Randall-Sundrum Dark Matter Portal Model
Symmetries, Amplitudes and Phenomenology

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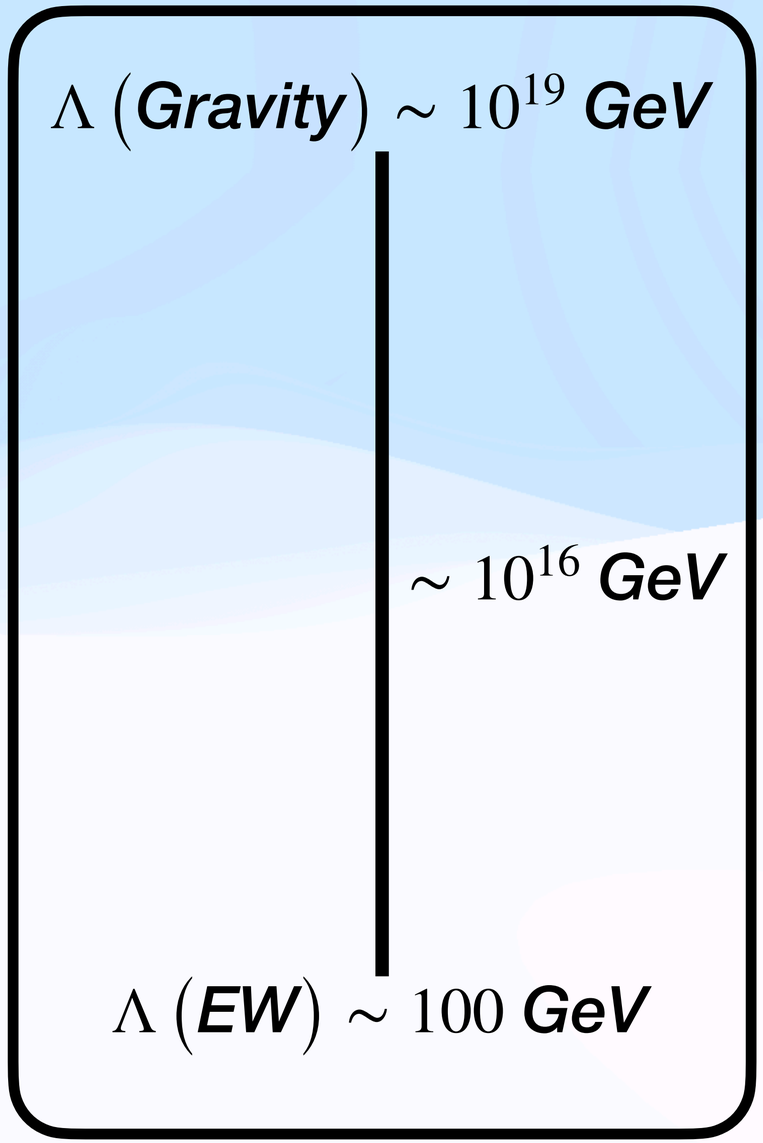
Randall-Sundrum Context

Some problems with SM:

- Hierarchy problem
- Highly fine tuned - Higgs mass
- Dark matters/dark energy unknowns

Randall-Sundrum models can answer* these and:

- Generate $\mathcal{O}(s)$ massive spin-2 EFT
- Be phenomenological probes for dark matter



General Relativity as Massless Spin-2

- **Perturbative** expansion about flat background by **spin-2 field**

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_4 h_{\mu\nu}$$

$$\kappa_4 = \frac{2}{M_{\text{Pl}}} \sim 10^{-19} \frac{\text{GeV}}{c^2}$$

- Einstein's GR is precisely massless spin-2 theory - ghost free to all orders!

$$S_{\text{EH}} = \frac{1}{\kappa_4^2} \int d^4x \sqrt{|\det g|}\, R = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \zeta^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \mathcal{O}\left(h^3\right) \right]$$

- Effective Field Theory with cutoff at $M_{\text{Pl}} \sim 10^{19}$ GeV
- Diffeomorphism invariance built in

Massive Gravity - Fierz-Pauli

$$S_{\mathbf{FP}} = \int d^4x \left\{ -\frac{1}{4} h^{\mu\nu} \zeta^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) - \frac{\kappa_4^2}{2} h_{\mu\nu} T^{\mu\nu} \right\}$$

- Mass term explicitly breaks diffeomorphism invariance
- Expect GR for $m \to 0$

