

# Production of $B_c$ States In Parton Showers

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In Collaboration With:

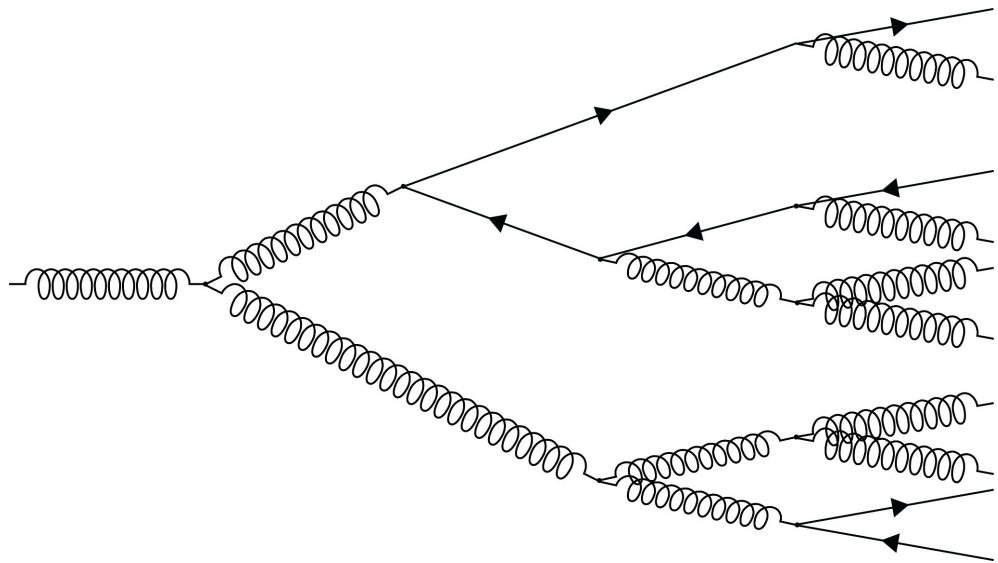
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PHENO 2026

May 11, 2026

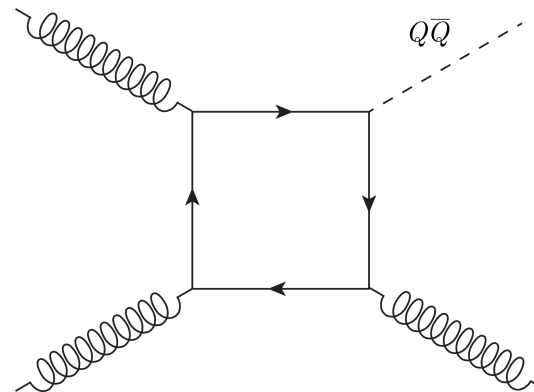
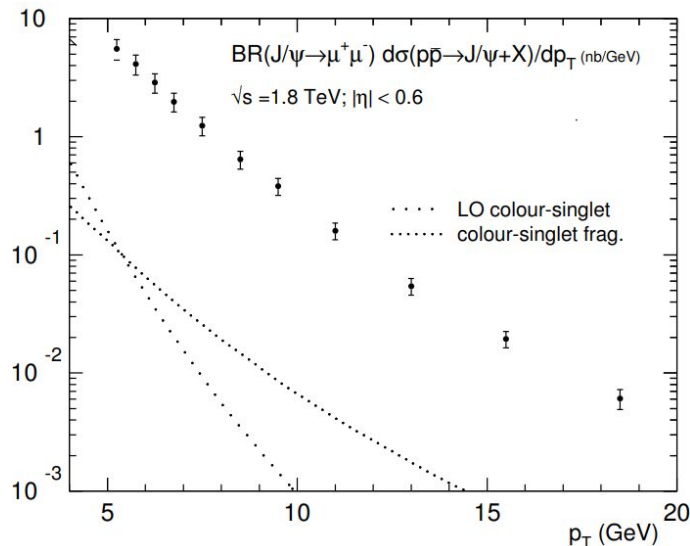


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# The Color-Singlet Model (CSM) for Quarkonia Production

Postulates:

- The cross-section for quarkonia (onia) production can be factorized into a perturbative piece and a nonperturbative piece.
- The production of the heavy quarks is calculated with perturbative QCD.
- The combination of the heavy quarks into a bound state is the nonperturbative piece.
- The spin and color of the quarks do not change during the combination.



