



The Entangled State from a cosmological Euclidean Wormhole

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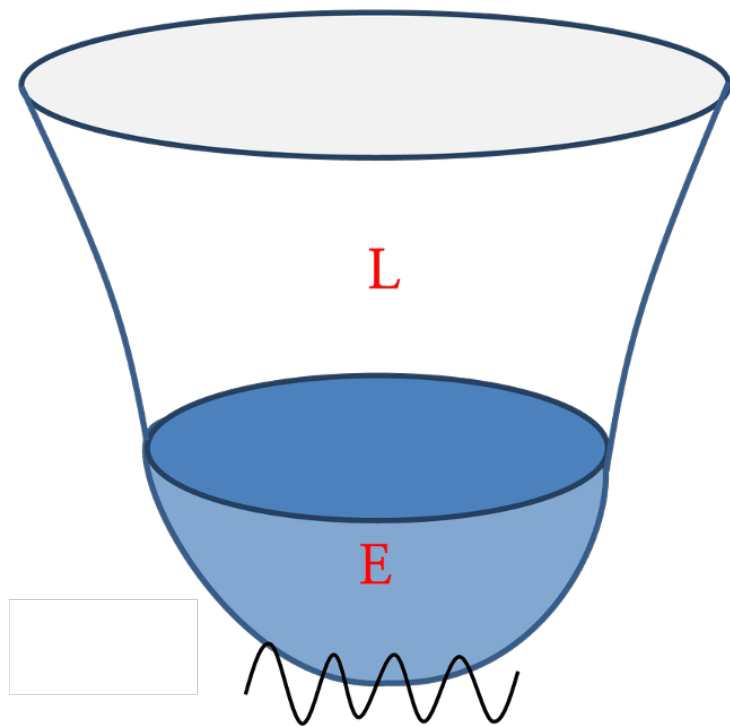
In collaboration with: Pisin Chen (NTU), Kuan-Nan Lin (APCTP), Dong-han Yeom (PNU)

arXiv: 2404.15450, 2505.02807, 2605.xxxxx

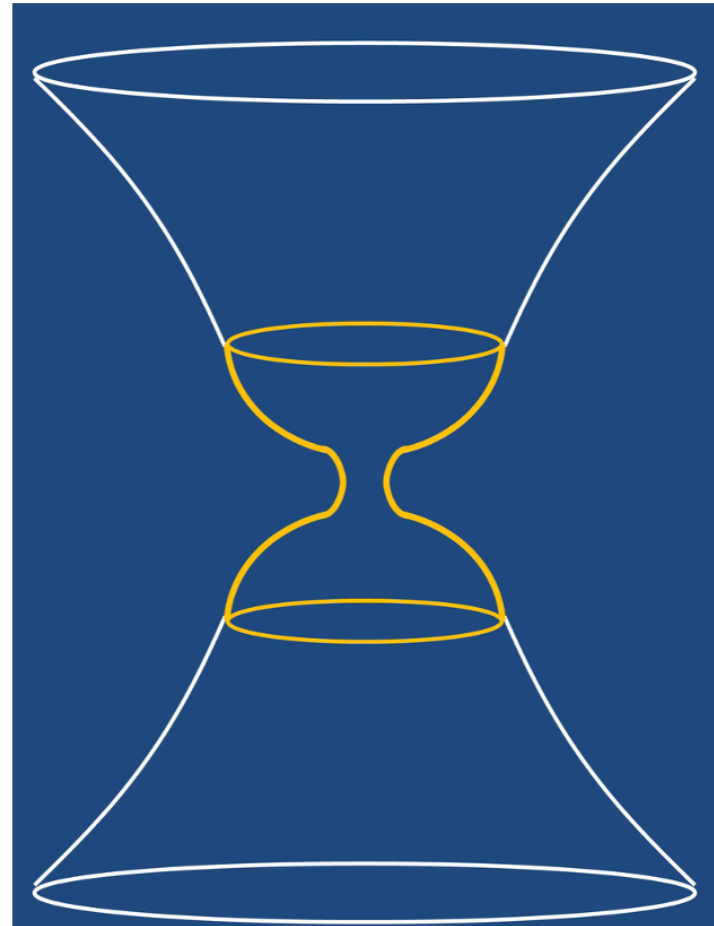
2026/5/11

The 2026 Phenomenology Symposium (PHENO)

dS (NB) instanton \Rightarrow Euclidean vacuum



Euclidean wormhole \Rightarrow ???



Outline

- Quantum cosmology – Euclidean path integral approach
- No Boundary Proposal & The Euclidean (Bunch-Davies) vacuum
- The Entangled State from a cosmological Euclidean Wormhole
- Summary

Quantum Cosmology – Euclidean path integral approach

The Wheeler-DeWitt equation: $\hat{H}\Psi = 0$

The Euclidean path integral is a solution to the Wheeler-DeWitt equation at the formal level:

$$\Psi [h_f, \Phi_f; h_i, \Phi_i] = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\Phi e^{-S_E[g_{\mu\nu}, \Phi]}$$

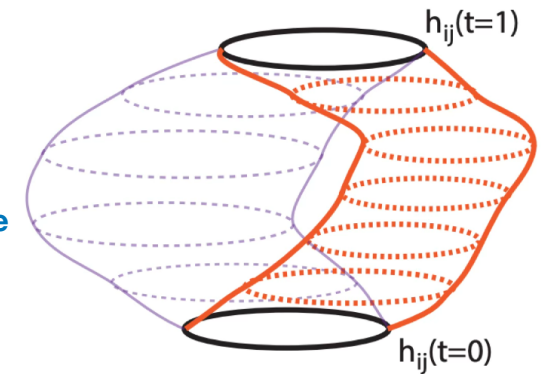
Steepest-descent approximation for practical computations:

$$\Psi [h_f, \Phi_f; h_i, \Phi_i] \approx \exp\left(-I[\bar{g}_{\mu\nu}, \bar{\phi}]\right) \int \mathcal{D}\phi e^{-I_m[\bar{g}_{\mu\nu}, \bar{\phi}; \phi]}$$

