Photon-Induced Triple Higgs Production in the Higgs Triplet Model at Muon Colliders

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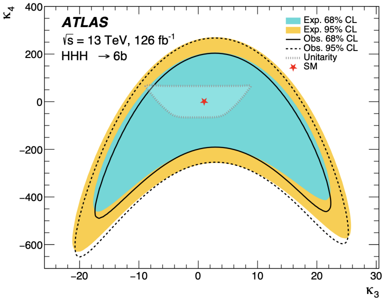
Motivation: Stress-testing the Standard Model

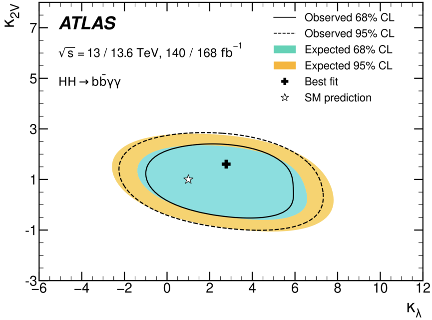
$$V_{SM} \supset \frac{M_h^2}{2} h^2 + \frac{1}{3!}\frac{3M_h^2}{v} h^3 + \frac{1}{4!}\frac{3M_h^2}{v^2} h^4$$

$$\lambda_{hhh} = \frac{3M_h^2}{v}, \quad \lambda_{hhhh} = \frac{3M_h^2}{v^2}$$

- Three/ Two Higgs production is crucial for testing the Higgs self-couplings (λ_{hhhh} and λ_{hhh}), which are responsible for determining the masses of elementary particles and the shape of the Higgs potential.
- The process $\gamma\gamma \to hhh$ is sensitive to λ_{hhhh} and λ_{hhh}, while $\gamma\gamma \to hh$ is sensitive to λ_{hhh}.
- Measuring λ_{hhh} and λ_{hhhh} directly is challenging, as it requires the simultaneous production of two or more Higgs bosons.
- These can be probed in a lepton collider, such as a muon collider.

Motivation: Current Limits from LHC





See `https://arxiv.org/pdf/2507.03495` and `https://arxiv.org/pdf/2411.02040`.

Motivation: Why do we need BSM?

- The Standard Model (SM) is remarkably successful but incomplete.
- It does not explain:
  - Dark matter and dark energy
  - Neutrino masses and oscillations
  - Gravity (no quantum gravity in SM)
- BSM theories aim to address these limitations and guide us toward a more fundamental theory.

- **Supersymmetry (SUSY)** – Introduces superpartners for SM particles, addresses hierarchy problem, provides dark matter candidates.
- **Higgs Triplet Model (HTM)** – Extends the Higgs sector with triplet scalars, can generate neutrino masses via type-II seesaw mechanism.
- **Two-Higgs-Doublet Models (2HDM)** – Add an extra Higgs doublet, allowing richer Higgs phenomenology and CP violation.
- **Singlet Extension of the SM** – Add an extra Higgs singlet.

Higgs Triplet Model

- The motivation for the Higgs Triplet Model (Type II Seesaw Model) stems from the observation that two doublets can be decomposed into a triplet and a singlet representation ($2 \otimes 2 = 3 \oplus 1$).

$$\Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$$

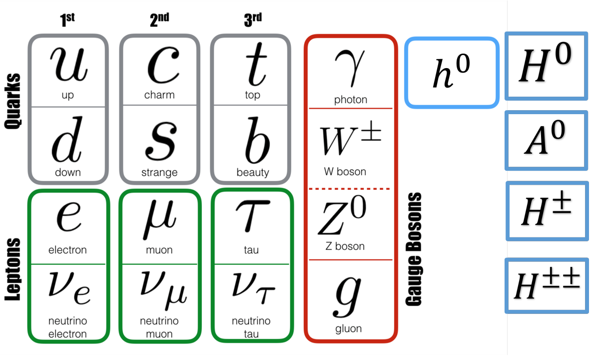
- The general scalar potential term ,$V(\Phi, \Delta)$,

$$\begin{aligned} V(\Phi, \Delta) = & -\mu_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \mu_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) \\ & + [\mu(\Phi^T i\sigma^2 \Delta^\dagger \Phi) + \mathrm{h.c}] + \lambda_1 (\Phi^\dagger \Phi) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2 \\ & + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 \Phi^\dagger \Delta \Delta \Phi \end{aligned}$$

