Global Extraction of Nuclear Electromagnetic Response Functions (R_L, R_T) and Comparisons to Nuclear Theory and Neutrino/Electron Monte Carlo Generators

1. Testing first principle nuclear theory predictions
2. Provide a platform for verification of electron and neutrino MC generators over the entire kinematic range of interest

A. Bodek[1], M. E. Christy[2], Z. Lin[1], G.-M. Bulugean[1],
A. M. Delgado[1], A. M. Ankowski[3], and J. T. Vidal[4]

[1]The University of Rochester, Rochester, NY, USA

[2]Thomas Jefferson National Accelerator Facility, Newport, VA, USA

[3]University of Wroclaw, Wroclaw, Poland

[4]Tel Aviv University, Israel

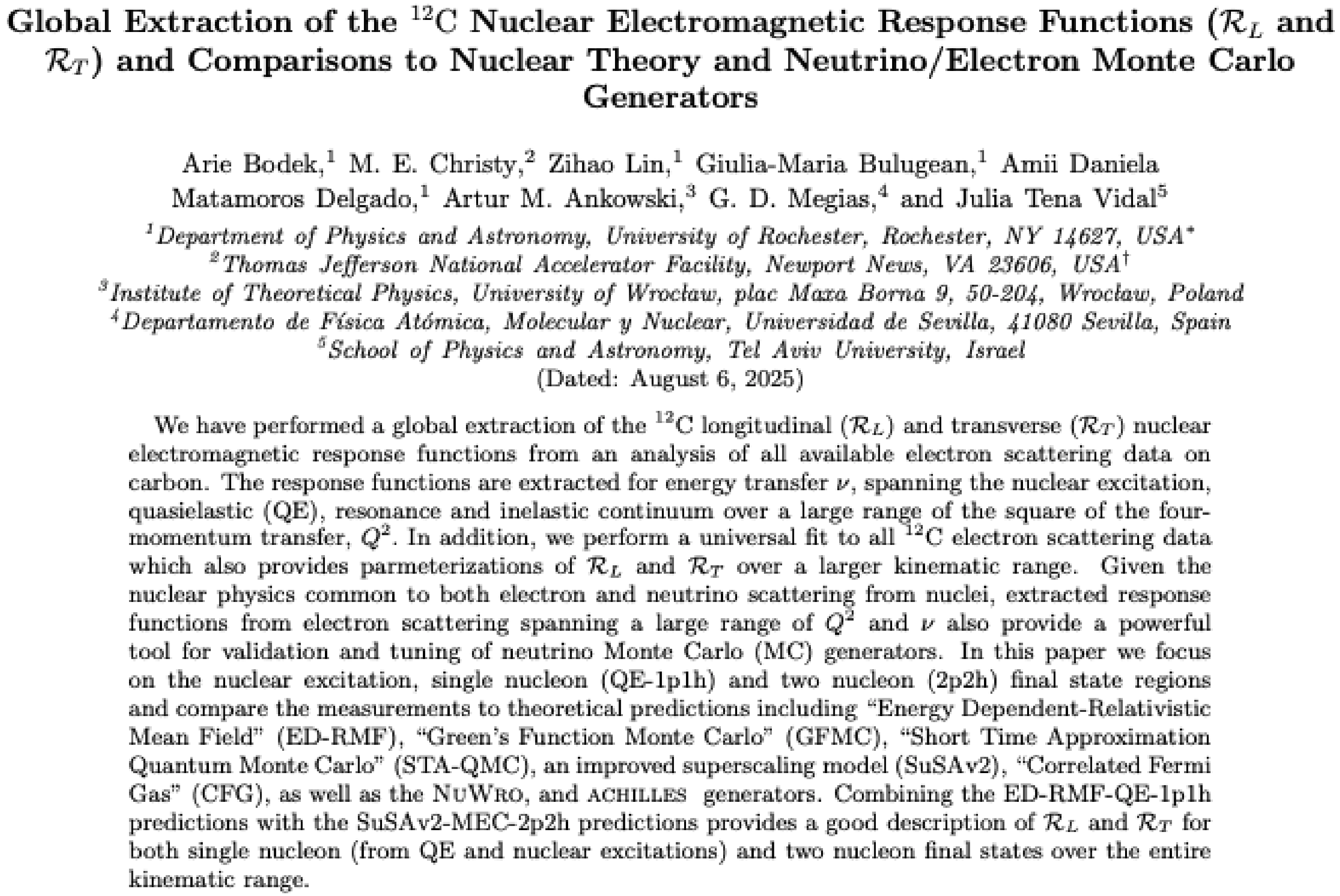
Presented by **Zihao Lin**

Pheno 2026, Pittsburgh

12 min talk + 3 min Q&A

This work was supported in part by the US Department of Energy award DE-SC0008475.

^{12}C analysis: https://arxiv.org/abs/2409.10637, in Phys. Rev. D review
^{40}Ca and ^{56}Fe analysis: in progress



Global Extraction of the ^{12}C Nuclear Electromagnetic Response Functions ($\mathcal{R}_L$ and $\mathcal{R}_T$) and Comparisons to Nuclear Theory and Neutrino/Electron Monte Carlo Generators

Arie Bodek,[1] M. E. Christy,[2] Zihao Lin,[1] Giulia-Maria Bulugean,[1] Amii Daniela Matamoros Delgado,[1] Artur M. Ankowski,[3] G. D. Megias,[4] and Julia Tena Vidal[5]

[1]*Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA**
[2]*Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA*†
[3]*Institute of Theoretical Physics, University of Wrocław, plac Maxa Borna 9, 50-204, Wrocław, Poland*
[4]*Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, 41080 Sevilla, Spain*
[5]*School of Physics and Astronomy, Tel Aviv University, Israel*

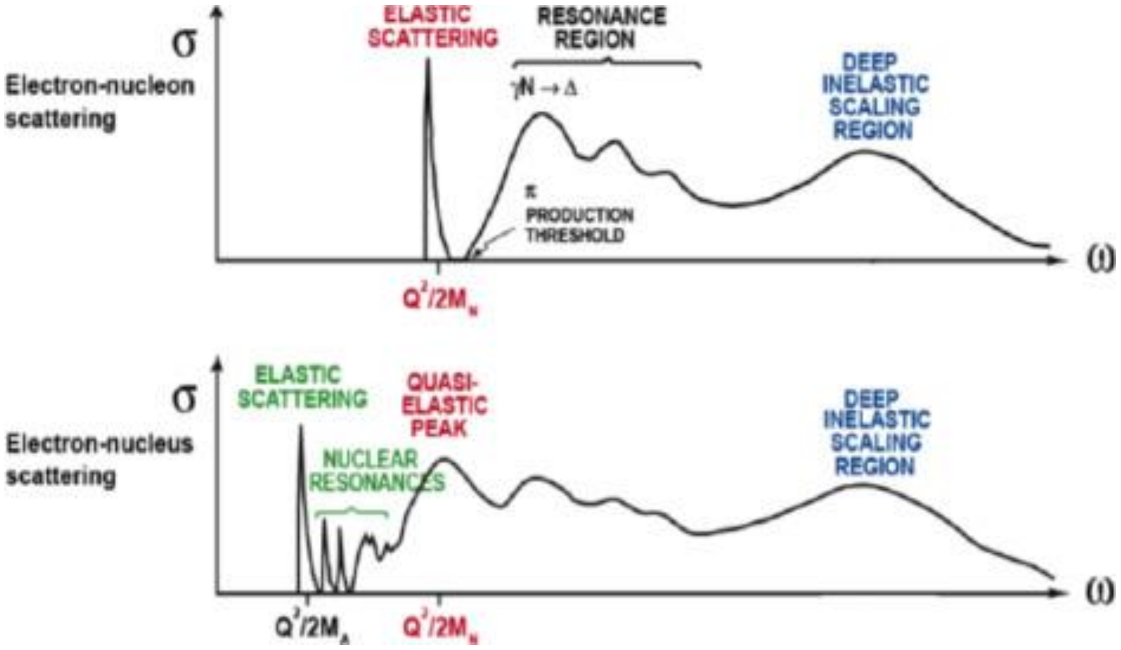
(Dated: August 6, 2025)