The **on-shell** solution
(**instanton**)

perturbations
related to QF in curved spacetime

$I_m[\bar{g}_{\mu\nu}, \bar{\phi}; \phi]$ is the quadratic action of the scalar field perturbations $\phi = \Phi - \bar{\phi}$



A figure from Lehnert (2023)

No Boundary proposal and the Euclidean vacuum

the Euclidean action: $t = -i\tau$

J. J. Halliwell and S. W. Hawking, PRD (1985)
R. Laflamme, PLB (1987)

$$S_E = \int \sqrt{+g} d^4x \left[\frac{R}{16\pi} + \frac{1}{2} (\partial_\mu \Phi)^2 + U(\Phi) \right],$$

a homogenous, isotropic, and closed metric:

$$ds_E^2 = \sigma^2 (d\tau^2 + a^2(\tau) d\Omega_3^2),$$

$$d\Omega_3^2 = d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$\Psi [h_f, \Phi_f; h_i, \Phi_i] \approx \exp(-I[\bar{g}_{\mu\nu}, \bar{\phi}]) \int \mathcal{D}\phi e^{-I_m[\bar{g}_{\mu\nu}, \bar{\phi}; \phi]}$$

instanton

$$\phi(\mathbf{x}, \tau) = \sum_{nlm} f_{nlm}(\tau) Q_{lm}^n(\mathbf{x})$$

$Q_{lm}^n(\mathbf{x})$: three-sphere harmonics. $f_{nlm}(\tau)$: Euclidean mode solutions

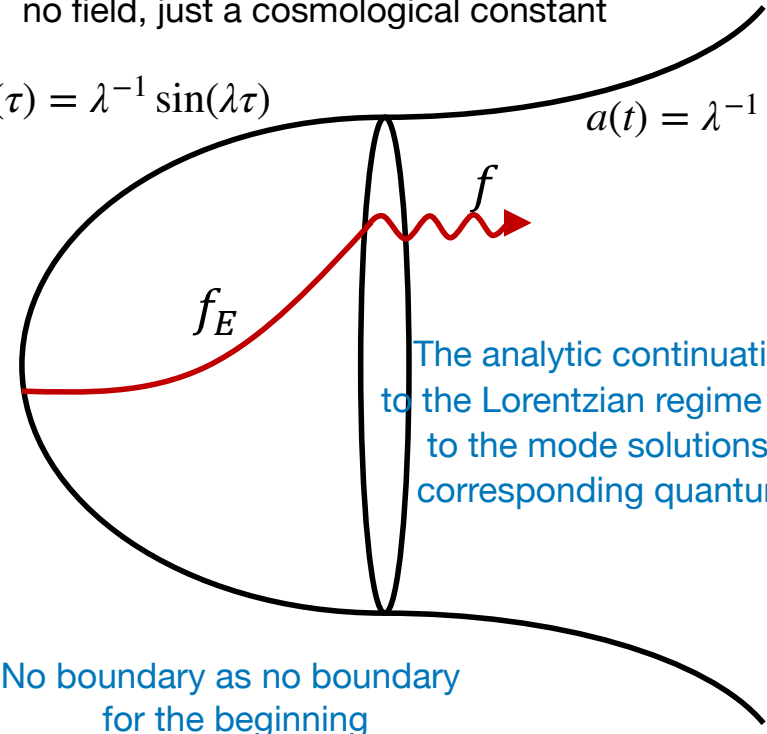
$$f_{nlm}'' + 3 \frac{a'}{a} f_{nlm}' - \left(m^2 + \frac{n^2 - 1}{a^2} \right) f_{nlm} = 0$$

the dS instanton

no field, just a cosmological constant

$$a(\tau) = \lambda^{-1} \sin(\lambda\tau)$$

$$a(t) = \lambda^{-1} \cosh(\lambda t)$$



The analytic continuation of f_n to the Lorentzian regime is related to the mode solutions of the corresponding quantum field.

No boundary as no boundary for the beginning

$$\Psi [h_f, \Phi_f; h_i, \Phi_i] \approx \exp \left(-I[\bar{g}_{\mu\nu}, \bar{\phi}] \right) \int \mathcal{D}\phi e^{-I_m[\bar{g}_{\mu\nu}, \bar{\phi}; \phi]}$$

$$\phi(\mathbf{x}, \tau) = \sum_{nlm} f_{nlm}(\tau) Q_{lm}^n(\mathbf{x})$$

$$\psi_n = \int \mathcal{D}f_n \exp \left[I_n(a, f_n) \right] \propto \exp \left[-I_{n(cl)}(a, f_n) \right] = \exp \left[\frac{-1}{2} a^3 f_n f_n' \Big|_{bdy} \right]$$

a quadratic action

$$\psi_n(\tilde{f}_n) \propto \exp \left[\frac{-1}{2} a^3 f_n f_n' \Big|_{\tau=\frac{\pi}{2\lambda}} \right] = \exp \left[\frac{-1}{2} \left(a^3 \frac{f_n'}{f_n} \Big|_{\tau=\frac{\pi}{2\lambda}} \right) \tilde{f}_n^2 \right]$$

