

# One Sum to Rule Them All

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Margarita Gavrilova

Based on the work with Y. Grossman, G. Papiri and S. Schacht  
arXiv:2602.22320 and ongoing work

**PHENO, Pittsburgh, May 12 2026**



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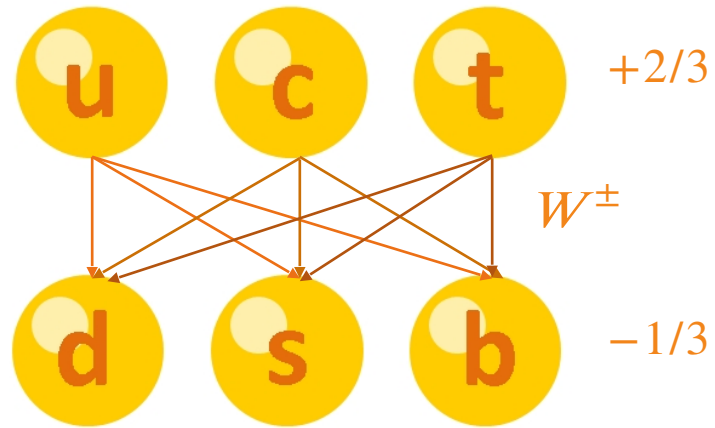
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This is a talk about **higher order**  
flavor sum rules for hadron decays  
in the **Standard Model**



Caltech



# The Challenge of Flavor Physics

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Number of theory parameters  $<$  number of observables

**This is challenging.** Due to non-perturbative QCD there are often more theory parameters than observables

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**Ways to approach the challenge:**

- Calculate the parameters (ex.: lattice, analytic approaches)
- Use symmetries to reduce the number of parameters

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derive Sum Rules

# Sum Rules: basic idea

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- Consider a set of two observables  $O_1$  and  $O_2$  which can be expressed in terms of two theory parameters  $t_1$  and  $t_2$  that encode strong interaction and two parameters  $\alpha_1$  and  $\alpha_2$  that encode weak physics

$$\begin{array}{l} \text{observables} \\ \left[ \begin{array}{l} O_1 \\ O_2 \end{array} \right] = \left[ \begin{array}{l} \alpha_1 t_1 \\ \alpha_2 t_2 \end{array} \right] \end{array} \begin{array}{l} \text{theory parameters} \\ \text{(strong interaction)} \end{array}$$

↑  
weak physics (CKM)

**Symmetry** of strong interaction:  $t_1 = t_2$   $\longrightarrow$  **sum rule:**  $\alpha_2 O_1 - \alpha_1 O_2 = 0$

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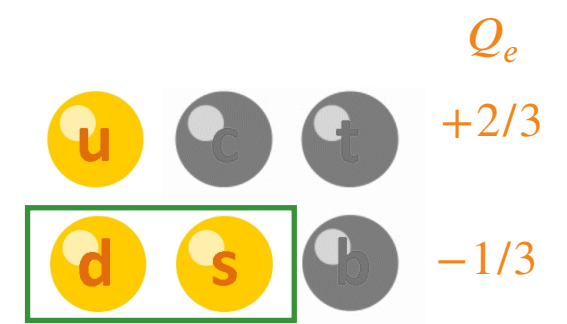
Symmetry of strong interaction:  $t_1 = t_2$   $\longrightarrow$  sum rule:  $\alpha_2 O_1 - \alpha_1 O_2 = 0$

More generally sum rules are relations of the following form

$$\sum_i \alpha_i O_i = 0$$

and observables of interest are decay rates. For example, we ask if we can relate  $\Gamma(D^0 \rightarrow \pi^+ \pi^-), \Gamma(D^0 \rightarrow \pi^+ K^-), \dots$

# Flavor symmetry



- $SU(3)$  flavor is an approximate symmetry of light quarks  $u, d, s$
- We focus on  $U$ -spin:  $SU(2)$  subgroup relating  $(d, s)$

Amplitudes of the processes related by flavor symmetry must be related!      symmetry  $\longrightarrow$  sum rules

- $U$ - spin is broken by  $\epsilon \sim \frac{\Delta m}{\Lambda_{QCD}} \sim O(30\%)$
- The breaking comes from quark mass differences

