Structure Formation with Dark Magnetohydrodynamics

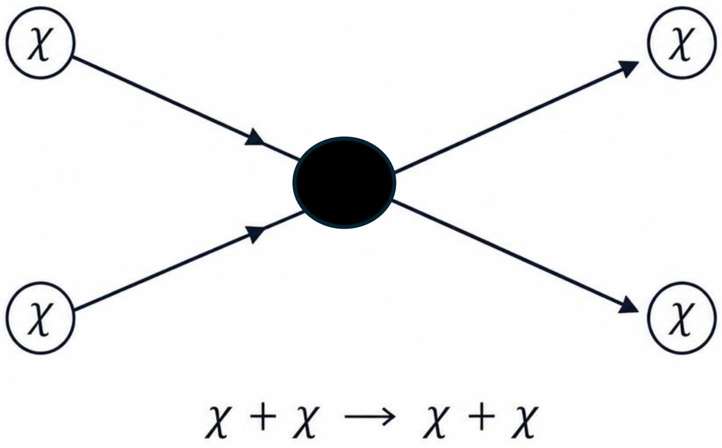
Pierce Giffin

Phenomenology 2026 Symposium

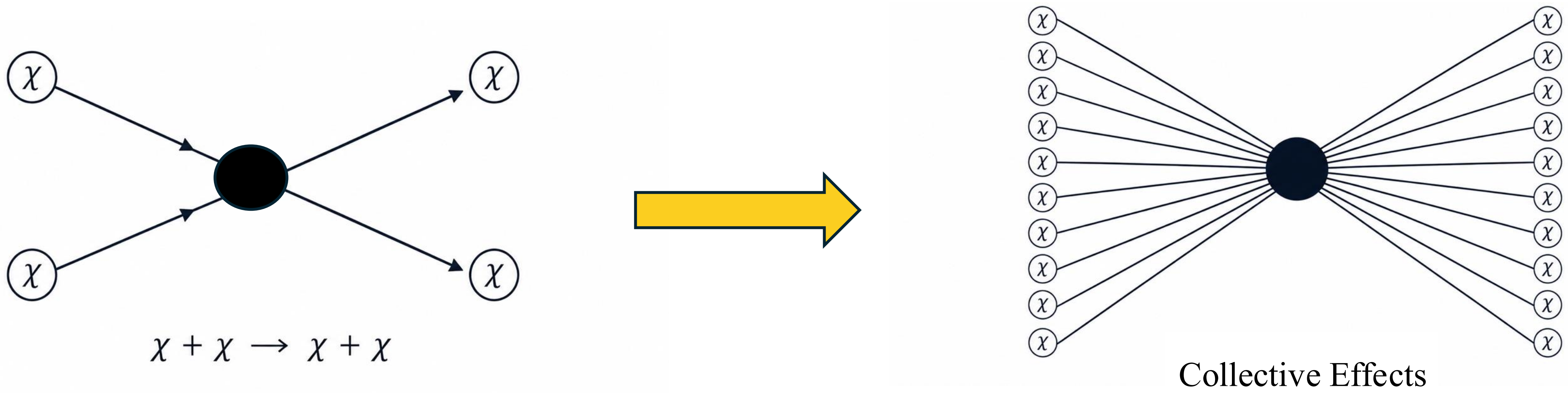
arXiv: 2511.15810

PG, Andrew Liu, Jeremias Boucsein, Akaxia Cruz,
Anirudh Prabhu, Stefano Profumo, M. Grant Roberts

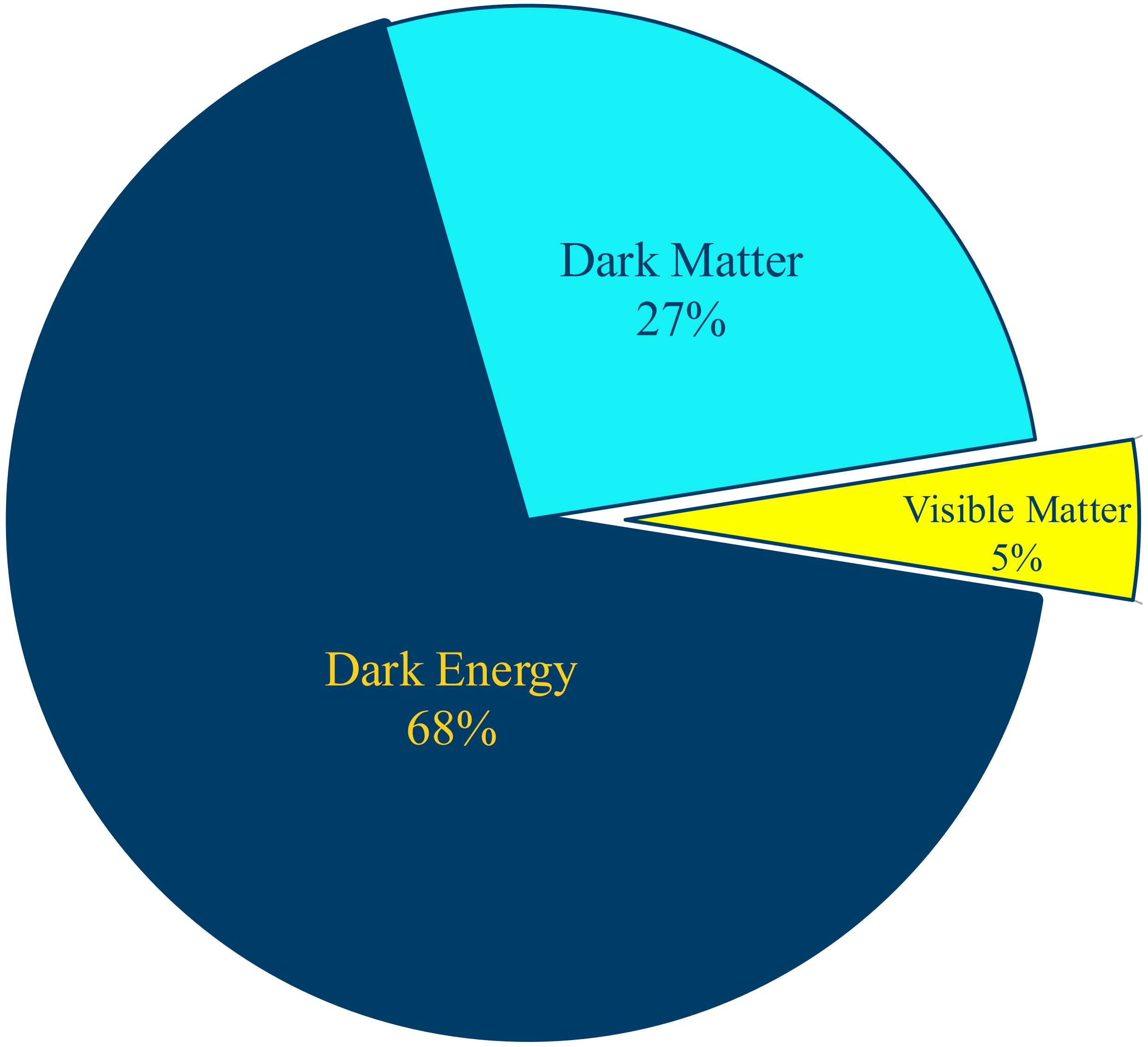
How Does Dark Matter Self-Interact?



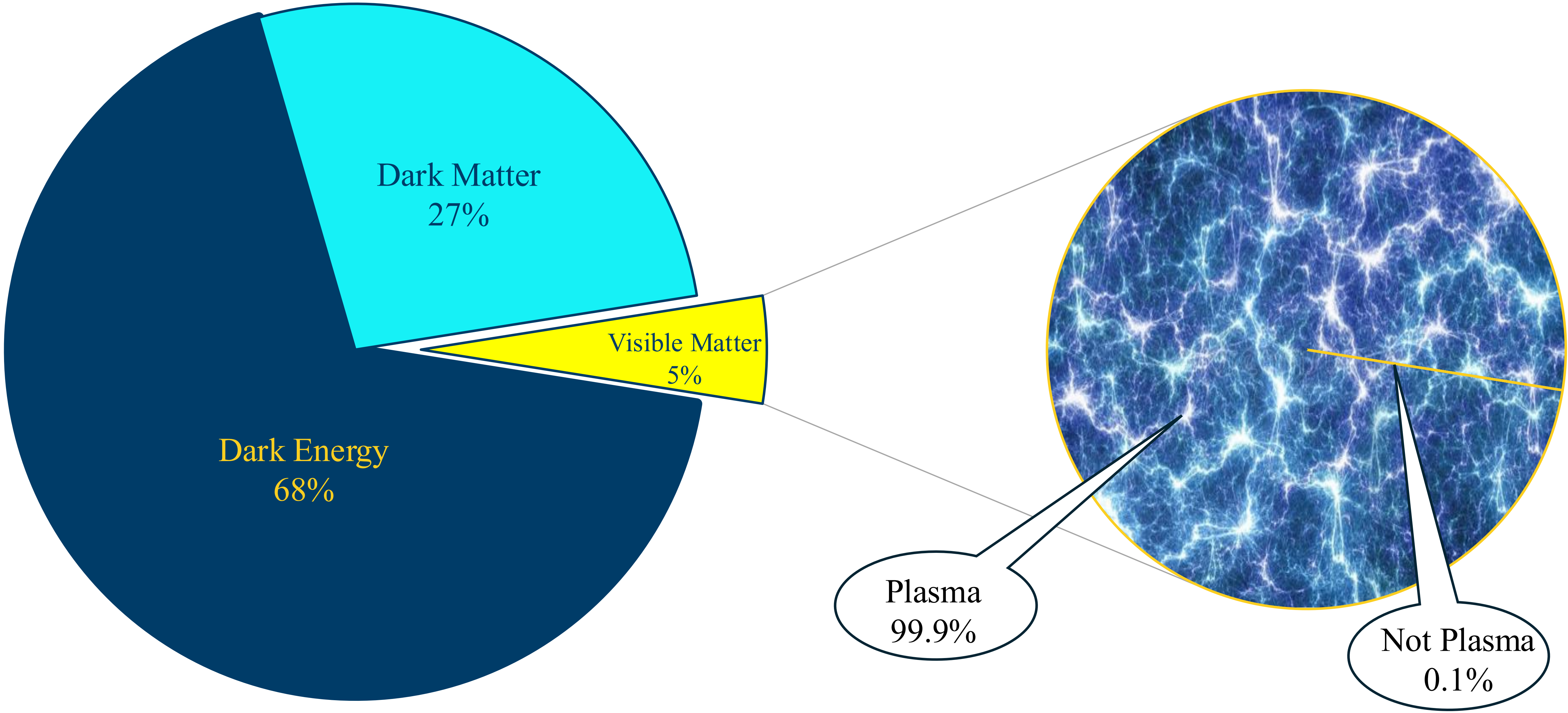
How Does Dark Matter Self-Interact?



