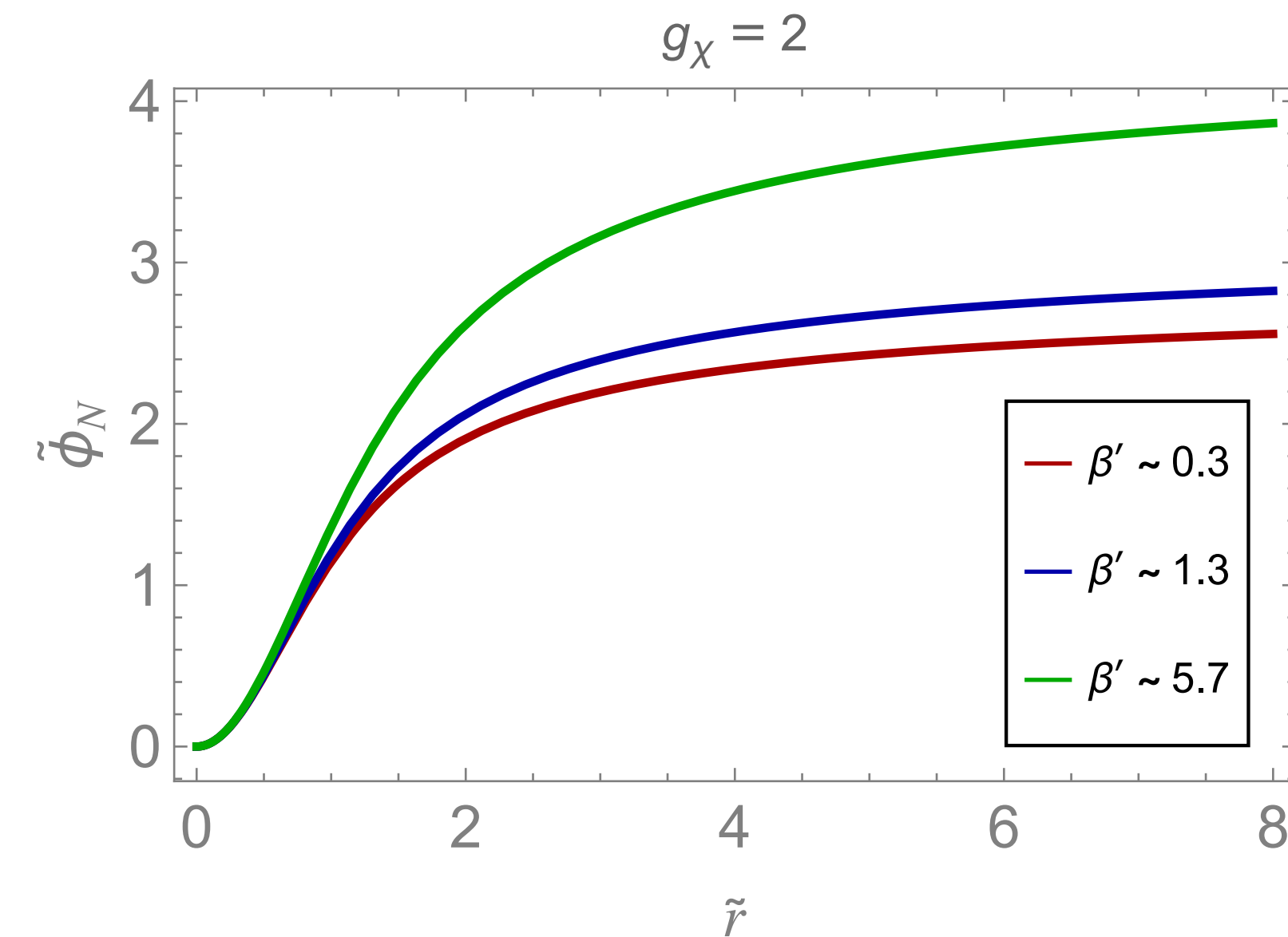
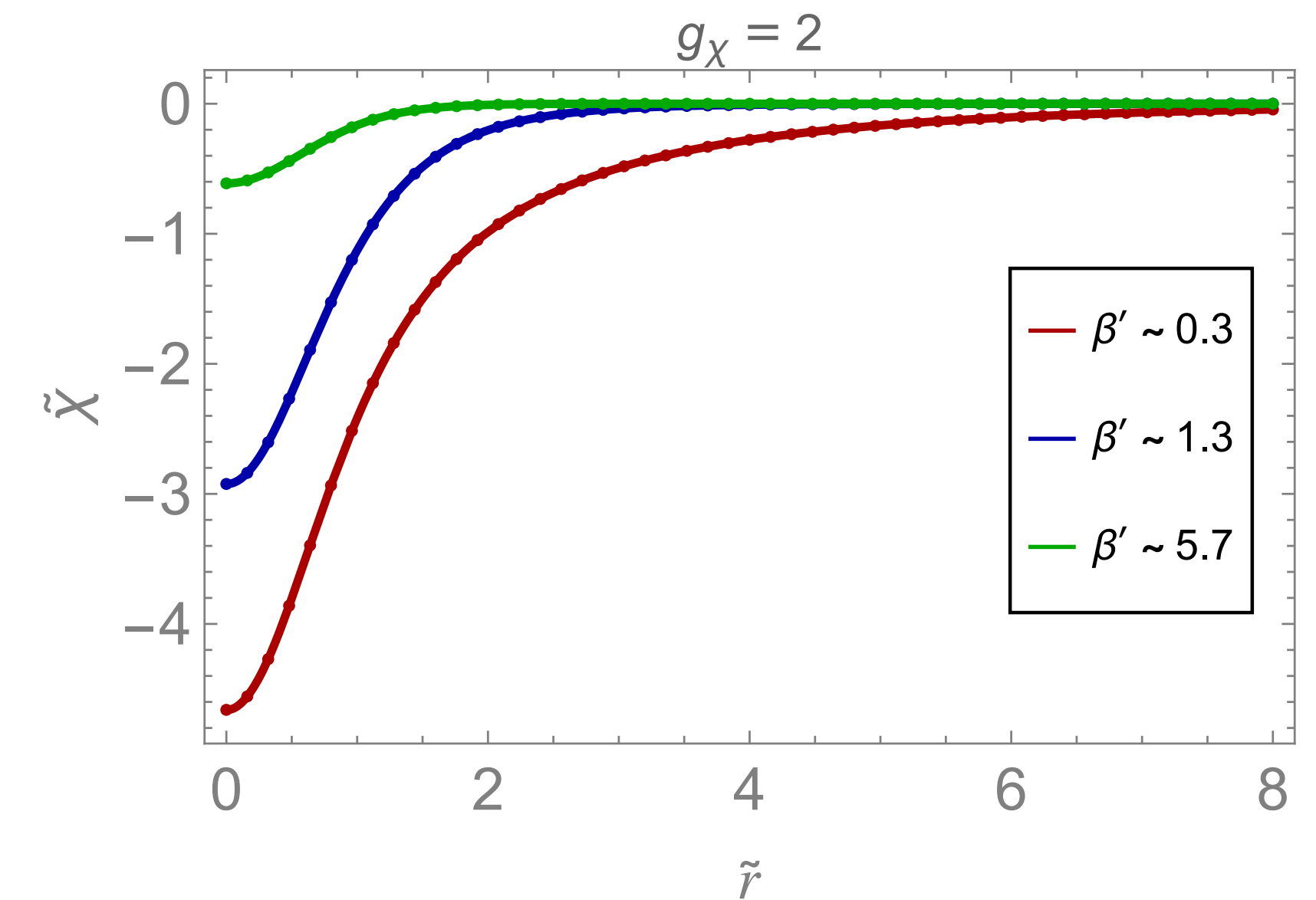
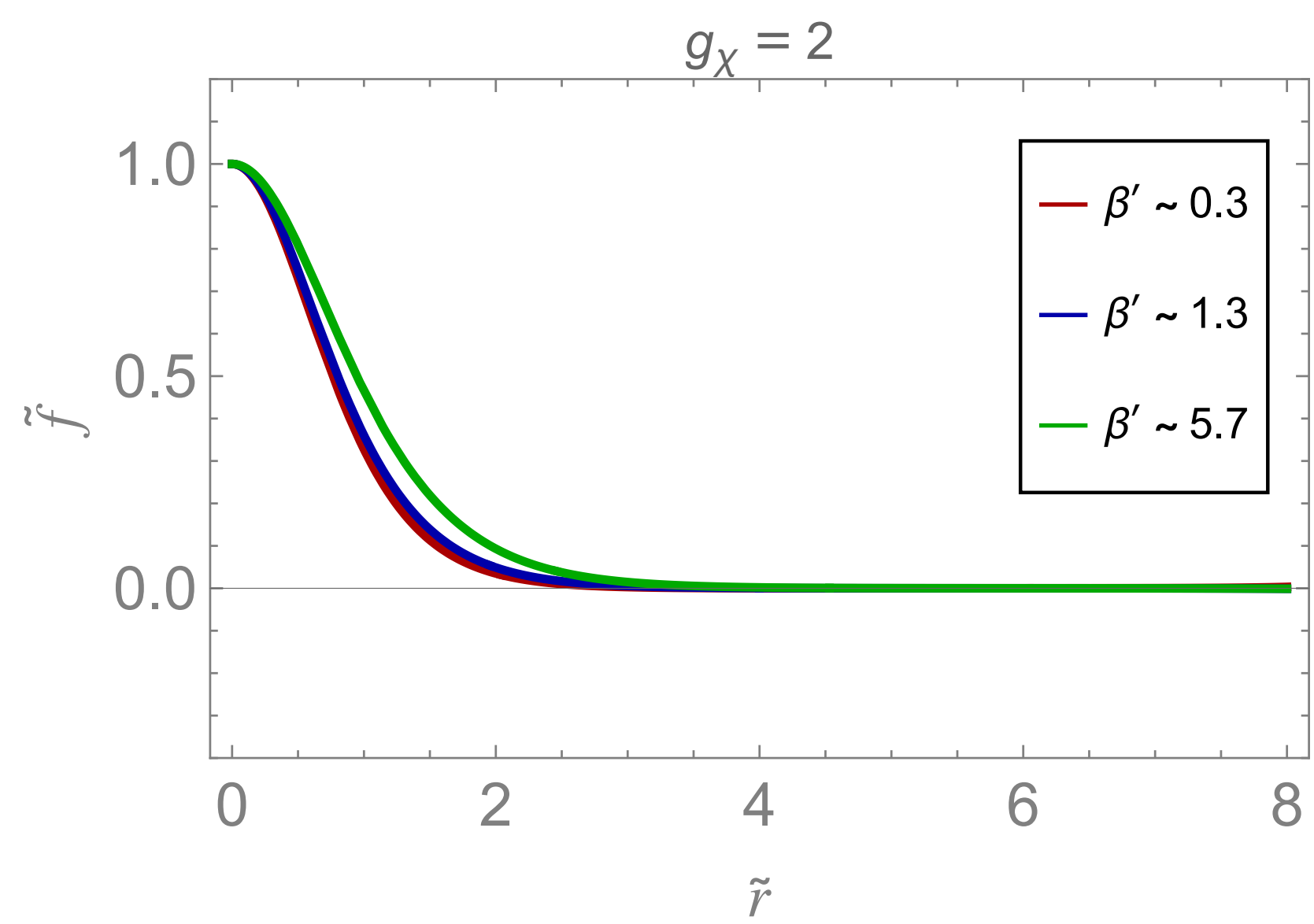
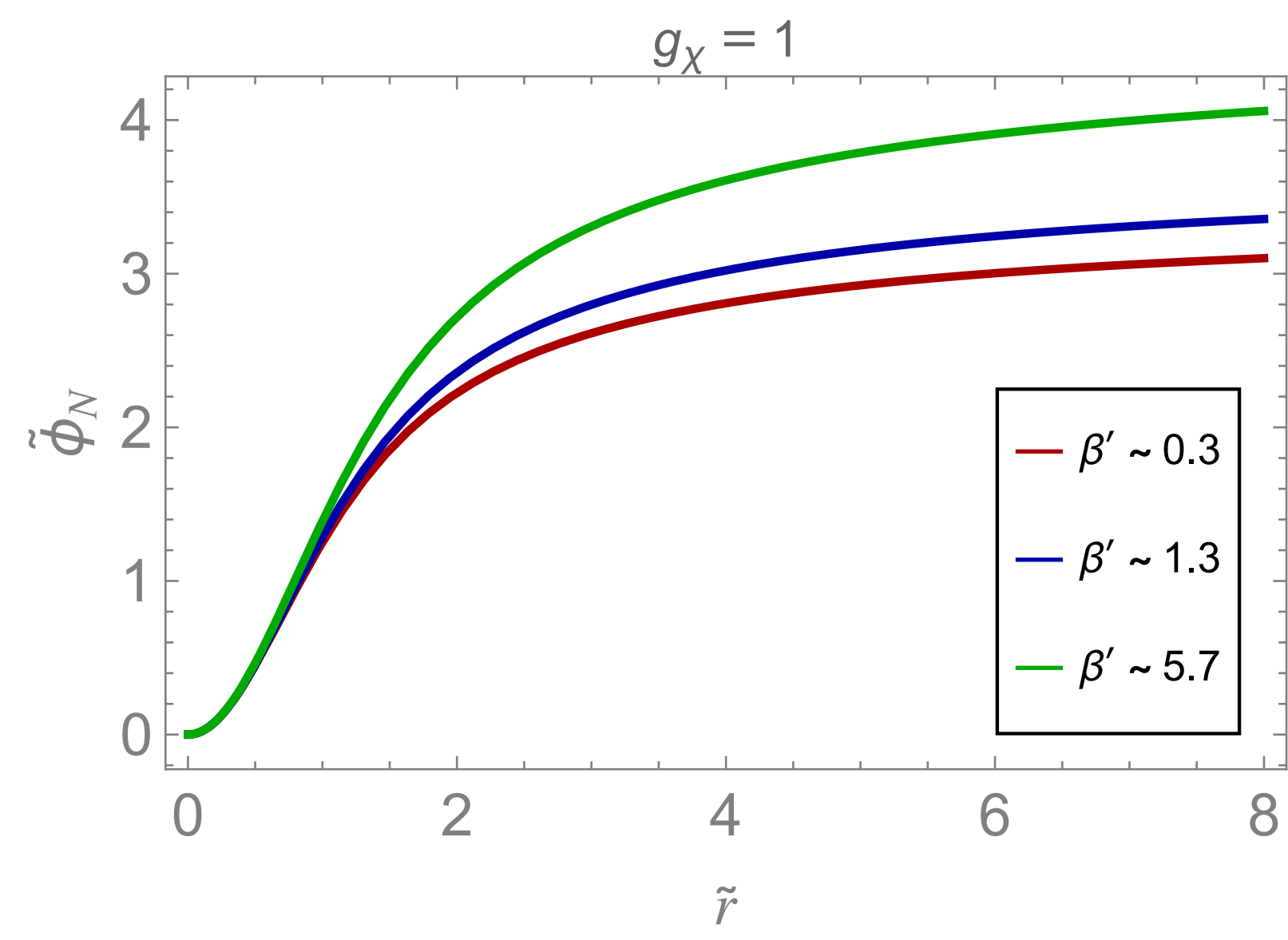
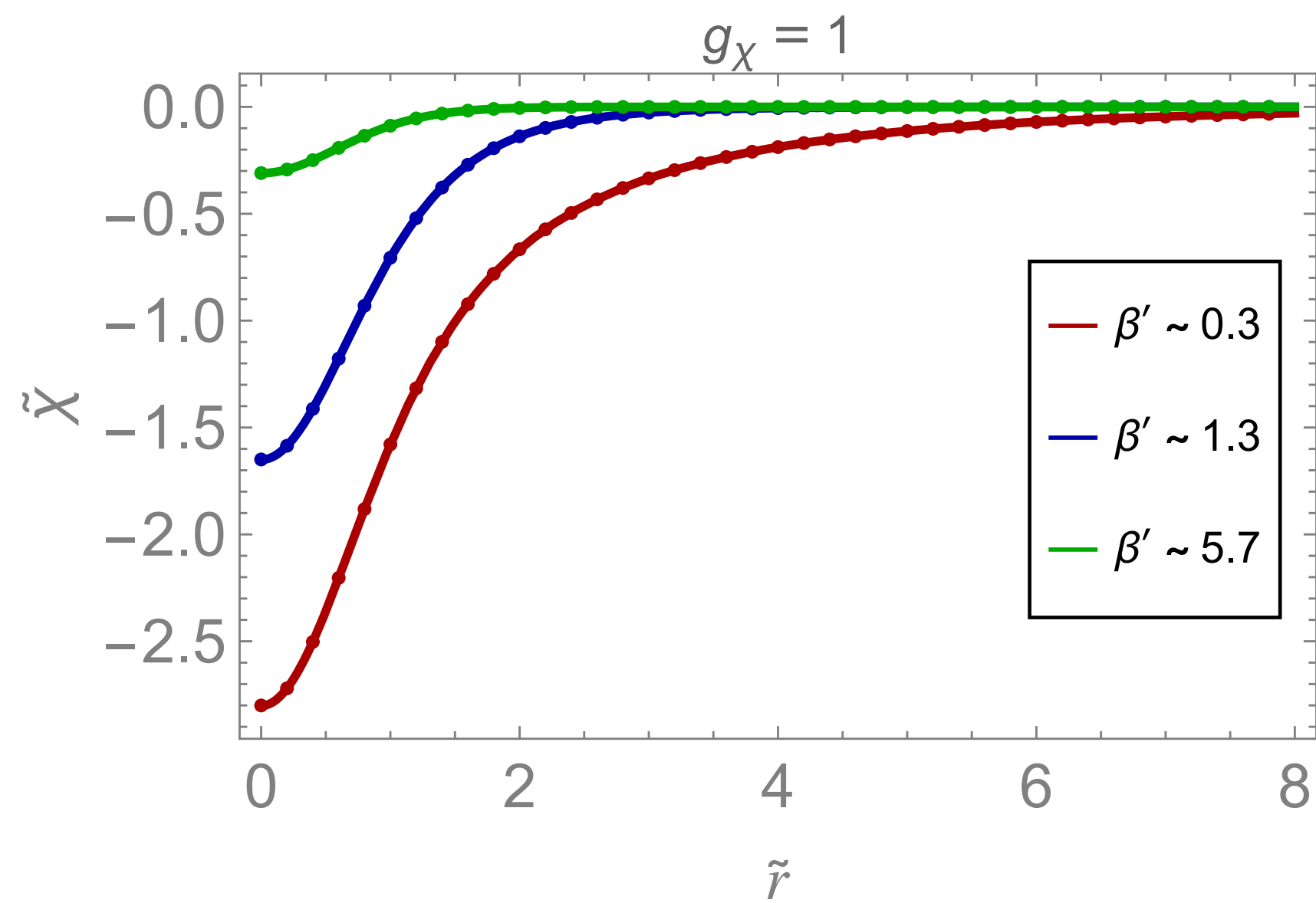
