Echoes from global cosmic strings

2604.15241

with Jeff Dror

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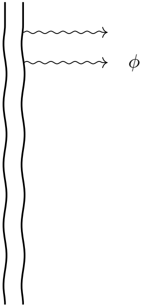
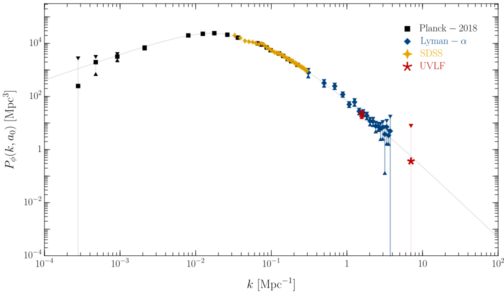
Motivation



Dark Matter

Sub-component



Emission

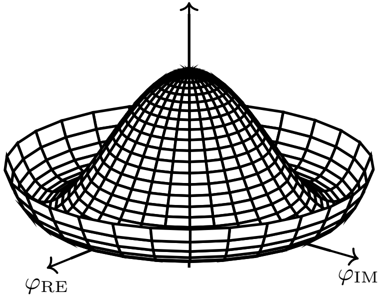
Non-relativistic evolution

Observations

Formation of strings

Spontaneous symmetry breaking

- U(1) global symmetry and a complex scalar $\varphi(x) = \frac{\rho(x)}{\sqrt{2}} e^{i\phi/f_a}$.



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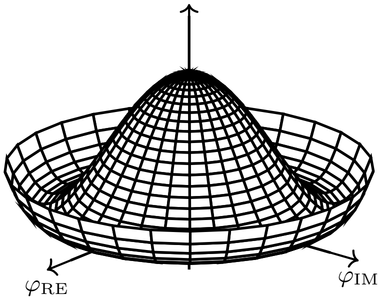
- Spontaneous symmetry breaking at scale f_a



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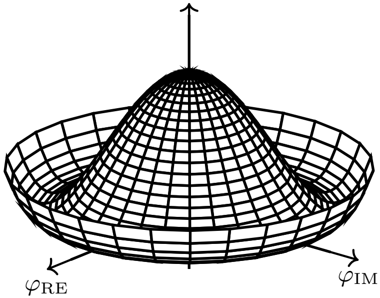
- ϕ is a pseudo Nambu-Goldstone boson



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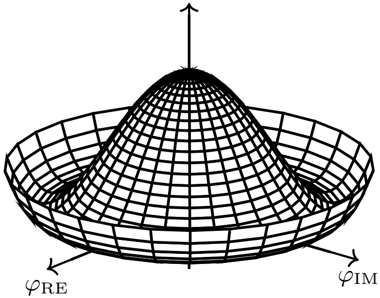

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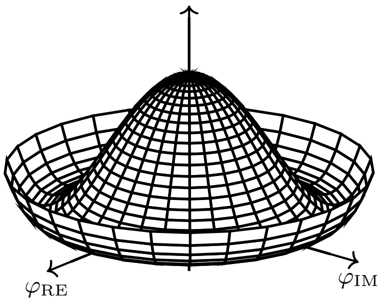
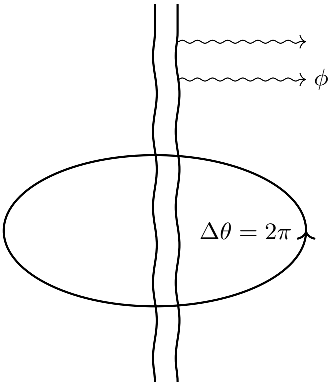
$$V(\phi) \approx \frac{1}{2} m_a^2(T)\phi^2$$

$$m_a(T) \sim T^{-n}$$



Formation of strings



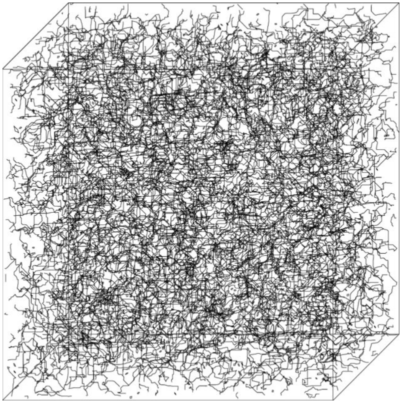


Figure 1: By Carlo Contaldi

String network collapse

$$m_a(a_m) = H(a_m)$$

Evolution of Nambu-Goldstone bosons

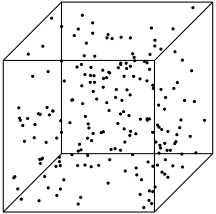
A cosmological footprint

1. What are the density fluctuations and the associated power spectrum of these axions?
2. Can we constrain the axion's mass and the symmetry breaking scale from cosmological observables?

Correlation functions

Treatment of the Scalar Field

$$N(\mathbf{k}) = f(\mathbf{k})\, d^3x\, \frac{d^3k}{(2\pi)^3}$$



Boddy et al; 2025

Foster et al; 2018

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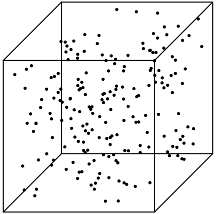

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Klein-Gordon equation

$$\ddot{\phi}_{\mathbf{k}} + \underbrace{\left(\mathbf{k}^2 + m_a^2\right)}_{\omega_{\mathbf{k}}^2} \phi_{\mathbf{k}} = 0$$

$$\phi_{\mathbf{k}} = \phi_0(\mathbf{k}) \sum_{j=1}^{N(\mathbf{k})} \cos(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\mathbf{x} + \delta_{\mathbf{k},j})$$

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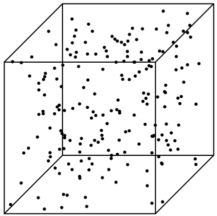

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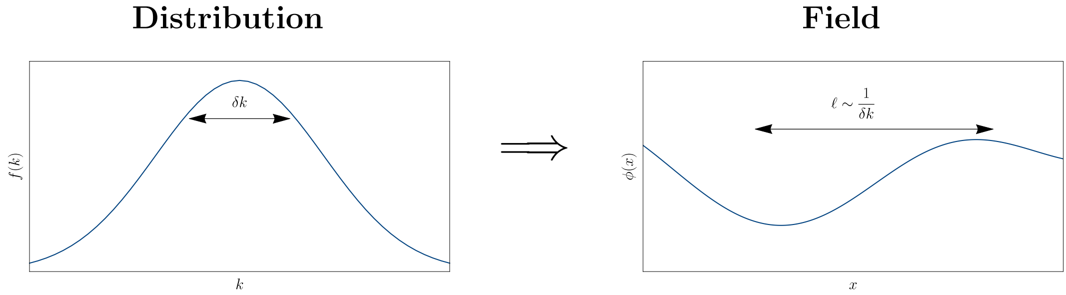
Sum over plane waves

$$\phi(t,\mathbf{x}) = \sum_{\mathbf{k}} \sqrt{\frac{f(\mathbf{k})\, d^3k}{(2\pi)^3 \omega_{\mathbf{k}}}}\, \alpha_{\mathbf{k}} \cos[\omega_{\mathbf{k}} t - \mathbf{k}\cdot\mathbf{x} + \delta_{\mathbf{k}}]$$

