WIMP Meets ALP: Coherent Freeze-out of Dark Matter

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in collaboration with Steven Ferrante and Maxim Perelstein

Outline of the talk

- Motivation
- Framework and dynamics
- Phenomenology
- Conclusions

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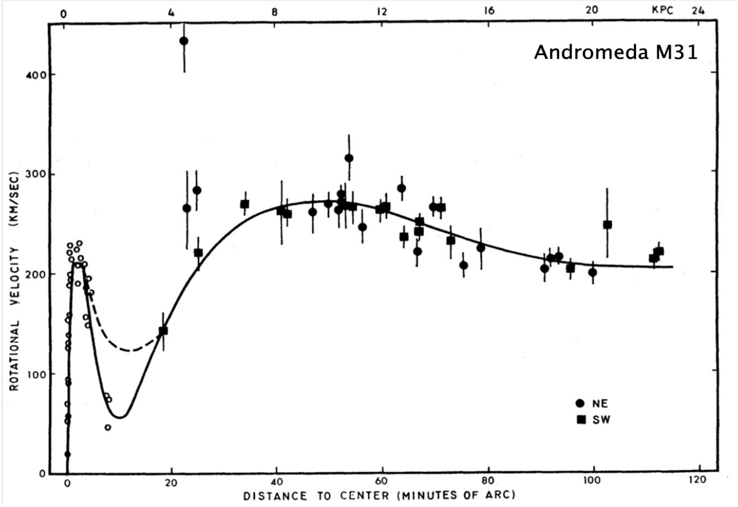
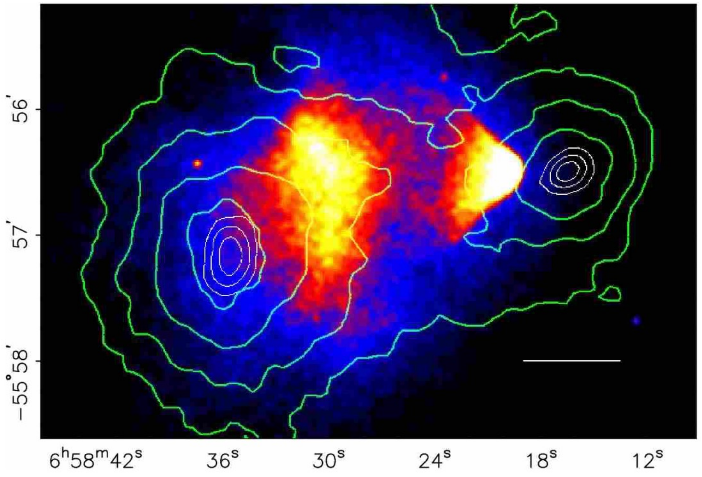
Evidence for dark matter

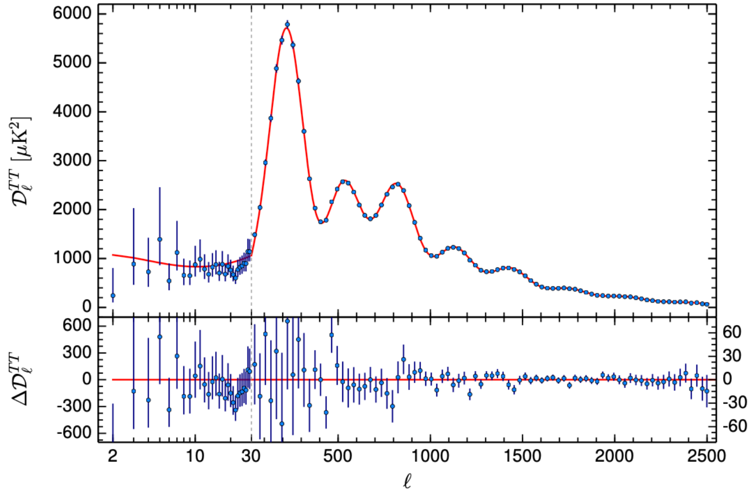
- A lot of evidence for DM from astrophysics and cosmology



Galaxy rotation curve



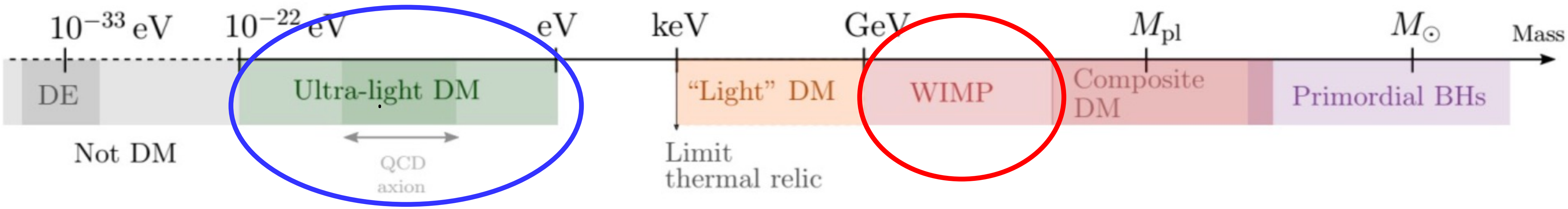
Bullet cluster



CMB anisotropies

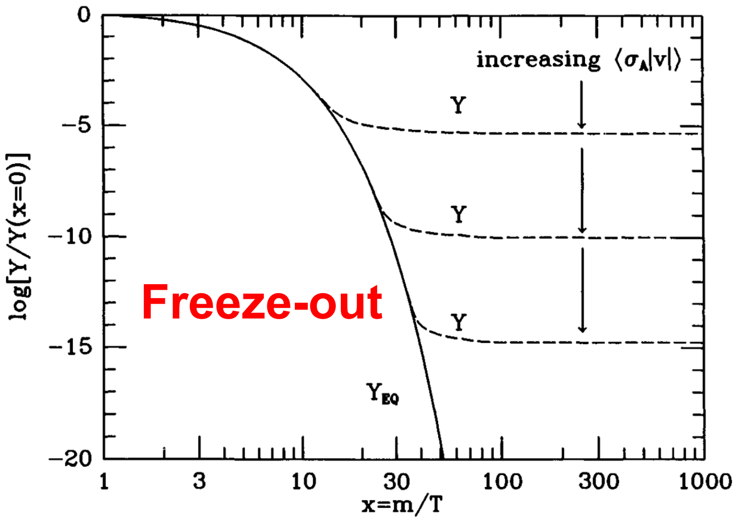
Particle nature of dark matter

- Yet, we know very little about the nature of dark matter



- For many reasons, **WIMP** and **ALP** are two leading candidates for particle dark matter -- they exhibit sharply **different** cosmological histories

Cosmic history of WIMP

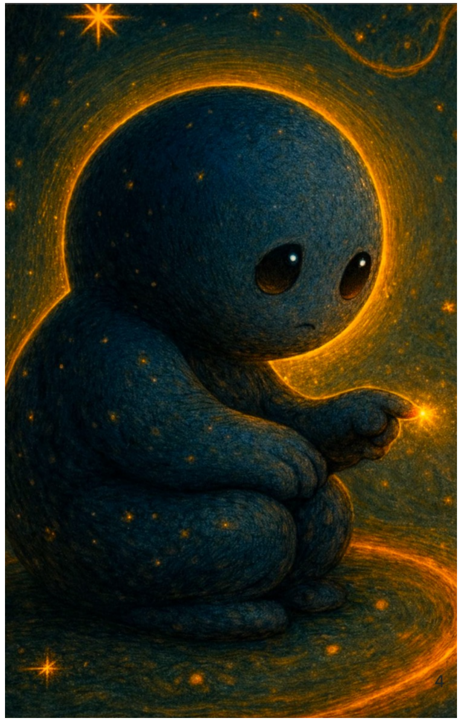


- Thermalized at high temperatures, decoupled later

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle\sigma v\rangle \left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2\right)$$

- Relic abundance insensitive to the initial condition

$$\frac{\Omega_\chi h^2}{0.12} \sim \left(\frac{x_{\mathrm{fo}}}{25}\right)\left(\frac{10^{-26}\mathrm{cm}^3/\mathrm{s}}{\langle\sigma v\rangle}\right)$$

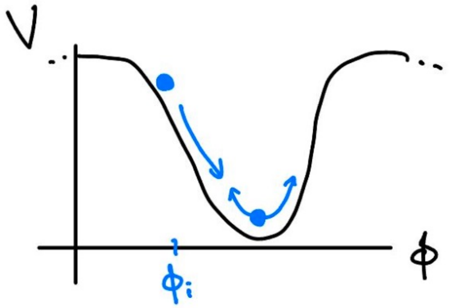


Pure weak-scale physics, "WIMP miracle"

Strong experimental constraints

Cosmic history of ALP

- Feeble coupling, never thermalized, described by a **homogeneous**, **classical** field



