

Searching for Spontaneous Cyclic Shift Symmetries

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About Me

5th year in HEP-th

- Interests: topological consequences in QFT, generalized sym.
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Advised by Matthew Reece

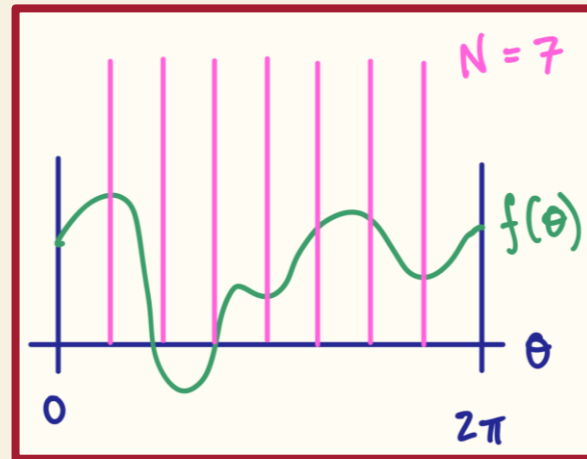
- Axions!
- Talk: 11:35 tomorrow



Key Mathematical Idea

For periodic $f(\theta)$:

*Summations of regularly shifted **copies of f***
can filter its harmonics



Visible via a Fourier decomposition



Key Mathematical Idea

$$\sum_{k=0}^{N-1} f\left(\theta - \frac{2\pi}{N}k\right) = \sum_{k=0}^{N-1} \sum_{\ell \in \mathbb{Z}} \hat{f}_\ell e^{i\theta\left(\ell - \frac{2\pi i}{N}k\right)}$$

$$= \sum_{\ell \in \mathbb{Z}} \hat{f}_\ell e^{i\theta\ell} \sum_{k=0}^{N-1} \omega_\ell^k$$

$$\omega_\ell := e^{\frac{2\pi i}{N}\ell}$$

$$= \sum_{\ell \in \mathbb{Z}} \hat{f}_\ell e^{i\theta\ell} \begin{cases} N & \text{if } \ell \text{ divides } N \\ \frac{1 - \omega_\ell^N}{1 - \omega_\ell} & \text{otherwise} \end{cases}$$



Hook's Construction (1802.10093)

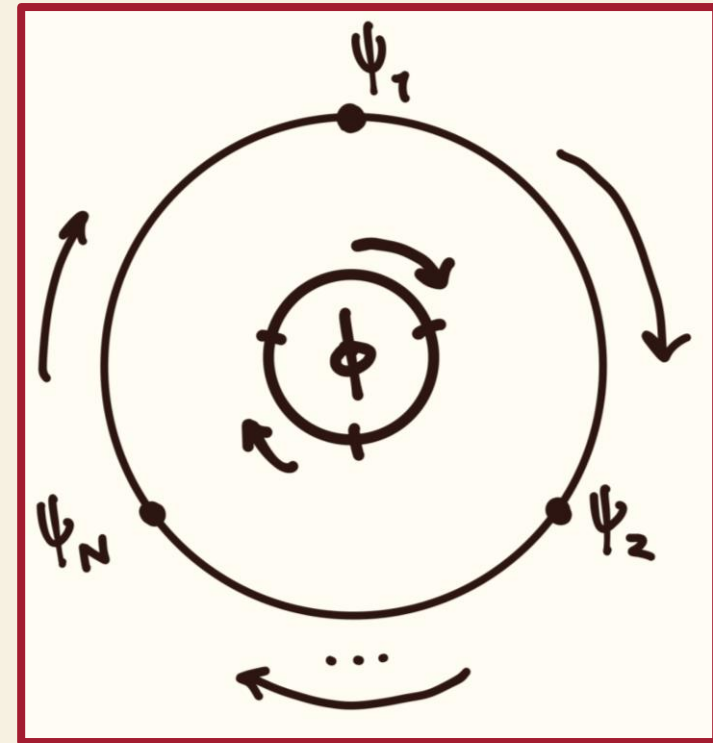
Ingredients:

1 periodic scalar ϕ

N identical species of fermion

Coupled symmetries:

$$\left. \begin{array}{l} \phi \rightarrow \phi + \frac{2\pi f}{N} \\ \psi_k \rightarrow \psi_{k+1} \end{array} \right\} \Rightarrow V(\phi) = \sum V_k(\phi)$$
$$\propto \sum f \left(\frac{\phi}{f} + \frac{2\pi}{N} k \right)$$



Hook's Construction (1802.10093)

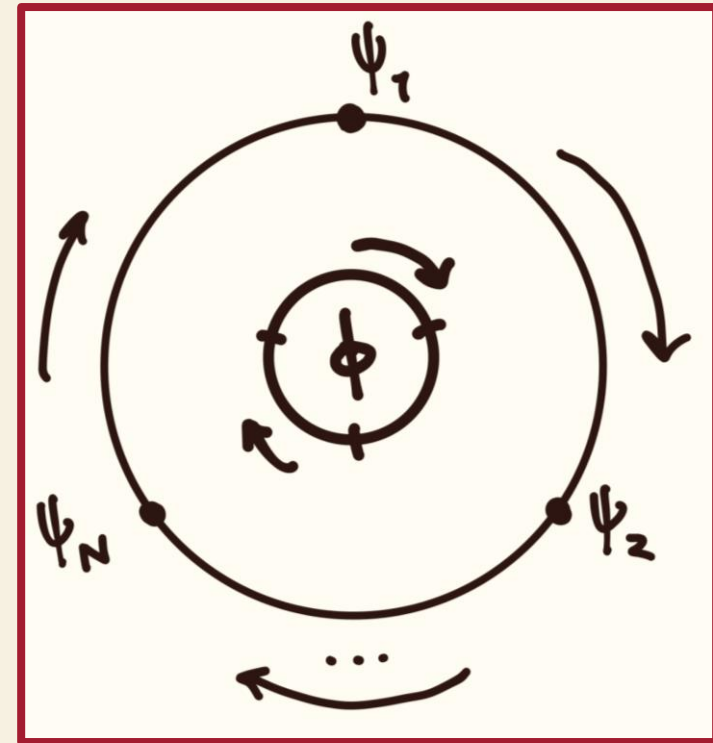
$$V(\phi) \propto \sum f \left(\frac{\phi}{f} + \frac{2\pi}{N} k \right)$$