From a Distribution to Correlations



Mapping momentum-space information to real-space correlations

Energy Density of the Scalar Field

Velocity Distribution

$$f(v) = \frac{n}{(2\pi v_0^2)^{3/2}} e^{-v^2/(2v_0^2)} \quad \Rightarrow \quad \delta k \sim m v_0$$

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$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}m_a^2\phi^2$$

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Two-Point Correlation $\langle \rho_\phi \rho_\phi \rangle$

$$\underbrace{\bar{\rho}_\phi^2}_{\text{Bkg}} + \underbrace{\langle \rho_\phi(t,\mathbf{x})\,\rho_\phi(t',\mathbf{x}')\rangle_+}_{\text{fast mode}} + \underbrace{\langle \rho_\phi(t,\mathbf{x})\,\rho_\phi(t',\mathbf{x}')\rangle_-}_{\text{slow mode}}$$

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Slow mode

Slow mode

$$\langle \rho_\phi(t,\mathbf{x})\rho_\phi(t',\mathbf{x}')\rangle_- = \bar{\rho}_\phi^2 \frac{\exp\left[-\frac{(mv_0\Delta\mathbf{x})^2}{1+\left(mv_0^2\Delta t\right)^2}\right]}{\left(1+\left(mv_0^2\Delta t\right)^2\right)^{3/2}}$$

Slow mode

Slow mode

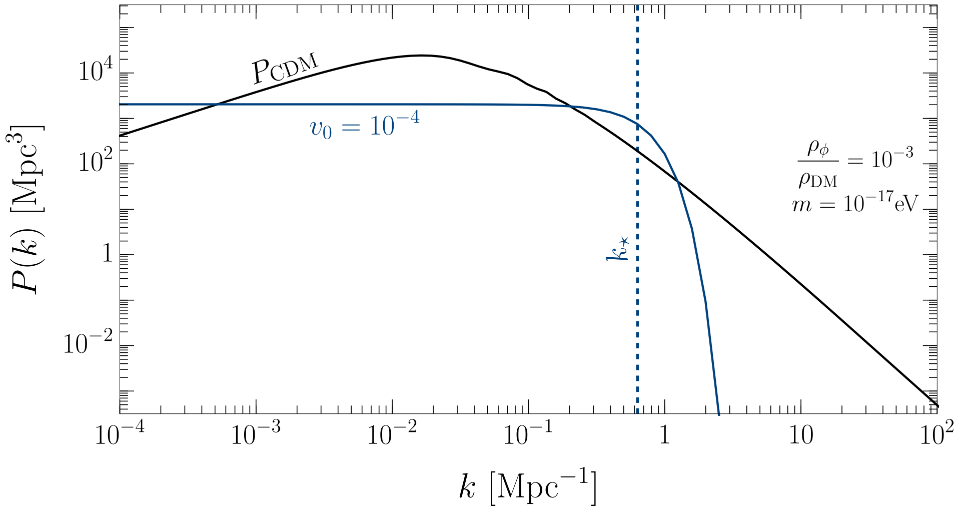
$$\langle \rho_\phi(t,\mathbf{x})\rho_\phi(t',\mathbf{x}')\rangle_- = \bar{\rho}_\phi^2 \frac{\exp\left[-\frac{(mv_0\Delta\mathbf{x})^2}{1+\left(mv_0^2\Delta t\right)^2}\right]}{\left(1+\left(mv_0^2\Delta t\right)^2\right)^{3/2}}$$

Equal time Power spectrum

$$P_-(k,t) \simeq \left(\frac{\bar{\rho}_\phi}{\bar{\rho}_{\rm DM}}\right)^2 \underbrace{\left(\frac{1}{mv_0}\right)^3}_{k_\star^{-3}} \underbrace{\exp\left[-\left(\frac{k}{2mv_0}\right)^2\right]}_{\mathcal{T}\left(\frac{k}{k_\star}\right)}$$

1. Suppression for $k \gg k_\star$
2. Constant for $k \ll k_\star$

Slow mode against data



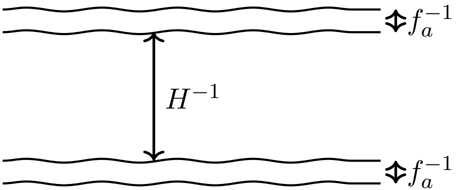
Cosmic string network

Cosmic strings

Effective tension

$$\mu = \frac{\text{Energy}}{\text{Length}} \simeq \pi f_a^2 \log\left(\frac{f_a}{H}\right)$$

Cosmic strings



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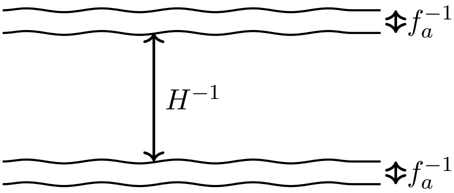

Effective tension

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Number of strings per Hubble volume (Gorghetto et al; 2018)

$$\xi(t) \xrightarrow{\text{Scaling regime}} \alpha \log\left(\frac{f_a}{H}\right)$$

Spectrum of Nambu-Goldstone modes

Energy density

$$\rho_s(t) = \frac{\xi(t)\mu}{t^2}$$

Spectrum of Nambu-Goldstone modes

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Rate of emission of Nambu-Goldstone modes

$$\text{For "free" strings} \rightarrow \xi(t) = \frac{t}{t_0},$$

$$\rho_s^{\text{free}} = \frac{\mu}{t t_0},$$

$$\Gamma = \dot{\rho}_s^{\text{free}}(t_0) - \dot{\rho}_s(t_0) \xrightarrow{\log \gg 1} 2H\rho_s$$

Spectrum of Nambu-Goldstone modes (cont.)

Energy density of the NB modes

$$\frac{1}{a^4(t)}\partial_t\left(a^4(t)\rho_\phi(t)\right) = \Gamma(t)$$

$$\rho_\phi(t) = \frac{1}{a^4(t)}\int^t dt' a^4(t')\Gamma(t')$$

Spectrum of Nambu-Goldstone modes (cont.)

