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PHYSICS

A Semiclassical Approach to Warm Inflation Non-Gaussianity

Phenomenology 2026 Symposium, University of Pittsburgh

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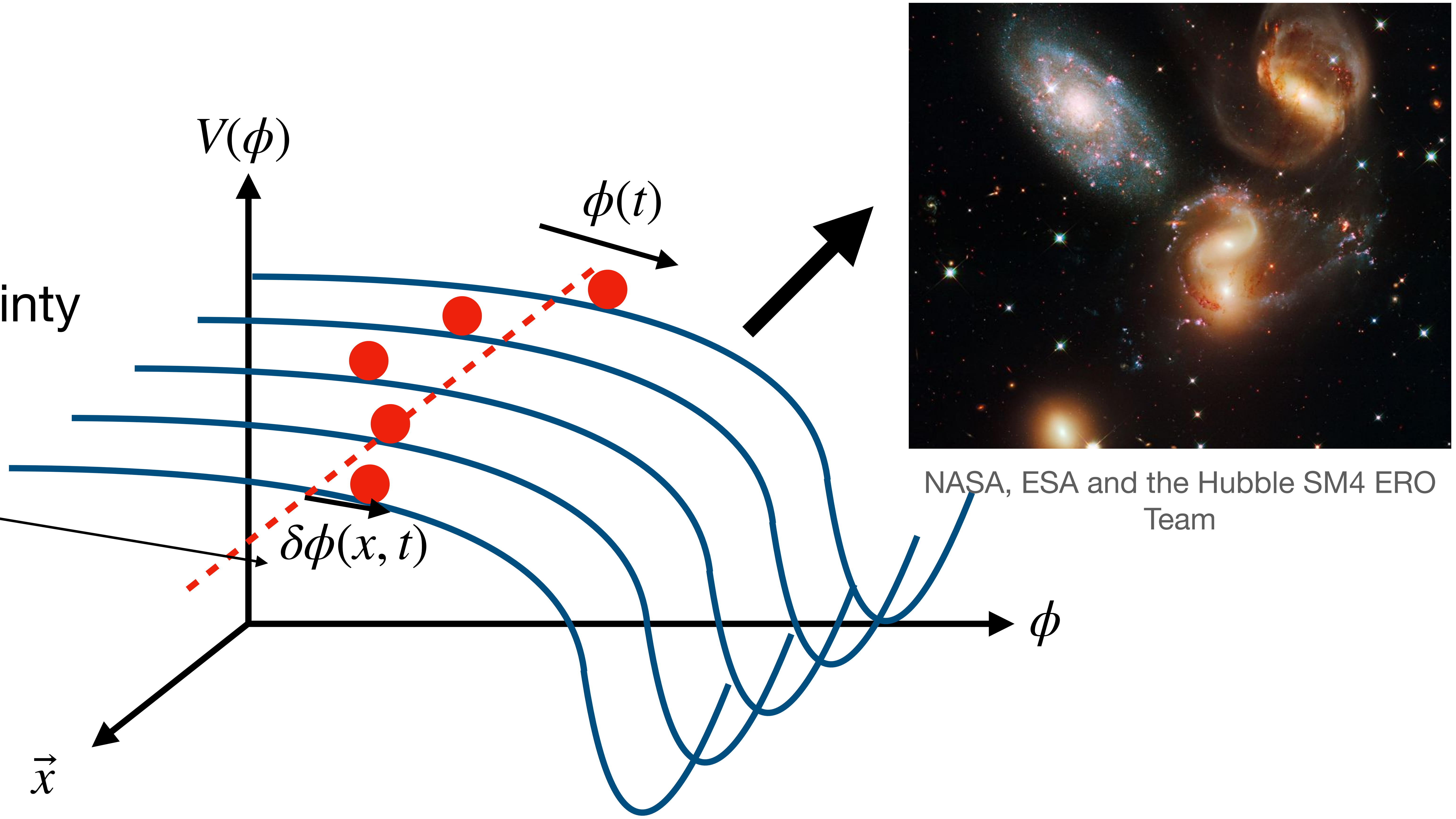
Based on arXiv:2505.07943 with Edward Broadberry, & Anson Hook
& arXiv:2606.xxxxx with Edward Broadberry, Anson Hook, & Kushan Panchal

Inflation says: Homogeneous and Flat Universe

But what seeds for all the structure?

Are these due to Heisenberg uncertainty
or
due to thermal fluctuations?