Massive Gravity - Fierz-Pauli

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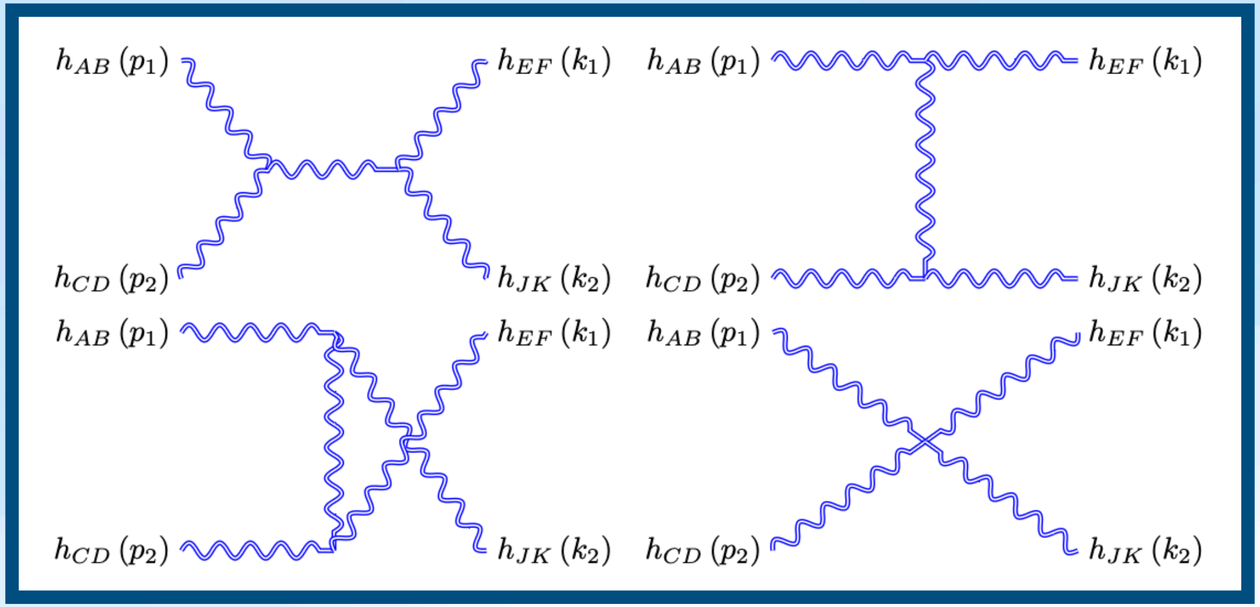
- Mass term explicitly breaks diffeomorphism invariance
- Expect GR for $m \to 0$ - **doesn't happen**
- Now have five degrees of freedom
- Longitudinal mode couples with trace of stress-energy tensor
- **van Dam-Veltman-Zakharov discontinuity**

Fierz-Pauli - Self-Scattering Amplitude

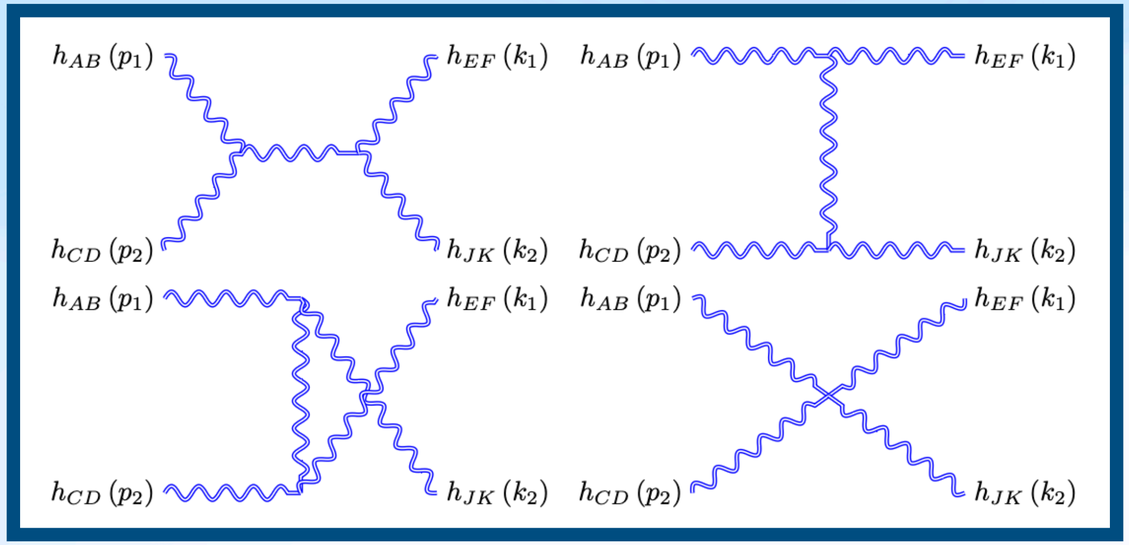


Amplitude Scales as $\mathscr{M} \sim \dfrac{s^5}{M_{\text{Pl}}^2 m^8}$

Effective Cutoff @ $\Lambda_{FP} = \left(M_{\text{Pl}}^2 m^8\right)^{1/5}$

dRGT - Self-Scattering Amplitude

Partial resolution: de Rham-Gabadadze-Tolley (dRGT) massive gravity model

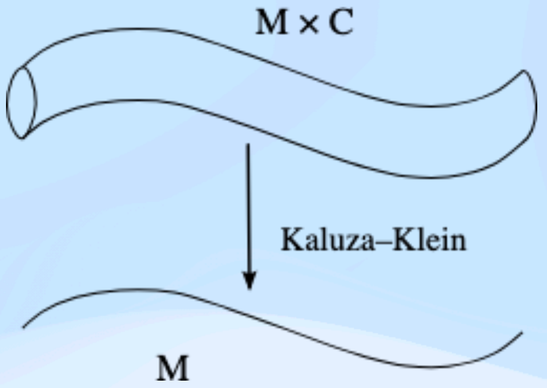


Amplitude Scales as $\mathscr{M} \sim \dfrac{s^3}{M_{\text{Pl}}^2 m^4}$

Effective Cutoff @ $\Lambda_{dRGT} = \left(M_{\text{Pl}}^2 m^4\right)^{1/3}$

Compactified Extra-Dimensional Model of Gravity

- **Kaluza-Klein** (1910/1920s) - unify **gravity** / **EM**
- **Introduces radion** - massless scalar field
- Extra-dim has **very small** extent $L \sim 10^{-30}$ cm
- **ONE Massless spin-2** state (the graviton - gravity)
- An **infinite tower of massive spin-2** states
- Also generate infinite tower of spin-1 and spin-0 fields
- **All spin-1** and **spin-0** (except **massless spin-0 - radion**) can be gauged
- Independent of underlying geometry - choose a phenomenologically relevant one

