Berry Phase in Axion Physics and Its Applications

Shuailiang Ge

KAIST & University of Chicago

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in collaboration with Qing-Hong Cao, Yandong Liu, and Jun-Chen Wang

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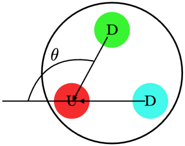
Axion: motivations

- Axion can solve the strong CP problem.

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}, \quad \theta_{\text{eff}} \to 0$$



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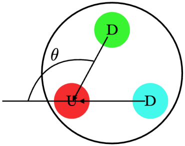
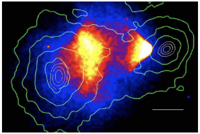


- Axions can be dark matter candidates.

$$a(t) \simeq a_0 \cos(m_a t), \quad \rho_a \simeq \frac{1}{2} m_a^2 a_0^2.$$



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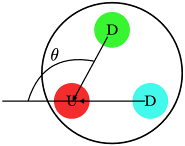

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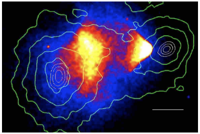
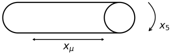


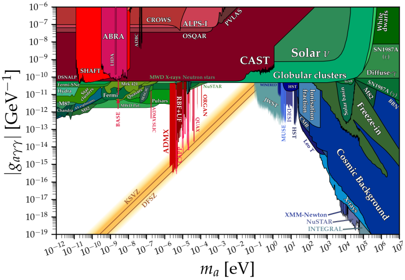
- Axions are predicted by string theory:

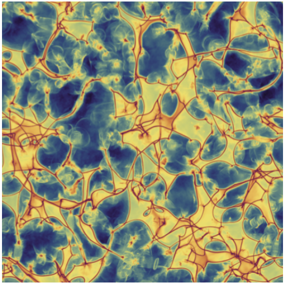
extra dimension + gauge fields $\to$ axions.



Axion: detection

- Axion theories have rich phenomena in cosmology and astrophysics, and large experimental efforts are devoted to search for them.
- $\mathcal{L}_{a\gamma\gamma} = \frac{1}{4}\frac{g_\gamma}{f} a F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \mathcal{L}_{aff} = -\frac{1}{2}\frac{g_f}{f}\partial_\mu a\, \bar{f}\gamma^\mu\gamma^5 f.$
- axion strings, axion domain walls...





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- Rotation of the linear polarization of CMB photons (Lue, Wang, Kamionkowski 1999; Minami, Komatsu 2020).
- Photons passing through axion topological defects: strings and walls (Agrawal, Hook, Huang 2019; Takahashi, Yin 2020; Jain, Long, Amin 2021).

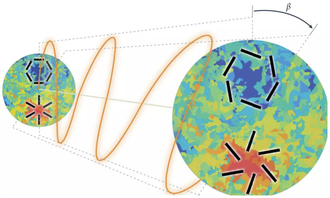
Cosmic birefringence

- The Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 + \frac{1}{4}\frac{g_\gamma}{f} a F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

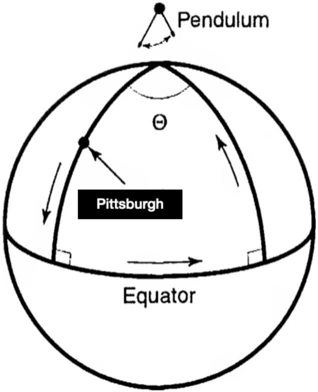
- Parity odd: $F\tilde{F} \to -F\tilde{F}$.
- The polarization rotation angle is

$$\beta = \frac{1}{2}\frac{g_\gamma}{f}(a_i - a_f)$$



Berry phase

- Cosmic birefringence is a demonstration of Berry phase.
- Classical analogy of Berry phase: parallel transport around a closed path can rotate a local frame.



