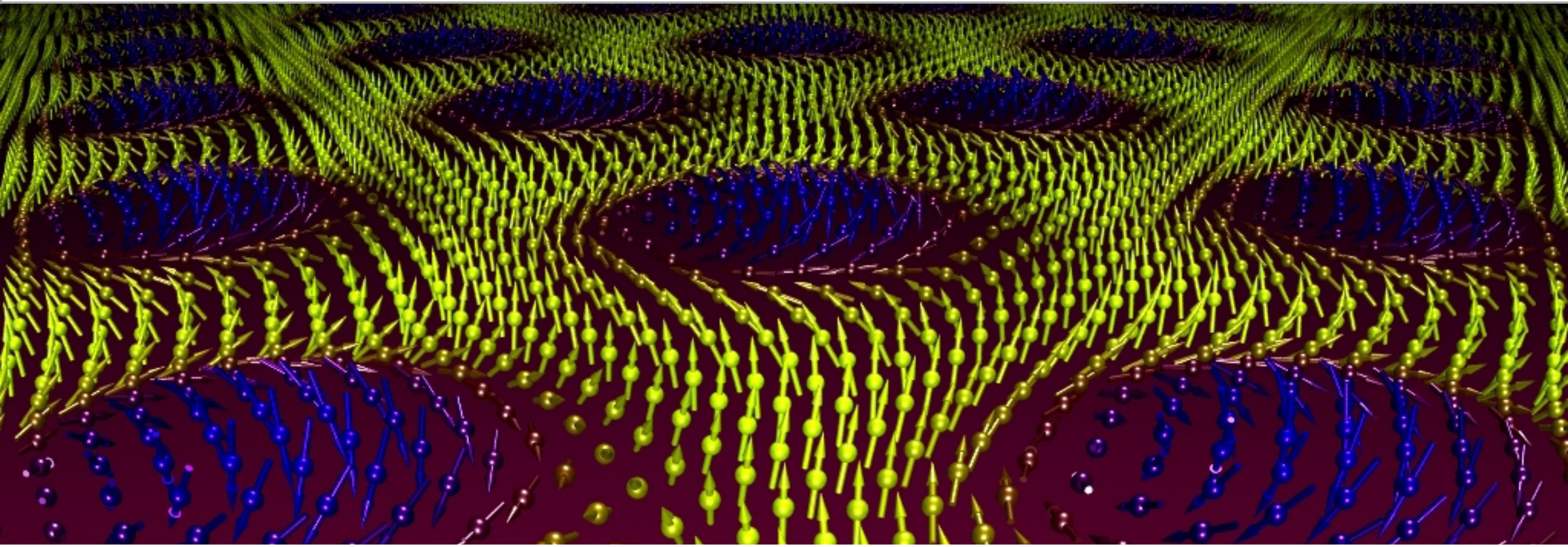
Cosmological Aspects of Axion-Monopole Interactions



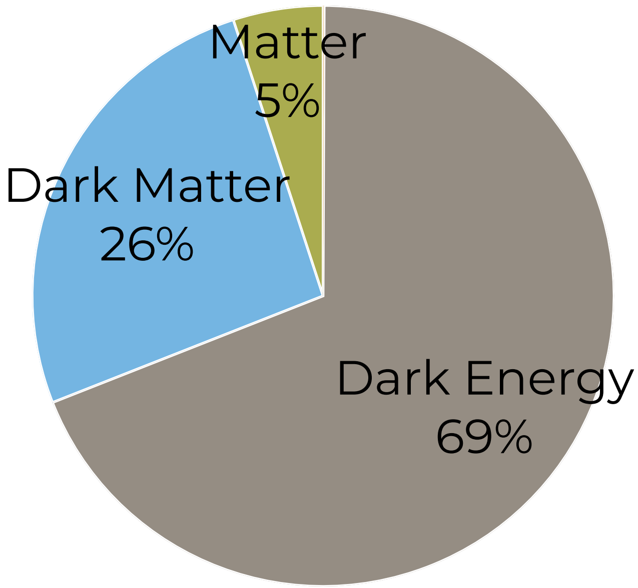
Hengameh Bagherian
EFI and Leinweber Institute, University of Chicago

Pheno 2026 Symposium | May, 2026

[H. Bagherian, M. Reece, L.-T. Wang, and H. Xiao, **arXiv**: 2026.XXXXX]

Dark sector today

- **Dark matter** candidates:
  WIMPS, **Axions**, PBH, etc.
- **Dark energy** candidates:
  Cosmological constant (Λ), quintessence (**light axion**, scalar field), etc.
- **Question**:
  Are these sectors **independent**?
  What if they are **interacting**?
  What if they have the same origin?
  DESI DR2 data gives hints of interaction!



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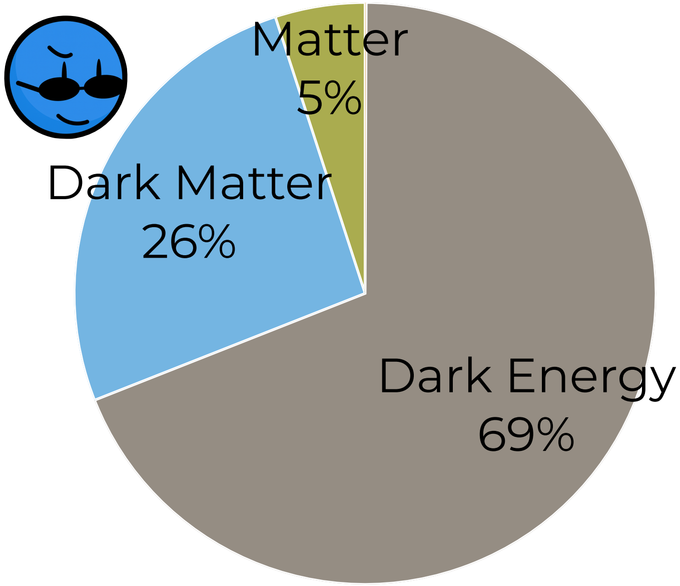

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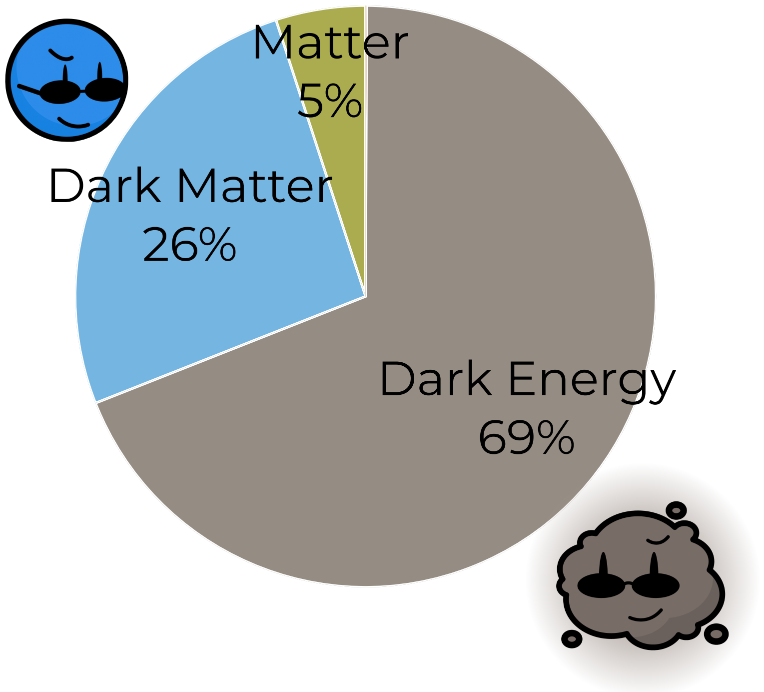