$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

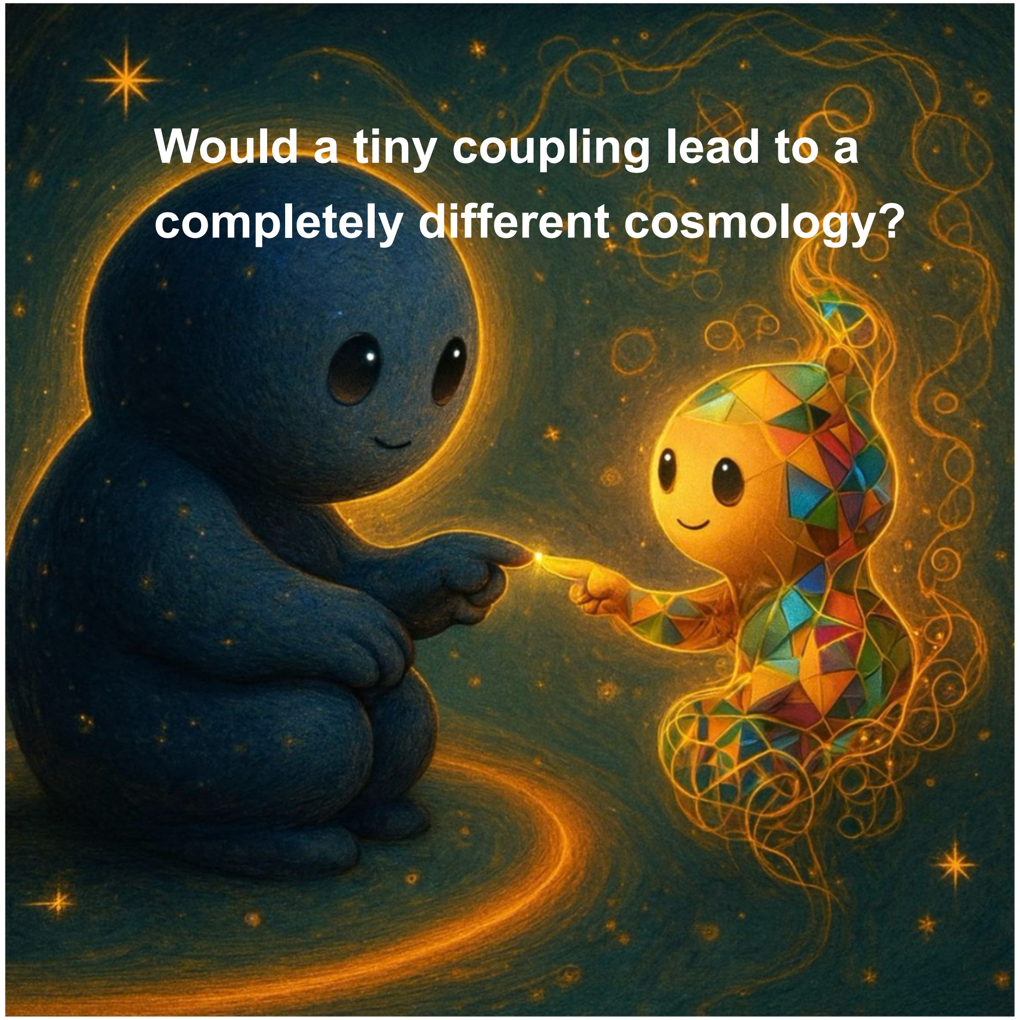
- Initial field value **misaligned**, relic abundance depend on both initial field value and mass

$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$$

Sensitive to the UV physics scale >> weak scale

What about WIMP + ALP?

$$\mathcal{L} =$$



WIMP Meets ALP: Coherent Freeze-Out

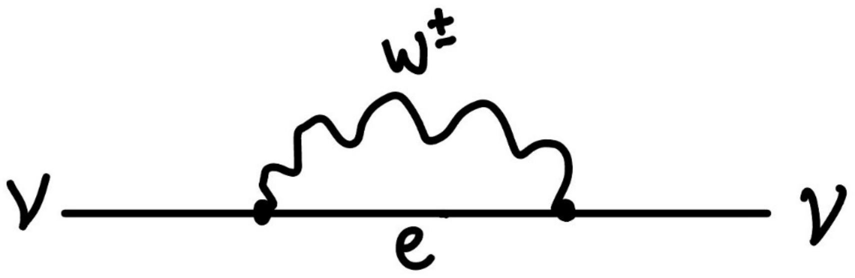
- Naively, you might think they would evolve independently, since ALP is **not** thermalized and momentum exchange is negligible

- However, physics are much richer thanks to the **coherent forward scattering** between these two sectors

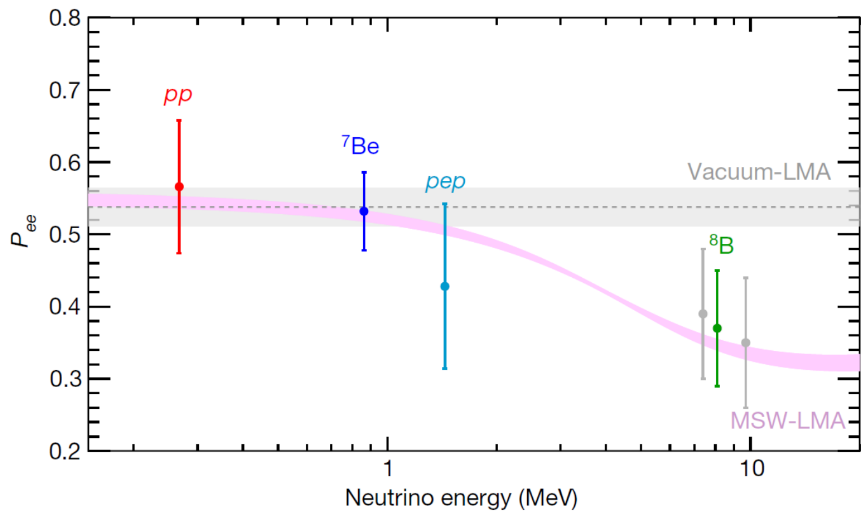
Coherent forward scattering

- No momentum exchange, but modify **dispersion relation** of scattering particles

- Example in the Standard Model: **MSW effect** of neutrino oscillation



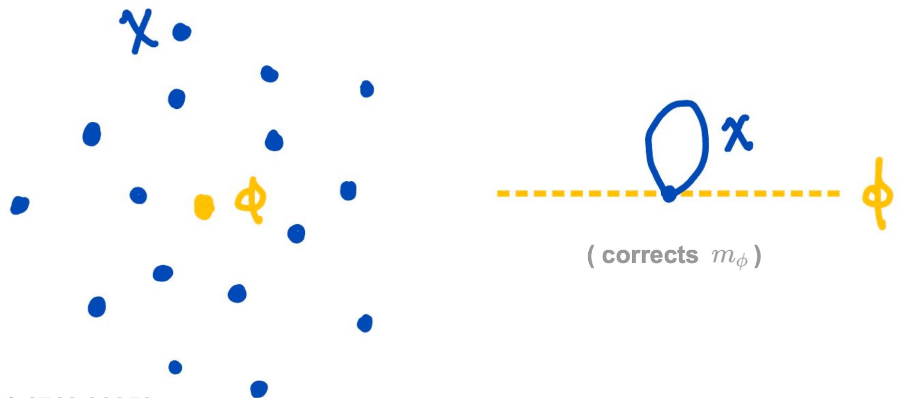
$$E_\nu \to E_\nu + \sqrt{2} G_F n_e$$



change neutrino oscillation probability by O(1), crucial for solving the solar neutrino missing puzzle

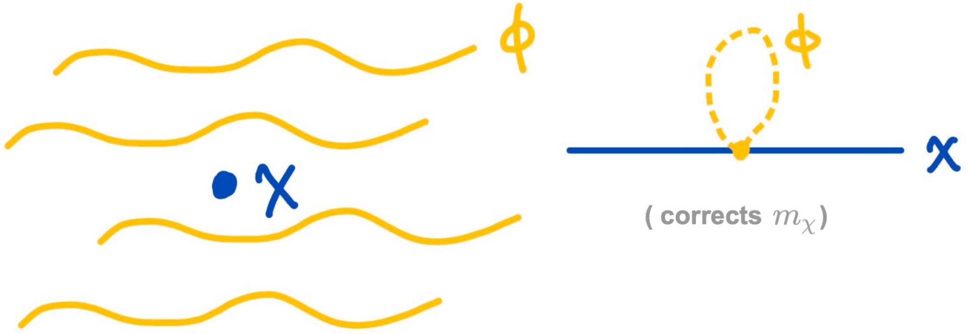
Coherent forward scattering for WIMP + ALP

- Due to coherent forward scattering, both of their dispersion relations are modified by the medium of the other one, schematically,



ALP interacts with WIMP thermal bath

$$m^2_{\phi,\text{eff}} = m^2_\phi + \delta m^2_\phi$$



WIMP interacts with ALP classical background

$$m_{\chi,\text{eff}} = m_\chi + \delta m_\chi$$

Coherent forward scattering for WIMP + ALP

- Due to coherent forward scattering, both of their dispersion relations are modified by the medium of the other one

- This effect is much more significant than one might naively expect. In the following, I will show it quantitatively via an explicit example

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- **Framework and dynamics**
- Phenomenology
- Conclusions

The setup

- We consider an effective **quadratic** coupling between a fermion χ (the WIMP) and a real (pseudo-)scalar ϕ (the ALP):

$$\mathcal{L} = \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

- This is the **leading non-derivative** coupling for a pNGB

- Motived by the QCD axion coupling to nucleons (at scales below Λ_{QCD})

including the sign!