- There are 7 physical Higgs states: $h^0, H^0, A^0, H^\pm, H^{\pm\pm}$
- The HTM generates neutrino masses without requiring a right-handed neutrino.

$$\mathcal{L} \supset Y_{jk} L_j^T C i\sigma_2 \Delta L_k + h.c.$$

Higgs Triplet Model

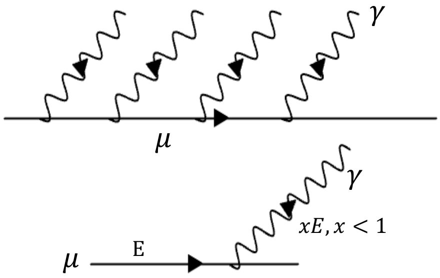


Muon Colliders

- A particle collider accelerates beams of particles to high energies and smashes them together to study fundamental physics.
- Muon colliders are ideal for next-gen discoveries due to their clean collisions and high energy potential.
- The U.S. and Europe are collaborating on muon collider development, with the U.S. playing a major role in the International Muon Collider Collaboration (IMCC).
- A future facility must deliver multi-TeV center-of-mass collisions and high-precision measurements.
- These capabilities are crucial for exploring new physics and for characterizing discoveries from the LHC or HL-LHC.
- To measure the tri-linear coupling at the LHC requires high luminosity.
- Muon colliders can reach higher center of mass energies than proton colliders ($\sqrt{s} \sim \mathcal{O}(10\,TeV)$) , which can increase the production rate of triple Higgs events.

(E. Asakawa, CO 2008)

Photon Emission off Muons



EPA and Iterative Solutions for QED

1. The case of collinear photon emission from an muon at leading order (LO) can be described using the equivalent photon approximation (EPA) or Weizsacker-Williams Approximation,

$$f_{\gamma,l}(x) \approx \frac{\alpha_e}{2\pi} P_{\gamma,l}(x) \ln \frac{E^2}{m_l^2}$$

2. Where the splitting function are, $P_{\gamma,l} = (1 + (1 - x^2))/x$ for $l \to \gamma$.

By solving iteratively the DGLAP equations, the approximate solutions for the PDF, $(LO + \mathcal{O}(\alpha_e^2 t^2))$,

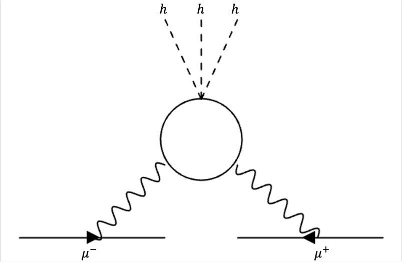
$$f_\gamma(x,t) = \frac{\alpha_e}{2\pi} t P_{vf}^f + \frac{1}{2}\left(\frac{\alpha_e t}{2\pi}\right)^2 [(P_\gamma^v + P_f^v) P_{vf}^f + I_{vfff}].$$

$$t = log\left(\frac{E^2}{M_\mu}\right), P_f^v = 3/2, P_f^v = -40/9, P_{vf}^f = \frac{1 + (1 - x)^2}{x}$$

$$I_{vfff} = \left(\frac{3}{2} + 2log(1 - x)\right) P_{vf}^f + \frac{(1 - x)(2x - 3)}{x} + (2 - x)log(x)$$

(F Garosi, CO 2023)

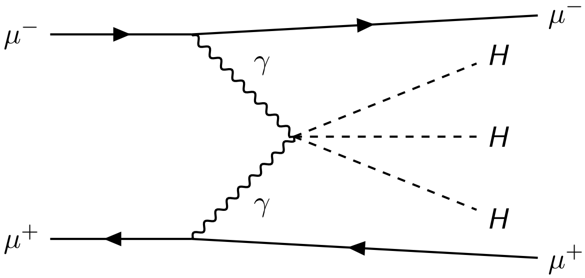
Why Study Triple Higgs (HHH) Production?



Key Insights

- **Higgs Self-Coupling**: Directly probes λ_{hhhh}—critical for verifying the Higgs potential shape.
- **New Physics**: Deviations from SM predictions could reveal BSM effects.
- **Electroweak Phase Transition**: Tests Higgs' role in cosmic inflation.
- **Ultimate SM Stress Test**: Rare process challenges experimental precision.