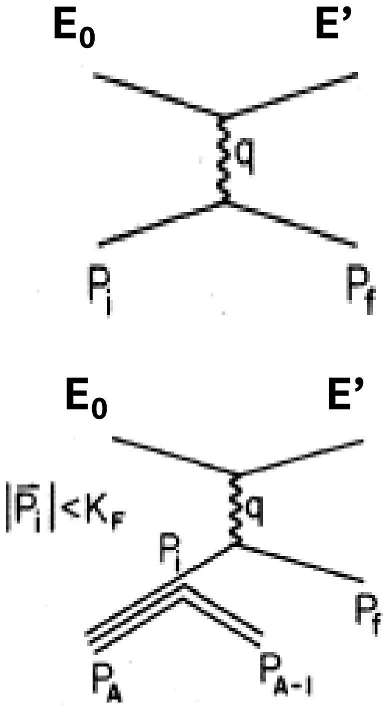
We have performed a global extraction of the ^{12}C longitudinal ($\mathcal{R}_L$) and transverse ($\mathcal{R}_T$) nuclear electromagnetic response functions from an analysis of all available electron scattering data on carbon. The response functions are extracted for energy transfer ν, spanning the nuclear excitation, quasielastic (QE), resonance and inelastic continuum over a large range of the square of the four-momentum transfer, Q^2. In addition, we perform a universal fit to all ^{12}C electron scattering data which also provides parmeterizations of $\mathcal{R}_L$ and $\mathcal{R}_T$ over a larger kinematic range. Given the nuclear physics common to both electron and neutrino scattering from nuclei, extracted response functions from electron scattering spanning a large range of Q^2 and ν also provide a powerful tool for validation and tuning of neutrino Monte Carlo (MC) generators. In this paper we focus on the nuclear excitation, single nucleon (QE-1p1h) and two nucleon (2p2h) final state regions and compare the measurements to theoretical predictions including "Energy Dependent-Relativistic Mean Field" (ED-RMF), "Green's Function Monte Carlo" (GFMC), "Short Time Approximation Quantum Monte Carlo" (STA-QMC), an improved superscaling model (SuSAv2), "Correlated Fermi Gas" (CFG), as well as the NuWro, and ACHILLES generators. Combining the ED-RMF-QE-1p1h predictions with the SuSAv2-MEC-2p2h predictions provides a good description of $\mathcal{R}_L$ and $\mathcal{R}_T$ for both single nucleon (from QE and nuclear excitations) and two nucleon final states over the entire kinematic range.

Introduction

Electron scattering off nucleon and nucleus:

G.T. Garvey et al. / Physics Reports 580 (2015) 1-45





- Given the nuclear physics common to both electron and neutrino scattering from nuclei, we can study electron scattering to validate and tune MC generators for electron and neutrino interactions.

Introduction

Descriptions of electron scattering differential cross section used in the literature:

- In terms of longitudinal and transverse virtual photon cross sections:

$$\frac{d\sigma}{d\Omega dE'} = \Gamma[\sigma_T(W^2, Q^2) + \epsilon\sigma_L(W^2, Q^2)],$$

where Γ is the flux of virtual photons, ϵ is the virtual photon polarization;

- In terms of structure functions:

$$\frac{d\sigma}{d\Omega dE'} = \sigma_M \left[\mathcal{W}_2(W^2, Q^2) + 2\tan^2\left(\frac{\theta}{2}\right)\mathcal{W}_1(W^2, Q^2)\right],$$

where $\sigma_M = \frac{4\alpha^2 E'^2}{Q^4}\cos^2\left(\frac{\theta}{2}\right)$ is the Mott cross section; $\mathcal{W}_1, \mathcal{W}_2$ are related to the $\mathcal{F}_1, \mathcal{F}_2$ structure functions as $\mathcal{F}_1 = M\mathcal{W}_1, \mathcal{F}_2 = \nu\mathcal{W}_2$, M is nucleon mass.

- In terms of longitudinal and transverse electromagnetic response functions $R_L(Q^2, \nu), R_T(Q^2, \nu)$:

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(Q^2, \nu) + \left(\tan^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{2\mathbf{q}^2}\right) R_T(Q^2, \nu)\right].$$

We use the R_L, R_T description.

Introduction

- The three descriptions can translate to each other:

$$R_T = \frac{2\mathcal{F}_1}{M} = \frac{K}{2\pi^2\alpha}\sigma_T,$$

$$R_L = \frac{\mathbf{q}^2}{Q^2}\frac{\mathcal{F}_L}{2Mx} = \frac{\mathbf{q}^2}{Q^2}\frac{K}{2\pi^2\alpha}\sigma_L,$$

where $K = \frac{2M\nu - Q^2}{2M}$, $x = \frac{Q^2}{2M\nu}$; $\mathcal{F}_L = \mathcal{F}_2\left(1 + \frac{4M^2x^2}{Q^2}\right) - 2x\mathcal{F}_1$ is called longitudinal structure function.

- Important quantities:

Longitudinal and transverse response functions R_L, R_T,
energy transfer ν,
4-momentum transfer squared Q^2,
3-momentum transfer $\mathbf{q}$ where $\mathbf{q}^2 = Q^2 + \nu^2$,
nuclear target mass M_A where $M_A = 11.178\ \mathrm{GeV/c^2}$ for $^{12}\mathrm{C}$,
final state invariant mass W where $W^2 = M^2 + 2M\nu - Q^2$,
excitation energy $E_x = \nu - \frac{Q^2}{2M_A}$.

Experimental Method: Rosenbluth Separation

- Rosenbluth reduced cross section[1]:

[1] J. Jourdan, Phys. Lett. B 353, 189 (1995)

$$\Sigma = \left(\frac{E_0}{E_0+V_{eff}}\right)^2 \frac{\mathbf{q}_{eff}^4}{4\alpha^2 E_{eff}'^{\,2}} \frac{1}{\cos^2\left(\frac{\theta}{2}\right)+2\left(\frac{\mathbf{q}_{eff}}{Q_{eff}}\right)^2 \sin^2\left(\frac{\theta}{2}\right)} \frac{d\sigma}{d\nu d\Omega}$$

$$= \epsilon R_L + \frac{1}{2}\left(\frac{\mathbf{q}}{Q}\right)^2 R_T$$

where $\epsilon = \left[1 + 2\left(1 + \frac{\nu^2}{Q^2}\right)\tan^2\left(\frac{\theta}{2}\right)\right]^{-1}$ is the virtual photon polarization.

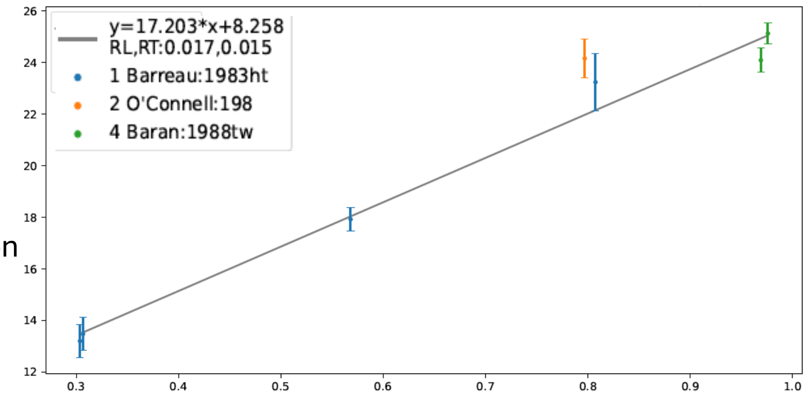
- Fit Σ against ϵ linearly in bins of $|\mathbf{q}|$ (or Q^2) and ν (or W^2, E_x), then

$$R_L = \text{slope}, R_T = 2\left(\frac{Q}{\mathbf{q}}\right)^2 \times \text{ intercept}.$$

$Q^2 = 0.093(\text{GeV/c})^2, \nu = 0.11\text{GeV}$

Σ

Reduced cross section



ϵ

Virtual photon-polarization

Experimental Method: Christy-Bodek Universal Fit

Updated universal fit (A. Bodek, E. Christy. Phys. Rev. C 106, L061305 (2022)):

