

Flavor physics @ the EIC with b-jet tagging

(or: “b-parity” for flavor physics @ the EIC & beyond)

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Based on:

- “Counting inclusive b-jets as an efficient probe of new physics”

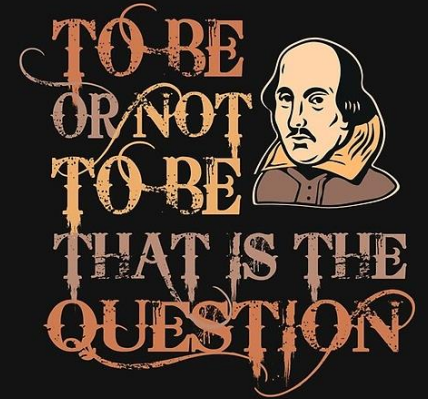
SBS (Technion), J. Wudka (UCR), PRL 2001 (hep-ph/9904365)

- “Flavor physics at the EIC with b-jet tagging”

SBS (Technion), J. Wudka (UCR), PRD 2026 (arxiv: 2601.03345)

+ work in progress

- 2 b or not 2 b ...



Counting **b-jets** as a method for probing new flavor physics of the 3rd gen.

Introducing "**b-parity**": a useful approx. quantum No. for collider experiments

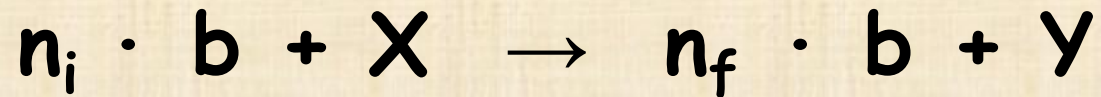
SBS, Wudka, PRL 2001 (hep-ph/9904365)

- Basic motivation:

find a simple (though perhaps not optimal)

PROCESS-INDEPENDENT way of detecting **ANY TYPE** of new flavor physics involving the 3rd gen. quarks

Consider the reaction:



- X , Y = light-quarks/jets, leptons, missing energy ...
- n_i , n_f = number of $(b + \bar{b})$ quark/jets (j_b)

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- X , Y = light-quarks/jets, leptons, missing energy ...
- n_i , n_f = number of $(b + \bar{b})$ quark/jets (j_b)

Key observation: in the SM, to a high accuracy

$$(-1)^{n_i} = (-1)^{n_f}$$

Or: defining **b-Parity** of a process to be:

$$b_p \equiv (-1)^{n_f - n_i} \quad \blacktriangleright \quad b_p \text{ (SM)} = + 1$$

the SM is “mostly” b_p -even ($b_p = + 1$)

(an odd # of b -quarks is rarely generated in a SM process !)

In the limit $V_{3j} , V_{j3} \rightarrow 0$ ($j \neq 3$) the SM acquires an additional global

$U(1)_b$ symmetry (“bottomness”)

which holds to any order in perturbation theory

$$V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \rightarrow (\lambda^2 \rightarrow 0) \rightarrow \begin{pmatrix} 1 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

* b_p -violation $|_{SM} \propto |V_{3j}|^2$ or $|V_{j3}|^2$ ($j \neq 3$)



$$\text{SM} : \frac{b_p = -1 \text{ processes}}{b_p = +1 \text{ processes}} \sim \lambda^4 \sim 10^{-3} \text{ at best !}$$



**SM irreducible backg to b_p -odd NP signals
is strongly suppressed !**

b-parity for collider experiments

The goal:

search for b_p -odd signatures of new flavor physics @ the EIC & FCC-ee

Consider the inclusive multiple *b*-jet production processes:

- **EIC** : $e + p/A \rightarrow n \cdot j_b + X$

$n = \#$ of $(b + \bar{b})$ - jets (j_b)

- **FCC-ee** : $e^+ + e^- \rightarrow n \cdot j_b + X$

b-parity for collider experiments

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Consider the inclusive multiple b-jet production processes:

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For these processes:

$b_p = (-1)^n$ is an approximate symmetry in the SM

(@ EIC: to the extent that the b-quark content in the proton can be neglected ...)

b-parity for collider experiments

The method - counting b-jets:

determine the b_p of the final state by simply counting # of b-jets

- relies primarily on b-tagging & purity of the sample
- does not require :

- The particular structure of the final state
- The identification of any particle BUT the b
- The identification of the charge of the b

b-parity for collider experiments

Leading potential background:

- (1) Purity: SM reducible backg due to light-jet mis-identification as a b-jet, i.e., **non-perfect purity of the sample**
- (2) b-tagging: SM reducible backg due to **non-perfect b-tagging** efficiency
- (3) off-diag CKM: SM irreducible backg due to $|V_{3j}|^2$ or $|V_{j3}|^2$ ($j \neq 3$), i.e., $\propto \lambda^4$

*** turns out that: (1) > (2) >> (3) !



SM is "almost" b_p -even ...

b-parity for collider experiments

Let (e.g., @ EIC):

$$\sigma_{nml} = \sigma(e + p/A \rightarrow n \cdot j_b + m \cdot j_c + l \cdot j_l + X)$$

X = leptons, photons &/or missing energy ...