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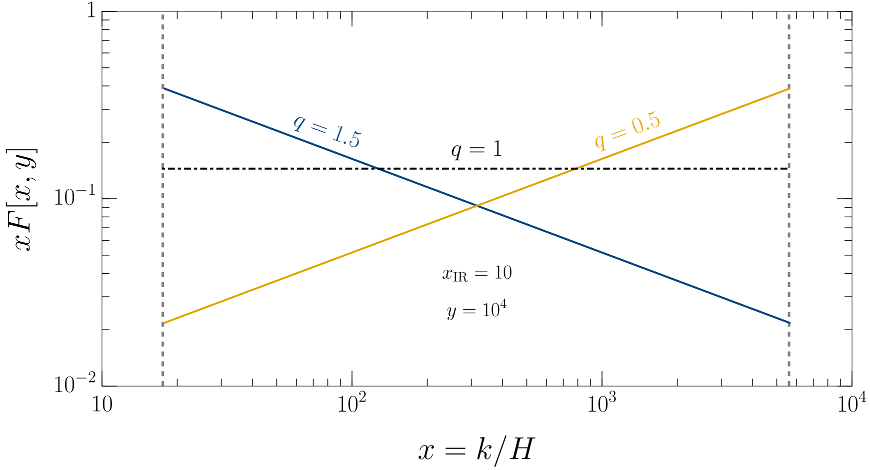
Differential rate

$$\Gamma(t)=\int dk\frac{\partial\Gamma}{\partial k}=\int dk\frac{\Gamma(t)}{H(t)}F\left[\frac{k}{H},\frac{f_a}{H}\right]$$

$$F[x,y]=\frac{N}{x^q},\qquad \underbrace{x_{\rm IR}}_{\mathcal{O}(10)}<x<y\qquad\left(\underbrace{x_{\rm IR}H}_{k_{\rm IR}}<k<f_a\right)$$

Shape of spectrum



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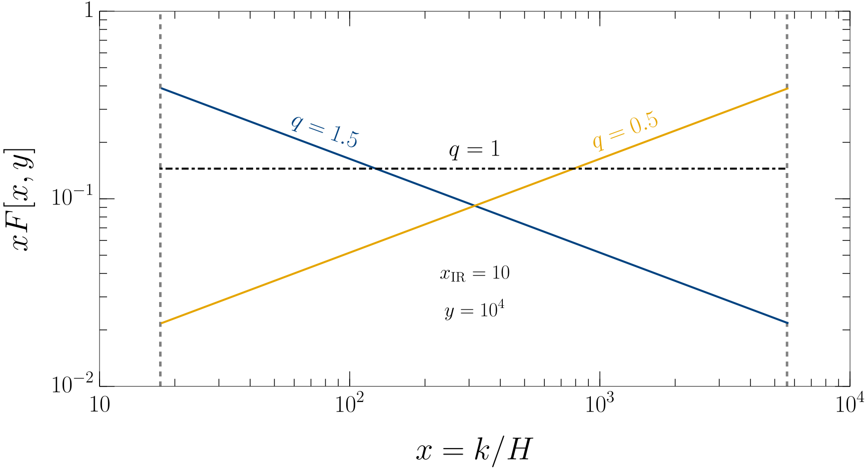

$$q(t) = 0.51 + 0.053 \log\left(\frac{f_a}{H(t)}\right)$$

(Gorghetto et al; 2020)

Energy spectra

The spectrum over comoving momenta

$$\frac{\partial \rho_\phi}{\partial k} = \frac{1}{a^4}\int^a da'(a')^2 \frac{\Gamma(a')}{H(a')} F\left[\frac{k}{a'H(a')}, \frac{f_a}{H(a')}\right]$$

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Characteristic momentum

$$k_\star \simeq k_{\rm IR} = x_{\rm IR} a_m H(a_m)$$

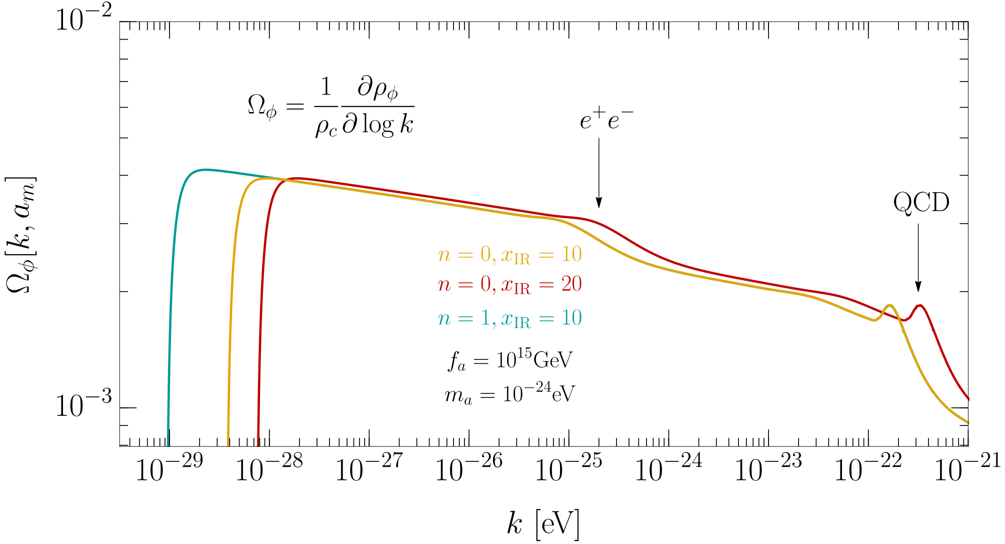
Characteristic momentum

Ultra-light particles

$$m(T) = m_a \left(\frac{T_c}{T}\right)^n, \qquad T_c = \sqrt{m_a f_a}$$

$$k_\star(n=0) \simeq 1\text{Mpc}^{-1} \left(\frac{m_a}{10^{-25}\text{eV}}\right)^{1/2}$$

The spectrum



The power spectrum

General form

Adiabatic + Isocurvature

$$P(a_0, k) = P_{\mathrm{adi}}(a_0, k) + D^2_{\mathrm{iso}}(a_0, k) P_{\mathrm{iso}}(k)$$

Determining the power spectrum

Step 1

$$\langle \rho_\phi(\mathbf{x})\rho_\phi(\mathbf{x}')\rangle_- = \frac{m_{\rm NR}^2}{a_{\rm NR}^6}\int \frac{d^3k d^3k'}{(2\pi)^6} f(\mathbf{k}) f(\mathbf{k}') \cos\left(\Delta\mathbf{x}\cdot\Delta\mathbf{k}\right)$$

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Step 2

$$f(k) = \frac{2\pi^2 a_m^4 \rho_c(a_m)}{k^4}\Omega_\phi(k, a_m)$$

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Step 3

$$P_-(k, a_{\rm NR}) = \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} \frac{\langle \rho_\phi(\mathbf{x}) \rho_\phi(0) \rangle_-}{\bar{\rho}_{\rm DM}^2(a_{\rm NR})}$$

The power spectrum

Power spectrum today

$$P_{-}(k, a_0) = D^2(a_0, k) \left(\frac{\bar{\rho}_\phi(a_0)}{\bar{\rho}_{\mathrm{DM}}(a_0)} \right)^2 k_\star^{-3} \mathcal{T}\left(\frac{k}{k_\star} \right)$$

The power spectrum

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Growth factor (Amin et al; 2025)

$$D^2(k, a_0) \approx \begin{cases} \left(\dfrac{a}{a_{\rm eq}} \right)^2 & k < k_{\rm J}(a_{\rm eq}) \\ \dfrac{k_{\rm J}(a_{\rm eq})}{k} \left(\dfrac{a}{a_{\rm eq}} \right)^2 & k > k_{\rm J}(a_{\rm eq}) \end{cases}$$

$$k_J(a) = \sqrt{\frac{3\Omega_{\rm DM}}{2} \frac{H_0 m_a}{k_\star} a^{1/2}}$$

$$\frac{k_J(a_{\rm eq})}{k_\star} \simeq 3 \times 10^{-3} \left(\frac{10}{x_{\rm IR}} \right)^2$$