NASA, ESA and the Hubble SM4 ERO Team

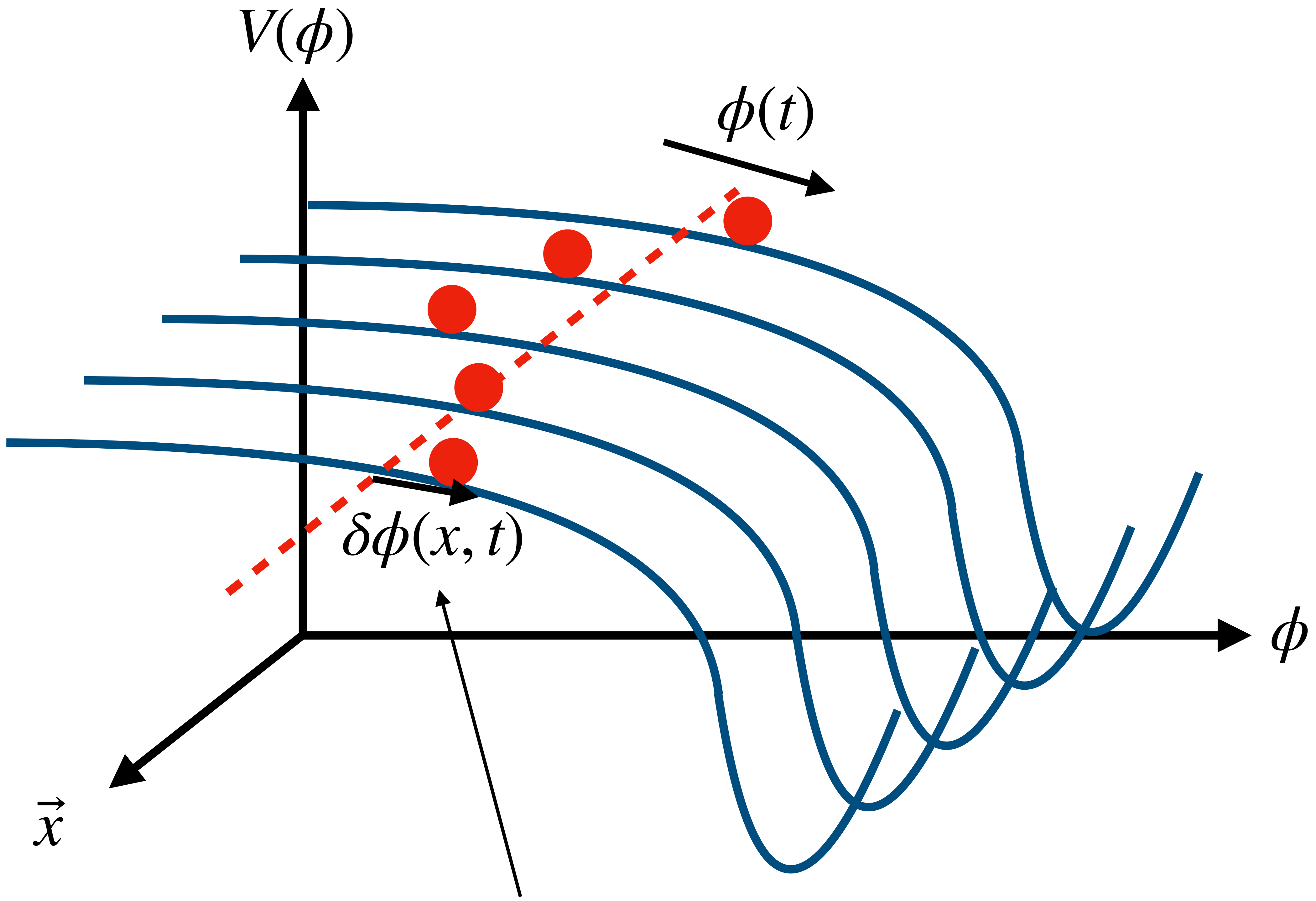
Let’s try to find out … Have to look into Non-Gaussianity.

Part I - Intro to warm inflation

What is Warm Inflation?

- The inflaton interacts with a bath of radiation.
- The radiation (field denoted by χ in this talk) acts like an additional source of friction, which we call γ.
- The interactions with the radiation also provide thermal fluctuations.
- The thermal fluctuations provide a classical origin for perturbations.

[1] A. Berera and L.-Z. Fang, Thermally induced density perturbations in the inflation era, Phys. Rev. Lett. 74 (1995) 1912–1915, [astro-ph/9501024].



Mostly Thermal Fluctuations
(instead of Quantum Fluctuations)

Framework

Including Interactions

The interaction acts like an additional source of friction on the inflaton

Friedmann (Einstein's) Equation: $$H^2 = \frac{1}{3M_{\rm pl}^2}\left(V(\phi) + \frac{1}{2}\dot{\phi}^2 + \rho_r\right)$$

Inflaton Equation of Motion: $$\ddot{\phi} + (3H+\gamma)\dot{\phi} + V'(\phi) = 0$$

Collision Term

Radiation Evolution Equation $\nabla_\mu T^{\mu\nu} = 0$: $$\dot{\rho}_r = -4H\rho_r + \gamma\dot{\phi}^2$$

This is a Boltzmann equation; Work with tree-level amplitudes

- There is now a stable steady state with $\dot{\rho}_r \approx 0$.

Part II - A simple model with chemical potential

Model Building

To understand the background equations, it would seem that we simply need to calculate the friction coefficient, γ.

- To do this we add an interaction between the inflaton, and some field(s) that will act as radiation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \mathscr{L}'_{\text{int}} \,.$$

- We will calculate the thermal expectation value

[3] J. Yokoyama and A. D. Linde, Phys. Rev. D 60, 083509 (1999). [arXiv:hep-ph/9809409].

$$\langle \mathscr{L}'_{\text{int}} \rangle = -\gamma\dot{\phi} + \boxed{V'_T(\phi, T)}$$

This is a problem

- We will find that the radiation sector develops a chemical potential

Our work!

[4] Broadberry, Hook and **SM** arXiv:2505.07943

$$\gamma\dot{\phi} \sim \boxed{\gamma_0}\dot{\phi}(1 - \boxed{c(\mu)});$$

Usually calculated

Dealing with Thermal Backreaction

Derivative Interactions

- To suppress the thermal back reaction it's natural to consider a derivative interaction of the form

$$\mathscr{L}_{\mathrm{int}} = \frac{1}{f}\partial_\mu \phi J^\mu \,.$$

Ensures backreaction is zero

- In the slow-roll background, $\dot{\phi} \approx \mathrm{const}$, this is a chemical potential for some non-conserved charge

$$\mathscr{L}_{\mathrm{int}} = \frac{\dot{\phi}}{f} J^0 \,.$$

- So, this should be balanced by the chemical potential generated due to the response of the bath!