Berry phase

- Berry phase in quantum mechanics. Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\left|\psi\right\rangle = H(\lambda(t))\left|\psi\right\rangle, \qquad H(\lambda(t))\left|\psi\right\rangle = E(\lambda(t))\left|\psi\right\rangle.$$

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- The Hamiltonian H is in general time-dependent. Suppose the parameters $\lambda(t)$ are slowly changing with time. Based on the adiabatic theorem, a system starting at $|\psi(\lambda(0))\rangle$ remains in the instantaneous state $|\psi(\lambda(t))\rangle$ up to a phase:

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$$\mathrm{e}^{i\alpha(t)}\left|\psi(\lambda(t))\right\rangle .$$

- Substituting it back into the Schrödinger equation, for $\lambda(t)$ finishing a closed circle C after time T, $\lambda(T) = \lambda(0)$, one gets

$$\alpha(C) = \oint_C \mathrm{d}\lambda_a\, i\left\langle \psi(\lambda)\right|\frac{\mathrm{d}}{\mathrm{d}\lambda_a}\left|\psi(\lambda)\right\rangle + \int_0^T \mathrm{d}t\,\left[-\frac{1}{\hbar}E(\lambda(t))\right] .$$

Berry phase

- One can define

$$\mathcal{A}^a = i \left\langle \psi(\lambda) | \frac{\mathrm{d}}{\mathrm{d}\lambda_a} | \psi(\lambda) \right\rangle$$

so that

$$\alpha(C) = \oint_C \mathrm{d}\lambda_a \, \mathcal{A}^a + \int_0^T \mathrm{d}t \, \left[-\frac{1}{\hbar} E_n(\lambda(t)) \right].$$

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- $\mathcal{A}^a$ is a gauge field. We can see this by making a transformation $|\psi(\lambda)\rangle \to \mathrm{e}^{i\gamma(\lambda)} |\psi(\lambda)\rangle$. The transformation of $\mathcal{A}^a$ is
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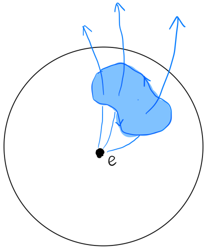
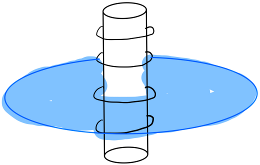
- The corresponding field strength is
$$F^{ab} = \partial^a \mathcal{A}^b - \partial^b \mathcal{A}^a.$$
$$\alpha_{\mathrm{Berry}}(C) = \oint_C \mathcal{A} = \int_S F.$$

Berry phase

- $\alpha_{\text{Berry}} = \oint \mathcal{A} = \int_S F$.
- Two typical cases with $\alpha_{\text{Berry}} \neq 0$. F integrated over the blue area.



“magnetic monopole”



solenoid / AB effect

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- This motivates us to derive effective Hamiltonians from the Lagrangian of axion-SM interactions.
- It turns out that in this way, the Berry phases arising in the axion-photon and axion-fermion interactions can be treated in a unified way.
- We start from the interaction terms:

$$\mathcal{L}_{a\gamma\gamma} = \frac{1}{4}\frac{g_\gamma}{f} a F_{\mu\nu}\tilde{F}^{\mu\nu}, \qquad \mathcal{L}_{aff} = -\frac{1}{2}\frac{g_f}{f}\partial_\mu a\, \bar{f}\gamma^\mu\gamma^5 f.$$

Effective Hamiltonians in axion physics

- We first focus on the axion-fermion interaction. The equation of motion is the Dirac equation. In the non-relativistic limit, the Dirac equation can be written as

$$i\partial_t \psi = H_{aff}\psi, \qquad H_{aff} = \frac{g_f}{2f}\left[\boldsymbol{\nabla} a + \partial_t a \frac{\mathbf{p}}{m_f}\right]\cdot \boldsymbol{\sigma}.$$

- $\mathbf{p}$ and m_f are respectively fermion momentum and mass.

