Cosmological Signals from Finite-Lifetime Domain Wall Networks

Fengwei Yang
University of Notre Dame
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In collaboration with Kylar Greene, Aurora Ireland, Gordan Krnjaic, Yuhsin Tsai (work in progress).

Introduction to Domain Wall

1. DW is a 2+1 D topological defect from a theory with a spontaneous discrete symmetry breaking.

E.g., Z_2 DW, a real scalar field ϕ with a double-well potential $V(\phi)$, with $\langle\phi\rangle = \pm$ v.

2. Wall tension: $\sigma = \int d\mathbf{x}_{\perp}\mathcal{H}(x) = \left[\frac{\text{Energy}}{\text{Area}^2}\right]$.

3. Important quantities for DW network: correlation length, energy density, collapse time.

ℓ_c ρ_{DW} η_c

[Kibble Mechanism]

T.W.B. Kibble, J. Phys. A. 1976

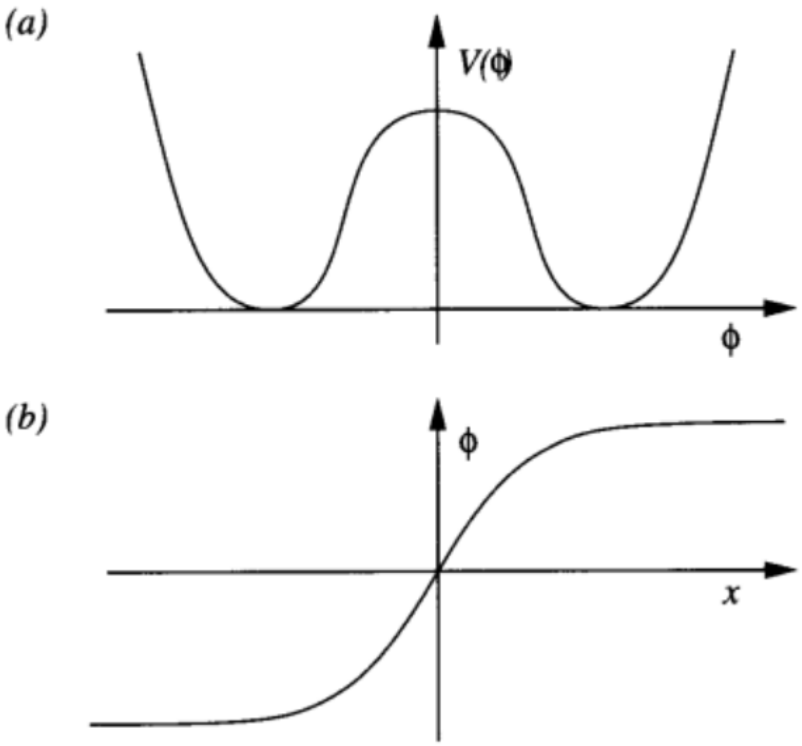


Figure credit to: *Cosmic strings and other topological defects,* 1994 Vilenkin & Shellard

Properties of DW network in scaling regime

1. Correlation length linearly grows with time

$$\ell_c = \xi\, \eta$$

ξ: correlation length parameter

e.g., Z_2 DW, $\xi = 0.63$, and RMS velocity $v_{\mathrm{r.m.s.}} = 0.5$ (RD).

2. DW (physical) energy density

$$\rho_{\mathrm{DW}} = \frac{\sigma}{a\ell_c} = \frac{\sigma}{\xi a \eta} \propto a^{-2}$$

consider $\rho_{\mathrm{DW}} \ll \rho_r$

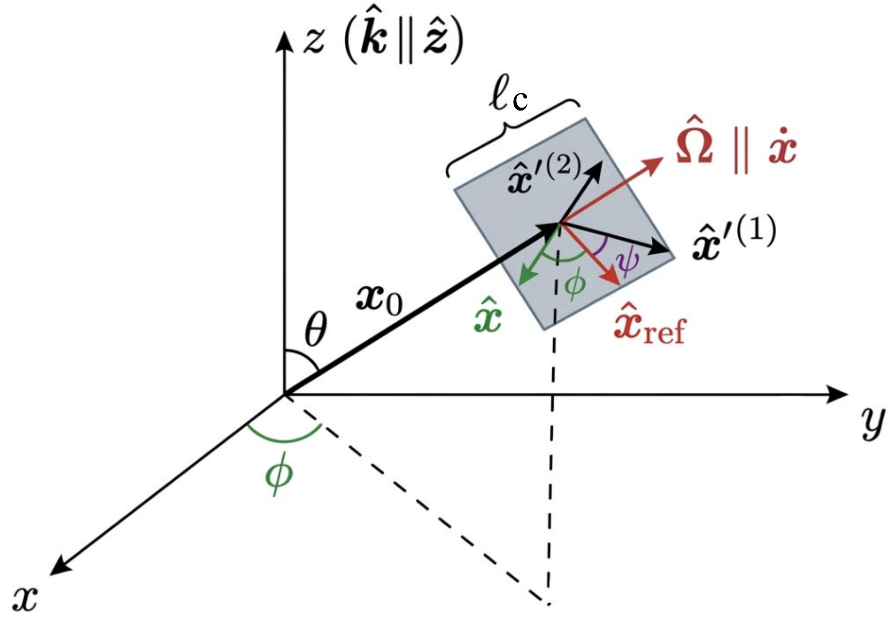
3. DW energy density fraction in terms of total energy density in RD

$$f_{\mathrm{DW}} \equiv \frac{\rho_{\mathrm{DW}}}{\rho_r} = f_{\mathrm{DW,ini}}\, a^2$$

f_{DW} continues growing since formation

Unconnected Segment Model

- analytical two-point function of DW stress-energy tensor $k \leq \ell_c^{-1}$



DW worldsheet trajectory

$$T^{\mu\nu} = \frac{\sigma}{\sqrt{-g}} \int d^3\zeta \delta^{(4)} \left[x^\mu - x^\mu_{\rm DW}(\zeta^c)\right] \left(\sqrt{-h} h^{cd} x^\mu_{,c} x^\nu_{,d}\right)$$