ϕ feels contributions from all N sectors

First N mode contributions cancel!

Axion potential lowest order becomes $\mathcal{O}(N)$

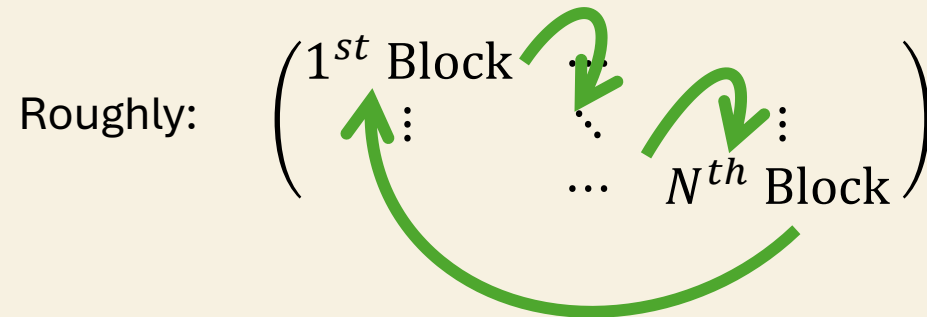
Exponentially flattened naturally*



Our Goal: Obtain Dynamically

Spontaneous breaking with **discrete** IR symmetries

$$SU(MN) \rightarrow SU(M)^N \rtimes \mathbb{Z}_N$$



Precedence: “Alice Strings” with 5-dim Higgs

$$SU(2) \rightarrow U(1) \rtimes \mathbb{Z}_2$$

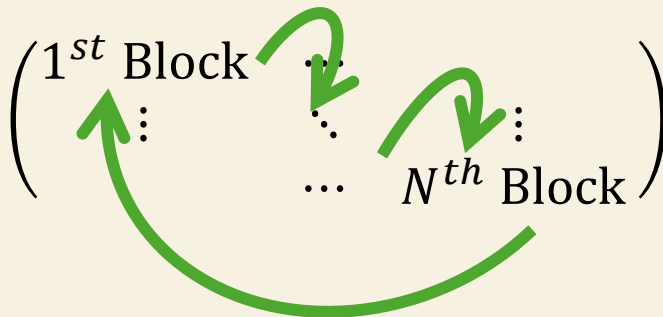


Our Goal: Obtain Dynamically

Spontaneous breaking with **discrete** IR symmetries

$$SU(MN) \rightarrow SU(M)^N \rtimes \mathbb{Z}_N$$

Roughly: $\left(\begin{array}{ccc} 1^{st} \text{ Block} & \vdots & \\ \vdots & \ddots & \\ \dots & & N^{th} \text{ Block} \end{array} \right)$

A diagram showing a block matrix with three rows and three columns. The top-left element is labeled '1st Block', the bottom-right is 'Nth Block', and the middle-right is 'Nth Block'. There are vertical ellipses in the first and third columns, and a horizontal ellipsis in the second row. Green arrows point from the '1st Block' to the 'Nth Block' in the second column, from the 'Nth Block' in the second column to the 'Nth Block' in the third column, and a large curved arrow from the 'Nth Block' in the third column back to the '1st Block' in the first column.

But why?

- Accessible UV physics vs engineered EFT
- Discrete gauge groups give defects



Strategy

- 1) Couple fermions to scalar h transforming under $SU(MN)$
- 2) Find vev stable under both cts. and discrete components

New engineering question: which representation?

Simplest choice: N -fold exterior power



Determinant Rep!



Strategy with Λ^N

Decompose

$$\mathbb{C}^{MN} \rightarrow \bigotimes_{k=0}^{N-1} \mathbb{C}^M$$

Let $h \in \Lambda^N(\mathbb{C}^{MN})$ interact with each gauge block:

$$h \sim \sum_{k=0}^{N-1} c^k (e_{k,1} \wedge \cdots \wedge e_{k,M})$$

Roughly: $\left(\begin{array}{ccc} 1^{st} \text{ "Block Baryon"} & \cdots & \\ \vdots & \ddots & \vdots \\ \cdots & N^{th} \text{ "Block Baryon"} & \cdots \end{array} \right)$



Outcomes with Λ^N

$N = 2$ case too simple! \Rightarrow symplecticC

Generally, correct continuous group!

$$\text{Diagonal } U: h \rightarrow \sum_{k=0}^{N-1} c^k \det(U_k) (e_{k,1} \wedge \cdots \wedge e_{k,M})$$

BUT $SU(M)$ & \mathbb{Z}_N are too strong:

$$SU(MN) \rightarrow SU(M)^N \rtimes S_N$$



Moving Forward

Different representations? → Maybe,
Skeptical due to general rep. classification

More than one SSB scalar → More likely

