Searching for Spontaneous Cyclic Shift Symmetries

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Phenomenology ’26

About Me

5th year in HEP-th

- Interests: topological consequences in QFT, generalized sym.
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Advised by Matthew Reece

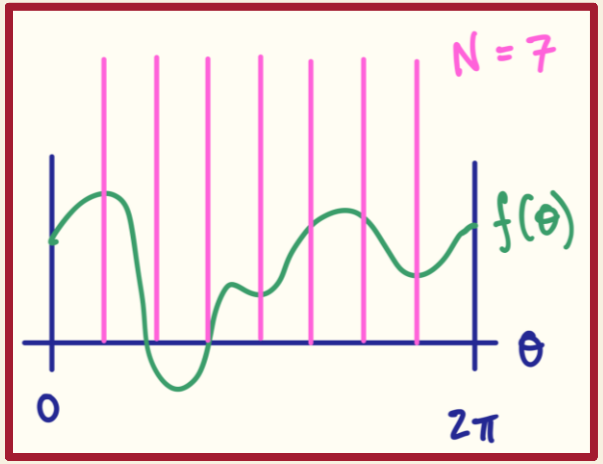
- Axions!
- Talk: 11:35 tomorrow

Key Mathematical Idea

For periodic $f(\theta)$:

Summations of <u>regularly</u> <u>shifted</u> **copies of *f***

can filter its harmonics



Visible via a Fourier decomposition

Key Mathematical Idea

$$\sum_{k=0}^{N-1} f\left(\theta - \frac{2\pi}{N}k\right) = \sum_{k=0}^{N-1}\sum_{\ell\in\mathbb{Z}} \hat{f}_\ell\, e^{i\theta\left(\ell - \frac{2\pi i}{N}k\right)}$$

$$= \sum_{\ell\in\mathbb{Z}} \hat{f}_\ell\, e^{i\theta\ell} \sum_{k=0}^{N-1} \omega_\ell^k \qquad \omega_\ell \coloneqq e^{\frac{2\pi i}{N}\ell}$$

$$= \sum_{\ell\in\mathbb{Z}} \hat{f}_\ell\, e^{i\theta\ell} \begin{cases} \mathrm{N} & \text{if } \ell \text{ divides } N \\ \dfrac{1-\omega_\ell^N}{1-\omega_\ell} & \text{otherwise} \end{cases}$$

Hook's Construction (1802.10093)

Ingredients:

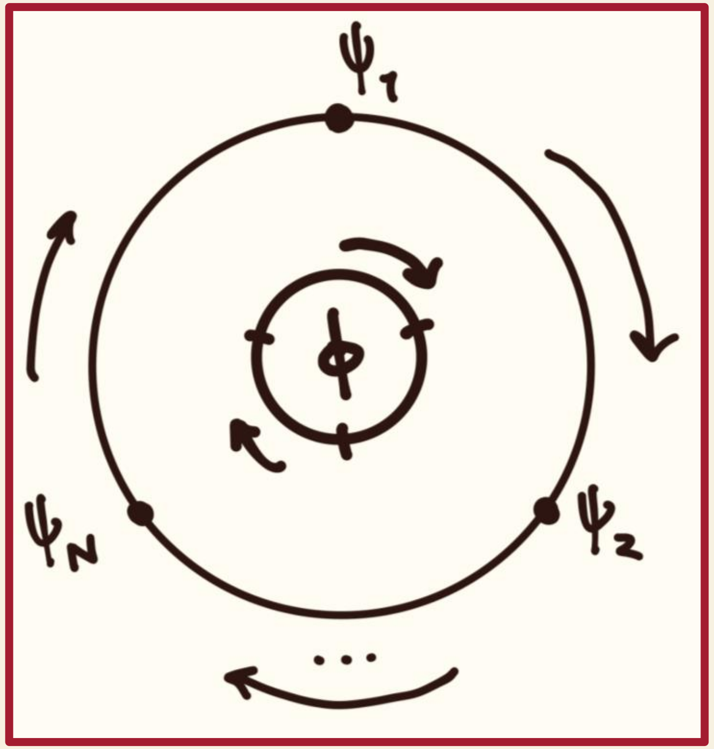
1 periodic scalar ϕ

N identical species of fermion

Coupled symmetries:

$$\left.\begin{aligned} \phi &\to \phi + \frac{2\pi f}{N} \\ \psi_k &\to \psi_{k+1} \end{aligned}\right\} \Rightarrow V(\phi) = \sum V_k(\phi)$$

$$\propto \sum f\left(\frac{\phi}{f} + \frac{2\pi}{N}k\right)$$



Hook's Construction (1802.10093)