The setup

- We consider an effective **quadratic** coupling between a fermion χ (the WIMP) and a real (pseudo-)scalar ϕ (the ALP):

$$\mathcal{L} = \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

- Possible UV origin: confinement of some dark SU(N), with χ being the dark nucleon and ϕ being the pNGB, analogous to the nucleon and the QCD axion

- In this talk, we take the EFT point of view and do not specify the UV origin of this coupling

Separation of scales

$$\mathcal{L} = \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

- We assume the hierarchy among three relevant scales

$$\Lambda \gg m_\chi \gg m_\phi \qquad\qquad m_\chi \sim \text{electroweak scale}$$

- The coupling is assumed to be small enough to prevent the ALP from ever being thermalized via its scattering with the WIMP

$$\Gamma \sim \frac{T^3}{\Lambda^2} \ll H \sim \frac{T^2}{M_{\rm Pl}} \quad \Rightarrow \quad \Lambda \gg \sqrt{T_{\rm RH} M_{\rm Pl}}$$

Effective potential at finite T

$$x \equiv m_\chi / T$$

$$\varphi \equiv \phi / \sqrt{m_\chi \Lambda}$$

$$\gamma \equiv |1 - \varphi^2/2|$$

- At finite T, the WIMP thermal bath modifies the ALP potential

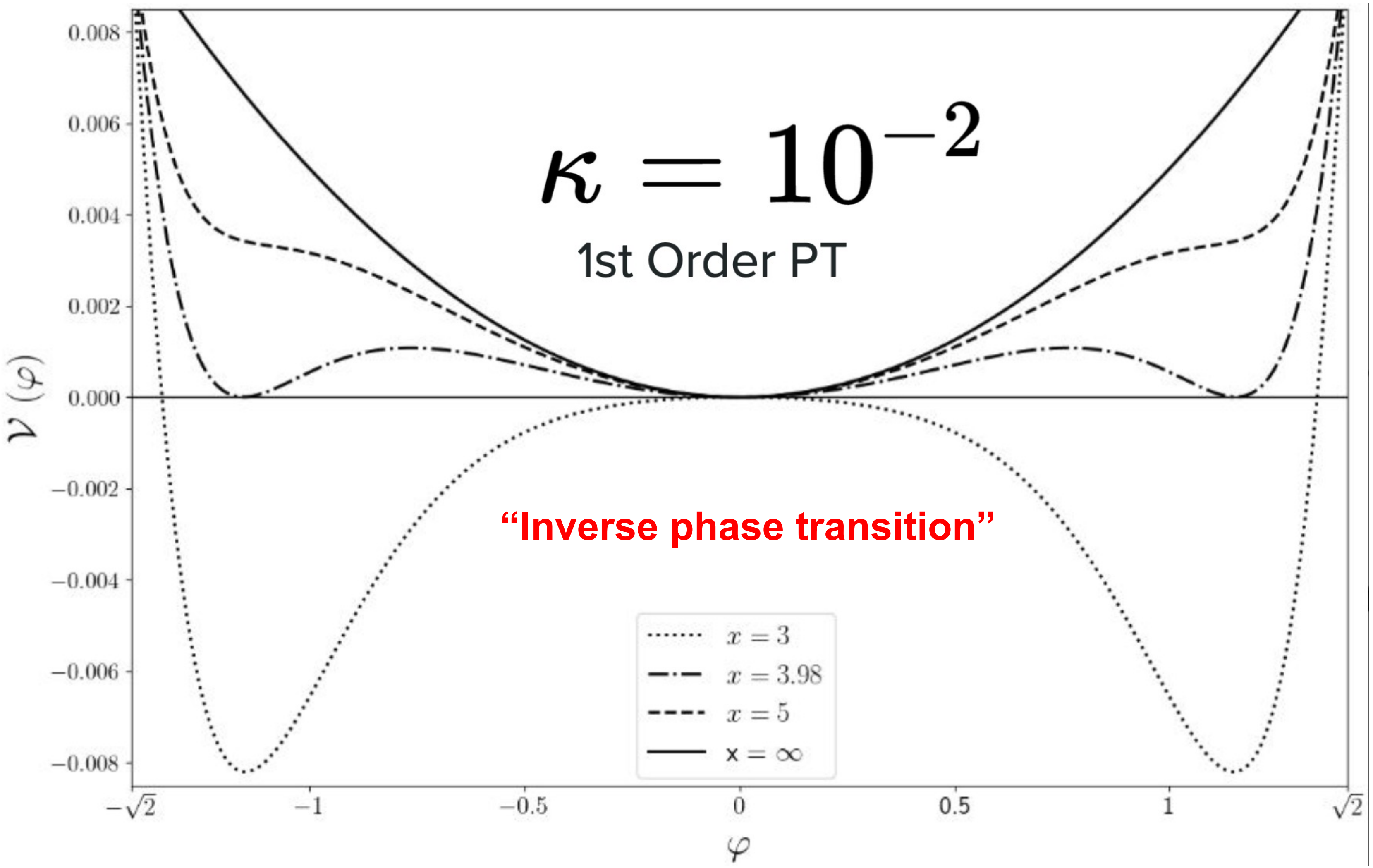
$$\mathcal{V}(\varphi, x) = \boxed{-\frac{\varphi^2}{2}\frac{\gamma^2 K_1(\gamma x)}{x}} + \boxed{\kappa \frac{\varphi^2}{2}}$$

$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

thermal mass term **bare mass term**

- **High-T:** symmetry is broken, ALP obtains a VEV, shifting the WIMP mass

  Low-T: symmetry is restored, VEV returns to zero, WIMP freezes out



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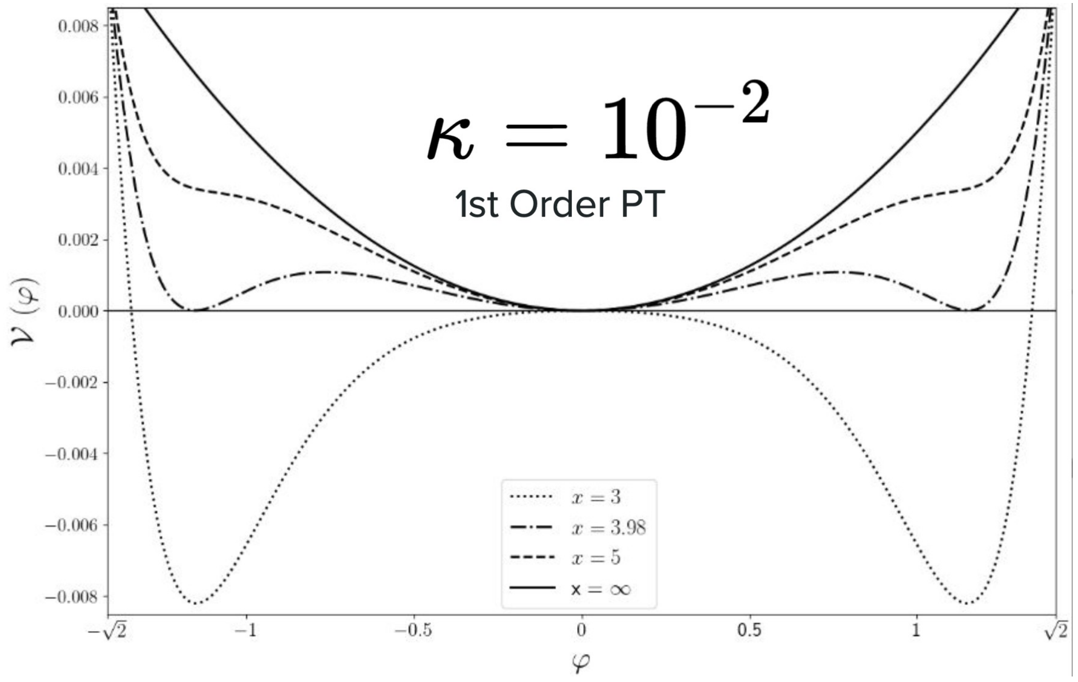
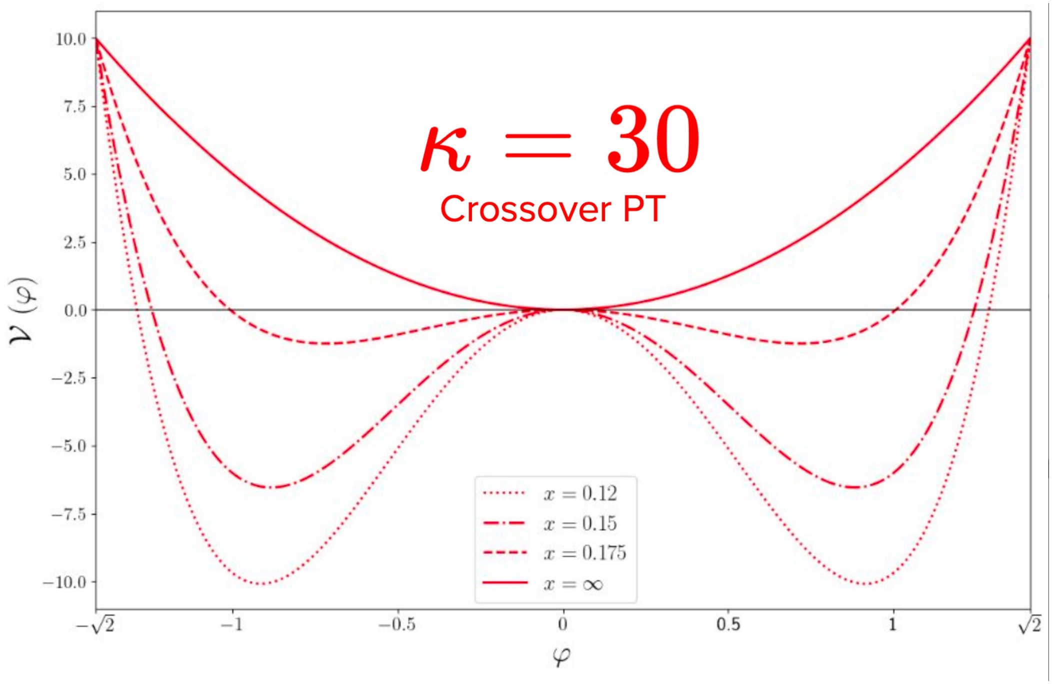
Orders of phase transition

