

Matter Unification and Lepton Flavor Violation

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Based on [arXiv : 2603.02313](#) with [Pavel Fileviez Perez](#)

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Main Goal

Discuss the possibility to test the idea of
Quark–Lepton Unification at the low scale
through Flavor Experiments.

Flavor Violation in the Standard Model

- ▶ V_{CKM} introduces flavor changing interaction in the quark Sector
- ▶ As neutrinos are massless, no flavor violation in the leptonic sector.
- ▶ Lepton flavor is conserved in SM ($U(1)_e \times U(1)_\mu \times U(1)_\tau$)
- ▶ Neutrino Oscillation tells us that Lepton Flavor symmetry is not a good symmetry of the nature.

Any Charged Lepton Flavor Violating Signature \rightarrow **New Physics.**

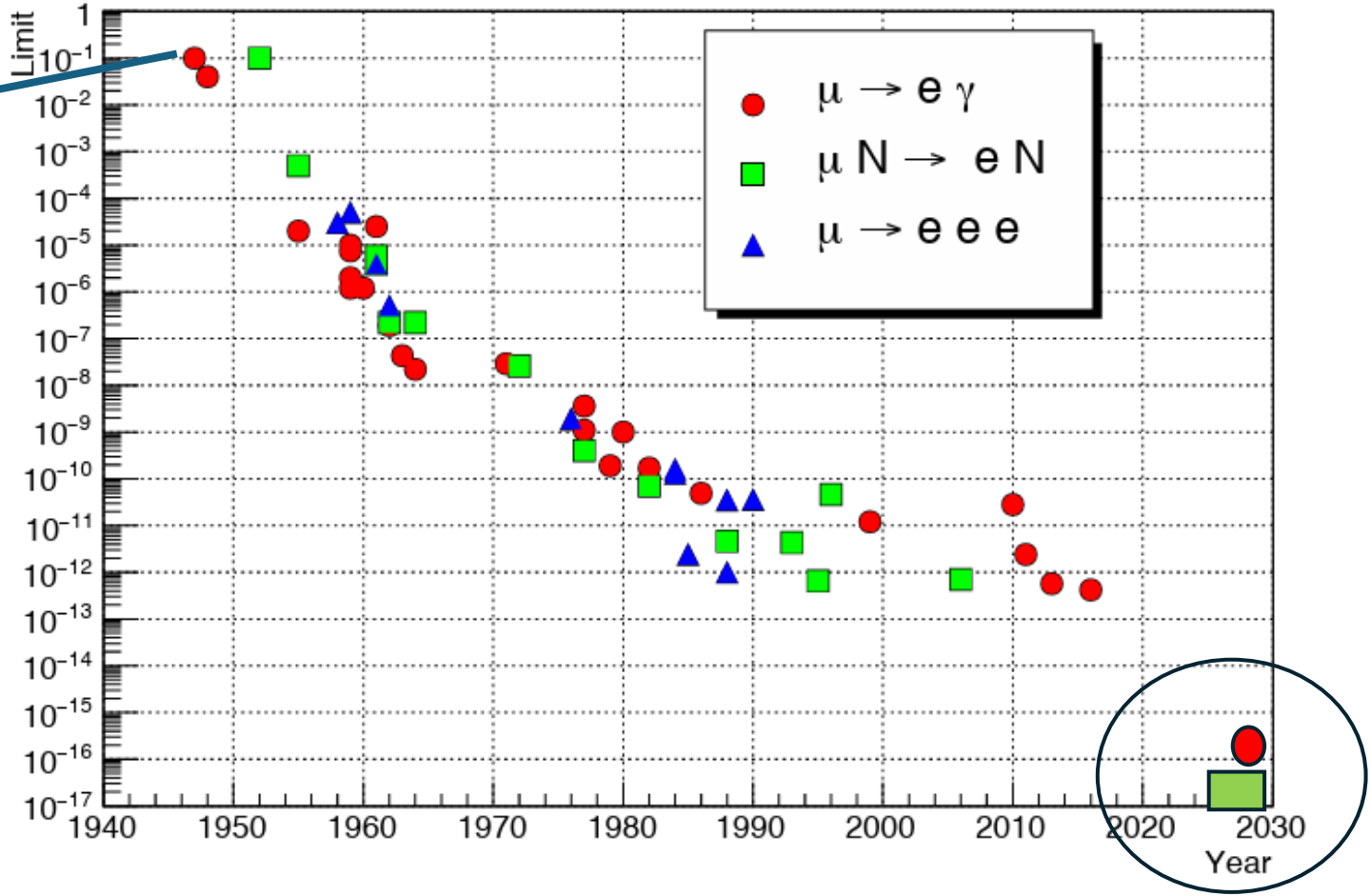
$$\mu \rightarrow e\gamma, \quad \mu + N \rightarrow e + N, \quad \dots$$

History of LFV Searches

Hincks and Pontecorvo (1947)
< 10 %

MEG II

Mu2e (FermiLab)



Quark Lepton Unification

- ▶ Why quarks experience strong force but leptons do not ?

Perhaps symmetry between Quarks and Leptons broken at high energy ?

- ▶ A mathematically elegant framework ? ...

Pati Salam Quark Lepton Unification

- ▶ Lepton number as a Fourth color , $SU(4)_c$

Broken at High Scale
 $SU(4) \rightarrow SU(3)_c \times U(1)_{B-L}$

$$\begin{array}{c}
 \text{SU(4) color} \\
 \xrightarrow{\hspace{1.5cm}} \\
 \left(\begin{array}{cccc}
 u_r & u_g & u_b & \nu \\
 d_r & d_g & d_b & e
 \end{array} \right)_L
 \end{array}
 \quad
 \left(u_r \quad u_g \quad u_b \quad \nu \right)_R$$

- ▶ Right – handed neutinos were first introduced to complete the multiplet.

