Gravitational Waves from Early–Universe Turbulent Sources

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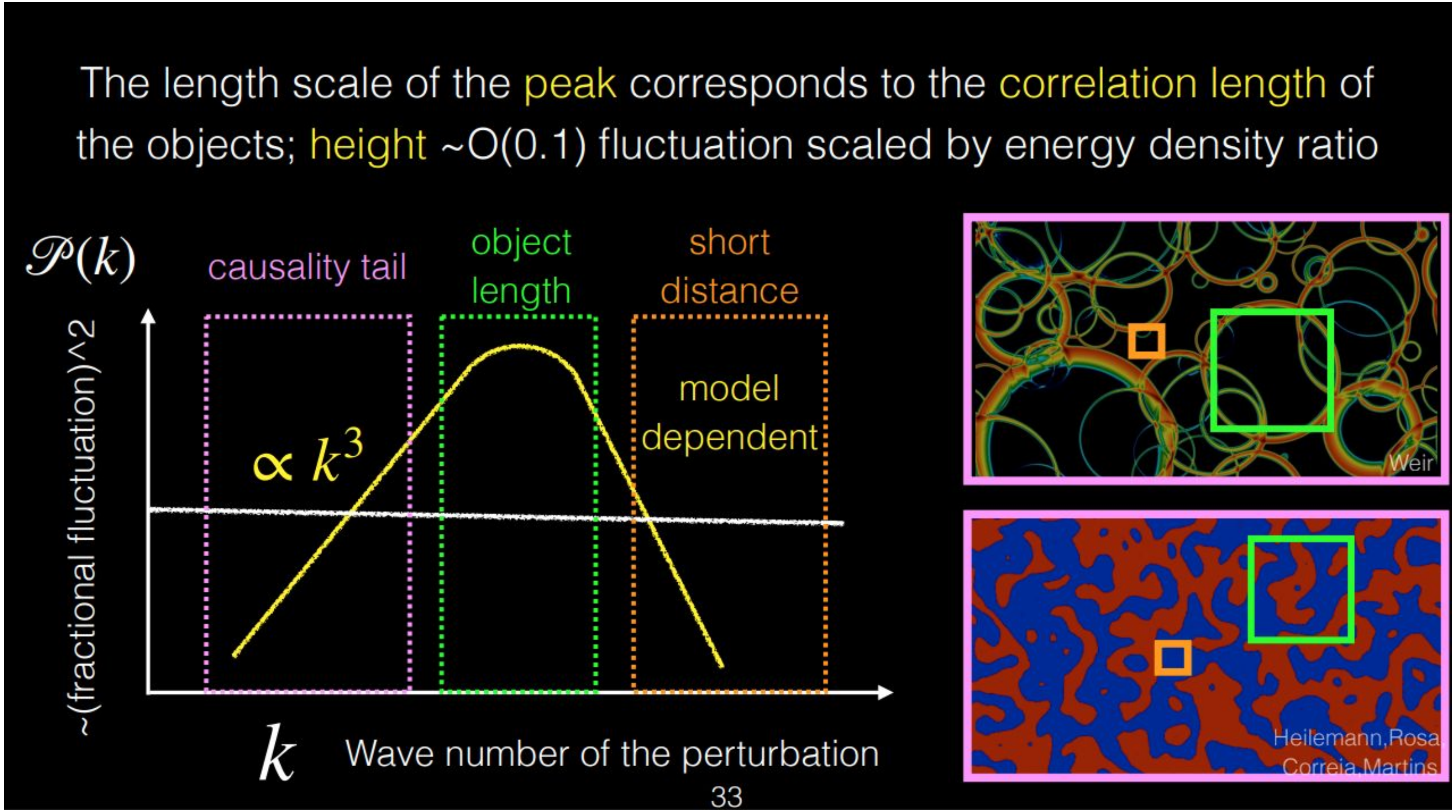
Introduction