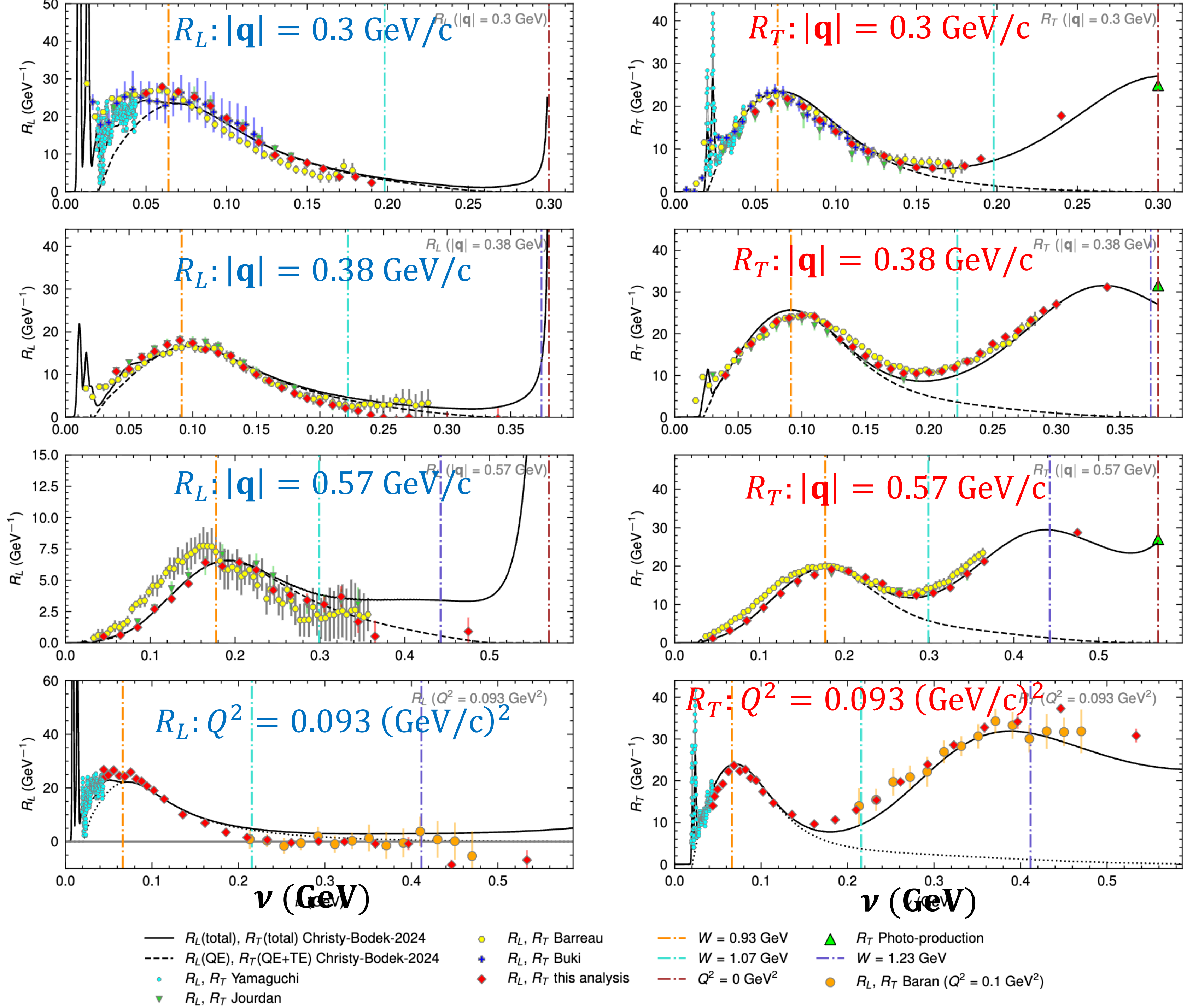
1. Fits for all kinematic regions against ~30 e-Carbon scattering and photo production data sets, ~10,000 cross-section measurements:
   Elastic, nuclear excitations, Quasi-Elastic, resonance and pion production, deep inelastic, photo-production at $Q^2 = 0$ (GeV/c)^2.
2. The fit alone is also a tool to evaluate MC predictions.

We use this fit in Rosenbluth separation:

1. Determine cross section **relative normalizations** and identify inconsistent experiment data.
2. Determine **bin-centering corrections** in individual R_L, R_T extractions, so the cross sections at various ϵ values are at the $\mathbf{q}$ (or Q^2) and ν bin center.

Momentum transfers are also corrected with **Coulomb corrections**: to account for the effective potential $V_{\text{eff}} = 3.1\text{MeV}$ in Carbon nucleus.

We extracted R_L, R_T at 18 fixed $|\mathbf{q}|$ values: $0.1 < |\mathbf{q}| < 2.78$ GeV/c, and at 18 fixed Q^2 values: $0 < Q^2 < 3.45$ (GeV/c)^2, both as functions of ν.
ν ranges from $\nu = 0$ GeV to the end of the resonance region where $W = 2.0$ GeV/c^2.



Compare extracted carbon R_L, R_T to previous extractions$^{2\sim6}$

$|q|$ bins: 0.3, 0.38, 0.57 GeV/c:
Jordan, Barreau, Buki
Q^2 bin: 0.093(GeV/c)^2 : Baran.

Christy-Bodek universal fit's total: black solid line;
QE only: black dashed line.

Our extractions: red scatters.

---

[2] J. Jourdan, Phys. Lett. B 353, 189 (1995) and J. Jourdan, Nucl. Phys. A 603, 117 (1996).

[3] A. Y. Buki et al., Eur. Phys. J. A 57, 288 (2021).

[4] A. Yamaguchi et al., Phys. Rev. C 3, 1750 (1971).

[5] D. T. Baran et al., Phys. Rev. Lett. 61, 400 (1988).

[6] P. Barreau et al., Nucl. Phys. A 402, 515 (1983).

Comparison to Theories and MC generators

We compared our R_L, R_T fit and extracted values to the following theories and MC generations[7~13].
(1b =1 body, 2b = 2 body, 1p1h = 1 nucleon final sate, 2p2h = 2 nucleons final state)

- 1st principle nuclear quantum MC predictions: both 1b and 2b currents

1. GFMC (Green's Function Monte Carlo), 1p1h only, limited **q** range
2. ED-RMF (Energy Dependent Relativistic Mean Field) 1p1h only, all **q** range
   Implemented in NEUT – we find good agreement with QE data
3. STA-QMC (Short Time Approximation Quantum Monte Carlo) 1p1h+2p2h, limited **q** range
4. ACHILLES 1b+2b, 1p1h only, limited **q** range

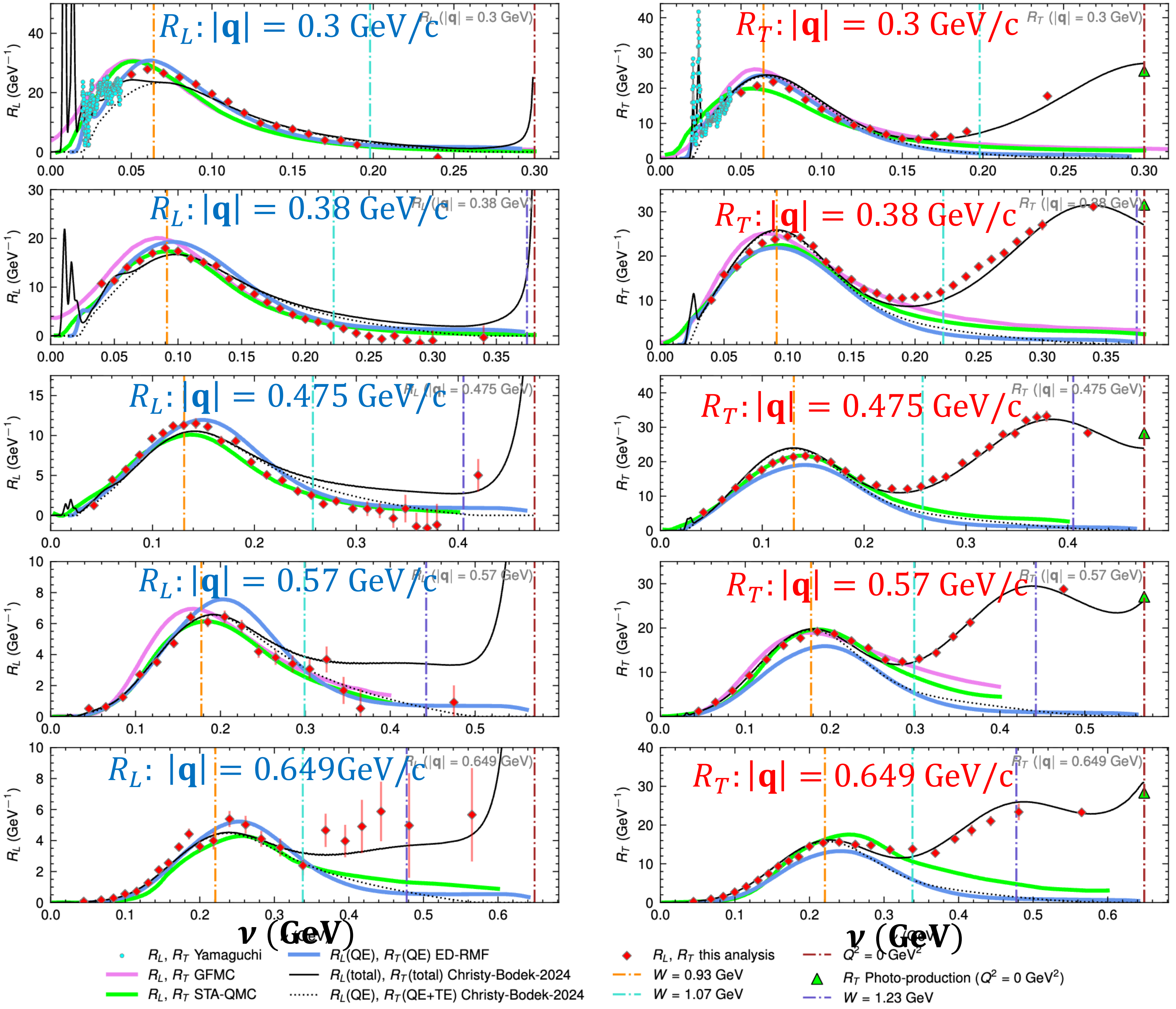
- Other theoretical approaches:

1. NuWro, 1b for 1p1h only in electron mode, 1p1h+2p2h for neutrino model, all **q** range
2. SuSAv2 (Improved Superscaling) 1b for 1p1h, 1b+2b for 2p2h, all **q** range

We gratefully thank the theorists that provide us their predictions!