Then, the probability (CSX) for detecting **precisely k b-jets** :

$$\overline{\sigma}_k = \sum_{u+v+w=k} P_n^u P_m^v P_l^w [\epsilon_b^u (1 - \epsilon_b)^{n-u}] [t_c^v (1 - t_c)^{m-v}] [t_j^w (1 - t_j)^{l-w}] \cdot \sigma_{nml} \cdot \delta_{u+v+w,k}$$

$$P_j^i \equiv \frac{i!}{j! (i-j)!}$$

New Physics: model independent approach

EFT prescription:

$$L = L_{SM} + \sum_{n=5}^{\infty} \frac{1}{\Lambda^{n-4}} \sum_i \alpha_i Q_i^{(n)}$$

Relevant NP parameter (dim.6): $\Lambda_{eff} = \frac{\Lambda}{\sqrt{\alpha}}$

e.g., $(\bar{L}R)(\bar{L}R)$ tensor operator: $Q_{lequ}^{(3)} = (\bar{\ell}^j \sigma_{\mu\nu} e_R) \varepsilon_{jk} (\bar{q}^k \sigma_{\mu\nu} u_R)$

more types of NP considered in: [SBS, J. Wudka, PRD 2026 \(arxiv: 2601.03345\)](#)

effective operator	Chirality (type)	Interactions
$Q_{\ell q}^{(3)}(1113) = (\bar{\ell}_1 \gamma_\mu \tau^I \ell_1) (\bar{q}_1 \gamma^\mu \tau^I q_3)$	$(\bar{L}L)(\bar{L}L)$ (vector)	$eetu, eebd, ev_e bu$
$Q_{lequ}^{(1)}(1131) = (\bar{\ell}_1^j e) \varepsilon_{jk} (\bar{q}_3^k u)$	$(\bar{L}R)(\bar{L}R)$ (scalar, tensor)	$eetu, ev_e bu$
$Q_{lequ}^{(3)}(1131) = (\bar{\ell}_1^j \sigma_{\mu\nu} e) \varepsilon_{jk} (\bar{q}_3^k \sigma_{\mu\nu} u)$	$(\bar{L}R)(\bar{L}R)$ (scalar, tensor)	$eetu, ev_e bu$
$Q_{\ell eq}(1131) = (\bar{\ell}_1^j e) (\bar{b} q_1^j)$	$(\bar{L}R)(\bar{R}L)$ (scalar, vector)	$eebd, ev_e bu$
$Q_{Hud}(13) = i (\tilde{H}^\dagger D_\mu H) (\bar{u} \gamma^\mu b) + \text{h.c.}$	$(\bar{R}R)$ (vector, fermion)	Wub
$Q_{Hq}^{(3)}(13) = i (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_1 \gamma^\mu \tau^I q_3)$	$(\bar{L}L)$ (vector, fermion)	Wub

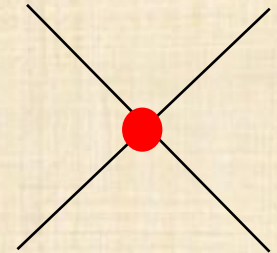
New Physics: model independent approach

$$Q_{lequ}^{(3)} = (\bar{\ell}^j \sigma_{\mu\nu} e_R) \varepsilon_{jk} (\bar{q}^k \sigma_{\mu\nu} u_R)$$

EIC , $E_{CM} \approx 140 \text{ GeV}$

$$(\bar{\nu}_{eL} \sigma_{\mu\nu} e_R) (\bar{b}_L \sigma_{\mu\nu} u_R) \quad (evbu - \text{tensor})$$

$$e u \rightarrow b \nu_e$$



SU(2)

$$e e \rightarrow t u$$

$$(\bar{e}_L \sigma_{\mu\nu} e_R) (\bar{t}_L \sigma_{\mu\nu} u_R) \quad (eetu - \text{tensor})$$

FCC-ee , $E_{CM} \approx 240 \text{ GeV}$

$$Q_{lequ}^{(3)} (1131)$$

$EIC : E_{CM} \sim 140 \text{ GeV}$

$(E_p=275 \text{ GeV} , E_e=18 \text{ GeV})$

$L=100 \text{ fb}^{-1}$

SBS, J. Wudka, PRD 2026 (arxiv: 2601.03345)

$$EIC : E_{CM} \sim 140 \text{ GeV}$$

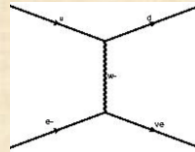
$$(E_p = 275 \text{ GeV}, E_e = 18 \text{ GeV})$$

CC single-jet production:

$$SM: e + p/A \rightarrow j_\ell + MET$$

light-jet

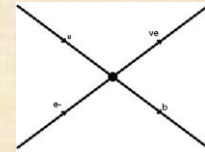
$$\sigma_{SM} \sim 13 \text{ pb} \quad (90\% \text{ from } e u \rightarrow d \nu_e)$$



$$NP: e + p/A \rightarrow j_b + MET$$

b-jet

$$\sigma_{NP} \sim 0.2 \text{ pb} \quad (\text{from } e u \rightarrow b \nu_e)$$



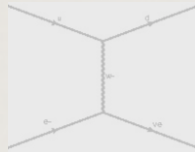
$$\bar{\sigma}_{1j_b}(\epsilon_b, t_j, \Lambda_{eff}) = t_j \cdot \sigma_{SM} + \epsilon_b \cdot \sigma_{NP}(\Lambda_{eff})$$