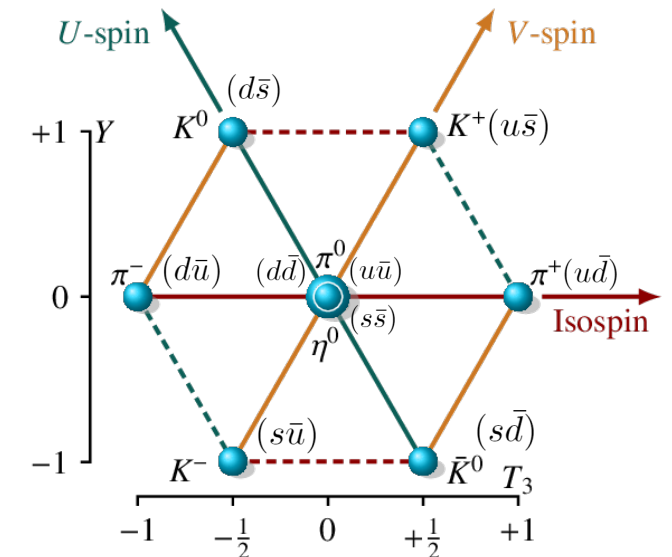
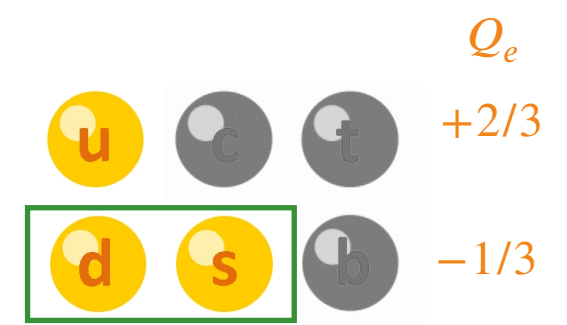


Image adapted from arXiv:1502.07089

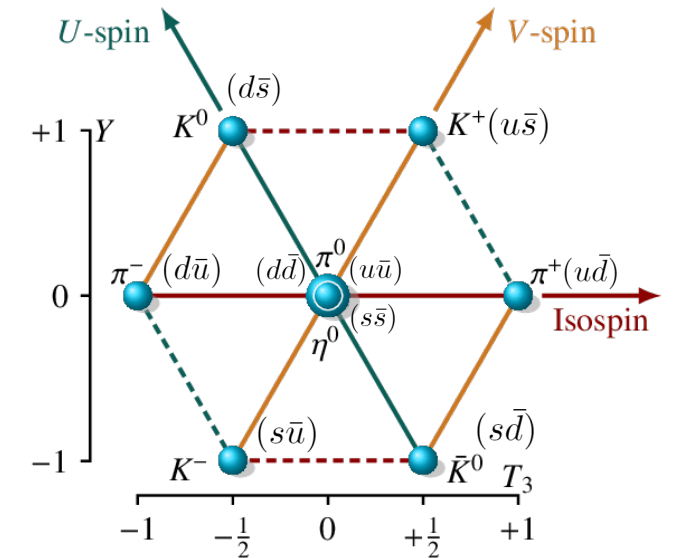
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Systematic expansion in the symmetry-breaking spurion allows us to derive sum rules that hold to **higher orders in  $\epsilon$** . This is the goal.

Image adapted from arXiv:1502.07089

# The long history of flavor sum rules

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- **Since the 1950s-60s:** **symmetry-limit amplitude and rate sum rules** are well understood [Shmushkevich 1955; Dushin & Shmushkevich 1956; MacFarlane, Mukunda & Sudarshan 1964; Pinski, MacFarlane & Sudarshan 1965; ...]
- **Since 1980s:** SU(3), isospin, and U-spin symmetry-limit relations became standard tools in weak phenomenology [Zeppenfeld 1981; Savage & Wise 1989; Gronau & London 1990; Lipkin, Rosner, ...]
- **2013-15:** **“accidental” second-order rate sum rules** were discovered in specific charm systems [Grossman & Robinson 2013; Gronau 2014, 2015]
- **2022-2024:** systematic framework for higher-order U-spin **amplitude sum rules**: all sum rules, to all orders in symmetry breaking, for arbitrary systems [MG, Grossman & Schacht 2022; MG & Schacht 2024]
- **2026:** first **general results for higher-order rate sum rules**; universal second-order charm master sum rule [MG, Grossman, Papiri & Schacht 2026]