Minimal Quark Lepton Unification

$$SU(4)_C \times SU(2)_L \times U(1)_R$$

Fermionic Fields

$$F_{qL} \sim \begin{pmatrix} u_r & u_g & u_b & \nu \\ d_r & d_g & d_b & e \end{pmatrix}_L \sim (4, 2, 0)$$

$$F_{uR} = (u_r \quad u_g \quad u_b \quad \nu)_R \sim (4, 1, 1/2)$$

$$F_{dR} = (d_r \quad d_g \quad d_b \quad e)_R \sim (4, 1, -1/2)$$

Higgs Fields

$$\chi = (\chi_r \quad \chi_g \quad \chi_b \quad \chi_0) \sim (4, 1, 1/2)$$

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \sim (1, 2, 1/2)$$

$$\Phi = \begin{pmatrix} \Phi_8 & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2 \sim (15, 2, 1/2)$$

Minimal Quark Lepton Unification

$$SU(4)_C \times SU(2)_L \times U(1)_R$$

Gauge Fields

$$V^\mu = \begin{pmatrix} G^\mu & X^\mu/\sqrt{2} \\ (X^\mu)^*/\sqrt{2} & 0 \end{pmatrix} + T_{15} V_{15}^\mu \sim (\mathbf{15}, \mathbf{1}, 0),$$

- Gives masses to the Vector Leptoquarks X and Z'

$$M_X^2 = \frac{1}{2} g_4^2 v_\chi^2$$

$$\frac{M_X^2}{M_{Z'}^2} = \frac{2}{3} \cos^2 \theta_s$$

v_χ Matter Unification Scale

Symmetry Breaking

$$SU(4)_C \times SU(2)_L \times U(1)_R$$

$$\downarrow \langle \chi_0 \rangle = v_\chi/\sqrt{2}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\downarrow \begin{aligned} \langle H_1 \rangle &= v_1/\sqrt{2} \\ \langle H_2 \rangle &= v_2/\sqrt{2} \end{aligned}$$

$$SU(3)_C \times U(1)_{EM}$$

Fermion Masses

- ▶ The Yukawa interactions for the fermions can be written as

$$-\mathcal{L}_Y = Y_1 \bar{F}_{qL} i\sigma_2 H_1^* F_{uR} + Y_2 \bar{F}_{uR} \Phi^\alpha F_{qL}^\beta \epsilon_{\alpha\beta} + Y_3 \bar{F}_{qL} H_1 F_{dR} + Y_4 \bar{F}_{qL} \Phi F_{dR} + \text{H.c.}$$

- ▶ The mass matrices for the SM fermions reads as

$$M_U = Y_1 \frac{v_1}{\sqrt{2}} - \frac{1}{2\sqrt{6}} Y_2^\dagger \frac{v_2}{\sqrt{2}}$$

$$M_D = Y_3 \frac{v_1}{\sqrt{2}} + \frac{1}{2\sqrt{6}} Y_4^\dagger \frac{v_2}{\sqrt{2}}$$

$$M_E = Y_3 \frac{v_1}{\sqrt{2}} - \frac{3}{2\sqrt{6}} Y_4^\dagger \frac{v_2}{\sqrt{2}}$$

Diagonalized

$$U_L^\dagger M_U U_R = M_U^{diag}$$

$$D_L^\dagger M_D D_R = M_D^{diag}$$

$$E_L^\dagger M_E E_R = M_E^{diag}$$

$$M_\nu^D = Y_1 \frac{v_1}{\sqrt{2}} + \frac{3}{2\sqrt{6}} Y_2^\dagger \frac{v_2}{\sqrt{2}}$$



Extreme fine tuning needed to generate small neutrino mass

Neutrino Masses

- ▶ To generate small neutrino masses naturally at a low scale, one needs to go beyond the Canonical Seesaw.

Canonical Seesaw Scale $\sim 10^{14-15}$ GeV

- ▶ **Inverse Seesaw** : Introducing three singlet left – handed fermionic fields , $S_L \sim (1,1,0)$

$$\mathcal{L} = Y_5 \overline{F_{Q_L}} \chi S_L + \frac{1}{2} \mu S_L^T C S_L + h.c.$$

- ▶ The renormalizable couplings of this model respect a global U(1) fermion matter symmetry.

$$U(1)_F \rightarrow F_{Q_L} = 1 \quad F_u = F_d = -1 \quad S_L = 1$$

$$\mu S_L^T C S_L$$



Softly Breaks the matter symmetry

Natural to consider this μ parameter much smaller.

Neutrino Masses

- ▶ In this case, the mass matrix for the neutrinos in the basis $(\nu \nu^c S)$ can be written as

$$(\nu \nu^c S) \begin{pmatrix} 0 & M_\nu^D & 0 \\ (M_\nu^D)^T & 0 & M_\chi^D \\ 0 & (M_\chi^D)^T & \mu \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ S \end{pmatrix}.$$

$$M_\chi^D = Y_5 v_\chi / \sqrt{2}.$$

$$M_\nu^D = Y_1 \frac{v_1}{\sqrt{2}} + \frac{3}{2\sqrt{6}} Y_2^\dagger \frac{v_2}{\sqrt{2}}$$

- ▶ In the limit $\mu \ll M_\nu^D \ll M_\chi^D$, the light neutrino mass is given by

$$m_\nu \approx \mu (M_\nu^D)^2 / (M_\chi^D)^2$$

- ▶ For $\mu \sim O(10^{-2})$ GeV, $M_\nu^D \sim 10^2$ GeV, then $M_\chi^D \sim 10^3$ TeV $\leftarrow 10^{14-15}$ GeV

↓
Low Scale Unification

Minimal Quark Lepton Unification Predictions

$$SU(4)_C \times SU(2)_L \times U(1)_R$$

- ▶ Majorana Neutrinos
- ▶ One Vector Leptoquark , X
- ▶ Two Scalar Leptoquark doublets , Φ_3 and Φ_4
- ▶ A Color Octet Scalar , Φ_8
- ▶ An Additional Higgs doublet , H_2

Rich Spectrum of New Fields

Flavor Violating Meson Decay

$$K_L \rightarrow \mu^\pm e^\mp$$

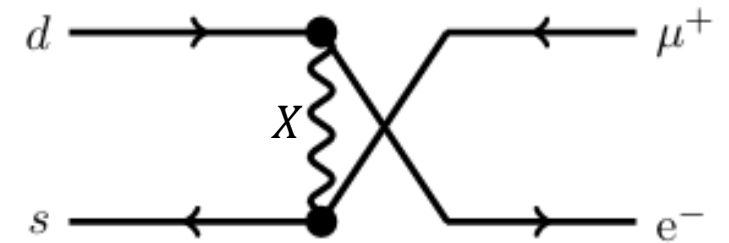
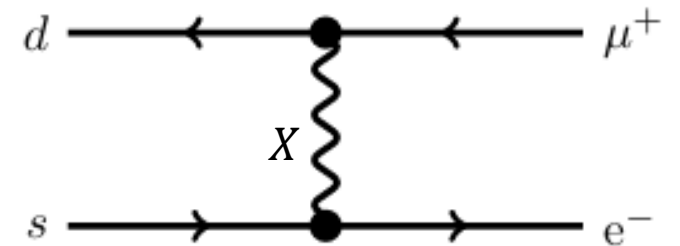
- ▶ Relevant interaction of vector Leptoquarks

$$\bar{d}_i e_j X_\mu: \quad ig_4/\sqrt{2} \gamma^\mu (V_L^{ij} P_L + V_R^{ij} P_R)$$