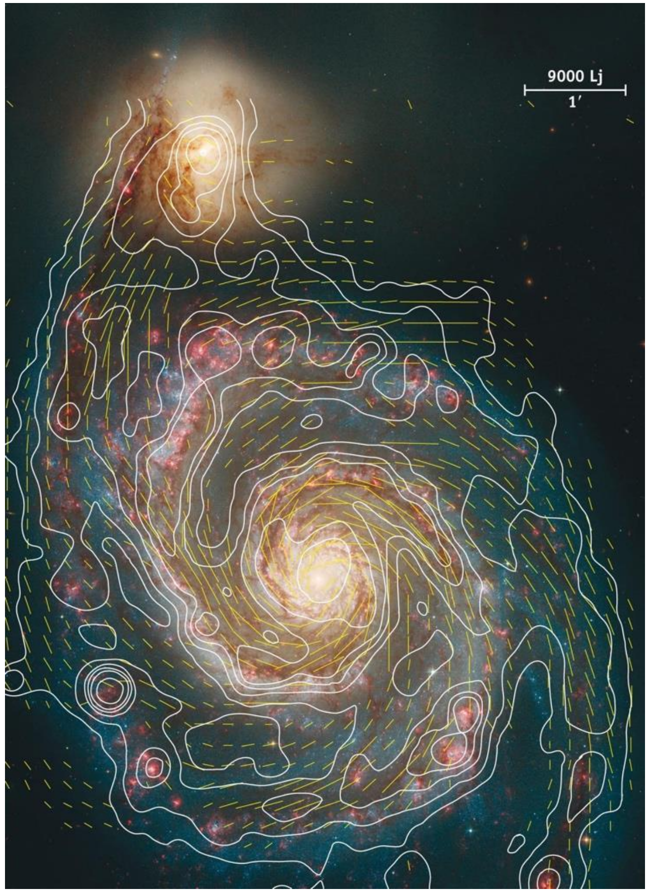
Why Collective Effects?



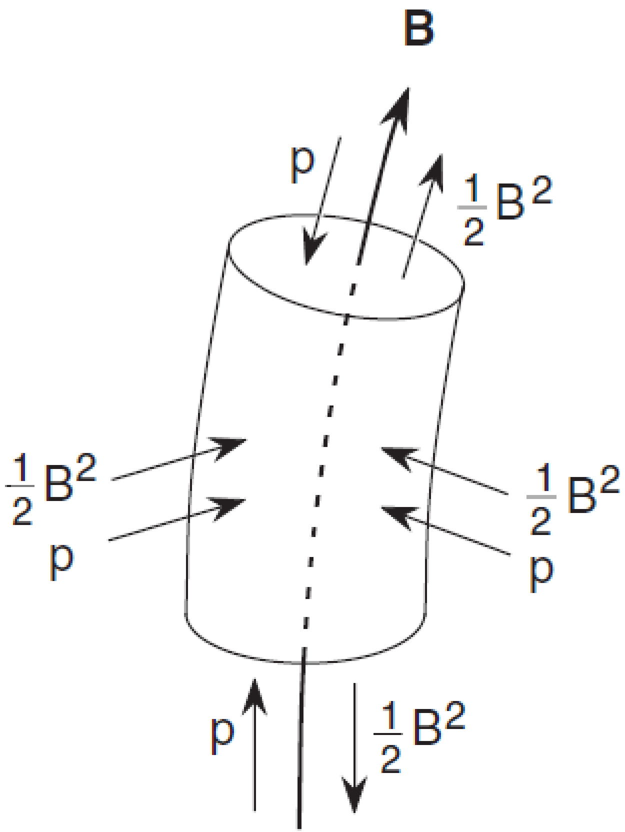
Why Collective Effects?



Probing Collective Effects with Gravity



Max Planck Institute



Principles of Magnetohydrodynamics by Hans Goedblood & Stefaan Poedts

- Can cosmic magnetic fields exist in the dark sector?
- How would the impact early structure formation?
- Can the dark matter Jeans length be altered by a large-scale dark magnetic field?

Plasma Dynamics with Gravity

Dark matter model: $\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \bar{\chi}(\gamma^\mu(i\partial_\mu - q_\chi A'_\mu) - m_\chi)\chi$

Maxwell's equations for dark electromagnetism:

$$\nabla \cdot \vec{E} = \rho_c \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\partial_t \vec{B} \quad \nabla \times \vec{B} = \vec{J} + \partial_t \vec{E}$$

Net force on each particle: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) - \nabla V$

Poisson's equation for gravity: $\nabla^2 V = 4\pi G \rho_m$

Taken to be negligible for collisionless dark matter as $\frac{q_\chi}{m_\chi} < 2 \times 10^{-14}\ \text{GeV}^{-1}$ (**PG**, W. DeRocco 2411.11958)

Evolve phase space distribution with Boltzmann's equation: $\partial_t f_s + \nabla_x f_s \cdot \vec{v} + \vec{a} \cdot \nabla_v f_s = C[f_s]$

$$\left(\partial_t - \frac{1}{m_s}\nabla V \cdot \nabla_x + \frac{q_s}{m_s}\left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \nabla_v + \vec{v} \cdot \nabla_x\right) f_s(\vec{x}, \vec{v}, t) = 0$$

Initial Conditions: $f_0(\vec{x}, \vec{v}) \propto e^{-v^2/2v_{th}^2} \qquad \vec{B}(\vec{x}) = B_0 \hat{z}$

$$\det(D) = 0$$

with

$$D_{xx} = k_\parallel^2 + \omega^2$$

$$D_{zz} = k_\perp^2 + \omega^2$$

$$D_{xz} = D_{zx} = k_\perp k_\parallel - k_\perp^2 - k_\parallel^2$$

$$D_{xy} = D_{yx} = -i\omega \sum_s \frac{\sqrt{\pi}}{2} \frac{v_{T,s} x_s^{-3}}{\lambda_{D,s}^2 k_\parallel} \sum_{n=-\infty}^{\infty} Z(\xi_{s,n}^-) n F_5(n)$$

$$D_{yz} = -i\omega \sum_s \frac{1}{\sqrt{2\pi}} \frac{v_{T,s} x_s^{-2} z_s^{-1}}{\lambda_{D,s}^2 k_\parallel} \sum_{n=-\infty}^{\infty} \xi_{s,n}^- Z(\xi_{s,n}^-) F_5(n)$$

$$D_{zy} = -i\omega \left(\left(\frac{\sum_p \sqrt{2\pi} \frac{q_p \rho_{0,p}}{m_s v_{T,p}}}{k^2 - \sum_d k_{J,d}^2} \right) \sum_s \frac{q_s \rho_{0,s}}{m_s v_{T,s}^2} \frac{x_s^{-2}}{2\sqrt{\pi} k_\parallel} \sum_{n=-\infty}^{\infty} Z(\xi_{s,n}^-) F_5(n) - \frac{1}{\lambda_{D,s}^2} \frac{x_s^{-2}}{\sqrt{2\pi} k_\parallel} \sum_{n=-\infty}^{\infty} (1 + \xi_{s,n}^- Z(\xi_{s,n}^-)) F_5(n) \right)$$

$$D_{yy} = -i\omega \sum_s \frac{\sqrt{\pi}}{2} \frac{v_{T,s} x_s^{-1}}{\lambda_{D,s}^2 k_\parallel} \sum_{n=-\infty}^{\infty} Z(\xi_{s,n}^-) F_7(n)$$

where

$$F_5(n) = \frac{(1 + 2|n|)\Gamma(|n| + \frac{3}{2})}{\Gamma(|n| + 2)}$$

$$F_7(n) = (32)4^{|n|}$$

Closure of MHD Equations

$$\nabla \cdot \vec{E} = \rho_c \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{U}) = 0 \qquad \rho_m \frac{d\vec{U}}{dt} = \vec{J} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$$

$$\frac{d}{dt}\left(\frac{p_\perp B^2}{\rho_m^3}\right) = 0 \qquad \frac{d}{dt}\left(\frac{p_\parallel}{\rho_m B}\right) = 0$$

Does not depend on charge or mass of dark matter particle

Closure of MHD Equations

$$\nabla \cdot \vec{E} = \rho_c \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{U}) = 0 \qquad \rho_m \frac{d\vec{U}}{dt} = \vec{J} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$$

$$\frac{d}{dt}\left(\frac{p_\perp B^2}{\rho_m^3}\right) = 0 \qquad \frac{d}{dt}\left(\frac{p_{||}}{\rho_m B}\right) = 0$$

Does not depend on charge or mass of dark matter particle

Typical choices of closure

$$\vec{E} + \vec{U} \times \vec{B} = \eta \vec{J} + \frac{1}{2}\frac{m_\chi^2}{q_\chi^2 \rho}\left[\partial_t \vec{J} + \nabla \cdot (\vec{U}\vec{J} + \vec{J}\vec{U})\right] + \frac{m_\chi}{q_\chi \rho} \nabla \cdot \left(\overleftrightarrow{P}_+ - \overleftrightarrow{P}_-\right)$$

Closure of MHD Equations

$$\nabla \cdot \vec{E} = \rho_c \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{U}) = 0 \qquad \rho_m \frac{d\vec{U}}{dt} = \vec{J} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$$

$$\frac{d}{dt}\left(\frac{p_\perp B^2}{\rho_m^3}\right) = 0 \qquad \frac{d}{dt}\left(\frac{p_{\|}}{\rho_m B}\right) = 0$$