Modified model

Boltzmann Equation for the friction term

Theoretical control over backreaction $\propto \alpha_1 \alpha_2$

$$\mathscr{L}_{\text{int}} = -\frac{\alpha_1}{4!}\chi^4 e^{4i\phi/f} - \frac{\alpha_2}{4!}\chi^4 e^{5i\phi/f} + c.c.$$

- The Boltzmann equation for the friction is slightly modified relative to the number density

Collision Term

$$\boxed{\frac{d\rho_r}{dt} = \left\{\alpha_1^2\left(4\frac{\dot\phi}{f} - 4\mu\right) + \frac{5}{4}\alpha_2^2\left(5\frac{\dot\phi}{f} - 4\mu\right)\right\}\frac{\dot\phi}{f}T^3 I_{\text{phase}}}$$

- Plugging in the chemical potential gives (check limits)

$$\boxed{\mu = \frac{|\alpha_1|^2 + \frac{5}{4}|\alpha_2|^2}{|\alpha_1|^2 + |\alpha_2|^2}\frac{\dot\phi}{f}}$$

Chemical Potential interactions when α_1 or α_2 is zero!

$$\boxed{\gamma = \frac{\alpha_1^2\alpha_2^2}{\alpha_1^2 + \alpha_2^2}\frac{T^3}{4f^2}I_{\text{phase}}}$$

Part III - Classical calculation of power spectrum

The Noise Term

- The equation of motion for the Inflaton including fluctuations is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) - \frac{\nabla^2\phi}{a^2} = \mathscr{L}'_{\rm int}(\phi)\,.$$

- The interactions with the radiation will provide random kicks to the inflaton analogous with the random kicks in Brownian motion.
- We define a "noise term" $\boxed{\xi(x) \equiv \mathscr{L}'_{\rm int} - \langle \mathscr{L}'_{\rm int}\rangle\,.}$
- In the zero-width limit, that is, when self-interactions in the bath is stronger than Hubble or scattering length $\sim (\lambda_\chi^2 T)^{-1} \ll H^{-1}$, for the χ-correlators,

$$\boxed{\langle \xi(x)\xi(y)\rangle = A\delta^{(4)}(x-y)\,.}$$

$$\mathscr{L}_{\rm self} \supset -\lambda_\chi |\chi|^4$$

Fluctuation-Dissipation Theorem

- If the friction is stronger, it is because the inflaton is “hit harder” every time it interacts with the radiation. (the fluctuation-dissipation theorem)

$$A = 2\gamma T$$

- In reality we found

[5] Mehrdad Mirbabayi and Andrei Gruzinov JCAP02(2023)012

$$A = 2\gamma_0 T \implies A = \frac{\boxed{2\gamma T}}{1 - \boxed{c(\mu)}}$$

Our work!

[4] Broadberry, Hook and **SM** arXiv:2505.07943

- Another way to calculate the fluctuations is to directly calculate in thermal field theory, so we did both

$$\boxed{A = \frac{T^4}{2f^2} I_{\text{phase}} \left(16\alpha_1^2 + 25\alpha_2^2\right)} \qquad \boxed{A = 2\gamma T \frac{(\alpha_1^2 + \alpha_2^2)(16\alpha_1^2 + 25\alpha_2^2)}{\alpha_1^2 \alpha_2^2}}$$

CMB Power Spectrum

- We calculate the CMB power spectrum by solving the equations of motion including the noise term as a Langevin equation.

$$D(\zeta, \delta\rho, \delta q)\zeta(x) = \xi(x) \implies \zeta_k(z) = \int_z^{\infty} dz' G_\zeta(z, z')\hat{\xi}_k(z')$$

- Where, the gauge invariant scalar curvature perturbation $\zeta(x) = \dfrac{\delta\phi}{\dot{\phi}}$.

- From this we can calculate the CMB power spectrum (c.f. cold inflation)

$$\langle \zeta_k(0)\zeta_{k'}(0)\rangle' = A\left(\frac{H}{\dot{\phi}}\right)^2 \frac{1}{k^3}F_2(Q)\,.$$

$$\boxed{A \propto \frac{\alpha^2 T^4}{f^2}}$$

$$F_2(Q) = k^3 \int_{-\infty}^{0} d\eta\, \eta^4 G_{\Phi_k}(0,\eta)^2$$

Part IV - Non-Gaussianity (NG) and its sources

Semiclassical Treatment

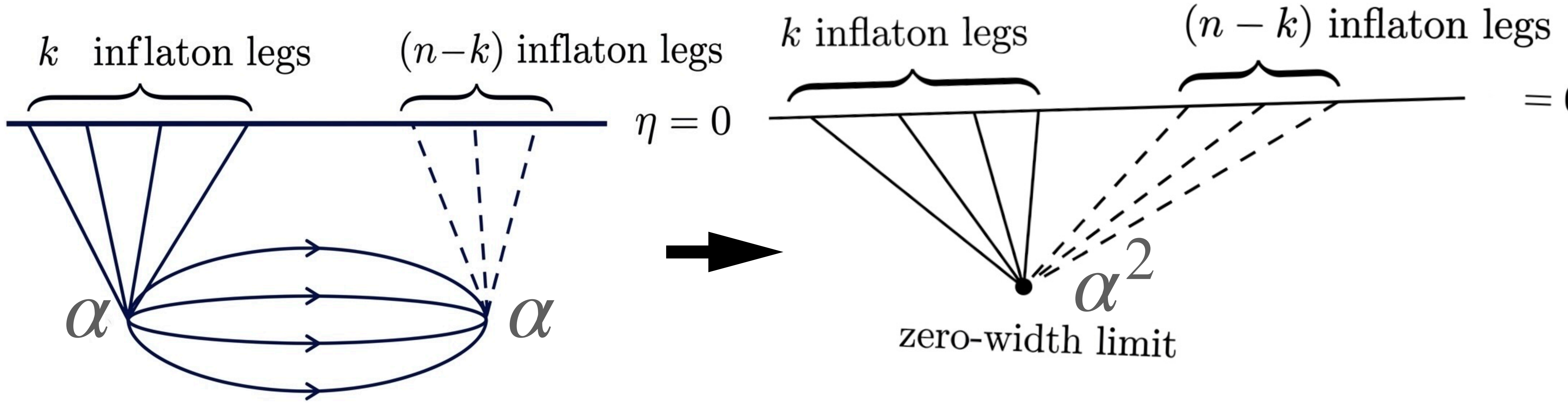
The n-point noise correlator

- In the zero-width limit for the χ-correlators, only the classical part survives.