- ▶ Unknown mixing matrices $V_L = D_L^\dagger E_L$, $V_R = D_R^\dagger E_R$

- ▶ The decay width can be written as

$$\Gamma(K_L^0 \rightarrow \mu^+ e^-) = \frac{F_K^2 m_{K_L}}{64\pi} \left(\left| C_S \frac{m_{K_L}^2}{m_d + m_s} - m_\mu C_V \right|^2 + \left| C_P \frac{m_{K_L}^2}{m_d + m_s} + m_\mu C_A \right|^2 \right) \times \left(1 - \frac{m_\mu^2}{m_{K_L}^2} \right)^2$$



- ▶ Wilson Coefficients C_S, C_P, C_V, C_A are function of the mixing matrices

Flavor Violating Meson Decay

$$K_L \rightarrow \mu^\pm e^\mp$$

► Experimental Limit

$$\text{Br}(K_L \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$$

► In the limit $V_L = V_R \sim \mathbb{1}$, $|C_K| = 1$

$$M_X \geq 2400 \text{ TeV}$$

► Freedom in the mixing matrix V_L and V_R

Symmetry Breaking scale can be $\mathcal{O}(100)$ TeV

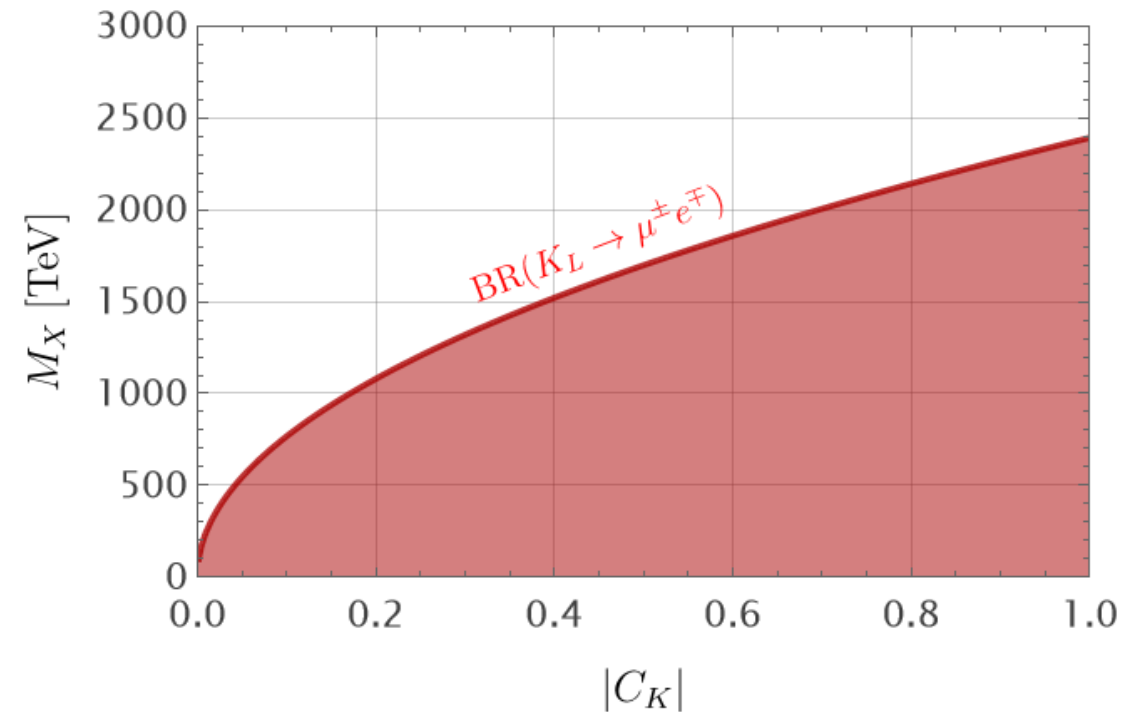


Fig: Red shaded region is excluded

$$C_K = V_{de}^* V_{s\mu} + V_{se}^* V_{d\mu}$$

Flavor Violating Meson Decay

- ▶ General form of V_L and V_R matrix when they are real matrix

$$V_{L,R} = V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix},$$

- ▶ Consider four simple scenerios

Case I : $\theta_{13} = \theta_{23} = 0$

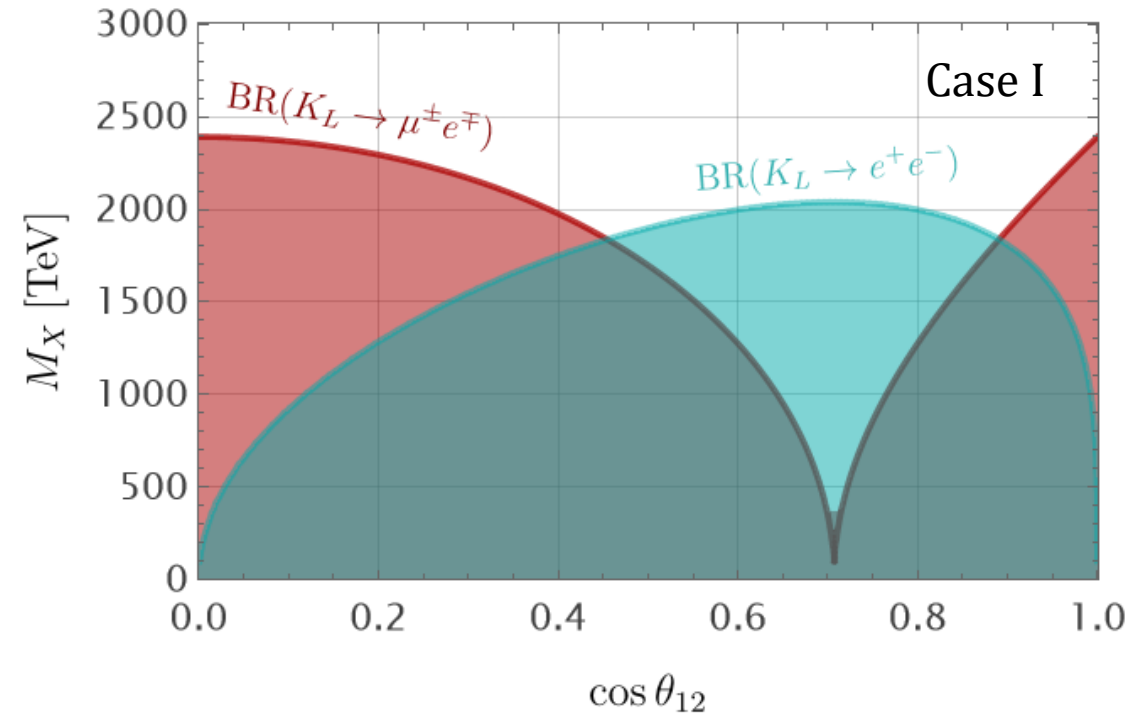