Does not depend on charge or mass of dark matter particle

Typical choices of closure

$$\vec{E} + \vec{U} \times \vec{B} = \eta \vec{J} + \frac{1}{2}\frac{m_\chi^2}{q_\chi^2 \rho}\left[\partial_t \vec{J} + \nabla \cdot (\vec{U}\vec{J} + \vec{J}\vec{U})\right] + \frac{m_\chi}{q_\chi \rho} \nabla \cdot \left(\overleftrightarrow{P}_+ - \overleftrightarrow{P}_-\right)$$

Ideal MHD

$$\vec{E} + \vec{U} \times \vec{B} = 0$$

- Assumes perfectly conducting fluid
- Insensitive to particle charge and mass

Resistive MHD

$$\vec{E} + \vec{U} \times \vec{B} = \eta \vec{J} \qquad \eta = \frac{m_\chi^2 \nu_\pm}{q_\chi^2 \rho}$$

- Assumes fluid has finite resistivity
- Individual particles experience high collisionality

Inertial MHD

$$\vec{E} + \vec{U} \times \vec{B} = \frac{1}{2}\frac{m_\chi^2}{q_\chi^2 \rho_m}\frac{\partial \vec{J}}{\partial t}$$

- Retain ideal MHD scenario for sufficiently large charges
- Decouple from magnetic fields for sufficiently low charges

Changes to the Effective Phase Speed

Evolution of dark matter density perturbations: $$\ddot{\delta}_{\boldsymbol{k}} + 2H\dot{\delta}_{\boldsymbol{k}} + \left[\frac{c_s^2(\mu)k^2}{a^2} - 4\pi G\bar{\rho}_m\right]\delta_{\boldsymbol{k}} = 0$$

Anisotropic phase speed: $$c_s^2(\mu) = c_{s,\parallel}^2\,\mu^2 + c_{s,\perp}^2(1-\mu^2), \qquad \mu \equiv \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{b}}$$

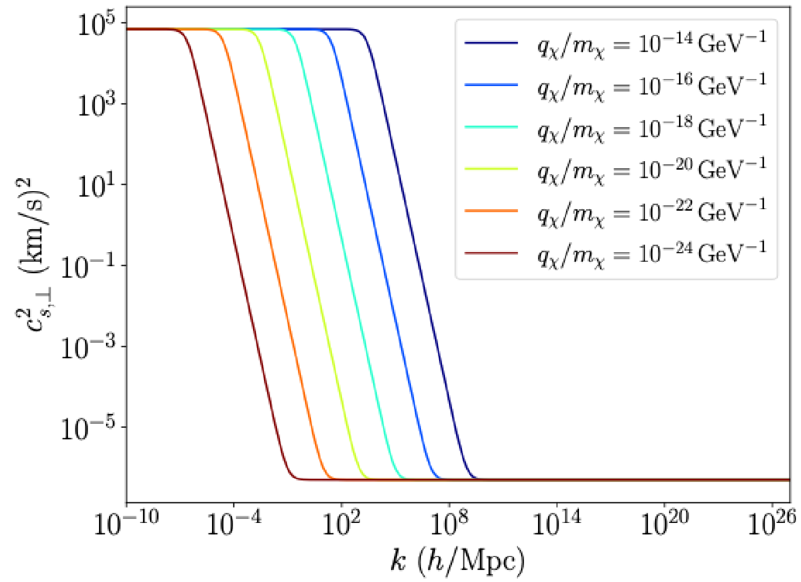
Altered classic Jeans length: $$\lambda_J^2(\mu, a) = \frac{\pi c_s^2(\mu)}{G\bar{\rho}_m(a)\,a^2}$$

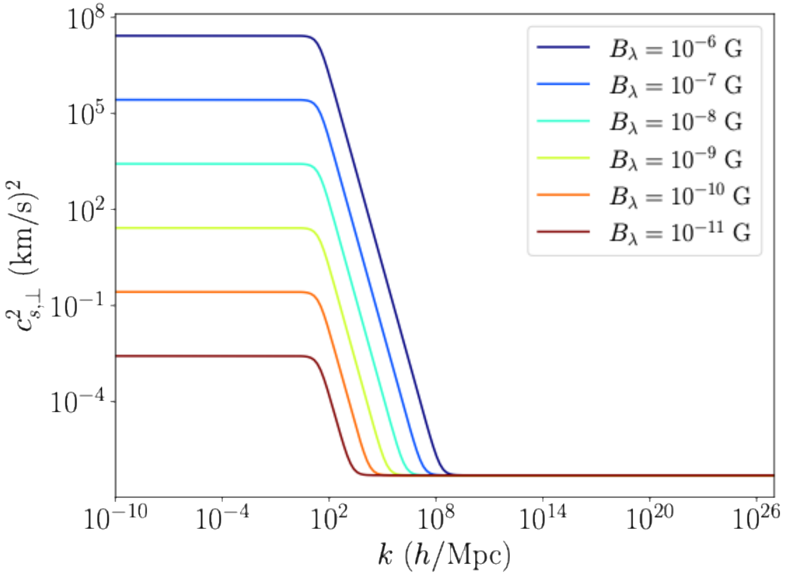
$$c_s^2(\mu,k,z) = 3v_{\rm th}^2\,\mu^2 + \left[\frac{1+\beta}{2}v_{\rm th}^2 + \beta\frac{B_{\rm com}^2}{\rho(z)}\left(\frac{k}{k_B}\right)^{n+3}(1+z)^4\right](1-\mu^2)$$

$$\beta = \frac{1}{1+\frac{k^2}{2\rho}\frac{m_\chi^2}{q_\chi^2}} \qquad 0<\beta<1$$

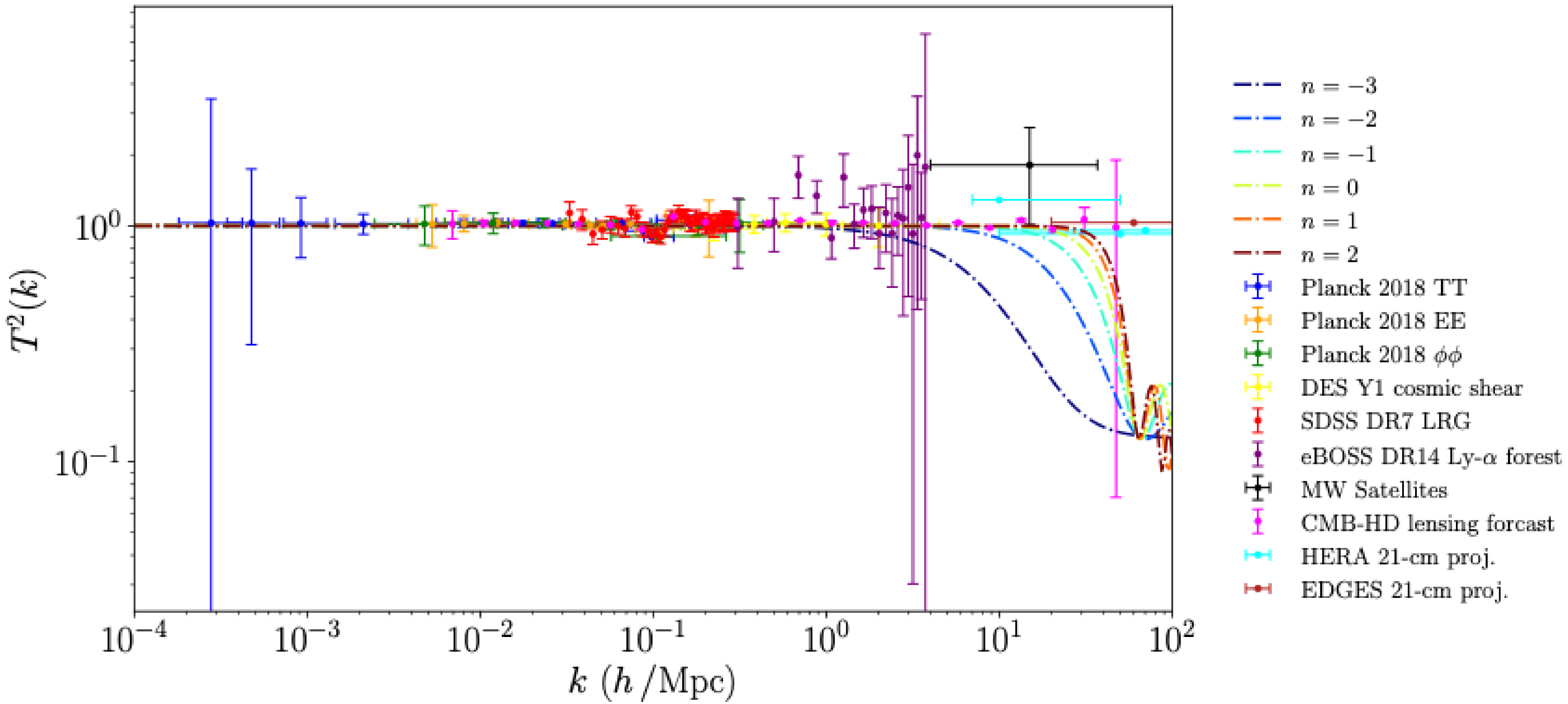
$$\frac{q_\chi}{m_\chi} \ll \frac{\sqrt{2\rho}}{k} \rightarrow \beta \ll 1 \rightarrow$$ Decouple from magnetic field (No plasma effects)

$$\frac{q_\chi}{m_\chi} \gg \frac{\sqrt{2\rho}}{k} \rightarrow \beta \sim 1 \rightarrow$$ Maximally coupled to magnetic field (Ideal MHD)