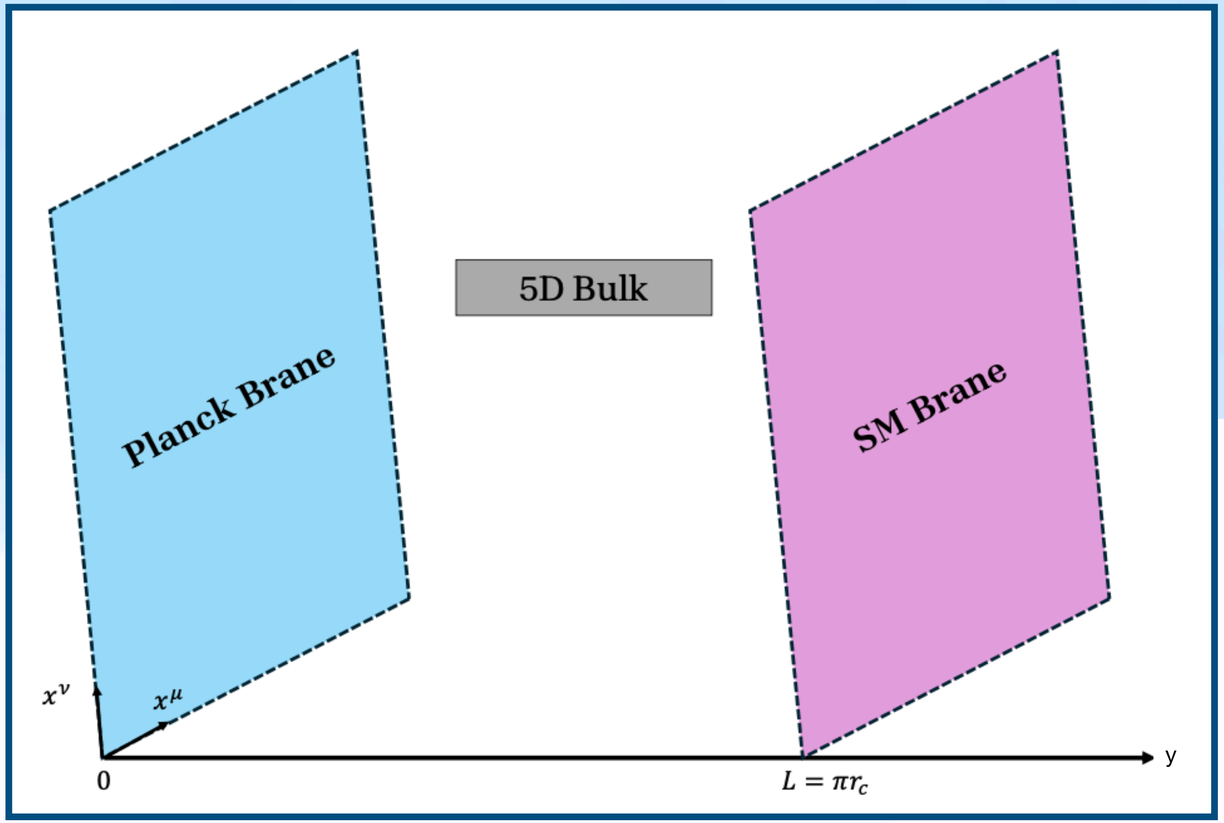


Geometric Higgs Mechanism

Image from: https://en.m.wikipedia.org/wiki/File:Kaluza_Klein_compactification.svg

Randall-Sundrum (RS) Models

- Warped 4D, flat extra-dim
- Two 4D branes @ $y = 0, \pi r_c$
- Fundamental 5D Planck mass M_5
- Bulk Λ, brane-localised λ_i cosmological constants
- Constraints on Λ, λ_i fixed by $\Lambda_{4D} = 0$
- Additional - Goldberger-Wise mechanism
  - Length fixed by VEVs of bulk scalar
  - Radion becomes massive



$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$S = -\frac{M_5^3}{4}\int d^4x \int_0^{\pi r_c} dy \sqrt{\left|\det G\right|}\,(\tilde{R} - 2\Lambda) + \int d^4x \sqrt{\left|\det G_i\right|}\,[\lambda_i]_{y=y_i}$$

RS Behaviour

- Motivated by Anti de-Sitter/Conformal Field Theory duality, choose $\Lambda > 0$