$$M_X > 1800 \text{ TeV}$$

Case II : $\theta_{13} = \pi/2, \theta_{23} = 0$

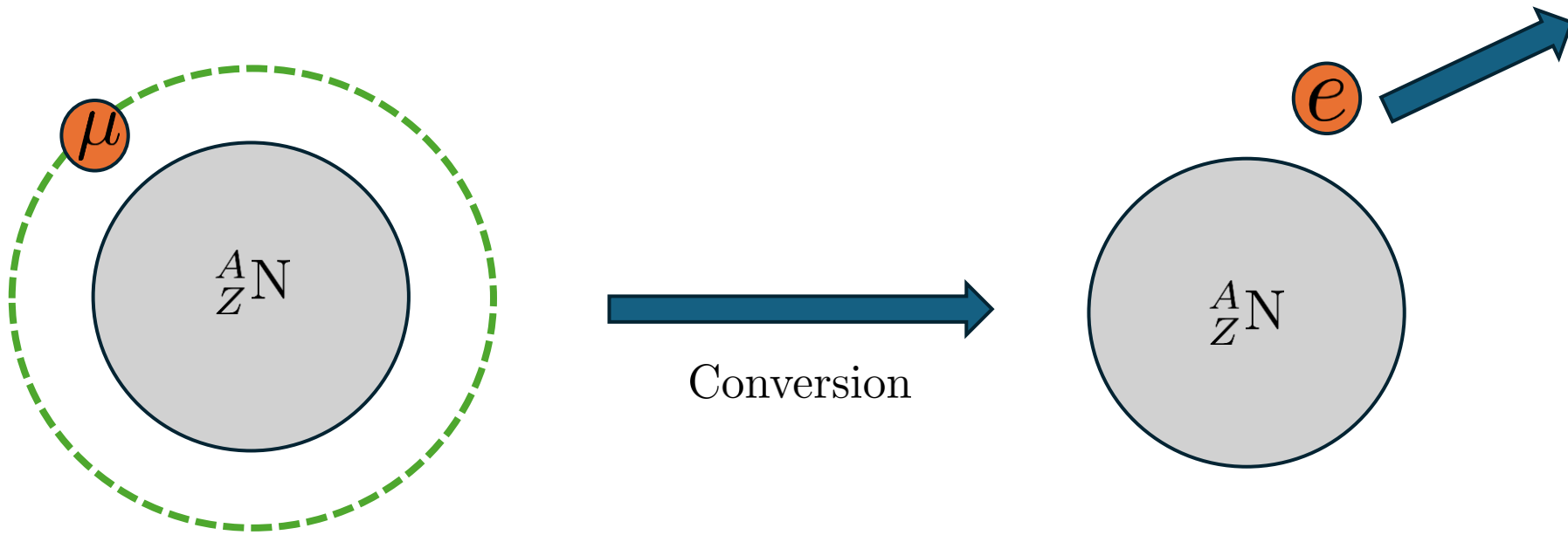
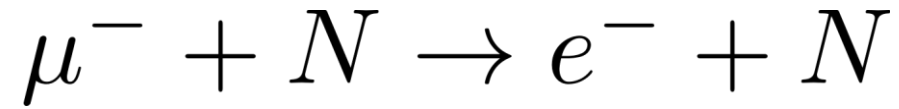
Case III : $\theta_{13} = 0, \theta_{23} = \pi/2$

Case IV : $\theta_{13} = \theta_{23} = \pi/2$

$K_L \rightarrow \mu^\pm e^\mp, e^\mp e^\pm$
Vanishes



LFV Process ($\mu \rightarrow e$ conversion)



- 1) Captured by the nucleus
- 2) Decays to $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$
- 3) Converts into an electron

Fig. Schematic representation of $\mu \rightarrow e$ conversion in Nuclei.

LFV Process ($\mu \rightarrow e$ conversion)

- ▶ The muon conversion rate mediated by the **vector Leptoquark, X** can be written as

$$\Gamma_{\mu \rightarrow e} = \frac{m_{\mu}^5}{16} \left[\left| 4 \left(\tilde{C}_{VR}^{(p)} V^{(p)} + \tilde{C}_{VR}^{(n)} V^{(n)} + \tilde{C}_{SR}^{(p)} S^{(p)} + \tilde{C}_{SR}^{(n)} S^{(n)} \right) \right|^2 + \left| 4 \left(\tilde{C}_{VL}^{(p)} V^{(p)} + \tilde{C}_{VL}^{(n)} V^{(n)} + \tilde{C}_{SL}^{(p)} S^{(p)} + \tilde{C}_{SL}^{(n)} S^{(n)} \right) \right|^2 \right],$$

- ▶ The Branching ratio can be written as

$$\text{BR}_{\mu \rightarrow e}(Z) = \frac{\Gamma_{\mu \rightarrow e}(A, Z)}{\Gamma_{\text{capt}}^{\mu}(A, Z)}$$