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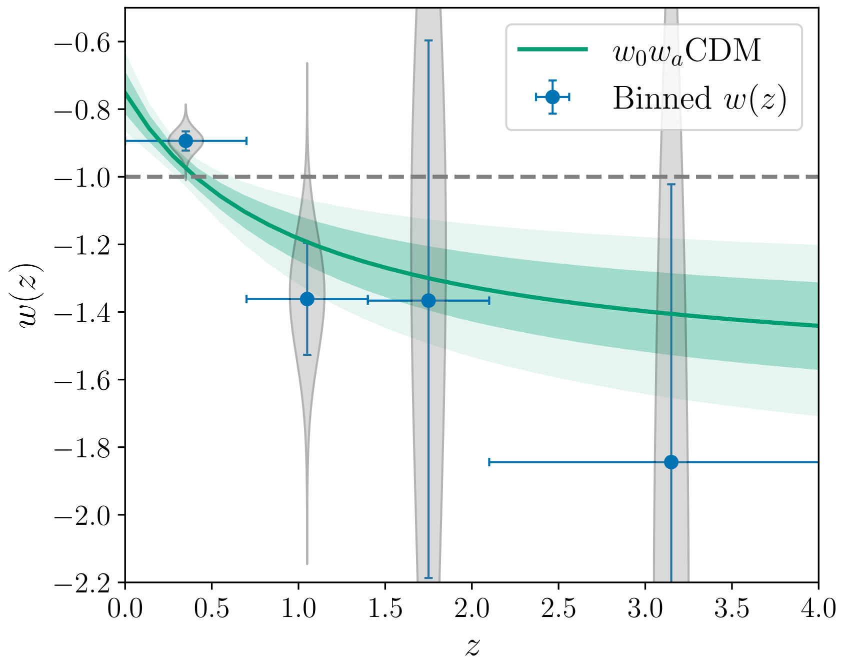

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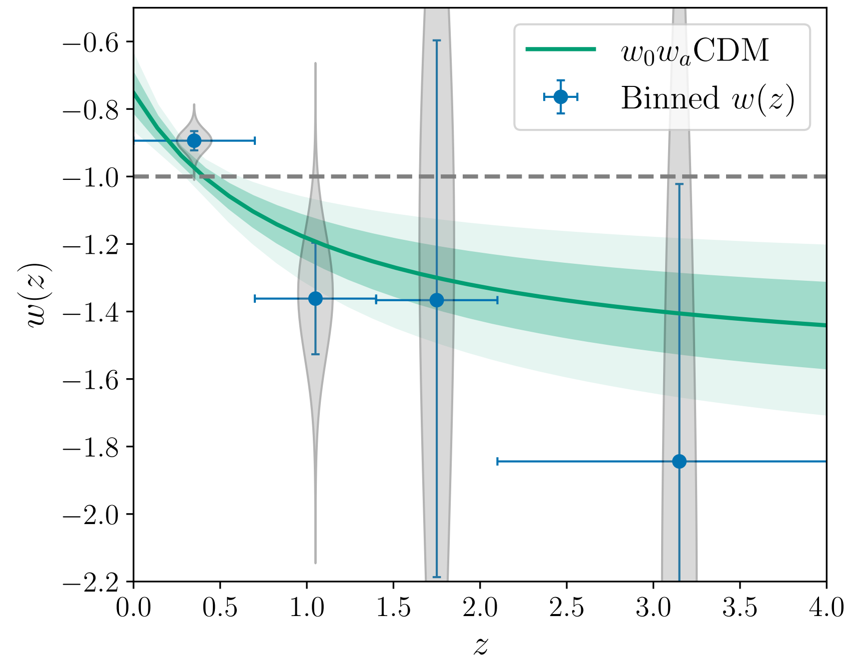

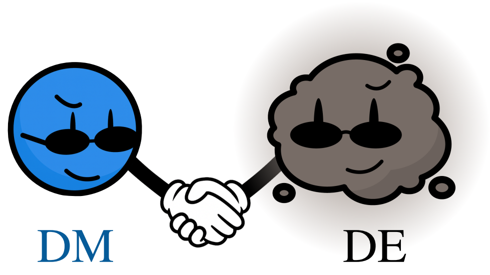

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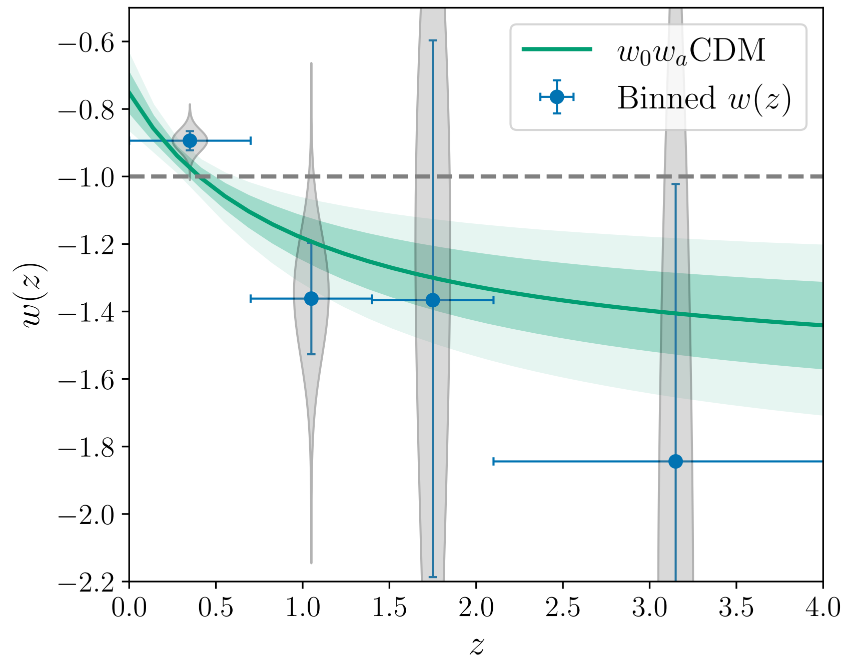
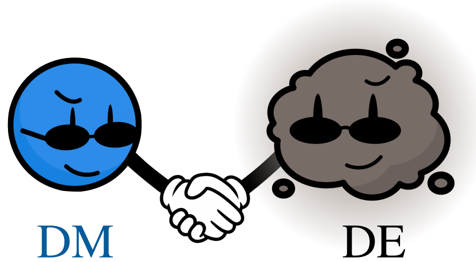




I claim **axions naturally** provide an **interacting dark sector today**!

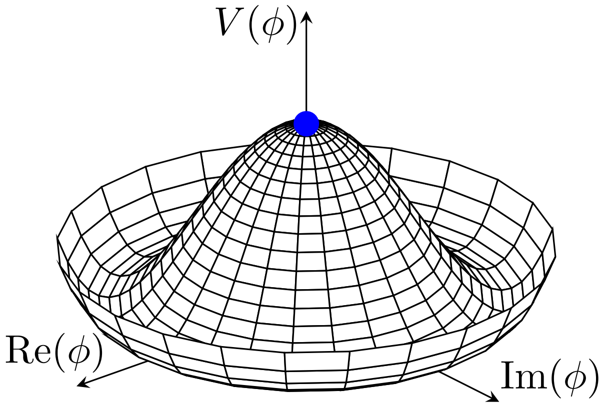
Axions can be both DM and DE

- Axions in this talk: a **scalar field** with **shift symmetry** + potential from **instanton effects**

- **Oscillating axions** with mass heavier than H_0 redshift as **dark matter**:
  $V(\theta) = m_a^2 f_a^2 (1 - \cos\theta)$
  $\ddot{\theta} + 3H\dot{\theta} + m_a^2 \sin\theta \simeq 0$

- **Ultra light axion** with mass smaller than H_0 behaves like **dark energy**

- Axions & Gauge Fields: **couple to monopoles** and detection signals (**birefringence**)



[Marsh, **Phys. Rept.** 643 (2016), 1-79]