EIC : $E_{CM} \sim 140 \text{ GeV}$ ($E_p=275 \text{ GeV}$, $E_e=18 \text{ GeV}$)

CC single-jet production:

SM: $e + p/A \rightarrow j_\ell + \text{MET}$
light-jet

$\sigma_{SM} \sim 13 \text{ pb}$ (90% from $e u \rightarrow d \nu_e$)



NP: $e + p/A \rightarrow j_b + \text{MET}$
b-jet

$\sigma_{NP} \sim 0.2 \text{ pb}$ (from $e u \rightarrow b \nu_e$)



$$\bar{\sigma}_{1j_b}(\epsilon_b, t_j, \Lambda_{eff}) = t_j \cdot \sigma_{SM} + \epsilon_b \cdot \sigma_{NP}(\Lambda_{eff})$$

NOTE:

- NC single-jet ($e + p/A \rightarrow j + e$) won't work: SM-backg too large ...
- di-jet CC production ($e + p/A \rightarrow 2j + \text{MET}$) less sensitive to NP
- no single top-quark production ($E_{CM} < m_t$) ...
- SM background processes yielding a charm-jet are negligible

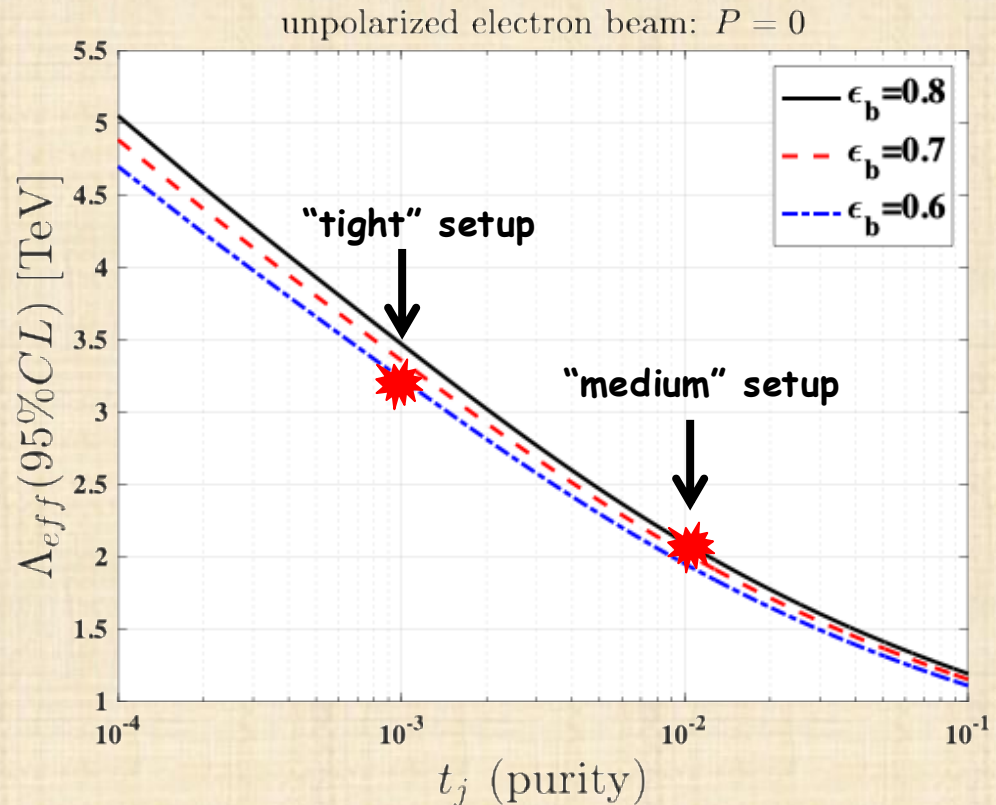
95% CL bound on the scale of NP Λ_{eff} :

$$Q_{lequ}^{(3)} = (\bar{\ell}^j \sigma_{\mu\nu} e_R) \epsilon_{jk} (\bar{q}^k \sigma_{\mu\nu} u_R)$$