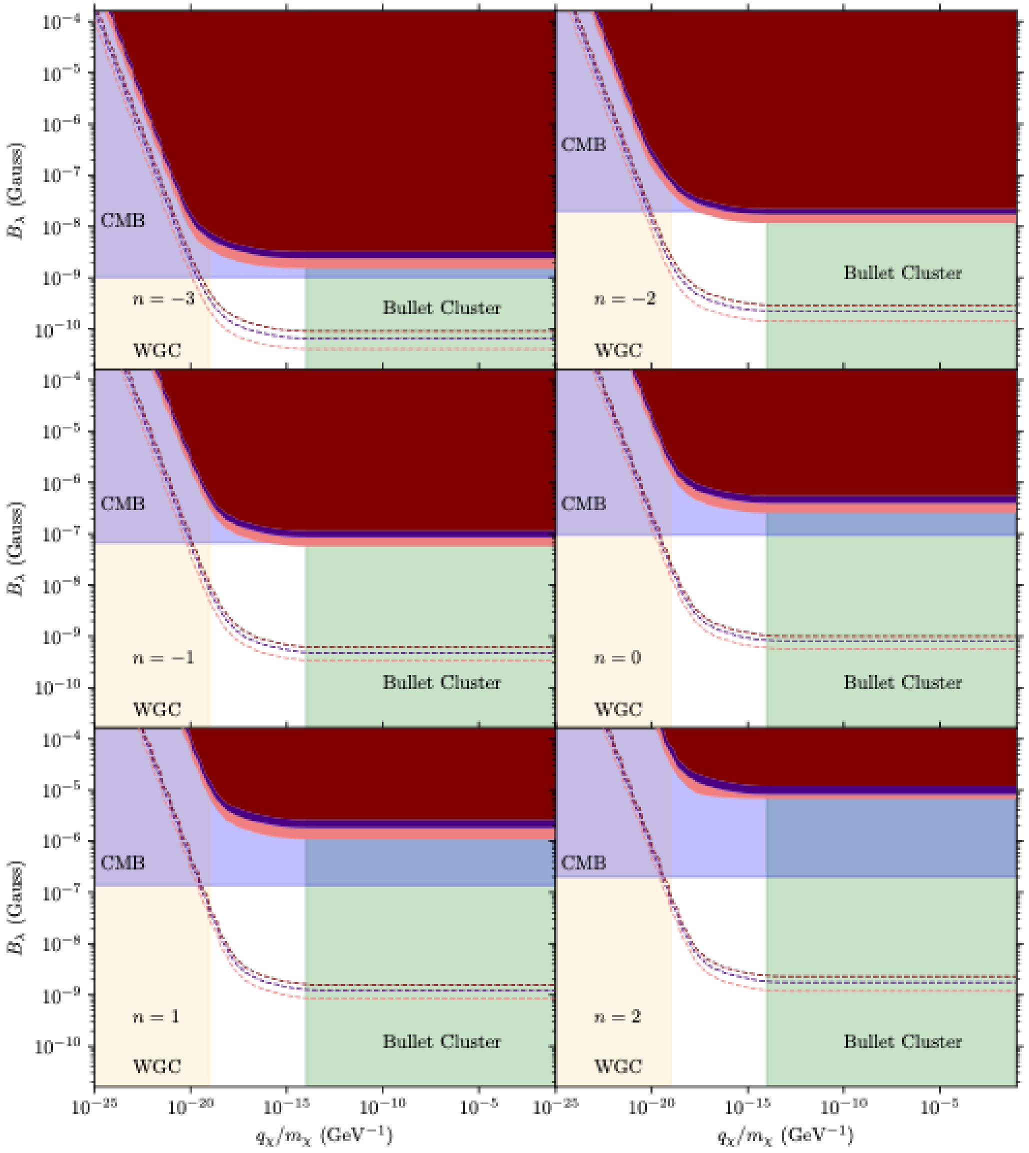
Linear Matter Power Spectrum



Constraints

Power spectrum of dark magnetic field: $P_B(k) = Ak^n e^{-k^2\ell^2}$

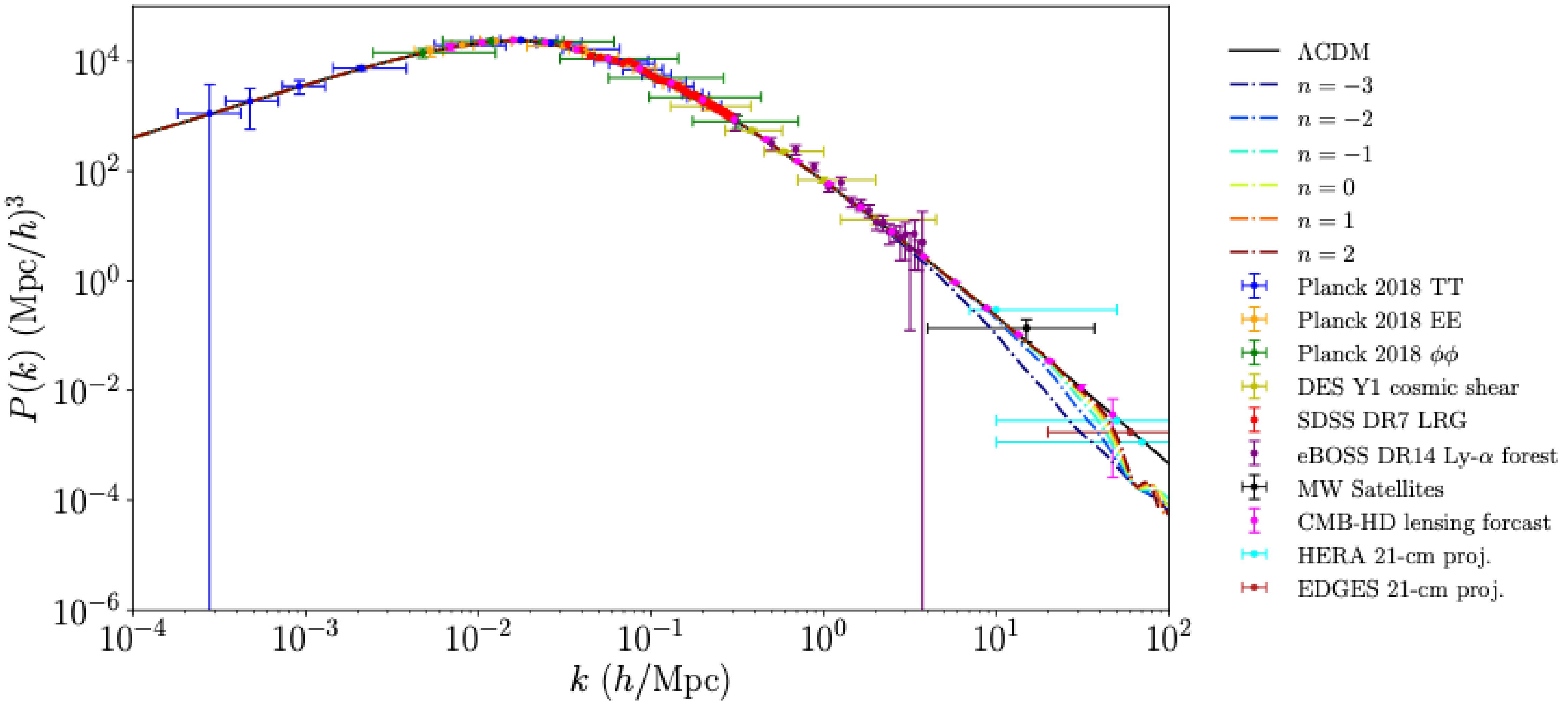
- Weak gravity conjecture implies that there cannot be any forces weaker than gravity
- Bullet Cluster constraints derived from PIC simulations of collisionless shocks (**PG**, William DeRocco 2411.11958)
- Tensor modes of the CMB strongly constrain standard model primordial magnetic fields through couplings to gravitational waves
  - Also applicable to dark primordial magnetic fields
- Current observations of the linear matter power (solid shading) spectrum cannot constrain large regions of new parameter space
- However future observations (dashed) may show unique signatures of dark primordial magnetic fields!



Summary

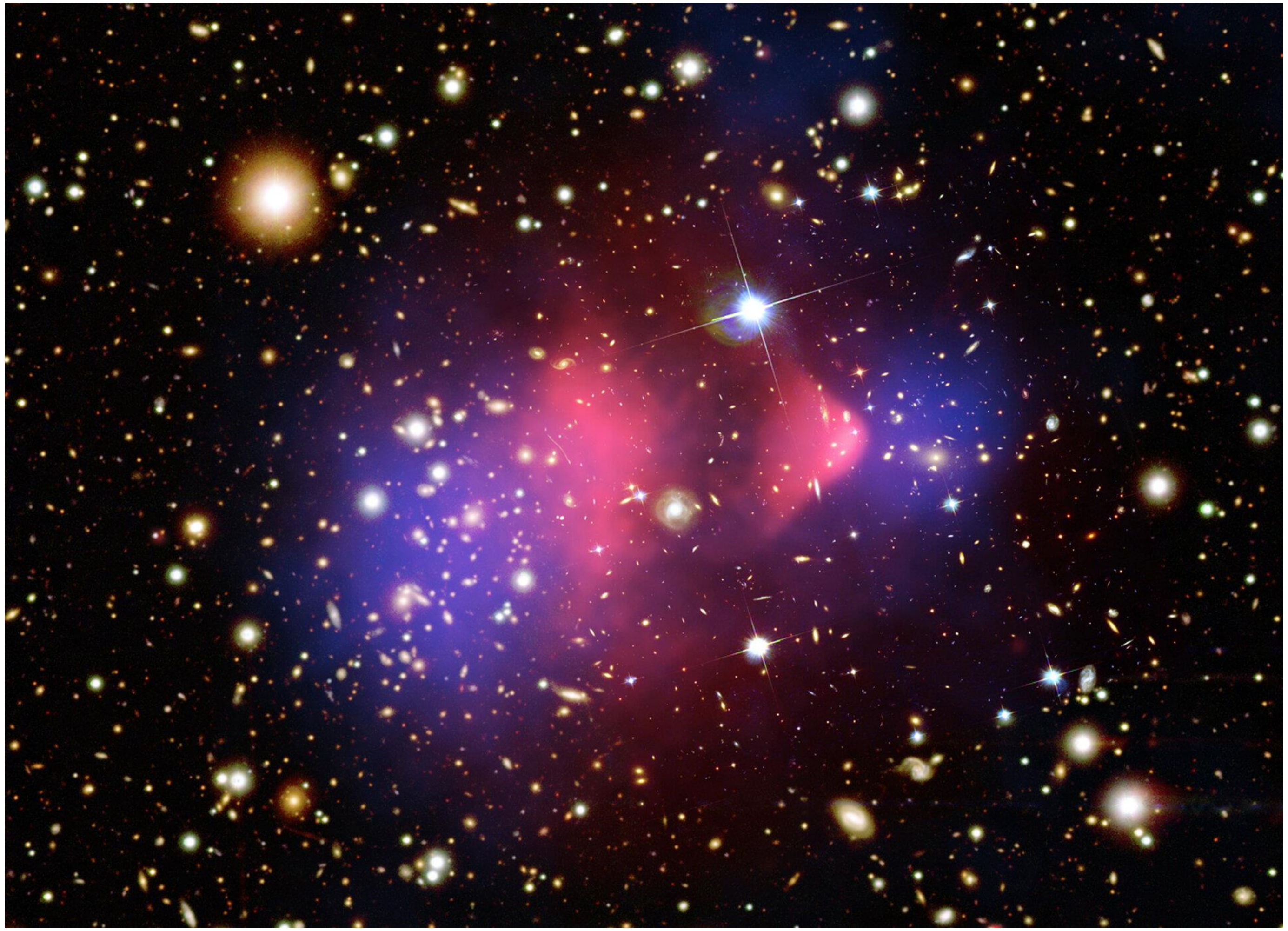
- SIDM is more than just two-to-two interactions. Collective effects can dramatically alter dynamics of dark sector models with long-range forces.
- Many techniques and results from plasma physics already allow us to probe simple models such as a secluded $U(1)_D$.
- Though large scale dark magnetic fields are tightly constrained by CMB measurements, upcoming observations such as CMB-HD lensing, HERA, and EDGES may be able to unconstrained parameter space.