---

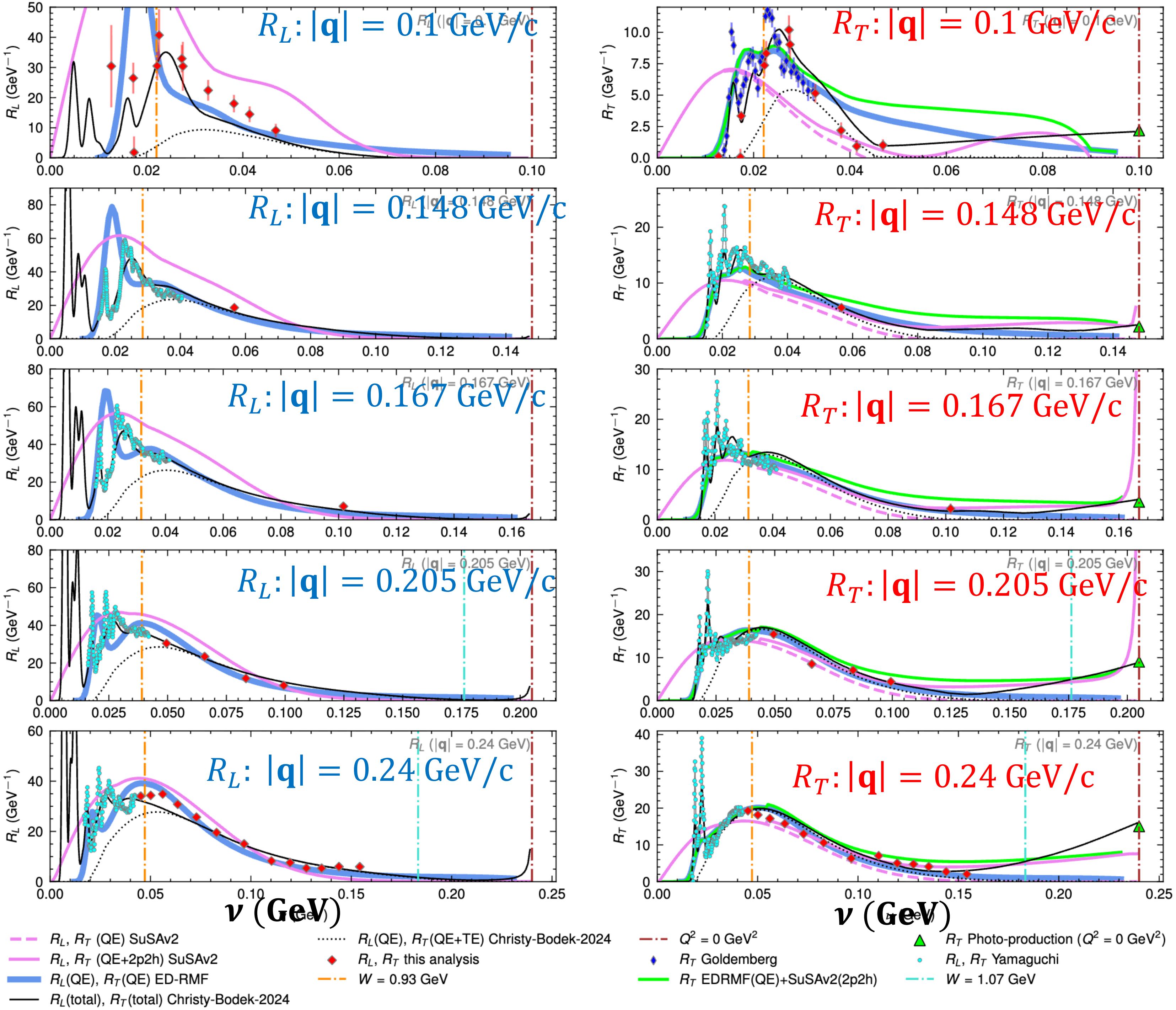
[7] T. Franco-Munoz et al., J. Phys. G: Nucl. Part. Phys. 52 025103 (2025)
[8] S. Pastore et al., Phys. Rev. C 101, 044612 (2020)
[9] J. Issacson et al., Phys. Rev. D 107, 033007
[10] T. Golan el al., Phys.Rev. C86 (2012) 015505
[11] R. Gonzalez-Jimenez et al., Phys. Rev. C 90, 035501 (2014)
[12] B. Bhattacharya et al., Phys. Rev. D 111, 096021 (2025)
[13] A. Lovato et al., Phys. Rev. Lett. 117, 082501 (2016)



Compare carbon R_L, R_T to ED-RMF, GFMC, STA-QMC

at $|\mathbf{q}|$ = 0.3, 0.38, 0.470, 0.570, 0.649 GeV/c

- All 3 predictions are for 1p1h single nucleon final states; all include contributions from 1b+2b currents.
- **GFMC** is computationally expensive, only available for $0.3 \le |\mathbf{q}| \le 0.57\ GeV$.
- **STA-QMC** is only valid for $0.3 \le |\mathbf{q}| \le 0.76\ GeV$.



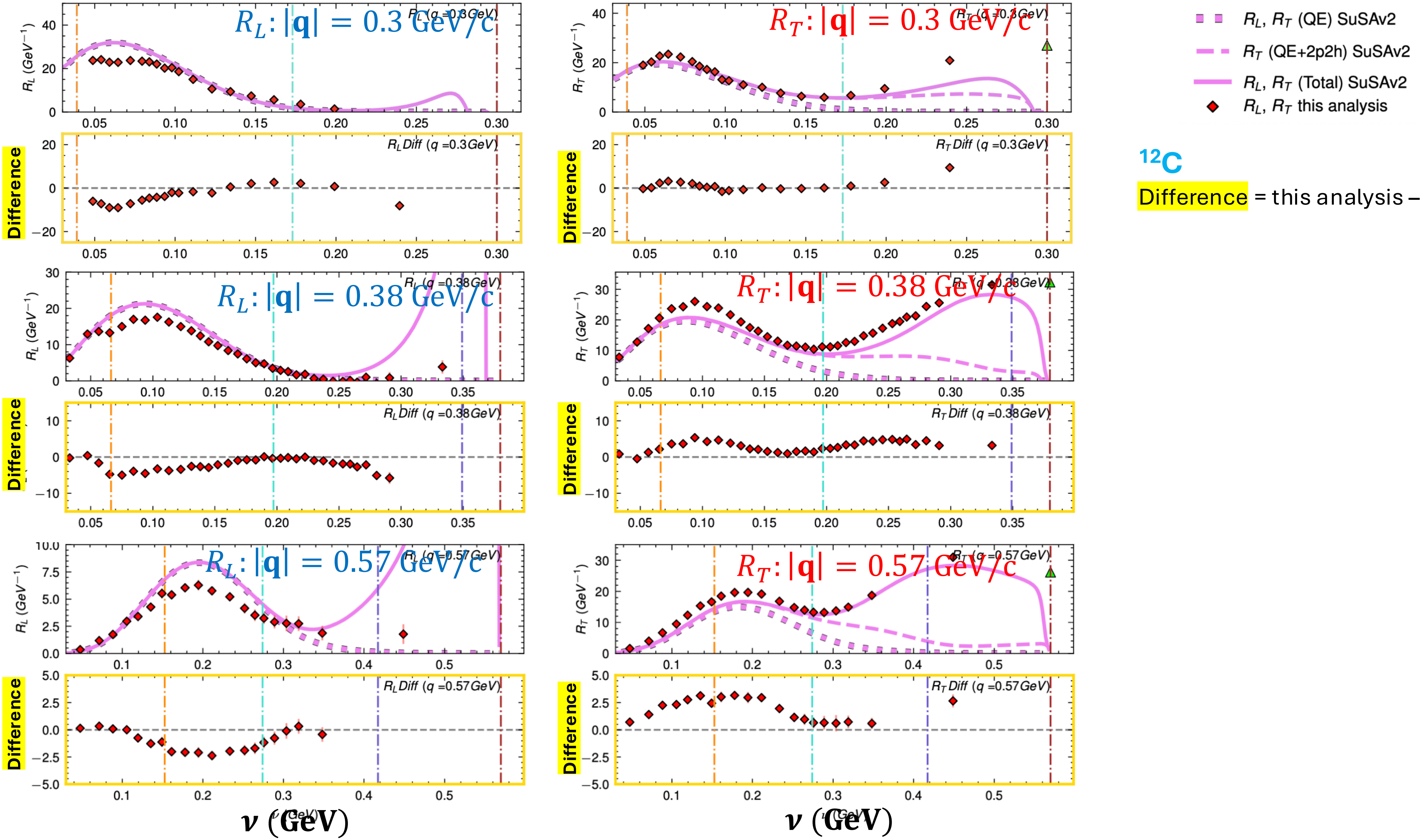
Compare carbon R_L, R_T to ED-RMF (1b+2b) and SuSAv2 (1b) at low q