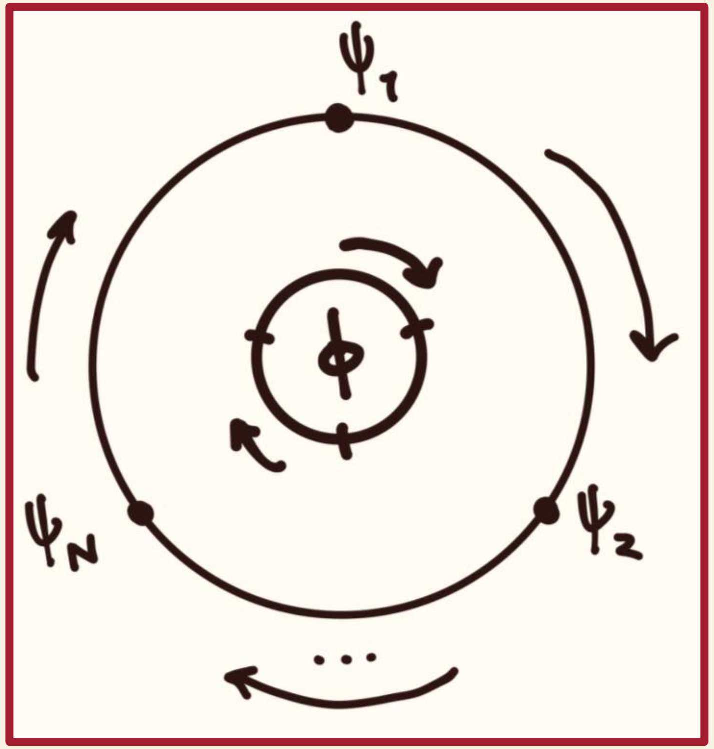
$$V(\phi) \propto \sum f\left(\frac{\phi}{f} + \frac{2\pi}{N}k\right)$$

ϕ feels contributions from all N sectors

First N mode contributions cancel!

Axion potential lowest order becomes $\mathcal{O}(N)$

Exponentially flattened naturally*

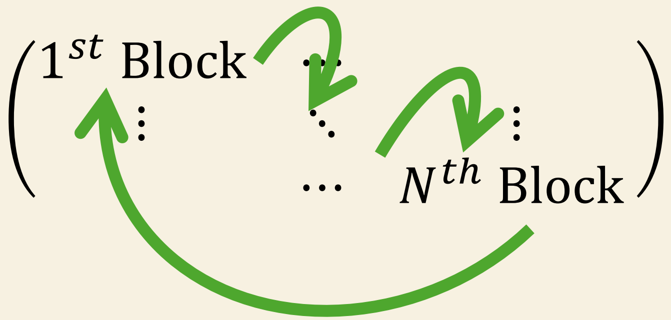


Our Goal: Obtain Dynamically

Spontaneous breaking with **discrete** IR symmetries

$$SU(MN) \rightarrow SU(M)^N \rtimes \mathbb{Z}_N$$

Roughly:



Precedence: “Alice Strings” with 5-dim Higgs

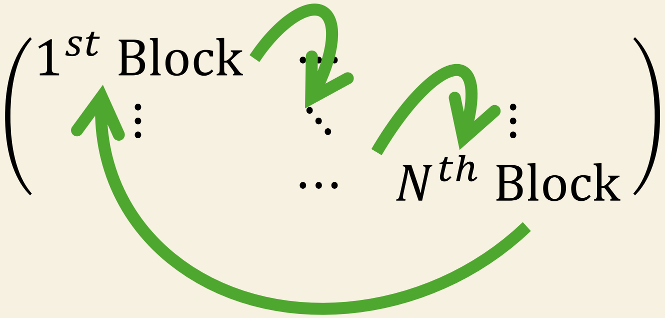
$$SU(2) \rightarrow U(1) \rtimes \mathbb{Z}_2$$

Our Goal: Obtain Dynamically

Spontaneous breaking with **discrete** IR symmetries

$$SU(MN) \to SU(M)^N \rtimes \mathbb{Z}_N$$

Roughly:



But why?

- Accessible UV physics vs engineered EFT
- Discrete gauge groups give defects

Strategy

1) Couple fermions to scalar h transforming under $SU(MN)$
2) Find vev stable under both cts. and discrete components

New engineering question: which representation?

Simplest choice: N-fold exterior power

↓

Determinant Rep!

Strategy with Λ^N

Decompose

$$\mathbb{C}^{MN} \to \bigotimes_{k=0}^{N-1} \mathbb{C}_k^M$$

Let $h \in \Lambda^N(\mathbb{C}^{MN})$ interact with each gauge block:

$$h \sim \sum_{k=0}^{N-1} c^k \left(e_{k,1} \wedge \cdots \wedge e_{k,M}\right)$$

Roughly: $\begin{pmatrix} 1^{st} \text{ "Block Baryon"} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & N^{th} \text{ "Block Baryon"} \end{pmatrix}$

Outcomes with Λ^N

$N = 2$ case too simple! ⇒ symplecticC

Generally, correct continuous group!

$$\text{Diagonal } U\text{: } h \to \sum_{k=0}^{N-1} c^k \det(U_k) \left(e_{k,1} \wedge \cdots \wedge e_{k,M}\right)$$

BUT $SU(M)$ & $\mathbb{Z}_N$ are too strong:

$$SU(MN) \to SU(M)^N \rtimes S_N$$

Moving Forward

Different representations? → Maybe,

Skeptical due to general rep. classification

More than one SSB scalar → More likely