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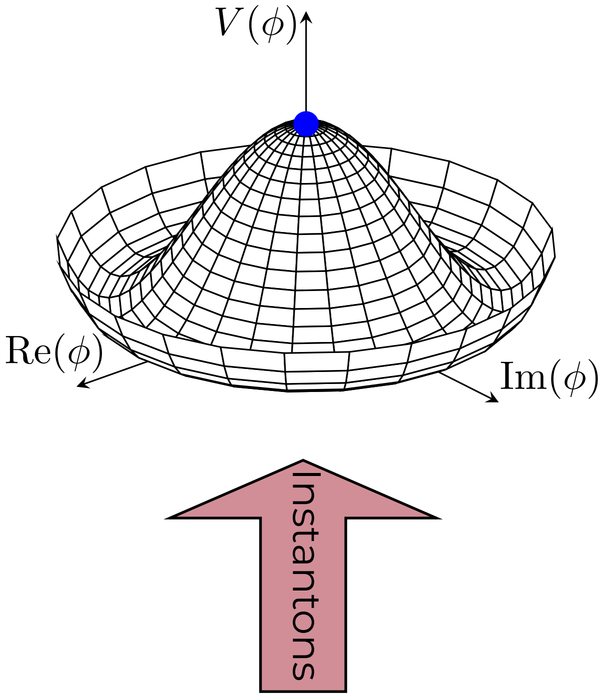

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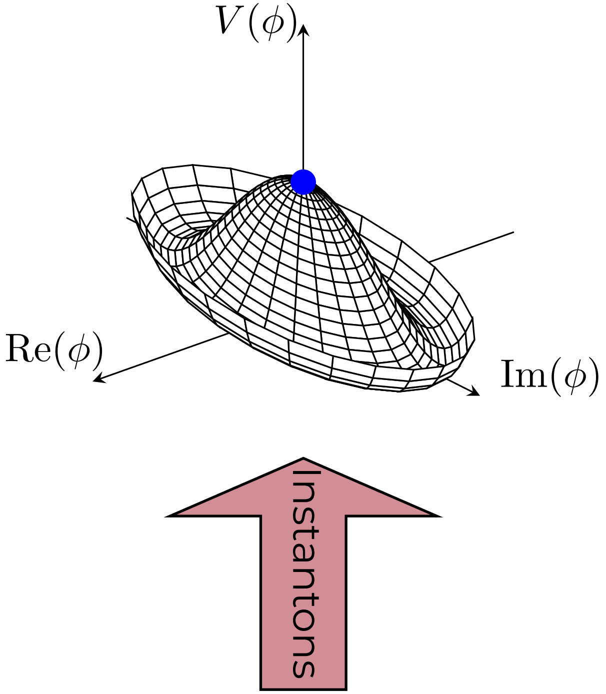

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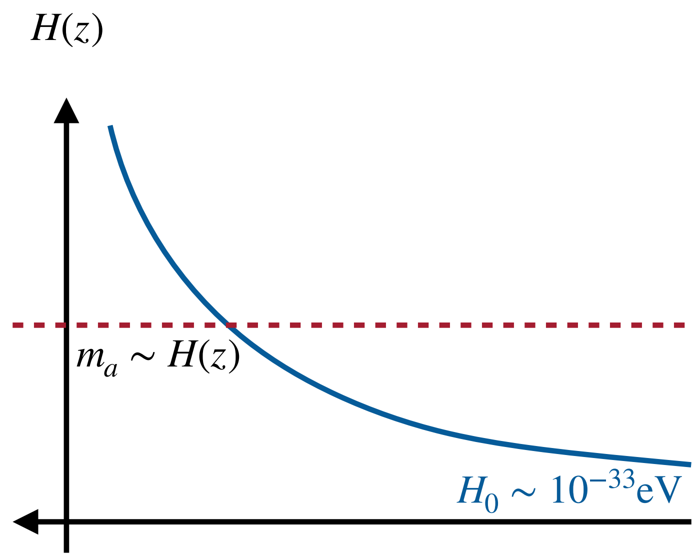

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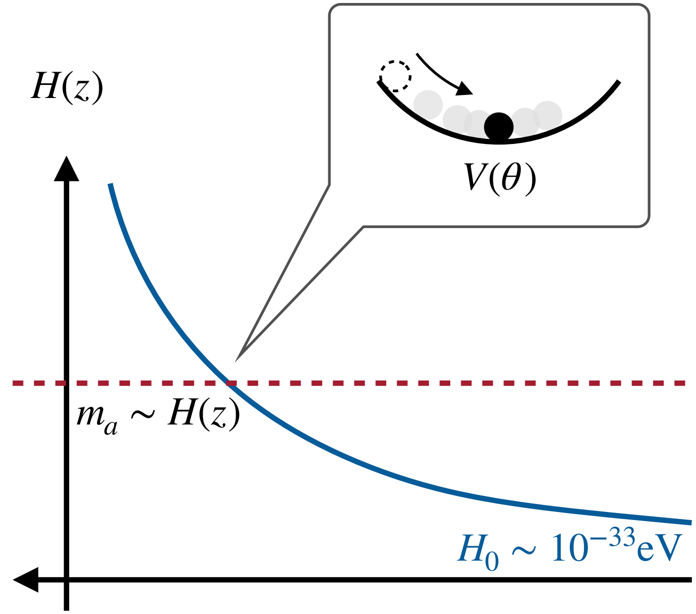

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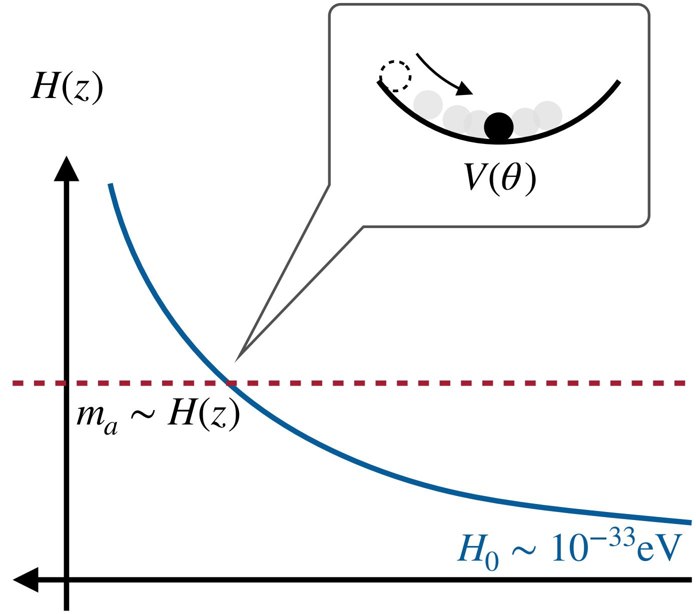