$$\text{NP: } e + p/A \rightarrow j_b + \text{MET}$$

Purity dominates the BG

analysis benefits more from
purity rather than from
higher b-tagging efficiency



$$\Lambda_{eff}(95\%CL) > 3.2 \text{ TeV ("tight")}$$

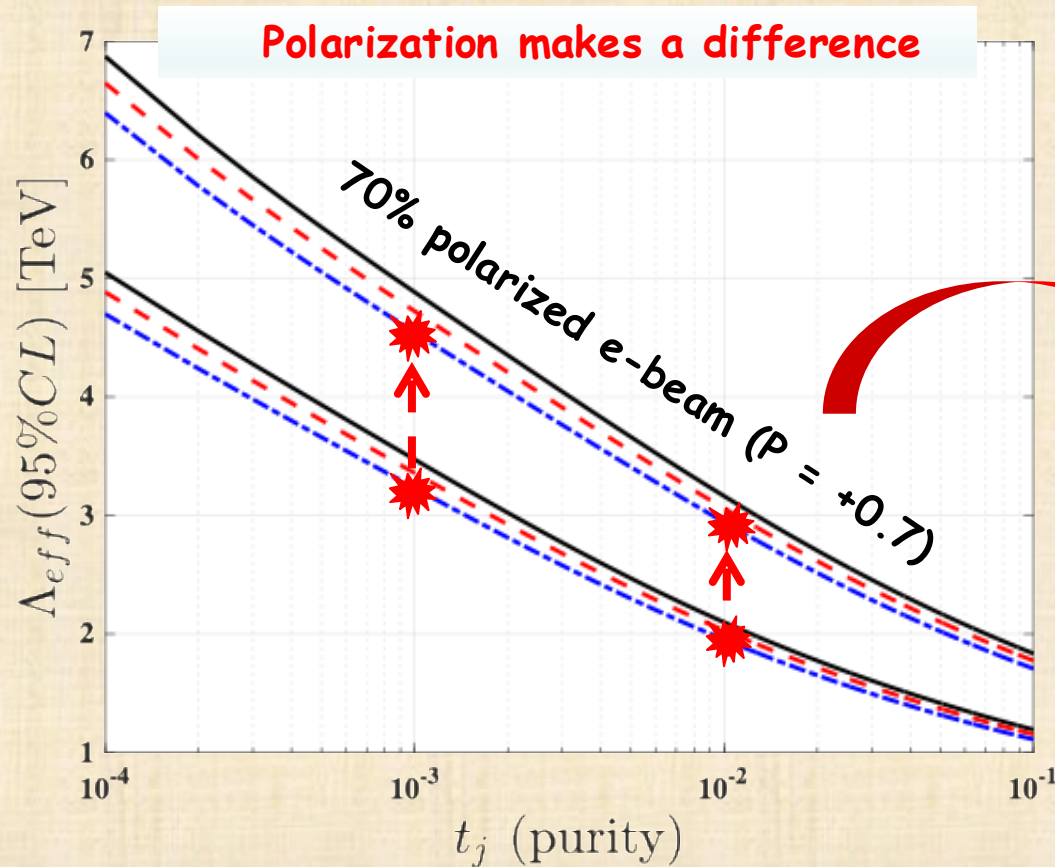
$$\Lambda_{eff}(95\%CL) > 2 \text{ TeV ("medium")}$$

Polarization of electron beam :

$$\sigma(P_e) = \frac{1}{2} [(1 - P_e) \cdot \sigma_- + (1 + P_e) \cdot \sigma_+]$$

$$\sigma_- = \sigma(P_e = -1)$$

$$\sigma_+ = \sigma(P_e = +1)$$



"tight" setup with 70% pol: $\Lambda_{eff}(95\%CL) > 30 \times E_{CM}(EIC)$

FCC-ee: $E_{CM} \sim 240 \text{ GeV}$

($E_{e^+,e^-} = 120 \text{ GeV}$)

Work in progress ...

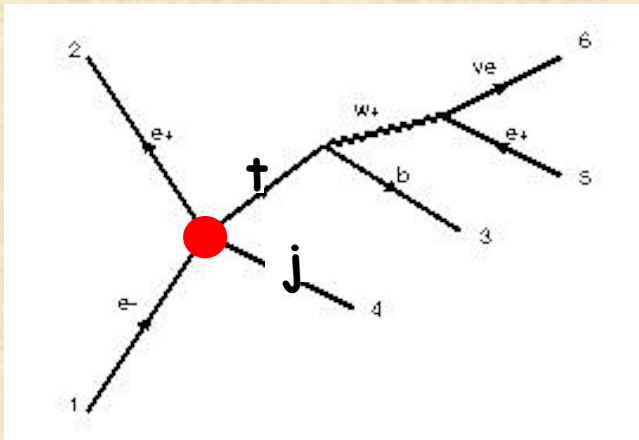
Example:

2-jets + charged lepton + MET:

NP(single-top prod):

$$e^+ e^- \rightarrow t j \rightarrow j_b + j + \ell + \text{MET}$$

($t \rightarrow bW$; $W \rightarrow \ell \nu_\ell$)

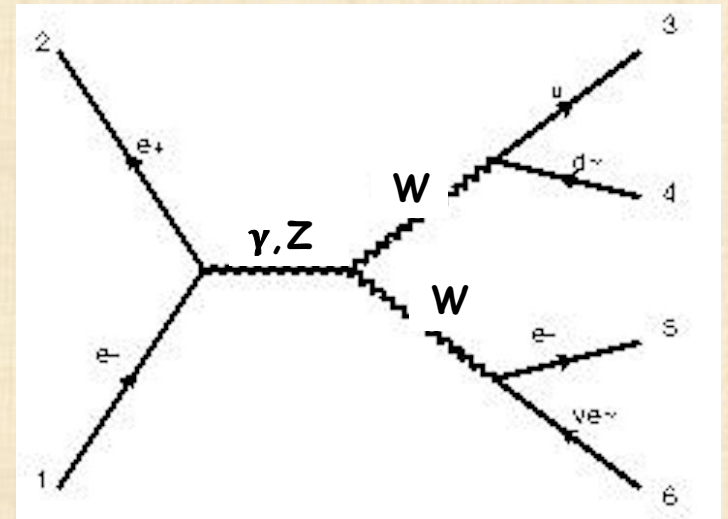


$$(\bar{e}_L \sigma_{\mu\nu} e_R)(\bar{t}_L \sigma_{\mu\nu} u_R): eetu - \text{tensor}$$