Triple Higgs Boson Production via Photon Fusion



Total cross sections $\mu^+\mu^- \to \gamma\gamma \to hhh$

Since the triple Higgs boson production via photon-photon collisions is a subprocess of $\mu^+\mu^-$ collisions at the muon collider, the total cross section can be obtained by,

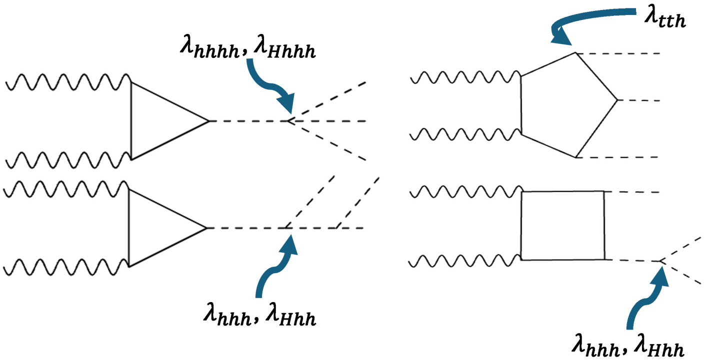
$$\sigma(s, \mu^+\mu^- \to \gamma\gamma \to \phi\phi) = \int_{\tau_0}^{1} d\tau \frac{d\mathcal{L}_{\gamma\gamma}}{d\tau} \hat{\sigma}(\hat{s} = \tau s, \gamma\gamma \to hhh),$$

along with the photon luminosity

$$\frac{d\mathcal{L}_{\gamma\gamma}}{d\tau} = \int_{\tau}^{1} \frac{dx}{x} f_{\gamma/\mu}(x) f_{\gamma/\mu}(\frac{\tau}{x}),$$

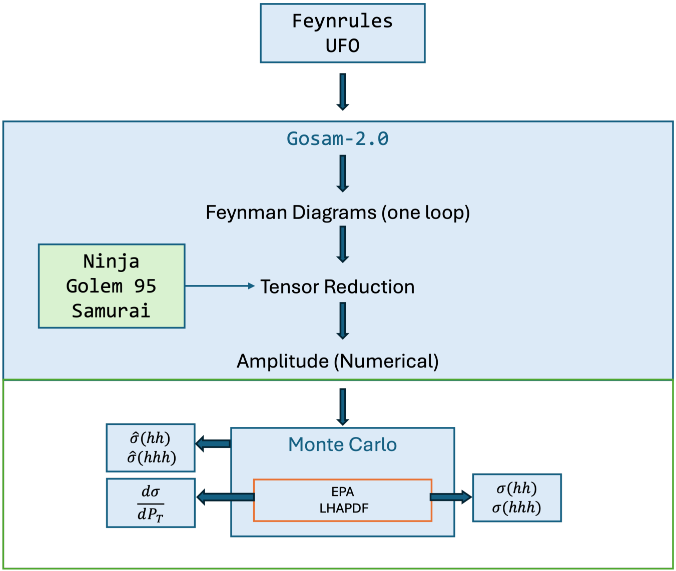
where $\tau_0 = 9M_h^2/\sqrt{s}$.

Feynman Diagrams

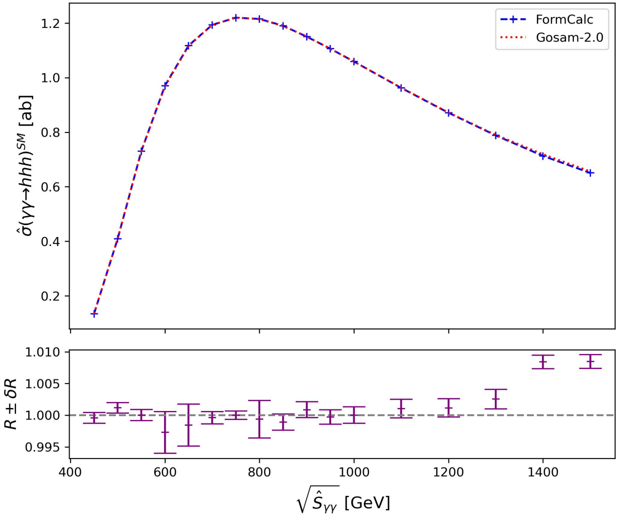


There are 5456 Feynman diagrams in $\gamma\gamma \to hhh$.

