Towards Quantum Asymptotic Velocity Domination

- The Polarized Gowdy Cosmologies -

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Outline

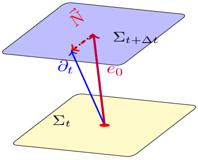
1. Einstein gravity in ADM formalism, its Carroll limit, and Asymptotic Velocity Domination (AVD).
2. Polarized Gowdy cosmologies: classical AVD, canonical quantization, and state-dependence.
3. AVD for quantum two-point functions and spatial gradient expansion.
4. Conclusions.

Based on Niedermaier–Sedighi Jafari, *Ann. Phys.* **490** (2026) 170492; arXiv:2509.13162.

Gravity in ADM formalism

We use the ADM parameterization of the metric:

$$g_{\mu\nu}(y)\, dy^\mu\, dy^\nu \;=\; -N^2\, dt^2 + \gamma_{ij}\,(dx^i + N^i\, dt)\,(dx^j + N^j\, dt),$$



where $y^\mu = (t, x^1, x^2, x^3)$, and $\gamma_{ij}(t,.)$ is the metric on Σ_t, $N(t,x)$ is the lapse, $N^i(t,x)$ is the shift vector.

Vacuum Einstein gravity has action:

$$S = \frac{1}{2\kappa}\int_{t_i}^{t_f} dt \int_\Sigma d^3x\, \sqrt{\gamma}\left\{ \frac{1}{4N}\, e_0(\gamma_{ij})\, G^{ijkl}(\gamma)\, e_0(\gamma_{kl}) + N R(\gamma) \right\}.$$

with $e_0 := \partial_t - \mathcal{L}_{\vec{N}}$ derivation transversal to Σ_t's, and $G^{ijkl}(\gamma) := \frac{1}{2}\left(\gamma^{ik}\gamma^{jl} + \gamma^{il}\gamma^{jk} - 2\gamma^{ij}\gamma^{kl}\right)$.

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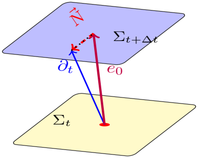

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ADM formalism – Velocity Domination

(Seemingly) ad-hoc simplification: Drop all dynamical spatial derivatives (but keep those in e_0). The resulting action S^{VD} describes a Carroll type gravity theory [Henneaux, 1979]:

$$S^{VD} = \frac{1}{2\kappa} \int_{t_i}^{t_f} dt \int_\Sigma d^3x \sqrt{\gamma} \left\{ \frac{1}{4N} e_0(\gamma_{ij}) \, G^{ijkl}(\gamma) \, e_0(\gamma_{kl}) \right\}.$$

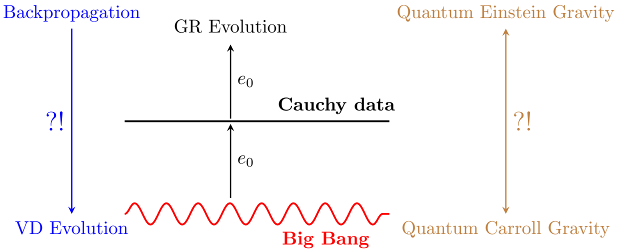
Derive the respective field equations by variation with respect to the fields:

- $\frac{\delta S}{\delta \gamma_{ij}}$: 6 Evolution Equations
- $\frac{\delta S}{\delta N_i}$: 3 Diffeomorphism Constraints
- $\frac{\delta S}{\delta N}$: 1 Hamiltonian Constraint

- $\frac{\delta S^{VD}}{\delta \gamma_{ij}}$: 6 ODEs w.r.t. e_0
- $\frac{\delta S^{VD}}{\delta N_i}$: Unchanged
- $\frac{\delta S^{VD}}{\delta N}$: No Spatial Gradients

Asymptotic Velocity Domination scenario

Consider "generic" cosmological spacetime for "suitable" foliation and gauge fixing.