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

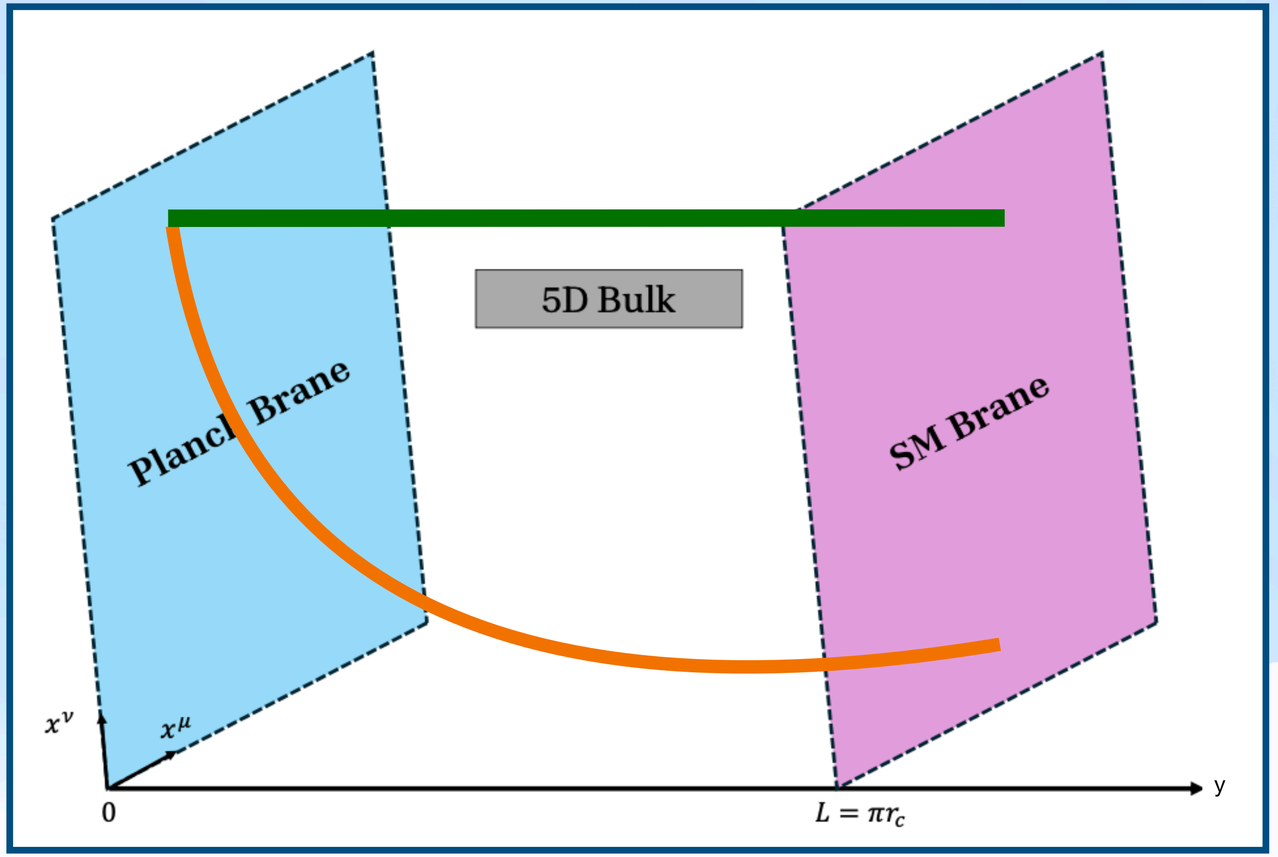
$$A(y) = ky,\ k = \sqrt{\frac{\Lambda}{3}}$$

- Graviton has **universal coupling @ M_{Pl}**
- **SM Higgs VEV** exponentially smaller

$$\frac{\langle\Phi\rangle_{\text{SM}}}{\langle\Phi\rangle_{\text{Pl}}} = e^{-k\pi r_c}$$