$$\begin{bmatrix} d \\ s \end{bmatrix} = \begin{bmatrix} |1/2, +1/2\rangle \\ |1/2, -1/2\rangle \end{bmatrix}, \quad \begin{bmatrix} \bar{s} \\ -\bar{d} \end{bmatrix} = \begin{bmatrix} |1/2, +1/2\rangle \\ |1/2, -1/2\rangle \end{bmatrix}$$

fundamental doublets under  $U$ -spin

# Example: $D^0 \rightarrow P^+ P^-$

$U$ -spin set of processes:

$$D^0 \rightarrow \pi^+ K^-, \quad D^0 \rightarrow K^+ \pi^-, \quad D^0 \rightarrow \pi^+ \pi^-, \quad D^0 \rightarrow K^+ K^-$$

Initial and final states:

$$D^0 = |c\bar{u}\rangle = |0,0\rangle, \quad P^+ = \begin{bmatrix} K^+ \\ \pi^+ \end{bmatrix} = \begin{bmatrix} |u\bar{s}\rangle \\ -|u\bar{d}\rangle \end{bmatrix} = \begin{bmatrix} |1/2, +1/2\rangle \\ |1/2, -1/2\rangle \end{bmatrix}, \quad P^- = \begin{bmatrix} \pi^- \\ K^- \end{bmatrix} = \begin{bmatrix} |d\bar{u}\rangle \\ |s\bar{u}\rangle \end{bmatrix} = \begin{bmatrix} |1/2, +1/2\rangle \\ |1/2, -1/2\rangle \end{bmatrix}$$

Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{U\text{-spin limit}} = f_{1,1} H_1^1 + f_{1,0} H_0^1 + f_{1,-1} H_{-1}^1$$

$$H_1^1 = (\bar{u}s)(\bar{d}c), \quad H_0^1 = \frac{(\bar{u}s)(\bar{s}c) - (\bar{u}d)(\bar{d}c)}{\sqrt{2}}, \quad H_{-1}^1 = -(\bar{u}d)(\bar{s}c)$$

$$f_{1,1} = V_{cd}^* V_{us} \sim \lambda^2, \quad f_{1,0} = \frac{V_{cs}^* V_{us} - V_{cd}^* V_{ud}}{\sqrt{2}} \sim \lambda, \quad f_{1,-1} = -V_{cs}^* V_{ud} \sim -1$$

(DCS)

(SCS)

(CF)

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**This is universal Hamiltonian for hadronic charm decays.**

Therefore all weak hadronic charm decay systems have the group-theoretical structure:

$$(I_1 \otimes \dots \otimes I_r) \otimes I_H, \quad I_H = 1$$

Here, we have:  $\frac{1}{2} \otimes \frac{1}{2} \otimes 1$

# Trivial Rate Sum Rules at $\mathcal{O}(\varepsilon)$

---

Amplitudes related to each other by the  $U$ -spin conjugation ( $d \leftrightarrow s$ ) are equal to each other:

$$A_\alpha = A_{\bar{\alpha}}$$

Gronau, arXiv: hep-ph/0008292

Ex.  $D^0 \rightarrow P^+P^-$ : up to corrections of  $\mathcal{O}(\varepsilon)$

$$A(D^0 \rightarrow \pi^+K^-) = A(D^0 \rightarrow K^+\pi^-), \quad A(D^0 \rightarrow \pi^+\pi^-) = A(D^0 \rightarrow K^+K^-)$$

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Then in the symmetry limit

$$\Gamma_\alpha = \int d\Pi |A_\alpha|^2, \quad \Gamma_{\bar{\alpha}} = \int d\Pi |A_{\bar{\alpha}}|^2$$

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$$\frac{\Gamma(D^0 \rightarrow \pi^+K^-)}{\Gamma(D^0 \rightarrow K^+\pi^-)} = 1 + \mathcal{O}(\varepsilon),$$