$$\mathscr{L}_{\rm int} \supset -\frac{\alpha}{4!}\chi^4 e^{4i(\frac{\dot{\phi}}{f}t+\frac{\delta\phi}{f})} + h.c$$

$$\langle \xi(x_1)\cdots\xi(x_n)\rangle \overset{l.o}{=}$$

$$\frac{2c_1T^4}{f^n}\left(\prod_{i=2}^{n}\delta^4(x_i-x_1)\right)\times\begin{cases}(4^n\alpha_1^2+5^n\alpha_2^2), & n \text{ even},\\ (5^n-4^n)\frac{\alpha_1^2\alpha_2^2}{\alpha_1^2+\alpha_2^2}\frac{u^\mu\partial_\mu\phi}{2fT}, & n \text{ odd}.\end{cases}$$



- NG in the noise is large, since $\frac{\langle\xi^n\rangle}{\langle\xi^2\rangle^{n/2}} \sim (\frac{\lambda_\chi^4}{\alpha\sqrt{2c_1}})^{n-2}$ which diverges since α is small and $\lambda_\chi \sim \mathcal{O}(1)$.

Non-Gaussianity due to Shot Noise

The n-point inflaton correlator

- Purely due to shot noise, the n-point correlator is given as,

$$\boxed{\langle \zeta_{k_1} \cdots \zeta_{k_n} \rangle' \overset{l.o}{=} 2c_1 T^4 \left(-\frac{H}{f\dot{\phi}} \right)^n \int_{-\infty}^{0} d\eta \eta^{4(n-1)} \prod_{i=1}^{n} G_{\Phi_{k_i}}(0,\eta) \times \begin{cases} (4^n \alpha_1^2 + 5^n \alpha_2^2), & n \text{ even}, \\ (5^n - 4^n) \dfrac{\alpha_1^2 \alpha_2^2}{\alpha_1^2 + \alpha_2^2} \dfrac{u^\mu \partial_\mu \phi}{2fT}, & n \text{ odd}. \end{cases}}$$

- The NG are small due to the central limit theorem, even if the NG in the noise term is large!

- All the higher n-point correlators $\dfrac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n/2}}$ are suppressed by $\sim \left(\dfrac{H}{T} \right)^{2(n-2)}$ $\Longrightarrow$ **Gaussian!**

- Odd-point correlators have an additional suppression of $\sim \left(\dfrac{\dot{\phi}}{fT} \right)$

Non-Gaussian Noise →Gaussian Inflaton Fluctuations

- Inflaton fluctuation $\delta\phi$ lengthscale $\sim \dfrac{1}{\sqrt{\gamma H}}$
- Noise ξ lengthscale (small scattering length) $\sim \dfrac{1}{\lambda_\chi^2 T}$
- So an inflaton fluctuation is sourced by $\sim \left(\dfrac{\lambda_\chi^2 T}{\sqrt{\gamma H}}\right) \sim \left(\dfrac{\lambda_\chi^2 f}{\alpha\sqrt{TH}}\right) \sim \mathcal{O}(10^2)$ independent noise sources.