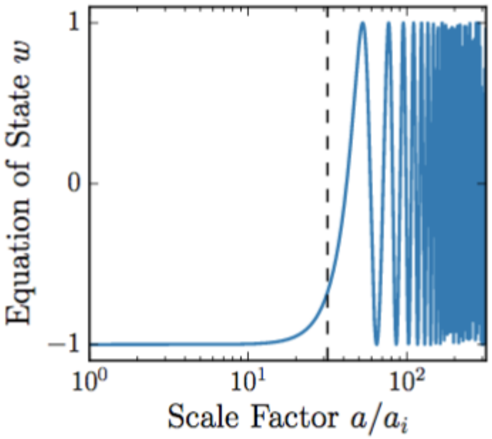

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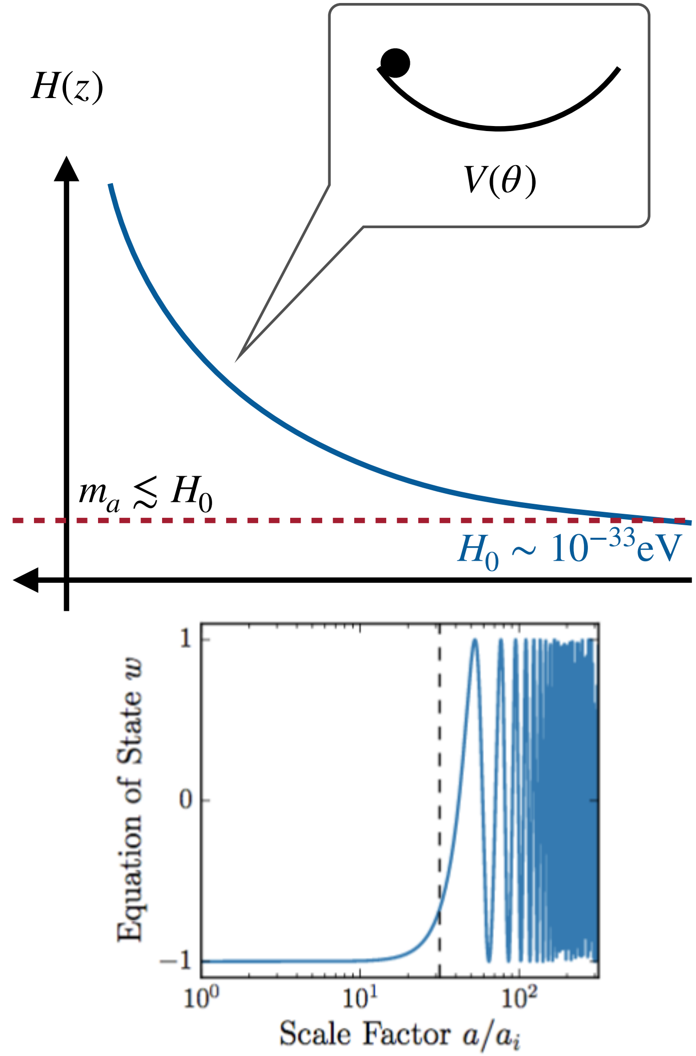

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Outline I

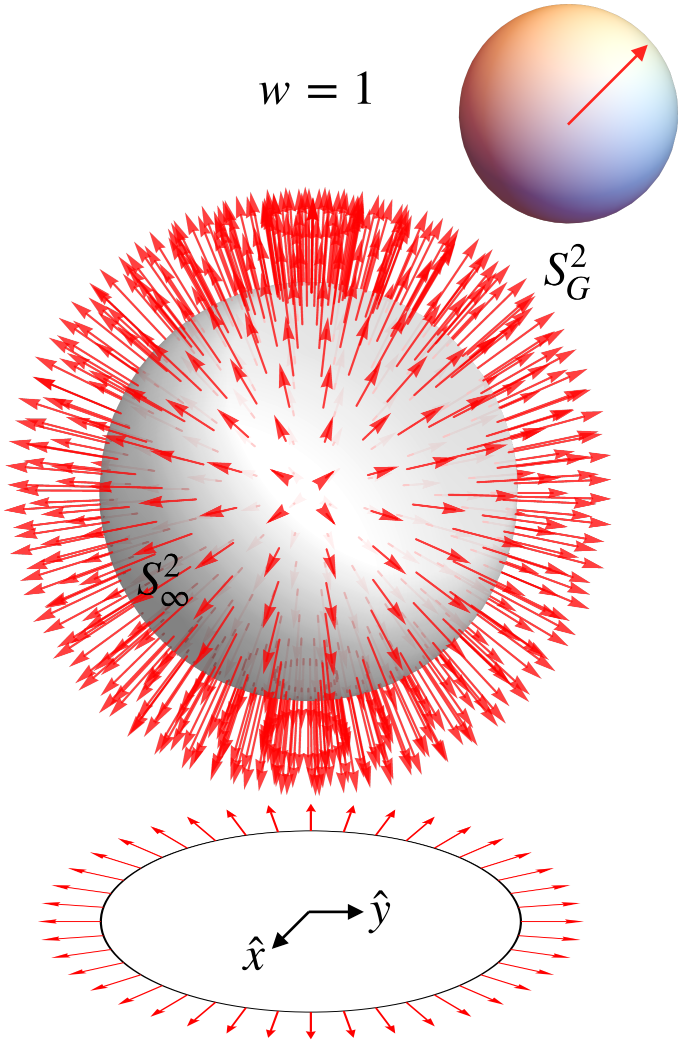
- **Axions & Gauge Fields**
  - **’t Hooft- Polyakov monopoles** as topological defects (**DM**)
  - **Witten effect** (Why axions couple to monopoles)
  - ’t Hooft-Polyakov **charge** + **mass** in axion background (**DM-DE interaction**)
- What is a ***natural*** DM-DE interaction?
- **Potential cosmological signatures**
  - Evolving DM-DE interaction (DESI)
  - Cosmic Birefringence (Ask me after the talk)

Minimal example: 't Hooft-Polyakov

't Hooft-Polyakov monopole arises in the **Georgi-Glashow model**:

$$\mathscr{L} \supset -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} + \frac{1}{2}(D_\mu\sigma^a)^2 + V(\sigma)$$

- Spontaneous breaking of $SU(2) \to U(1)$
- Higgs triplet σ^a gets a vev to minimize $V(\phi) = -\lambda(\sigma^a\sigma^a - v^2)^2$
- Any finite energy configuration obeys $\sigma^a\sigma^a \to v^2$ (this is S^2_G) at $|\vec{r}| \to \infty$ (which is S^2_∞)
- Monopole solutions exists bc $\pi_2(S^2_G) = Z$
- Monopole **charge**: $q_M = 4\pi/e_D\, w\, (\in Z)$
- Monopole **mass**: $m_M = v\,|q_M|$



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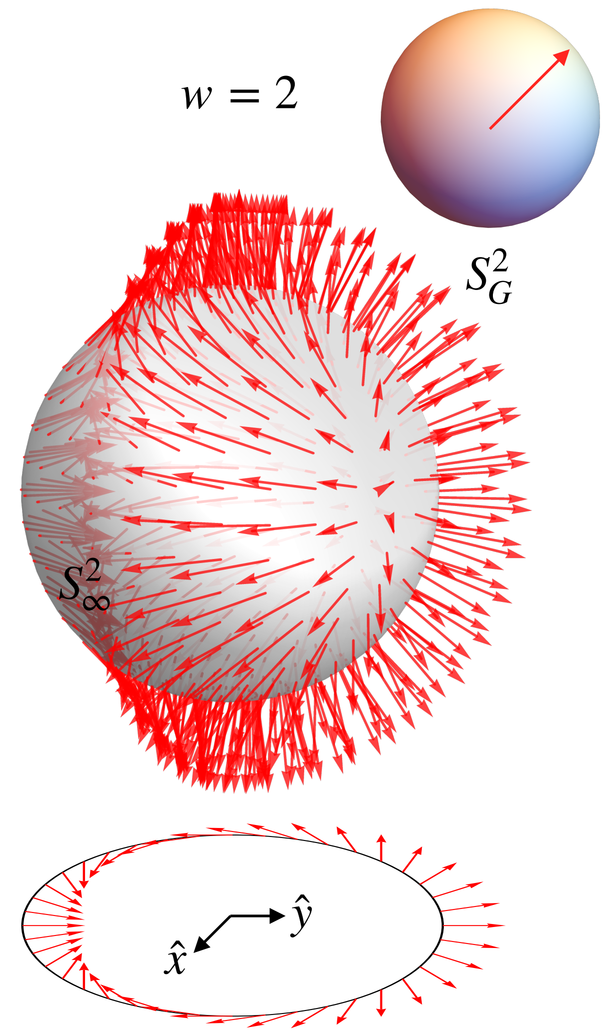