- ▶ The Parameters S and V depend on the Nucleus

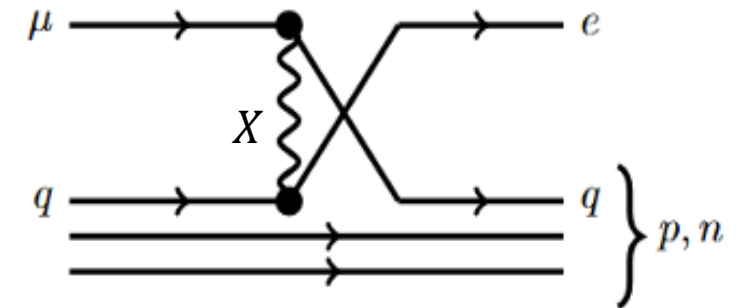
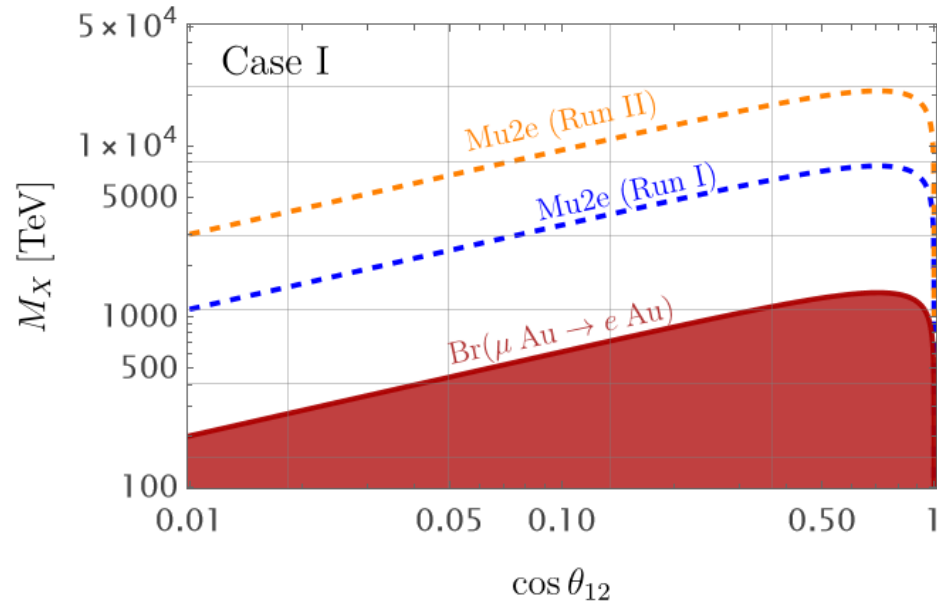


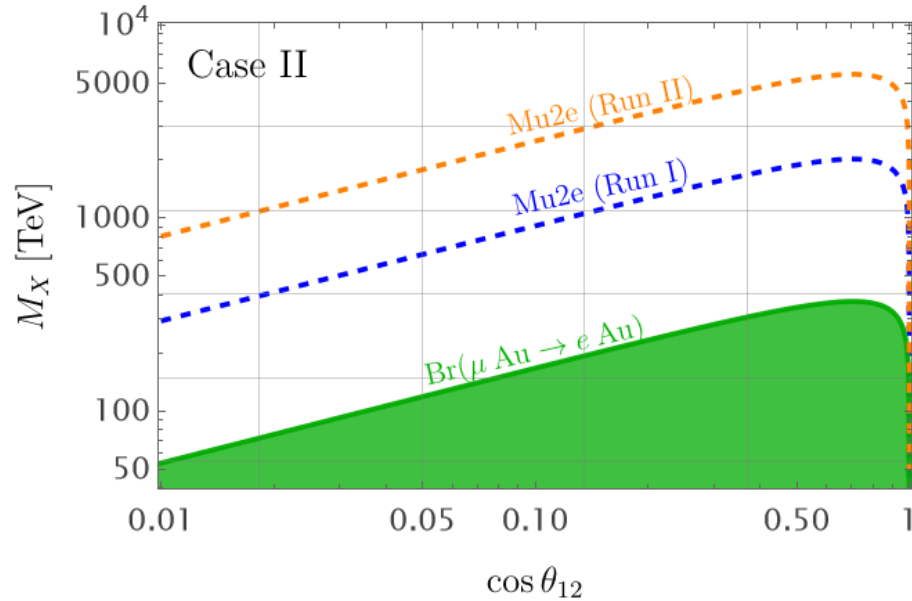
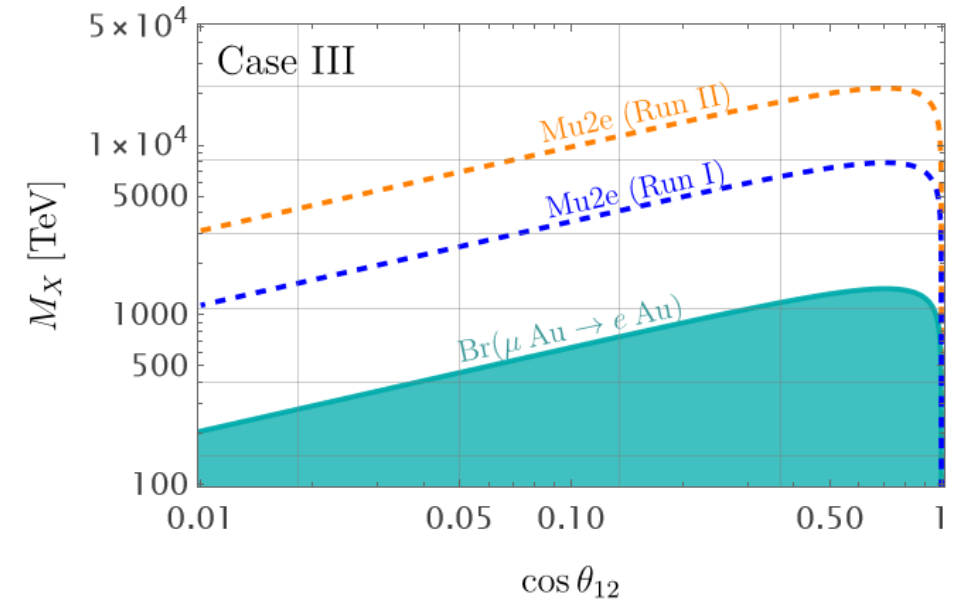
Fig. Feynman graph for $\mu \rightarrow e$ conversion

LFV Process ($\mu \rightarrow e$ conversion)