*Int.J.Mod.Phys.A* 21 (2006) 3857-3916  
e-Print: [hep-ph/0602091](https://arxiv.org/abs/hep-ph/0602091)

# The NRQCD Approach

- Non-Relativistic Quantum Chromodynamics (NRQCD) involves a perturbative expansion in both the strong coupling  $\alpha_s$  and relative velocity between the heavy quarks  $v$ .
- Works as an approximation due to the heavy quark rest mass in relation to their kinetic energy in the bound state.
- Physical onia states can be expanded into nonphysical Fock states at higher orders of  $v$ :

$$|QQ\bar{Q} [^{2S+1}L_J]\rangle = \mathcal{O}(1) |QQ\bar{Q} [^{2S+1}L_J^{(1)}]\rangle + \mathcal{O}(v) |QQ\bar{Q} [^{2S+1}L_J^{(8)}]g\rangle + \mathcal{O}(v^2) |QQ\bar{Q} [^{2S+1}L_J^{(8)}]gg\rangle + \dots$$

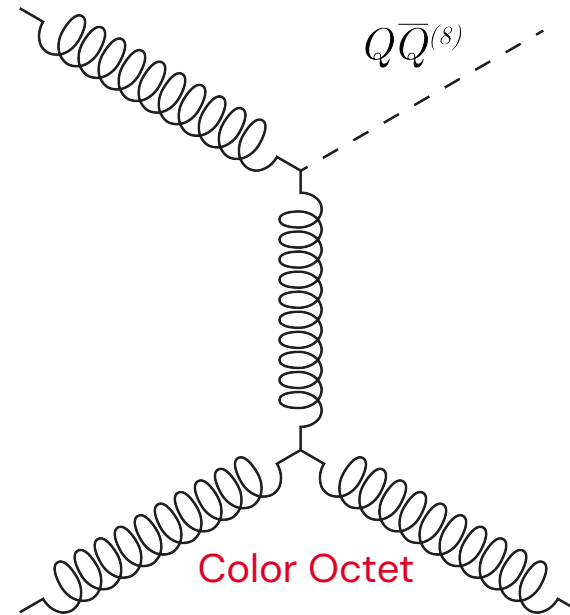
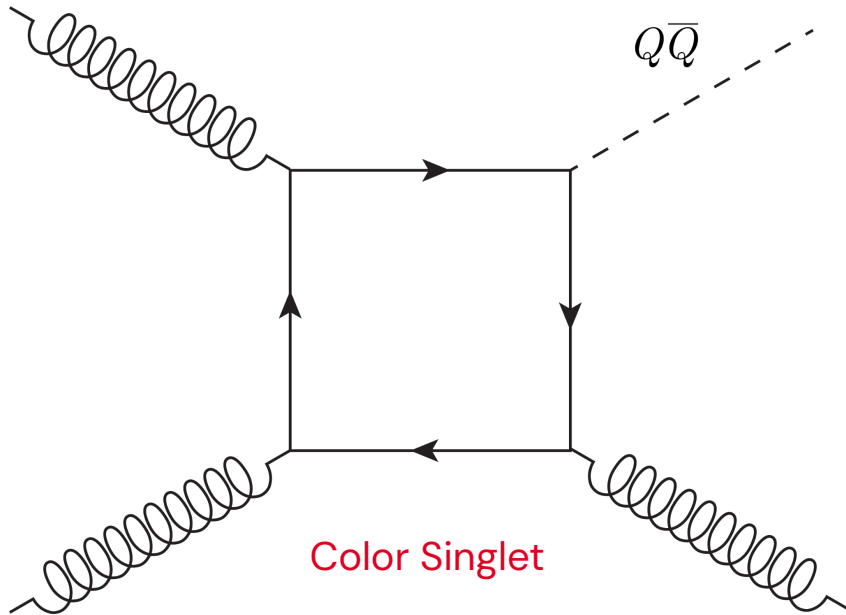
- Like the CSM, NRQCD factorizes the production cross-section into a perturbative and nonperturbative part:

$$d\hat{\sigma}(a_1 a_2 \rightarrow QQ\bar{Q} [^{2S+1}L_J] X) = \sum_{S'} \sum_{L'} \sum_{J'} \sum_{C'} \underbrace{\langle \mathcal{O}_{QQ\bar{Q}[^{2S+1}L_J]} [^{2S'+1}L_{J'}^{(C')}] \rangle}_{\text{Nonperturbative LDME}} \underbrace{d\hat{\sigma}(a_1 a_2 \rightarrow QQ\bar{Q} [^{2S'+1}L_{J'}^{(C')}] X)}_{\text{Perturbative QCD}}$$

- The nonperturbative part, referred to as the long-distance matrix element (LDME), must be determined empirically

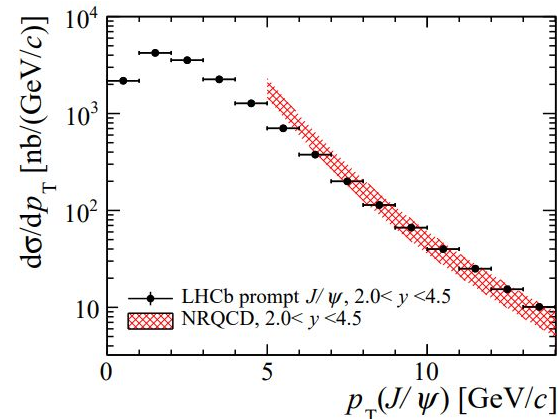
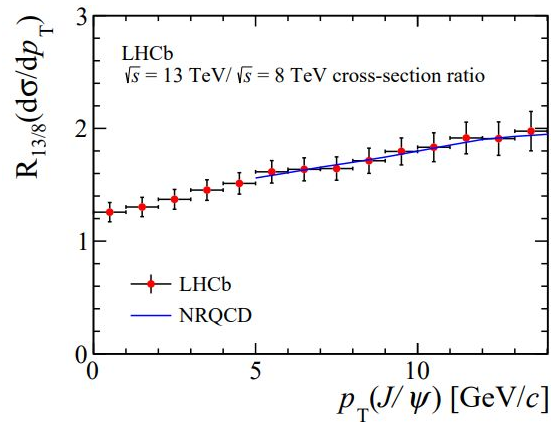
# The Color Octet Mechanism

- The CSM required heavy quarks not change color or spin during formation of the bound state.
- NRQCD allows for changing of heavy quark spin/color, leading to production of nonphysical color octet states
- Color octet states evolve into physical onium states with the emission of gluons



# NRQCD Predictions

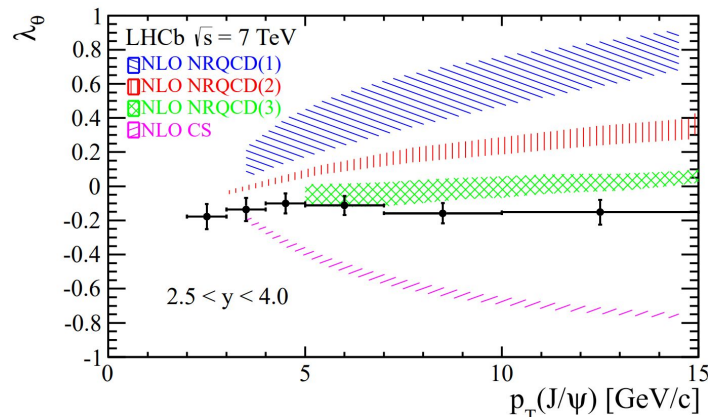
- NRQCD is able to predict total onia production rates more accurately than the CSM



*JHEP* 10 (2015) 172, *JHEP* 05 (2017) 063

e-Print: [1509.00771](https://arxiv.org/abs/1509.00771)