one boundary

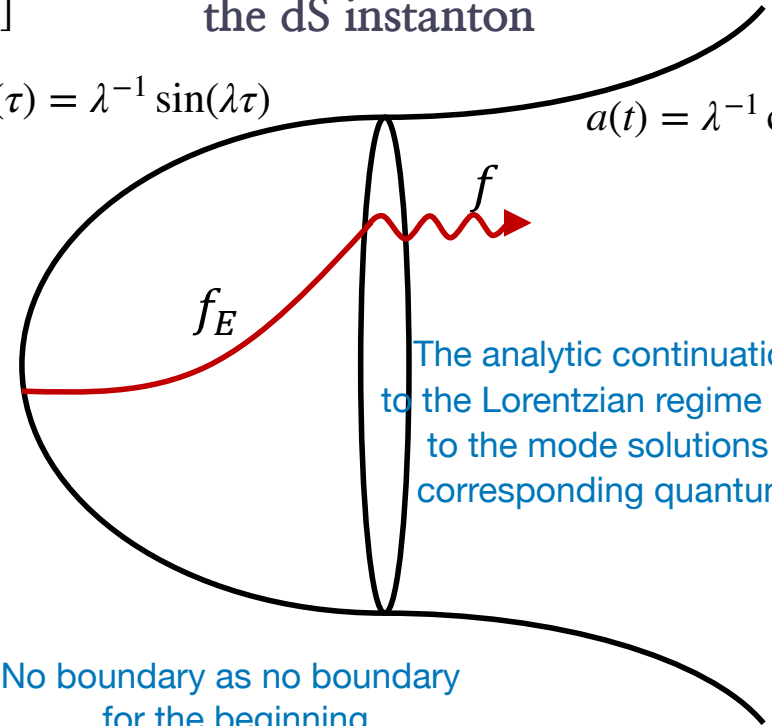
\tilde{f}_n denotes the quantity at the $\tau = \frac{\pi}{2\lambda}$ boundary

$$f_n'' + 3 \frac{a'}{a} f_n' - \left(m^2 + \frac{n^2 - 1}{a^2} \right) f_n = 0$$

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$$\psi_n(\tilde{f}_n) \propto \exp\left[\frac{-1}{2}a^3 f_n f_n' \Big|_{\tau=\frac{\pi}{2\lambda}}\right] = \exp\left[\frac{-1}{2}\left(a^3 \frac{f_n'}{f_n} \Big|_{\tau=\frac{\pi}{2\lambda}}\right) \tilde{f}_n^2\right]$$

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\tilde{f}_n denotes the quantity at the $\tau = \frac{\pi}{2\lambda}$ boundary

$$f_n'' + 3\frac{a'}{a}f_n' - \left(m^2 + \frac{n^2 - 1}{a^2}\right)f_n = 0$$

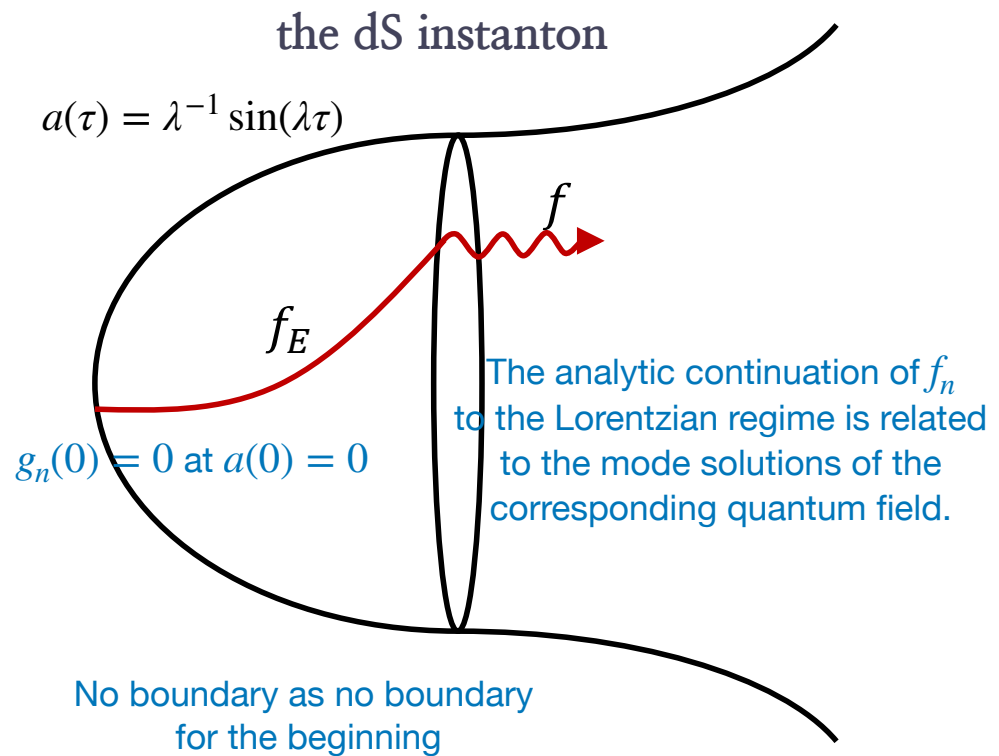
$$f_n(\tau) = A_n g_n(\tau) + B_n d_n(\tau)$$