- Higher KK modes couple to SM as

$$\Lambda_{\text{TeV}} = M_{\text{Pl}}e^{-k\pi r_c}$$

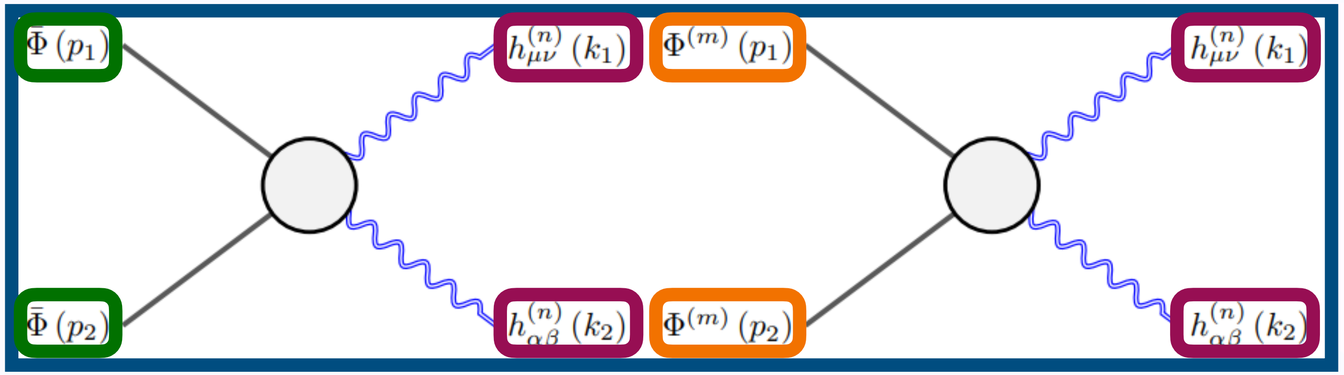


- Now want to investigate UV behaviour of this model

Scattering Amplitudes with Matter

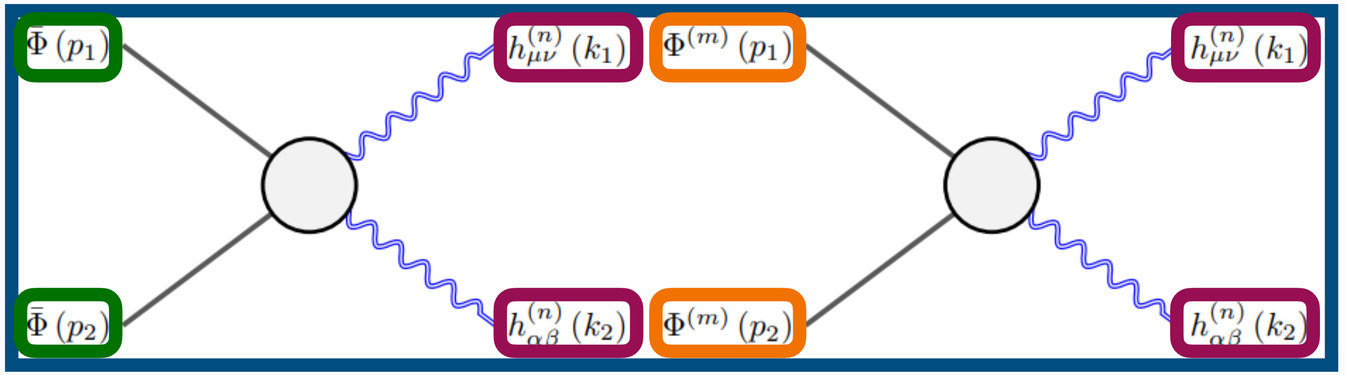
Phys. Rev. D 109, 015033 arXiv:2311.00770

- Produce **on-shell KK modes** from **brane-localisesd** or **bulk** matter
- Takeaways:
  - We define **sum-rules** as a series of rules that relate the summation of propagating KK modes to external on-shell KK modes.
  - For all (brane or bulk) matter species the combination of these sum-rules eliminate the bad high energy behaviour.
  - The **leading high energy behaviour** of the amplitude is always $\mathscr{M} \propto s/\Lambda$



Gauge Implications

- Previous result was in unitary gauge - try ’t-Hooft Feynman-like gauge
- Generates unphysical vector and scalar KK modes - KK mass degeneracy
- $N = 2$ SUSY relationship between spin-2/spin-1/scalar KK
- High energy limit of amplitude appears as though scattering scalar KK modes, not spin-2 KK modes
- Behaviour resembles a Goldstone-Equivalence theorem…



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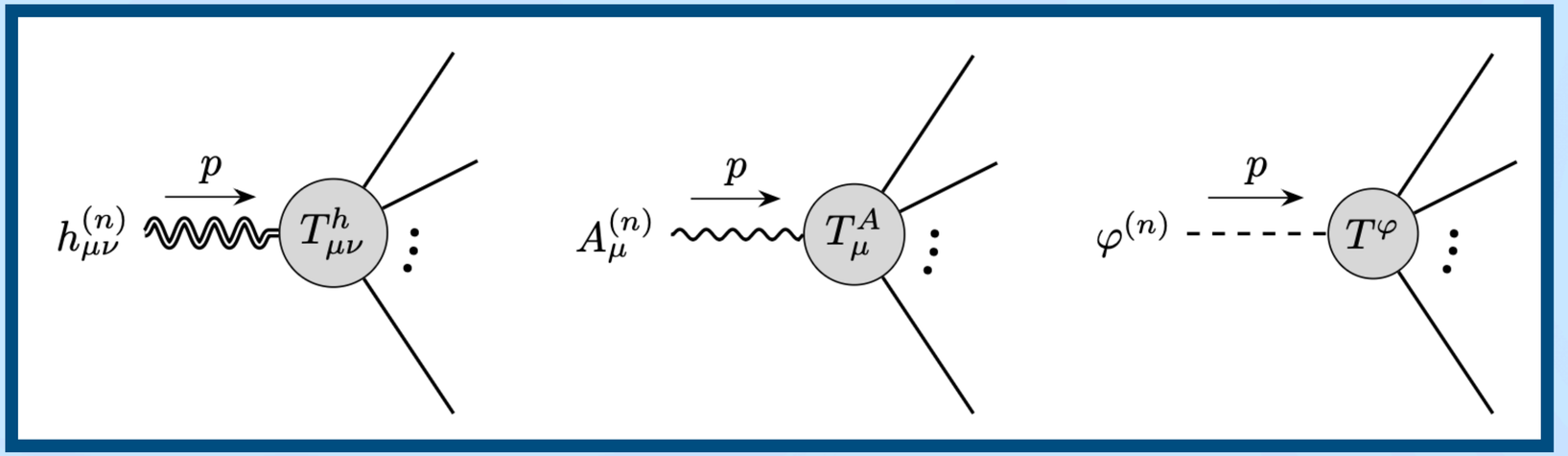
Symmetries and Equivalence theorems

Phys. Rev. D 109, 075016 arXiv:2312.08576

- The compactification is a ‘soft’ breaking of the 5D diffeomorphism
- Residual 4D differomorphism invariance → equivalence theorem
- Exchange
  - (Unitary) **Spin-2 helicity-1** KK mode → (’t-Hooft-Feynman) **spin-1 vector** KK mode
  - (Unitary) **Spin-2 helicity-0** KK mode → (’t-Hooft-Feynman) **scalar** KK mode
- The **gauge choice and mass degeneracy** produce Ward identities
- Ward identities give transparent power counting for scattering processes