F.T.

$$\Theta_{\mu\nu}(\eta, \mathbf{k}) = \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} T_{\mu\nu}(\eta, \mathbf{x})$$

Unequal-time correlation function (UETC):

$$\langle \Theta^*_{\mu\nu} \Theta_{\mu\nu} \rangle = \int \frac{d^3\mathbf{x}_0}{V_c} \int \frac{d^2\hat{\mathbf{\Omega}}}{4\pi} \int_0^{2\pi} \frac{d\psi}{2\pi} \Theta_{\mu\nu}(\eta_1, \mathbf{k}) \Theta^*_{\mu\nu}(\eta_2, \mathbf{k}')$$

ensemble average

metric defined on the DW worldsheet

$$h_{cd} = a^2 \begin{pmatrix} -(1-\dot{\mathbf{x}}^2) & 0 & 0 \\ 0 & \mathbf{x}'^{(1)^2} & 0 \\ 0 & 0 & \mathbf{x}'^{(2)^2} \end{pmatrix}$$

Transverse gauge, featureless wall segments

Why Metastable DW?

- Cosmology:
    - To solve DW problem: finite lifetime DW consistent with existing CMB constraints.
    - Open windows for late-time observations that can probe high k modes; perturbations sourced by stable DW will dominate at the latest time ~ low k, so CMB places the most stringent constraint.
- Particle physics:
    - The underlying symmetry is only approximate.
    - The phase transition with larger symmetry breaking scale (UV physics) can be probed.

Curvature and metric perturbations: $k \leq \ell_c^{-1}$

$$\tilde{\rho}_{\rm DW}(\eta, k) = a^{-2}\Theta_{00}(\eta, k) \Rightarrow \zeta = -H\frac{\delta\rho_{\rm DW}}{\dot{\bar{\rho}}_{\rm tot}}$$ spatially flat gauge

$$h_\lambda'' + 2\frac{\mathcal{H}}{k}h_\lambda' + h_\lambda = \frac{16\pi G}{k^2}\varepsilon_\lambda^{ij}\Theta_{ij}(y, \mathbf{k})$$

Curvature and metric perturbation power spectrum in RD:

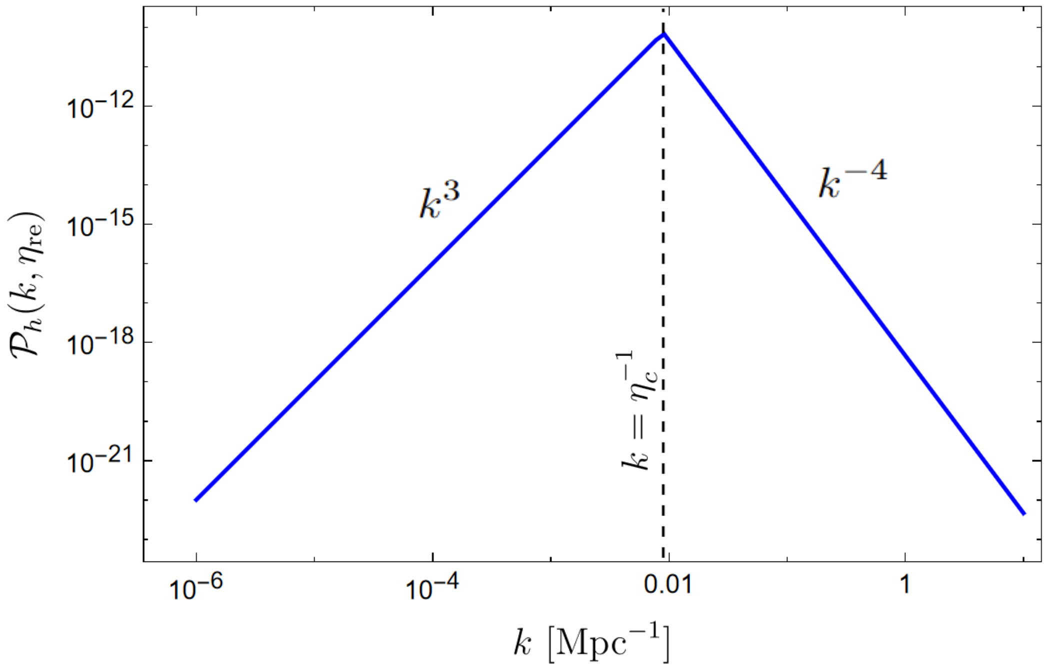
$$\mathcal{P}_\zeta^{\rm DW}(\eta, k) = \frac{1}{32\pi^2}\gamma^2 f_{\rm DW,ini}^2 (k\xi\eta_{\rm PT})^3 \left(\frac{g_*}{g_{*,\rm PT}}\right)^{-\frac{1}{3}} \left(\frac{\eta}{\eta_{\rm PT}}\right)^7$$

correlation length at PT

$$\frac{\mathcal{P}_h^{\rm DW}(\eta, k)}{\mathcal{P}_\zeta^{\rm DW}(\eta, k)} = 0.675$$

Focusing on reenter time: $$\eta_{\rm re} = \begin{cases} k^{-1}, & \text{reenter before } \eta_c\,, \\ \eta_c\,, & \text{reenter after } \eta_c\,. \end{cases}$$