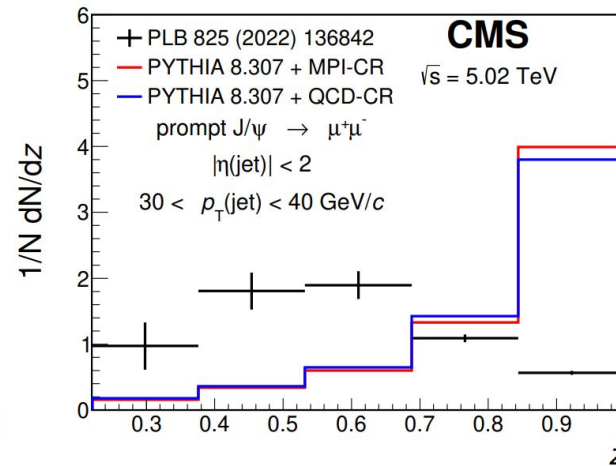
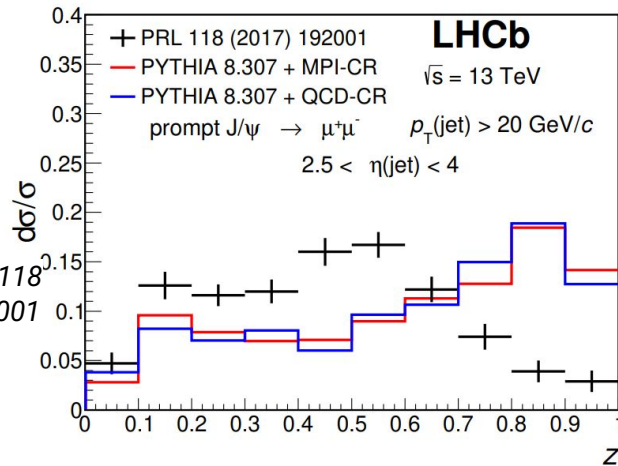
- NRQCD struggles to predict onia polarization and isolation



*Eur.Phys.J.C* 73 (2013) 11, 2631  
e-Print: [1307.6379](https://arxiv.org/abs/1307.6379) [hep-ex]

# NRQCD Predictions

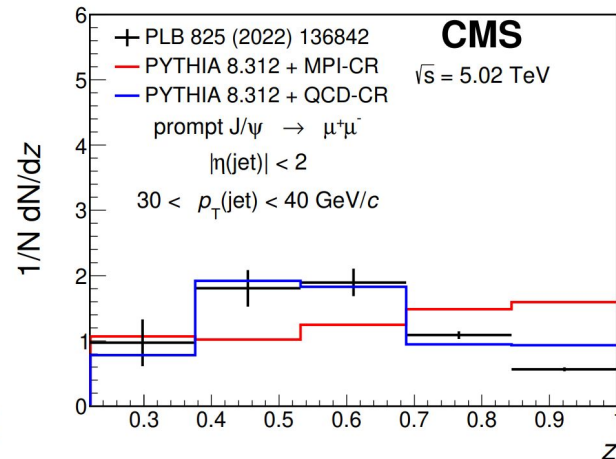
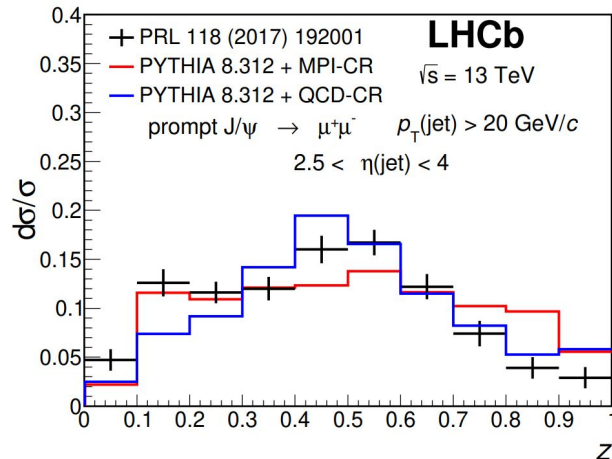
- Hard-process NRQCD struggles to predict onia isolation:



Phys.Rev.Lett. 118  
(2017) 19, 192001  
e-Print:  
[1701.05116](https://arxiv.org/abs/1701.05116)

Phys.Lett.B 825 (2022)  
136842  
e-Print: [2106.13235](https://arxiv.org/abs/2106.13235)