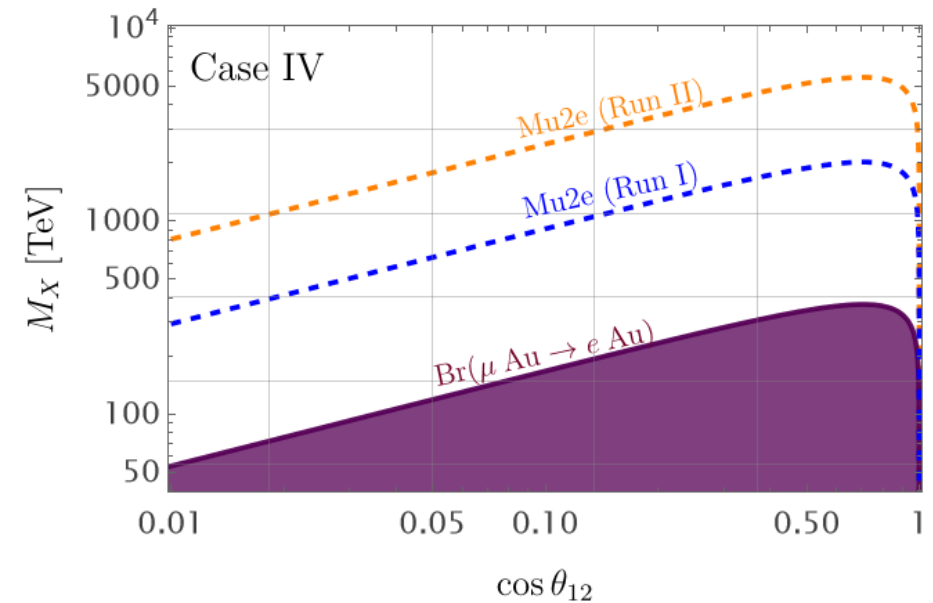
$\mu \rightarrow e$ reach

10^4 TeV



$\mu \rightarrow e$ reach

5×10^3 TeV



LFV Processes and Constraints

- Stronger Constraints on M_X or Symmetry Breaking Scale

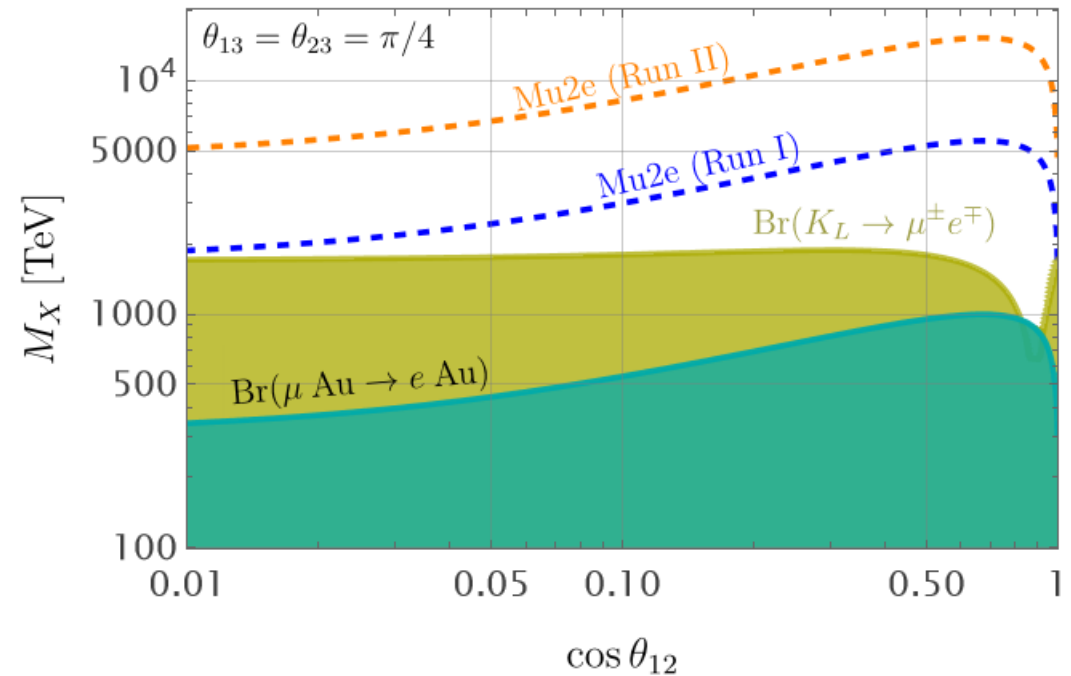
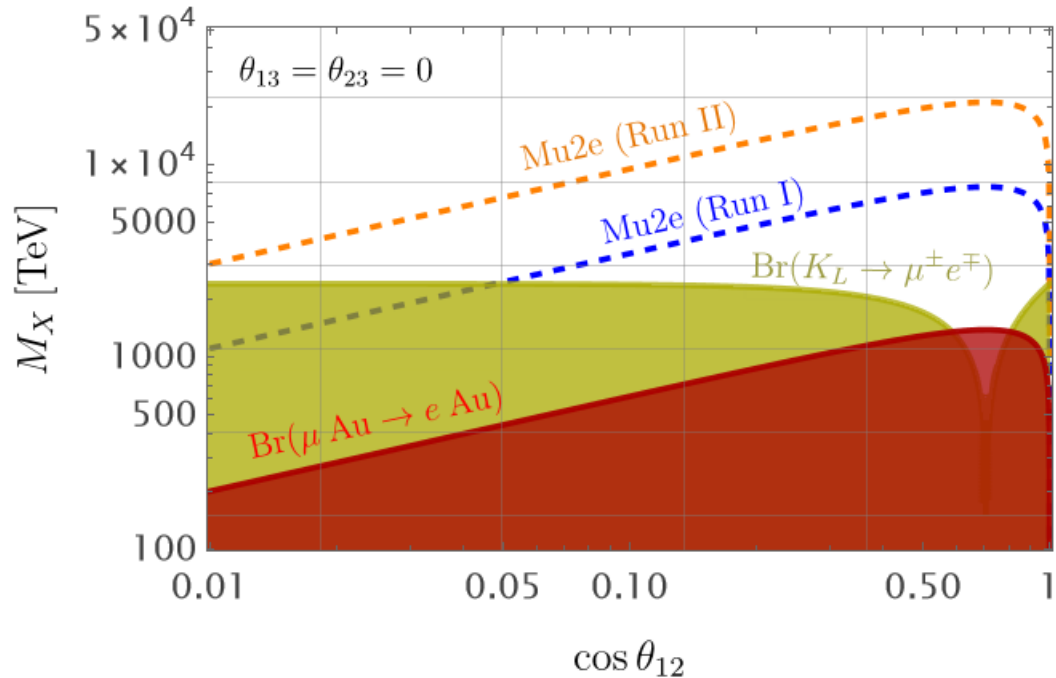