$$\mathcal{V}(\varphi, x) = -\frac{\varphi^2}{2}\frac{\gamma^2 K_1(\gamma x)}{x} + \kappa\frac{\varphi^2}{2}$$

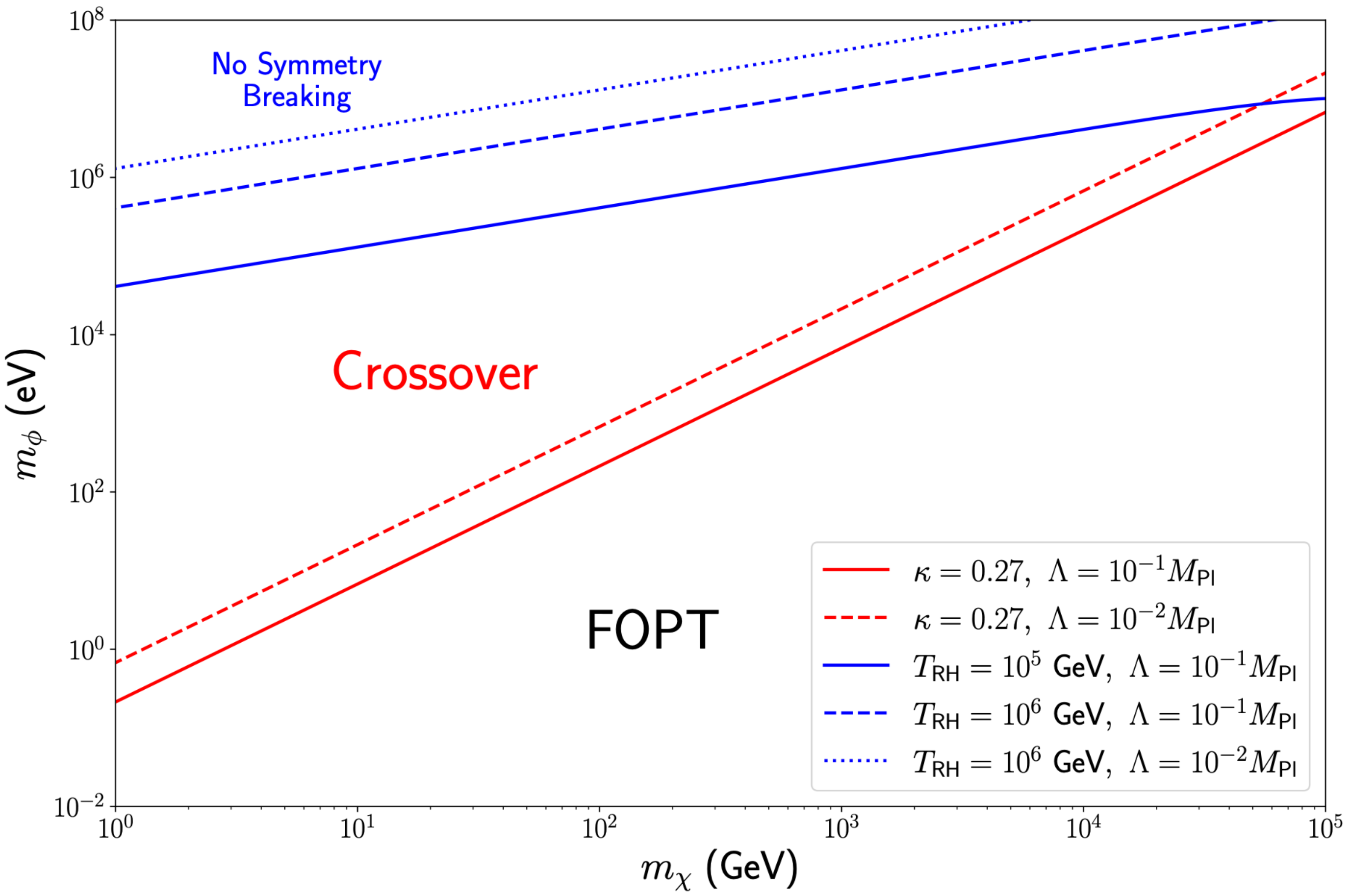
$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

- Using Ginzburg-Landau theory, we find $\kappa_c \sim 0.27$ splits FOPT and crossover





Classification of the parameter space



$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

$$\kappa \gtrsim 0.27: \ \text{Crossover}$$

$$\kappa \lesssim 0.27: \ \text{FOPT}$$

Dynamics and phenomenology are very different in these two regimes

Equation of motion for ALP

$$x \equiv m_\chi / T$$

$$\varphi \equiv \phi / \sqrt{m_\chi \Lambda}$$

$$\gamma \equiv |1 - \varphi^2/2|$$

- The EOM is non-linear due to the backreaction

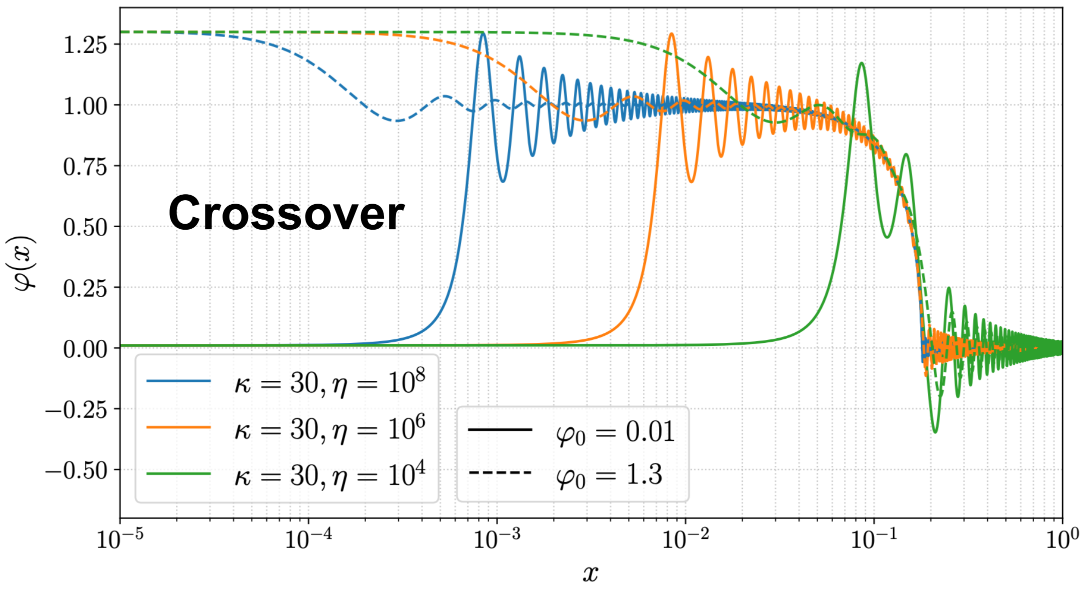
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi, T) = 0$$