- **1980s**: Witten (1984), Hogan (1986) - FOPTs produce GWs.
- **1992-1994**: Kosowsky, Turner & Watkins; Kamionkowski, Kosowsky & Turner - Envelope approximation, analytical parametrization framework.
- **2002**: Kosowsky, Mack & Kahniashvili - Turbulence as a GW source.
- **2007-2009**: Gogoberidze, Kahniashvili & Kosowsky; Caprini, Durrer & Servant - MHD refinements, analytic templates.
- **2010s-today**: Hindmarsh et al.; Roper Pol et al.; Auclair et al. - Numerical simulations.

Slide from Yuhsin Tsai, this Morning's Presentation



From Cosmological Perturbation Theory to GWs

$$h''_A + 2\mathcal{H}h'_A + k^2 h_A = 16\pi G a^2 \sigma_A$$

- **LHS:** Wave equation for first-order TT tensor perturbations with Hubble friction in conformal variables.
- **RHS:** TT anisotropic stress of relevant process which stirs cosmic fluids.

Parameters of the Model

- α : Strength of the phase transition.
- β : Inverse duration of the phase transition.
- T_* : Temperature of the phase transition.
- κ : Efficiency of energy transfer.
- v_w : Wall velocity of the bubbles.

$$\beta^2 h \propto 16\pi G a^2 \sigma$$

$$\dot{h} \propto 16 \frac{\pi G a^2 \sigma}{\beta}$$

With simple dimensional analysis one can generate a rough estimate of the total energy density of the GW radiation and the relevant frequency scales for the signal.
[Caprini & Figueroa (2018)][Kosowsky et. al 1992]

$$\rho_{GW} \propto \frac{1}{32\pi G} \langle \dot{h}\dot{h} \rangle$$

$$f \sim 1.6 \times 10^{-5} [\mathrm{Hz}] \left(\frac{\beta}{H_*}\right) \left(\frac{g_*}{100}\right)^{1/6} \left(\frac{T_*}{100[\mathrm{GeV}]}\right)$$

$$h^2 \Omega_{GW} = 1.6 \times 10^{-5} \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\kappa\alpha}{1+\alpha}\right)^2$$

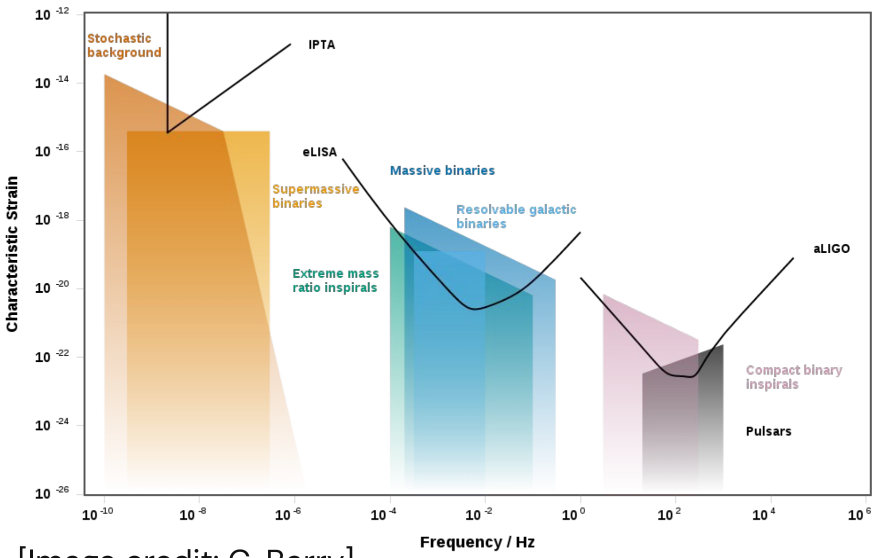
Parameters of the Model

With these equations one can determine the relevance for different physical processes.