Fig. Limits and Possible reach on Vector Leptoquark, X Mass

Scalar Leptoquarks

Scalar Fields

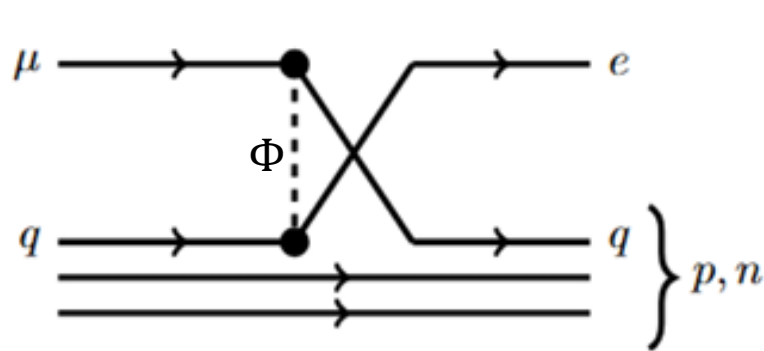
$$\Phi = \begin{pmatrix} \Phi_8 & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2$$

$$\Phi_3 \sim (\bar{3}, 2, -1/6)_{SM} \quad \Phi_4 \sim (3, 2, 7/6)_{SM}$$

$$\Phi_3 = \begin{pmatrix} \Phi_3^{1/3} \\ \Phi_3^{-2/3} \end{pmatrix} \quad \Phi_4 = \begin{pmatrix} \Phi_3^{5/3} \\ \Phi_3^{2/3} \end{pmatrix}$$

- Yukawa interaction for Φ_4

$$-\mathcal{L} \supset Y_2 \bar{u}_R \phi_4^{5/3} e_L - Y_2 \bar{u}_R \phi_4^{2/3} \nu_L + Y_4 \bar{u}_L \phi_4^{5/3} e_R + Y_4 \bar{d}_L \phi_4^{2/3} e_R + \text{H.c.}$$

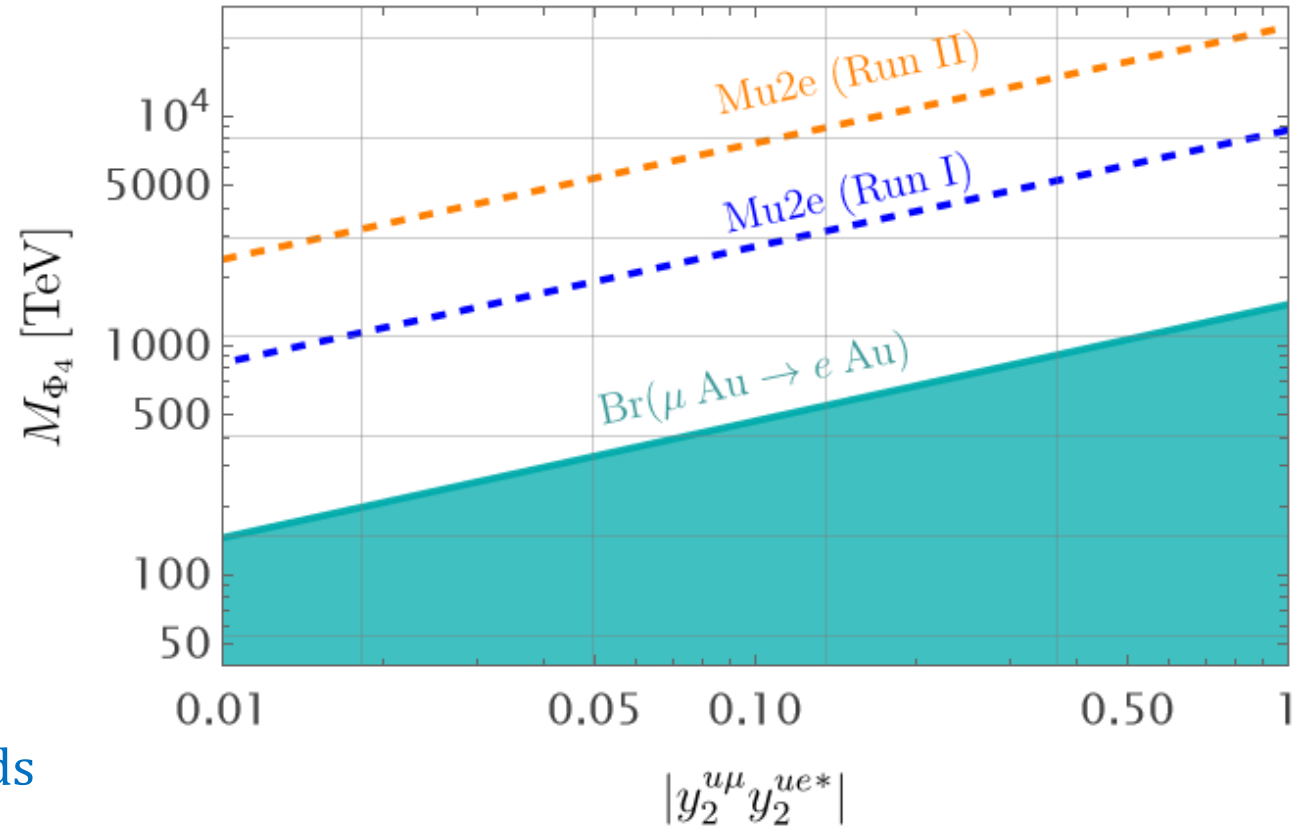


Dominant contribution to $\mu \rightarrow e$
via the Yukawa interaction of Y_2

Fig. Feynman graph for $\mu \rightarrow e$ conversion

LFV Constraints on Scalar Leptoquarks

- ▶ Depending on the Yukawa coupling, $\mu \rightarrow e$ can exclude upto 1000 TeV.
- ▶ Mu2e Experiment can reach upto 10^4 TeV.
- ▶ Much stronger than the collider bounds



Summary

- ▶ We discuss the idea of Quark Lepton unification at low scale .
- ▶ To achieve low scale unification naturally , neutrino masses are generated via inverse seesaw mechanism.
- ▶ We discuss the meson decays and show that symmetry breaking scale can be much lower than 10^3 TeV when the freedom of mixing angles are considered.
- ▶ It is striking that the Mu2e experiment at Fermilab can test the idea of quark-lepton unification at the low scale .
- ▶ **Final takeaway :** Precision Flavor experiments may provide the first window into the unification of matter.