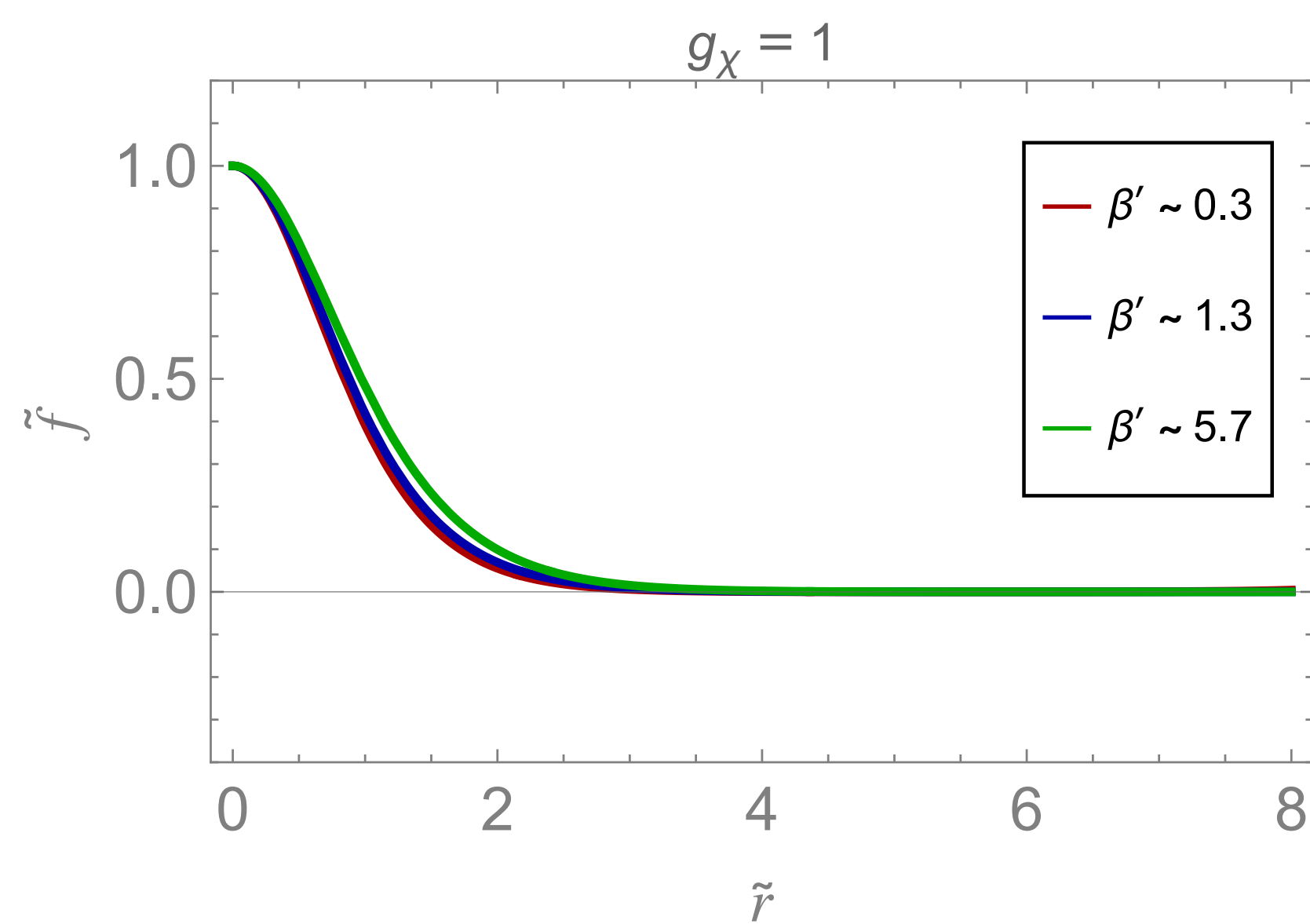
Disclaimer

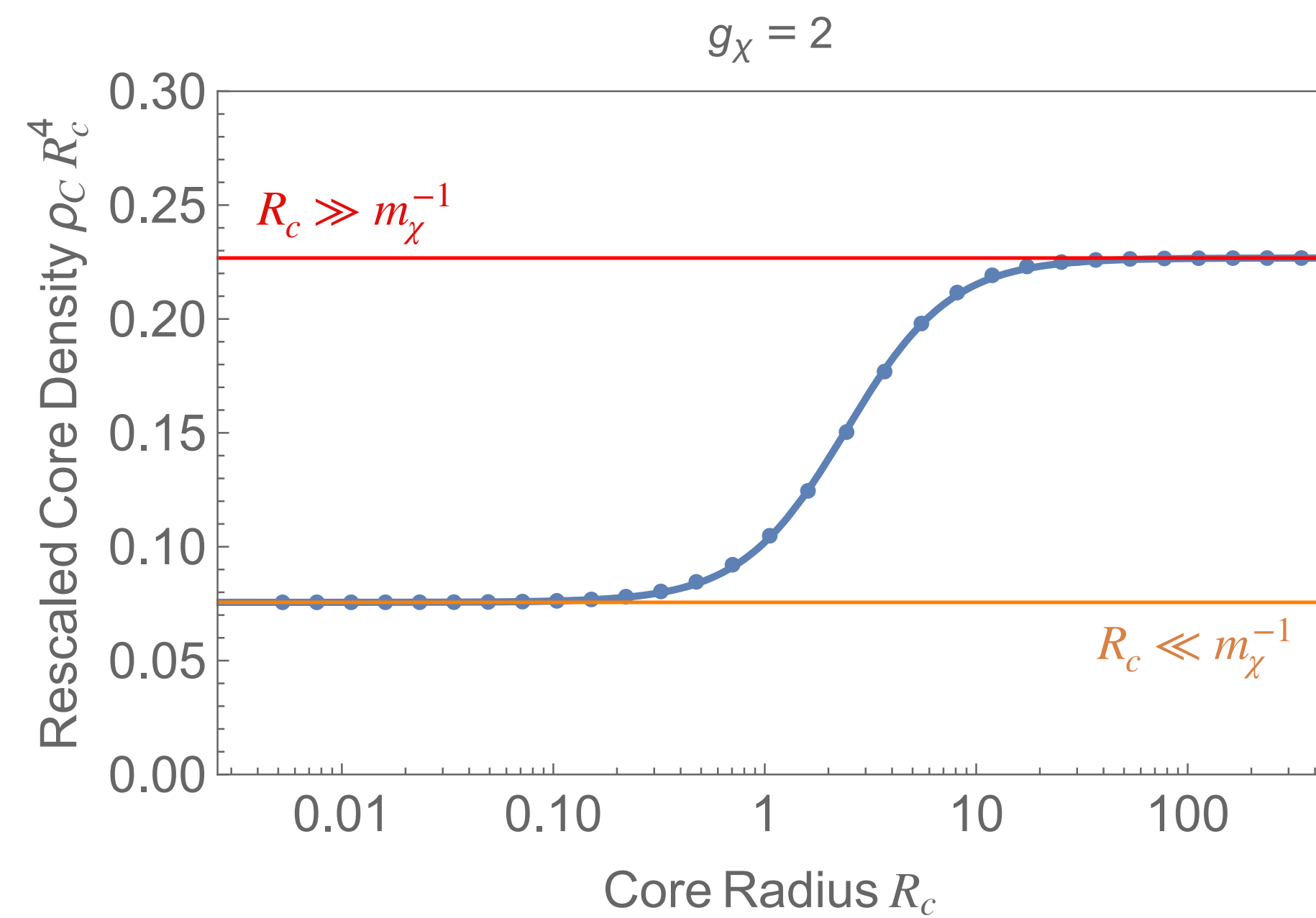
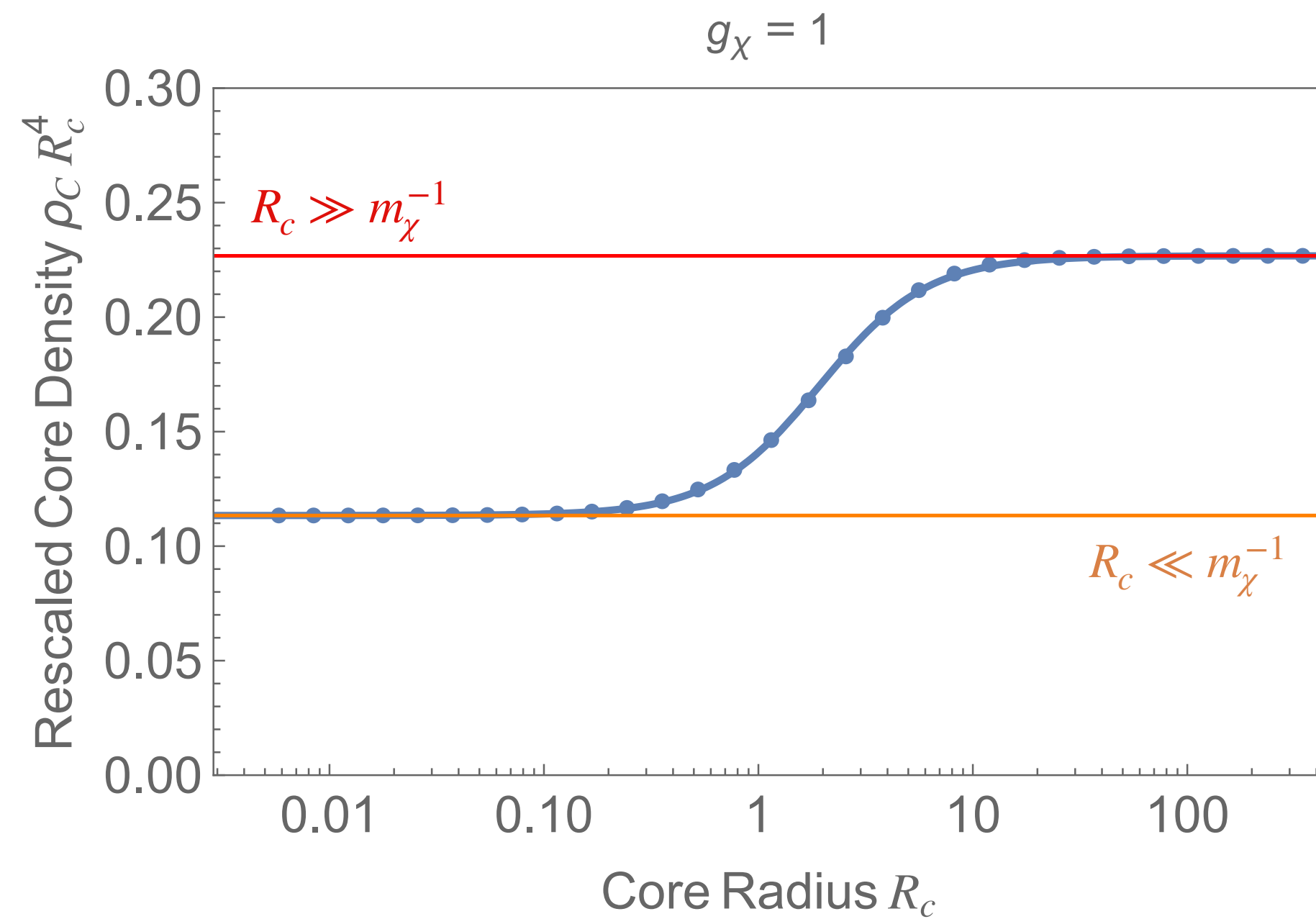
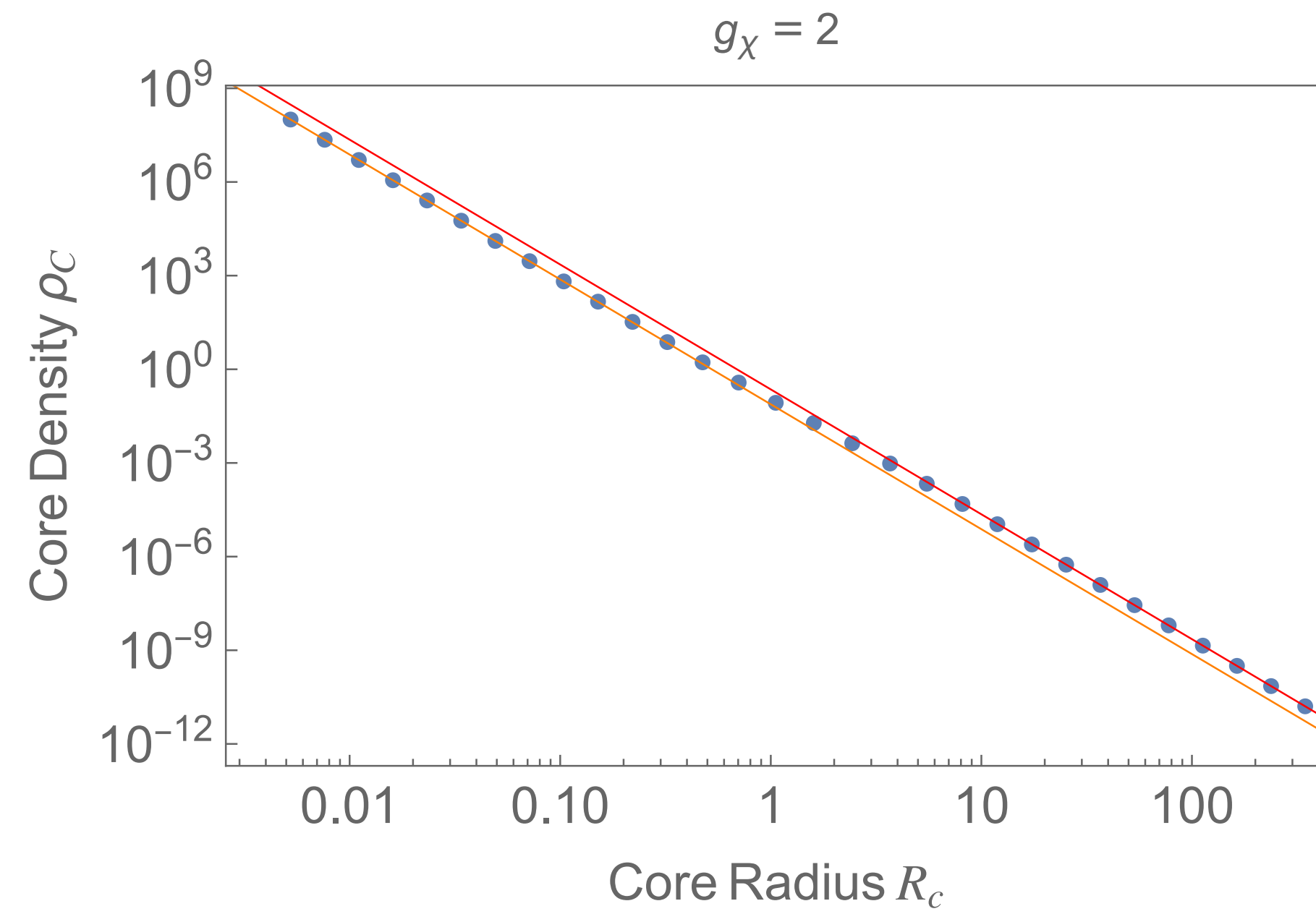