Effective Hamiltonians in axion physics

- Heuristically, we express the axion-photon system analogously by dealing with the Maxwell equations. One has

$$\partial_t \mathbf{E}_0 = \dot{a}\,\frac{g_\gamma}{2f}\frac{\mathbf{k}}{|\mathbf{k}|} \times \mathbf{E}_0, \qquad \mathbf{E}(t) = \mathbf{E}_0(t)\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{x}).$$

- It can be further written as

$$i\partial_t \psi = H_{a\gamma\gamma}\psi, \qquad \psi \equiv (E_{0,1}, E_{0,2}, E_{0,3})^T,$$

where

$$H_{a\gamma\gamma} = \frac{g_\gamma}{2f}\dot{a}\frac{\mathbf{k}\cdot\mathbf{S}}{|\mathbf{k}|}.$$

- S are the spin-1 matrices:

$$(S^\beta)_{\alpha\delta} = i\epsilon^{\alpha\beta\delta}, \quad [S^\alpha, S^\beta] = i\epsilon^{\alpha\beta\delta}S^\delta, \quad (S^1)^2+(S^2)^2+(S^3)^2 = 2.$$

- Note that now the equations of motion for the axion-fermion and axion-photon systems share the same form

$$i\partial_t \psi = H\psi, \qquad H(t) = \mathbf{V}(t) \cdot \mathbf{j},$$

where

$$\mathbf{V}(t) = \frac{g_f}{2f}\left[\boldsymbol{\nabla} a + \partial_t a \frac{\mathbf{p}}{m_f}\right] \quad \text{or} \quad \frac{g_\gamma}{2f}\dot{a}\frac{\mathbf{k}}{|\mathbf{k}|}, \qquad \mathbf{j} = \boldsymbol{\sigma} \text{ or } \mathbf{S}.$$

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- The change of $\mathbf{V}$ as a function of time will induce Berry phase.
- Scenario I: magnitude $|\mathbf{V}|$ is changing, i.e. the axion field is changing while the fermion or photon momentum is fixed.
- Scenario II: direction of $\mathbf{V}$ is changing, i.e. the direction of fermion or photon momentum is changing while the axion field is fixed.

Scenario I: axion field is varying

- In the axion-photon case, consider right- and left-handed circular polarization states

$$\phi_R(0) = \frac{(1, i)^T}{\sqrt{2}}, \qquad \phi_L(0) = \frac{(1, -i)^T}{\sqrt{2}}.$$

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- A linearly polarized photon, as a superposition of right- and left-handed modes, acquires a rotation angle

$$|\alpha_{\mathrm{Berry}}| .$$

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$$a/f \sim a/f + 2\pi, \qquad \text{parameter space: } \mathbb{S}^1.$$

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- The Berry phase is genuine only if

$$a(T) - a(0) = 2\pi n \cdot f,$$

which gives

$$|\alpha_{\mathrm{Berry}}| = g_\gamma \pi n.$$

Scenario I: axion field is varying

- A similar result can be obtained in the axion-fermion case. An electron with helicity $j_z = \pm 1/2$ traveling in the axion background acquires

$$\alpha_{\mathrm{Berry}} = j_z v^{-1} \frac{g_f}{f} [a(T) - a(0)], \qquad j_z = \pm \frac{1}{2}.$$

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- v is velocity of axion in the rest frame of electron. The genuine Berry phase is thus

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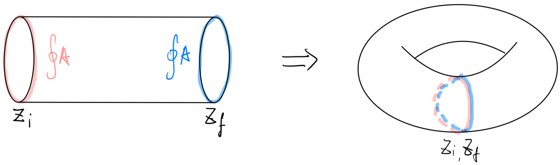
- An electron, as a superposition of left- and right-handed modes, travels in the axion background. Its spin will rotate by

$$|\alpha_{\rm Berry}| = v^{-1} g_f \pi n.$$

Scenario I: axion field is varying

- In the axion case, the Berry phase can be written as

$$\alpha_{\text{Berry}} = \pm c \left[\oint \mathcal{A}(z_f) - \oint \mathcal{A}(z_i) \right] = \pm c \int_{\text{torus}} F = \pm c \cdot 2\pi n.$$



- Differed by a large gauge transformation.
- This is especially clear in the extra-dimensional axion where $a/f = \oint \mathcal{A}$.