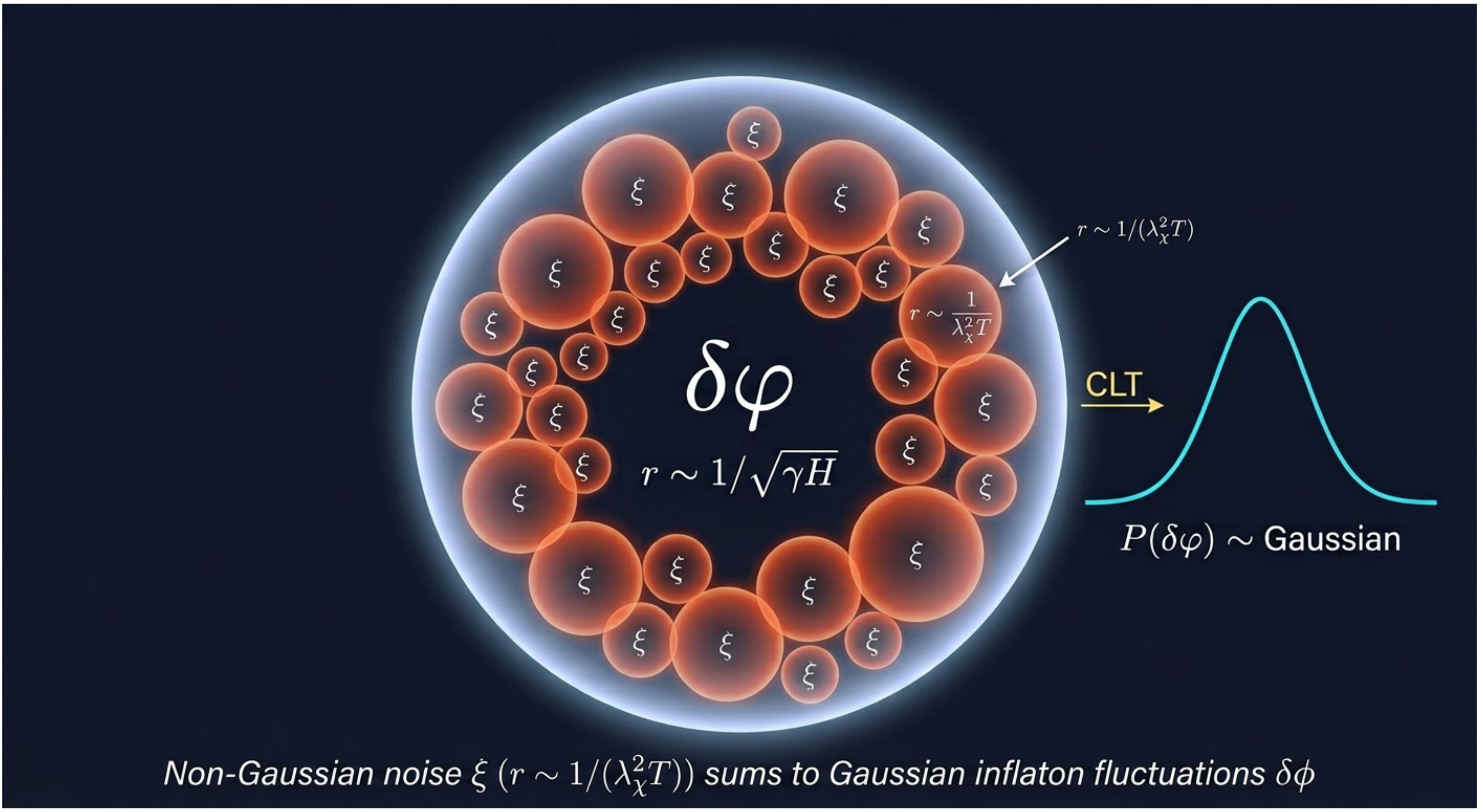


Image Credit: Perplexity AI

Other Sources of NG

- The second-order cosmological perturbations.
- The correction to the friction and the noise terms due to higher order corrections to the friction term (this vanishes for the odd orders in $\dot{\phi}/(fT)$), and they could be important, since $\dot{\phi}/(fT) \sim 0.5$

$$\gamma \to \gamma(1+c(\frac{\dot{\phi}}{fT})^2+\mathcal{O}((\frac{\dot{\phi}}{fT})^4)) \qquad \xi \to \xi(1+c'(\frac{\dot{\phi}}{fT})^2+\mathcal{O}((\frac{\dot{\phi}}{fT})^4))$$

- The corrections to the noise term due to density perturbations in the bath lead to local temperature fluctuations.

$$\xi \to \xi(1+\frac{\mathcal{E}}{2}+\mathcal{O}(\mathcal{E}^2))$$

Part V - Conclusions

Conclusion

Summary

- Wrote down a perturbative model that realises warm inflation, while allowing us to calculate the n-point correlations explicitly.
- The thermal bath develops a chemical potential, which acts to (partially) cancel the thermal friction, thereby modifying the fluctuation-dissipation theorem.
- Showed the semiclassical approach to calculate NG in the leading order.
- Argued that the NG are small because of the Central Limit Theorem.
- We are in the process of calculating the lineshapes, which will hint towards classical fluctuations in the inflaton.

Thanks for Listening

Backup Slides

Constraints

Spectral tilt

- To derive the spectral tilt recall $dN = Hdt$, and during slow-roll

$$H^2 \approx \frac{V(\phi)}{3M_{\mathrm{Pl}}^2} \implies \frac{d\log H}{dN} = -\epsilon_V$$

$$\dot{\phi} \approx \frac{V(\phi)'}{3H(1+Q)} \implies \frac{d\log\dot{\phi}}{dN} = \epsilon_V - \eta_V - \frac{Q}{1+Q}\frac{d\log Q}{dN}$$

- To get the final term we have to use

$$\begin{gathered} \rho_r \propto T^4 \\ \gamma \propto T^3 \\ 4H\rho_r = \gamma\dot{\phi}^2 \end{gathered} \quad \to \frac{d\log Q}{dN}, \frac{d\log T}{dN} = f(\epsilon_V, \eta_V)$$

Constraints

Spectral tilt

- We then simply take the derivative

$$\frac{d}{dN}\log\left(A\left(\frac{H}{\dot{\phi}}\right)^2 F_2(Q)\right).$$

- We plug in all the results from the previous page, to find,

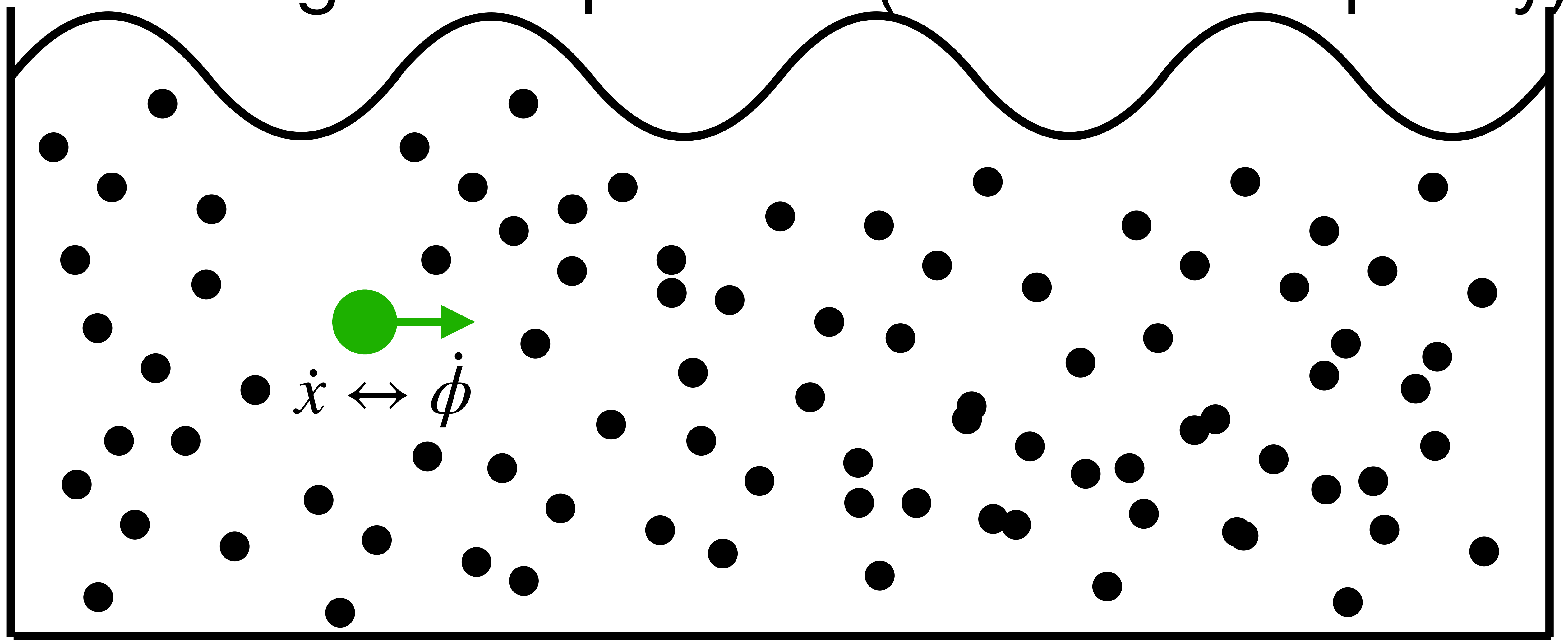
$$\boxed{\frac{d\log\Delta_r}{dN}=\frac{2-Q}{1+7Q}\epsilon-6\frac{1+Q}{1+7Q}\eta+\frac{d\log F_2}{d\log Q}\frac{1+Q}{1+7Q}(10\epsilon-6\eta)}$$