- EWPT ~ 100 GeV
- QCDPT ~ 150 MeV

$$f \sim 1.6\times10^{-5}[\mathrm{Hz}]\left(\frac{\beta}{H_*}\right)\left(\frac{g_*}{100}\right)^{1/6}\left(\frac{T_*}{100[\mathrm{GeV}]}\right)$$

$$h^2\Omega_{GW} = 1.6\times10^{-5}\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2$$

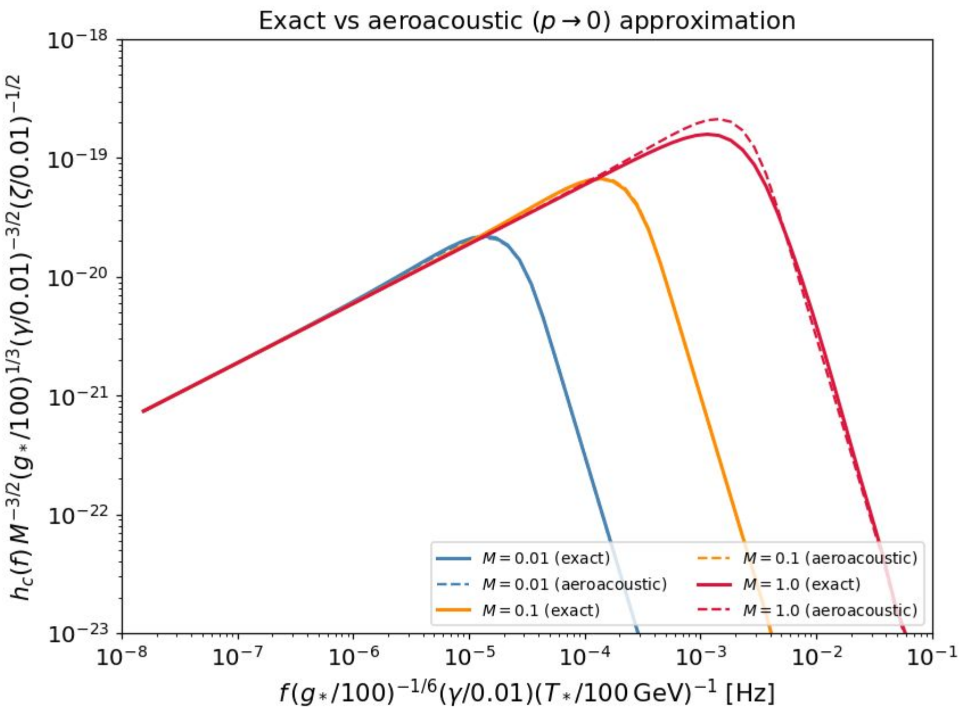


[Image credit: C. Berry]

Gravitational Waves from MHD

Case I: Stationary turbulence

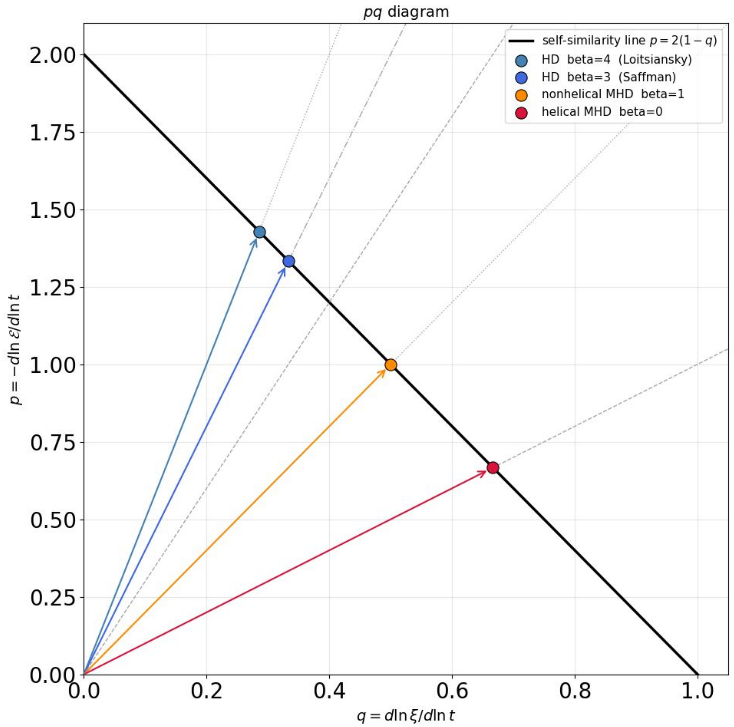
The modeling assumes fully developed stationary **Kolmogorov turbulence** and with **Kraichnan decoration** functions. [Gogoberidze et. al. 2007]



