The Geometry of SMEFT

From Field-Space Geometry to Collider Signatures

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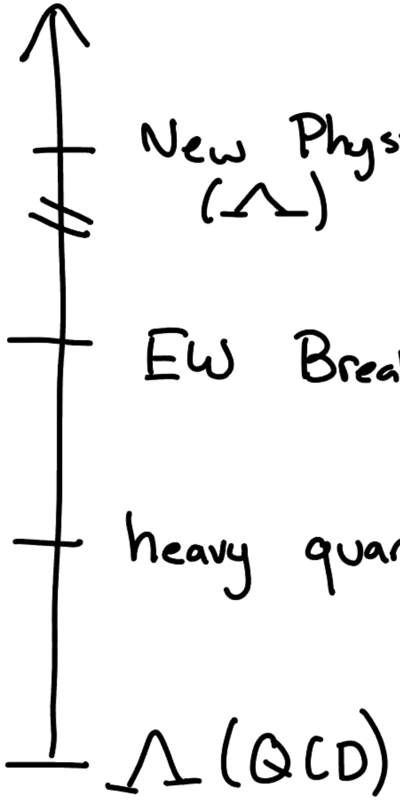
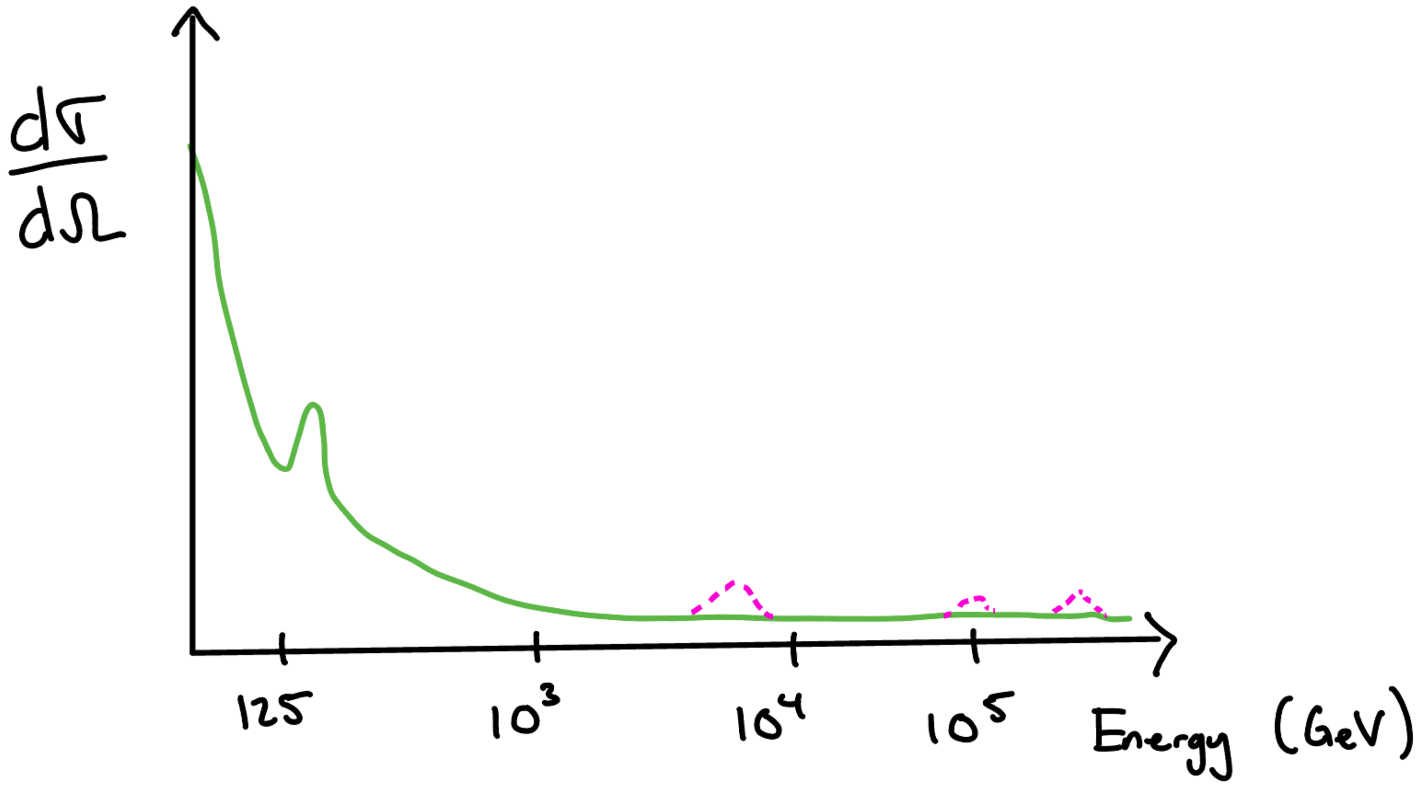
In collaboration with Rachel Houtz & Mia West