The Witten effect

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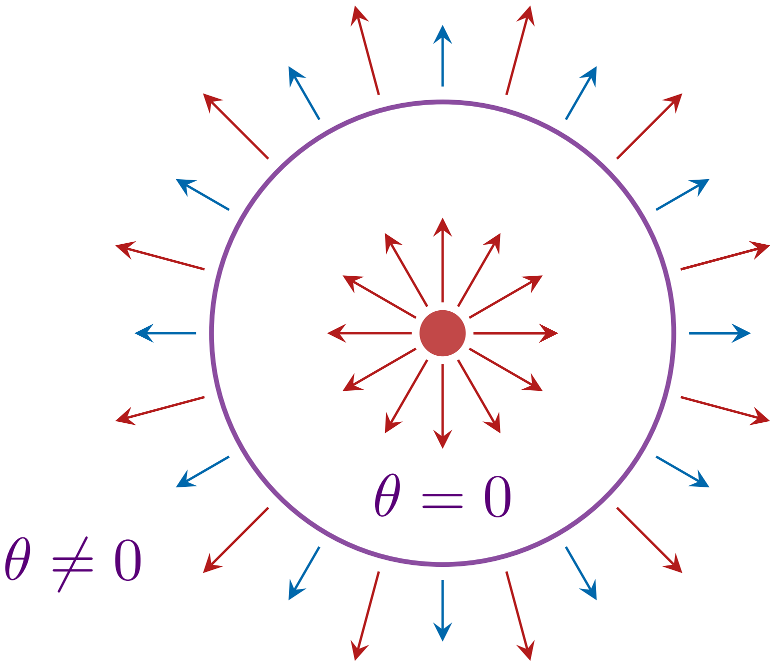
Monopoles in the presence of a non-zero θ term ($\mathscr{L} \supset \theta F\tilde{F}$) **acquire *electric charge***:

- Put a magnetic monopole inside a region, surrounded by $\theta \neq 0$
- Monopole magnetic field **induces an electric field** when θ changes.
- Take size $\rightarrow 0$ limit, because electric charge is independent of the inner region size.



$$\vec{\nabla} \cdot \vec{E} = -\frac{\alpha_D}{2\pi}(\vec{\nabla}\theta \cdot \vec{B})$$

$$q_E = -\frac{\alpha_D}{2\pi}\theta\, q_M$$

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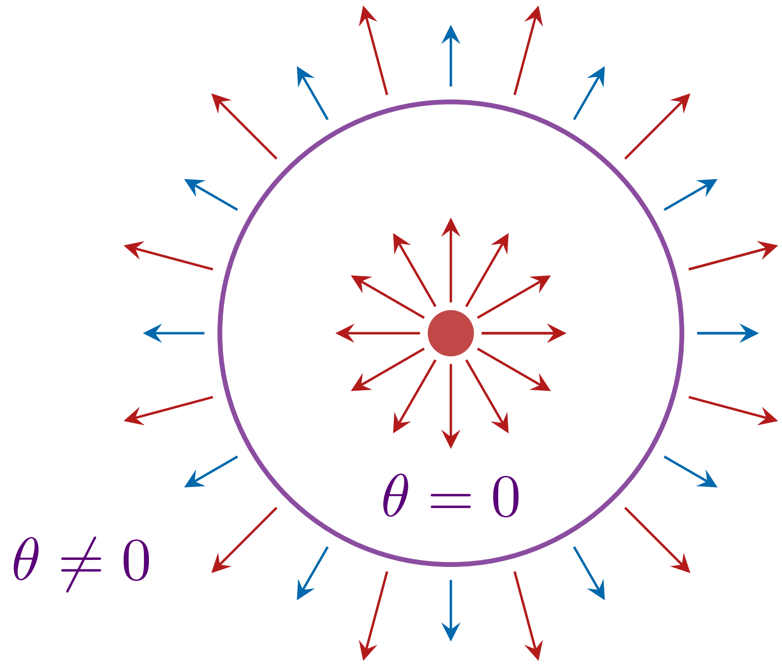

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How does this effect **generalize** to **’t Hooft-Polyakov monopoles**?

't Hooft-Polyakov charge + mass

- In the presence of θ, monopoles acquire electric charge and **become a tower of dyons** (Witten effect):

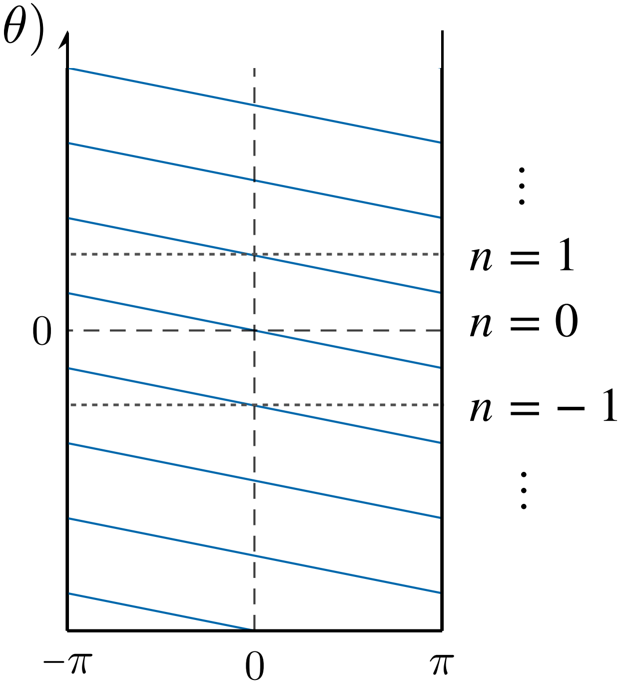
$$q_E = \alpha_D \, q_M \, (n - \frac{\theta}{2\pi})$$

- The electric charge changes the dyon masses in a θ-dependent way:

$$m_D = v\sqrt{q_M^2 + q_E^2} \sim m_M \, (1 + \frac{\alpha_D^2}{2}(n - \frac{\theta}{2\pi})^2)$$

- This is equivalent to axion having a potential

$$V(\theta) = \Lambda^4(1 - \cos\theta) + \rho_M \frac{\alpha_D^2}{2}(n - \frac{\theta}{2\pi})^2$$



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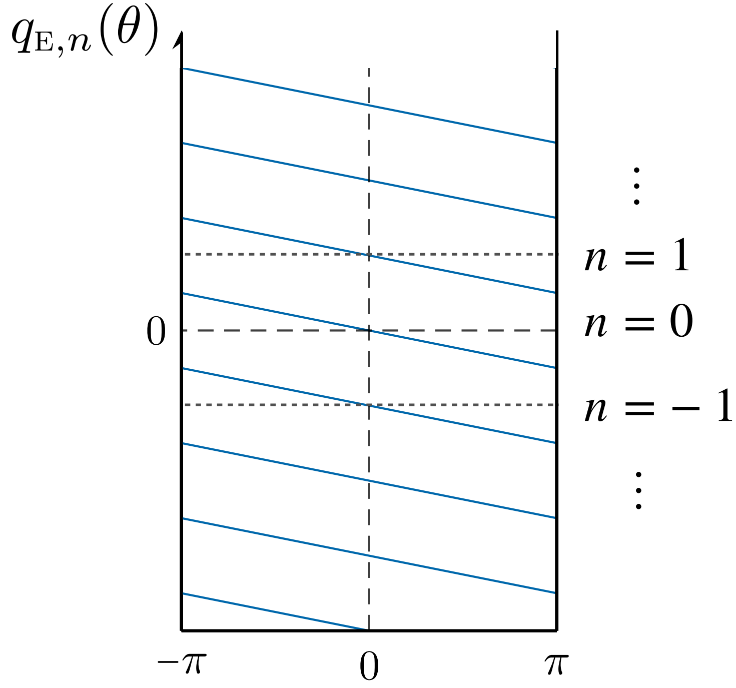