ED-RMF is the only 1b+2b theory with 1p1h predictions at low **q**, also has prediction for nuclear excitation region.

SuSAv2 doesn't have 2b current contributions to 1p1h, so it overpredicts R_L and underpredicts R_T.

SuSAv2's 1p1h R_L needs quenching at low **q**; R_T needs enhancement.

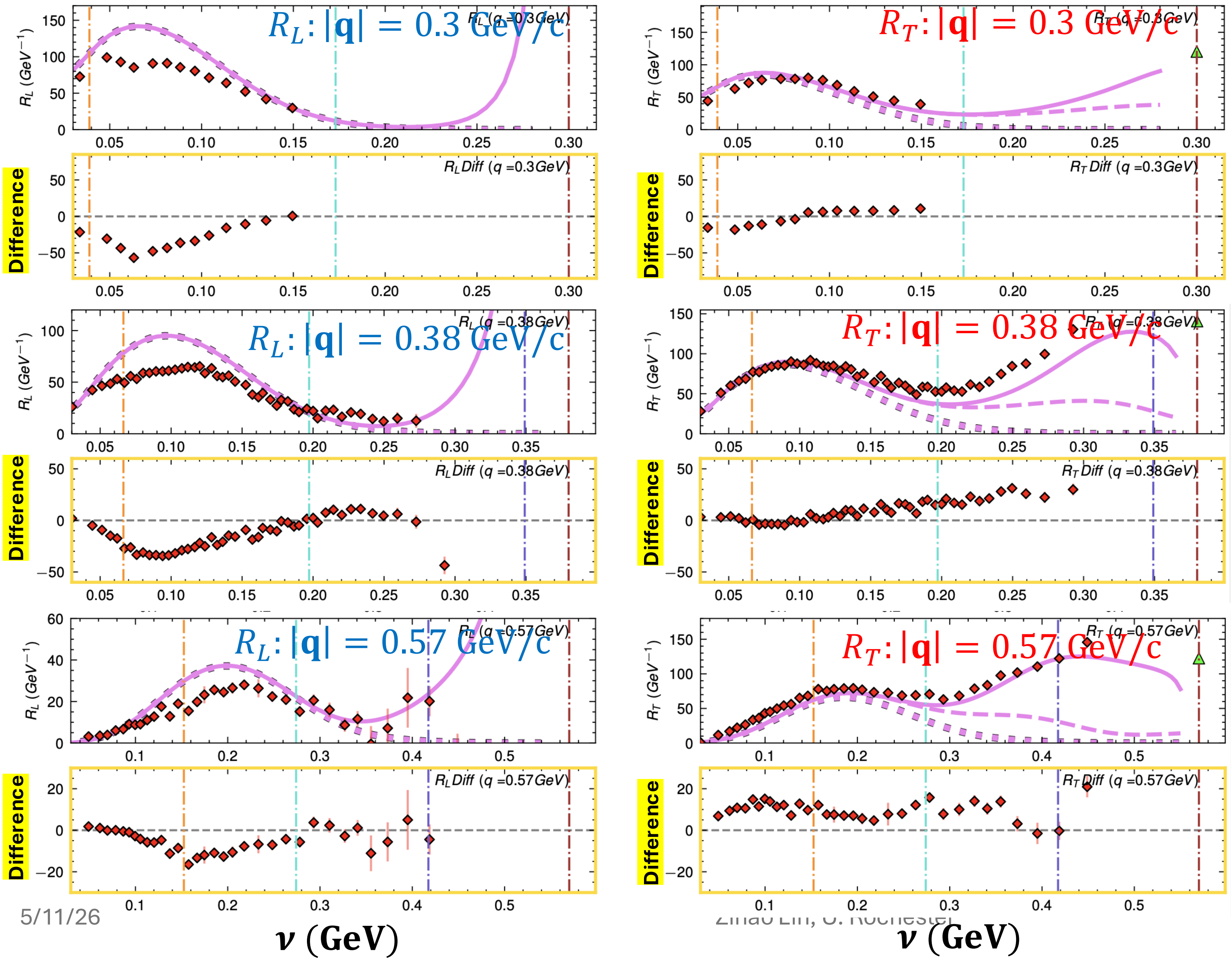
SuSAv2 carbon R_L, R_T comparison difference plots at $|\mathbf{q}|$=0.3, 0.38, 0.57 GeV/c (Preliminary)



^{12}C

Difference = this analysis – SuSAv2

SuSAv2 iron R_L, R_T comparison difference plots at $|\mathbf{q}|$=0.3, 0.38, 0.57 GeV/c (Preliminary)