Probing the Unknown

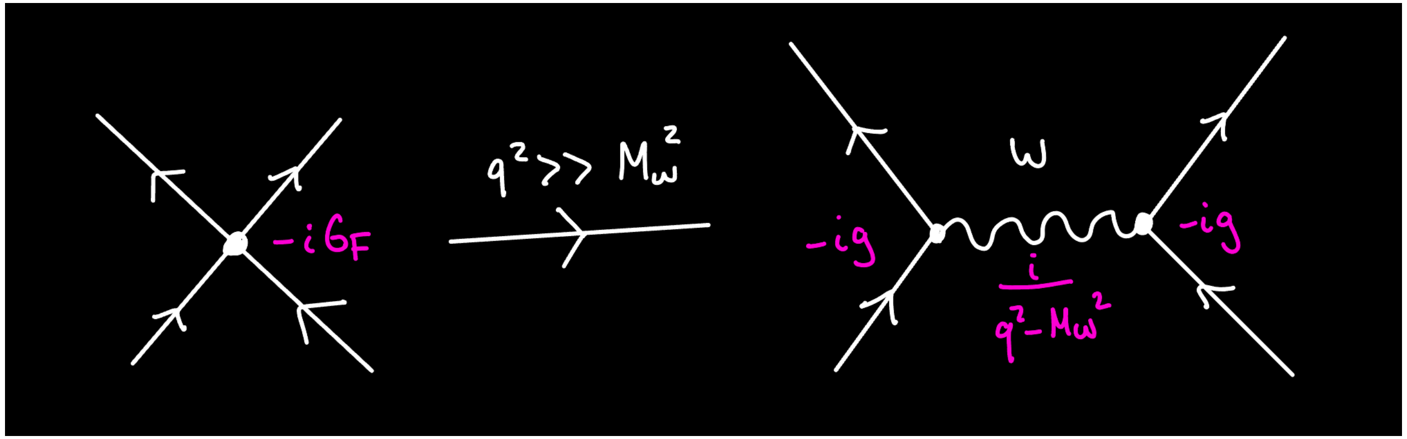




What is an EFT?

A systematic, order-by-order expansion that parametrizes the effects of unknown higher-scale physics in a model-independent way.

$$\mathscr{L} = \mathscr{L}_{SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_i^{(8)} + \cdots$$