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Symmetry-limit  $U$ -spin  
sum rules

one  $\mathcal{O}(\varepsilon^2)$  sum rule

all hadronic  
charm decays

# One Sum to Rule Them All

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- For every irrep  $I_i > 1/2$  in the system, there exist at least one  $\mathcal{O}(\varepsilon^2)$  rate sum rule.
- All weak hadronic charm decay systems have the group-theoretical structure

$$(I_1 \otimes \cdots \otimes I_r) \otimes I_H, \quad I_H = 1$$

- For hadronic charm decays, applying this result to the weak-Hamiltonian triplet,  $I_H = 1$ , gives

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$$\frac{\text{sum of CF and DCS CKM-free rates}}{\text{sum of SCS CKM-free rates}} = 1 + \mathcal{O}(\varepsilon^2)$$

Ex.  $D^0 \rightarrow P^+P^-$ :

$$\frac{\Gamma(D^0 \rightarrow \pi^- K^+) + \Gamma(D^0 \rightarrow K^- \pi^+)}{\Gamma(D^0 \rightarrow K^- K^+) + \Gamma(D^0 \rightarrow \pi^- \pi^+)} = 1 + \mathcal{O}(\varepsilon^2)$$

# Testing the Sum Rules

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- symmetry-limit test

$$\frac{\Gamma(i \rightarrow f)}{\Gamma(\bar{i} \rightarrow \bar{f})} = 1 + \mathcal{O}(\varepsilon)$$

- second-order test

$$R_H(U - \text{spin system}) \equiv \frac{\sum_{\text{CF,DCS}} \Gamma}{\sum_{\text{SCS}} \Gamma} = 1 + \mathcal{O}(\varepsilon^2)$$

# Fully measured systems

$D^0 \rightarrow P^- P^+$

Decay	BR	CKM	Conjugate decay	BR	CKM
$D^0 \rightarrow \pi^- \pi^+$	$(1.453 \pm 0.024) \cdot 10^{-3}$	SCS	$D^0 \rightarrow K^- K^+$	$(4.08 \pm 0.06) \cdot 10^{-3}$	SCS
$D^0 \rightarrow \pi^- K^+$	$(1.362 \pm 0.025) \cdot 10^{-4}$	DCS	$D^0 \rightarrow K^- \pi^+$	$(3.945 \pm 0.030)\%$	CF