The issue of regularity of at $a(0) = 0$

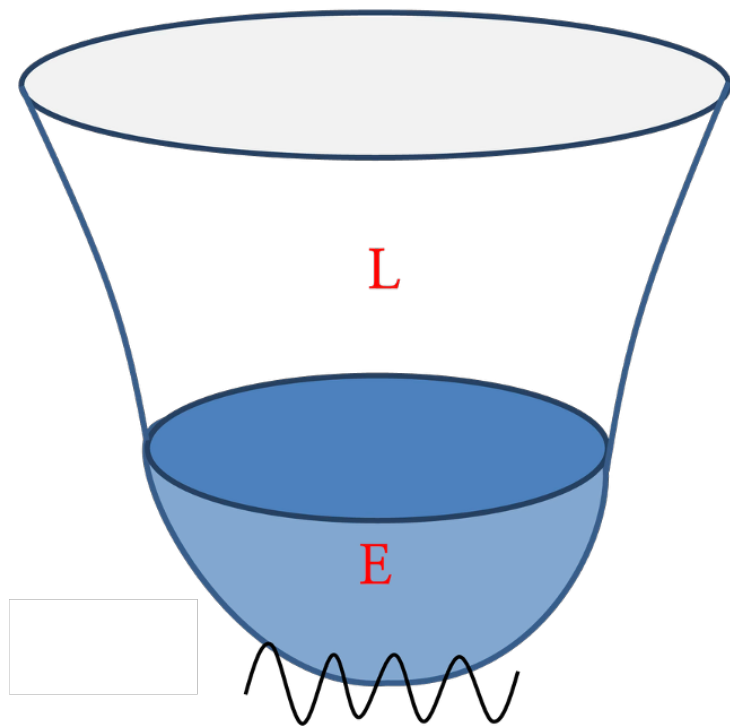
There is only one unique solution that is not divergent there

The analytic continuation of this special mode solution defines

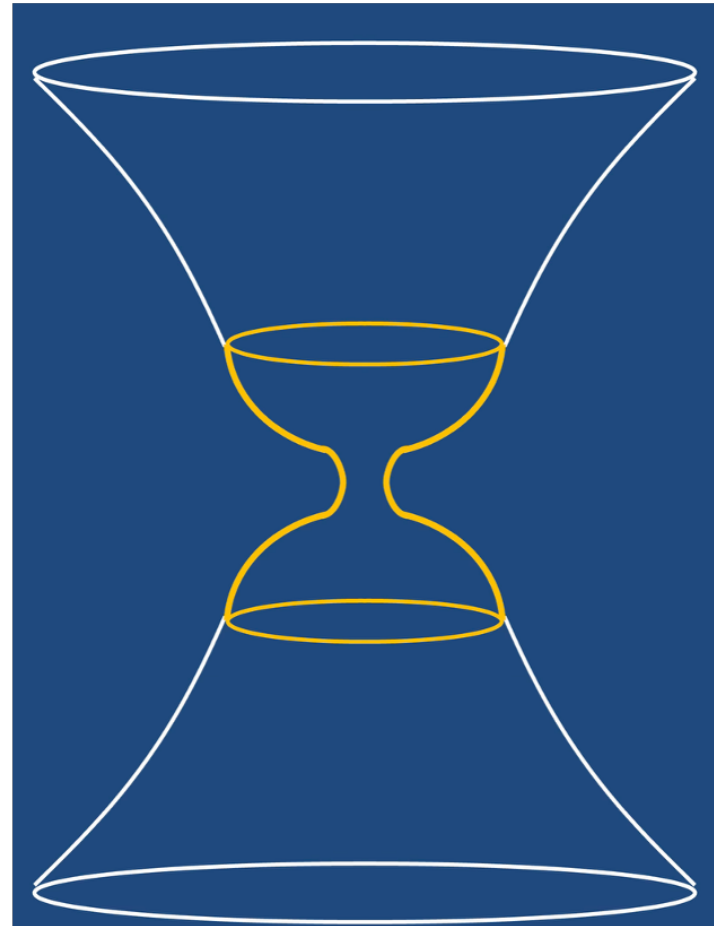
Euclidean (Bunch-Davies) vacuum

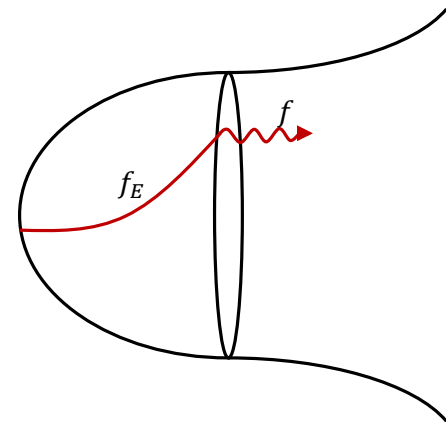
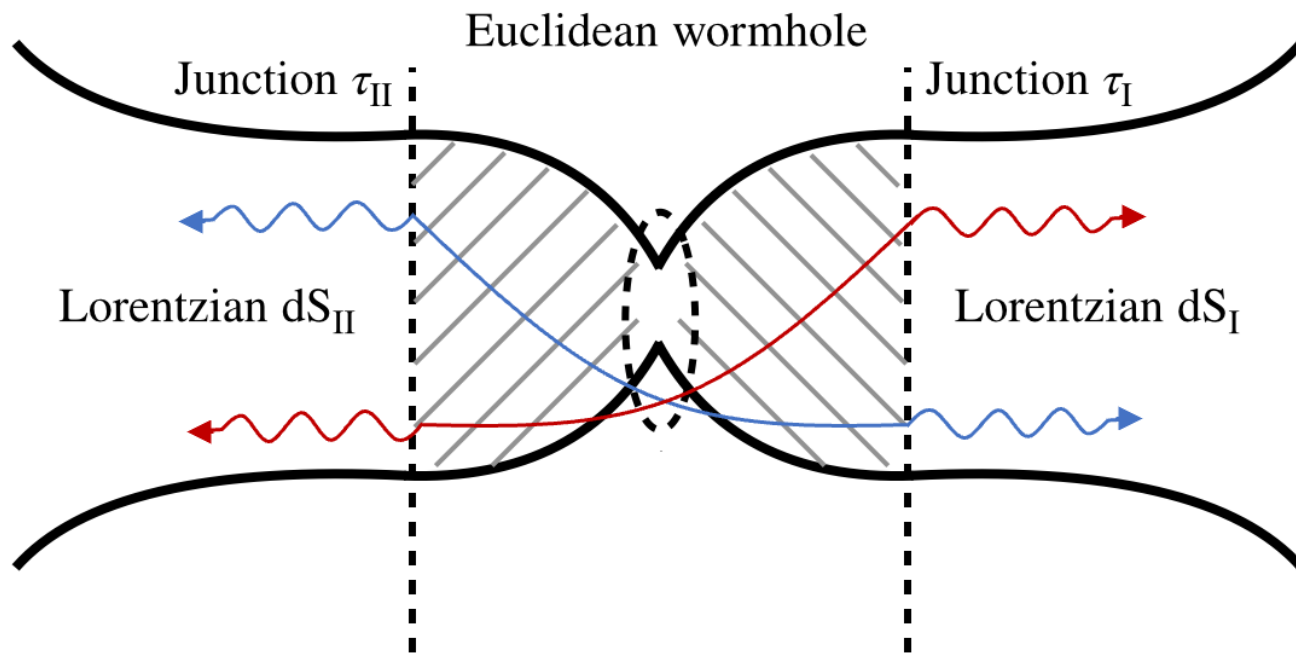