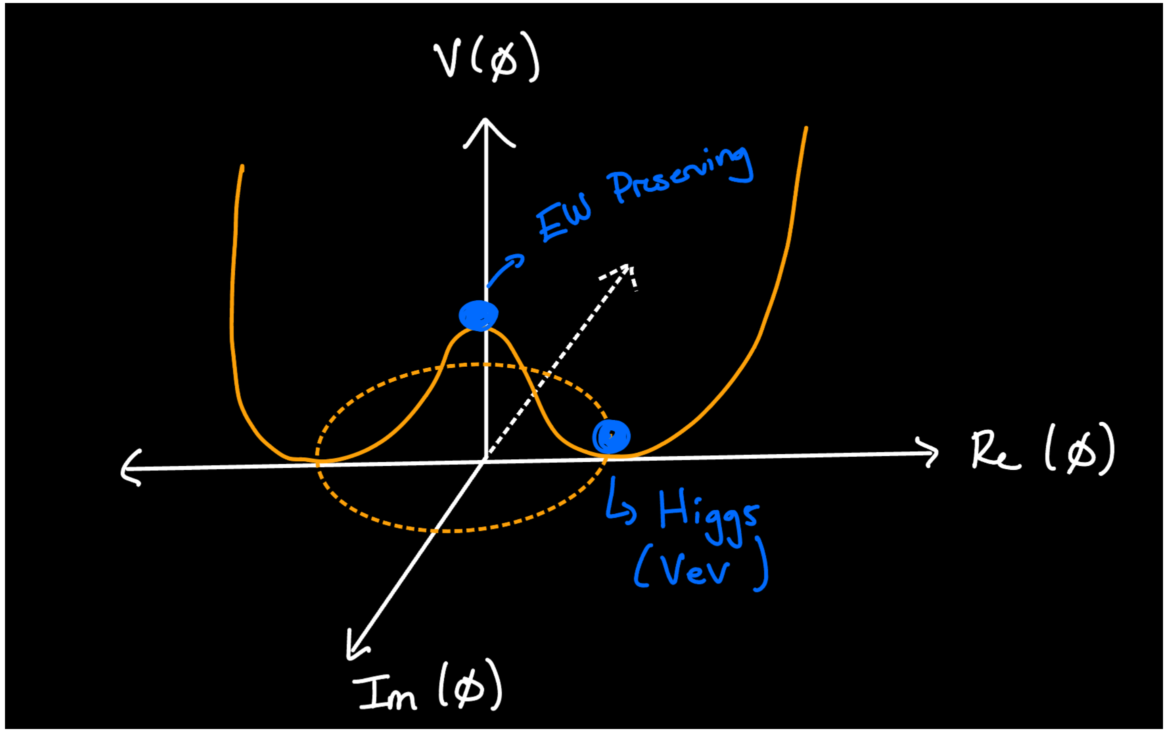


Challenges of Generic EFTs

(1) Basis Redundancy:

- Same physics different field choices.

(2) Different choices of field redefinition can make different “things” manifest



How to Fix

Field Space redefinitions keep on-shell S-matrix elements invariant. (By LSZ Formulation)

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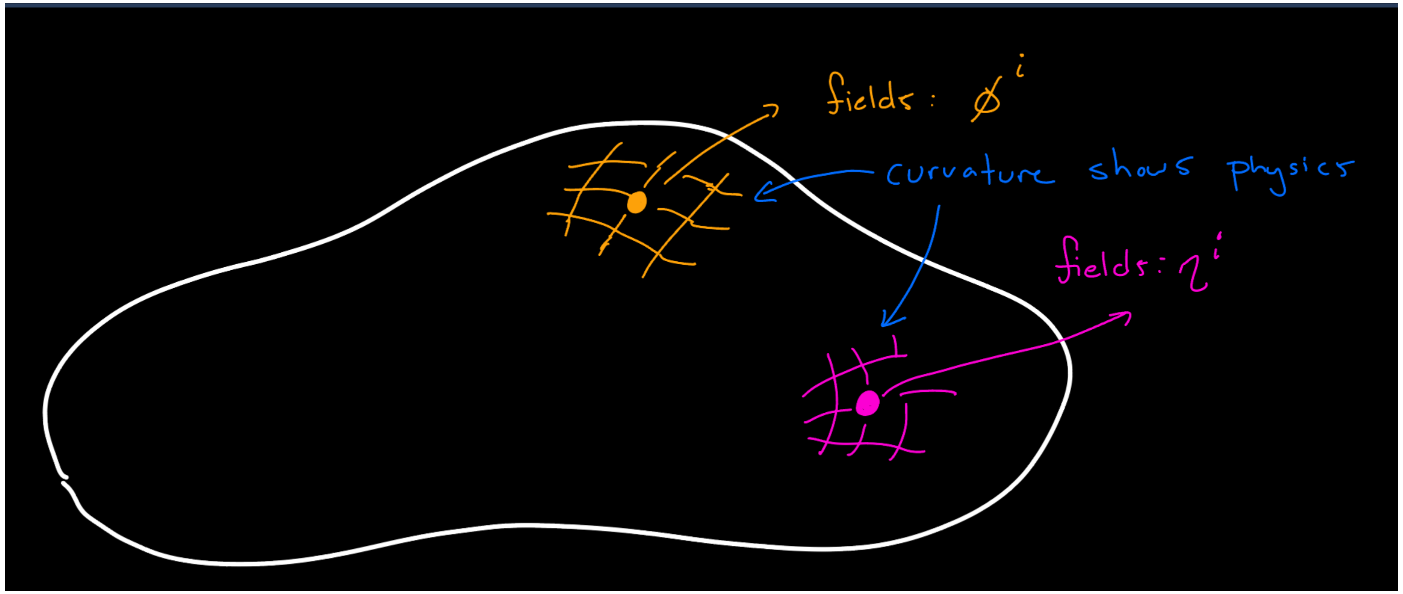
Field Space redefinitions keep on-shell S-matrix elements invariant. (By LSZ Formulation)