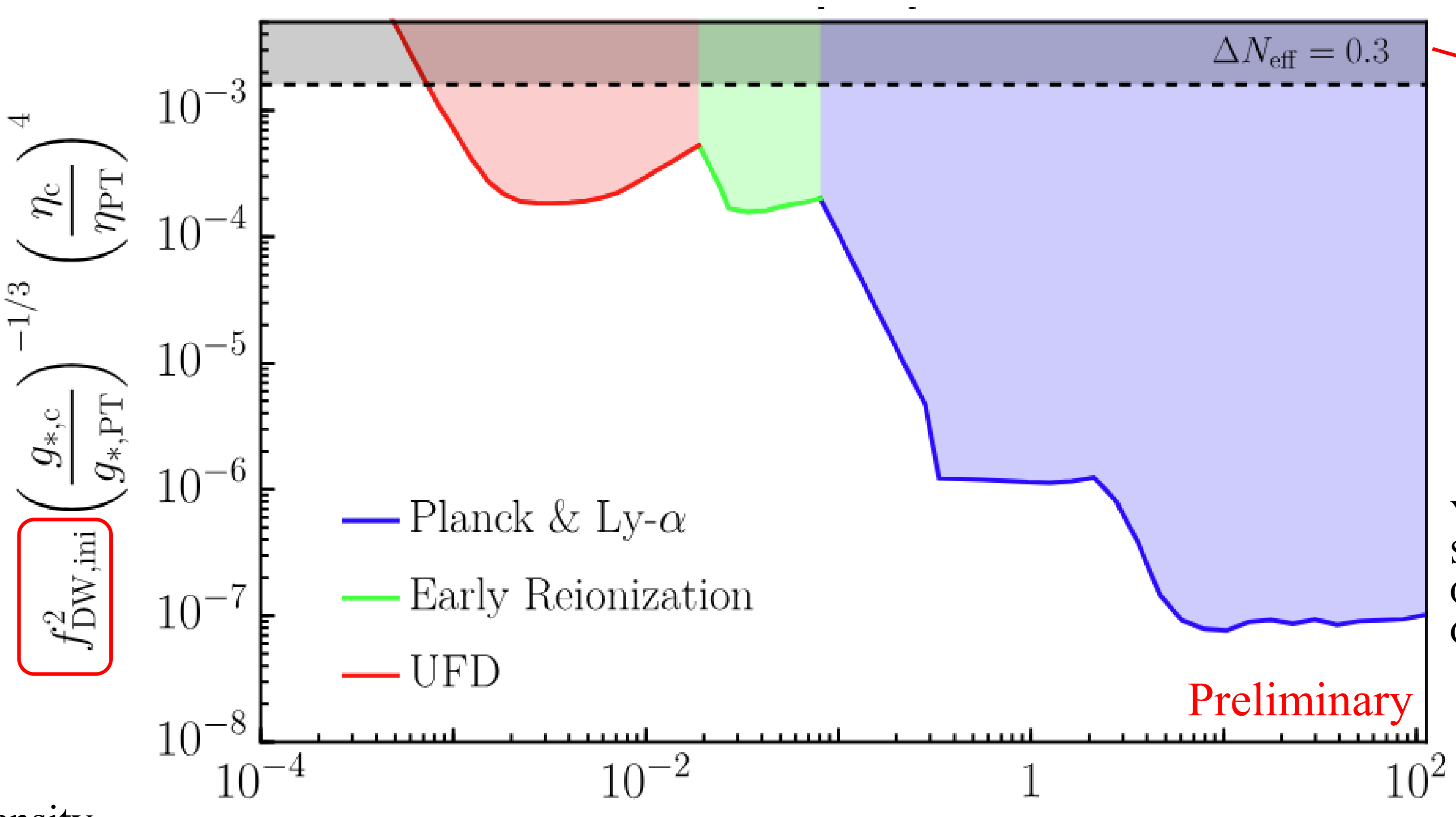
Peaks at $k = \eta_c^{-1}$



Curvature power spectrum constraints

$\propto$ The peak amplitude of

$\mathcal{P}_\zeta(k)$



DW network collapse time

After DW collapses, all energy convert to dark radiation or GW.

$f_{\rm DW,ini}$:
DW energy density fraction at PT

Y. Akrami et al. A&A 2020 (Planck);
S. Bird et al. MNRAS 2011 (Lyα);
Qin et al. 2506.13858 (Early Rein.);
Graham & Ramani, PRD 2024 (UFD)

Case study 1: Z_2 DW

A real scalar field ϕ with a double-well potential:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{4}\lambda\left(\phi^2 - \frac{v_a^2}{2}\right)^2$$

Wall tension:

$$\sigma = \int_{-\infty}^{\infty} dz\, T_0^0(z) = \int dz (\partial_z \phi)^2 = \frac{\sqrt{\lambda}}{3} v_a^3$$

DW energy density fraction at PT:

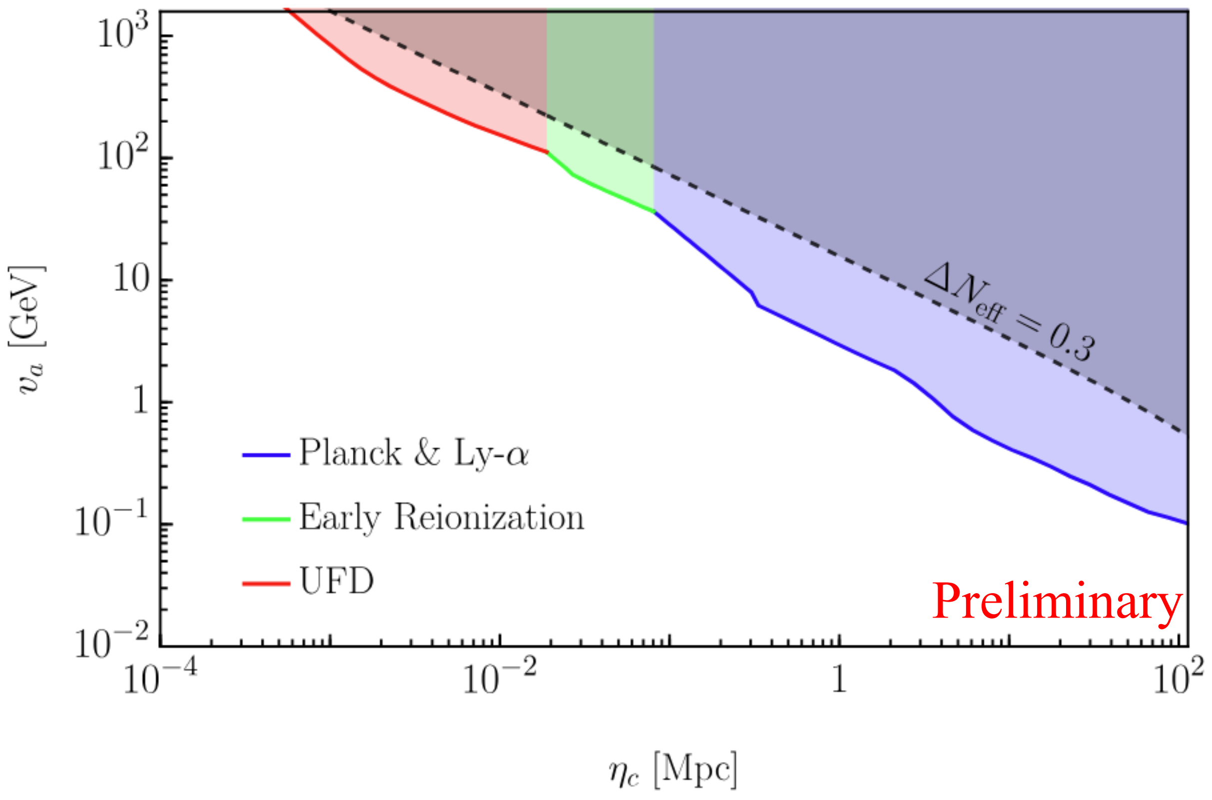
$$f_{\rm DW,ini} \approx 7.3 \times 10^{-21} \left(\frac{0.63}{\xi}\right)\left(\frac{10^2}{g_*}\right)^{\frac{1}{2}}\left(\frac{v_a}{\rm GeV}\right)$$

(assuming $\lambda = 1$)

correlation length parameter

symmetry breaking scale

Shaded regions are excluded.



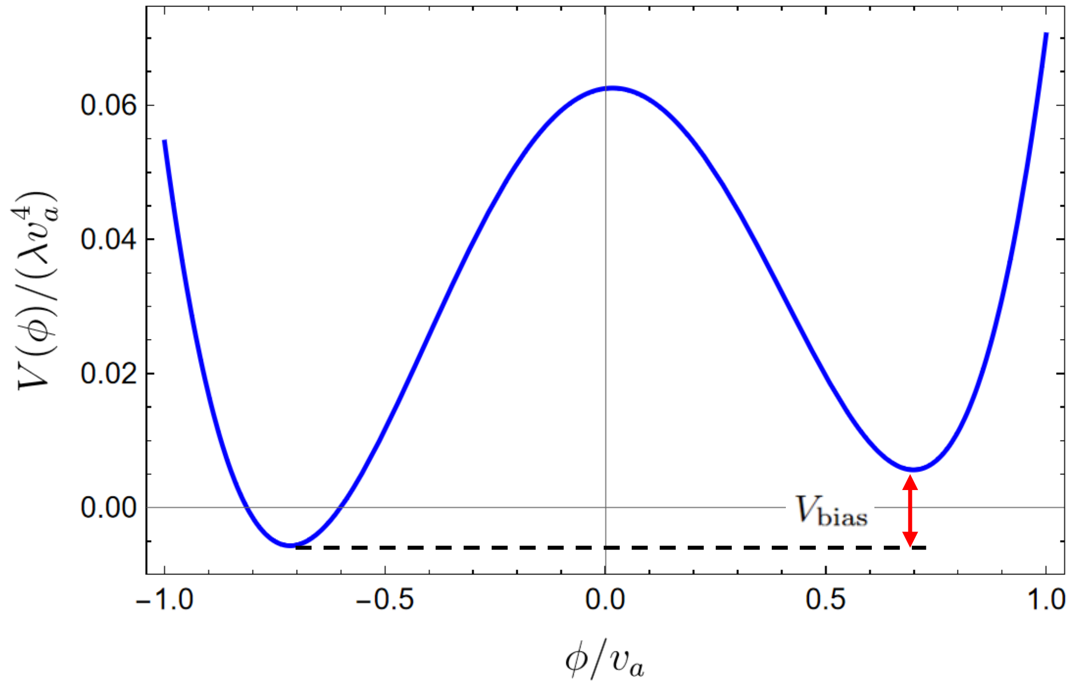
P_ζ constraints on (v_a, ϵ)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{4}\lambda\left(\phi^2 - \frac{v_a^2}{2}\right)^2 - V_{\not{\text{sym}}}$$

Explicit symmetry breaking term

$$V_{\not{\text{sym}}} = \epsilon^3 v_a^3 \phi$$

$$V_{\text{bias}} = V_{\not{\text{sym}}}(\phi = v_a/\sqrt{2}) - V_{\not{\text{sym}}}(\phi = -v_a/\sqrt{2}) = \sqrt{2}\epsilon^3 v_a^4$$