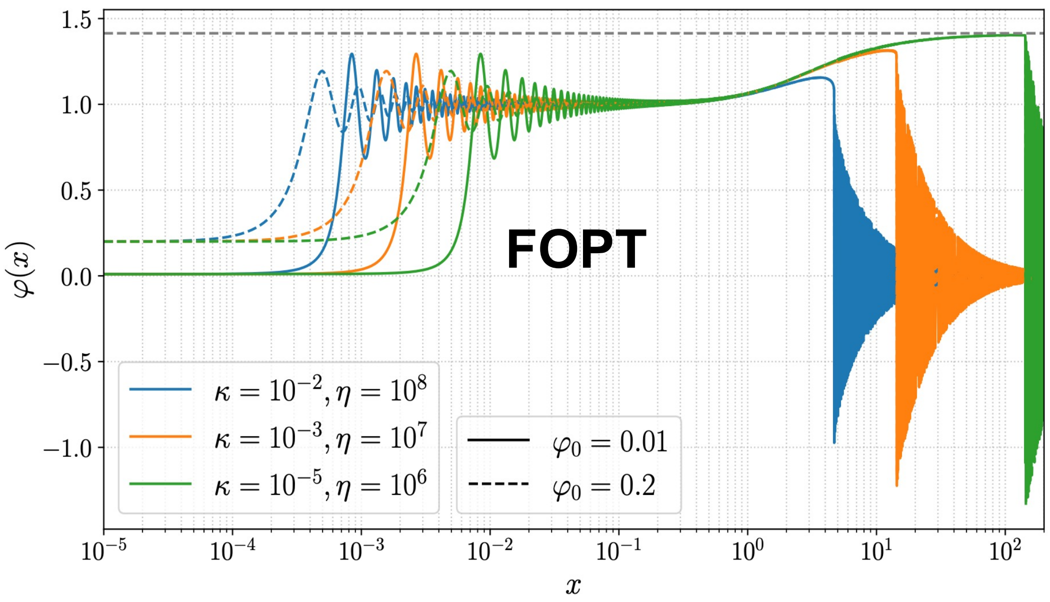
$$\varphi'' + \frac{2}{x}\varphi' - \eta x \left(1 - \frac{\varphi^2}{2}\right)\left[\left(1 - \varphi^2\right) K_1(\gamma x) + \frac{\varphi^2}{2}\gamma x K_0(\gamma x)\right]\varphi + \eta\kappa x^2 \varphi = 0$$

- The EOM depends only on two dimensionless parameters:

$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3} \qquad \eta \sim \frac{M_{\rm Pl}^2}{m_\chi \Lambda}$$

Solutions of the EOM





- Crossover regime: ALP evolution is **nearly adiabatic**, decouples earlier, does not affect freeze-out
- FOPT regime: ALP evolution is **non-adiabatic**, decouples later, affects WIMP freeze-out

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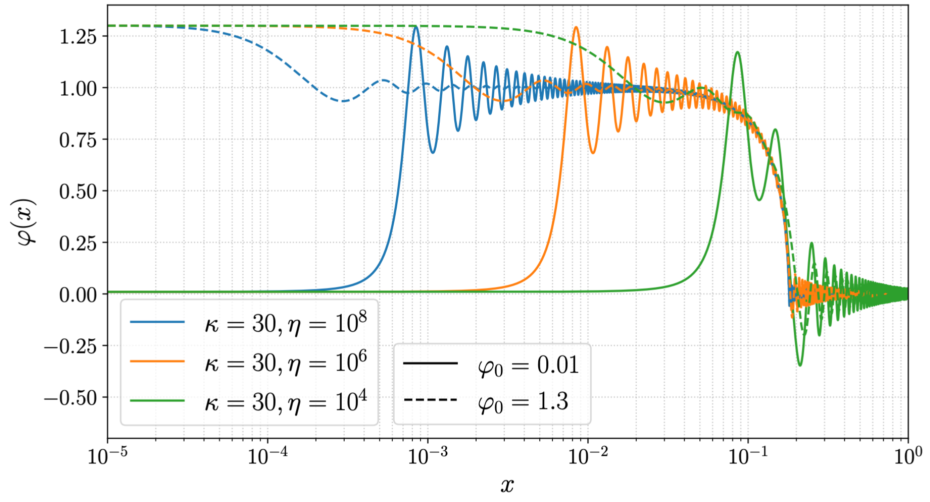
Crossover regime

(both WIMP and ALP contribute to dark matter)

Evolution of the ALP field



- In this regime, the symmetry is restored earlier at $x \ll 25$, so freeze-out is not affected

- The EOM in this regime is given by

$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

$$\varphi'' + \frac{2}{x}\varphi' - \eta\left(1 - \kappa x^2\right)\varphi + \eta\,\varphi^3 = 0$$

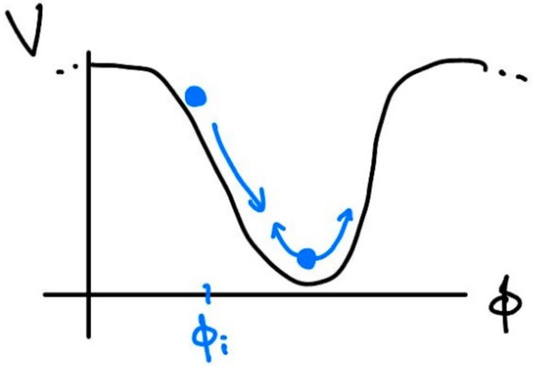
$$\eta \sim \frac{M_{\rm Pl}^2}{m_\chi \Lambda}$$

  - for $x \ll 1/\sqrt{\eta}$: the ALP field is frozen by Hubble friction
  - for $1/\sqrt{\eta} \lesssim x \lesssim 1/\sqrt{\kappa}$: it evolves toward the vacuum at $\varphi_* = 1$ **wash out the initial dependence**
  - for $x \gtrsim 1/\sqrt{\kappa}$: symmetry is restored and the field relaxes back toward 0

Relic abundance of ALP

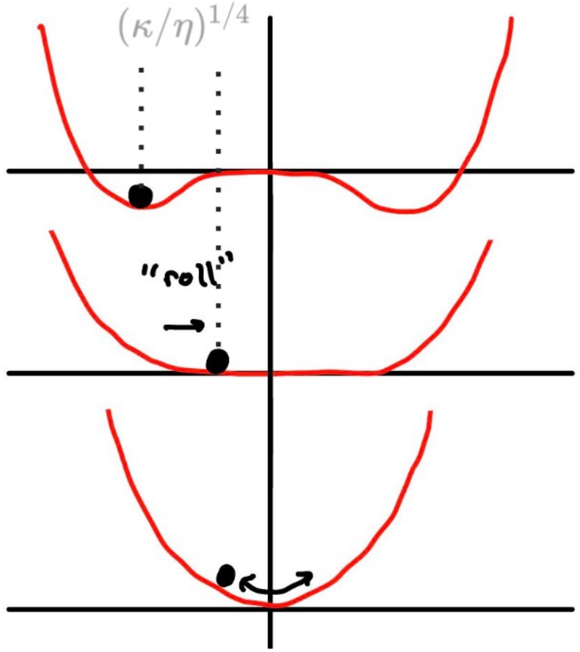
- Usual misalignment: φ_i ~ UV dependent

$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$$

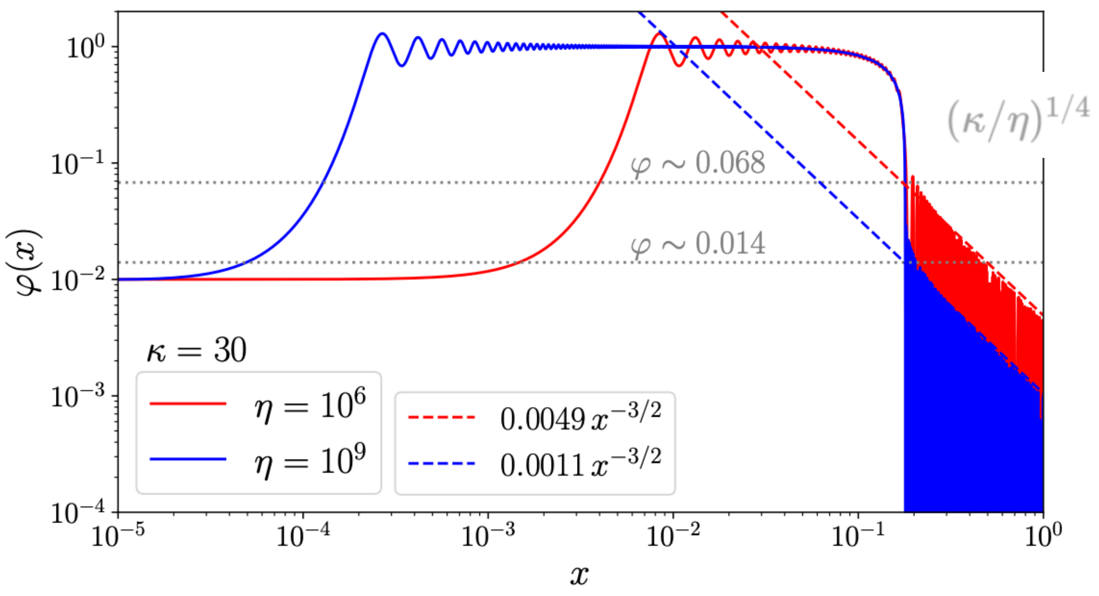


- Our case: $\varphi_i \sim \varphi_* (\kappa/\eta)^{1/4}$ **fixed by thermal dynamics**