Example



$$\mathscr{M}^h = \epsilon^{\mu\nu}\left(p\right) T^h_{\mu\nu}$$

$$\mathscr{M}^A = \epsilon^{\mu}\left(p\right) T^A_{\mu}$$

$$\mathscr{M}^\phi = T^\phi$$

Mass degeneracy between helicity modes
ensures the following Ward Identities hold

$$ip^\nu T^h_{\mu\nu} - \frac{1}{\sqrt{2}} m_n T^A_\mu = 0$$

$$-\frac{1}{2} m_n \eta^{\mu\nu} T^h_{\mu\nu} + \frac{i}{\sqrt{2}} p^\mu T^A_\mu + \sqrt{\frac{3}{2}} m_n T^\phi = 0$$

$$\frac{1}{\sqrt{6}} T^h_{\mu\nu}\left(\eta^{\mu\nu} + \frac{2p^\mu p^\nu}{m}\right) = T^\phi$$

Helicity-2 sub-amplitude related to Goldstone scalar sub-amplitude

Phys. Rev. D 109, 075016
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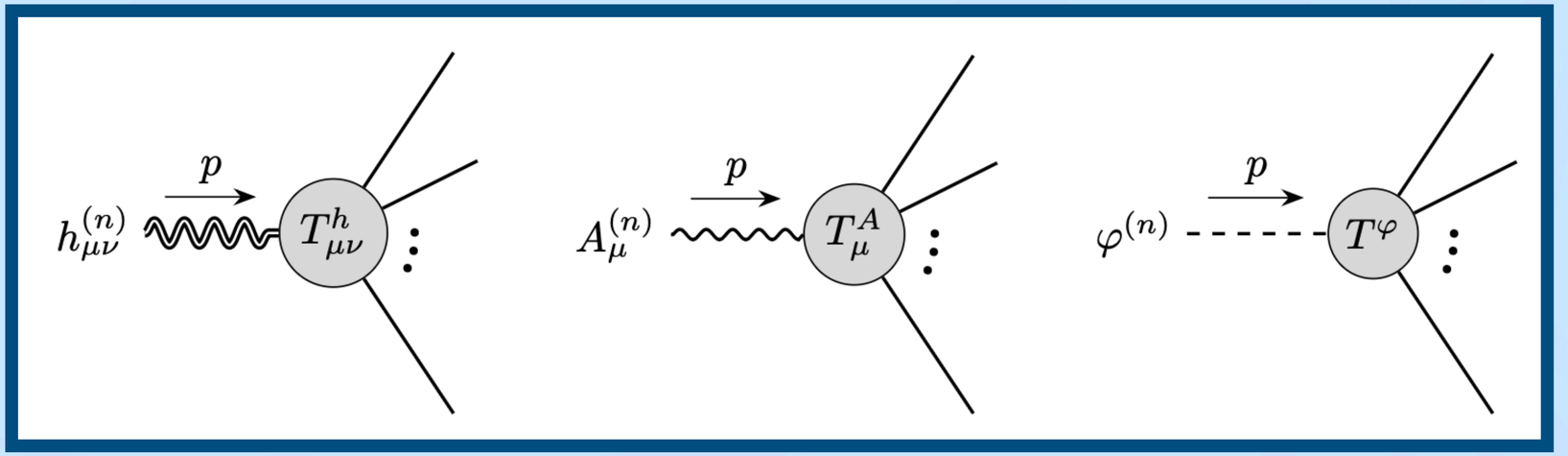
Example



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$$\mathscr{M}^\phi = T^\phi$$

High energy scattering of massive longitudinal spin-2 KK mode has same behaviour as scalar field

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_0 = T^\phi + \mathcal{O}\left(E^0\right)$$

Massive spin-2 modes in unitary gauge are eaten Goldstones in 't-Hooft Feynman gauge

$$\frac{1}{\sqrt{6}} T^h_{\mu\nu}\left(\eta^{\mu\nu} + \frac{2p^\mu p^\nu}{m}\right) = T^\phi$$