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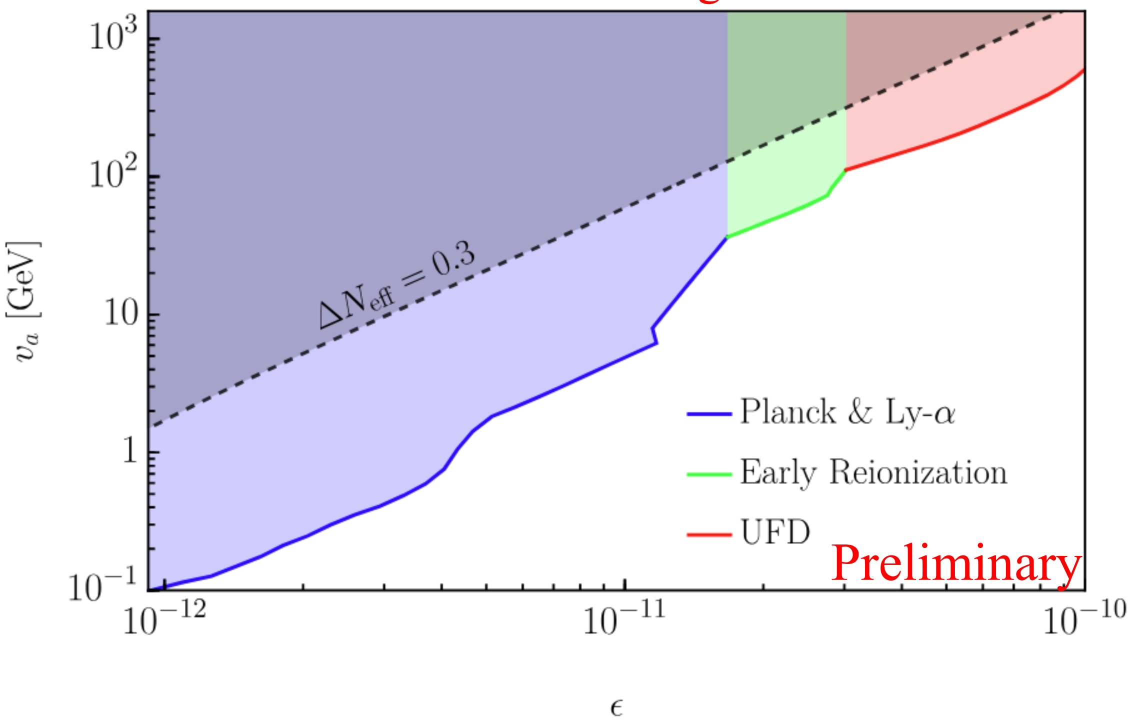


Case study 2: axion DW bounded by strings

Axion field $a = v_a\theta$ with a cosine potential:

$$\mathcal{L}_\theta = \frac{1}{2}v_a^2(\partial_\mu\theta)^2 - \frac{m_a^2 v_a^2}{N^2}(1-\cos(N\theta))$$

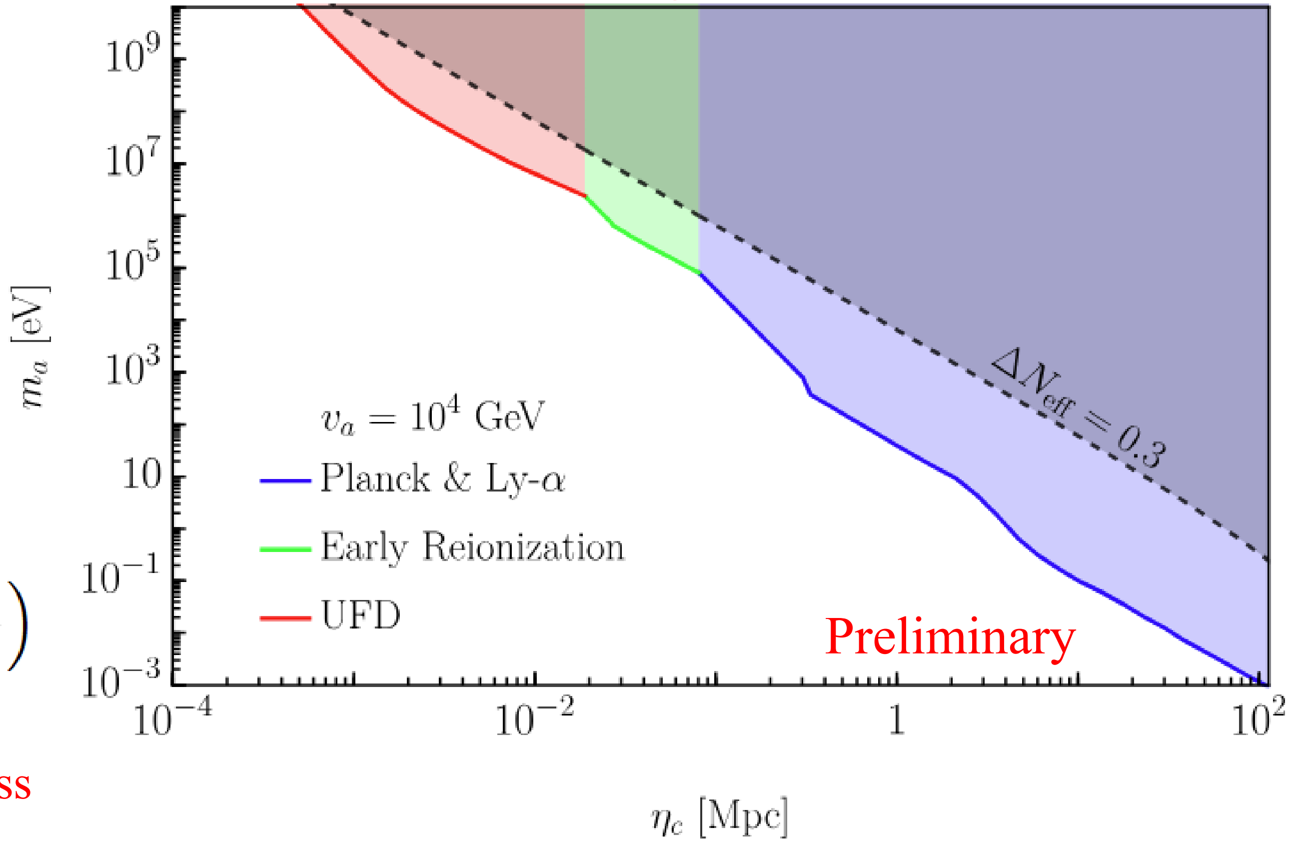
$$\sigma = \frac{8 m_a v_a^2}{N^2}$$

N: DW number, consider $N = 2$ here with a biased term.