SM:

$$e^+ e^- \rightarrow 2 j + \ell + \text{MET}$$

e.g., from $ee \rightarrow WW$; $W \rightarrow u d / \ell \nu_\ell$

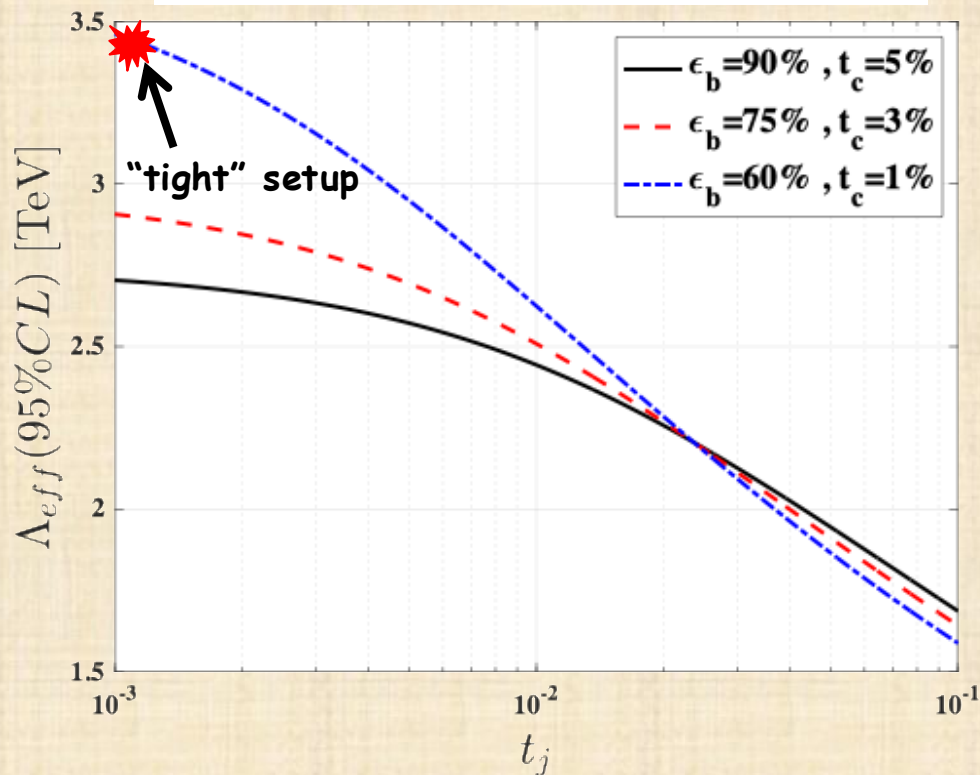


95% CL bound on the scale of NP :

$$Q_{lequ}^{(3)} = (\bar{\ell}^j \sigma_{\mu\nu} e_R) \varepsilon_{jk} (\bar{q}^k \sigma_{\mu\nu} u_R)$$



$$(\bar{e}_L \sigma_{\mu\nu} e_R) (\bar{t}_L \sigma_{\mu\nu} u_R): \text{eetu} - \text{tensor}$$



- Charm mis-identification makes a difference
- analysis benefits more from purity rather than from higher b-tagging efficiency

$$\Lambda_{eff}(95\%CL) > 3.5 \text{ TeV ("tight")}$$

Sensitivity with kinematics:

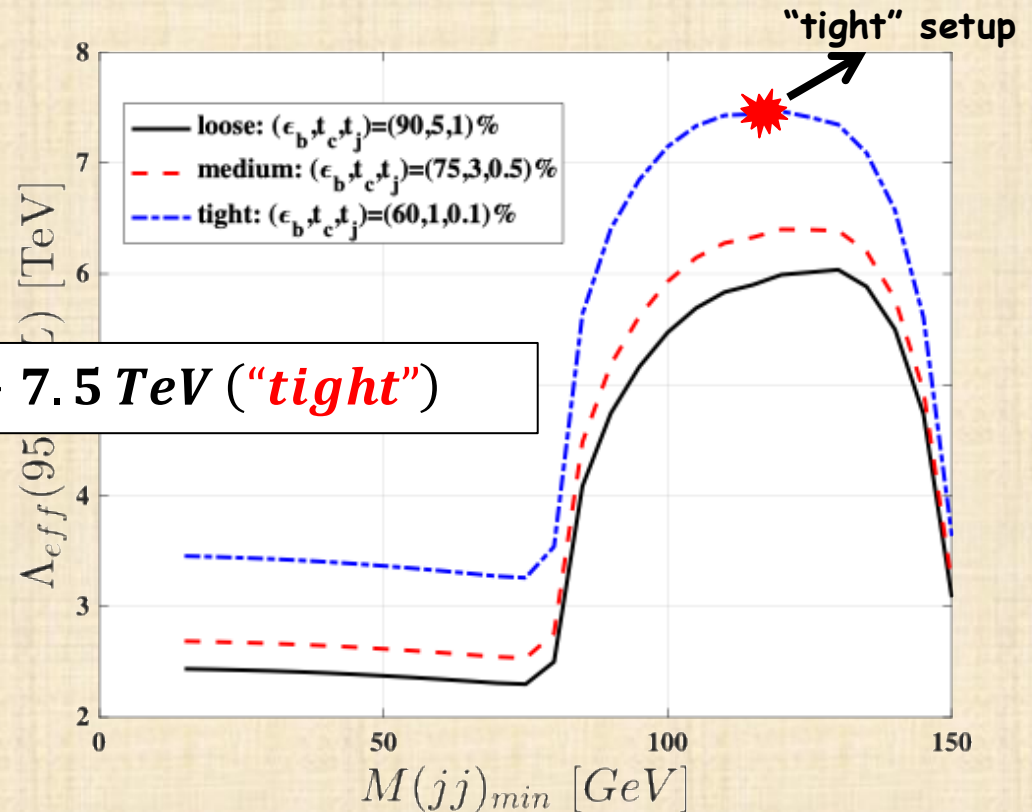
$$Q_{lequ}^{(3)} = (\bar{\ell}^j \sigma_{\mu\nu} e_R) \varepsilon_{jk} (\bar{q}^k \sigma_{\mu\nu} u_R)$$