■ ■ R_L, R_T (QE) SuSAv2

▬ ▬ R_T (QE+2p2h) SuSAv2

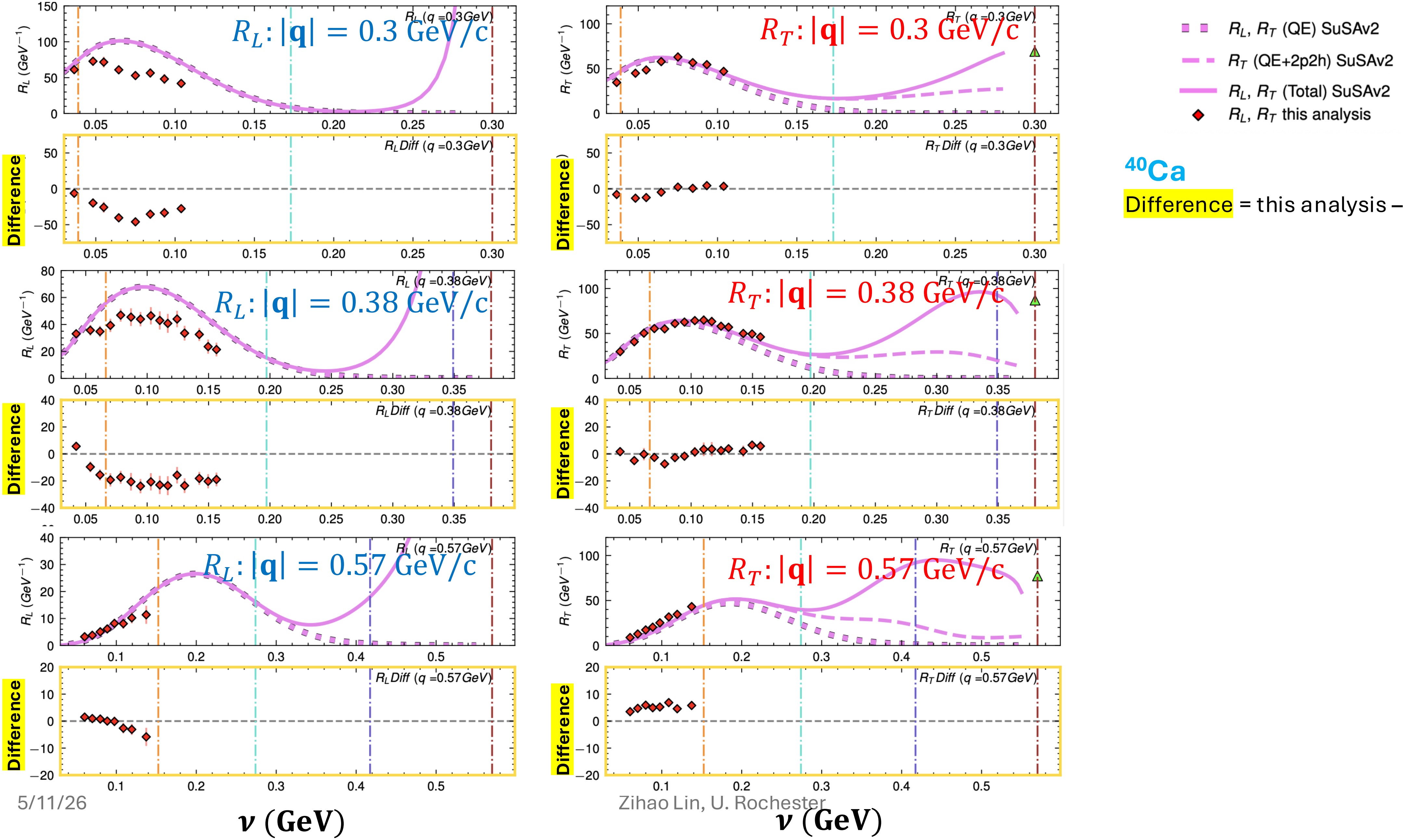
▬ R_L, R_T (Total) SuSAv2

◆ R_L, R_T this analysis

^{56}Fe

Difference = this analysis – SuSAv2

SuSAv2 calcium R_L, R_T comparison difference plots at |**q**|=0.3, 0.38, 0.57 GeV/c (Preliminary)



^{40}Ca

Difference = this analysis – SuSAv2

Conclusion

The R_L and R_T extractions cover a large kinematic range. The values are in good agreement with the Christy-Bodek Universal fit to all cross-section values. The universal fit covers an even larger kinematic range.

The fit provides a simple way to validate electron and neutrino MC generators over a large kinematic range.

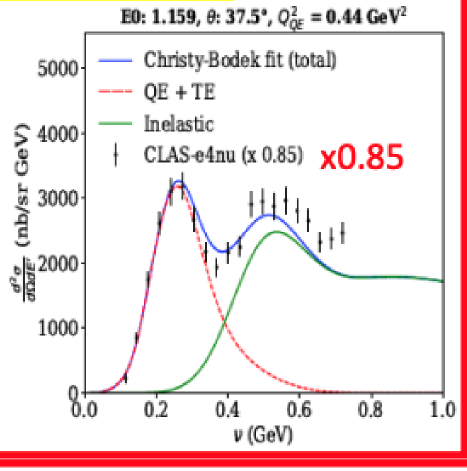
- We compared to GFMC, STA-QMC, ED-RMF's theoretical predictions of R_L and R_T in QE region.
- ED-RMF has the best description of data overall and is available for all values of $|\mathbf{q}|$ (or Q^2) and ν.
- SuSAv2 doesn't includes effects of 2b currents in 1p1h, so it overpredict R_L and underpredict R_T; need corrections.
- More to see at https://arxiv.org/abs/2409.10637; investigation of inelastic processes and ^{40}Ca, ^{56}Fe analyses are under way.

Thank you!

Back up slides start next.

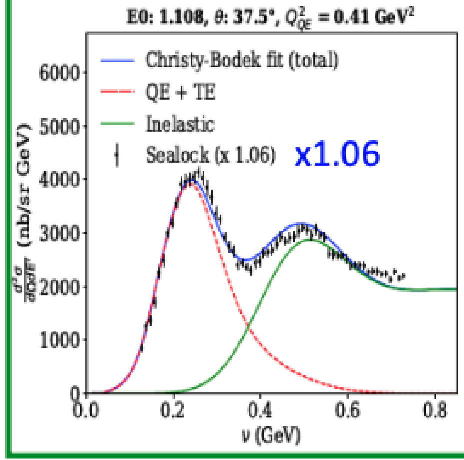
Use fit to **remove some of the 30 experiments which are** inconsistent with world's data

CLAS-e4nu:23 Norm=0.85
Experiment found error after
informed about this



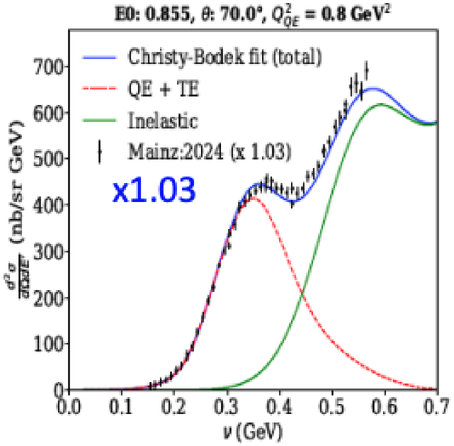
x0.85

Sealock:89
Norm=1.06 OK

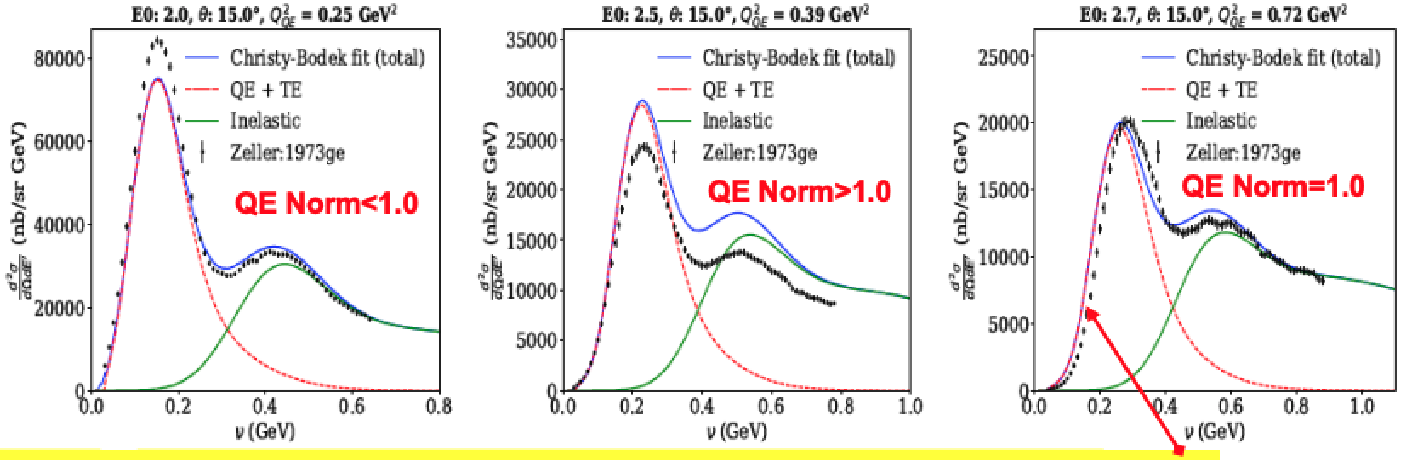


x1.06

Mainz: 2024
Norm=1.03 OK



x1.03



QE Norm<1.0

QE Norm>1.0

QE Norm=1.0

Zeller:73 – Normalizations inconsistent between 3 Energies, ν shift
Also inconsistent normalization between QE and Δ(1238) resonance

Experimental Corrections

- Dataset relative normalizations: ensure that the datasets are consistent with each other in cross-section magnitude (small corrections close to 1).
  - Normalization factors are estimated using Christy-Bodek universal fit[1,2].
- Coulomb corrections: account for the effective electrostatic potential in nucleus.
  - $V_{\text{eff}} = 3.1\text{MeV}$ for Carbon;
  - $E_0 \rightarrow E_{0,\text{eff}} = E_0 + V_{\text{eff}}; E' \rightarrow E'_{\text{eff}} = E' + V_{\text{eff}}$;
  - $Q^2, \mathbf{q}, W^2, \epsilon \rightarrow Q^2_{\text{eff}}, \mathbf{q}_{\text{eff}}, W^2_{\text{eff}}, \epsilon_{\text{eff}}$.