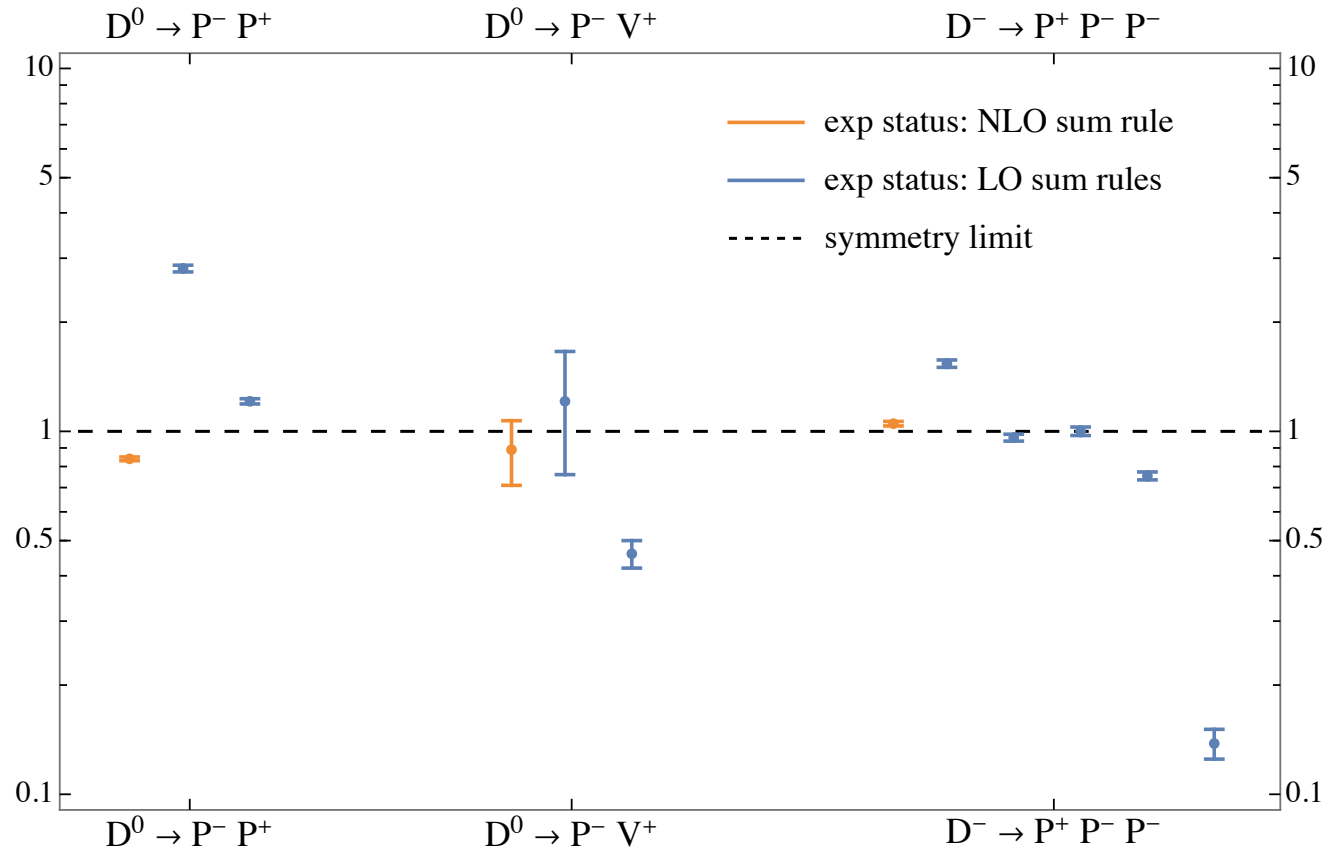
$D^0 \rightarrow P^- V^+$

Decay	BR	CKM	Conjugate decay	BR	CKM
$D^0 \rightarrow K^- \rho^+$	$(11.2 \pm 0.7)\%$	CF	$D^0 \rightarrow \pi^- K^{*+}$	$3 \times (1.28 \pm 0.47) \cdot 10^{-4} \dagger$	DCS
$D^0 \rightarrow K^- K^{*+}$	$3 \times (1.55 \pm 0.10) \cdot 10^{-3} (S = 1.3) \dagger$	SCS	$D^0 \rightarrow \pi^- \rho^+$	$(1.01 \pm 0.05)\%$	SCS

$D^- \rightarrow P^+ P^- P^-$

Decay	BR	CKM	Conjugate decay	BR	CKM
$D^- \rightarrow \pi^+ K^- \pi^-$	$(4.91 \pm 0.09) \cdot 10^{-4}$	DCS	$D_s^- \rightarrow K^+ \pi^- K^-$	$(5.45 \pm 0.08) \cdot 10^{-2}$	CF
$D^- \rightarrow K^+ K^- K^-$	$(6.14 \pm 0.11) \cdot 10^{-5}$	DCS	$D_s^- \rightarrow \pi^+ \pi^- \pi^-$	$(1.090 \pm 0.014) \cdot 10^{-2}$	CF
$D^- \rightarrow K^+ \pi^- \pi^-$	$(9.38 \pm 0.16) \cdot 10^{-2}$	CF	$D_s^- \rightarrow \pi^+ K^- K^-$	$(1.293 \pm 0.027) \cdot 10^{-4}$	DCS
$D^- \rightarrow K^+ K^- \pi^-$	$(9.68 \pm 0.18) \cdot 10^{-3}$	SCS	$D_s^- \rightarrow \pi^+ \pi^- K^-$	$(6.23 \pm 0.10) \cdot 10^{-3}$	SCS
$D^- \rightarrow \pi^+ \pi^- \pi^-$	$(3.27 \pm 0.09) \cdot 10^{-3}$	SCS	$D_s^- \rightarrow K^+ K^- K^-$	$(2.18 \pm 0.20) \cdot 10^{-4}$	SCS

# Fully measured systems



Grossman & Robinson (2013)