$$(\bar{e}_L \sigma_{\mu\nu} e_R) (\bar{t}_L \sigma_{\mu\nu} u_R): \text{eetu} - \text{tensor}$$

$M(jj)_{\min}$ cut in:

$e^+ e^- \rightarrow 2 j + \ell + \text{MET}$



$\Lambda_{eff}(95\%CL) > 7.5 \text{ TeV}$ ("tight")

"tight" setup with kinematics: $\Lambda_{eff}(95\%CL) > 30 \times E_{CM}(ee)$!



- **b-parity (b_p):** counting b-jets as a simple and sensitive probe of new flavor physics in 3rd gen. quark sector

look for a final state with an odd # of b-jets ...

- **process independent:** does not necessarily require the particular structure and identification of final state
- **model independent:** b_p will "capture" ANY type of new flavor physics that generates 3rd \rightarrow 2nd & 3rd \rightarrow 1st gen transitions
- **Relies on:**
 - Purity of the b-jets sample
 - negligible b-quark content in the initial state - problematic for LHC ...



- possible applications for future searches:

- single jet events @ EIC
- 2,4,6 jets events @ FCC-ee

- Expected sensitivity for such searches:

$$\Lambda_{eff}(95\%CL) > 30 \times E_{CM}(collider) !$$

Thank you!

Backups

Observability:

define $\xrightarrow{\text{Luminosity}}$ acceptance

$$\bullet N_k = L \cdot A \cdot \overline{\sigma}_k \Rightarrow \# \text{ of events with } k \text{ b-jets}$$

uncertainties:

- statistical: $\Delta_{stat} = \sqrt{N_k}$
- systematic: $\Delta_{sys} = N_k \cdot \delta_s$
- theory: $\Delta_{the} = N_k \cdot \delta_t$

e.g.,

$$A = 80\%, \delta_s = 2\%, \delta_t = 2\%$$

$$L = 100 \text{ fb}^{-1} \text{ (EIC)}, L = 2000 \text{ fb}^{-1} \text{ (FCC-ee)}$$

• Total uncertainty:

$$\Delta = \sqrt{\Delta_{stat}^2 + \Delta_{sys}^2 + \Delta_{the}^2}$$

then, significance of the signal ($N_{SD} = \# \text{ of SD}$):

$$\left| N_k - N_k^{(SM)} \right| > N_{SD} \cdot \Delta$$

"Proof":

SM's FC transitions (generation transitions) are due to charged current interactions \propto off diagonal CKM's:

$$V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \rightarrow (\lambda^2 \rightarrow 0) \rightarrow \begin{pmatrix} 1 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In the limit $\lambda^2 \rightarrow 0$ ($V_{3j} = V_{j3} \rightarrow 0$, $j \neq 3$) 3rd gen. do not mix with others



Conservation of 3rd family fermion number

In addition:

Fast top decay + $\text{BR}(t \rightarrow bW) \sim 1$

$(\# \text{ of } t) = (\# \text{ of } b)$

$t \text{ "=" } b \text{ for this purpose}$



**The experimentally approximate conserved flavor number
is carried only by the b-quarks !**



In the limit $V_{3j}, V_{j3} \rightarrow 0$ ($j \neq 3$) the SM acquires an additional global

$U(1)_b$ symmetry ("bottomness")

which holds to any order in perturbation theory

b-parity for collider experiments

The strategy:

define

- ϵ_b = b-tagging efficiency
- t_c = probability of mis-identifying a c-jet for a b-jet
- t_j = probability of mis-identifying a light-quark (or gluon) jet for a b-jet

recall: backg $\in \epsilon_b < 1$ & $t_c, t_j \neq 0$ (impurity of the sample)

Let:

$$\sigma_{nml} = \sigma(e^+e^- \text{ or } e + p/A \rightarrow n \cdot j_b + m \cdot j_c + l \cdot j_l + X)$$

FCC-ee.

EIC

X = leptons, photons &/or missing energy ...

Then, the probability (CSX) for detecting **precisely k b-jets** :

$$\overline{\sigma}_k = \sum_{u+v+w=k} P_n^u P_m^v P_l^w [\epsilon_b^u (1 - \epsilon_b)^{n-u}] [t_c^v (1 - t_c)^{m-v}] [t_j^w (1 - t_j)^{l-w}] \cdot \sigma_{nml} \cdot \delta_{u+v+w,k}$$

$$P_j^i \equiv \frac{i!}{j! (i-j)!}$$

Bounds ...