AVD Scenario:

When backpropagated to the Big Bang, “almost every” cosmological solution of the Einstein equations becomes a solution of the corresponding (much simpler) velocity-dominated system. Proven for several important cases, including Gowdy cosmologies.

AVD for polarized Gowdy

Metric Ansatz:

$$g^{PG}_{\mu\nu}(X)dX^\mu dX^\nu = e^\sigma\left(-dt^2 + d\zeta^2\right) + te^{-\phi}(dy^1)^2 + te^{\phi}(dy^2)^2,$$

where σ and ϕ are functions of $t \geq 0$ and $\zeta \in [0, 2\pi]$ (periodic).

Full Einstein Eqns	**Velocity-Dom. Eqns**	
$\partial_t\left(t\partial_t\phi\right) - t\partial_\zeta^2\phi = 0$	$\partial_t\left(t\partial_t\varphi\right) = 0$	Evolution
$\partial_t\sigma = \frac{1}{2}t\left[(\partial_t\phi)^2 + (\partial_\zeta\phi)^2\right]$	$t\partial_t\varsigma = \frac{1}{2}\left(t\,\partial_t\varphi\right)^2$	Ham. Constr.
$\partial_\zeta\sigma = t\partial_t\phi\,\partial_\zeta\phi$	$\partial_\zeta\varsigma = t\,\partial_t\varphi\,\partial_\zeta\varphi$	Diffeo. Constr.

Asymptotic Velocity Domination (AVD): Proven rigorously for polarized Gowdy in *Isenberg & Moncrief (1990)*.

For almost all solutions

$$(\phi, \sigma) \xrightarrow{t \to 0^+} (\varphi, \varsigma).$$

Conversely, one can reconstruct (ϕ, σ) from (φ, ς).

Fourier decomposition and canonical quantization

- **Strategy:**

  Since σ and ς are determined by ϕ and φ, respectively, the main focus is on the **one-to-one correspondence** between $\phi(t,\zeta)$ and $\varphi(t,\zeta)$. Moreover, as their defining wave equations are linear, the problem can naturally be reformulated in terms of spatial **Fourier transforms**.

- **Full Gowdy System:**

$$\boxed{\phi(\tau,\zeta) = T_0(\tau)a_0 + T_0(\tau)^* a_0^* + \sum_{n\neq 0}\Big\{T_n(\tau)e^{in\zeta}a_n + T_n(\tau)^* e^{-in\zeta}a_n^*\Big\}\,,}$$

  where we define $\tau := \ln(t/t_0)$, (Big bang for $\tau \to -\infty$) and

$$\left[\partial_\tau^2 + e^{2\tau}(t_0 n)^2\right]T_n(\tau) = 0, \quad (\partial_\tau T_n)T_n^* - (\partial_\tau T_n)^* T_n = -i.$$

- **Velocity-Dominated System:**

$$\boxed{\varphi(\tau,\zeta) = T_0(\tau)a_0 + T_0(\tau)^* a_0^* + \sum_{n\neq 0}\Big\{\mathrm{t}_n(\tau)e^{in\zeta}a_n + \mathrm{t}_n(\tau)^* e^{-in\zeta}a_n^*\Big\}\,,}$$

  where $\partial_\tau^2 \mathrm{t}_n = 0$, i.e. the t_n's are **linear** in τ.

Origin of state-dependence

1. Solutions T_n and $\mathfrak{t}_n$ are non-unique but in one-to-one correspondence. Take a_n, a^*_{-n} to be the same in both systems.
2. A choice of $T_n(\tau)$ determines the choice of coefficients $a^T_n, (a^T_{-n})^*$ for given ϕ and φ.
3. For canonical quantization, promote a^T_n and $(a^T_n)^*$ to Fock space operators.
4. A choice of $a^T_n, (a^T_{-n})^*$ then determines a choice of Fock vacuum, where $a^T_n \left|0_T\right\rangle = 0$.