Methodology: Taking advantage of automation



hhh production within the SM



(M. Chiesa, CO 2021)

Input Set

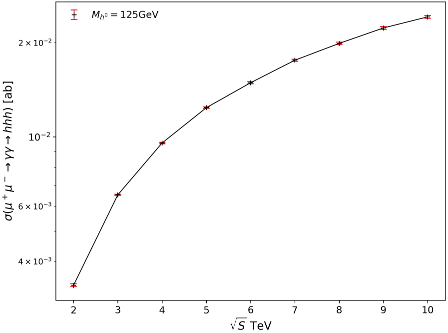
Scenario	α	M_{h^0}	M_{H^0}	M_{A^0}	$M_{H^{\pm\pm}}$	$M_{H^\pm}$
I	0	125	376.5000	376.5124431	290.5	336.26207047
II	$\pi/4$	125	126.0204	125.5153850	240.0	191.5110709

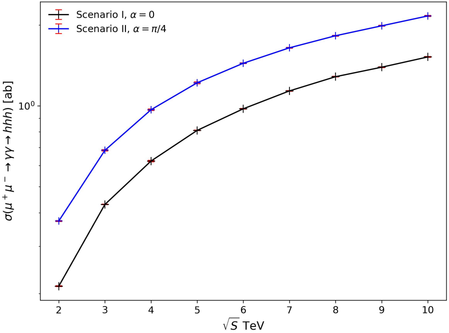
Table: For Scenario I, we have chosen $\epsilon_P = 4.248 \times 10^{-3}$, whereas for Scenario II, it is 3.2822×10^{-3} when evaluating the singly charged Higgs mass.

Scenario	λ	λ_1	λ_2	λ_3	λ_4	μ
I	0.51	2.7	5.61	-5.61	1.89	3.31
II	0.52	1.382	2.514	-2.514	-1.382	0.368

Table: The calculated Lagrangian parameters.

hhh production within the HTM





σ^{HTM}/σ^{SM}

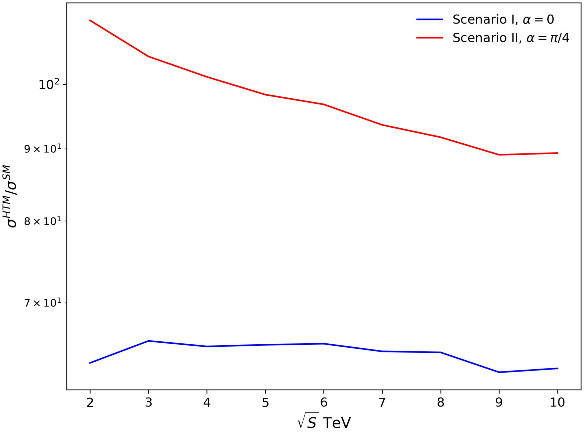
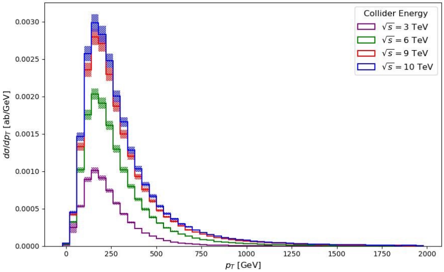


Figure: The ratio of σ^{HTM}/σ^{SM} as a function of center-of-mass energy shows a significant enhancement in the cross sections for both scenarios compared to the SM.

Analysis of Kinematic Distributions in the SM



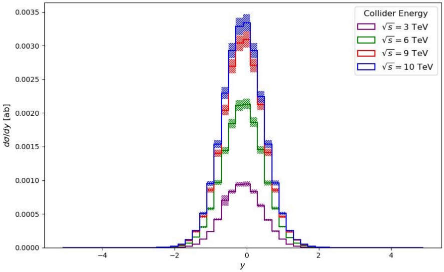


Figure: Higgs transverse-momentum (left) and rapidity (right) distributions for three-Higgs production via photon-induced processes in $\mu^+\mu^-$ collisions within the SM.