Effective operator	Chirality (type)	Interactions
$\mathcal{Q}_{\ell q}^{(3)}(1113) = (\bar{\ell}_1 \gamma_\mu \tau^I \ell_1)(\bar{q}_1 \gamma^\mu \tau^I q_3)$	$(\bar{L}L)(\bar{L}L)$ (vector)	$eet_u, eebd, ev_e bu$
$\mathcal{Q}_{\ell equ}^{(1)}(1131) = (\bar{\ell}_1^j e) \epsilon_{jk} (\bar{q}_3^k u)$	$(\bar{L}R)(\bar{L}R)$ (scalar, tensor)	$eet_u, ev_e bu$
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$\mathcal{Q}_{\ell edq}(1131) = (\bar{\ell}_1^j e) (\bar{b} q_1^j)$	$(\bar{L}R)(\bar{R}L)$ (scalar, vector)	$eebd, ev_e bu$
$\mathcal{Q}_{Hud}(13) = i(\bar{H}^\dagger D_\mu H)(\bar{u} \gamma^\mu b) + \text{H.c.}$	$(\bar{R}R)$ (vector, fermion)	Wub
$\mathcal{Q}_{Hq}^{(3)}(13) = i(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_1 \gamma^\mu \tau^I q_3)$	$(\bar{L}L)$ (vector, fermion)	Wub

Drell-Yan processes @ LHC:

$$b u \rightarrow e \nu_e \text{ \& \ } b d \rightarrow e e \Rightarrow \Lambda_{\text{eff}} \gtrsim 3\text{-}7 \text{ TeV}$$

(depending on opt)

B decays:

$$b \rightarrow u e \nu_e \text{ \& \ } b \rightarrow d e e \Rightarrow \Lambda_{\text{eff}} \gtrsim O(10 \text{ TeV})$$

Single top prod. @ LEP2:

$$e e \rightarrow t u \Rightarrow \Lambda_{\text{eff}} \gtrsim O(1 \text{ TeV})$$

tt prod. @ LHC followed by FC top decay:

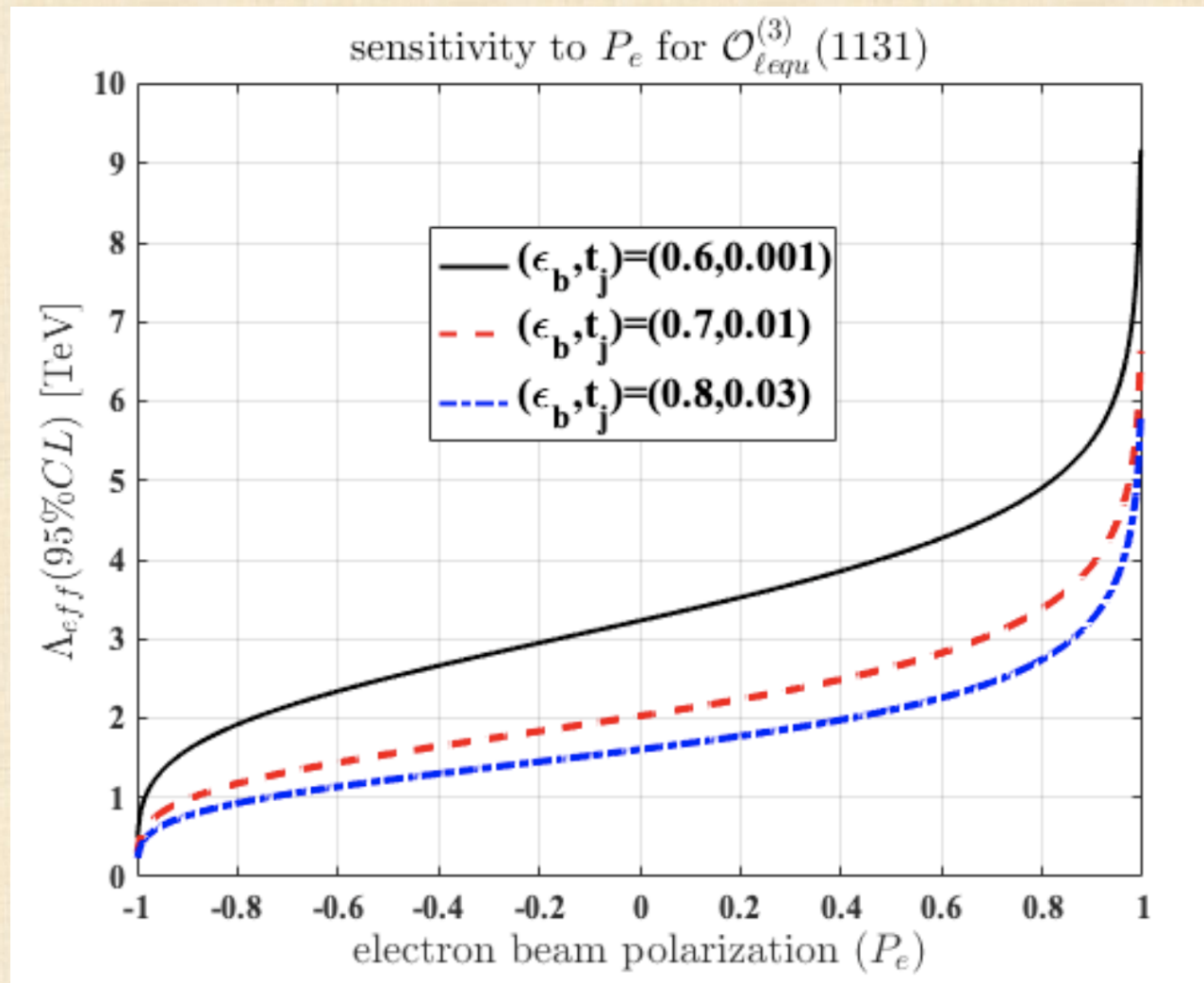
$$p p \rightarrow t t; t \rightarrow u e e \Rightarrow \Lambda_{\text{eff}} \gtrsim O(1 \text{ TeV})$$