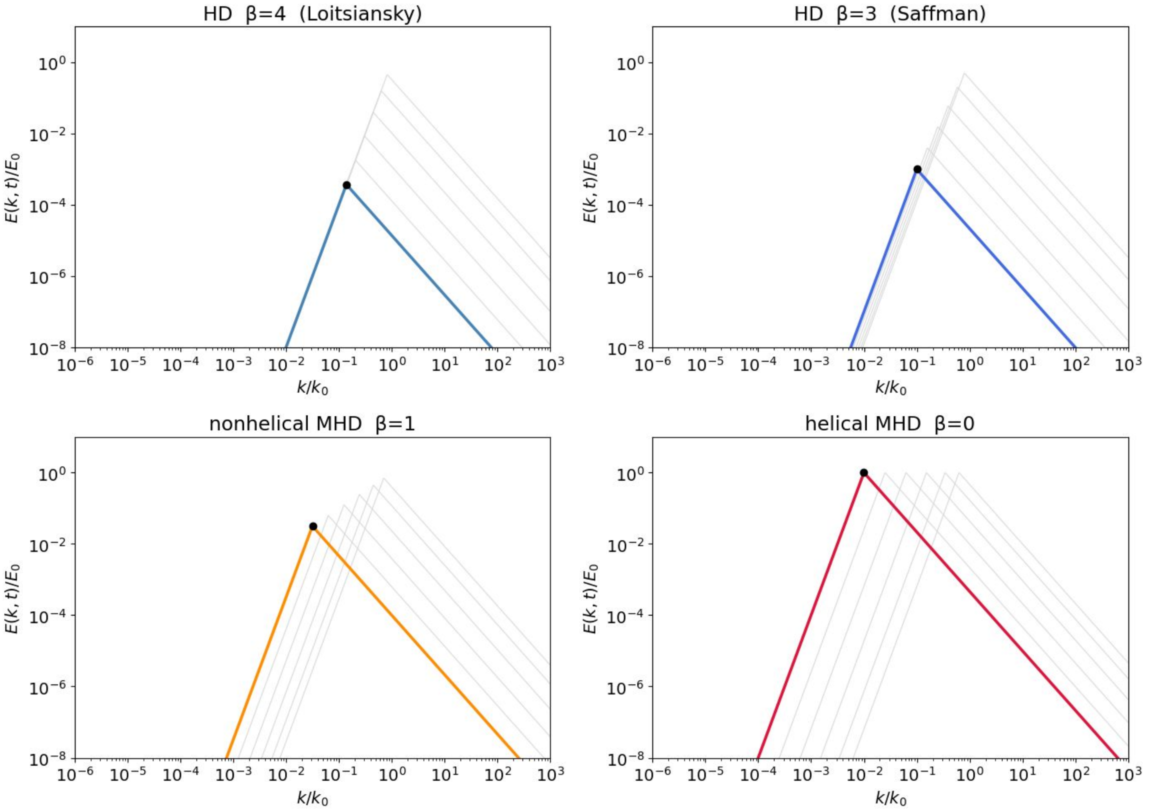
Gravitational Waves from MHD

Case II: Decaying turbulence

To introduce decaying we impose analytical ansatz of power law decay of both energy and coherence length scale. [Brandenburg & Kahniashvili 2016]