$$f_{\rm DW,ini} = 1.7\times 10^{-28} N^{-2}\left(\frac{0.63}{\xi}\right)\left(\frac{10^2}{g_*}\right)^{\frac{1}{2}}\left(\frac{m_a}{\rm eV}\right)$$

axion mass

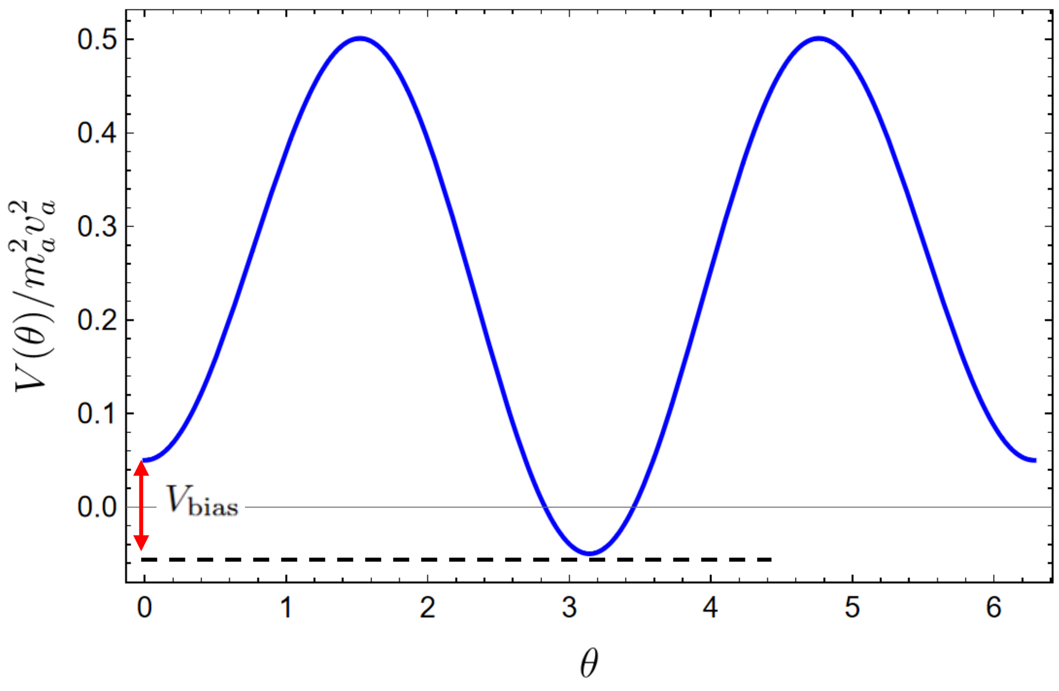
Shaded regions are excluded.



P_ζ constraints: axion DW

$$\mathcal{L}_\theta = \frac{1}{2} v_a^2 (\partial_\mu \theta)^2 - \frac{m_a^2 v_a^2}{N^2}(1 - \cos(N\theta)) \qquad V_{\text{sym}} = \epsilon_a m_a^2 v_a^2 \cos\left(\tfrac{1}{2} N\theta\right)/N^2$$

$$V_{\text{bias}} = V_{\text{sym}}(\theta = 0) - V_{\text{sym}}(\theta = \pi) = \epsilon_a m_a^2 v_a^2 / 2$$