dS (NB) instanton \Rightarrow Euclidean vacuum



Euclidean wormhole \Rightarrow ???



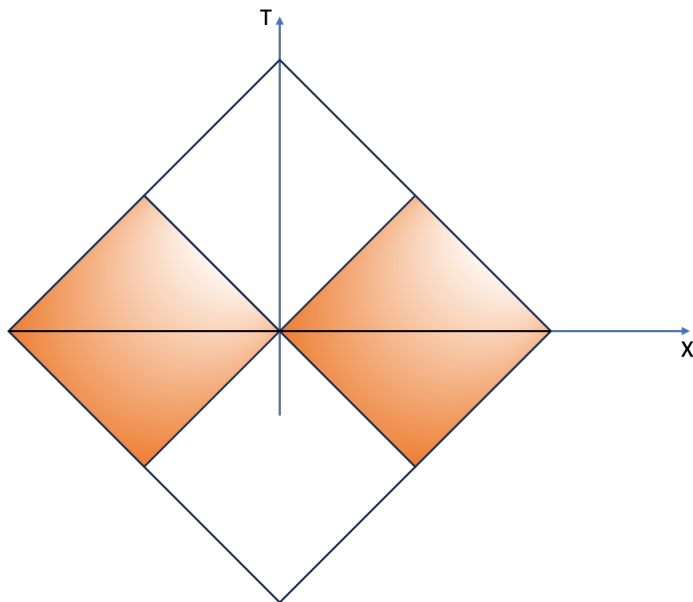


The Unruh effect and the thermofield double state

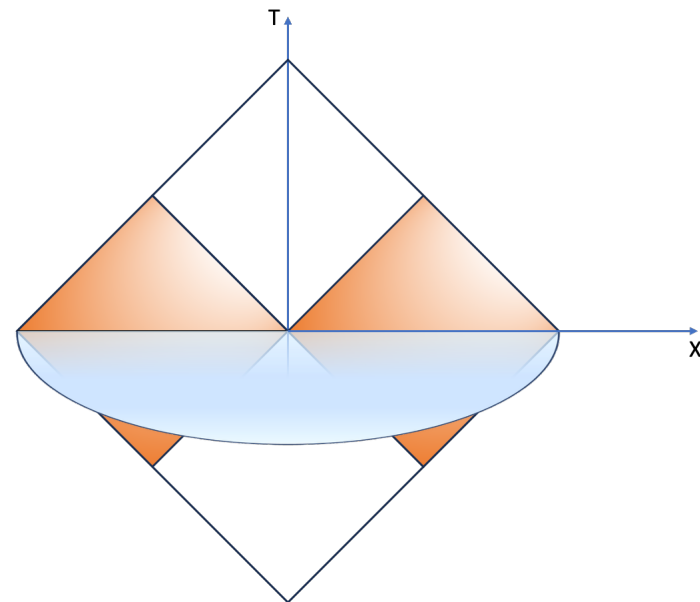
W.G.Unruh; PRD (1976)

W. Israel; PLA (1976)

R. Laflamme; NPB (1989)



The Penrose diagram of the Minkowski spacetime and the Rindler wedges



(a 3D figure)

The Euclidean SHO example:

R. Laflamme; NPB (1989)

$$S_E = \frac{m}{2} \int_{\tau_{\text{II}}}^{\tau_{\text{I}}} d\tau [\dot{x}^2(\tau) + \omega^2 x^2(\tau)]$$

$$K_E(x_f, x_0; \tau_f, \tau_0) = \int_{x(\tau_0)=x_0}^{x(\tau_f)=x_f} \mathcal{D}x e^{-S_E/\hbar} = \left(\frac{-1}{2\pi\hbar} \frac{\partial^2 S_{E(cl)}}{\partial x_f \partial x_i} \right)^{1/2} \exp\left(\frac{-1}{\hbar} S_{E(cl)} \right) \quad (\text{works for free fields})$$

$$\frac{S_{E(cl)}}{m\omega} = \frac{(1+z^2)(x_{\text{I}}^2 + x_{\text{II}}^2) - 4zx_{\text{I}}x_{\text{II}}}{2(1-z^2)}; \quad z = e^{-\beta\omega/2} \quad S_{E(cl)} \text{ is the on-shell action; } \beta/2 = \tau_{\text{I}} - \tau_{\text{II}}$$