Linear Matter Power Spectrum



Current Constraints

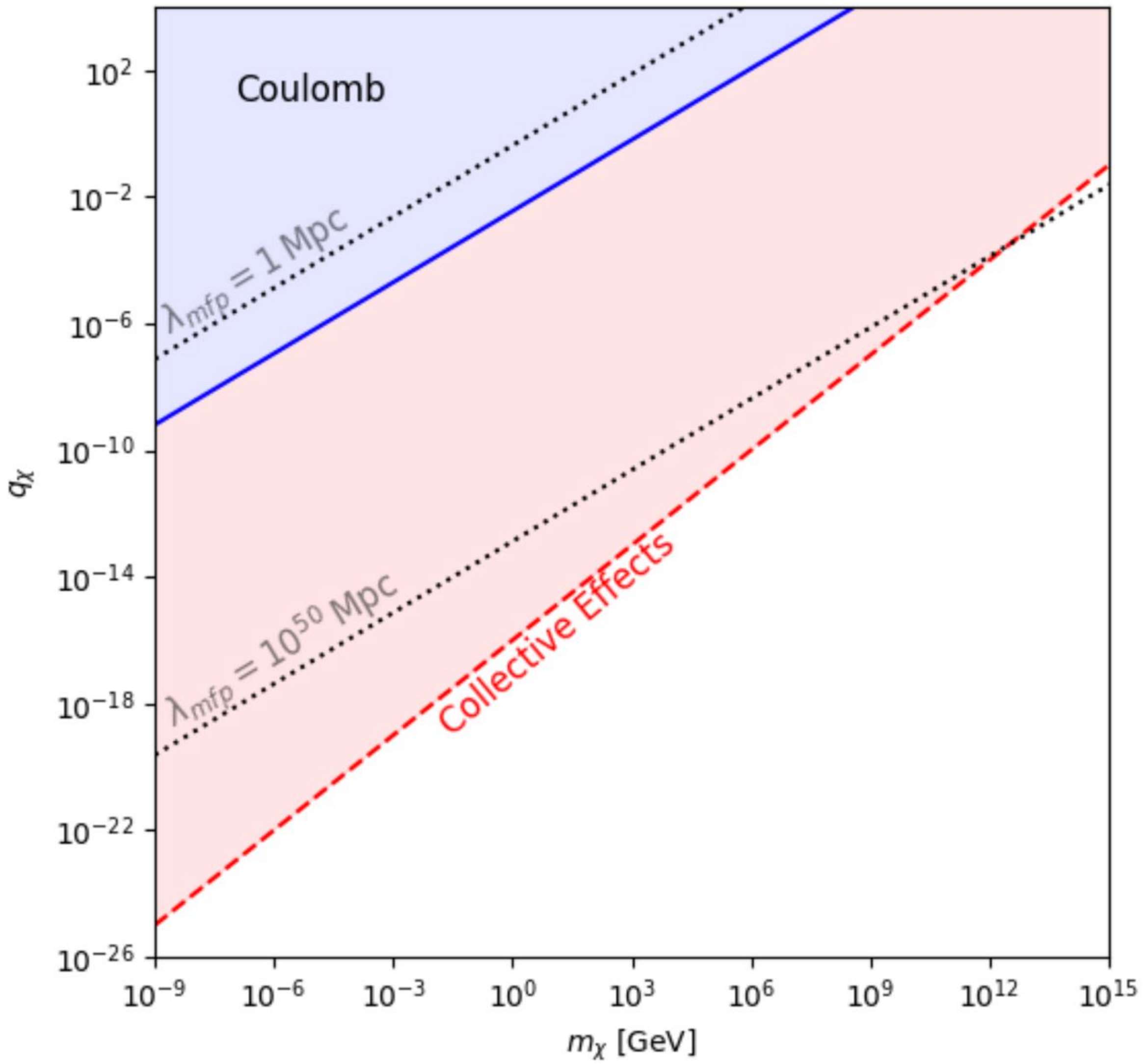
- Some of the strongest 2→2 constraints come from dissociative cluster mergers such as the Bullet Cluster
  - $\sigma/m \lesssim 1\ \mathrm{cm}^2/\mathrm{g}$
- Main Observables
  - Evaporation of dark matter halo
  - Offset of dark matter and standard model centers



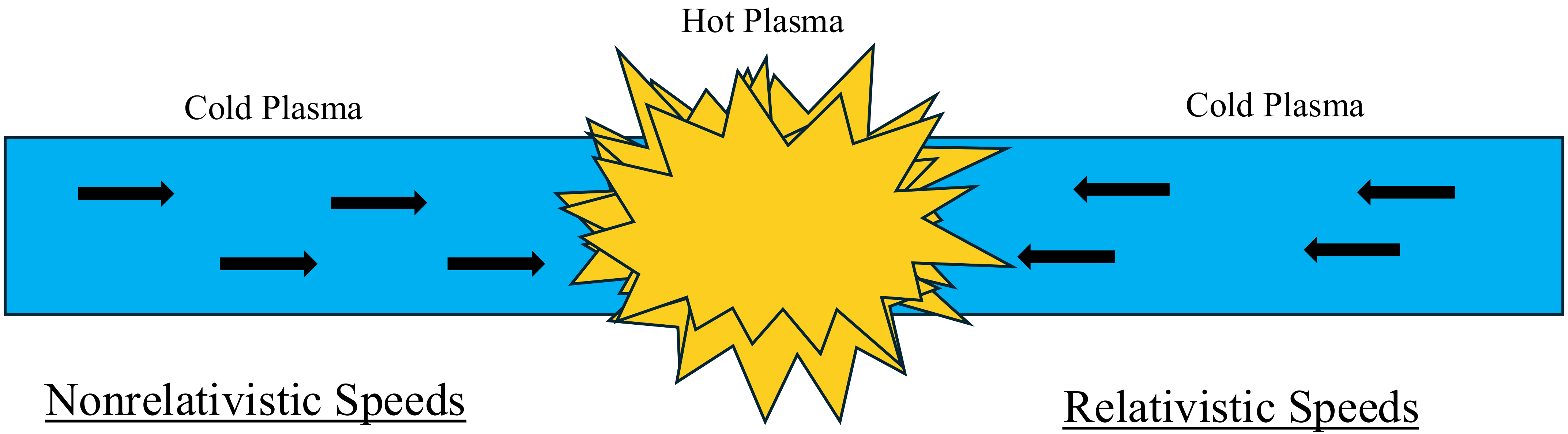
Collisionless Regime

- Size of Bullet Cluster ~ 100 kpc
- Mean free path of dark matter

$$\lambda \sim 300\,\mathrm{kpc}\left(\frac{v_{rel}}{0.01c}\right)^4\left(\frac{q_\chi}{q_e}\right)^{-4}\left(\frac{m_\chi}{\mathrm{GeV}}\right)^3\left(\frac{\rho_\chi}{0.01\ \mathrm{GeV/cm^3}}\right)$$