Two-point function from initial value kernel

Full Gowdy: Define the symmetric matrix two-point function as

$$\mathcal{W}^s(\tau,\tau',\zeta-\zeta') := \begin{pmatrix} 1 & \partial_{\tau'} \\ \partial_\tau & \partial_\tau\partial_{\tau'} \end{pmatrix} \langle 0_T|\,\phi(\tau,\zeta)\phi(\tau',\zeta') + \phi(\tau',\zeta')\phi(\tau,\zeta)\,|0_T\rangle \,.$$

Get all symmetric two-point functions of ϕ and $\pi := \partial_\tau\phi$. Then

$$\mathcal{W}^s(\tau,\tau',\zeta-\zeta') = \int_0^{2\pi}\frac{d\zeta_0}{2\pi}\int_0^{2\pi}\frac{d\zeta_0'}{2\pi} D(\tau,\zeta;\tau_0,\zeta_0)\,\mathcal{W}^s(\tau_0,\tau_0,\zeta_0-\zeta_0')\,D(\tau',\zeta';\tau_0,\zeta_0')^T.$$

Here $D(\tau,\zeta;\tau_0,\zeta_0)$ is a state-independent kernel with explicitly known Fourier decomposition.

VD Gowdy: The VD counterpart of $\mathcal{W}^s$ is much simpler and carries a linear τ,τ' dependence pointwise in $\zeta-\zeta'$

$$\mathfrak{W}^s(\tau,\tau',\zeta-\zeta') = \begin{pmatrix} 1 & \tau-\tau_0 \\ 0 & 1 \end{pmatrix} \mathfrak{W}^s(\tau_0,\tau_0,\zeta-\zeta') \begin{pmatrix} 1 & 0 \\ \tau'-\tau_0 & 1 \end{pmatrix}.$$

Conditions on the state

The state-dependence of $\mathcal{W}^s(\tau, \tau', \zeta - \zeta')$ is carried by the initial data kernel on which we impose three conditions:

Positivity: The Fourier coefficients $Z_n(\tau_0)$ are positive definite 2×2 matrices.

$$\mathcal{W}^s(\tau_0, \tau_0, \zeta - \zeta') = \sum_{n \in \mathbb{Z}} e^{in(\zeta - \zeta')} Z_n(\tau_0).$$

Time consistency: The τ_0-dependence of $Z_n(\tau_0)$ is such that $\mathcal{W}^s(\tau, \tau', \zeta - \zeta')$ does not depend on the chosen reference time τ_0.

Hadamard-type: For large n behavior of $Z_n(\tau_0)$ is

$$Z_n(\tau_0) \asymp \left. \begin{pmatrix} \frac{2}{x} - \frac{1}{4x^3} & -\frac{1}{x} + \frac{3}{8}\frac{1}{x^3} \\ -\frac{1}{x} + \frac{3}{8}\frac{1}{x^3} & \frac{x}{2} + \frac{9}{16}\frac{1}{x} - \frac{105}{256}\frac{1}{x^3} \end{pmatrix} \right|_{x = |n| e^{\tau_0}} + O(n^{-5}) \,.$$

Gradient map

The mode functions in the full and VD Gowdy systems entering the expansions of $\phi(\tau,\zeta)$ and $\varphi(\tau,\zeta)$, respectively, are related by

$$\begin{pmatrix} T_n(\tau) \\ \partial_\tau T_n(\tau) \end{pmatrix} = I^{\mathrm{grad}}(|n|e^{\tau}) \begin{pmatrix} \mathfrak{t}_n(\tau) \\ \partial_\tau \mathfrak{t}_n(\tau) \end{pmatrix}, \quad n \in \mathbb{Z},$$

$$I^{\mathrm{grad}}(x) := \begin{pmatrix} J_0(x) & U_0(x) \\ -xJ_1(x) & -xU_1(x) \end{pmatrix} =: \mathbb{1} + \sum_{k\geq 1} I_k\, x^{2k},$$

with $U_k(x) := (\pi/2)Y_k(x) - \ln(|x|e^{\gamma_E}/2)J_k(x)$, $k = 0, 1$. All matrix elements have absolutely convergent Taylor expansions in x^2. With $x^2 = n^2 e^{2\tau}$ the n^2 powers correspond to spatial gradients in position space.