The Mehler Formula:

$$\frac{1}{\sqrt{1-z^2}} \exp\left[\frac{4xyz - (1+z^2)(x^2 + y^2)}{2(1-z^2)} \right] = e^{-(x^2+y^2)/2} \sum_{\mu=0}^{\infty} \frac{z^\mu}{2^\mu \mu!} H_\mu(x) H_\mu(y), \quad H_\mu \text{ is the Hermite polynomial}$$

$$e^{-S_{E(cl)}} \propto \sum_{\mu=0}^{\infty} e^{-\mu\beta\omega/2} \frac{e^{-\beta\omega/4}}{2^\mu \mu!} e^{-m\omega x_{\text{I}}^2/2} \times H_\mu\left(\sqrt{m\omega}x_{\text{I}}\right) e^{-m\omega x_{\text{II}}^2/2} H_\mu\left(\sqrt{m\omega}x_{\text{II}}\right),$$

$$|\Omega\rangle = \prod_i |\Omega_i\rangle = \prod_i \frac{1}{|\alpha_i|} \sum_{\mu=0}^{\infty} \left(\frac{\bar{\beta}_i}{\bar{\alpha}_i} \right)^\mu |\mu_i\rangle_{\text{I}} |\mu_i\rangle_{\text{II}}, \quad \{\alpha_i, \beta_i\} \text{ are the Bogoliubov coefficients between the global and local modes}$$

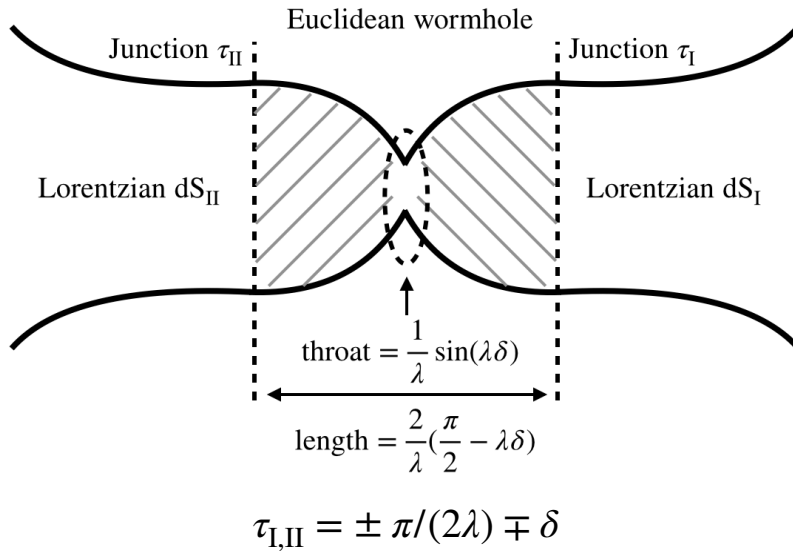
Entanglement between pair-created twin universes with opposite time arrows

should leave a birthmark on CMB spectrum

e-Print: 2505.02807

P. Chen, WL, K. Lin, D. Yeom

the Klebanov-Susskind-Banks (KSB) wormhole
(NPB; 1989)



$$a(\tau) = \lambda^{-1} \sin [\lambda(|\tau| + \delta)]$$

$$S_{E(cl)} = \frac{1}{2} a^3 f' f \Big|_{\tau_{II}}^{\tau_I}$$

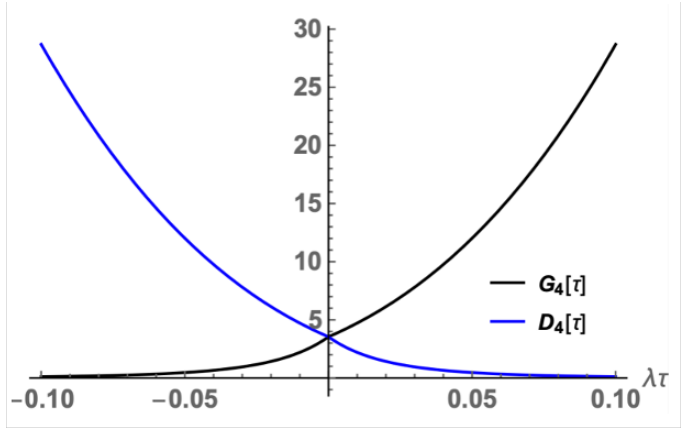
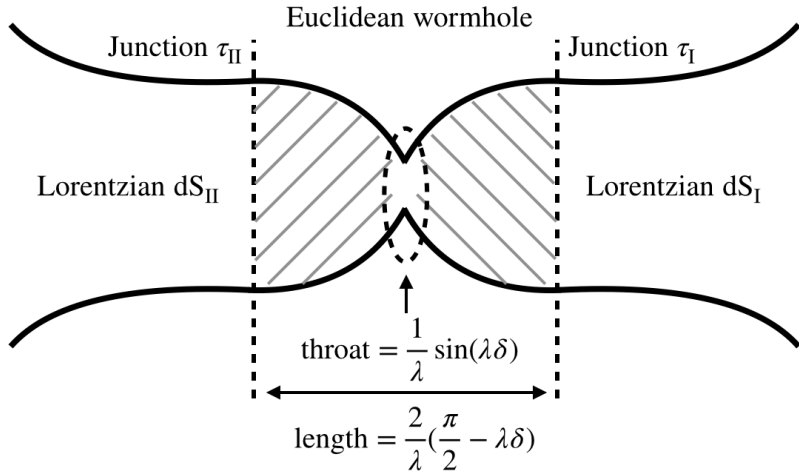
$$g(\tau) = C \frac{[n + \cos(\lambda(\tau + \delta))] \sin^{n-1}(\lambda(\tau + \delta))}{\cos^{2n}(\lambda(\tau + \delta)/2)}$$

$$d(\tau) = \bar{C} \frac{[n - \cos(\lambda(\tau + \delta))] \sin^{n-1}(\lambda(\tau + \delta))}{\sin^{2n}(\lambda(\tau + \delta)/2)}$$

$$C = \frac{e^{in\pi/2}}{2^n \sqrt{2n(n^2 - 1)}}$$

e-Print: 2404.15450

P. Chen, WL, K. Lin, D. Yeom



$$G_n(\tau) = \Theta(\tau)r_n g(\tau) + \Theta(-\tau)r_n^{-1}d(-\tau),$$

$$D_n(\tau) = \Theta(\tau)r_n^{-1}d(\tau) + \Theta(-\tau)r_n g(-\tau),$$

$$S_{E(cl)} = \frac{1}{2}a^3 \dot{f}f \Big|_{\tau_{II}}^{\tau_I}$$

$$-S_{E(cl)} = \frac{4zF_I F_{II} - (1+z^2)(F_I^2 + F_{II}^2)}{2(1-z^2)} \quad z_n = \frac{1}{r_n^2} = \frac{\beta_n}{\alpha_n}$$