$\epsilon^{\mu\nu}_0$

Helicity-2 sub-amplitude related to Goldstone scalar sub-amplitude

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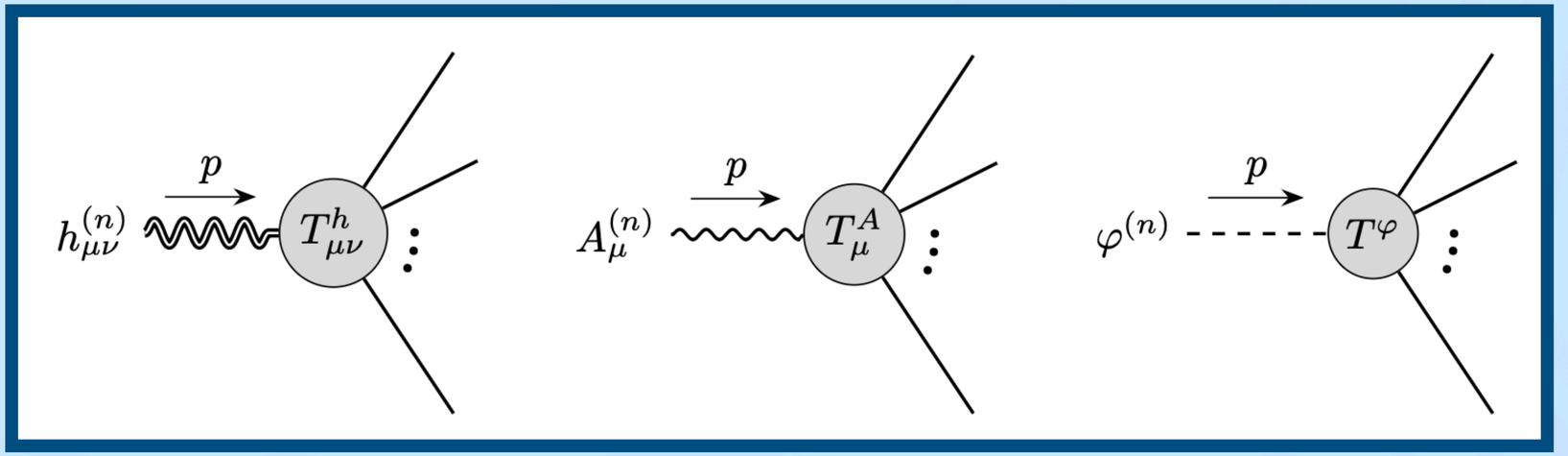
Results



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$$\mathscr{M}^\phi = T^\phi$$

- Generalises to **any number** of **external spin-2** fields
- AND can include **propagators** in framework
- Confirms unitary gauge result (previous work)
- No need for complex sum-rules in this formalism

Phys. Rev. D 109, 075016
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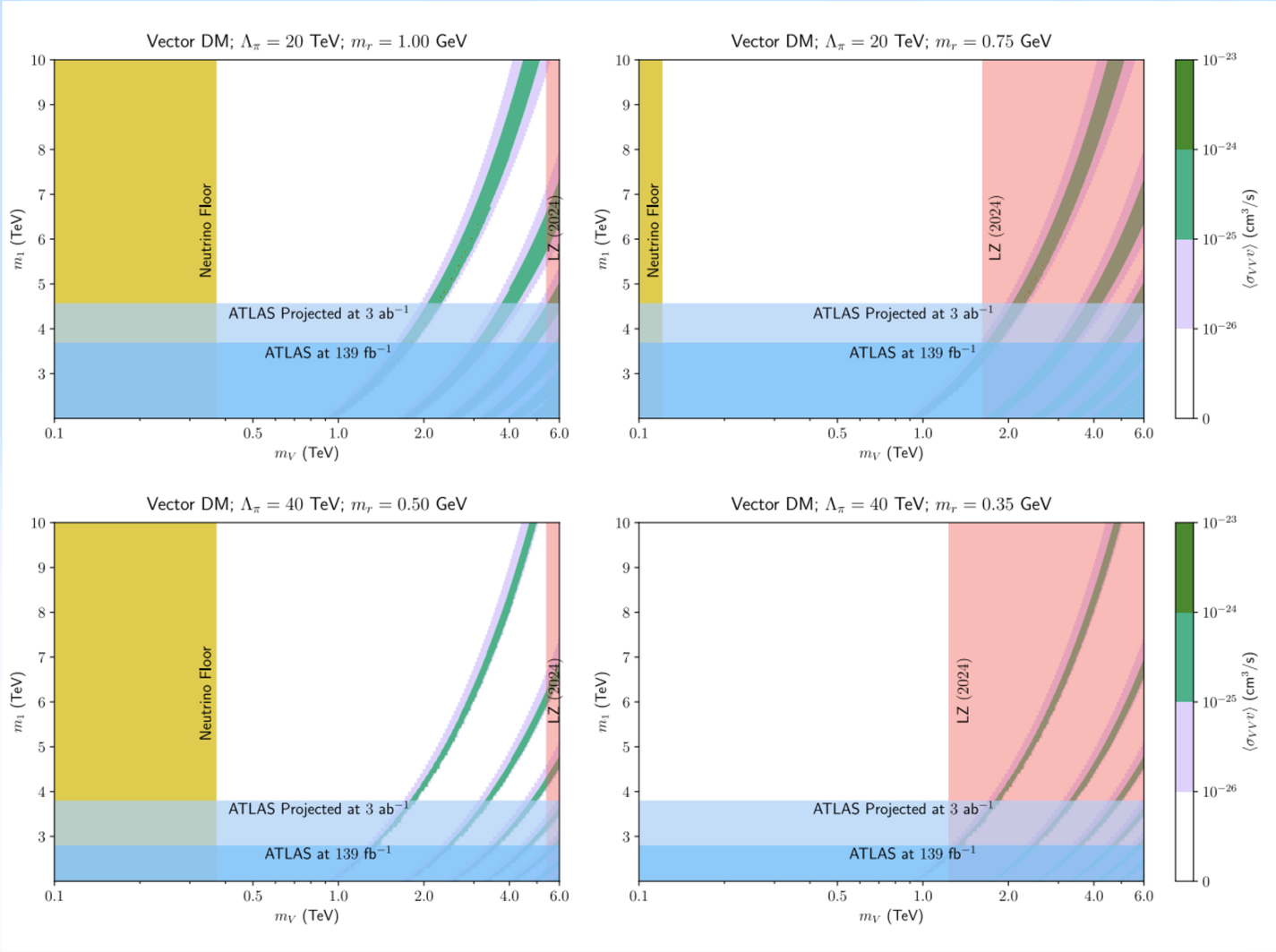
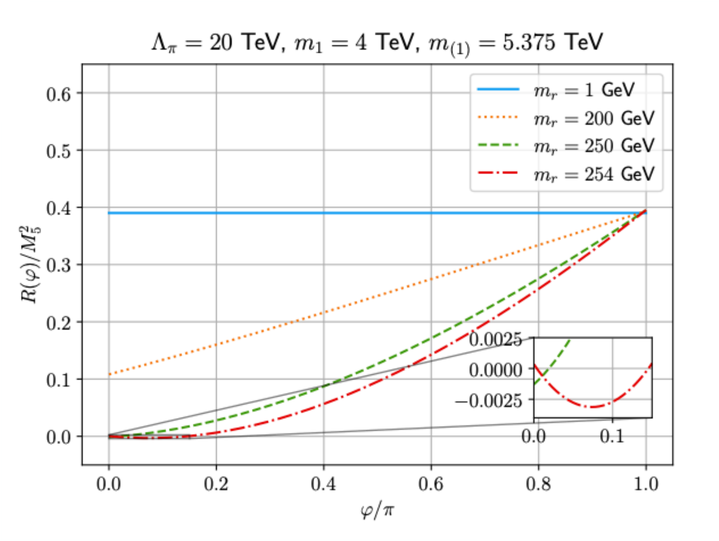
DM Phenomenology - Light Radion