MG, Y. Grossman, S. Schacht, arXiv:2602.22320

# Systems with missing data

$D^0 \rightarrow V^- P^+$

$D^0 \rightarrow K^{*-} \pi^+$	$3 \times (1.82 \pm 0.30)\% (S = 2.2)^\dagger$	CF	$D^0 \rightarrow \rho^- K^+$	$< (3.06 \pm 0.16) \cdot 10^{-4} \ddagger$	DCS
$D^0 \rightarrow K^{*-} K^+$	$3 \times (5.54 \pm 0.37) \cdot 10^{-4} \dagger$	SCS	$D^0 \rightarrow \rho^- \pi^+$	$(5.15 \pm 0.26) \cdot 10^{-3}$	SCS

Second-order master sum rule allows to derive prediction for the missing BR:  $\mathcal{B}(D^0 \rightarrow \rho^- K^+) = (2.08 \pm 0.30) \times 10^{-4}$

$C_b^+ \rightarrow L_b^+ P^+ P^-$

Decay	BR	CKM	Conjugate decay	BR	CKM
$\Lambda_c^+ \rightarrow \Sigma^+ K^- K^+$	$(3.6 \pm 0.4) \cdot 10^{-3}$	CF	$\Xi_c^+ \rightarrow p \pi^- \pi^+$	?	DCS
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^- \pi^+$	$(4.54 \pm 0.20)\%$	CF	$\Xi_c^+ \rightarrow p K^- K^+$	?	DCS
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^- K^+$	$(2.04 \pm 0.26) \cdot 10^{-3}$	SCS	$\Xi_c^+ \rightarrow p K^- \pi^+$	$(6.2 \pm 3.0) \cdot 10^{-3}$	SCS
$\Lambda_c^+ \rightarrow p K^- \pi^+$	$(6.35 \pm 0.25)\%$	CF	$\Xi_c^+ \rightarrow \Sigma^+ \pi^- K^+$	?	DCS
$\Lambda_c^+ \rightarrow p K^- K^+$	$(1.08 \pm 0.05) \cdot 10^{-3}$	SCS	$\Xi_c^+ \rightarrow \Sigma^+ \pi^- \pi^+$	$(1.4 \pm 0.8)\%$	SCS
$\Lambda_c^+ \rightarrow p \pi^- \pi^+$	$(4.67 \pm 0.24) \cdot 10^{-3}$	SCS	$\Xi_c^+ \rightarrow \Sigma^+ K^- K^+$	$(4.3 \pm 2.5) \cdot 10^{-3}$	SCS
$\Lambda_c^+ \rightarrow p \pi^- K^+$	$(1.13 \pm 0.17) \cdot 10^{-4}$	DCS	$\Xi_c^+ \rightarrow \Sigma^+ K^- \pi^+$	$(2.7 \pm 1.2)\%$	CF

# Systems with missing data

Decay	BR	CKM	Conjugate decay	BR	CKM
$\Xi_c^0 \rightarrow \Xi^- \pi^+$	$(1.43 \pm 0.27)\%$	CF	$\Xi_c^0 \rightarrow \Sigma^- K^+$	?	DCS
$\Xi_c^0 \rightarrow \Sigma^- \pi^+$	?	SCS	$\Xi_c^0 \rightarrow \Xi^- K^+$	$(3.9 \pm 1.1) \cdot 10^{-4}$	SCS
$\Xi_c^0 \rightarrow p K^-$	?	SCS	$\Xi_c^0 \rightarrow \Sigma^+ \pi^-$	?	SCS
$\Xi_c^0 \rightarrow \Sigma^+ K^-$	$(1.8 \pm 0.4) \cdot 10^{-3}$	CF	$\Xi_c^0 \rightarrow p \pi^-$	?	DCS
$\Xi_c^0 \rightarrow \Delta^- \pi^+$	?	DCS	$\Xi_c^0 \rightarrow \Omega^- K^+$	?	CF
$\Xi_c^0 \rightarrow \Sigma^{*-} \pi^+$	?	SCS	$\Xi_c^0 \rightarrow \Xi^{*-} K^+$	?	SCS
$\Xi_c^0 \rightarrow \Xi^{*-} \pi^+$	?	CF	$\Xi_c^0 \rightarrow \Sigma^{*-} K^+$	?	DCS