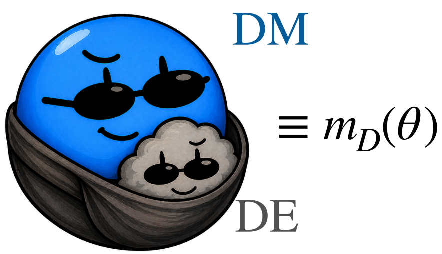

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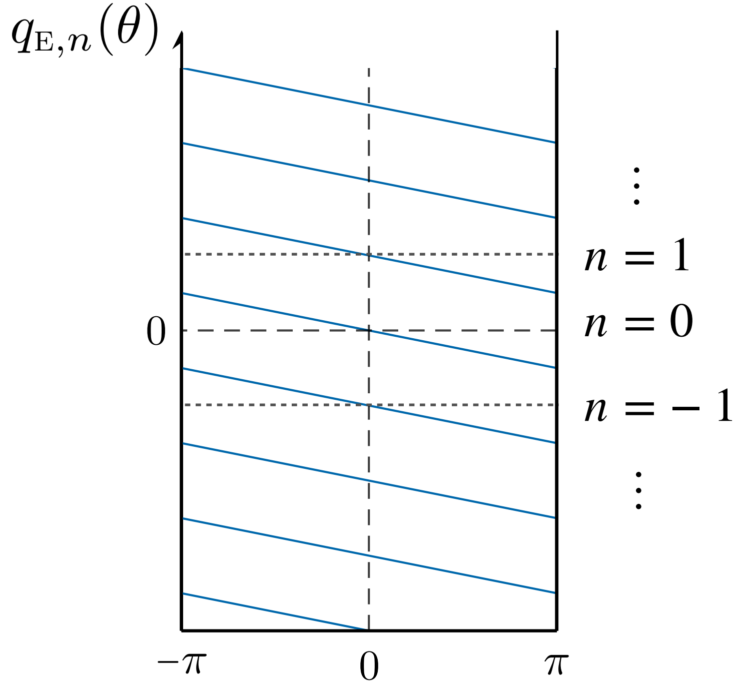

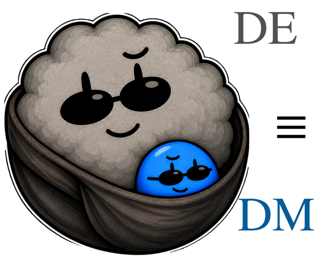

$\equiv V(\theta) \supset V_{\text{induced}}(\rho_M)$

Outline II

- The **magnetic monopoles** are a theoretically well-motivated **DM candidate**. It is a generic consequence of spontaneous symmetry breaking of gauge theories when the vacuum manifold has nontrivial topology.
- They **interact naturally** with **dynamic axion field (DE)**

- What is a ***natural*** DM-DE interaction?
- **Potential cosmological signatures**
  - Evolving DM-DE interaction (DESI)
  - Cosmic Birefringence (Ask me after the talk)

DM-DE Interactions

Dynamic DE models with scalar potential $V(\theta)$ &

- DM particle χ with θ-dependent mass, $m_\chi(\theta) \gg m_\theta$

  →**Suffer from a naturalness problem:**

  Loops of χ give a **Coleman-Weinberg correction** to θ-potential that need to stay small: $\Delta V'_{\rm CW}(\theta) \ll V'(\theta)$

- *Assume "Interesting physics"*:

  χ-density induced effective potential dominates over bare θ-potential, $V(\theta) \sim V_{\chi-{\rm ind}}(\rho_\chi)$

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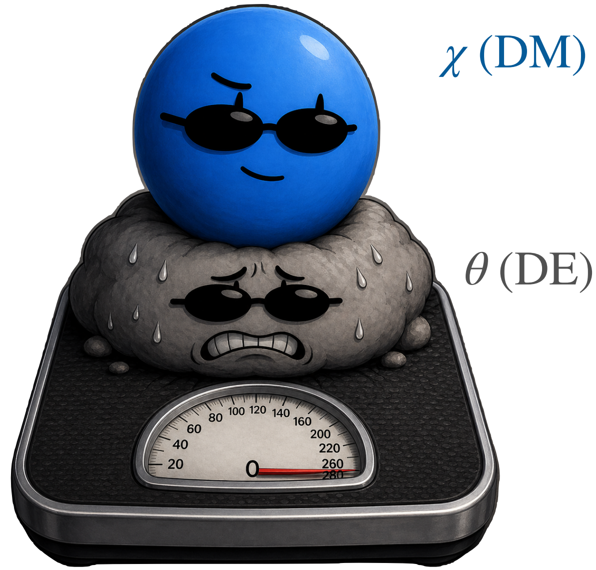

$$V(\theta) \sim V_{\chi-\mathrm{ind}}(\rho_\chi) = m_\chi(\theta)\langle n_\chi \rangle$$

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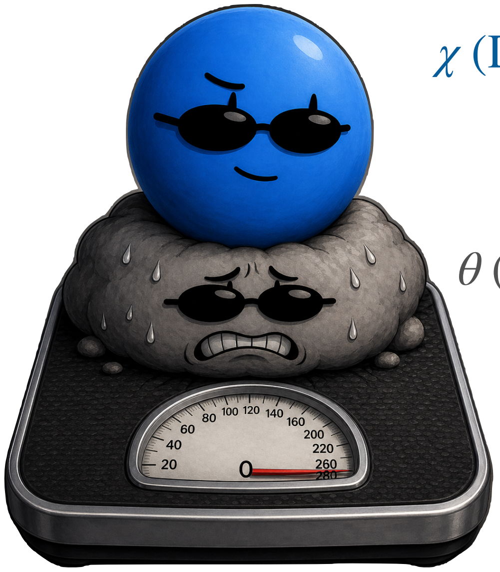
χ (DM)

θ (DE)



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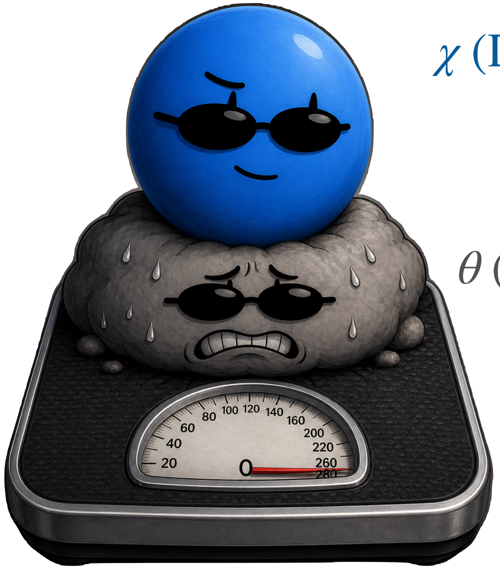

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χ (DM)

θ (DE)

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- DM particle χ with θ-dependent mass, $m_\chi(\theta) \gg m_\theta$

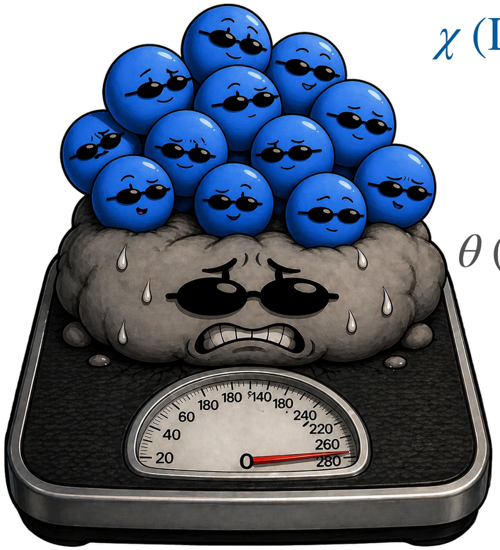
  → **Suffer from a naturalness problem:**

  Loops of χ give a **Coleman-Weinberg correction** to θ-potential that need to stay small: $\Delta V'_{\mathrm{CW}}(\theta) \ll V'(\theta)$

- *Assume "Interesting physics"*:

  χ-density induced effective potential dominates over bare θ-potential,

  $V(\theta) \sim V_{\chi-\mathrm{ind}}(\rho_\chi)$



χ (DM)

θ (DE)

$$V(\theta) \sim V_{\chi-\mathrm{ind}}(\rho_\chi) = m_\chi(\theta)\langle n_\chi \rangle$$

$$\Delta V_{\mathrm{CW}} \sim \frac{1}{16\pi^2}\Lambda^2 m_\chi^2(\theta)$$

$$\Delta V'_{\mathrm{CW}}(\theta) \ll V'_{\chi-\mathrm{ind}}(\rho_\chi) \to m_\chi^3 \ll \langle n_\chi \rangle$$

What kinds of model can **solve this naturalness problem**?