Backup Slides

Backup Slides

- ▶ The SM hypercharge Y is given by $Y = R + \sqrt{6}/3 T_4$
- ▶ $T_4 = \frac{1}{2\sqrt{6}} \text{diag} (1,1,1, -3)$, normalized SU(4) generator

Collider Constraints on Scalar Leptoquarks (Φ_3)

► Collider Signatures

$$p p \rightarrow \phi_3^{1/3} \left(\phi_3^{1/3} \right)^* \rightarrow \bar{b} \nu (b \bar{\nu})$$

$$p p \rightarrow \phi_3^{-2/3} \left(\phi_3^{-2/3} \right)^* \rightarrow \bar{b} \tau (b \bar{\tau})$$

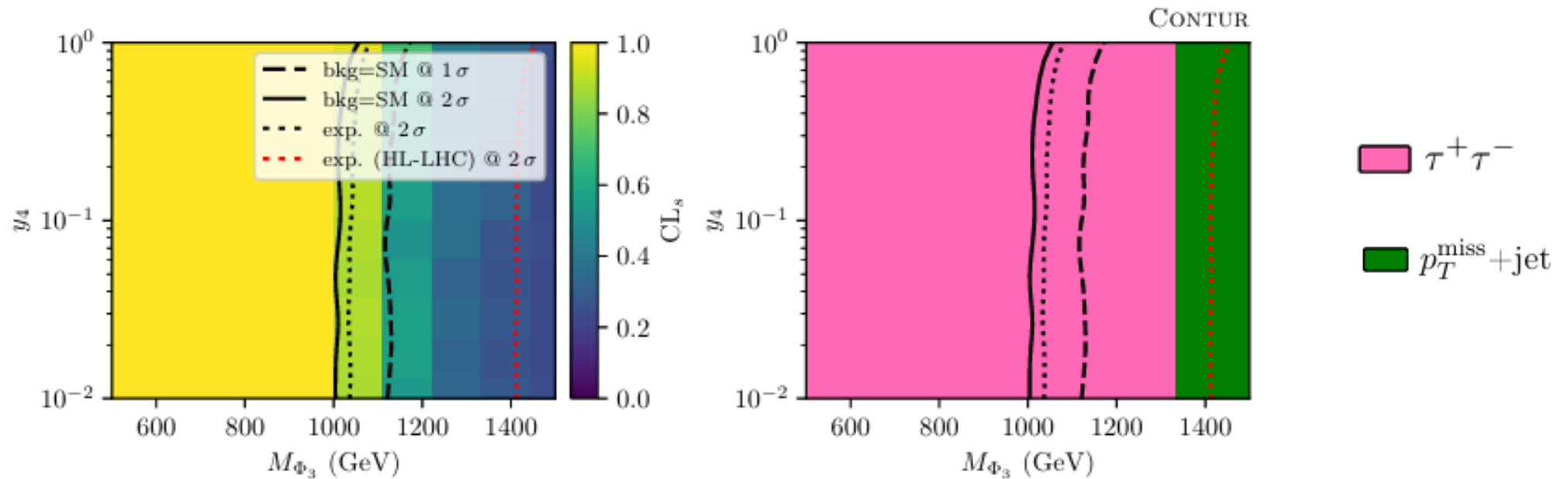


Fig. Exclusion Plot for Φ_3

► Exclusion comes from the di - τ measurements

$$M_{\Phi_3} > 1\text{TeV}$$

Collider Constraints on Scalar Leptoquarks (Φ_4)

► Collider Signatures

$$p p \rightarrow \phi_4^{2/3} \left(\phi_4^{2/3} \right)^* \rightarrow \bar{\tau} b \left(\tau \bar{b} \right) \quad p p \rightarrow \phi_4^{5/3} \left(\phi_4^{5/3} \right)^* \rightarrow \bar{\tau} t \left(\tau \bar{t} \right)$$

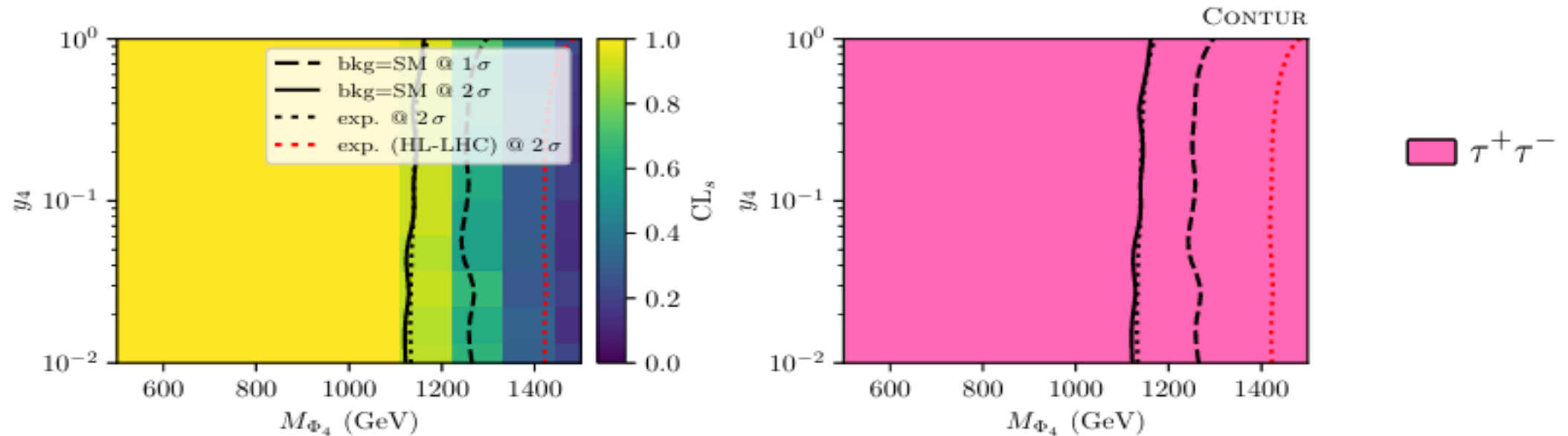


Fig. Exclusion Plot for Φ_4

► Exclusion comes from the di – τ measurements $M_{\Phi_4} > 1.2 \text{ TeV}$

Backgrounds for $\mu \rightarrow e$ conversion

- ▶ Decay in Orbit (DIO) : $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$
- ▶ Radiative π capture : $\pi N \rightarrow \gamma N^*$ and $\gamma \rightarrow e^+ e^-$
- ▶ Radiative muon capture
- ▶ Cosmic Rays
- ▶