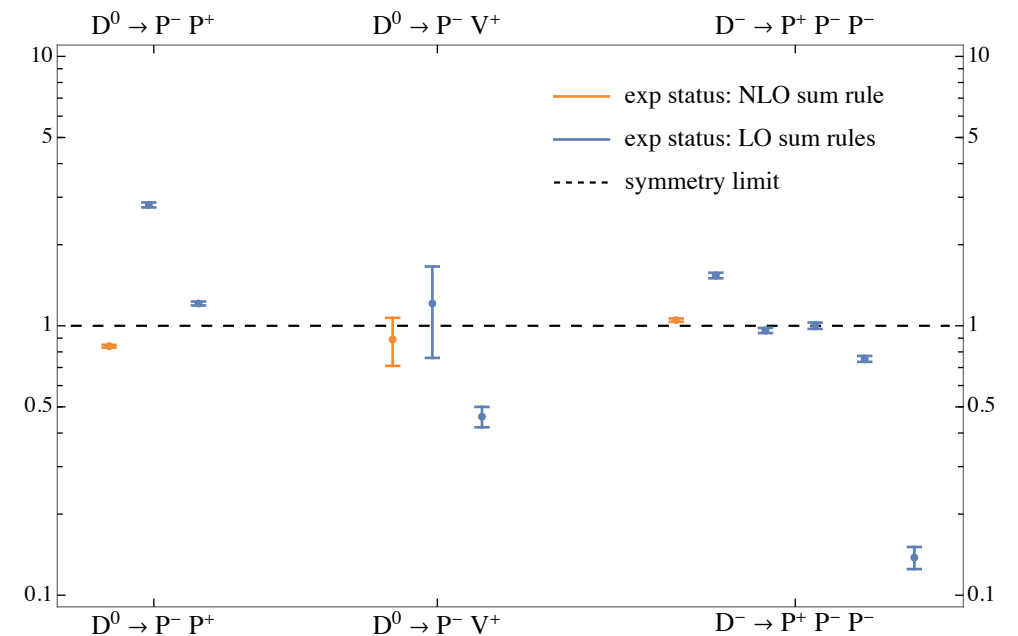
$$\frac{1}{2} \otimes \frac{3}{2} \otimes 1$$

Two irreps  $> 1/2 \longrightarrow$  **two  $\mathcal{O}(\varepsilon^2)$  sum rules!**

# Takeaway

- Second-order master sum rule in charm follows from a few **simple, general observations**
- These ideas can be applied **beyond charm, beyond U-spin, beyond weak decays**
- In data **second-order sum rules can work strikingly well**, even when symmetry-limit relations appear badly broken

$$\frac{\text{sum of CF and DCS CKM-free rates}}{\text{sum of SCS CKM-free rates}} = 1 + \mathcal{O}(\varepsilon^2)$$



# Outlook

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- Accumulate more second-order sum rules  
→ study patterns in symmetry breaking
- Learn from data  
→ possible probes of underlying dynamics, perhaps even new physics
- Crucial open question: what do  $\mathcal{O}(\varepsilon)$ ,  $\mathcal{O}(\varepsilon^2)$  actually mean?

# Backup

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# What do $\mathcal{O}(\varepsilon)$ , $\mathcal{O}(\varepsilon^2)$ mean?

---

At the fundamental level,  $U$ -spin **breaking comes from quark-mass difference**

$$\Delta m \equiv m_s - m_d, \quad \varepsilon \equiv \frac{\Delta m}{(\text{some energy scale})} \sim 30\%$$

Then, typically, one assumes

$$\mathcal{O}(\varepsilon) \sim 30\%, \quad \mathcal{O}(\varepsilon^2) \sim 10\%$$

In this talk, however,

$$\mathcal{O}(\varepsilon) \propto \Delta m$$

$$\mathcal{O}(\varepsilon^2) \propto \Delta m^2$$

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Then, typically, one assumes

~~$\mathcal{O}(\varepsilon) \sim 30\%$ ,  $\mathcal{O}(\varepsilon^2) \sim 10\%$~~

Precise meaning **can depend** on the **process** and specific **observable** considered!

In this talk, however,

$$\mathcal{O}(\varepsilon) \propto \Delta m$$

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