Axions on a Hyperbolic Ride

Geometric suppression of CMB isocurvature and a blue-tilted spectrum

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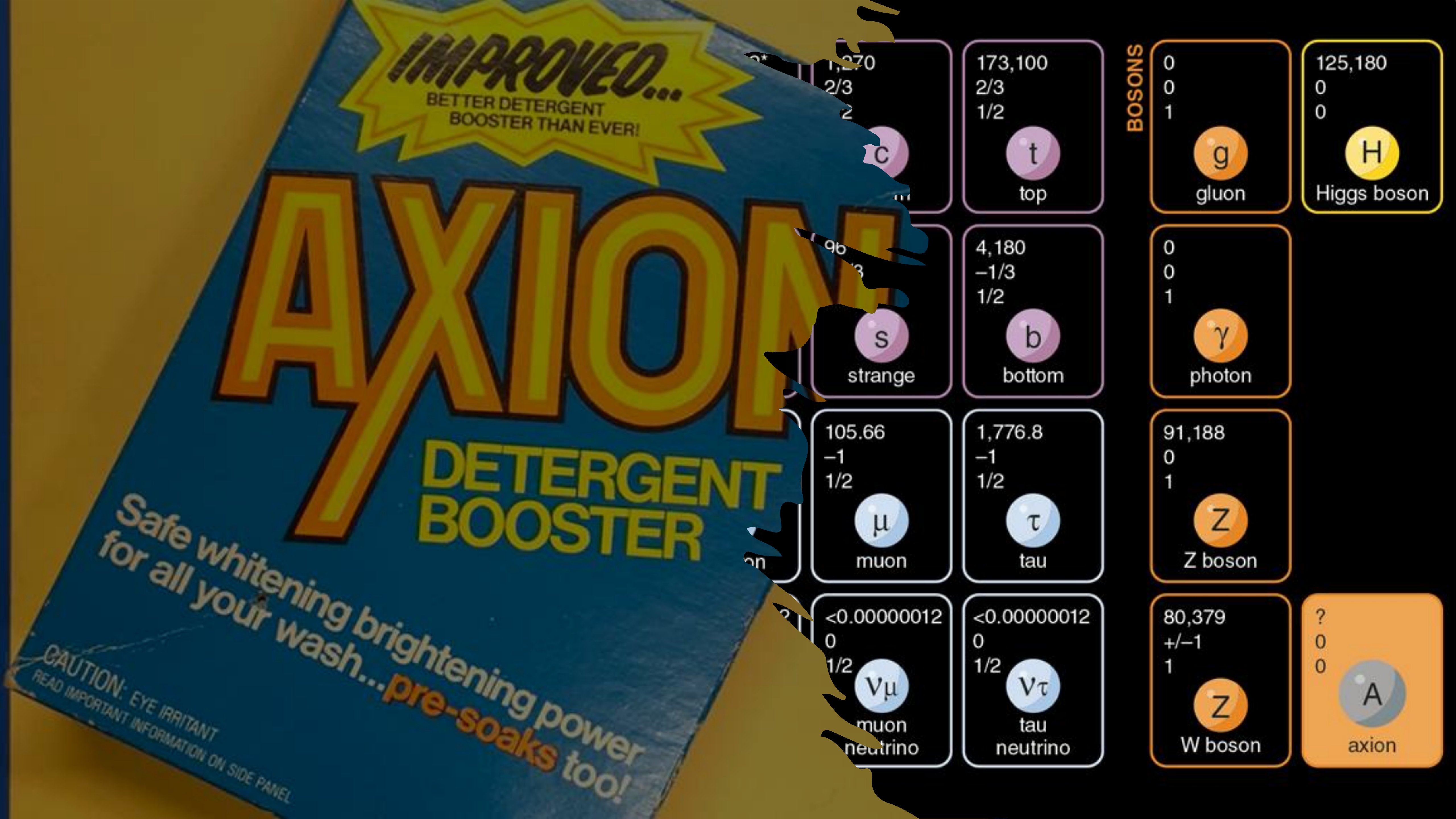