The power spectrum

Relic density

$$\bar{\rho}_\phi(a_0) = a_{\rm NR}^4 \rho_c(a_{\rm NR}) m_{\rm NR} \int_{k_{\rm IR}}^{\infty} \frac{dk}{k^2}\, \Omega_\phi(k)$$

The power spectrum

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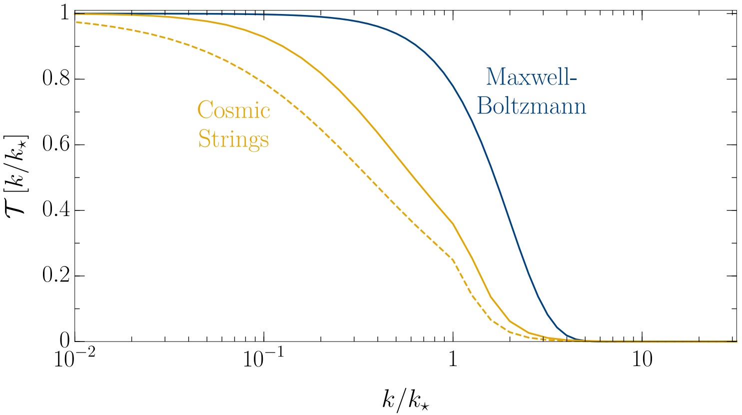
Transfer function

$$\mathcal{T}(k) = \frac{1}{2k\mathcal{N}} \int_{k_{\rm IR}}^{\infty} \frac{dp}{p^3}\Omega_\phi(p) \int_{|k-p|}^{k+p} \frac{d\ell}{\ell^3}\Omega_\phi(\ell), \quad \mathcal{T}(k\to 0) = 1$$

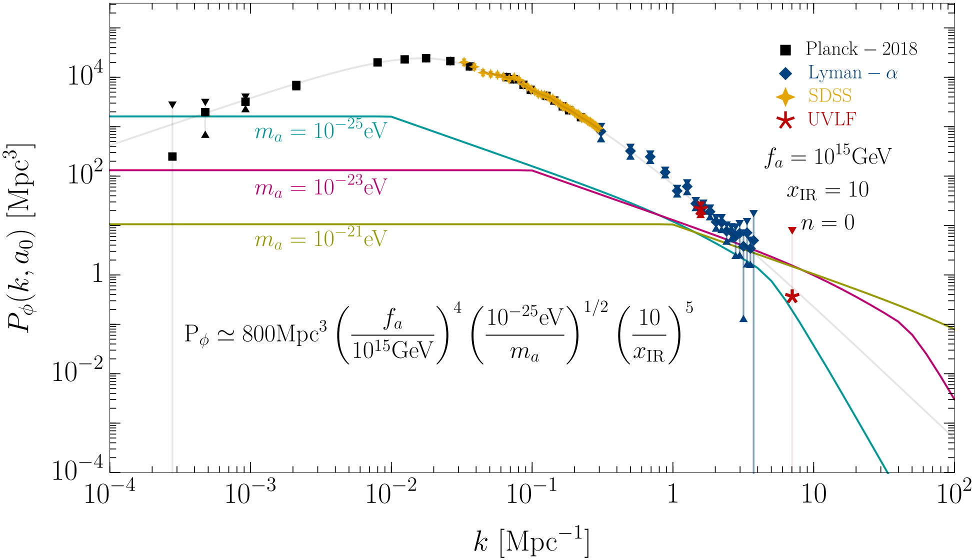
Analytic approximations

Assumption

$$\Omega_\phi(k, a_m) = \begin{cases} 0 & k < k_{\rm IR} \\ \text{const.} & k \geq k_{\rm IR} \end{cases}$$



The power spectrum against data



Constraints

Likelihood method

The likelihood function

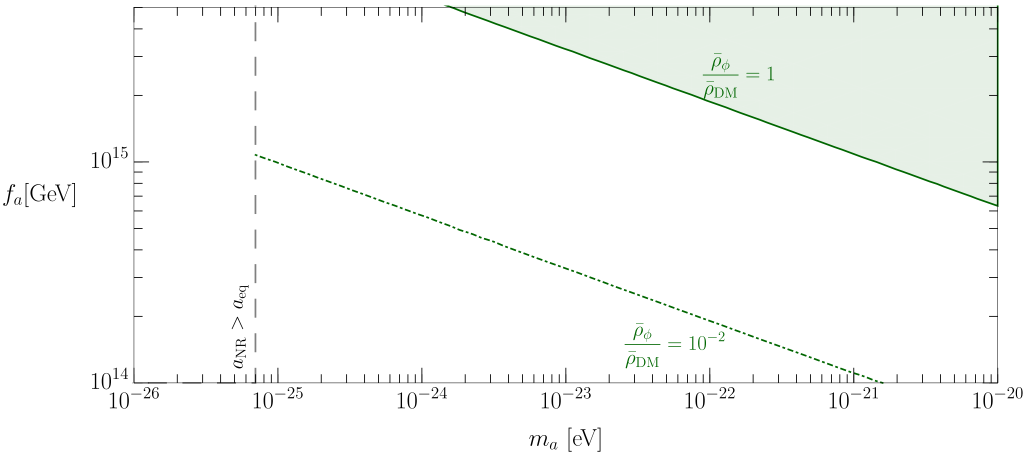
$$\mathcal{L}_{\mathrm{S+B}} \propto \exp\left[-\sum_i \frac{(P_{\mathrm{data}}(k_i) - P_{\mathrm{CDM}}(k_i) - P_\phi(k_i))^2}{2\sigma(k_i)^2}\right]$$

$$P_{\mathrm{CDM}}(k) = \frac{18\pi^2}{25} A_\Phi \left(\frac{a_{\mathrm{eq}}}{H_0^2 \Omega_M}\right)^2 k\, \mathcal{T}_{\mathrm{CDM}}^2(k) \left(\frac{k}{k_*}\right)^{n_s - 1}$$

Condition for constraint

$$2 \log\left(\frac{\mathcal{L}_{\mathrm{S+B}}}{\mathcal{L}_{\mathrm{B}}}\right) > 2.71$$