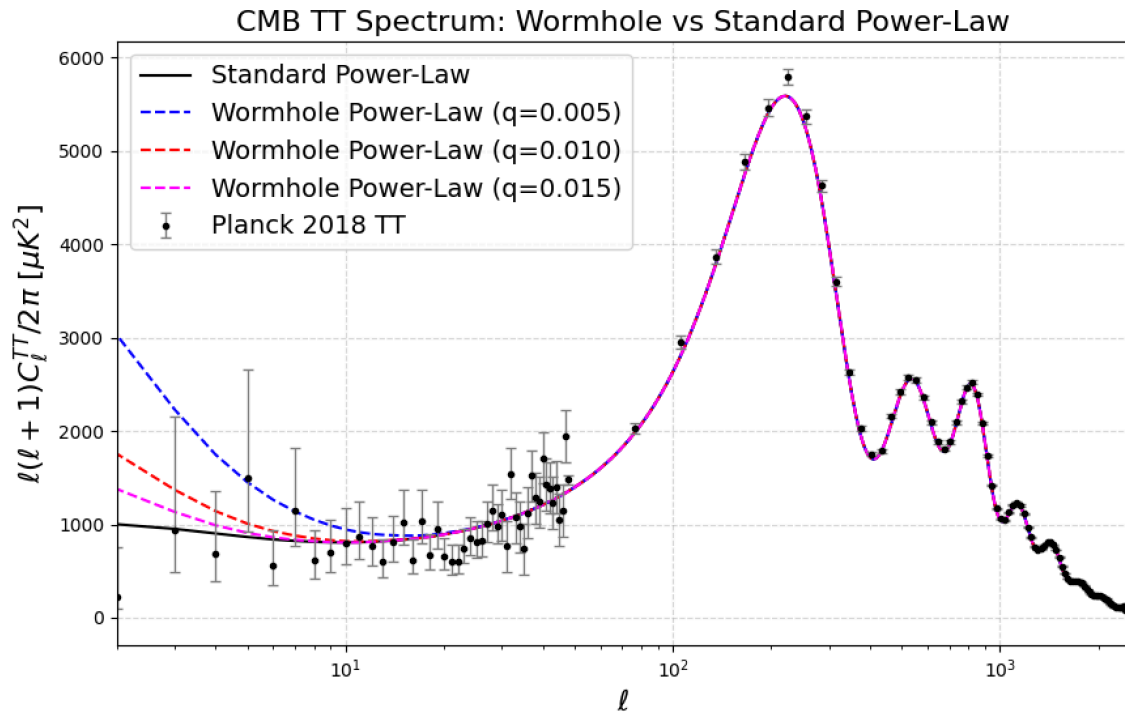
$$\frac{1}{\sqrt{1-z^2}} \exp\left[\frac{4xyz - (1+z^2)(x^2 + y^2)}{2(1-z^2)}\right] = e^{-(x^2+y^2)/2} \sum_{\mu=0}^{\infty} \frac{z^\mu}{2^\mu \mu!} H_\mu(x) H_\mu(y),$$

Continuity of the Euclidean basis function: $G(0_+) = G(0_-)$

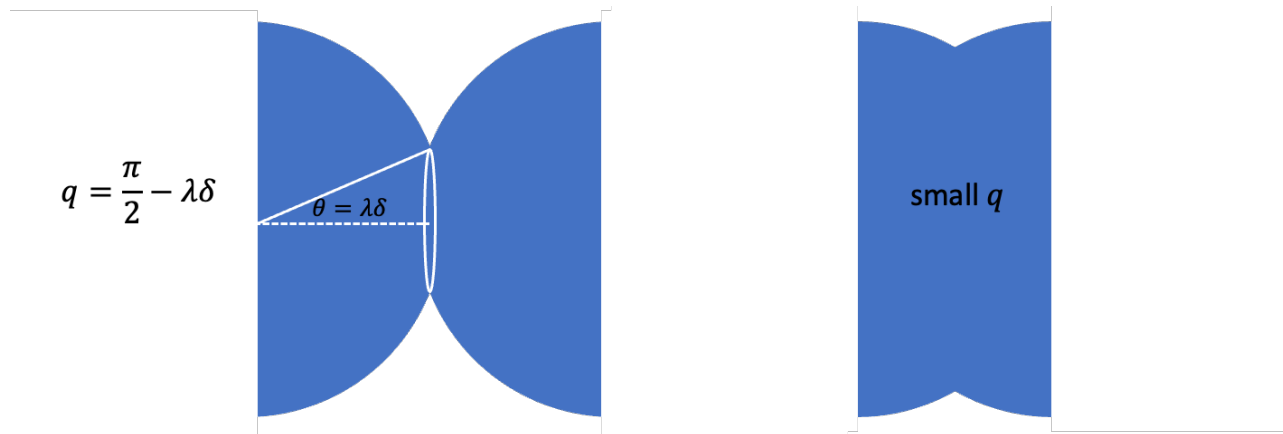
$$z_n = \frac{1}{r_n^2} = \left(\frac{n + \cos \lambda\delta}{n - \cos \lambda\delta}\right) \tan^{2n} \left(\frac{\lambda\delta}{2}\right)$$