Beam Instabilities



<u>Nonrelativistic Speeds</u>

- Electrostatic forces
- Longitudinal modes
- Exponential growth rate

$$\Gamma \sim \omega_p^{-1}$$

<u>Relativistic Speeds</u>

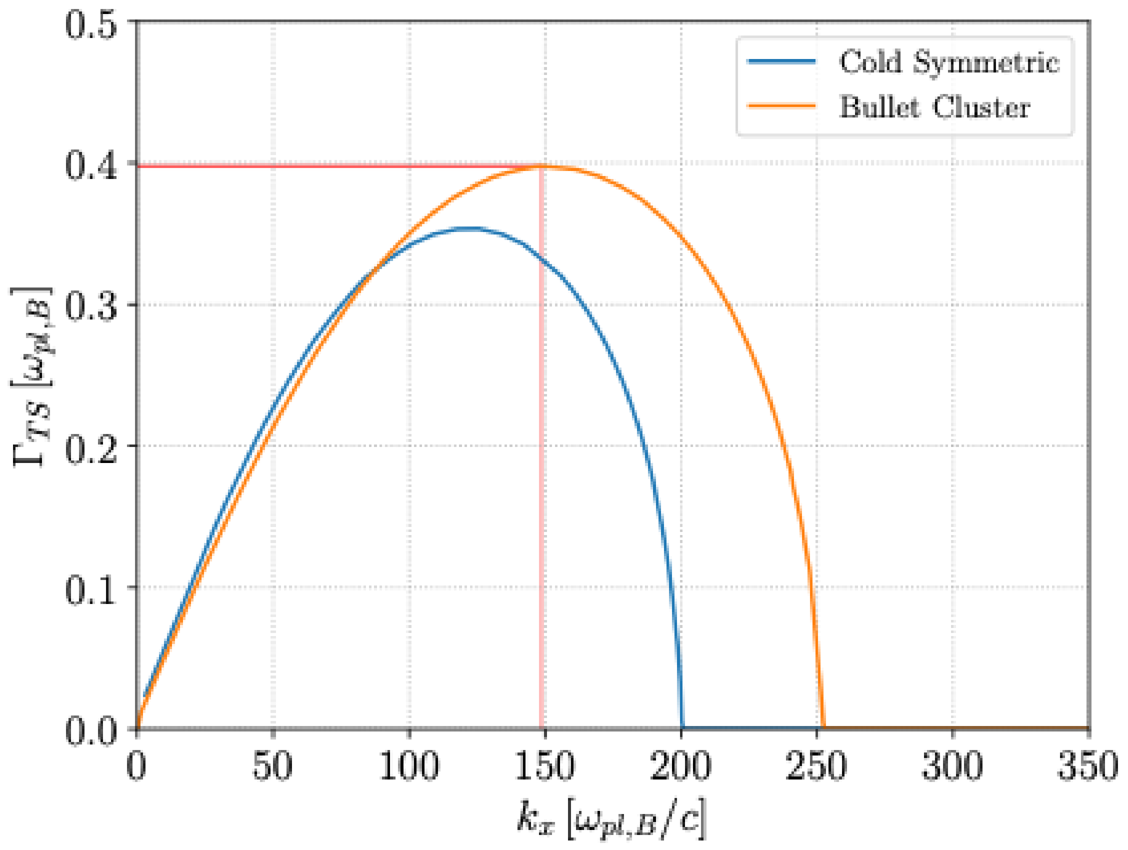
- Electromagnetic forces
- Transverse modes
- Exponential growth rate

$$\Gamma \sim v_{\mathrm{rel}} \omega_p^{-1}$$

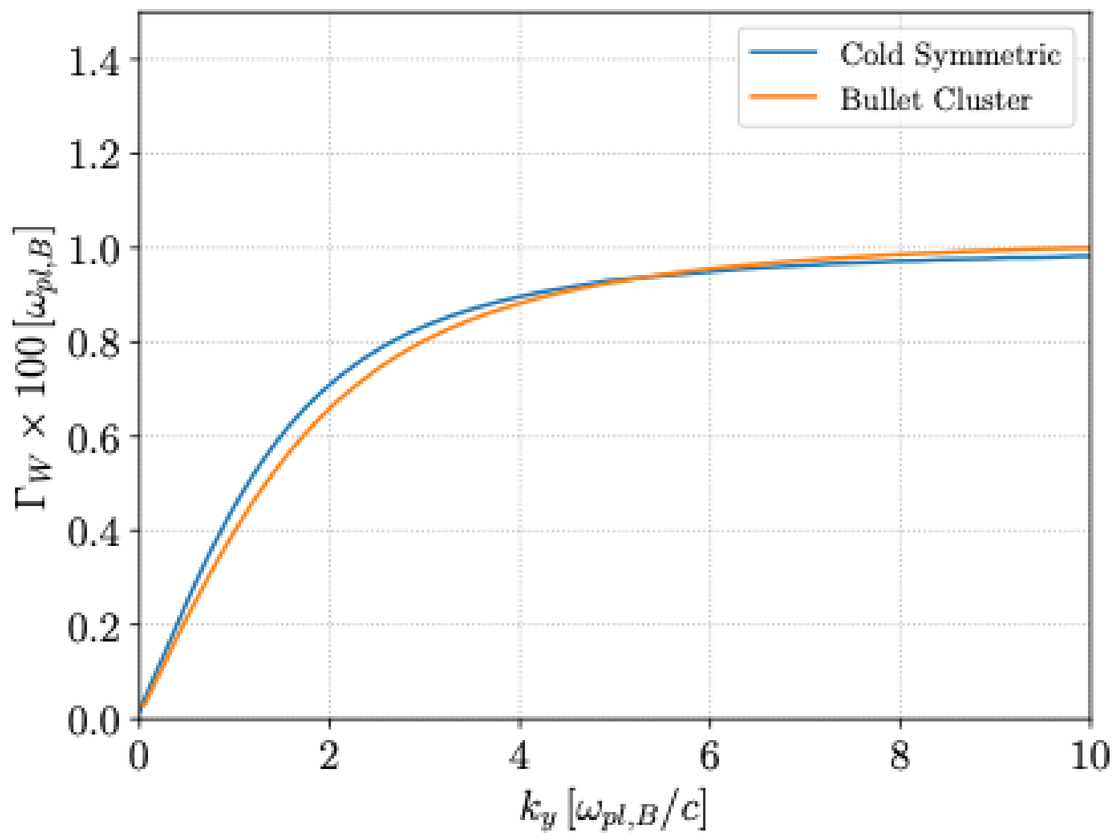
Linear Regime	Non-Linear Regime
Perturbations are small	Perturbations are large
Small timescales	Long timescales
Predict unstable modes and their growth rates	Require sophisticated simulations
Rough estimates of saturation time and mechanisms	Unstable modes may couple or saturate

Linear Regime

Longitudinal Electrostatic Mode



Transverse Electromagnetic Mode



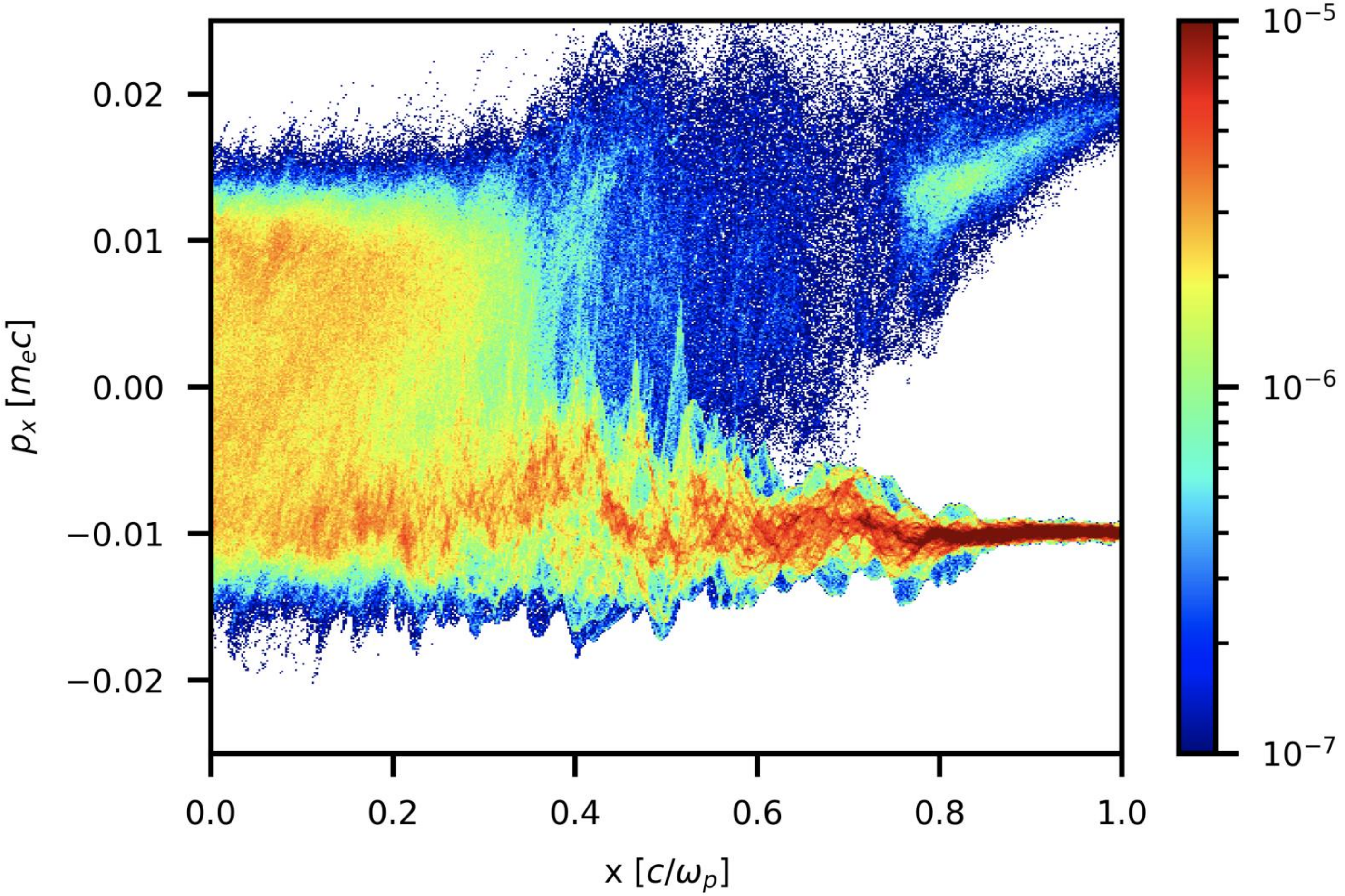
Non-Linear Regime

- Plasma frequency: $$\omega_\chi = \sqrt{\frac{4\pi q_\chi^2 n_{0,\chi}}{m_\chi}} = \frac{q_\chi}{m_\chi}\sqrt{4\pi\rho_\chi}$$