Analysis of Kinematic Distributions in the HTM

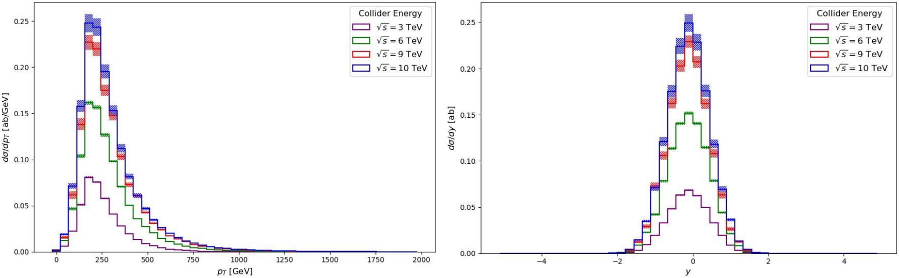


Figure: Higgs transverse-momentum (left) and rapidity (right) distributions for three-Higgs production via photon-induced processes in $\mu^+\mu^-$ collisions within the HTM under scenario I.

Analysis of Kinematic Distributions in the HTM

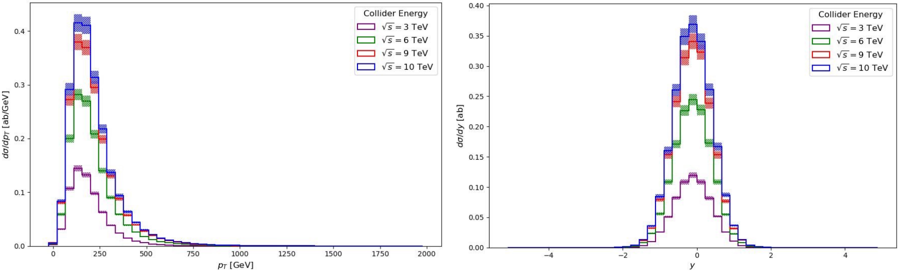


Figure: Higgs transverse-momentum (left) and rapidity (right) distributions for three-Higgs production via photon-induced processes in $\mu^+\mu^-$ collisions within the HTM under scenario II.

Scalar Couplings

Coupling	Scenario I ($\alpha = 0$)	Scenario II ($\alpha = \pi/4$)
$\lambda_{hhhh}^{HTM}(\alpha)$	$\approx 4 \times \lambda_{hhhh}^{HTM}(\alpha = \pi/4)$	$\approx \frac{1}{4} \times \lambda_{hhhh}^{HTM}(\alpha = 0)$
$\lambda_{hhhH}^{HTM}(\alpha)$	≈ 0	$\gg \lambda_{hhhH}^{HTM}(\alpha = 0)$
$\lambda_{hhH}^{HTM}(\alpha)$	≈ 0	$743 \times \lambda_{hhH}^{HTM}(\alpha = 0)$
$\lambda_{hhhh}^{HTM}(\alpha)$	$0.98 \times \lambda_{hhhh}^{SM}$	$\frac{1}{4} \times \lambda_{hhhh}^{SM}$
$\lambda_{hhh}^{HTM}(\alpha)$	$0.98 \times \lambda_{hhh}^{SM}$	$0.353 \times \lambda_{hhh}^{SM}$

Event counts

$\sqrt{s}$ (TeV)	Luminosity [ab^{-1}]	$N_{\text{Scenario I}}$	$N_{\text{Scenario II}}$
2	0.4	0	0
3	1	0	1
4	2	2	2
5	3	3	4
6	4	4	6
7	5	6	9
8	7	9	13
9	9	13	18
10	10	16	22

$$\mathcal{L} \sim (\sqrt{s}/10\ \mathrm{TeV})^2 \times 10\ \mathrm{ab}^{-1}$$

- Event counts (N) for three Higgs production in $\mu^+\mu^-$ colliders. The event count obtained by $N = \sigma\mathcal{L}$.
- For the SM, $N \ll 1$

Summary

- We have calculated the cross sections of $\sigma(\mu^+\mu^- \to \gamma\gamma \to hhh)$ for both the SM and the HTM using the Equivalent Photon Approximation.
- In the HTM, heavy Higgs couplings significantly enhance the cross section: $\sigma^{\text{HTM}}/\sigma^{\text{SM}} > 90$ (Scenario II, $\alpha = \pi/4$) and > 48 (Scenario I, $\alpha = 0$).
- The couplings hH^+H^- and $hH^{++}H^{--}$ affect these calculations in the HTM.
- Muon colliders can detect signals in all HTM scenarios, while the SM signal remains unobservable.
- Work submitted for peer-review (See `https://arxiv.org/abs/2503.05923`).