[1] A. Bodek et al., Phys. Rev. C 106, L061305 (2022)
[2] A. Bodek et al., Phys. Rev. C 107, 054309 (2023)

Experimental Corrections

- Bin-centering Corrections: bin-centering corrections account for the small differences between Q^2_{eff} and Q^2_{center} of the binned cross-sections.
  - For cross-section i with Q^2_{eff} and ν, Rosenbluth quantity Σ_i is multiplied with

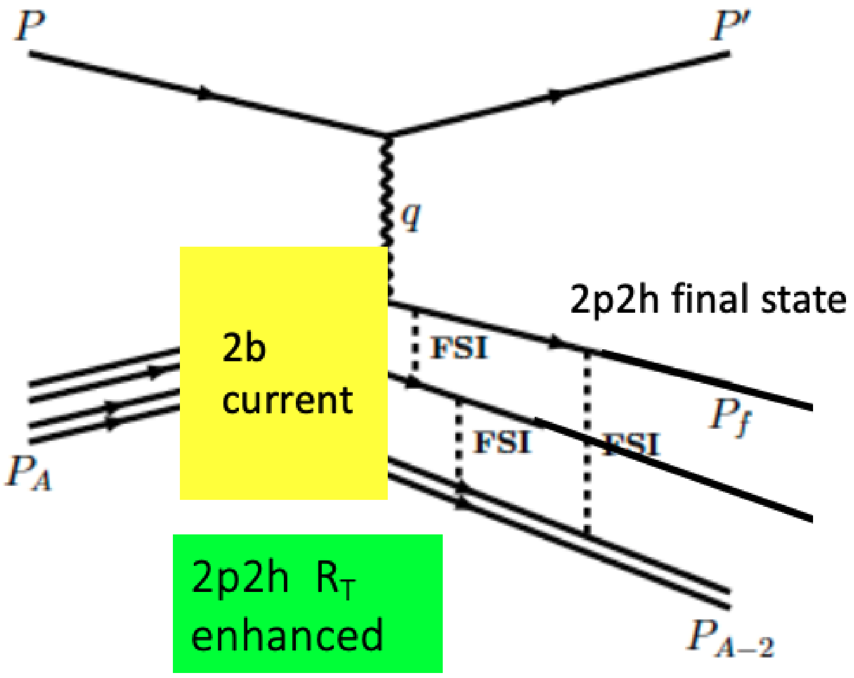
$$C_i = \frac{\epsilon_{\text{center}}\mathcal{R}_L^{\text{fit}}(Q^2_{\text{center}},\nu)+\frac{1\,\mathbf{q}^2_{\text{center}}}{2\,Q^2_{\text{center}}}\mathcal{R}_T^{\text{fit}}(Q^2_{\text{center}},\nu)}{\epsilon_{\text{eff}}\mathcal{R}_L^{\text{fit}}(Q^2_{\text{eff}},\nu)+\frac{1\,\mathbf{q}^2_{\text{eff}}}{2\,Q^2_{\text{eff}}}\mathcal{R}_T^{\text{fit}}(Q^2_{\text{eff}},\nu)},$$

where $\mathcal{R}_L^{\text{fit}}, \mathcal{R}_T^{\text{fit}}$ are estimated by Christy-Bodek universal fit (an iterative process).

  - After bin-centering correction, we can assume that $\Sigma_i\left(Q^2_{\text{eff}}, \nu\right)$ is at Q^2_{center}.

Importance of 2 body currents for 1p1h and 2p-2h

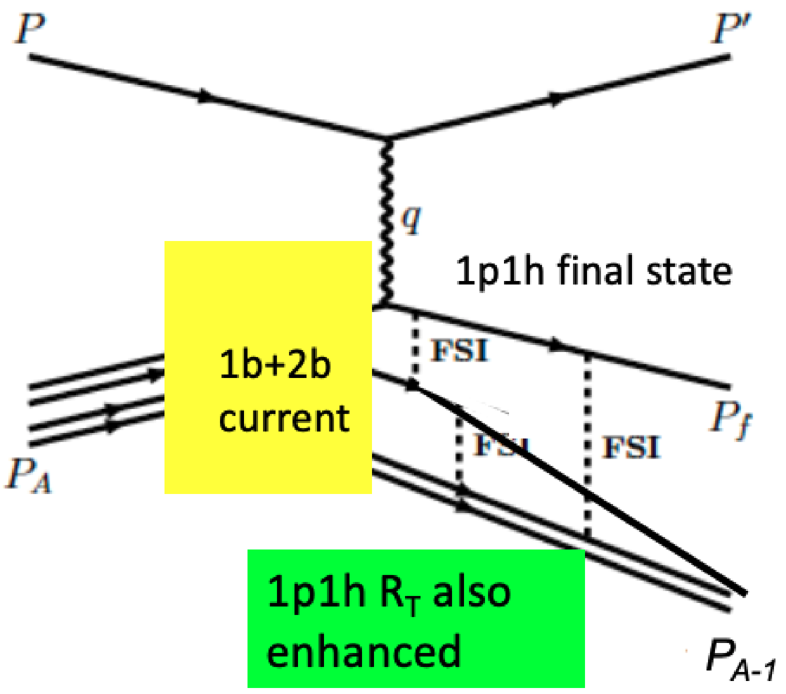
- ➔ 2b currents enhance R_T for 2p2h final states, but also enhance R_T for QE 1p1h final state (2nd nucleon is captured and does not leave).
- ➔ 2b calculations which only model 2p2h final states **are missing a significant 1p1h component** (**1b+2b currents interfere**).
- → Therefore, theories which do not include 2b currents for 1p1h processes **will underpredict R_T** and overpredict R_L at the QE peak