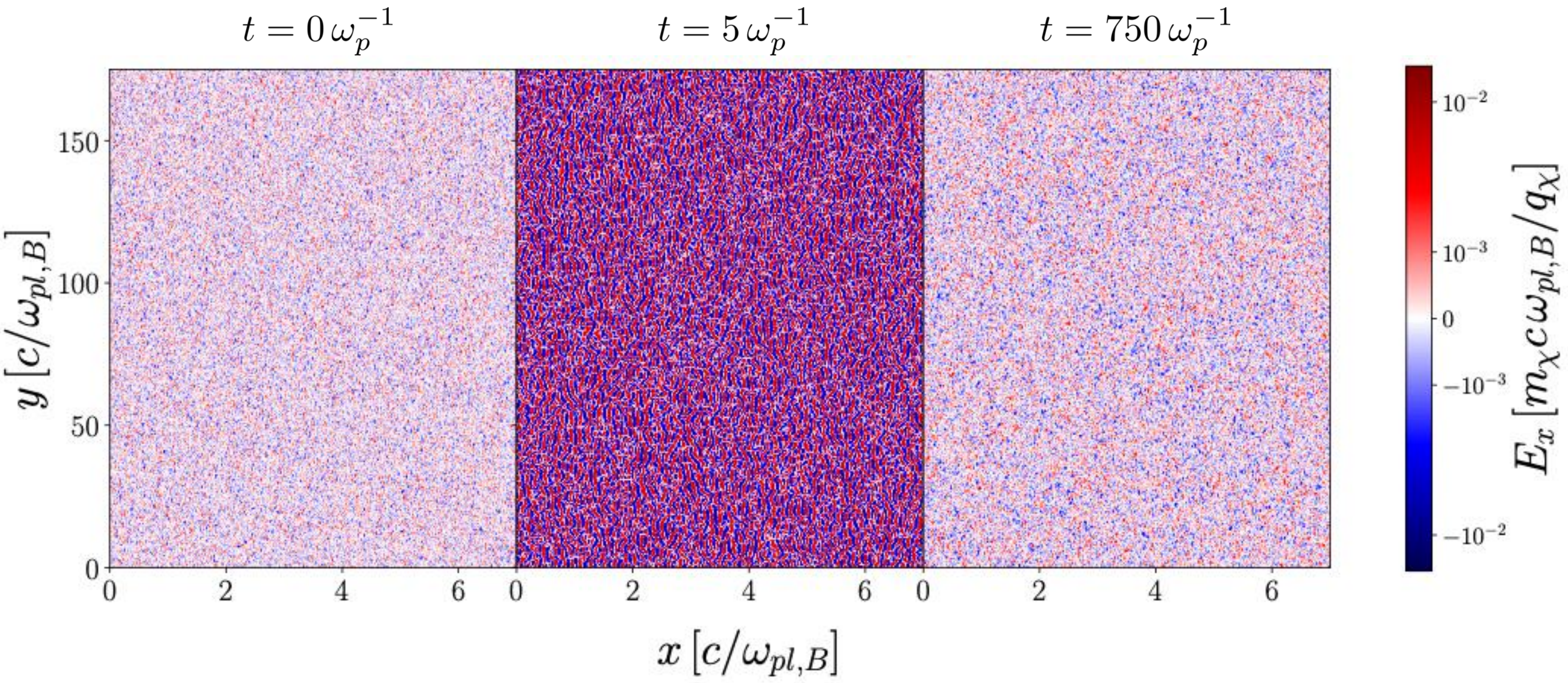
- "Smilei is a Particle-In-Cell code for plasma simulation. Open-source, collaborative, user-friendly and designed for high performances on super-computers, it is applied to a wide range of physics studies: from relativistic laser-plasma interaction to astrophysics."

Smilei)