IMPROVED...
BETTER DETERGENT BOOSTER THAN EVER!
AXION
DETERGENT BOOSTER
Safe whitening brightening power for all your wash...pre-soaks too!
CAUTION: EYE IRRITANT
READ IMPORTANT INFORMATION ON SIDE PANEL
173,100
2/3
1/2
t
top
4,180
–1/3
1/2
b
bottom
s
strange
105.66
–1
1/2
μ
muon
1,776.8
–1
1/2
τ
tau
<0.00000012
0
1/2
νμ
muon neutrino
<0.00000012
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1/2
ντ
tau neutrino
BOSONS
0
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1
g
gluon
125,180
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H
Higgs boson
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1
γ
photon
91,188
0
1
Z
Z boson
80,379
+/–1
1
Z
W boson
?
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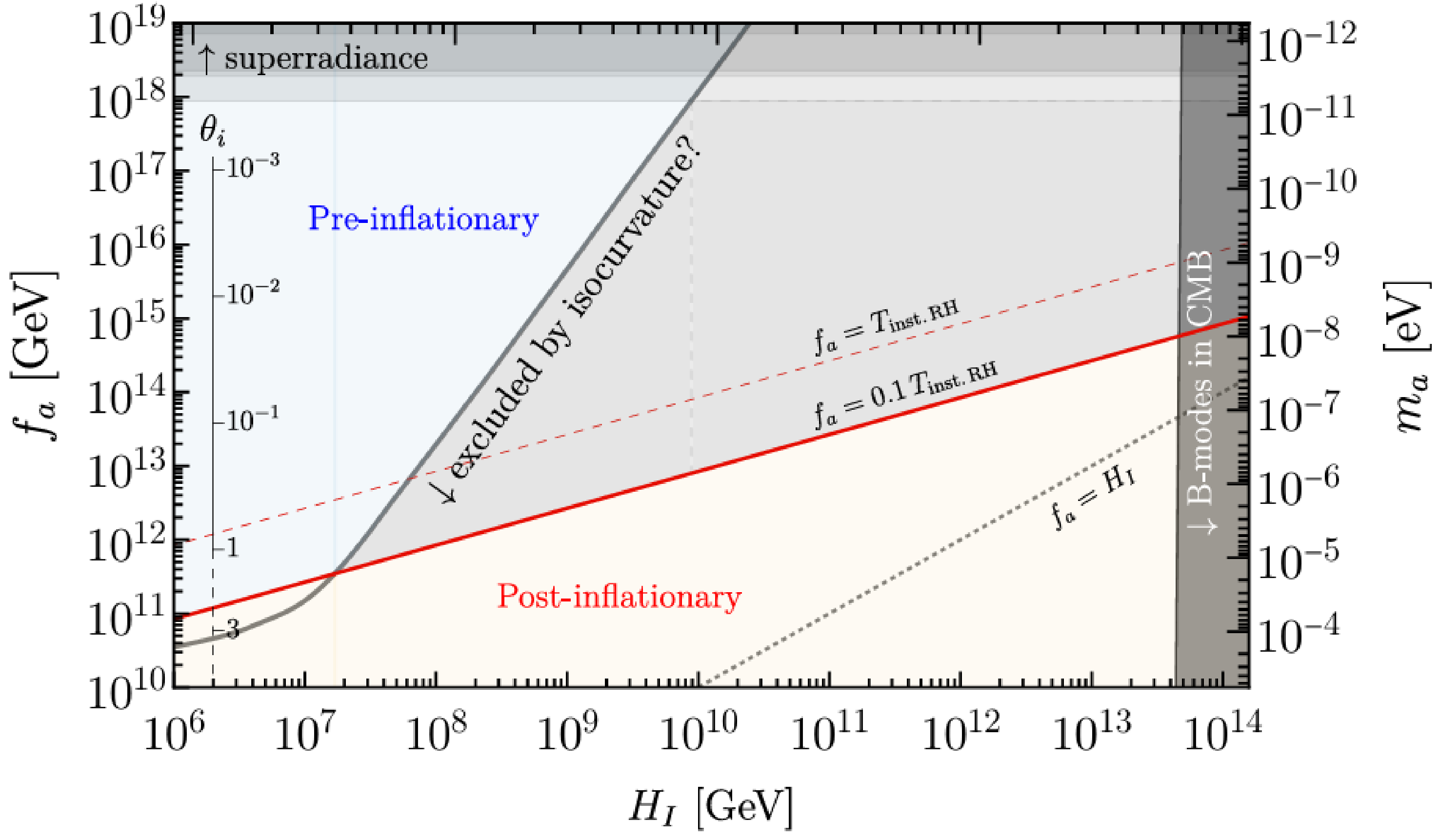
0
A
axion

QCD The axion isocurvature problem

Revisiting Isocurvature Bounds on the Minimal QCD Axion 2506.03348

Peter W. Graham,[a,b] Davide Racco[a,c,d]



$$\delta\theta_* \simeq \frac{H_{\rm inf}}{2\pi f_*}$$

$$S_{\rm CDM} \equiv \frac{\delta\rho_a}{\rho_a} \simeq 2\frac{\delta\theta_*}{\theta_i}$$

$$S_{\rm CDM} \lesssim 10^{-5}$$

Usual escape routes

Heavy axion

$m_\theta \gtrsim H_{\text{inf}}$ damps fluctuations.

But often requires explicit PQ breaking or axion-quality tuning.