Brownian Motion

Fluctuation-Dissipation

- The situation is analogous to Brownian motion, in which a pollen grain is moving in a fluid, and constantly knocking into the fluid molecules.
- We model this random motion as a "Langevin equation" (in 1D for simplicity)

$$m\ddot{x}(t) = -\gamma\dot{x}(t) + \xi(t)$$



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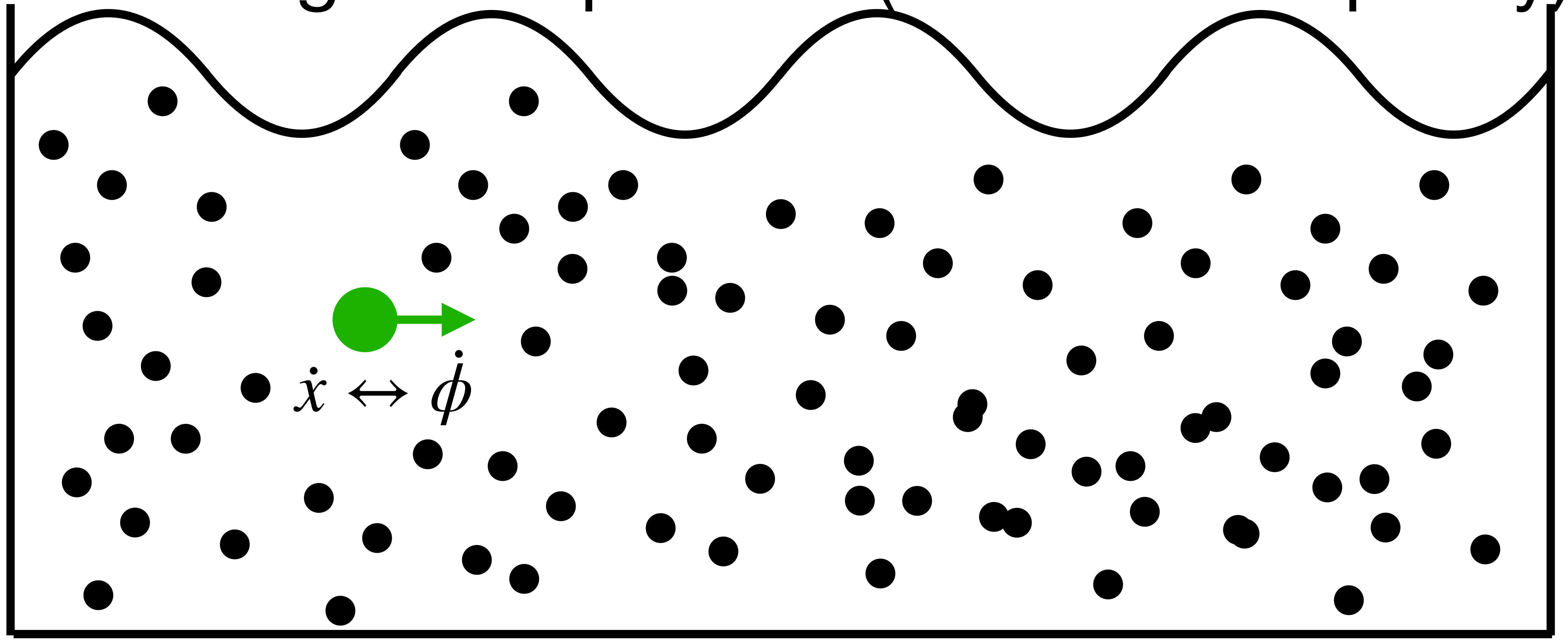
$$m\ddot{x}(t) = \boxed{-\gamma\dot{x}(t)} + \xi(t)$$

The fluid will net provide some friction that slows down a moving pollen grain.