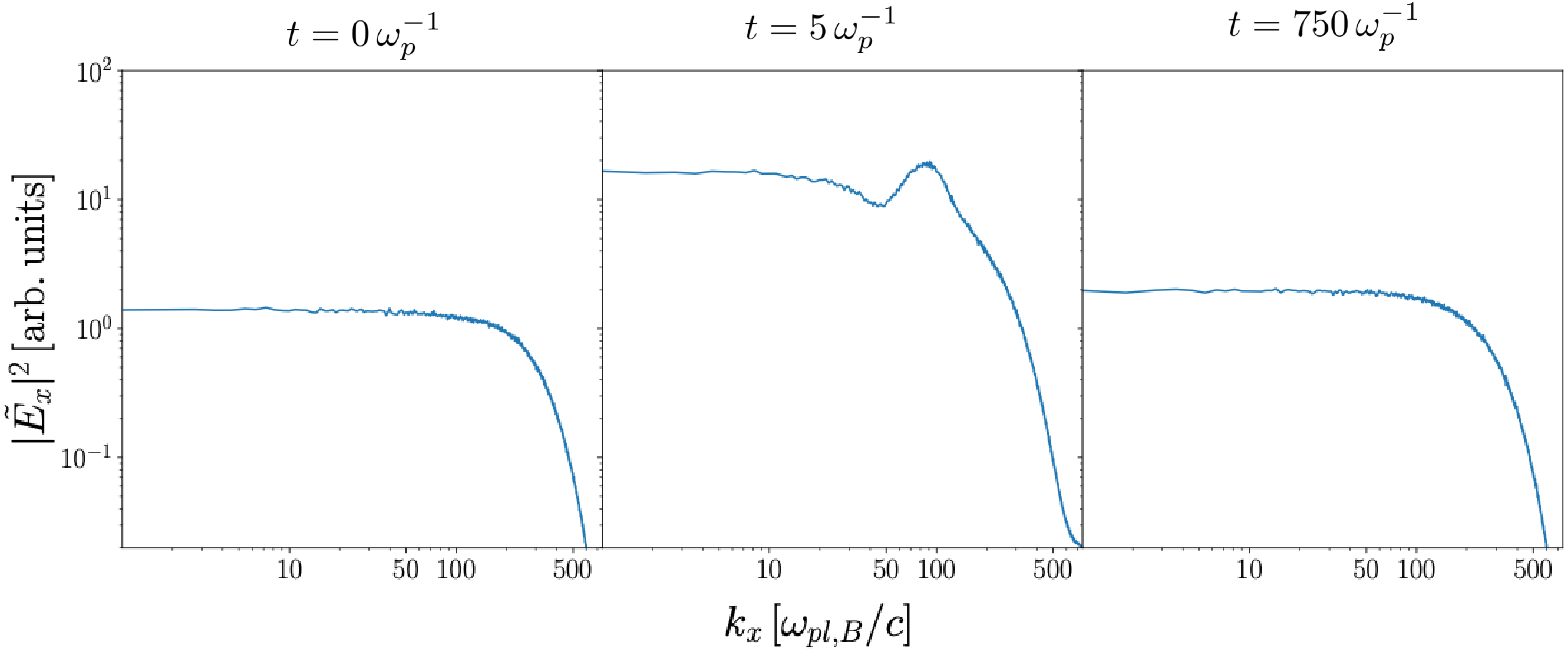
Plasma Shocks



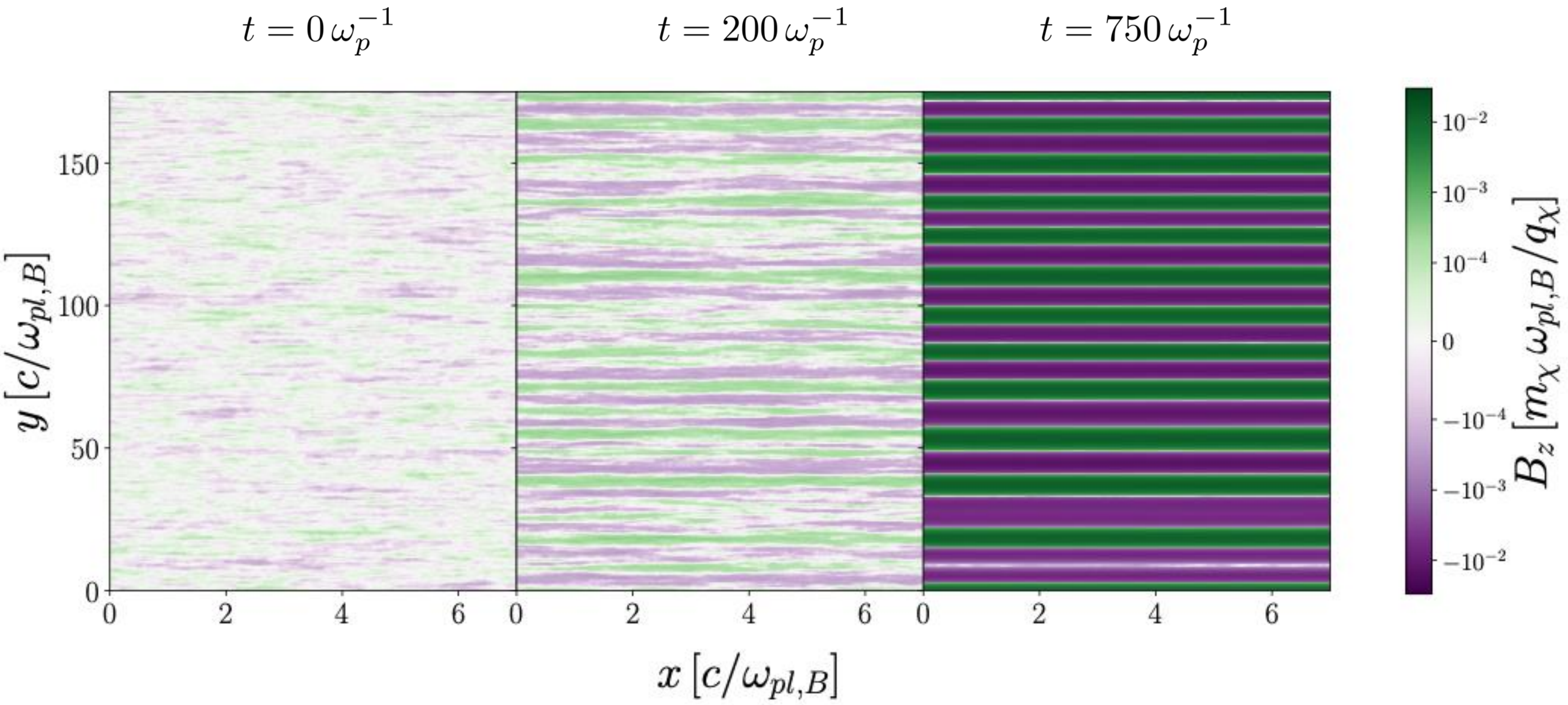
Simulation Results



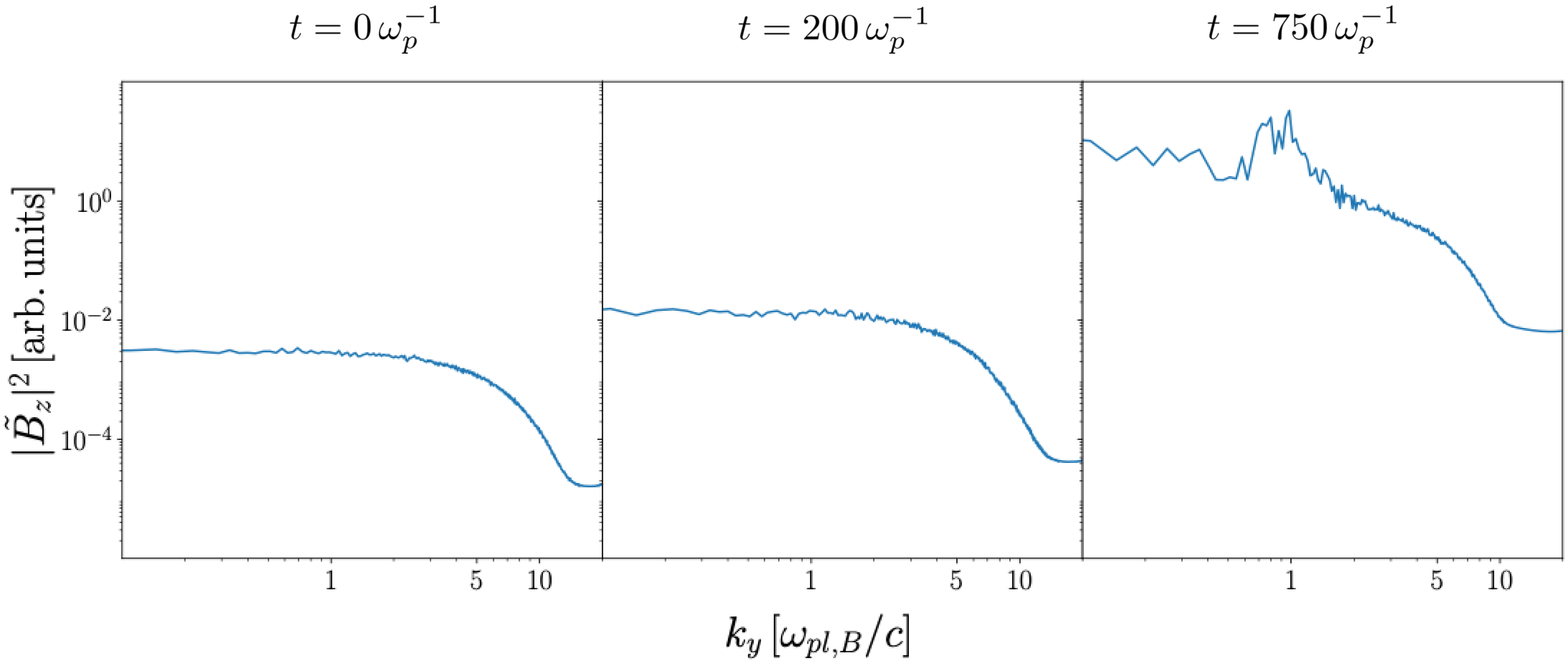
Simulation Results



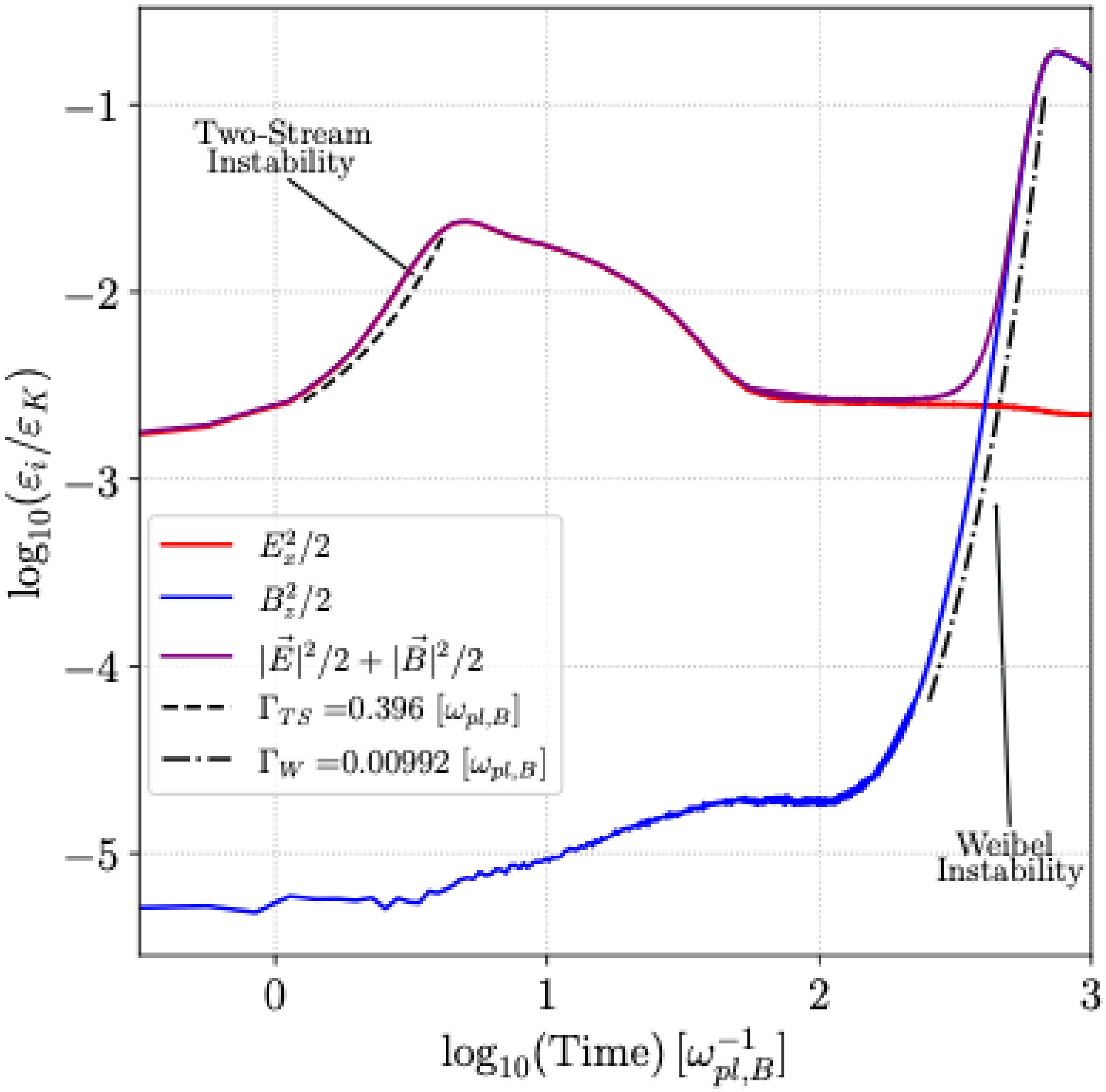
Simulation Results



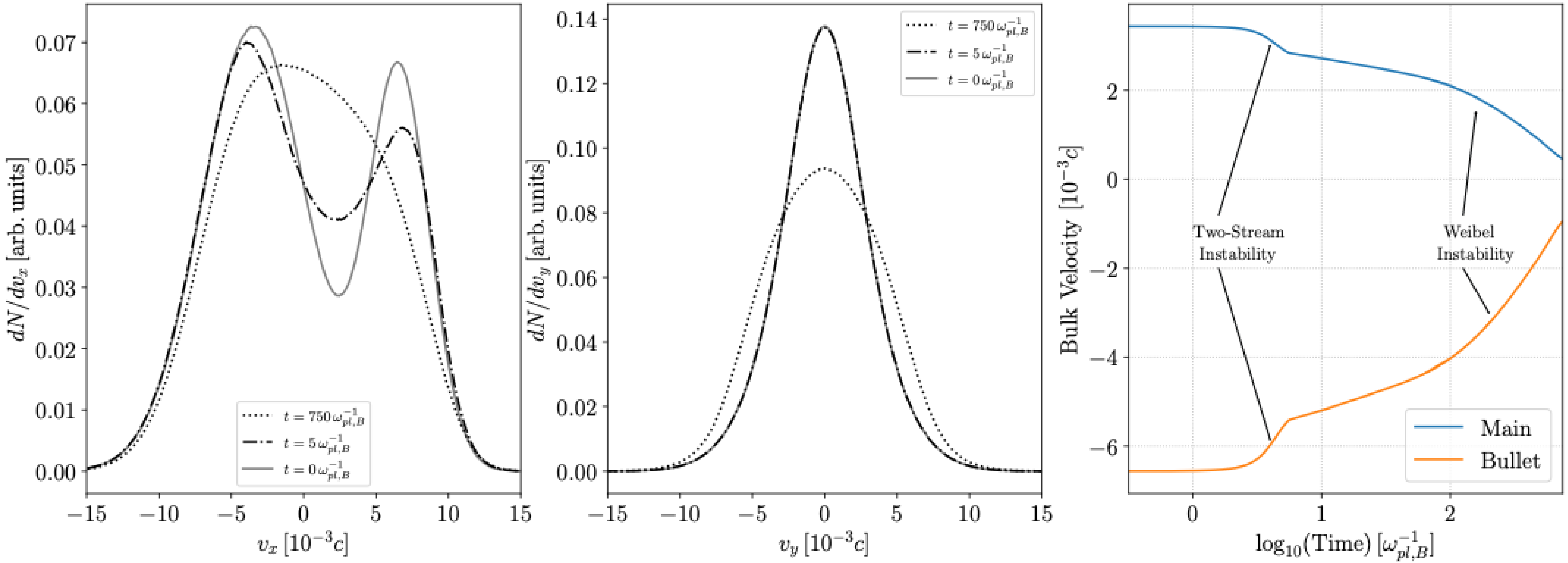
Simulation Results



Simulation Results



Simulation Results



After tracking the fraction of particles that have undergone a significant change in momentum, we can determine an effective cross-section

$$\sigma/m = -(0.33\,\mathrm{g/cm^2})^{-1}\log(1-p)$$