Large f_* (non-dynamical)

$$\delta\theta_* \propto H_{\text{inf}}/f_*.$$

Requires extreme radial displacement. Limited due to post-inflationary resonances.

Large f_* (dynamical rolling)

$$\delta\theta_* \propto H_{\text{inf}}/f_*(t).$$

Generates a blue-tilted isocurvature spectrum. Unfeasible for quartic PQ-potential.

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Core setup: non-flat PQ field-space

Write the complex PQ field in radial/angular variables:

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2}(\partial R)^2 - \frac{1}{2}f^2(R)(\partial\theta)^2 - V(R)\right]$$

$$d\sigma^2 = dR^2 + f^2(R)d\theta^2$$

Flat geometry:

$$f(R) = R$$

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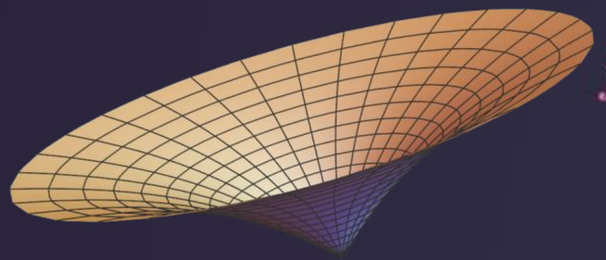
Hyperbolic metric = exponential lever arm



$$f(R) = L\sinh\left(\frac{R}{L}\right) \simeq \frac{L}{2}e^{R/L} \qquad (R \gg L)$$

L is the curvature of the field-space geometry.

$$\frac{\delta\theta_{\rm hyp}}{\delta\theta_{\rm flat}} \sim \frac{1/f_{\rm hyp}(R)}{1/R} \sim \frac{R}{L\sinh(R/L)} \ll 1$$

A modest radial displacement in curvature units, R/L, gives exponential isocurvature suppression.

Hyperbolic metrics are quite common in literature:
alpha-attractor models of inflation, SUGRA, Poincare disk, SU(1,1)/U(1)

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Geometry also gives scale dependence

The canonical angular mode feels a time-dependent effective mass as $R(t)$ slowly rolls.

$$u_k'' + \left[k^2 + a^2 m_{\text{eff}}^2 - \frac{a''}{a}\right] u_k = 0$$

$$m_{\text{eff}}^2 = \frac{f_{,R}}{f} V_{,R} - \frac{f_{,RR}}{f} \dot{R}^2 \xrightarrow{\textit{flat-metric}} m_{\text{eff}}^2 = \frac{V_{,R}}{R}$$

Different k-modes can freeze out with different suppression.

$\frac{f_{,R}}{f}$ comparatively very large for HYPERBOLIC.

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Observability: not too little, not too much

$$\Delta_\theta^2 \sim \left(\frac{H_{\rm inf}}{f(R)}\right)^2 \sim \left(\frac{H_{\rm inf}}{L}\right)^2 e^{-2R/L}$$

- **Large** R/L: exponential suppression kills isocurvature signal completely
- **Small** R/L: spectrum approaches ordinary scale-invariant isocurvature
- **Intermediate regime** $R/L \sim \mathcal{O}(10)$: CMB-safe on large-scale but becomes observable on small-scales

Hyperbolic geometry opens a detectable window rather than simply eliminating isocurvature.