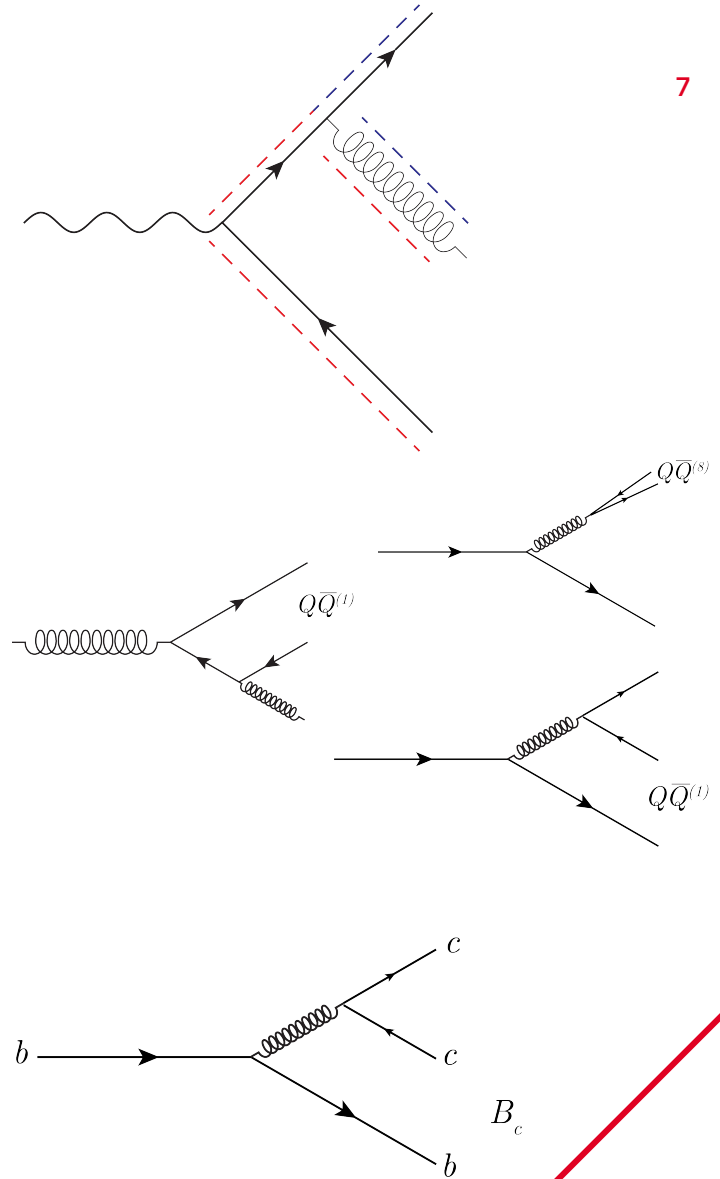
- NRQCD parton shower fragmentation better agrees with data:



Eur.Phys.J.Plus 140 (2025) 6,  
544,  
e-Print: [2506.15205](https://arxiv.org/abs/2506.15205) [hep-ph]

# Quarkonia In the Parton Shower

- Parton showers model the successive emission of quarks and gluons from color dipoles in a particle collision event.
- In recent years Pythia8 and Herwig7 have implemented onia production in their parton showers.
- These models allow for quarks and gluons to fragment into both color singlet and color octet onia states within the parton shower.
- In this work in progress, we are expanding the Pythia onia shower to allow for bound states of different flavored quarks, i.e.  $B_c$  states.



Pythia Onia Shower:  
*Eur.Phys.J.C* 84 (2024) 4, 432  
e-Print: [2312.05203](https://arxiv.org/abs/2312.05203)

Herwig Onia Shower:  
e-Print: [2508.06307](https://arxiv.org/abs/2508.06307)

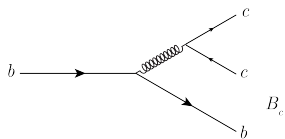
# Implementation in the Pythia Parton Shower

- The probability of a parton branching to two other partons are given by the kernels of the DGLAP equations, referred to as the QCD splitting functions or splitting kernels.