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$



Relic abundance of ALP



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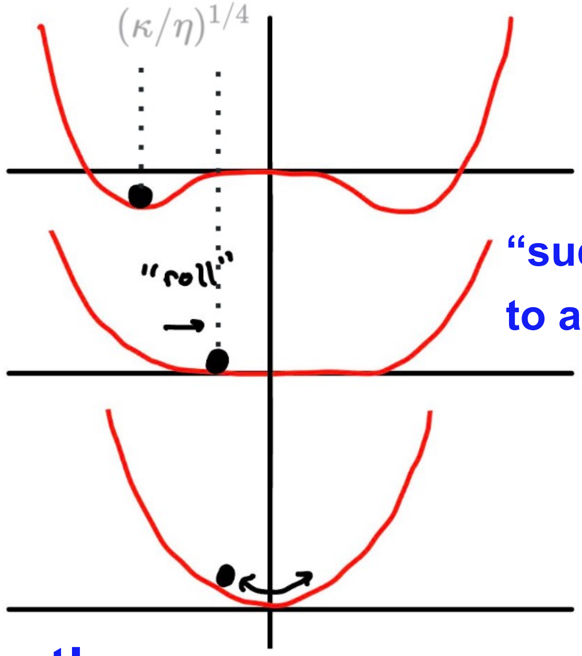


"sudden drop" due to adiabatic invariant

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$

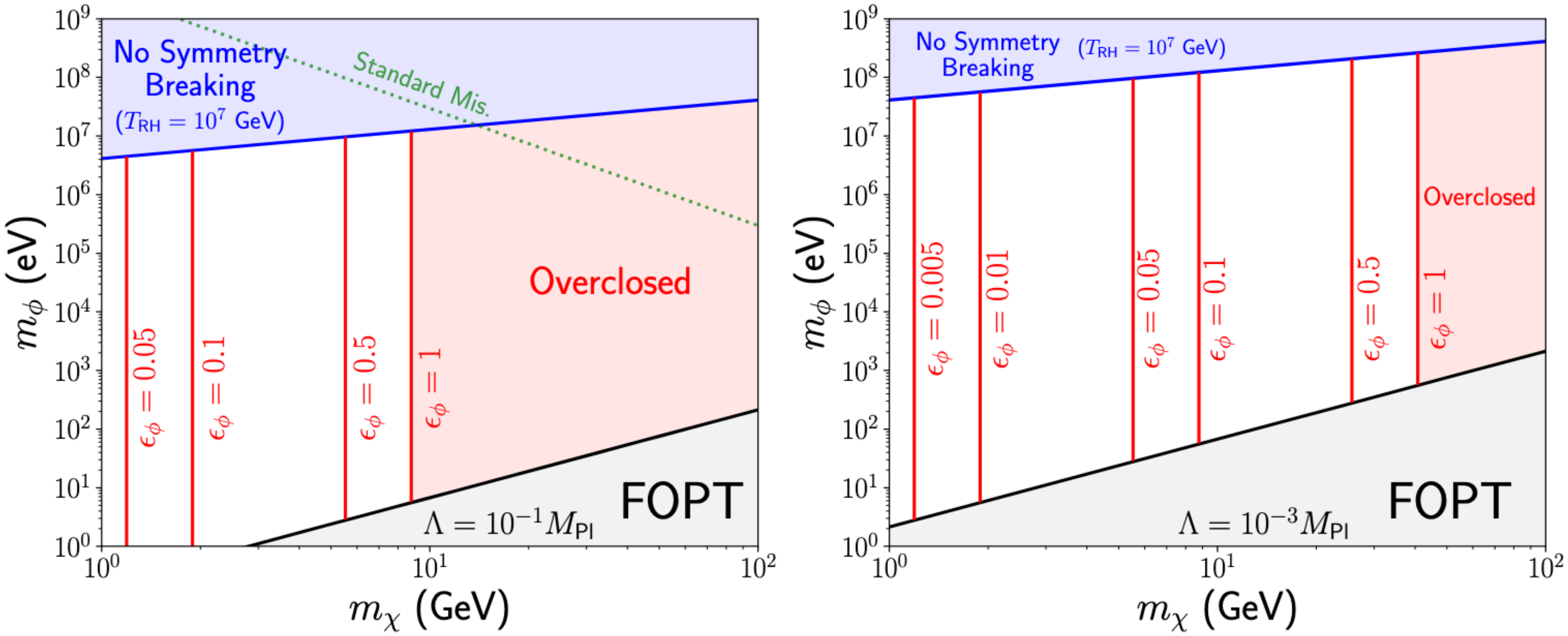
$$\Omega_\phi \approx 0.3 \left(\frac{m_\chi}{10\ \mathrm{GeV}}\right)^{3/2} \left(\frac{\Lambda}{0.1 M_{\mathrm{Pl}}}\right)^{1/2}$$

Surprisingly, the ALP mass also drops out!

The "ALP miracle"

$$\Omega_\phi \approx 0.3 \left(\frac{m_\chi}{10\ \mathrm{GeV}}\right)^{3/2} \left(\frac{\Lambda}{0.1 M_{\mathrm{Pl}}}\right)^{1/2}$$

- For $m_\chi \sim$ weak scale, $\Lambda \sim$ Planck scale, the ALP obtains correct relic abundance
- This is largely insensitive to both the initial ALP field value and the ALP mass

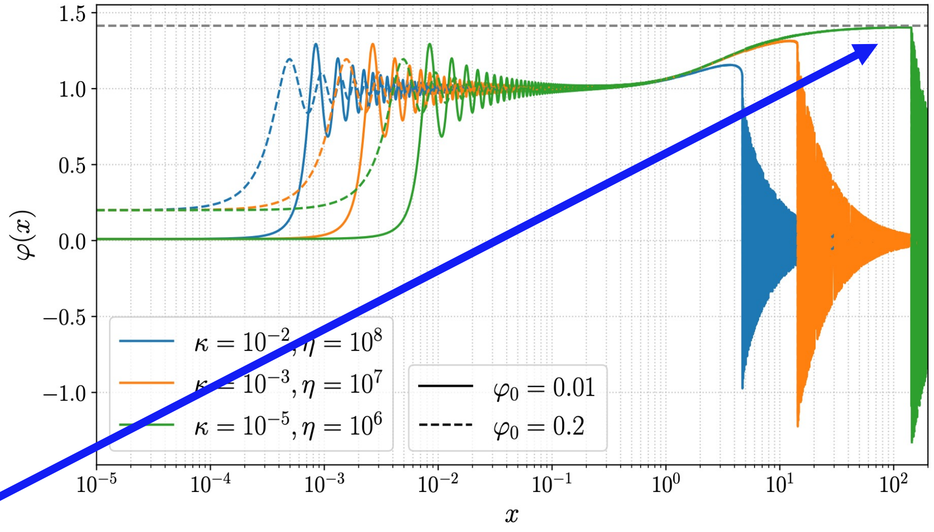


$$\epsilon_\phi \equiv \Omega_\phi / \Omega_{\mathrm{DM}}$$

First-order phase transition regime

(only WIMP contributes to dark matter)

Evolution of the vacuum



- Given the potential, for $\kappa \ll 1$ and $x \gg 1$, the symmetry-breaking vacuum evolves as

$$\varphi_* = \sqrt{2\left(1 - \frac{c_1}{x}\right)}$$

$c_1 \approx 1.33$ is the root of $c_1 K_0(c_1) = K_1(c_1)$

- The evolving vacuum reduces the effective WIMP mass as temperature decreases

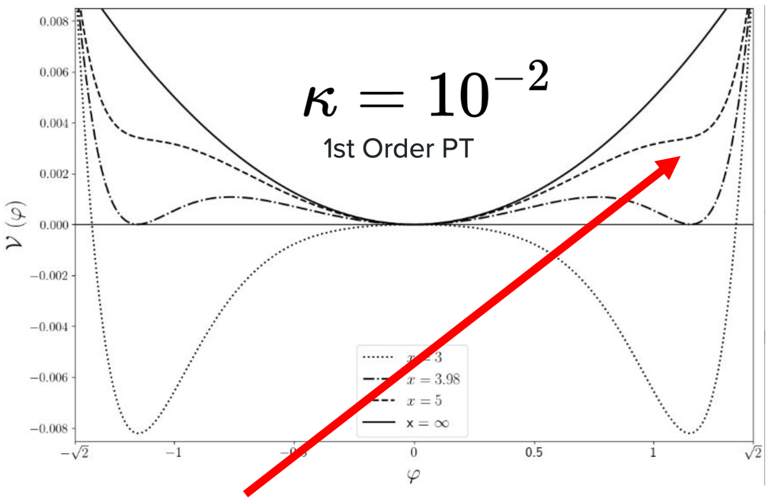
$$m_{\chi,\mathrm{eff}} = \left(1 - \varphi_*^2/2\right) m_\chi \sim T$$

- This makes the WIMP stay in equilibrium even at $x \equiv \frac{m_\chi}{T} \gg O(25)$

$$n_{\chi,\mathrm{eq}} \sim m_{\chi,\mathrm{eff}}^2 T K_2(m_{\chi,\mathrm{eff}}/T) \sim T^3$$

No Boltzmann suppression at large x!