Gradient expansion for two-point functions

Fourier modes of Full and VD two-point functions are related by

$$\mathcal{W}^s_n(\tau,\tau') = I^{\rm grad}(|n|e^{\tau})\,\mathfrak{W}^s_n(\tau,\tau')\,I^{\rm grad}(|n|e^{\tau'})^T\,, \quad n \neq 0.$$

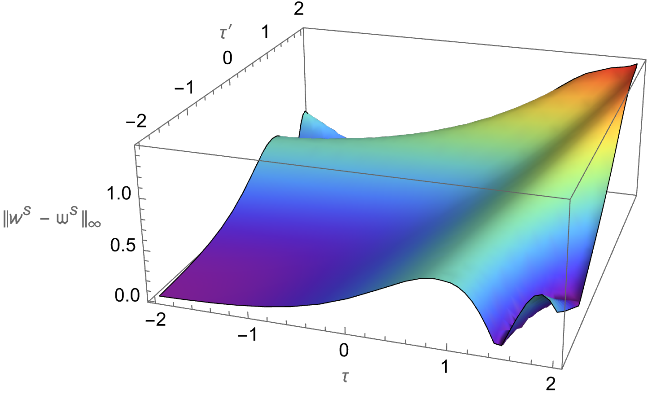
In position space need spatial averaging. Obtain

Result. For admissible states the matrix two-point function $\mathcal{W}^s(\tau,\tau',\zeta-\zeta')$ admits upon averaging with $f, g \in C^\infty(S^1)$ test functions a series expansion of the form

$$\int \frac{d\zeta}{2\pi}\frac{d\zeta'}{2\pi} f(\zeta)g(\zeta')\,\mathcal{W}^s(\tau,\tau',\zeta-\zeta') = \int \frac{d\zeta}{2\pi}\frac{d\zeta'}{2\pi} f(\zeta)g(\zeta')\mathfrak{W}^s(\tau,\tau',\zeta-\zeta')$$
$$+ \sum_{l\geq 1}\sum_{k=0}^{l} e^{2k\tau}e^{2(l-k)\tau'} \int \frac{d\zeta}{2\pi}\frac{d\zeta'}{2\pi} f(\zeta)g(\zeta')(\partial_\zeta\partial_{\zeta'})^l I_k\,\mathfrak{W}^s(\tau,\tau',\zeta-\zeta')I^T_{l-k}\,.$$

Here, I_k are the numerical 2×2 matrices in the expansion of $I^{\rm grad}(x)$ in powers of x^2. All terms in the expansion are spatial gradients of the two-point function $\mathfrak{W}^s(\tau,\tau',\zeta-\zeta')$ of the VD system, which is linear in τ,τ'. The series is uniformly convergent for all $\tau,\tau' < -\delta$, $\delta > 0$ and $\tau-\tau'$ bounded.

Difference $\|\mathcal{W}^s - \mathfrak{W}^s\|_\infty$ in Bunch-Davies vacuum



Bunch-Davies vacuum: $T_n(\tau) = \frac{\sqrt{\pi}}{2} H_0^{(2)}(|n|e^\tau)$.

Conclusions

- Polarized Gowdy cosmologies are simple laboratory to probe quantum aspects of asymptotic velocity domination.
- Two-point functions in admissible states of the quantum Gowdy scalar tend toward their velocity-dominated counterpart near the Big Bang. Conversely, they admit a convergent expansion in terms of spatial derivatives of the velocity dominated ones.
- The two-point functions considered also govern the two-point functions of the integrands of Dirac observables for the system. Quantum AVD carries over to them.
- Many technical details glossed over here; see *Ann. Phys.* **490** (2026) 170492; arXiv:2509.13162.

Beyond the polarized case: Dirac observables regular at the Big Bang and their Carroll/VD limit: Niedermaier–Sedighi Jafari, arXiv:2602.03663.

Thank you!