$$|\alpha_n|^2 - |\beta_n|^2 = 1$$

$$|\beta_n|^2 = 1/(r_n^4 - 1)$$



$$P(n) = P_{\text{BD}}(n) \left(1 + 2 |\beta_n|^2 \right)$$



Inevitable entanglement from cosmological Euclidean wormhole

P. Chen, WL, K. Lin, D. Yeom (In preparation)

The perturbations on a Euclidean wormhole as Euclidean parametric oscillator

$$K_E(q_f, q_i; \tau_f, \tau_i) = \left(\frac{-1}{2\pi\hbar} \frac{\partial^2 S_{E(cl)}}{\partial q_f \partial q_i} \right)^{1/2} \exp\left(\frac{-1}{\hbar} S_{E(cl)} \right)$$

$$K_E(q_f, q_i; \tau_f, \tau_i) = \sqrt{\frac{W_E}{2\pi\hbar P_E}} \exp\left[\frac{-1}{2\hbar P_E} \left((u_{Ei} v'_{Ef} - u'_{Ef} v_{Ei}) q_f^2 + (u_{Ef} v'_{Ei} - u'_{Ei} v_{Ef}) q_i^2 - 2W_E q_i q_f \right) \right]$$

What is the condition of the mapping between $K_E(q_f, q_i; \tau_f, \tau_i)$ and the Mehler Formula?

$$\frac{1}{\sqrt{1-z^2}} \exp\left[\frac{4xyz - (1+z^2)(x^2+y^2)}{2(1-z^2)} \right] = e^{-(x^2+y^2)/2} \sum_{\mu=0}^{\infty} \frac{z^\mu}{2^\mu \mu!} H_\mu(x) H_\mu(y),$$

Summary

- The perturbations on a Euclidean wormhole instanton leads to entangled initial state of the perturbations between the two Lorentzian universes.
- The relation is a generalization of the thermofield double state. Due to the non-trivial Euclidean geometry, the entangled initial state on one side deviates from a thermal distribution.
- This pair-universe entanglement enhances the long-wavelength modes corresponding to the low l range of the CMB power spectrum.

Thank you!

Supplementary

Regularity at $a(0) = 0$

$$\ddot{f}_n = -3\frac{\dot{a}}{a}\dot{f}_n + \left(\frac{n^2 - 1}{a^2} + \mu^2\right)f_n$$

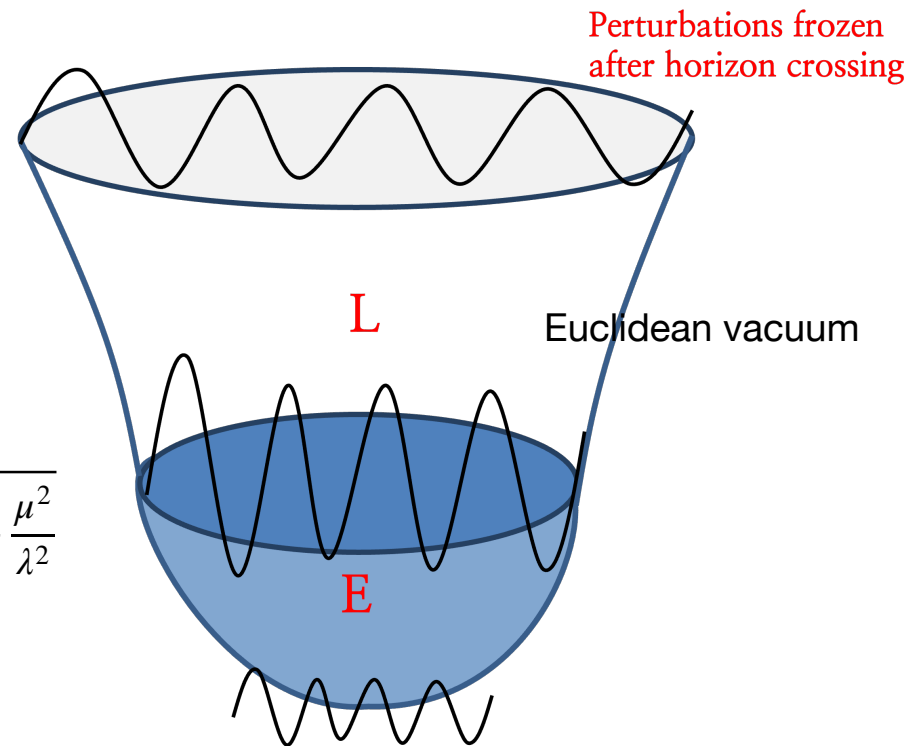
$$a(\tau) = \lambda^{-1} \sin(\lambda\tau)$$

$$\dot{f}_1(\tau = 0) = 0 \quad \text{and} \quad f_{n \geq 2}(\tau = 0) = 0 \quad \text{at} \quad \tau = 0$$

The mode solutions of the Euclidean vacuum

$$f_n(\tau) = D_n (\zeta - \zeta^2)^{(n-1)/2} {}_2F_1(n - \nu, n + \nu + 1; n + 1; \zeta), \quad \nu = -\frac{1}{2} + \sqrt{\frac{9}{4} - \frac{\mu^2}{\lambda^2}}$$

$$\zeta \equiv (1 - \cos(\lambda\tau))/2$$



Euclidean Wormholes

P. Chen, Y. Hu and D. Yeom JCAP (2017)

P. Chen, D. Yeom, EPJ (2018)

Background

$$\dot{a}^2 - 1 + a^2 \left(-\dot{\phi}^2 + V(\phi) \right) = 0,$$

$$\ddot{a} + 2a\dot{\phi}^2 + aV = 0,$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{2}\frac{dV}{d\phi} = 0$$

e.g.

$$V(\phi) = V_0$$

$$\frac{d\phi}{d\tau} = i\frac{A}{a^3},$$

$$\dot{a}^2 = 1 - a^2 - \frac{A^2}{a^4}$$