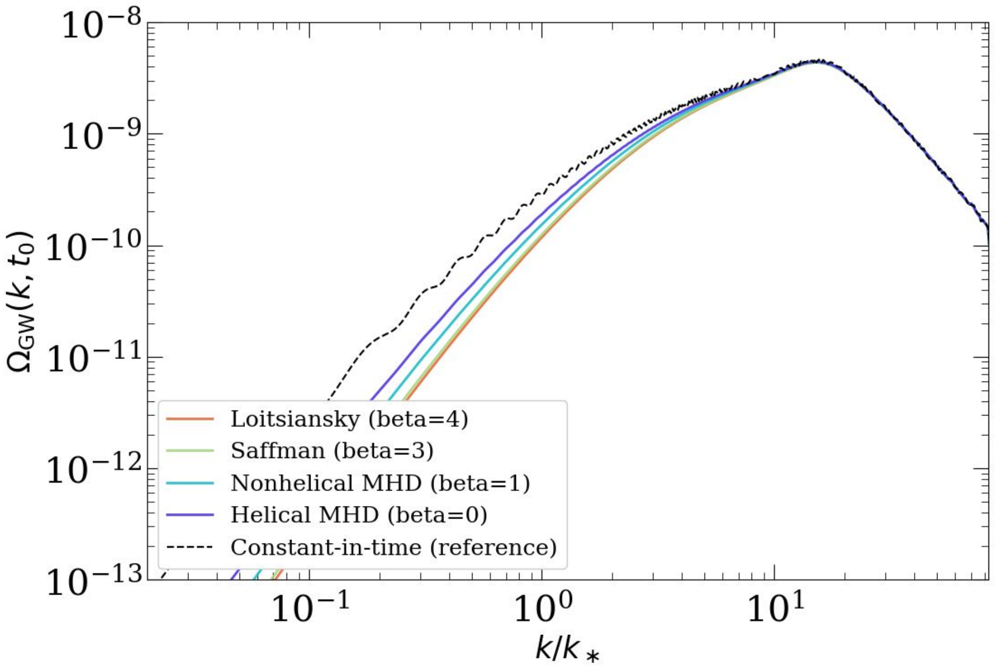
Gravitational Waves from MHD



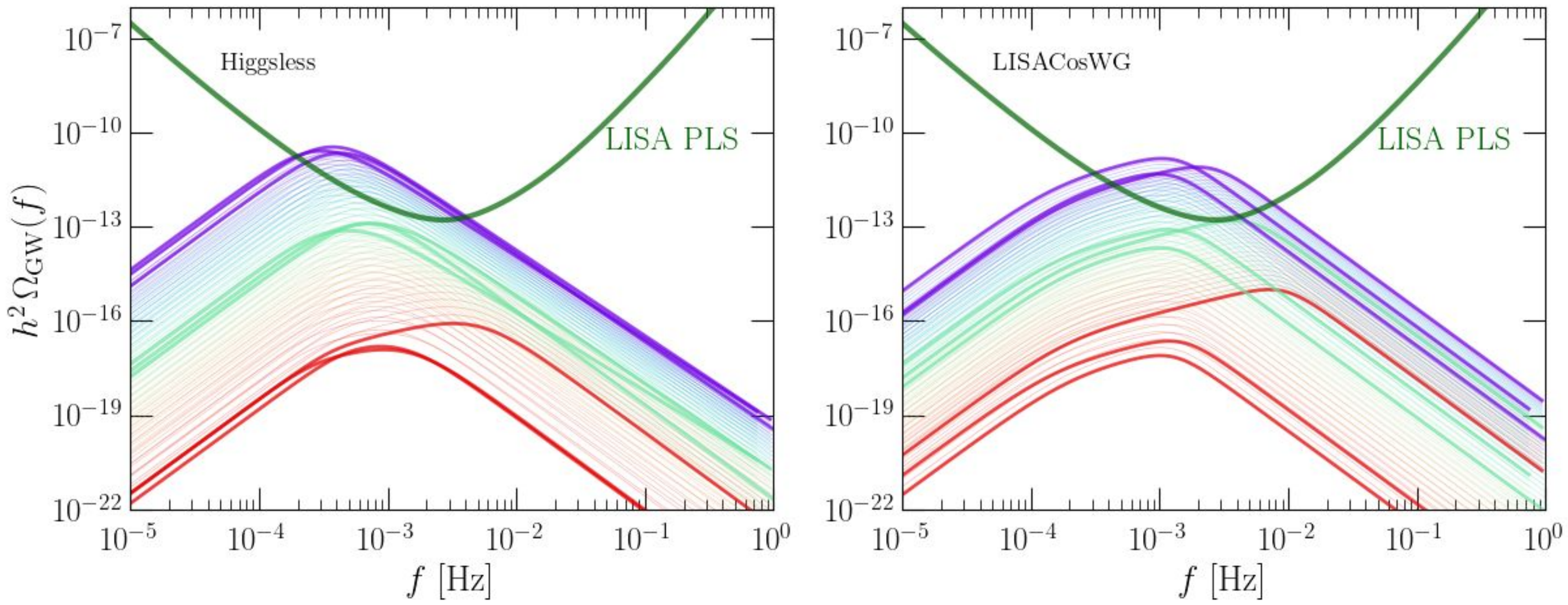
Gravitational Waves from MHD

Case II: Decaying turbulence

To introduce decaying we impose analytical ansatz of power law decay of both energy and coherence length scale. [A. Roper Pol et. al 2025]



Gravitational Waves from Numerical Simulations
CosmosGW
(Soon on Github: https://github.com/CosmoGW)



Conclusions

Two routes, both incomplete
- analytical: transparent and cheap, but closure dependent
- numerical: full physics, but expensive and code dependent

CosmoGW, paper this month, release in two weeks
- one hub for Pencil Code, Cosmos Lattice, Higgsless
- common analytical pipeline, consistent redshifting
- simulations and models on the same footing

Outlook
- feed simulated spectra into the analytical pipeline, test where closures break