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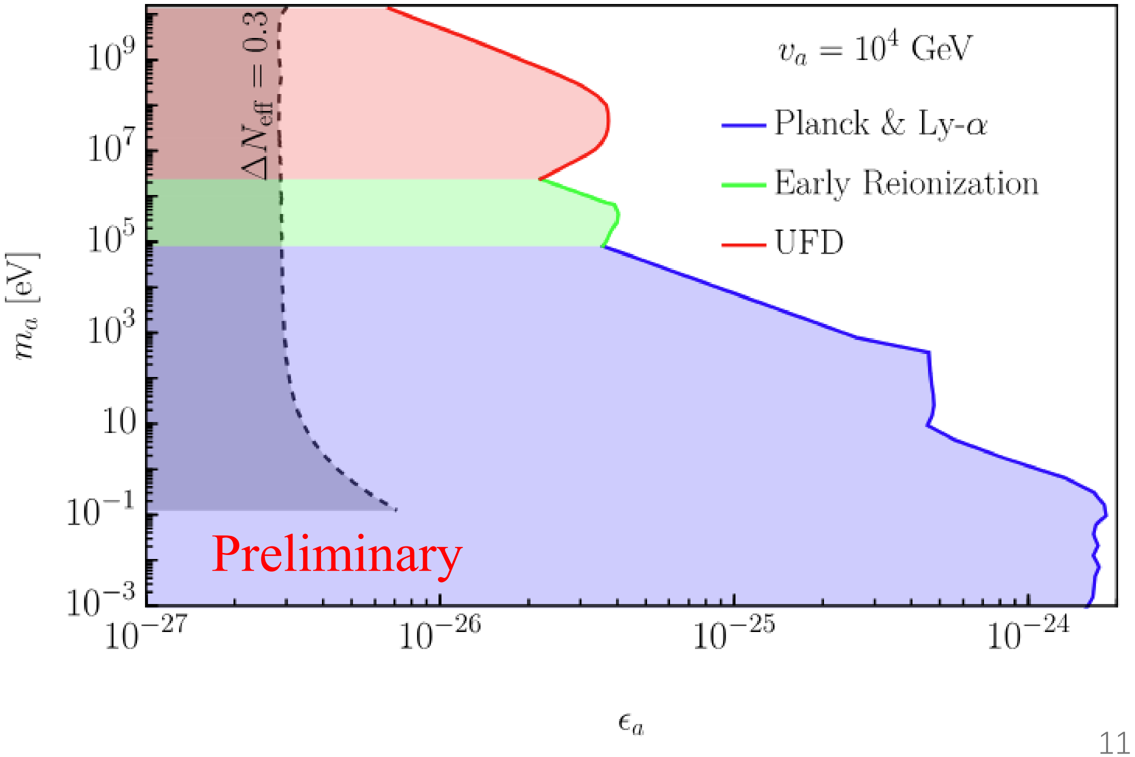
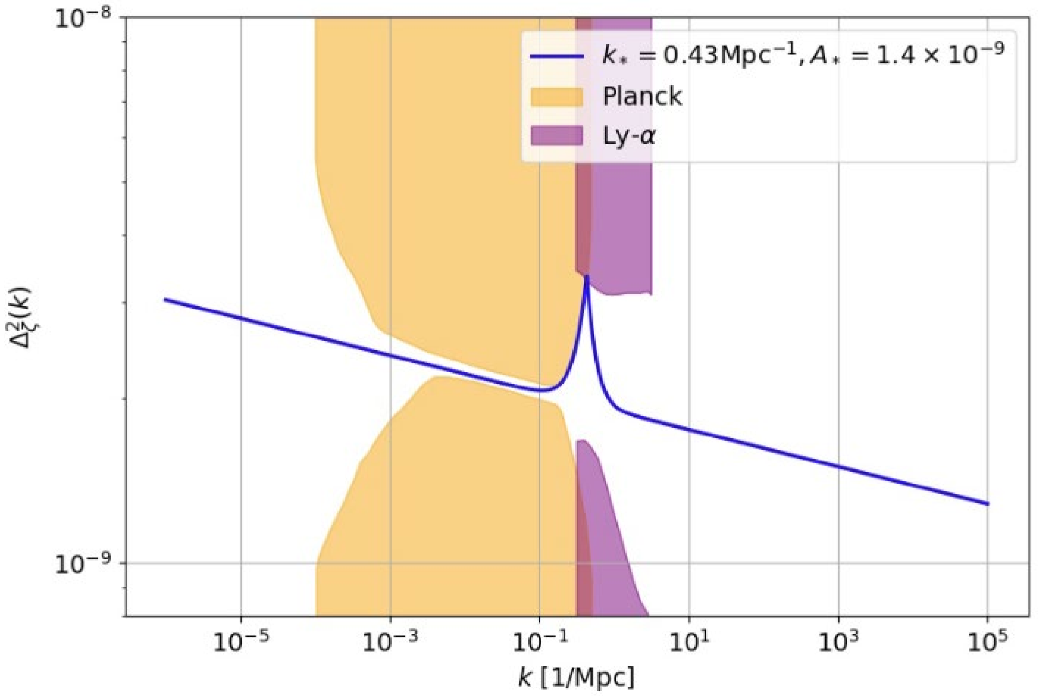
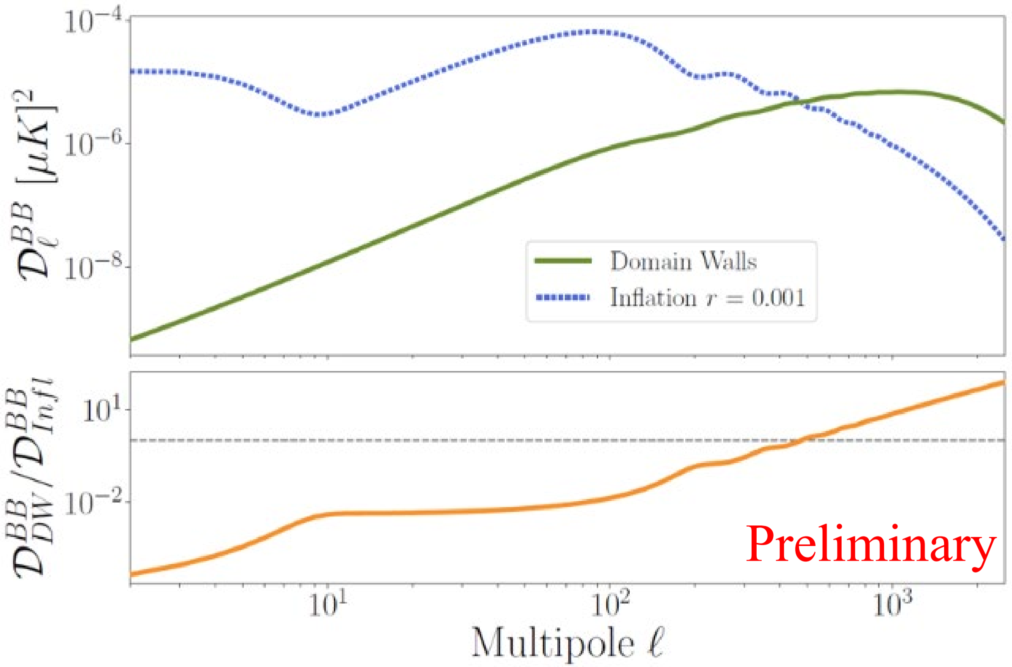


Projected CMB B-mode signals





Given curvature perturbation constraint
→ maximum amplitude in metric perturbation
→ CMB B-mode spectrum

$$\frac{\mathcal{P}_h^{\mathrm{DW}}(\eta, k)}{\mathcal{P}_\zeta^{\mathrm{DW}}(\eta, k)} = 0.675$$

Conclusion

1. We study the cosmological perturbations from finite-lifetime DWs.

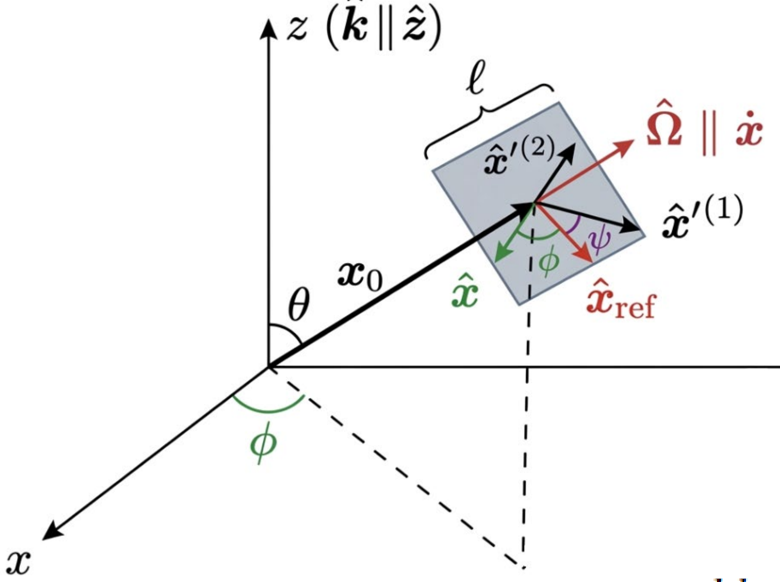
2. Derive the model-independent constraints on DW energy density fraction at collapse time. Case studies on Z_2 DW and axion DW.