Constraints for $n = 0$



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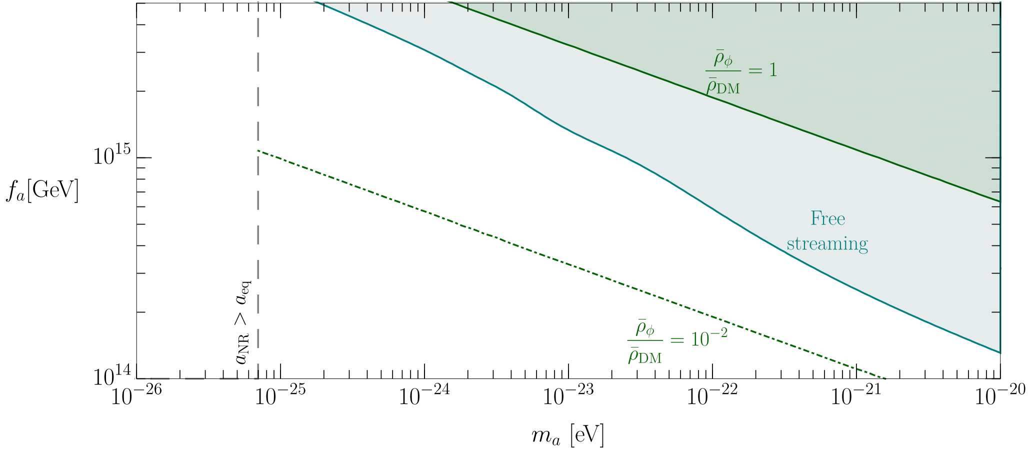

$\lambda_{\rm fs}(k,t) = \int^t \frac{dt}{v(a)}$ $\quad P_{\rm CDM}(k) T^2_{\rm fs}(k,a)$ $\quad T_{\rm fs} = \frac{\bar\rho_\phi}{\bar\rho_{\rm DM}} \,{\rm sinc}\left(k\lambda_{\rm fs}(k,a)\right)$ $\quad$ Amin et al; 2024

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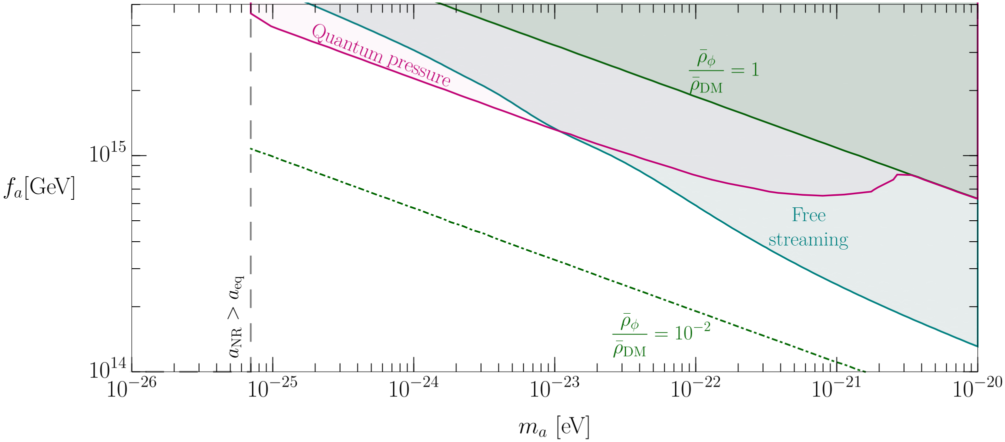
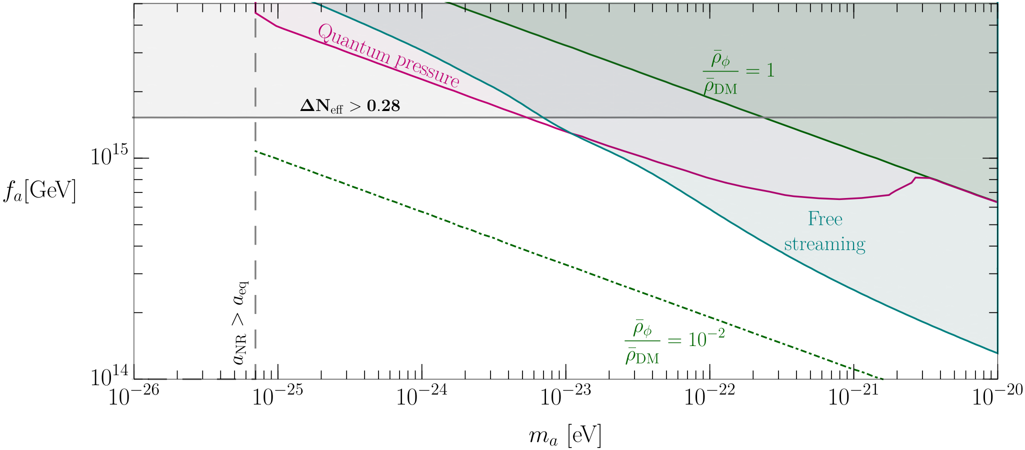


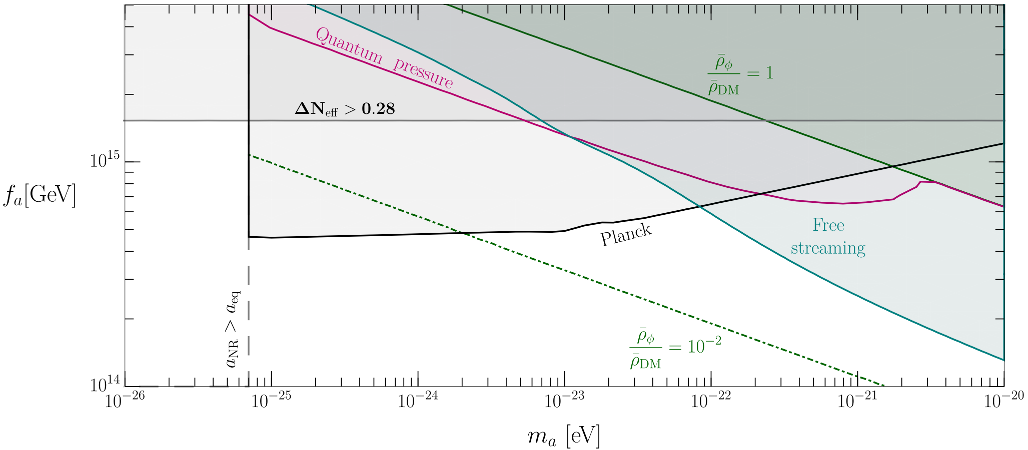
$k_Q \simeq 7\,\mathrm{Mpc}^{-1}\left(\frac{m_a}{10^{-22}\mathrm{eV}}\right)^{1/2}$ Kobayashi et al ; 2017

Constraints for $n = 0$



$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \frac{\bar{\rho}_\phi}{\bar{\rho}_\gamma}$$

Constraints for $n = 0$



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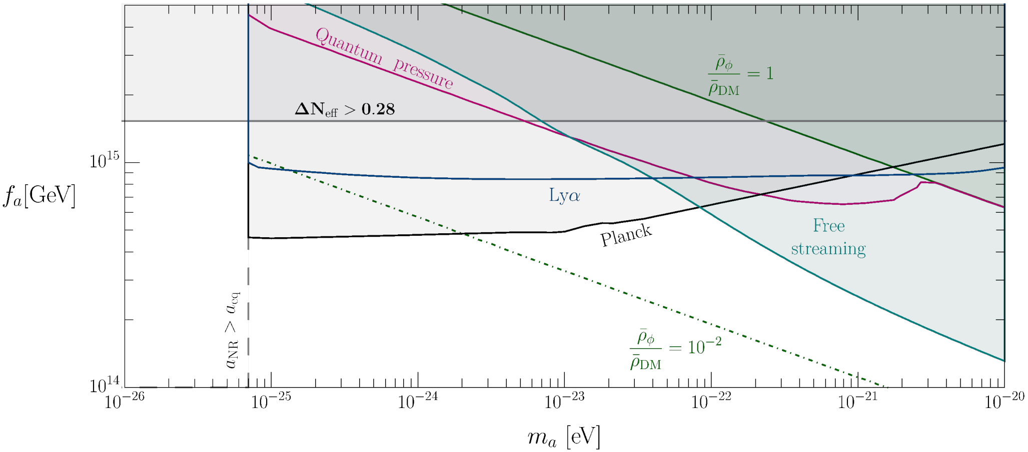