Scenario I: axion field is varying

- Two key features for Berry phases arising in axion physics.
- Axion is a pseudoscalar. Under parity,

$$F\tilde{F} \to -F\tilde{F}, \qquad \bar{f}i\gamma^5 f \to -\bar{f}i\gamma^5 f.$$

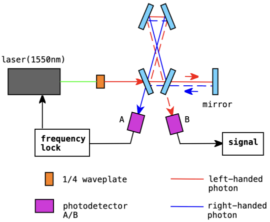
- Axion is periodic:

$$a/f \sim a/f + 2\pi.$$

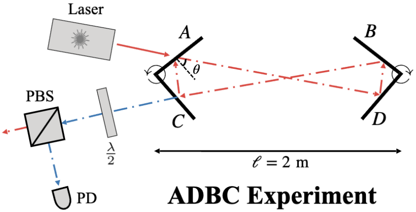
  The parameter space is $\mathbb{S}^1$.

Scenario I: experiments

- Birefringence-cavity experiments essentially measure the Berry phase of photons traveling in the axion background: rotation of the linear polarization angle.
- Examples: (Obata, Fujita, Michimura 2018; Liu et al 2018; Oshima et al 2023; Pandey, Hall, Evans 2024).



(Obata, Fujita, Michimura 2018)

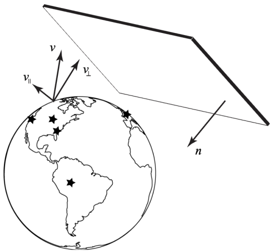


(Liu et al 2018)

Scenario I: experiments

- Interaction between axion domain wall and fermions.
- Example: an axion domain wall passing through the Earth (Pospelov et al 2012).
- The rotation angle of electron spin is

$$|\alpha_{\rm Berry}| = v^{-1} g_f \pi.$$



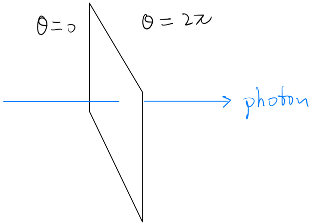
(Pospelov et al 2012)

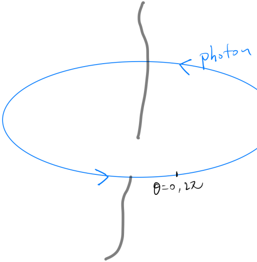
Scenario I: experiments

- Cosmic birefringence induced by axion domain walls and axion strings. (Agrawal, Hook, Huang 2019; Takahashi, Yin 2020; Jain, Long, Amin 2021).
- The rotation of the linearly-polarized photon angle is

$$|\alpha_{\rm Berry}| = g_\gamma \pi,$$

a genuine Berry phase.





Scenario I: experiments

- The genuine Berry phase is

$$|\alpha_{\text{Berry}}| = g_\gamma \pi n.$$

  By measuring $|\alpha_{\text{Berry}}|$, one can determine g_γ. In general g_γ is $\mathcal{O}(1)$, not suppressed by the axion decay constant f.
- g_γ gives valuable information about the global structure of the Standard Model gauge group.
- Global structure of SM gauge group (e.g., Tong 2017):

$$\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_p}, \qquad p = 1, 2, 3, 6.$$

Scenario I: experiments

- Axion interactions with the SM gauge fields:

$$\frac{K_3}{32\pi^2}\frac{a}{f}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}+\frac{K_2}{32\pi^2}\frac{a}{f}F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}+\frac{K_1}{16\pi^2}\frac{1}{36}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}.$$

- After electroweak symmetry breaking:

$$\frac{K_3}{32\pi^2}\frac{a}{f}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}+\frac{E}{16\pi^2}\frac{a}{f}F_{\mathrm{EM},\mu\nu}\tilde{F}^{\mu\nu}_{\mathrm{EM}},$$

  where

$$E=\frac{1}{36}(K_1+18K_2),\qquad K_3=2N=N_{\mathrm{DW}}.$$

- The axion periodicity $a/f\sim a/f+2\pi$ constrains E and N, with different results for $p=1,2,3,6$.

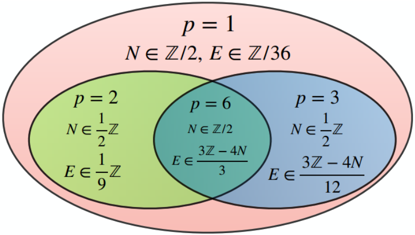
Scenario I: experiments

- The dimensionless coupling

$$g_\gamma = \frac{\alpha_{\rm EM}}{\pi}(E - 1.92N)$$

(Cortona et al 2026).