Monodromic DM-DE Interactions

Dynamical DE models (at least one is true):

- Extremely fine tuned $V(\theta)$
- **Natural suppression** of CW-corrections
- $n_\chi \gg m_\chi^3$ and $m_\chi \ll \mathrm{eV}$:

  No fermions, χ is **wave dark-matter**

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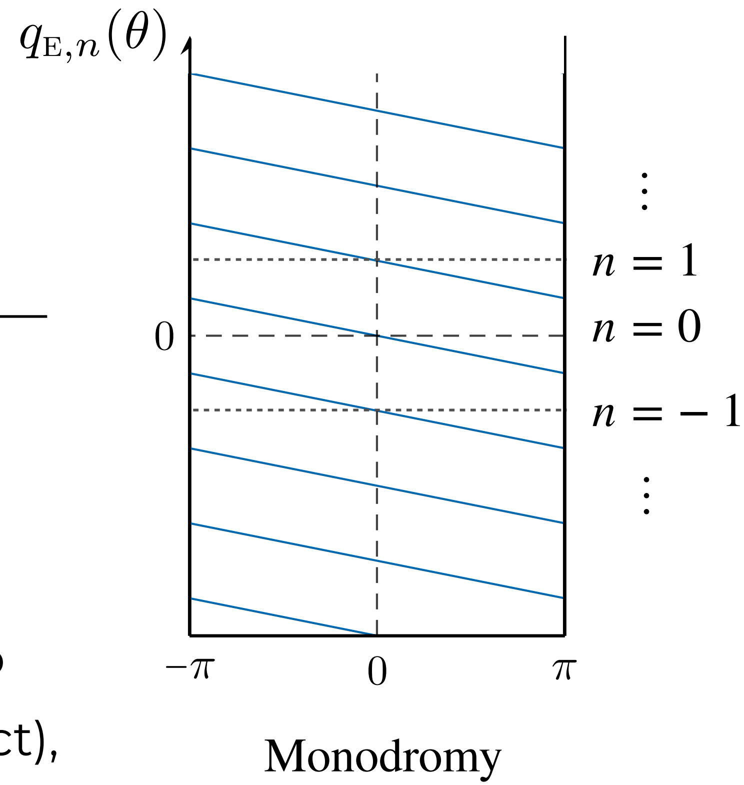

Few models with CW-suppression:

- Dark magnetic dyons (DM)
- Light axion (DE)
- An infinite tower of dyon states **shifted into each other when** $\theta \to \theta + 2\pi f$ (Witten effect), leads to a CW potential scaling $\exp(-2\pi/\alpha_{\mathrm{D}})$, [M. Reece, etal., **arXiv**: 2105.09950]

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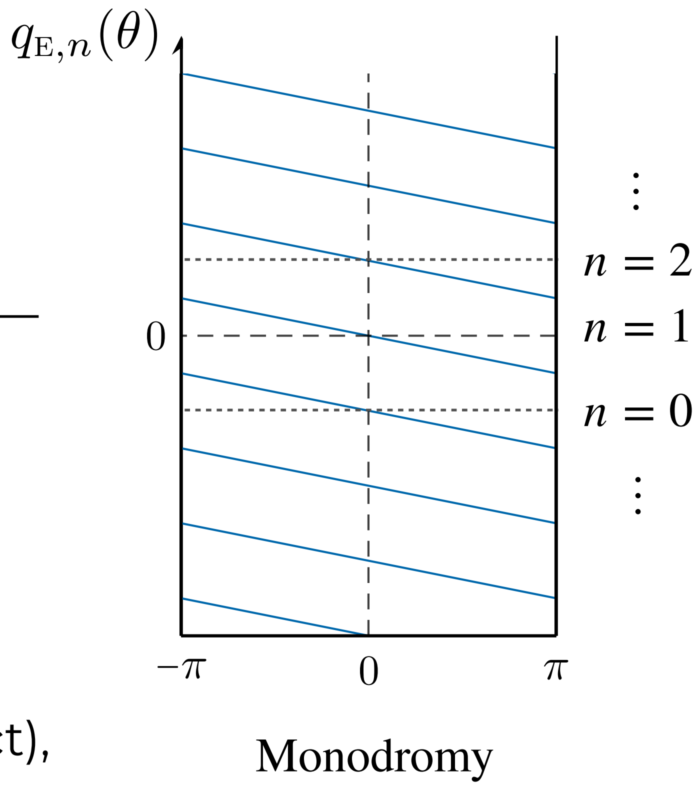

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Outline III

- The **magnetic monopoles** are a theoretically well-motivated **DM candidate**.
- They **interact naturally** with **dynamic axion field (DE)**
    - Summing over tower of dyon states **exponentially suppresses CW-corrections (Monodromy)**
- **Potential cosmological signatures**
    - Evolving DM-DE interaction (DESI)
    - Cosmic Birefringence (Ask me after the talk)

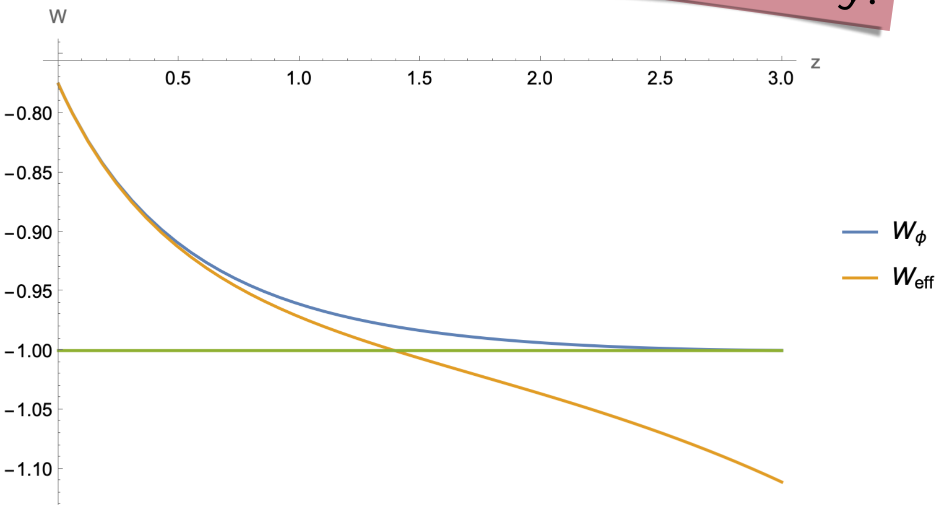
Effective equation of state

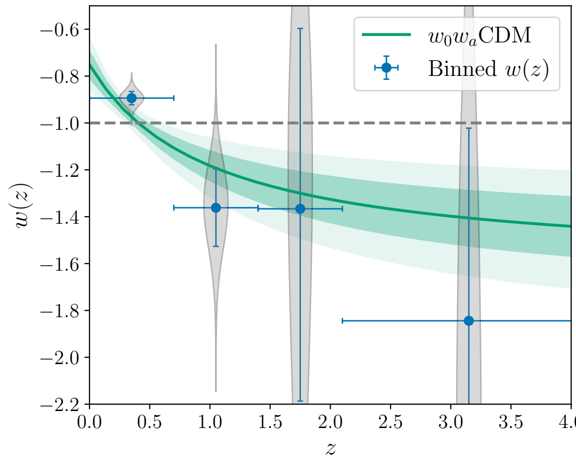
Preliminary!