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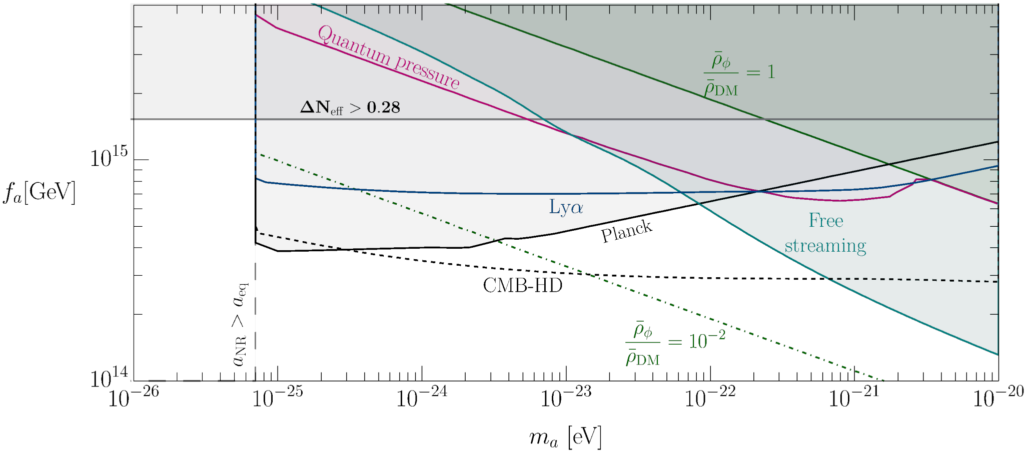

Summary and future directions

Summary

- Cosmic strings emit pseudo Nambu–Goldstone bosons
- These can contribute a sub-component of dark matter after becoming non-relativistic
- Described by a scalar field with spectrum peaked at k_{IR}
- Power spectrum: white noise for $k \ll k_{\mathrm{IR}}$, decays as k^{-4} for $k \gg k_{\mathrm{IR}}$
- Constraints on the (m_a, f_a) plane from cosmological observables.

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Future directions

- Dedicated statistical analysis for each observable
- Relativistic evolution for $m_a \lesssim 7 \times 10^{-26}\,\mathrm{eV}$ $(n = 0)$
- Temperature-dependent mass evolution for $a_c > a_{\rm eq}$
- Constraints for $a_m > a_{\rm eq}$ including CMB anisotropies

Scalar in expanding space

Klein-Gordon

$$\ddot{\phi}_{\mathbf{k}} + 3H\dot{\phi}_{\mathbf{k}} + \phi_{\mathbf{k}}\left(m^2 + \frac{|\mathbf{k}|^2}{a^2}\right) = 0,$$

Frequency

$$\omega_{\mathbf{k}}(t) = \sqrt{m^2(t) + \frac{\mathbf{k}^2}{a^2(t)}}$$

WKB solution

$$\phi_{\mathbf{k}}(x) = \frac{\phi_{\mathbf{k},0}}{a^{3/2}} \sum_{j=1}^{N_{\mathbf{k}}} \cos\left[\int^t \omega_{\mathbf{k}}(t')\, dt' - \mathbf{k}\cdot\mathbf{x} + \varphi_{\mathbf{k},j}\right],$$

Scalar in expanding space cont.

Perform sum

$$\phi_{\mathbf{k}}(x) = \sqrt{\frac{N_{\mathbf{k}}}{2a^3(t)}}\, \phi_{\mathbf{k},0}\, \alpha_{\mathbf{k}} \cos\left[\int^t \omega_{\mathbf{k}}(t')\, dt' - \mathbf{k}\cdot\mathbf{x} + \varphi_{\mathbf{k}}\right],$$

$\alpha_{\mathbf{k}}$ sampled from Rayleigh distribution

$$P(\alpha_{\mathbf{k}}) = \alpha_{\mathbf{k}} e^{-\alpha_{\mathbf{k}}^2/2}$$

Fix amplitude by $\langle \rho_{\phi,\mathbf{k}} \rangle = \frac{\omega_{\mathbf{k}} N_{\mathbf{k}}}{d^3 x a^3}$

$$\langle \rho_{\phi,\mathbf{k}} \rangle = \left\langle \frac{1}{2}\left(\dot{\phi}_{\mathbf{k}}^2 + m^2 \phi_{\mathbf{k}}^2 + \frac{(\nabla \phi_{\mathbf{k}})^2}{a^2}\right)\right\rangle = \frac{N_{\mathbf{k}}}{2a^3}\, \phi_{\mathbf{k},0}^2\, \omega_{\mathbf{k}}^2.$$

Final Result

Amplitude

$$\phi_{\mathbf{k},0} = \sqrt{\frac{2}{d^3x\,\omega_{\mathbf{k}}(t)}}.$$

Scalar field

$$\phi_{\mathbf{k}}(x) = \sqrt{\frac{f(\mathbf{k})\,d^3k}{(2\pi)^3 a^3 \omega_{\mathbf{k}}}}\,\alpha_{\mathbf{k}} \cos\left[\int^t \omega_{\mathbf{k}}(t')\,dt' - \mathbf{k}\cdot\mathbf{x} + \delta_{\mathbf{k}}\right].$$

Sum over plane waves

$$\phi(x) = \sum_{\mathbf{k}} \phi_{\mathbf{k}}(x)$$

Correlation functions

$$F_k = \frac{f(k)}{\sqrt{a^3(t)\,\omega_{\mathbf{k}}(t)\,a^3(t')\,\omega_{\mathbf{k}}(t')}}$$

Correlations

$$\langle \dot{\phi}\phi' \rangle = -\int \frac{d^3k}{(2\pi)^3} F_k\,\omega_{\mathbf{k}}(t)\sin\left[\int_{t'}^{t}\omega_{\mathbf{k}} - \mathbf{k}\cdot\Delta\mathbf{x}\right]$$

$$\langle \phi\nabla\phi' \rangle = -\int \frac{d^3k}{(2\pi)^3} F_k\,\mathbf{k}\sin\left[\int_{t'}^{t}\omega_{\mathbf{k}} - \mathbf{k}\cdot\Delta\mathbf{x}\right]$$

$$\langle \dot{\phi}\dot{\phi}' \rangle = \int \frac{d^3k}{(2\pi)^3} F_k\,\omega_{\mathbf{k}}(t)\omega_{\mathbf{k}}(t')\cos\left[\int_{t'}^{t}\omega_{\mathbf{k}} - \mathbf{k}\cdot\Delta\mathbf{x}\right]$$

$$\langle \dot{\phi}\nabla\phi' \rangle = -\int \frac{d^3k}{(2\pi)^3} F_k\,\omega_{\mathbf{k}}(t)\mathbf{k}\cos\left[\int_{t'}^{t}\omega_{\mathbf{k}} - \mathbf{k}\cdot\Delta\mathbf{x}\right]$$

$$\langle \partial_i\phi\partial_j\phi' \rangle = \int \frac{d^3k}{(2\pi)^3} F_k\,k_ik_j\cos\left[\int_{t'}^{t}\omega_{\mathbf{k}} - \mathbf{k}\cdot\Delta\mathbf{x}\right]$$