3. Projected B-mode signal can be sizable while consistent with the current curvature perturbation constraints.

Back-up slides

Unconnected Segment Model of DW

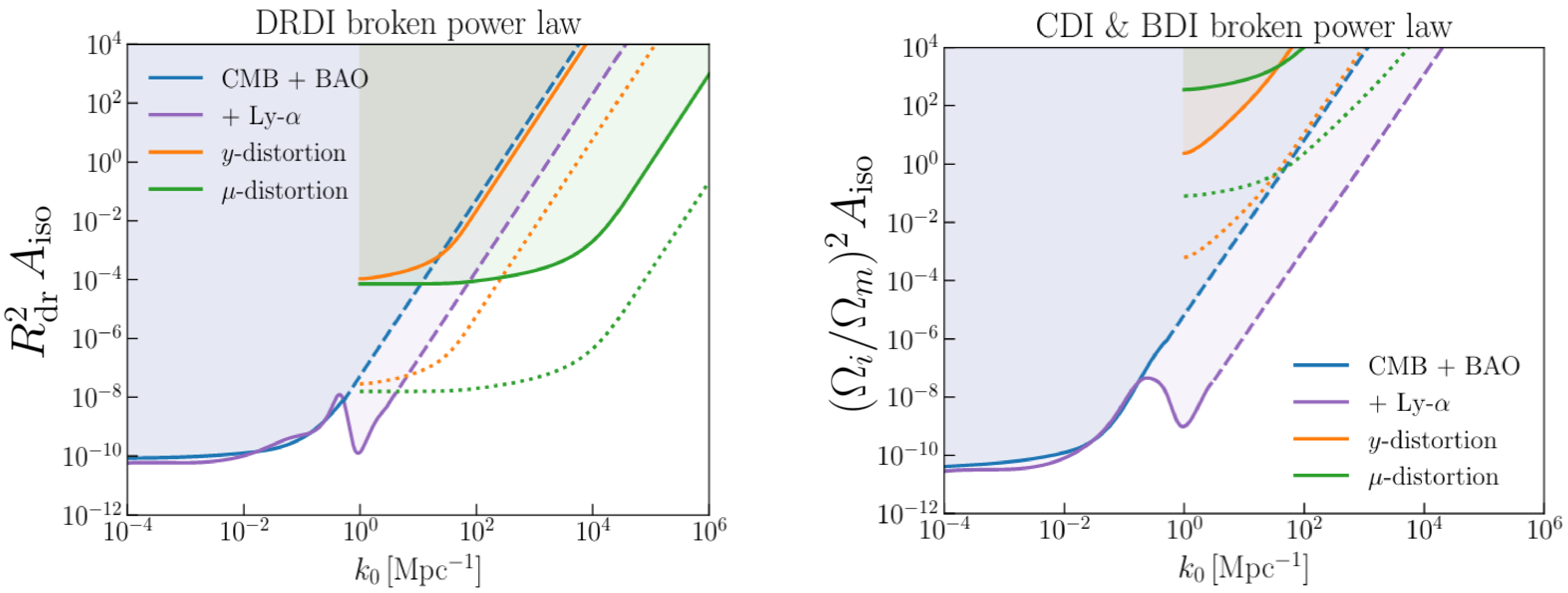
- USM → analytical two-point function of DW stress-energy tensor in low k



$$\Theta_{00}(\eta, \mathbf{k}) = -a\sigma\gamma e^{-i\mathbf{k}\cdot(\mathbf{x}_0 + v\eta\dot{\hat{\mathbf{x}}})} \frac{2\sin(\frac{kl}{2}\hat{x}_z'^{(1)})}{k\hat{x}_z'^{(1)}} \frac{2\sin(\frac{kl}{2}\hat{x}_z'^{(2)})}{k\hat{x}_z'^{(2)}},$$

$$\Theta_{ij}(\eta, \mathbf{k}) = \Theta_{00}(\eta, \mathbf{k})[v^2\dot{\hat{x}}_i\dot{\hat{x}}_j - \gamma^{-2}(\hat{x}_i'^{(1)}\hat{x}_j'^{(1)} + \hat{x}_i'^{(2)}\hat{x}_j'^{(2)})],$$

Isocurvature perturbation constraints



The upper bounds on dark radiation and CDM isocurvature perturbations. Buckley et al. 2502.20434

DW sourced cosmological perturbations

RD background:

$$\mathcal{P}_{\zeta,h}^{\mathrm{DW}}(k) \approx c \cdot 10^{-13} \left(\frac{\xi}{0.63}\right)^3 \left(\frac{f_{\mathrm{DW,ini}}}{10^{-5}}\right)^2 \gamma^2 \left(\frac{g_*}{g_{*,\mathrm{PT}}}\right)^{-\frac{1}{3}}$$
$$\times \begin{cases} (k\eta_{\mathrm{PT}})^{-4}, & \text{if } \eta_c^{-1} \le k \le (\xi\eta_{\mathrm{PT}})^{-1}, \\ (k\eta_{\mathrm{PT}})^3 \left(\frac{\eta_c}{\eta_{\mathrm{PT}}}\right)^7, & \text{if } k < \eta_c^{-1}. \end{cases}$$

Growth in time: $[k\,\ell_c(\eta)]^3 \left(\frac{\eta}{\eta_{\mathrm{PT}}}\right)^4$