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Axion Isocurvature Spectrum

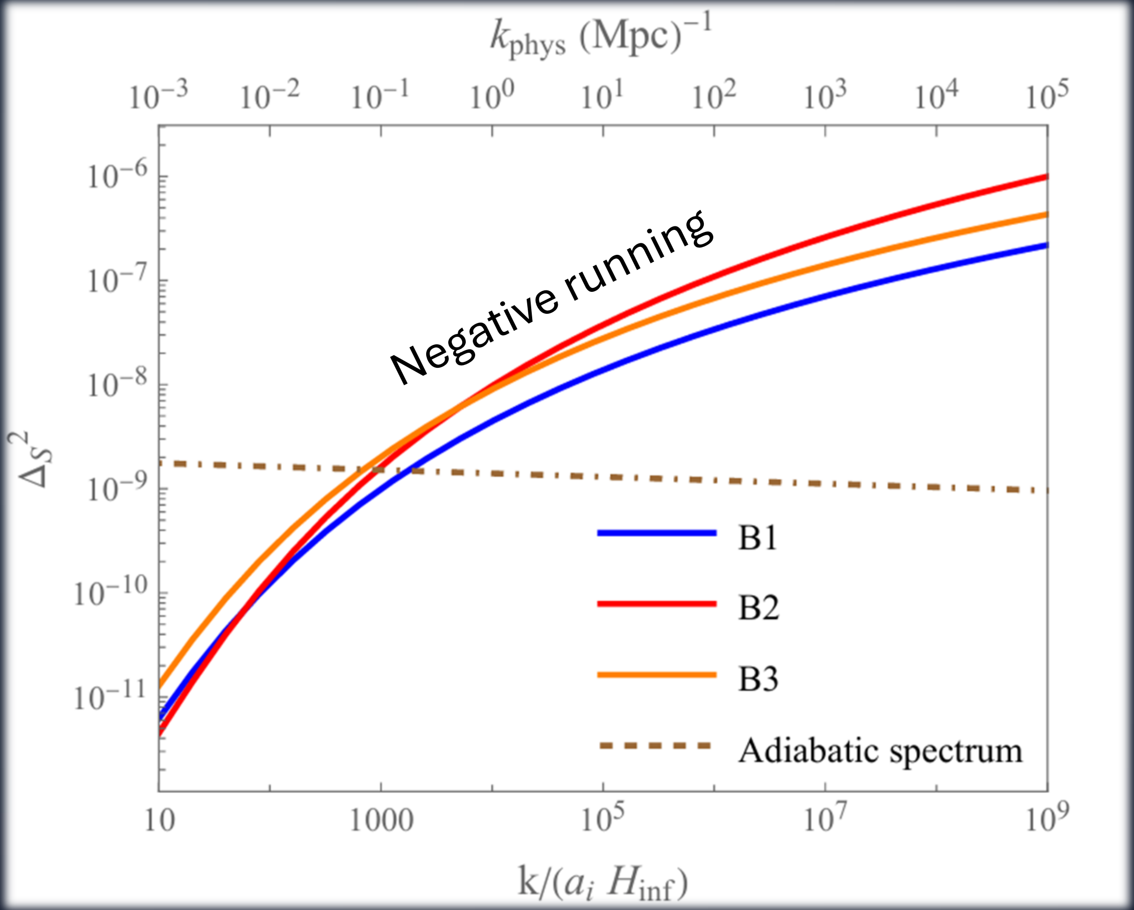
"Observable" blue-tilted spectrum with a characteristic running

$$V(R) = \frac{\lambda}{4}\left(R^2 - f_{\rm PQ}^2\right)^2$$

	λ	$f_a/H_{\rm inf}$	$R_i/H_{\rm inf}$	$N_{\rm inf}$	$L/H_{\rm inf}$
B1	2×10^{-7}	10^1	1.5×10^3	55	1.1×10^2
B2	4×10^{-6}	10^1	3.5×10^2	55	2.1×10^1
B3	2×10^{-9}	10^3	1.5×10^4	55	1.1×10^3

TABLE I. Benchmark parameter choices.

$$R/L \sim \mathcal{O}(10)$$



Parameter-space message

Standard lore:

$$H_{\text{inf}} \sim 10^{13}\,\text{GeV}, \qquad f_a \sim 10^{14} - 10^{16}\,\text{GeV}$$

is very hard for pre-inflationary QCD axion dark matter.

Hyperbolic field-space can make this region viable by suppressing CMB isocurvature geometrically.

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Phenomenological template

A useful data-facing form is:

$$\mathcal{P}_S(k) = \mathcal{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_I(k_*)-1+\frac{1}{2}\alpha_I \ln(k/k_*)+\cdots}$$

$$n_I,\ \alpha_I \quad \Longleftrightarrow \quad \text{geometric lever arm } \xi = R/L$$

$$n_I(k) = 4 - 2\sqrt{9/4 - x(k)}, \qquad x(k) = x(k_*)\left[1 + \frac{2}{3}\left(\frac{x(k_*)}{\xi_*}\right)\ln\left(\frac{k}{k_*}\right)\right]^{-3/2}$$

The scale dependence becomes a probe of PQ target-space curvature ξ_*.

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Distinct signature

Flat-metric prediction

$$m_{\delta a}^2 \propto \ln\left(\frac{k}{k_*}\right)^{-1}$$

All monomial potentials always give a **-1** exponent

Hyperbolic-metric prediction

$$m_{\delta a}^2 \propto \ln\left(\frac{k}{k_*}\right)^{-3/2}$$

The extra **-1/2** is a geometric fingerprint

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Take-home message

$$d\sigma^2 = dR^2 + L^2 \sinh^2(R/L) d\theta^2$$

Hyperbolic PQ geometry suppresses isocurvature during inflation.

$$m_{\rm eff}^2 = \frac{f_{,R}}{f} V_{,R} - \frac{f_{,RR}}{f} \dot{R}^2$$

sources a time-dependent mass for without explicit PQ-breaking

Generates a blue-tilted, observable small-scale spectrum.

Running of the spectrum directly probes the hyperbolic parameter R/L.

Beyond isocurvature...

- Tachyonic production of axion dark matter **(releasing next week)**
- A new axionic early-dark-energy model.
- Kination models.
- Hyerbolic Baryogenesis.