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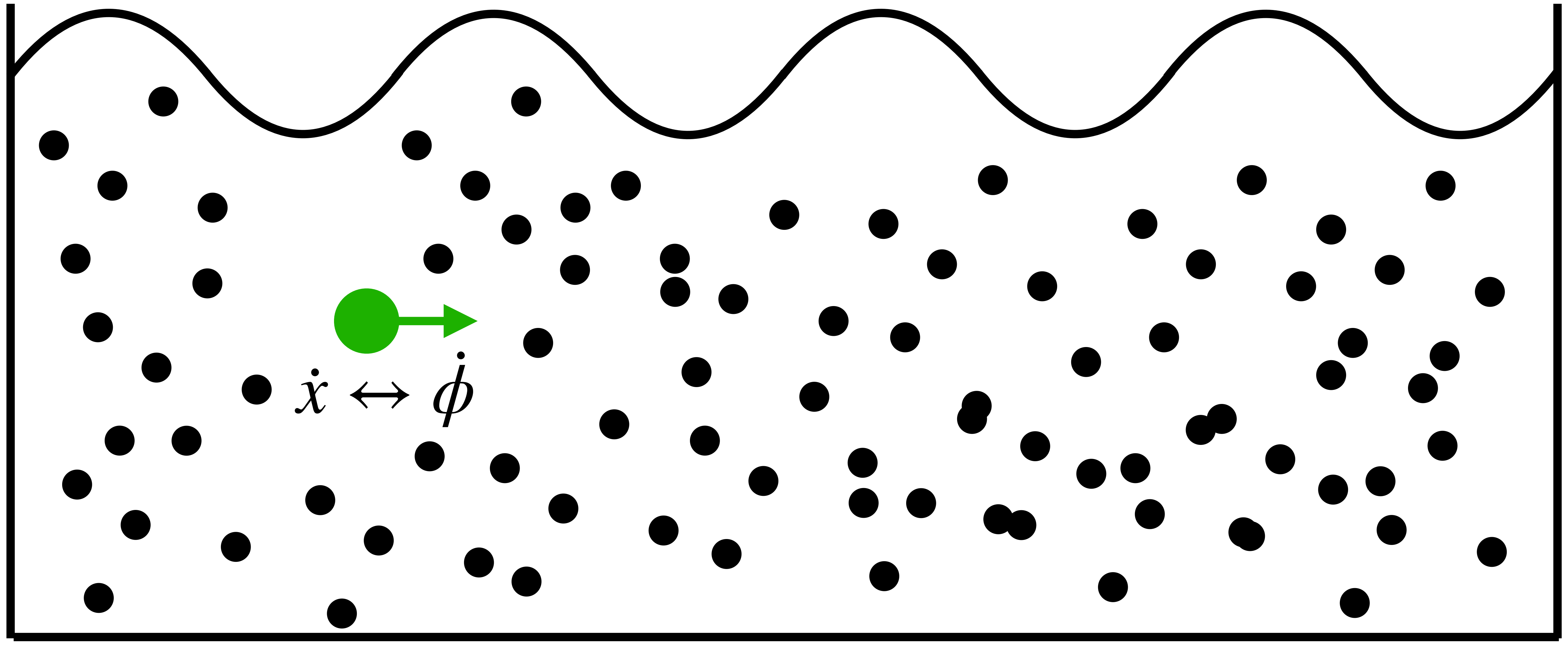
$$m\ddot{x}(t) = -\gamma\dot{x}(t) + \boxed{\xi(t)}$$

The pollen grain is knocked in a random direction every time it collides with a molecule in the fluid.



Brownian Motion

Fluctuation-Dissipation

- By the central limit theorem the average force follows a Gaussian distribution

$$\langle \xi(t)\xi(t')\rangle \equiv A\delta(t-t')\,.$$

- We can formally solve the Langevin equation by defining $v(t) = \dot{x}(t)$ (setting $m = 1$ for simplicity)

$$v(t) = e^{-\gamma t}\int_0^t dt' e^{\gamma t'}\xi(t') \implies \langle v^2(t)\rangle = \frac{A}{2\gamma}\left(1 - e^{-2\gamma t}\right)$$

Brownian Motion

Fluctuation-Dissipation

- If I did this with an ensemble of pollen grains, after a long time they should equilibrate with the fluid.

- By the equipartition theorem

$$\langle v^2(t) \rangle \rangle \to \frac{A}{2\gamma} = T\,.$$

- This gives the correct relationship between the friction and the fluctuations

$$A = 2\gamma T\,.$$

The Friction Term

The Classical Analogy to Boltzmann Equation

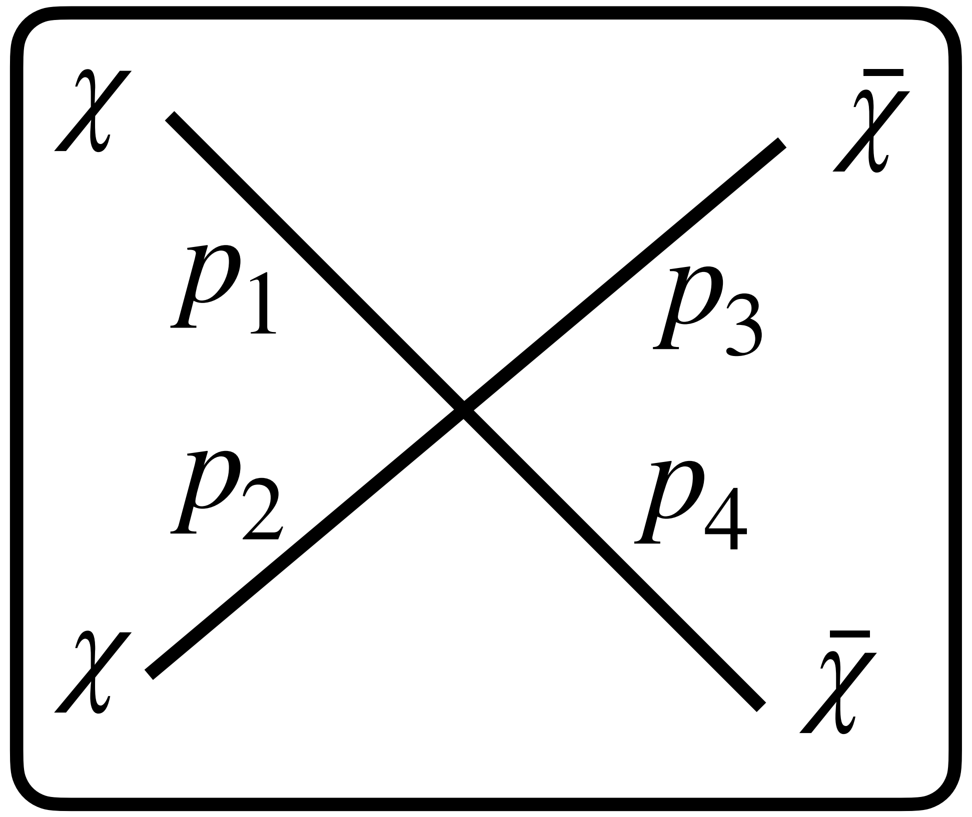
- Recall the background equation for the radiation bath