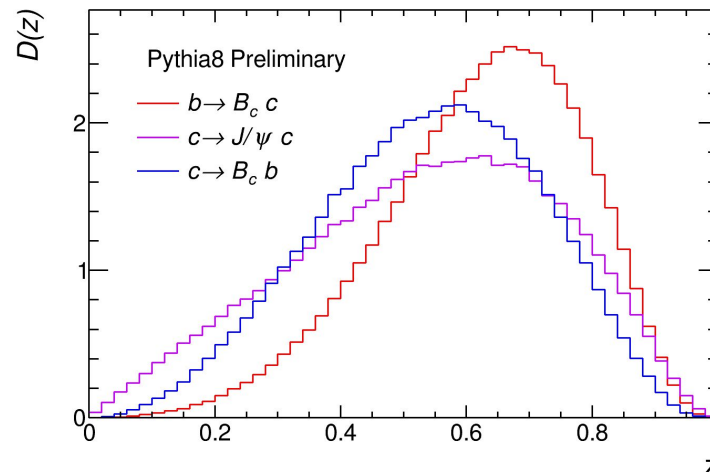
$$\frac{\partial f_i(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s^2(Q^2)}{2\pi} \sum_j \int_x^1 \frac{dz}{z} P_{ij} \left( \frac{x}{z} \right) f_j \left( \frac{x}{z}, Q^2 \right)$$

- During shower evolution, at a given energy scale, a heavy quark may fragment into a  $B_c$  state with some probability.
- The splitting weight is then calculated at a randomly sampled  $z$  at the virtuality of the color dipole.
- The splitting probability of  $B_c$  generated in the parton shower match the analytic form of the fragmentation functions.

$$z = \frac{E_{\text{onia}}}{E_{\text{parton}}}$$



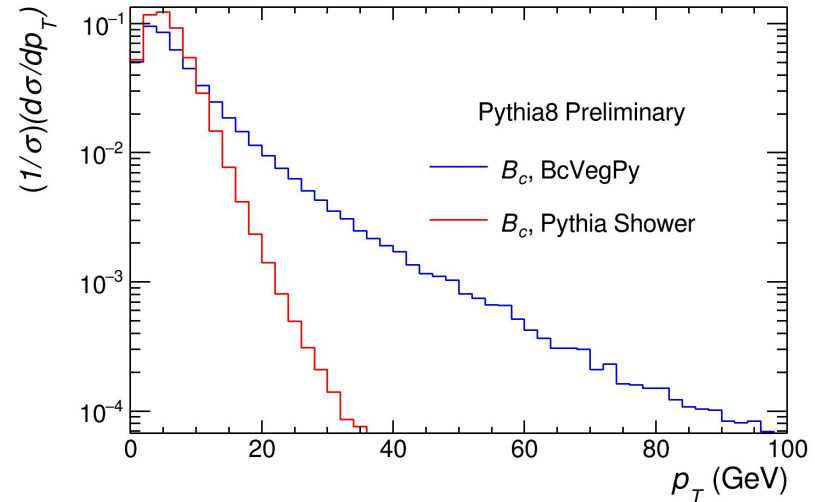
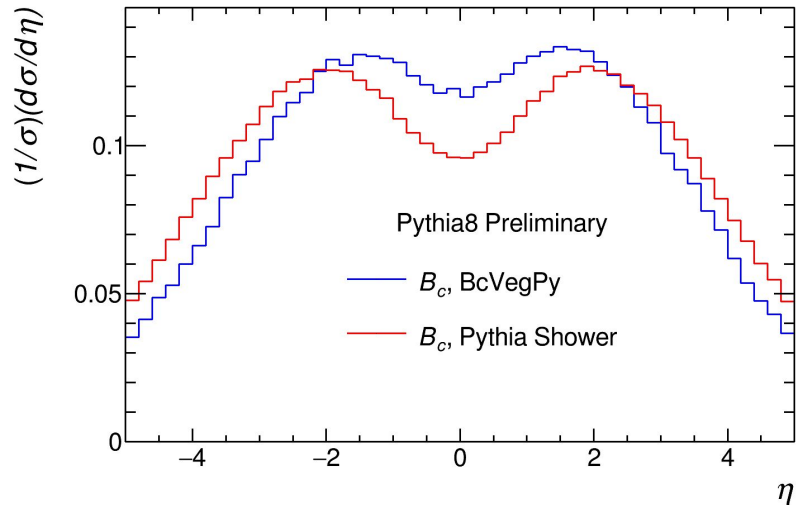
$$D(z) \propto P_{ij}(z)$$