Coherent freeze-out



$x_{\rm cfo} \sim \kappa^{-1/3}$

- The freeze-out is controlled by ALP dynamics

- We define $x_{\rm cfo}$ as the time when the local minimum disappears

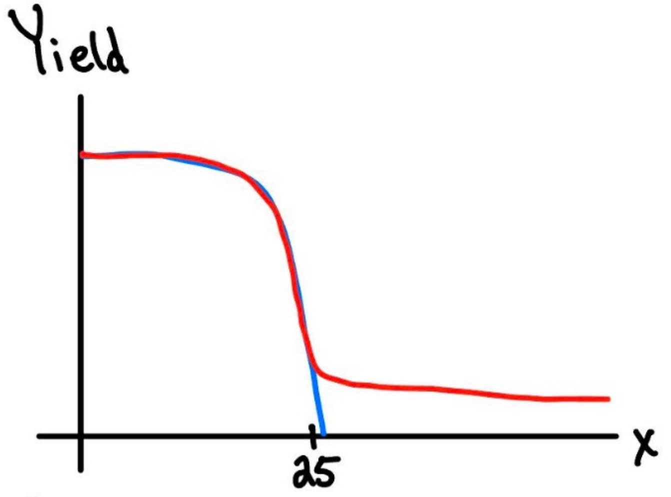
- For $x > x_{\rm cfo}$, ALP rolls back to the origin, $m_{\chi,\rm eff}$ rapidly tends to m_χ, driving WIMP out of equilibrium --- the beginning of **coherent freeze-out**

Coherent freeze-out

- The freeze-out is controlled by ALP dynamics

- We define x_{cfo} as the time when the local minimum disappears

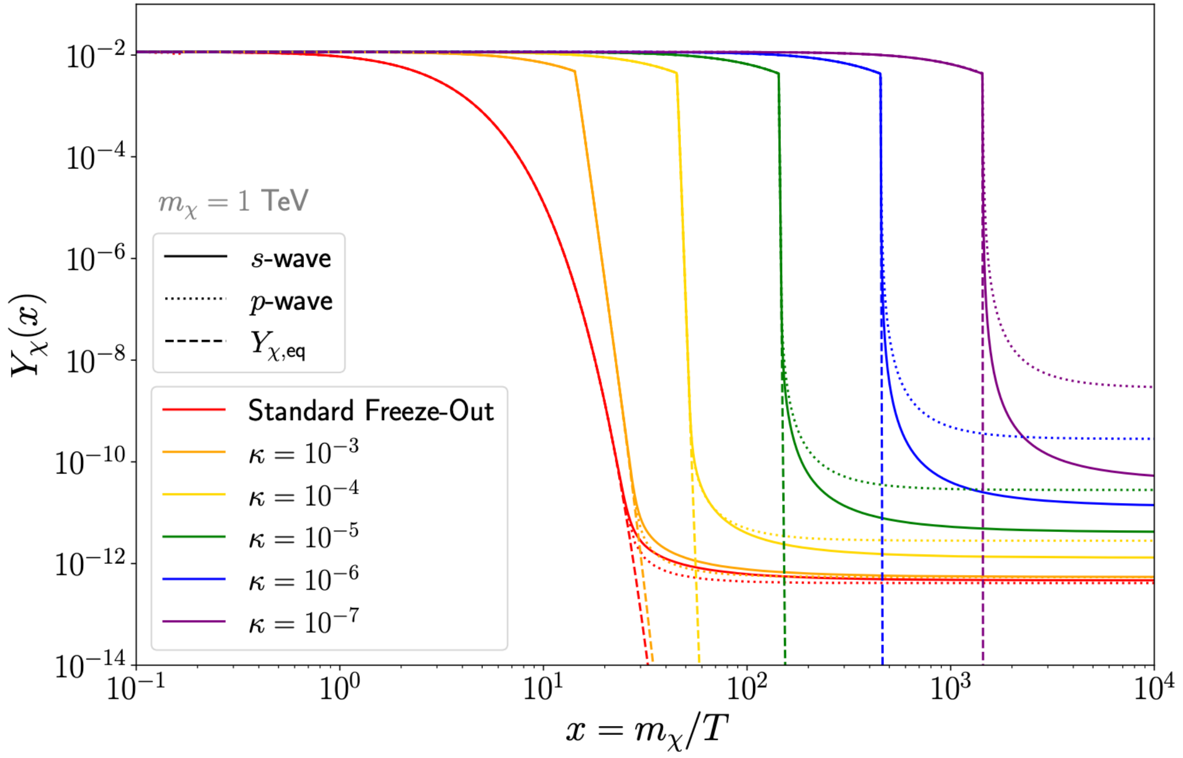
- For $x > x_{\mathrm{cfo}}$, ALP rolls back to the origin, $m_{\chi,\mathrm{eff}}$ rapidly tends to m_χ equilibrium --- the beginning of **coherent freeze-out**



- Normal freeze-out: triggered by universe expansion, typically occurs at $x_{\mathrm{fo}} \sim 25$
  Coherent freeze-out: triggered by phase transition, can occur at $x_{\mathrm{cfo}} \gg 25$

Relic abundance of coherent freeze-out

- Because freeze-out is delayed, the relic abundance is enhanced thanks to less redshift between freeze-out and present day



Relic abundance of coherent freeze-out

- Because freeze-out is delayed, the relic abundance is enhanced thanks to less redshift between freeze-out and present day

- Quantitively, we find

$$\frac{Y^{\rm cfo}_{\chi,\infty}}{Y^{\rm fo}_{\chi,\infty}} = \frac{14}{9}\frac{x_{\rm cfo}}{x_{\rm fo}}\frac{\sigma_0^{\rm fo} + 3\sigma_1^{\rm fo}/x_{\rm fo}}{\sigma_0^{\rm cfo} + 21\sigma_1^{\rm cfo}/(10x_{\rm cfo})}$$

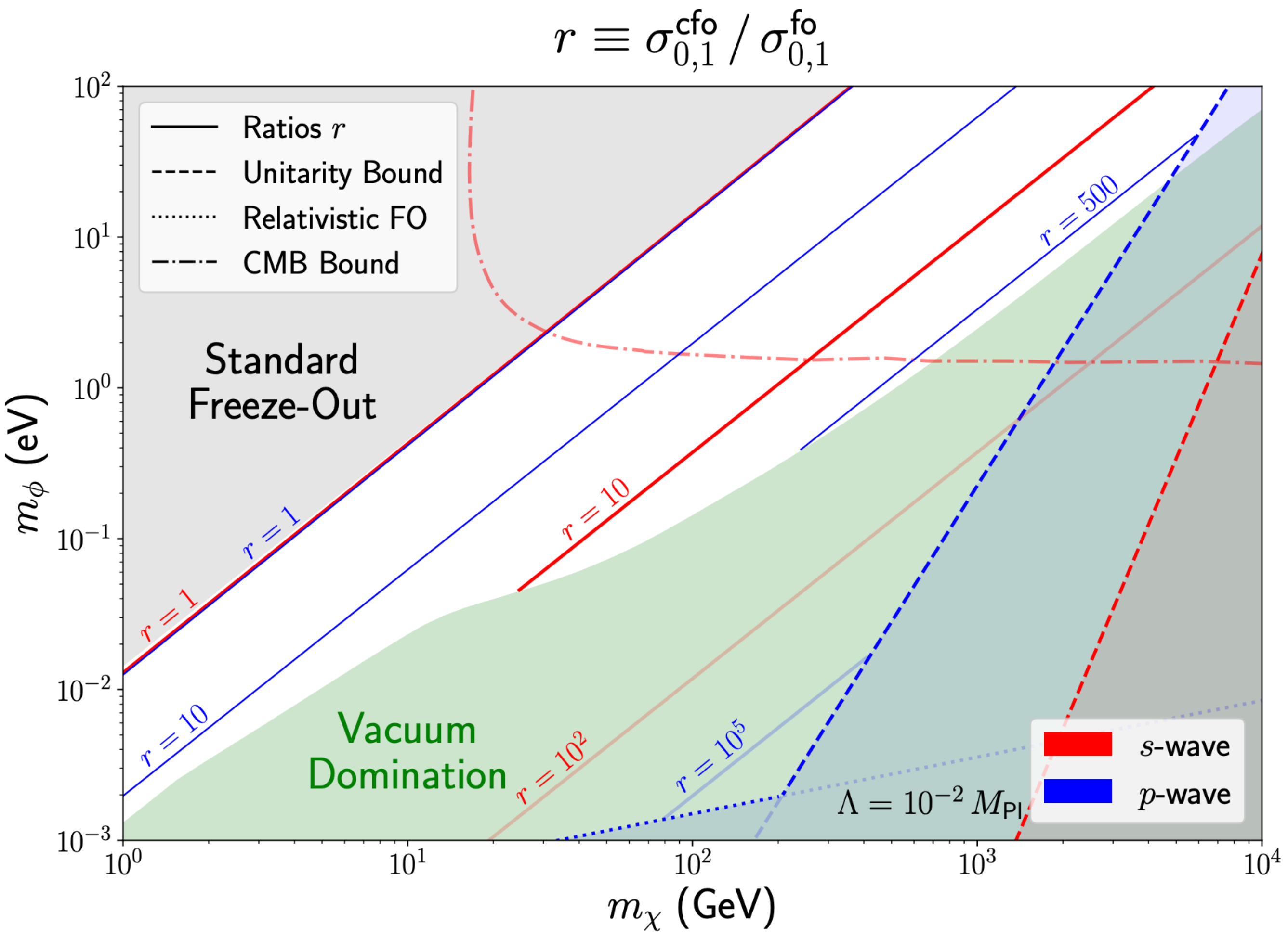
- To get the same yield, we have

s-wave: $\sigma_0^{\rm cfo}/\sigma_0^{\rm fo} \sim x_{\rm cfo}/x_{\rm fo}$

p-wave: $\sigma_1^{\rm cfo}/\sigma_1^{\rm fo} \sim (x_{\rm cfo}/x_{\rm fo})^2$

- So, the new mechanism allows for an **enhanced** WIMP cross section!