- Constraints on E and N for different p. (Cordova, Hong, Wang 2023; Choi et al 2023; Reece 2023).



Scenario II: photon or fermion momentum is varying

- Recall that

$$i\partial_t \psi = H\psi, \qquad H(t) = \mathbf{V}(t)\cdot\mathbf{j},$$

where

$$\mathbf{V}(t) = \frac{g_f}{2f}\left[\nabla a + \partial_t a \frac{\mathbf{p}}{m_f}\right] \quad \text{or} \quad \dot{a}\,\frac{g_\gamma}{2f}\frac{\mathbf{k}}{|\mathbf{k}|}, \qquad \mathbf{j} = \boldsymbol{\sigma} \text{ or } \mathbf{S}.$$

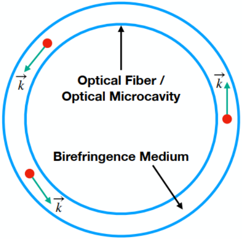
- Scenario II: direction of **V** is changing, i.e. the direction of fermion or photon momentum is changing while the axion field is fixed.

Scenario II: photon or fermion momentum is varying

- Photon-ring experiment. Photons propagate circularly in the xy plane within the fiber with momentum

$$\mathbf{k}(t) = (n\omega\cos(\Omega t),\ n\omega\sin(\Omega t),\ 0).$$

- $\Omega = 1/(nR)$: photon angular velocity. ω: photon energy. R: loop radius. n: optical fiber refractive index.
- The photon polarization is in the z direction.
- Similar to the proton-ring case (Graham et al 2020).



Scenario II: photon or fermion momentum is varying

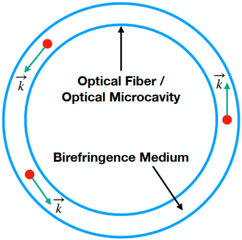
- The accumulated dynamical phase:

$$\alpha_{\rm dyn} = \frac{t_{\rm exp}\left[\frac{\rho_{\rm DM} g_\gamma^2 \sin^2(m_a t_0+\varphi)}{2n^2 f^2} - \frac{\omega\chi}{2n^2}\left(\Omega - \frac{\omega\chi}{2n^2}\right)\right]}{\sqrt{\frac{\rho_{\rm DM} g_\gamma^2 \sin^2(m_a t_0+\varphi)}{2n^2 f^2} + \left(\Omega - \frac{\omega\chi}{2n^2}\right)^2}}.$$

- Birefringent material with dielectric permittivity tensor

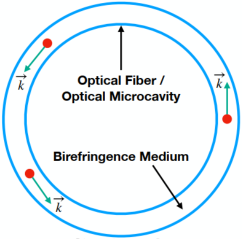
$$\epsilon = \begin{pmatrix} n^2 & i\chi & 0 \\ -i\chi & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix} = n^2 I - \chi S^z.$$

- Resonance condition: $\Omega = \omega\chi/(2n^2)$.



Scenario II: photon or fermion momentum is varying

- This gives
$$\alpha_{\rm dyn} = \frac{g_\gamma \sqrt{\rho_{\rm DM}} \sin(m_a t_0 + \varphi)}{\sqrt{2}\, nf} t_{\rm exp}.$$
- At the resonance condition, Berry phase is maximized:
$$\alpha_{\rm Berry} = -2\pi j_z.$$
- But we are not directly measuring Berry phase. We are measuring the dynamical phase.
- Aiming for detecting ultralight axion field: $m_a \lesssim 10^{-16}$ eV.



Summary

- Berry phase arises both in axion-photon and axion-fermion systems.
- Two scenarios. Scenario I: the closed loop in parameter space is axion field $\mathbb{S}^1$. Scenario II: the closed loop is in parameter space of fermion or photon momentum.
- Several experiments, including birefringent cavities, domain-wall effects on electron spin, proton rings, etc., can be understood through the lens of the Berry phase.
- Measuring a genuine Berry phase is helpful to understand the global structure of the Standard Model gauge group.
- Photon-ring experiment is proposed based on Scenario II.

Thank you