Fragmentation Functions Derived in:  
*Phys.Rev.D* 48 (1993) 11, R5049  
 e-Print: [hep-ph/9305206](https://arxiv.org/abs/hep-ph/9305206)  
*Phys.Rev.D* 50 (1994) 5664-5675  
 e-Print: [hep-ph/9405348](https://arxiv.org/abs/hep-ph/9405348)

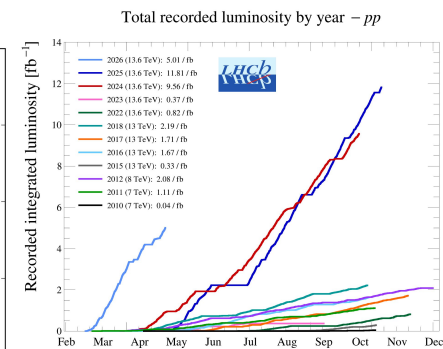
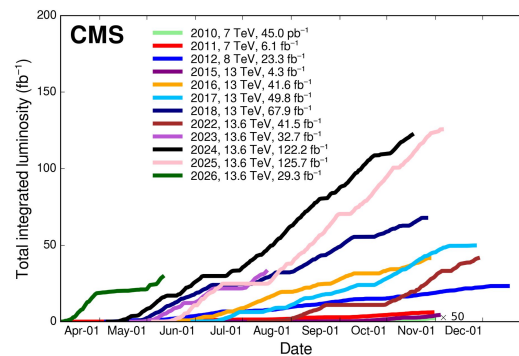
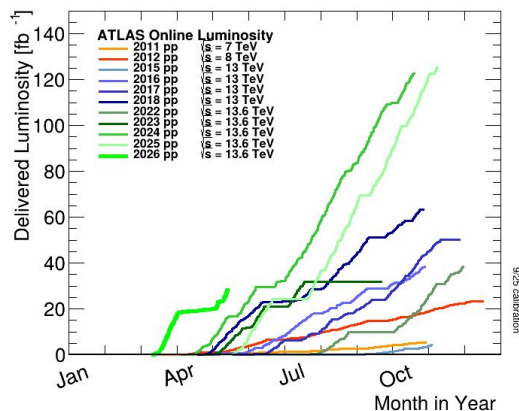
# Kinematic Predictions

- To test the predictions of kinematic variables from the  $B_c$  shower, we made comparisons with the program BcVegPy.
- BcVegPy is a fortran-written event generator specializing in the production of  $B_c$  mesons in the hard process.
- To interface with Pythia, a wrapper was written to generate the hard process using BcVegPy, and then pass this to Pythia8, where the rest of the event treatment (beams, parton shower, hadronization, decays, etc.) is performed.



# Comparisons to Data

- Much is still to be measured in the  $B_c$  meson family
- Shorter lifetime relative to other B-mesons, amongst other factors, makes the  $B_c$  harder to detect at LHCb.
- Belle2 doesn't operate at a high enough center-of-mass energy to produce  $B_c$  mesons.
- No polarization measurement has been made of the  $B_c^*$
- No isolation measurement has been made of the  $B_c$ .
- LHC experiments are currently sitting on more data than they have ever taken with the finalization of run 3, which could mean more  $B_c$  analyses could be published in the near future.
- The HiLumi LHC will bring even more data and  $B_c$  mesons produced, which will help fill the knowledge gap in the production of these mesons.
- Pythia will be ready to simulate these events when the time comes.



# Conclusions

- Modeling quarkonia production has been slowly refined over the years, with NRQCD currently leading the way in terms of prediction power.
- NRQCD is not perfect, and thus parton shower onia production has been developed in order to resolve discrepancies with experimental data.
- The Pythia quarkonia shower has been expanded to include production of  $B_c$  states
- Much is still unknown about  $B_c$  mesons, but the HiLumi LHC could bring more knowledge.
- When the time comes, Pythia will be ready as a prediction mechanism for  $B_c$  observables.