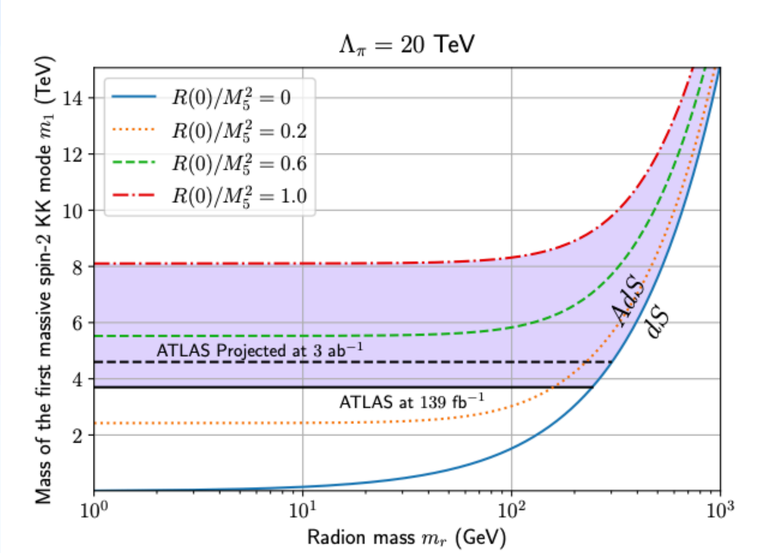
- Localise DM on TeV brane
- Vector DM - viable gap in parameter space
- Lilac and green regions
  - Equals or exceeds requirement for correct DM relic density
- Low backreaction from GW
  - $m_r \sim \mathcal{O}(1\ \text{GeV})$

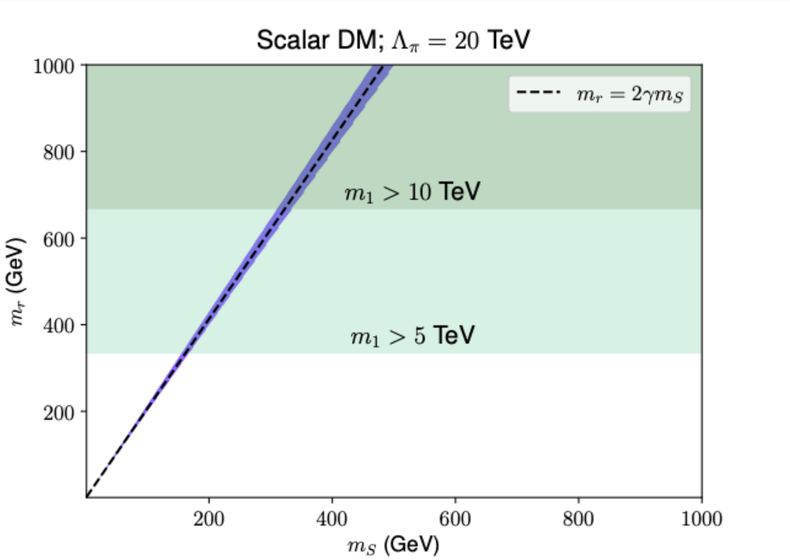


DM Phenomenology - Heavy Radion

- Localise DM on TeV brane, but high backreaction from GW $m_r \sim \mathcal{O}\,(100\ \text{GeV})$
- Change for AdS to dS - new constraint'
- Scalar and vector DM viable on resonance in a narrow band ($\Lambda = 20$ TeV)
- Annihilation to radion (decays to b-quarks) rules out $m_{\text{DM}} \approx 5 - 80$ GeV







Next Steps

Multibrane Model

- Introduce another hierarchy
- Changes to mass spectrum
- Investigate self scattering behaviour
- Preliminary findings:
  - There is no $\mathcal{O}\left(s^3\right)$ contribution to the amplitude
  - Leading order behaviour is of $\mathcal{O}\left(s^2\right)$