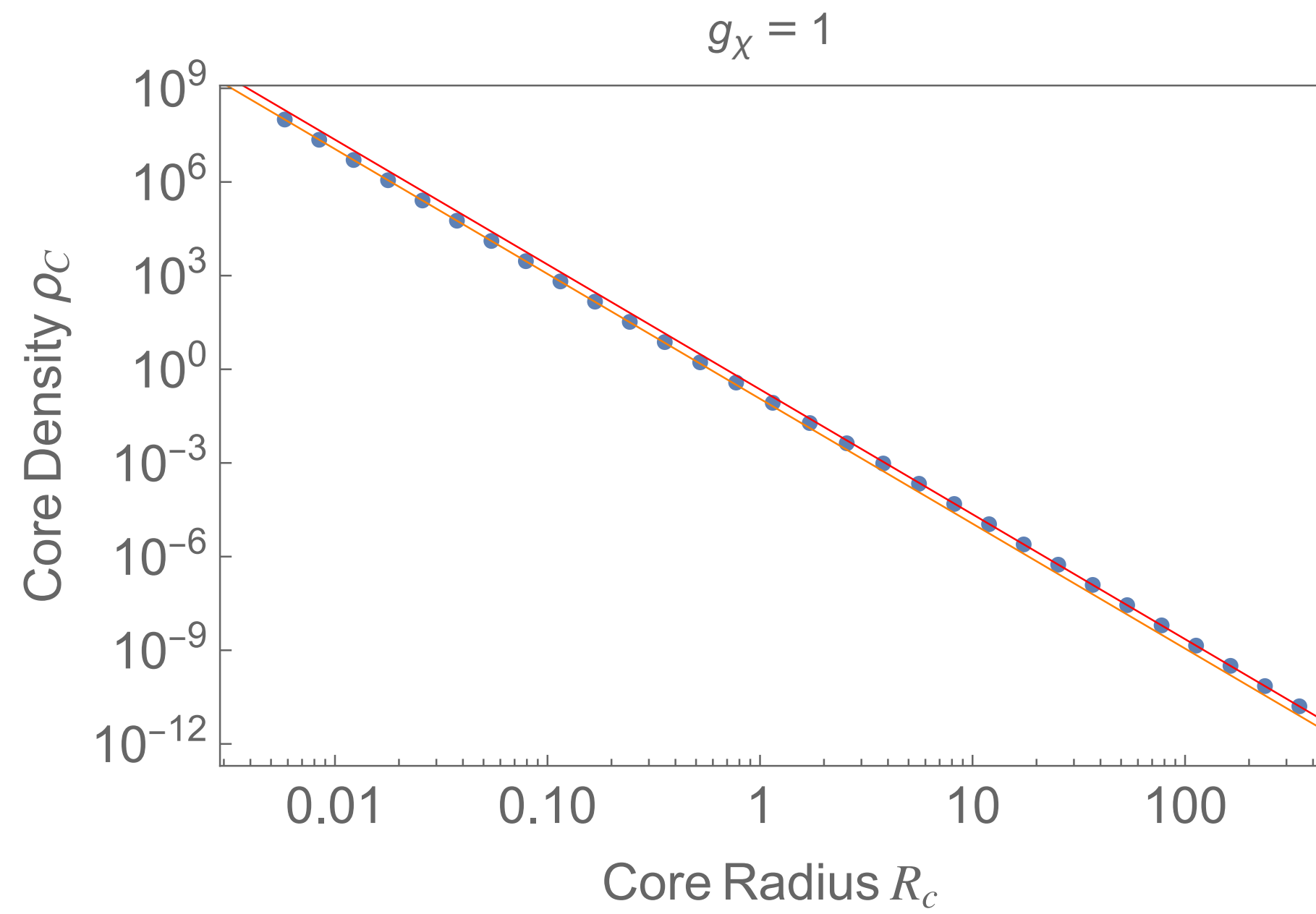
FITTING THE PLOTS: SINGLE MEDIATOR

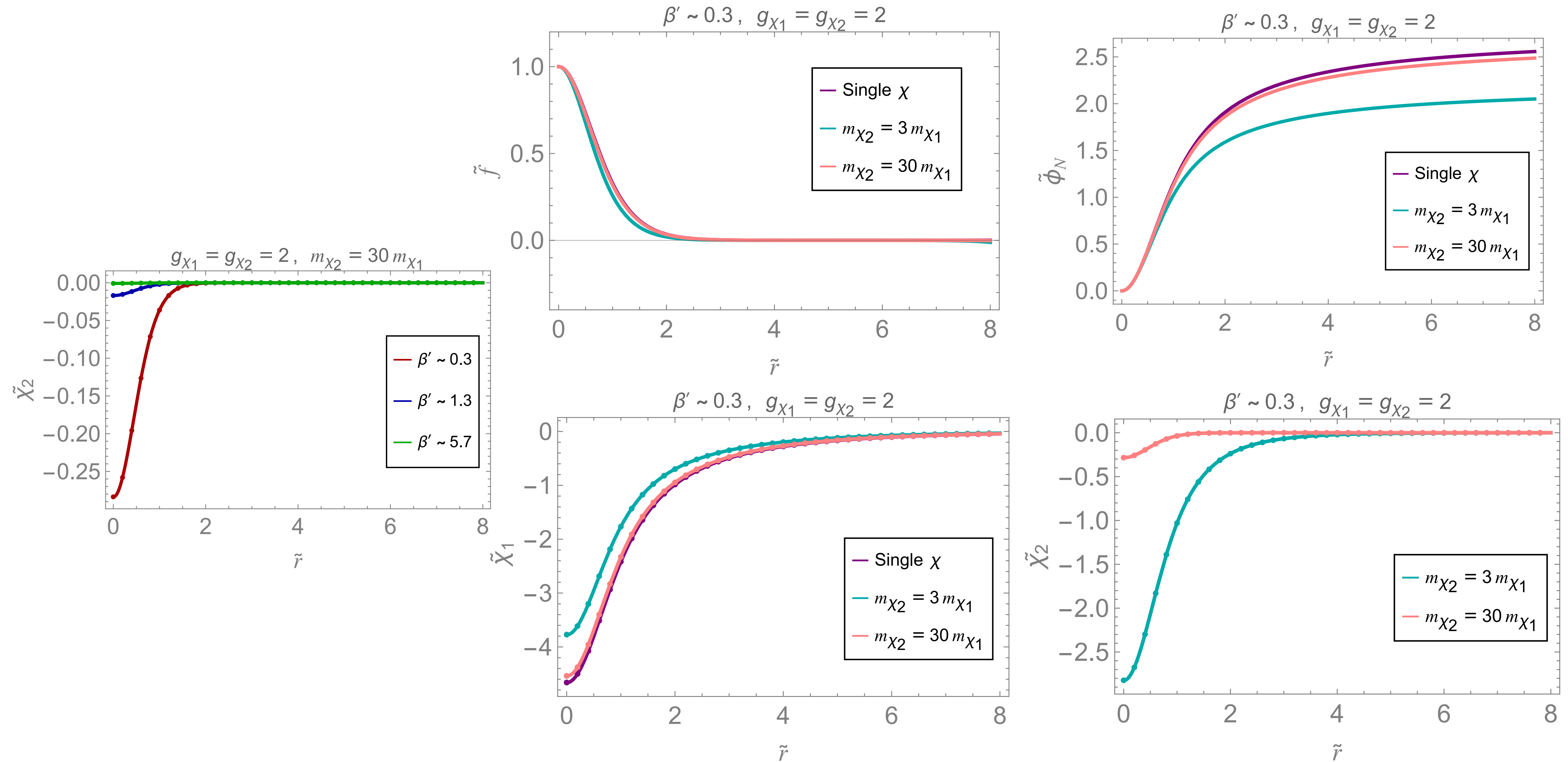
Interpolating function between the $R_c \ll m_\chi^{-1}$ and $R_c \gg m_\chi^{-1}$ asymptotes yields

$$(\rho_c R_c^4)_{\text{fit}} = \frac{\zeta}{(1 + g_\chi) G m_\phi^2} \left[1 + \frac{g_\chi}{1 + a(R_c m_\chi)^{-p}} \right]$$

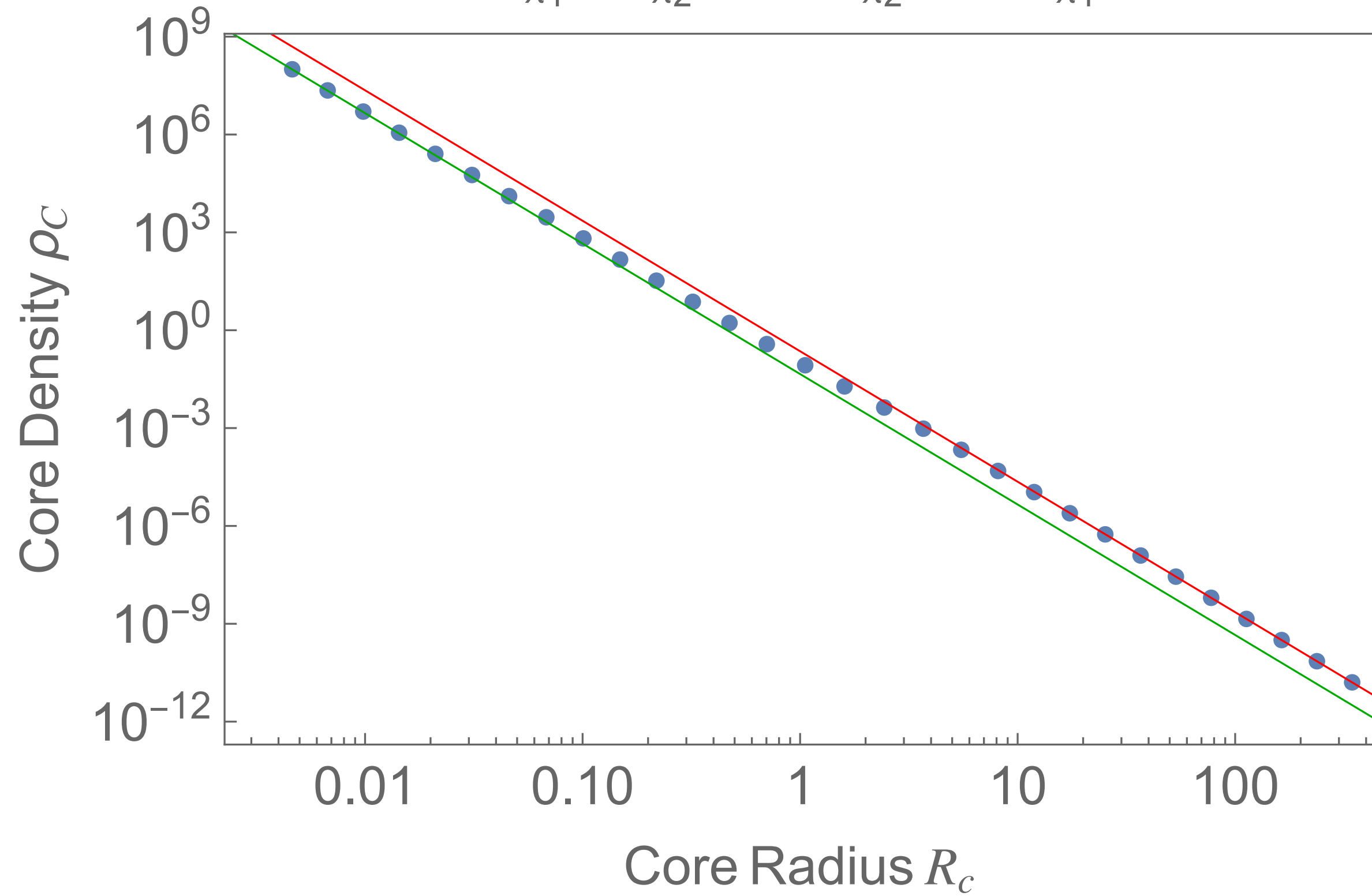
with $a, p > 0$



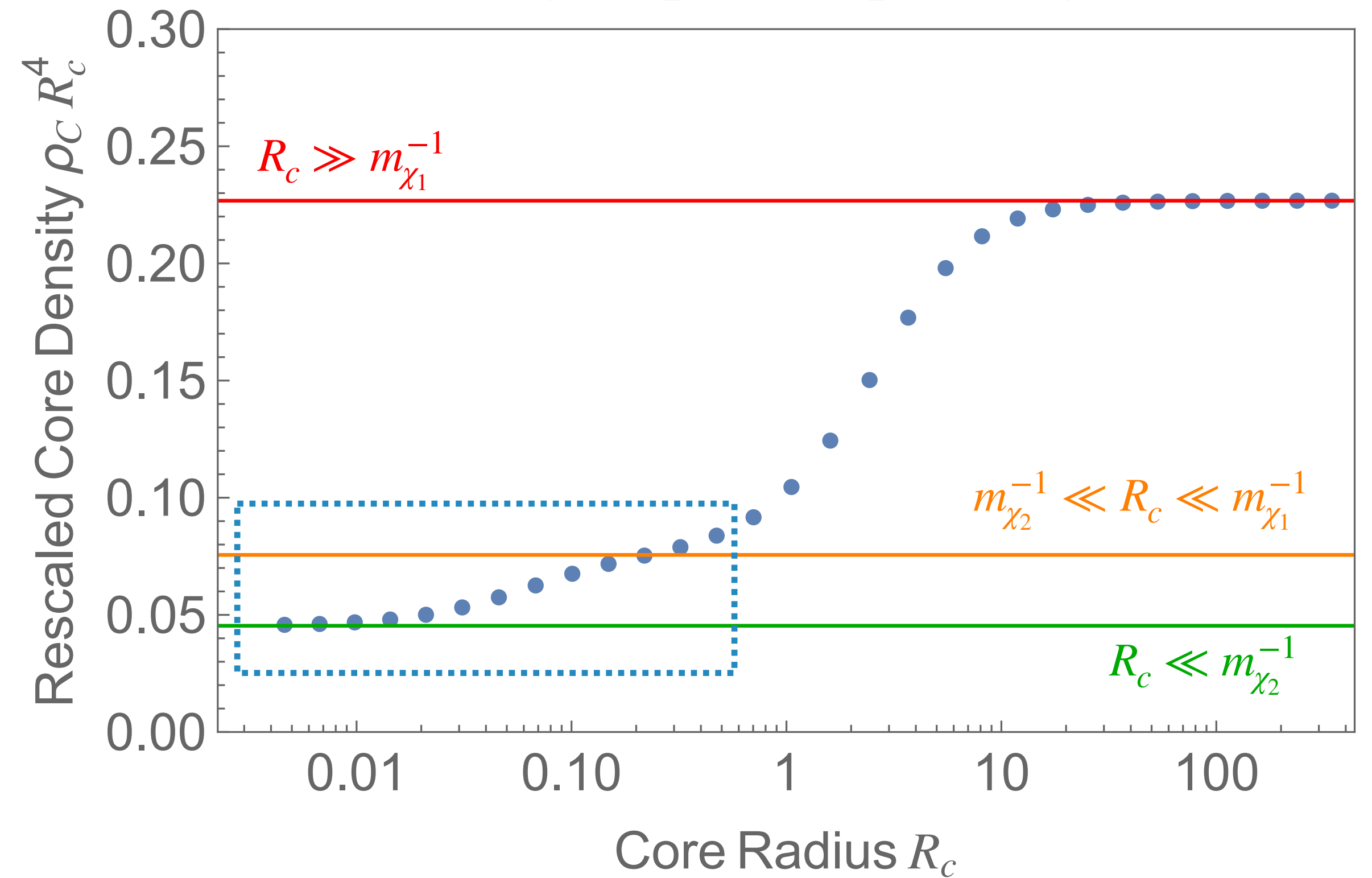




$$g_{\chi_1} = g_{\chi_2} = 2, \quad m_{\chi_2} = 30 m_{\chi_1}$$



$$g_{\chi_1} = g_{\chi_2} = 2, \quad m_{\chi_2} = 30 m_{\chi_1}$$



CONCLUSIONS & FUTURE WORK

- ▶ Alteration in core density versus core radius relation
 - ▶ Cross-over region softens the slope
 - ▶ Multiple mediator case is interesting to look at to fit the observational data better
- ▶ Results **not** enough to claim $q = 4 \rightarrow q \approx 1.3$ for $\rho_c \propto 1/R_c^q$
- ▶ Large g_χ case is interesting!
- ▶ ϕ oscillations can produce χ waves
- ▶ Possible residual effects in larger scales if $m_\chi^{-1} \sim kpc$

THANKS!

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