2p2h final state

2b current

2p2h R_T enhanced

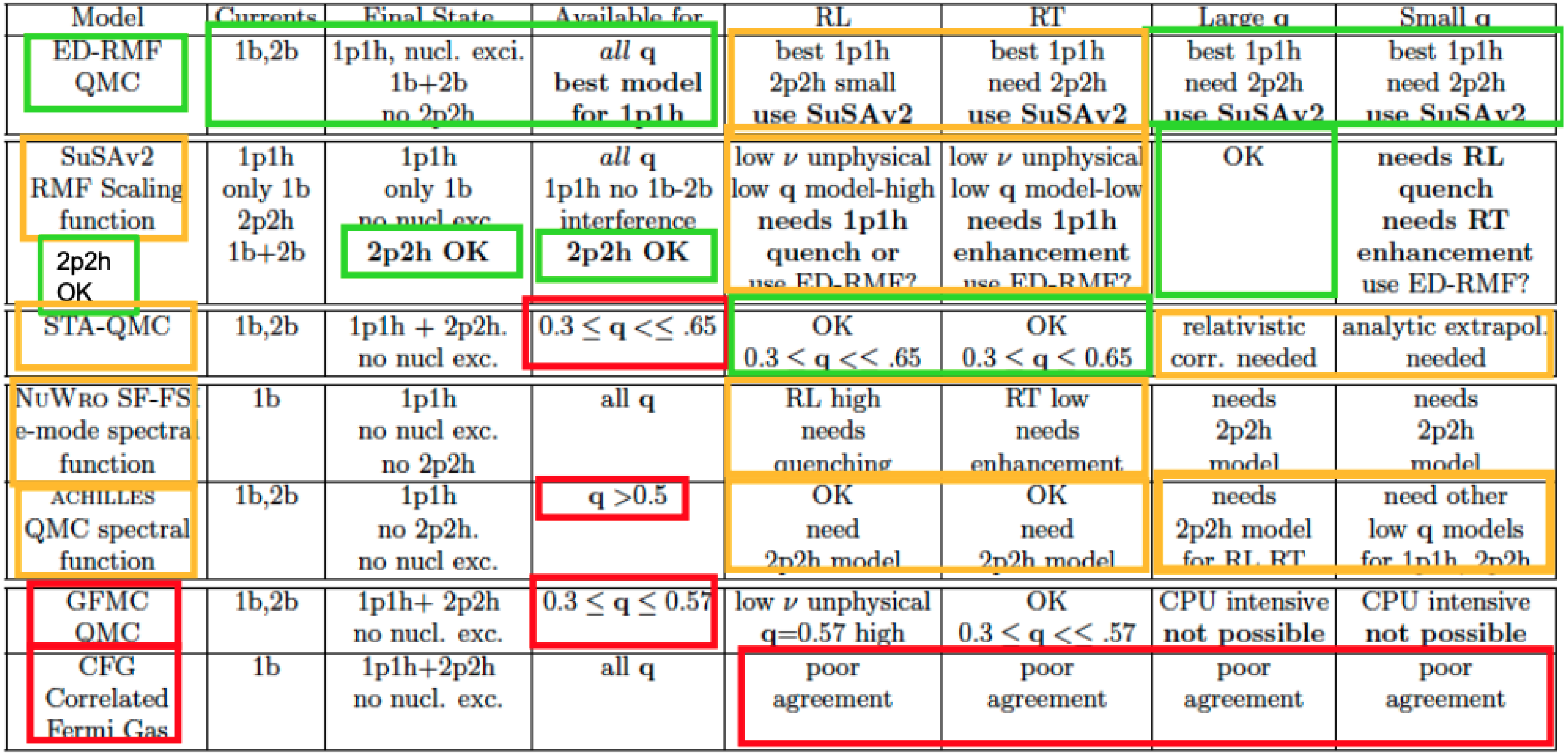


1p1h final state

1b+2b current

1p1h R_T also enhanced

Test: 7 theoretical approaches- Summary table



Model	Currents	Final State	Available for	RL	RT	Large q	Small q
ED-RMF QMC	1b,2b	1p1h, nucl. exci. 1b+2b no 2p2h	*all* q **best model** **for 1p1h**	best 1p1h 2p2h small **use SuSAv2**	best 1p1h need 2p2h **use SuSAv2**	best 1p1h need 2p2h **use SuSAv2**	best 1p1h need 2p2h **use SuSAv2**
SuSAv2 RMF Scaling function 2p2h OK	1p1h only 1b 2p2h 1b+2b	1p1h only 1b no nucl exc. **2p2h OK**	*all* q 1p1h no 1b-2b interference **2p2h OK**	low ν unphysical low q model-high **needs 1p1h** **quench or** use ED-RMF?	low ν unphysical low q model-low **needs 1p1h** **enhancement** use ED-RMF?	OK	**needs RL** **quench** **needs RT** **enhancement** use ED-RMF?
STA-QMC	1b,2b	1p1h + 2p2h. no nucl exc.	$0.3 \leq q << \leq .65$	OK $0.3 < q << .65$	OK $0.3 < q < 0.65$	relativistic corr. needed	analytic extrapol. needed
NuWro SF-FSI e-mode spectral function	1b	1p1h no nucl exc. no 2p2h	all q	RL high needs quenching	RT low needs enhancement	needs 2p2h model	needs 2p2h model
ACHILLES QMC spectral function	1b,2b	1p1h no 2p2h. no nucl exc.	q >0.5	OK need 2p2h model	OK need 2p2h model	needs 2p2h model for RL RT	need other low q models for 1p1h, 2p2h
GFMC QMC	1b,2b	1p1h+ 2p2h no nucl. exc.	$0.3 \leq q \leq 0.57$	low ν unphysical q=0.57 high	OK $0.3 < q << .57$	CPU intensive **not possible**	CPU intensive **not possible**
CFG Correlated Fermi Gas	1b	1p1h+2p2h no nucl. exc.	all q	poor agreement	poor agreement	poor agreement	poor agreement

Table III: A summary of comparisons of ^{12}C $\mathcal{R}_L(q,\nu)$ and $\mathcal{R}_T(q,\nu)$ to theoretical predictions (q units are in GeV).

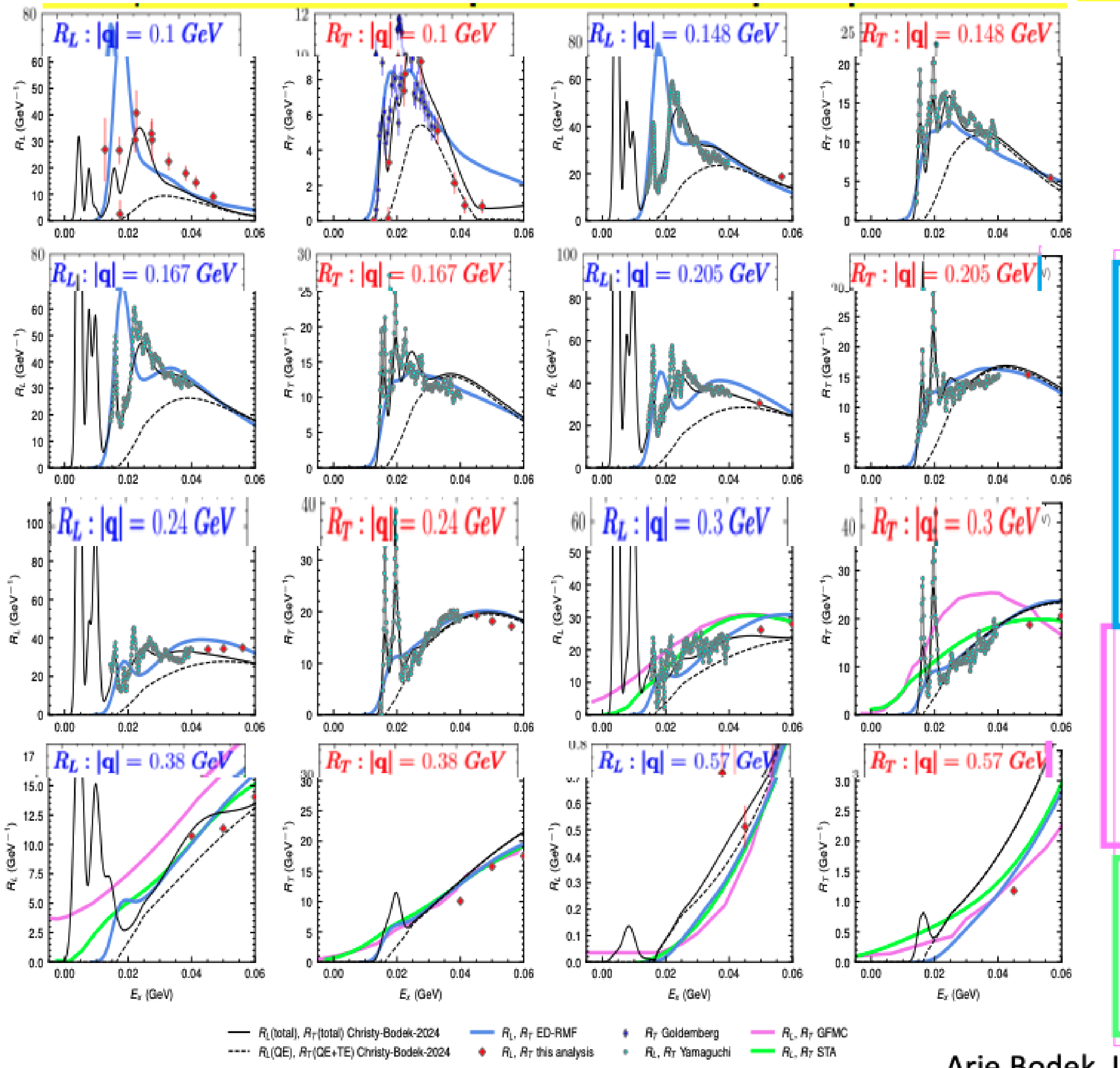
🟩 **Useful for neutrino MC generators in present state**

🟧

🟥 **Problematic in present state**

At present, only. ED-RMF, NuWRo and SuSAv2 make predictions at all q. So all the other models can only be used to compare to these model in a limited range of q. .

Example 5 of 18 values of **q for the three 1st principle Quantum MC predictions**



Comparison in Nuclear excitation Region
(|**q**| Bins)

ED-RMF, available for all |q|, has good agreement with data in QE and Ex region (is now implemented in NEUT generator). ED-RMF Only theory that predicts nuclear excitations on average

GFMC (1b+2b), 1p1h is computationally expensive, only available for $0.3 \le |\mathbf{q}| \le 0.57\ GeV$.

STA-QMC (1b+2b) 1p1h+2p2h is only valid for $0.3 \le |\mathbf{q}| \le 0.76\ GeV$.

29

Arie Bodek, U. Rochester

References and Acknowledgements

We thank Noah A. Steinberg for providing the predictions from the ACHILLES MC generator. We thank Lorenzo Andreoli for providing the predictions from STA-QMC. We thank Sam Carey for providing the predictions from CFG, and Tania Franco Munoz, José Manuel Udías Moinelo and Raúl González Jiménez for clarification of the predictions of ED-RMF.

Research supported in part by the Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177 (Jefferson Lab) and by the U.S. Department of Energy under University of Rochester grant number DE-SC0008475. The work of A. M. Ankowski is partly supported by the Polish National Science Centre under grant UMO-2021/41/B/ST2/02778. The work of Julia Tena Vidal is supported by a European Research Council ERC grant (NeutrinoNuclei, 101078772).

G. D. Megias' work is supported by the Spanish Ministerio de Ciencia, Innovación y Universidades and ERDF (European Regional Development Fund) under contracts PID2020-114687GB-100 and PID2023-146401NB-I00, by the European Union’s Horizon 2020 Research and Innovation Programme under the HORIZON-MSCA-2023-SE-01 JENNIFER3 grant agreement no.101183137, by the Junta de Andalucia (grant No.~FQM160) and by University of Tokyo ICRR’s Inter-University Research Program FY2024 (Ref.~2024i-J-001)