A model with bench mark values:
$m_a = 1.5\,H_0\,, \alpha_D \sim \mathcal{O}(0.1), f_a \sim \mathcal{O}(M_{\rm Pl})$

- A local observer who does not assume DM-DE interaction will attribute the change on the DM density to an **effective component of DE**

$$w_{\rm eff} = \frac{w_\phi}{1 + (\frac{A(\phi)}{A(\phi_0)} - 1)\frac{\rho_{DM}(0)}{a^3 \rho_\phi}}$$





© [DESI DRII, **arXiv**: 2503.14738]

Summary

- The **magnetic monopoles** are a theoretically well-motivated **DM candidate.**
- They interact naturally with **dynamic axion field (DE)**
  - Summing over infinite dyon states, **exponentially suppresses CW-corrections (Monodromy)**
- The model has potential **cosmological signatures** with **natural benchmark parameters,** including DESI and Birefringence.

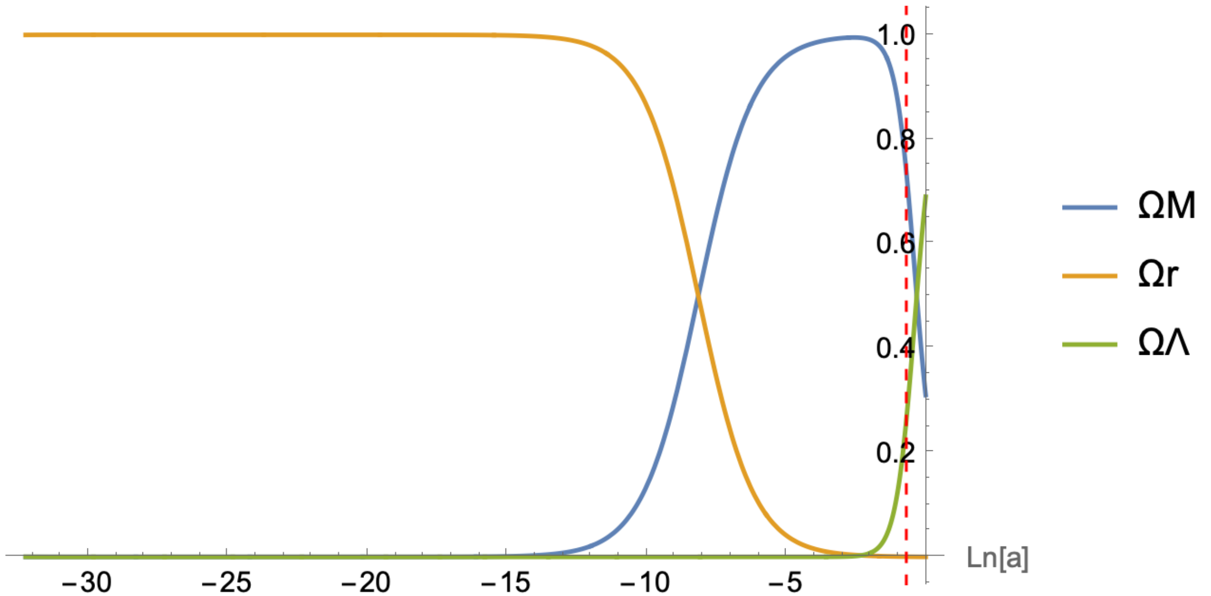
Thank you!

EOM and cosmic evolution

A model with bench mark values:

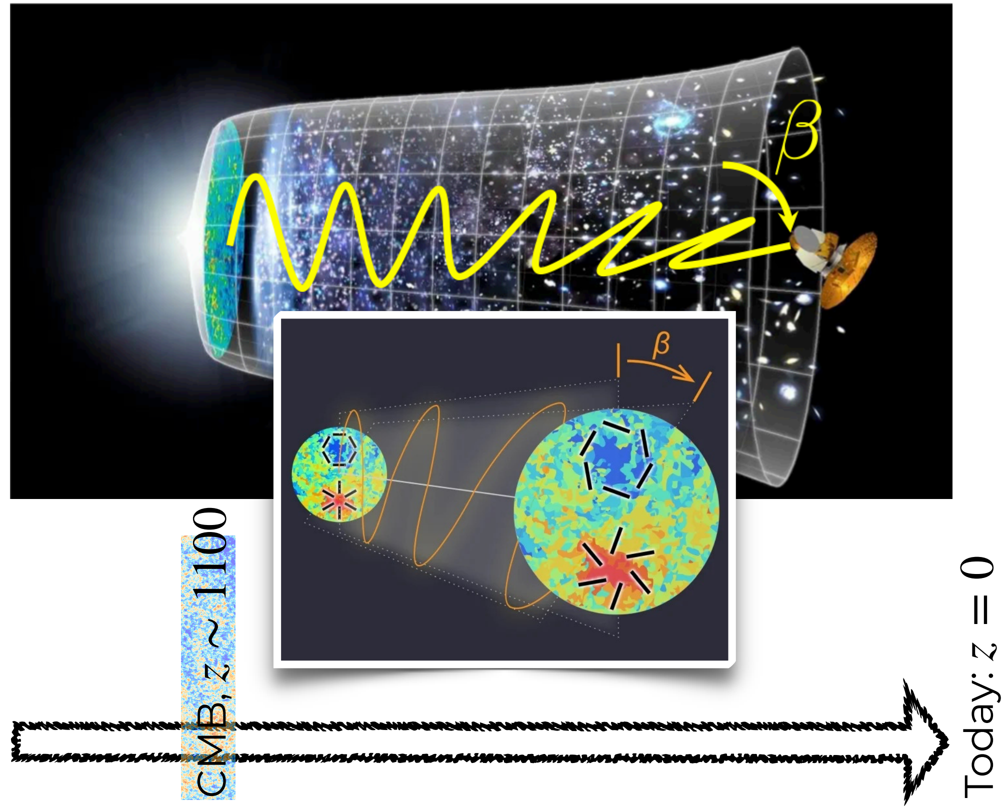
$$m_a = 1.5\, H_0\,, \alpha_D \sim \mathcal{O}(0.1), f_a \sim \mathcal{O}(M_{\rm Pl})$$

Dark Energy (Axion)	Dark Matter (Monopole)
$\rho_\theta = \Lambda^4(1-\cos\theta) + \dot{\theta}^2 f_a^2/2$	$\rho_M = \frac{\rho_M^0}{a^3}(1+\frac{\alpha_D^2}{8\pi^2}(\theta-2\pi n)^2)$



On-going: Axion DE & Birefringence

- Birefringence signal: Rotation of the photon polarization vector
- Turning on the coupling to photons: $g_{\theta\gamma}\,\theta F\tilde{F}$
- The **Birefringence angle** is computed via: $2\beta = g_{\theta\gamma}\,\Delta\theta$ (remember $c_{L/R} \equiv \dfrac{\omega_{L/R}}{k} = \sqrt{1 \pm \dfrac{g_{\theta\gamma}\dot{\theta}}{k}}$)
- To get $\beta_{\text{Planck}} \sim 0.35$ deg, we need $\alpha_{\text{vis}} \sim 0.1$



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- To get $\beta_{\rm Planck} \sim 0.35$ deg, we need $\alpha_{\rm vis} \sim 0.1$

Axions rotate polarization:

$$\ddot{A}_k^{L/R} + \omega_{L/R}^2 A_k^{L/R} = 0$$

$$c_{L/R} \equiv \frac{\omega_{L/R}}{k} = \sqrt{1 \pm \frac{g_{\theta\gamma}\dot{\theta}}{k}}$$