Slow and fast modes

Slow and fast modes

$$\langle \rho\rho' \rangle_{\pm} = \frac{1}{4} \int \frac{d^3k\, d^3k'}{(2\pi)^6} F_k F_{k'} A^{\pm}_{\mathbf{k}\mathbf{k}'}$$
$$\times \cos\left[\int_{t'}^{t} (\omega_{\mathbf{k}} \pm \omega_{\mathbf{k}'}) - (\mathbf{k} \pm \mathbf{k}') \cdot \Delta\mathbf{x} \right]$$

Approximation

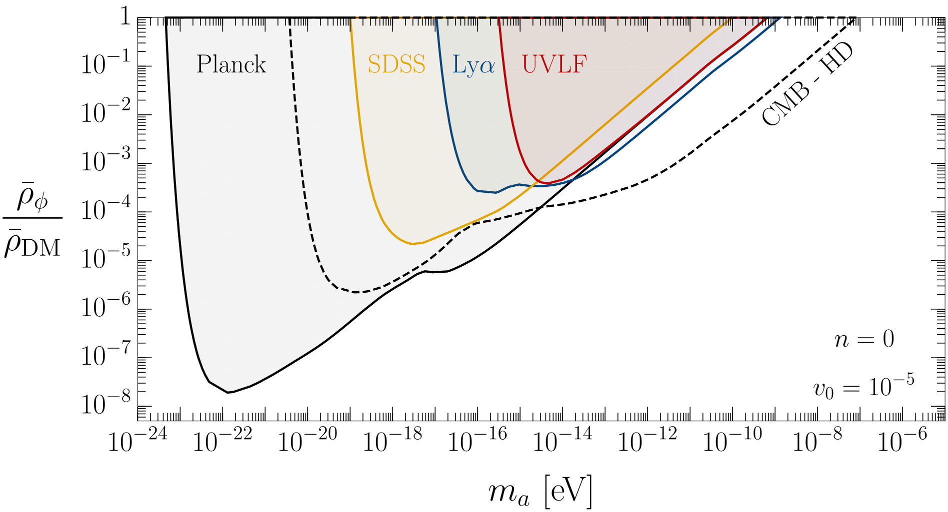
$$A^{+}_{\mathbf{k}\mathbf{k}'} \simeq \frac{|\mathbf{k} + \mathbf{k}'|^4}{4\, a^2(t) a^2(t')}$$
$$A^{-}_{\mathbf{k}\mathbf{k}'} \simeq 4\, m^2(t) m^2(t')$$

Slow and fast modes: coefficients

Coefficients

$$
\begin{aligned}
A^{\pm}_{\mathbf{k}\mathbf{k}'} = {} & m^2(t)m^2(t') + \omega_{\mathbf{k}}(t)\omega_{\mathbf{k}}(t')\omega_{\mathbf{k}'}(t)\omega_{\mathbf{k}'}(t') \\
& \mp m^2(t')\omega_{\mathbf{k}}(t)\omega_{\mathbf{k}'}(t) \mp m^2(t)\omega_{\mathbf{k}}(t')\omega_{\mathbf{k}'}(t') \\
& + \mathbf{k}\cdot\mathbf{k}' \left[\frac{\omega_{\mathbf{k}}(t)\omega_{\mathbf{k}'}(t) \mp m^2(t)}{a^2(t')} + \frac{\omega_{\mathbf{k}}(t')\omega_{\mathbf{k}'}(t') \mp m^2(t')}{a^2(t)} \right] \\
& + \frac{(\mathbf{k}\cdot\mathbf{k}')^2}{a^2(t)a^2(t')}
\end{aligned}
$$

Maxwell-Boltzmann constraints



Slow mode details

Correlations in terms of Ω_ϕ

$$\langle \rho_\phi(\mathbf{x})\rho_\phi(\mathbf{x}')\rangle = 4m_{\rm NR}^2 a_{\rm NR}^2 \rho_c^2(a_{\rm NR})\left[\int \frac{dk}{k^2}\Omega_\phi(k)\,\mathrm{sinc}(k|\Delta\mathbf{x}|)\right]^2$$

Identity for power spectrum calculation

$$\int_0^\infty dx x^2\,\mathrm{sinc}(kx)\,\mathrm{sinc}(\ell x)\,\mathrm{sinc}(px) = \frac{\pi}{4kp\ell}\theta(p-|k-\ell|)\theta(k+\ell-p).$$

Power spectrum

$$P_\phi(k, a_{\rm NR}) = 8\pi^2\left(\frac{\bar\rho_\phi(a_0)}{\bar\rho_{\rm DM}(a_0)}\right)^2 k_\star^{-3}\mathcal{T}(k)$$

Transfer function analytic approxmation

$k < 2k_{\mathrm{IR}}$

$$\mathcal{T}(y) \simeq \frac{5}{8y^5(1+y)}\Big[y^5 + y^4 - 2y^3 + 6y^2 + 12y(1-\log(1+y)) - 12\log(1+y)\Big]$$

$k > 2k_{\mathrm{IR}}$

$$\mathcal{T}(y) \simeq \frac{-5}{4y^5}\left[\frac{12y-8y^3}{y^2-1} + 3\log\left[\frac{y+1}{y-1}\frac{2y+1}{2y-1}\frac{2y^2+y-1}{2y^2-y-1}\right]\right]$$

Analytical density fraction

$\Omega_\phi = $ const for $k > k_{\rm IR}$

$$\frac{\bar{\rho}_\phi}{\bar{\rho}_{\rm DM}} = 6.8 \times 10^{-3} \left(\frac{f_a}{10^{15}{\rm GeV}} \right)^2 \left(\frac{m_a}{10^{-25}{\rm eV}} \right)^{1/2} \left(\frac{10}{x_{\rm IR}} \right)$$