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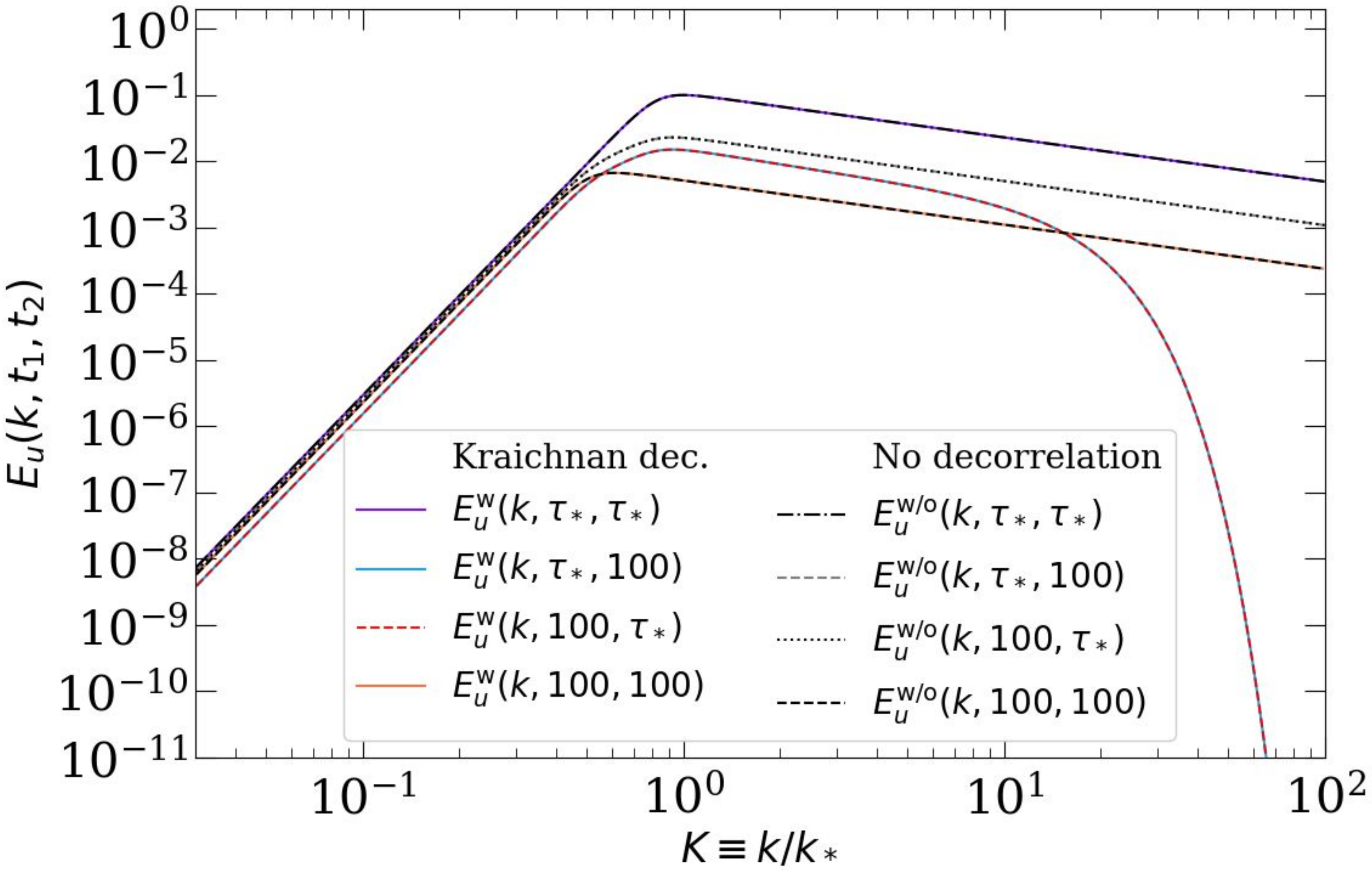

10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$E_u(k, t_1, t_2)$
10^{-1}
10^{0}
10^{1}
10^{2}
$K \equiv k/k_*$
Kraichnan dec.
$E_u^{\rm w}(k, \tau_*, \tau_*)$
$E_u^{\rm w}(k, \tau_*, 100)$
$E_u^{\rm w}(k, 100, \tau_*)$
$E_u^{\rm w}(k, 100, 100)$
No decorrelation
$E_u^{\rm w/o}(k, \tau_*, \tau_*)$
$E_u^{\rm w/o}(k, \tau_*, 100)$
$E_u^{\rm w/o}(k, 100, \tau_*)$
$E_u^{\rm w/o}(k, 100, 100)$

Backup

- **FOPTs Produce GWs 1980s** — Witten (1984), Hogan (1986)
- **Envelope Approximation, Analytical Parametrization Framework** — Kosowsky, Turner & Watkins (1992); Kamionkowski, Kosowsky & Turner (1994)
- **Turbulence as a GW Source** — Kosowsky, Mack & Kahniashvili (2002)
- **MHD Refinements, Analytic Templates** — Gogoberidze, Kahniashvili & Kosowsky(2007,2008) Caprini, Durrer & Servant (2009)
- **Numerical Simulations** — Hindmarsh et al.; Roper Pol et al.; Auclair et al. (2010–present)