Sensitivity to other opt. @ EIC

TABLE III. The expected 95% CL bounds on the effective scale $\Lambda_{\text{eff}} = \Lambda/\sqrt{\alpha}$ for the operators in Table I. Numbers are given for the three b -tagging efficiency setups: loose, medium, and tight and for unpolarized and +70% [$P_e = +0.7$, see Eq. (14)] polarized incoming electron beam. See also text.

95% CL bounds on $\Lambda_{\text{eff}}(Q)$ (TeV) with electron beam polarization $P_e = 0(+0.7)$			
Q	Tight setup $(\epsilon_b, t_j) = (0.6, 0.001)$	Medium setup $(\epsilon_b, t_j) = (0.7, 0.01)$	Loose setup $(\epsilon_b, t_j) = (0.8, 0.03)$
$Q_{\ell q}^{(3)}(1113)$	2.6 (2.4)	1.7 (1.6)	1.3 (1.3)
$Q_{\ell \text{equ}}^{(1)}(1131)$	1.0 (1.4)	0.6 (0.9)	0.5 (0.8)
$Q_{\ell \text{equ}}^{(3)}(1131)$	3.2 (4.6)	2.0 (3.1)	1.6 (2.5)
$Q_{\ell \text{edq}}^{(1)}(1131)$	1.0 (1.4)	0.6 (0.9)	0.5 (0.8)
$Q_{\text{Hud}}(13)$	1.3 (1.2)	0.8 (0.7)	0.6 (0.6)
$Q_{\text{Hq}}^{(3)}(13)$	2.3 (2.1)	1.5 (1.4)	1.2 (1.1)

Effect of pol. @ EIC



quite remarkable (although not realistic) that with $P_e = 100\%$ $\Lambda_{eff}(95\% CL) \rightarrow 10$ TeV

New flavor physics in top sector @ FCC-ee

TABLE III: The potential multi-jets signatures (final states) which result from the dim.6 FC 4-fermi effective operators of Table II, corresponding to the different final states $e^+e^- \rightarrow tj$, $tj + V$, where $V = \gamma, Z, W$.

	$t \rightarrow bW(\rightarrow jj)$	$t \rightarrow bW(\rightarrow \ell\nu)$
$e^+e^- \rightarrow tj$	$4j$ (1b)	$2j + \ell + \cancel{E}_T$ (1b)
$e^+e^- \rightarrow tj + \gamma$	$4j + \gamma$ (1b)	$2j + \gamma + \ell + \cancel{E}_T$ (1b)
$e^+e^- \rightarrow tj + Z(\rightarrow q\bar{q}, q \neq b)$	$6j$ (1b)	$4j + \ell + \cancel{E}_T$ (1b)
$e^+e^- \rightarrow tj + Z(\rightarrow b\bar{b})$	$6j$ (3b)	$4j + \ell + \cancel{E}_T$ (3b)
$e^+e^- \rightarrow tj + Z(\rightarrow \ell^+\ell^-)$	$4j + 2\ell$ (1b)	$2j + 3\ell + \cancel{E}_T$ (1b)
$e^+e^- \rightarrow tj + Z(\rightarrow \nu\bar{\nu})$	$4j + \cancel{E}_T$ (1b)	$2j + \ell + \cancel{E}_T$ (1b)
$e^+e^- \rightarrow tj + W(\rightarrow q\bar{q}', q, q' \neq b)$	$6j$ (1b)	$4j + \ell + \cancel{E}_T$ (1b)
$e^+e^- \rightarrow tj + W(\rightarrow \ell^-\bar{\nu})$	$4j + \ell + \cancel{E}_T$ (1b)	$2j + 2\ell + \cancel{E}_T$ (1b)

We will analyze below the effects of the operators $Q_{\ell e q u}^{(1,3)}$, which contribute to all the channels considered in Table III...

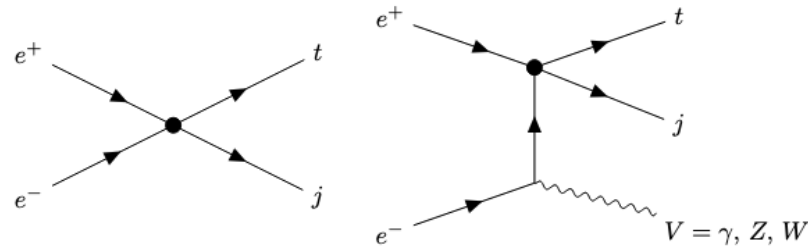


FIG. 1: Representative Feynman diagrams for $e^+e^- \rightarrow tj$ and $e^+e^- \rightarrow tj + V$, where $V = \gamma, Z$ or W .