Enhancement of annihilation cross section



s-wave: enhanced up to 30

p-wave: enhanced up to 10^3

This might open the indirect detection window for p-wave DM

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Conclusions

- Even a tiny (Planck-suppressed) coupling between WIMP and ALP can substantially modify their cosmological histories through coherent forward scattering

- This dramatically change the predicted properties of WIMP and ALP dark matter particles, such as their masses and annihilation cross sections

- This in turn leads to important consequences for targeted experimental searches for particle dark matter

Thank you for your attention!

Backup slides

Modification of dispersion relation

- Both masses are shifted due to the coherent forward scattering inside the other medium

$$m^2_{\phi,\mathrm{eff}} = m^2_\phi - \frac{\langle\bar{\chi}\chi\rangle_T}{\Lambda}$$

$$m_{\chi,\mathrm{eff}} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

$$\langle\bar{\chi}\chi\rangle_T = g_\chi \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{m_{\chi,\mathrm{eff}}}{E_\mathbf{k}} f_\chi(\mathbf{k})$$

$g_\chi = 4$ (or 2) for Dirac (or Majorana)

f_χ is the phase-space distribution

- The dynamics of the two sectors are therefore **coupled**

The full one-loop thermal potential

$$V_{\rm full}(\phi, T) = -\frac{g_\chi}{2\pi^2} T^4 \int_0^\infty {\rm d}y\, y^2 \log\left[1 + e^{-\sqrt{y^2 + m_{\chi,\rm eff}^2/T^2}}\right] + \frac{1}{2} m_\phi^2 \phi^2 \equiv \frac{g_\chi}{2\pi^2} m_\chi^4 \mathcal{V}_{\rm full}(\varphi, x)$$

$$m_{\chi,\rm eff} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

$$\mathcal{V}_{\rm full}(\varphi, x) = -\frac{1}{x^4} \int_0^\infty {\rm d}y\, y^2 \log\left[1 + e^{-\sqrt{y^2 + \gamma^2 x^2}}\right] + \kappa \frac{\varphi^2}{2}$$

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

EOM with the full potential

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

$$\eta \equiv \frac{g_\chi}{2\pi^2} \frac{1}{1.66^2 g_*} \frac{M_{\rm Pl}^2}{m_\chi \Lambda}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi, T)}{\partial \phi} = 0$$

$$\varphi'' + \frac{2}{x}\varphi' - \eta\, x\, \varphi\, \mathrm{sgn}\left(1 - \frac{\varphi^2}{2}\right) \sum_{n=1}^{\infty} \left[\frac{(-1)^{n-1}}{n} \gamma^2 K_1(n\gamma x)\right] + \eta\, \kappa\, x^2 \varphi = 0$$

Dark matter in the FOPT regime

- Freeze-out of WIMP is delayed in a *dynamical* way, allowing for orders of magnitude enhancement of its annihilation cross section while still yielding the correct relic abundance

- In this regime, however, ALP only behaves as a spectator field. In order not to overclose the Universe, it must decay to radiation after WIMP freeze-out. Therefore, dark matter consists solely of the WIMP in the FOPT regime

More calculations in the crossover regime

- Symmetry is restored at $x_c \equiv 1/\sqrt{\kappa}$

$$\varphi'' + \frac{2}{x}\varphi' - \eta\left(1 - \kappa x^2\right)\varphi + \eta\,\varphi^3 = 0$$

- The adiabatic approximation holds when $\eta\left(1 - \kappa x^2\right) \gg 1/x^2$

- We define $x_1 = x_c + \delta x$ the time when adiabaticity is restored, i.e., when effective mass = Hubble friction

$$\left|\eta\left(1 - \kappa x_1^2\right)\right| \;\equiv\; 1/x_1^2$$

- For $\eta \gg \kappa$ (which always holds), the loss of adiabaticity is extremely brief $\delta x/x_c \approx \kappa/(2\eta)$

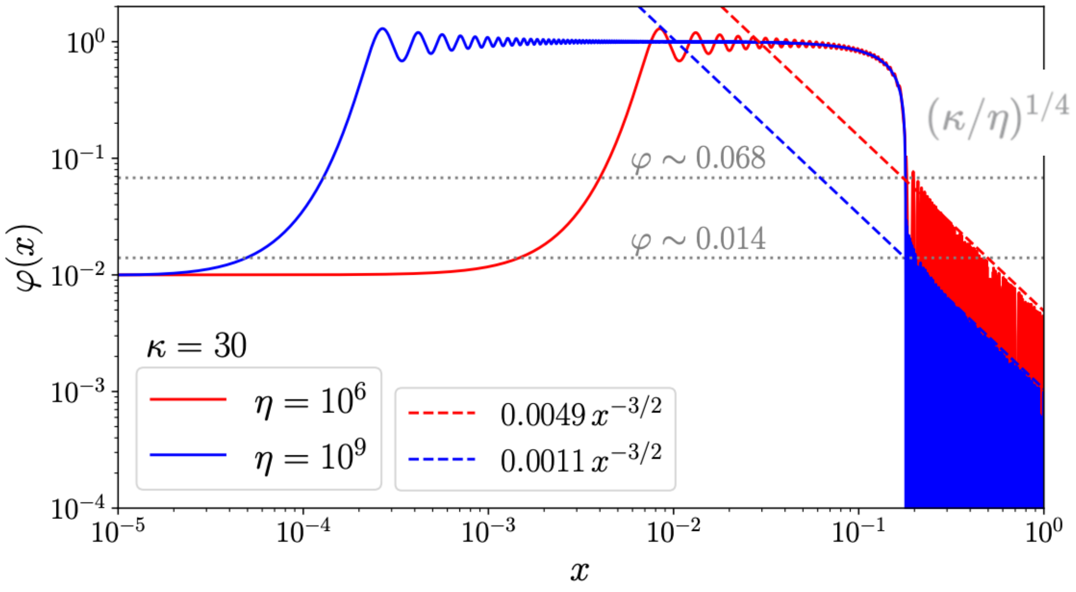
- For $x > x_1$, we have adiabatic invariant

$$m_{\phi,\mathrm{eff}}(x)\phi^2(x)\,a^3(x) = m_{\phi,\mathrm{eff}}(x_1)\phi^2(x_1)\,a^3(x_1)$$

More calculations in the crossover regime

- Near $x_c \approx x_1$, the field undergoes a sudden drop due to the adiabatic condition

$$\frac{\phi^2(x_0)\,a^3(x_0)}{\phi^2(x_1)\,a^3(x_1)} = \frac{H(x_1)}{m_\phi} \approx \frac{H(x_c)}{m_\phi} = \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \ll 1$$



- So, the present-day energy density is given by

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$

More calculations in the crossover regime

- So, the present-day energy density is given by

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$

$$\begin{aligned}
\rho_\phi(x_0) &= m_\phi^2 \times 2m_\chi \Lambda \times \frac{H(x_c)}{m_\phi} \times \frac{x_c^3}{x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= m_\phi^2 \times 2m_\chi \Lambda \times \frac{1.66\sqrt{g_*(x_c)}\, m_\chi^2}{M_{\rm Pl}\, m_\phi} \times \frac{x_c}{x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= \cancel{m_\phi^2} \times 2m_\chi \Lambda \times \frac{1.66\sqrt{g_*(x_c)}\, m_\chi^2}{M_{\rm Pl}\, \cancel{m_\phi}} \times \frac{\pi}{2\sqrt{3}} \left(\frac{g_\chi}{2\pi^2}\right)^{1/2} \frac{m_\chi^{3/2}}{\Lambda^{1/2} \cancel{m_\phi} x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= 1.66\sqrt{\frac{g_\chi g_*(x_c)}{6}}\, \frac{g_{*S}(x_0)}{g_{*S}(x_c)}\, \frac{m_\chi^{3/2} \Lambda^{1/2} T_0^3}{M_{\rm Pl}} \,.
\end{aligned}$$

ALP mass precisely drops out in the relic abundance!