Growth factor suppression

Dynamical time scale

$$t_D \sim H^{-1} \sim \frac{1}{\sqrt{H\overline{\rho}}}$$

Time for field to change by $\mathcal{O}(1)$

$$\Delta t \sim \frac{k^{-1}}{v_a}$$

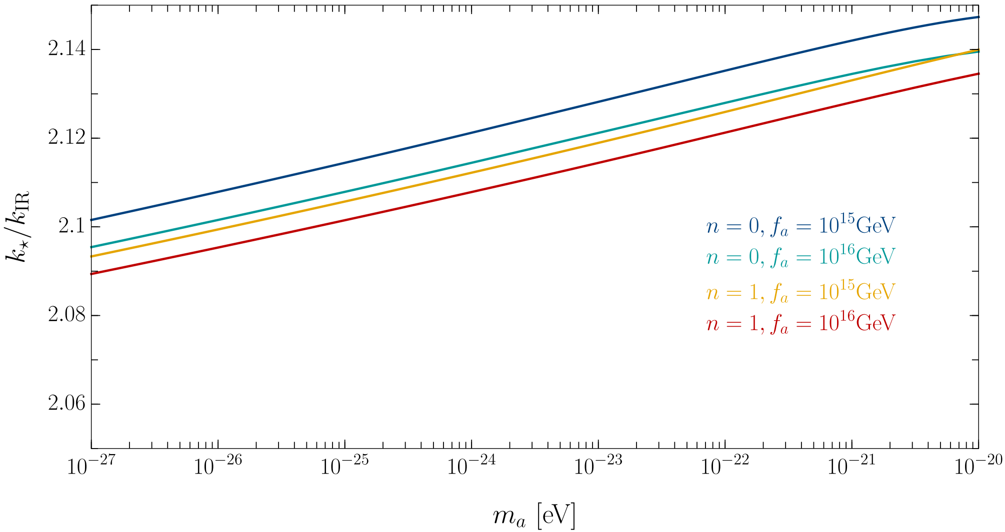
Number of field oscillations

$$N = \frac{t_D}{\Delta t}$$

Amplitude that gravity probes suppressed by $\sqrt{N}$

$$\frac{f_{\mathrm{DM}}\delta_\phi}{\sqrt{N}} \sim \frac{1}{\sqrt{k}}$$

$k_\star$



2.14
2.12
2.1
2.08
2.06
$k_\star/k_{\rm IR}$
10^{-27}
10^{-26}
10^{-25}
10^{-24}
10^{-23}
10^{-22}
10^{-21}
10^{-20}
m_a [eV]
$n = 0,\, f_a = 10^{15}$GeV
$n = 0,\, f_a = 10^{16}$GeV
$n = 1,\, f_a = 10^{15}$GeV
$n = 1,\, f_a = 10^{16}$GeV

Quantum pressure

Density fractions

$$\delta_m = F\delta_\phi + (1-F)\delta_c$$

Evolution of perturbations

$$\ddot{\delta}_{\phi\mathbf{k}} + 2H\dot{\delta}_{\phi\mathbf{k}} + \frac{c_s^2 k^2}{a^2}\delta_{\phi\mathbf{k}} - \frac{3}{2}H^2\delta_{m\mathbf{k}} = 0, \qquad c_s^2 = \frac{k^2}{4a^2m^2}$$

$$\ddot{\delta}_{c\mathbf{k}} + 2H\dot{\delta}_{c\mathbf{k}} - \frac{3}{2}H^2\delta_{m\mathbf{k}} = 0$$

$F = 1$

$$\delta_{m\mathbf{k}} \propto e^{2ic_s k/aH}$$

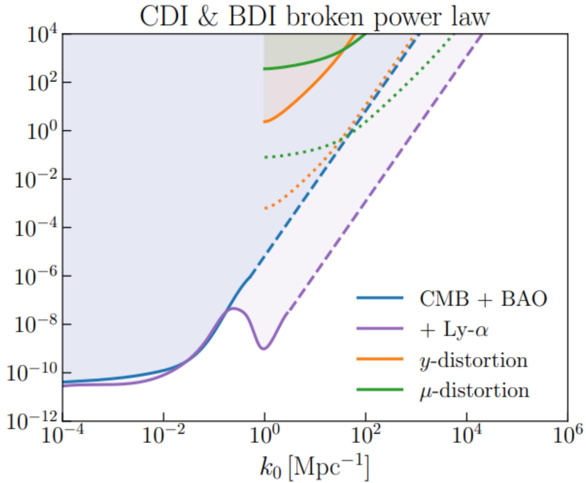
for $k > k_Q$

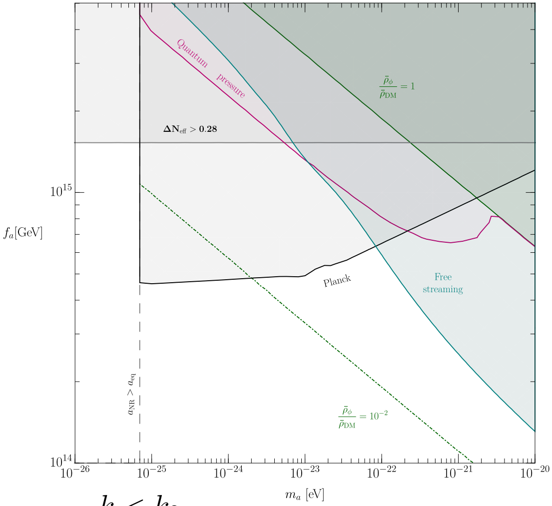
$0 < F < 1$

$$\delta_{m\mathbf{k}} \propto a^{n_+}$$

for $k > k_Q, n_+ = \frac{-1+\sqrt{25-24F}}{4}$

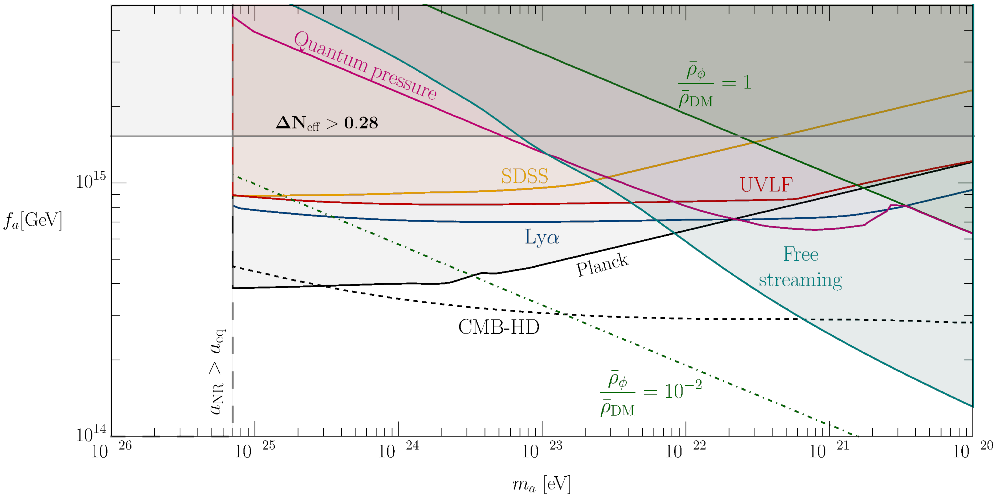
Comparison to literature





$$P_{iso}(k) = 2\pi^2 \frac{A_{iso}}{k_0^3} \begin{cases} 1, & k < k_0 \\ \left(\frac{k_0}{k}\right)^3, & k > k_0 \end{cases}$$

All the constraints for $n = 0$



All the contstraints for $n = 1$