$$\dot{\rho}_r + 4H\rho_r = \gamma\dot{\phi}^2\,.$$

- This looks like the Boltzmann equation for ρ_r, the friction is the collision term.
- The collision term is dominated by two-two scattering.

$$\boxed{\chi(p_1) + \chi(p_2) \rightleftharpoons \bar{\chi}(p_3) + \bar{\chi}(p_4)}$$

Number Violating Interactions



$$\mathscr{L}_{\text{int}} = -\frac{\alpha}{4!}\chi^4 e^{4i\frac{\dot{\phi}}{f}t}$$

Why does the friction still vanish?

- To get a term proportional to $\dot{\phi}^2$ we Taylor expand in

$$\boxed{\frac{\dot{\phi}}{Tf} \ll 1.}$$

- We pull out a term, and the rest of the integral can be solved numerically

$$\frac{d\rho_r}{dt} = \alpha^2 I_{\text{phase}} T^3 \frac{\dot{\phi}}{f}\left(\cancel{4\frac{\dot{\phi}}{f}} - 4\mu\right).$$

Goes to zero at chemical eqm

$$\boxed{\dot{\rho}_r \propto \gamma = 0}$$ Oops!

Collision Term

- The collision term is a standard two-two scattering collision term.

[7] S. Dodelson and F. Schmidt, Modern Cosmology. Academic Press, 2020.

$$\frac{d\rho_r}{dt} = 4\frac{\dot{\phi}}{f}\alpha^2 \int d\Pi_{12} \int d\Pi_{34} (2\pi)^4 \delta^3(\vec{p}_{\mathrm{tot}})\delta\boxed{(E_1 + E_2 - E_3 - E_4 - 4\dot{\phi}/f)}$$

$$\times\, (\bar{f}_3 \bar{f}_4 (1 + f_1)(1 + f_2) \boxed{-} f_1 f_2 (1 + \bar{f}_3)(1 + \bar{f}_4))$$

The process $\bar{\chi}\bar{\chi} \to \chi\chi$ injects energy into the bath, whereas the reverse direction removes energy.

$$\boxed{\chi(p_1) + \chi(p_2) \rightleftharpoons \bar{\chi}(p_3) + \bar{\chi}(p_4)}$$

$$f_i = \frac{1}{\exp[\beta(E_i - \mu)] - 1}$$

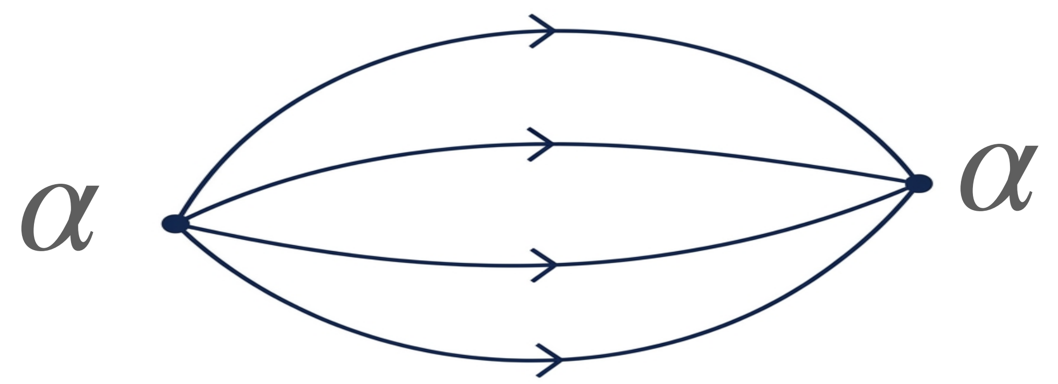
$$\bar{f}_i = \frac{1}{\exp[\beta(E_i + \mu)] - 1}$$

Naive Calculation (!)

At the EOM level

- Proceeding with the definition of the “noise term”, $\xi(x) \equiv \mathscr{L}'_{\text{int}} - \langle \mathscr{L}'_{\text{int}} \rangle$.
- Then the 2-point noise correlator is given as, $\langle \xi(x)\xi(y) \rangle \overset{l.o}{=} \langle \mathscr{L}'_{\text{int}}(x)\mathscr{L}'_{\text{int}}(y) \rangle \sim \mathcal{O}(\alpha^2)$

$$\mathscr{L}'_{\text{int}} \supset -\frac{4i}{f}\frac{\alpha}{4!}\chi^4 e^{4i\frac{\dot{\phi}}{f}t} + h.c$$



- Then the 3-point correlator is given as,

$$\langle \xi(x)\xi(y)\xi(z) \rangle \overset{l.o}{=} \langle \mathscr{L}'_{\text{int}}(x)\mathscr{L}'_{\text{int}}(y)\mathscr{L}'_{\text{int}}(z) \rangle \sim \mathcal{O}(\alpha^4)$$