(1) Fields → Coordinates on curved field space manifold.

(2) The physics is encoded in the geometry.



Scalar Lagrangian in Cartesian (“Warsaw”)

Custodial Symmetry limit: $SU(2)_L \times SU(2)_R \sim O(4)$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{bmatrix} \quad \sim \quad \vec{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix}$$

Promote to Global Symmetry (Recover gauge interactions covariantly)

See e.g [R. Alonso, E. Jenkins & A. Manohar: 1605.03602]

Scalar Lagrangian in Cartesian (“Warsaw”)

$$\mathscr{L} = \partial_\mu \phi^a G_{ab} \partial^\mu \phi^b + V(\phi)$$

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where $G_{ab} = A\left(\frac{\phi \cdot \phi}{\Lambda^2}\right)\delta_{ab} + B\left(\frac{\phi \cdot \phi}{\Lambda^2}\right)\frac{\phi_a \cdot \phi_b}{\Lambda^2}$

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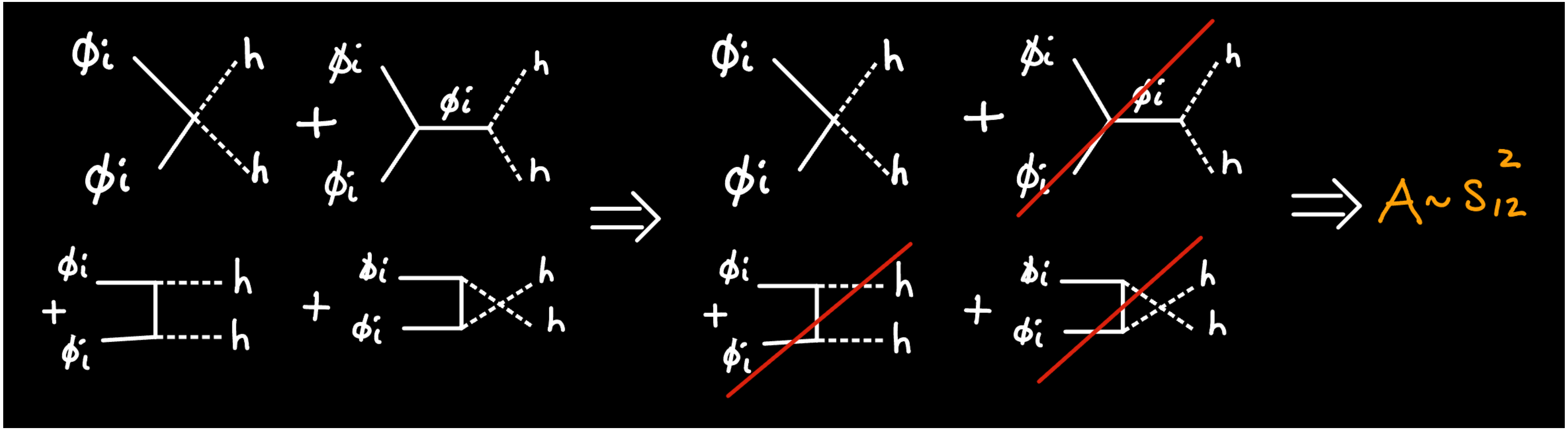
where A and B are functions that encode the NP effects:

$$A\left(\frac{\phi \cdot \phi}{\Lambda^2}\right) = 1 + a_1 \frac{\phi \cdot \phi}{\Lambda^2} + \cdots \ ; \ B\left(\frac{\phi \cdot \phi}{\Lambda^2}\right) = b_1 + b_2 \frac{\phi \cdot \phi}{\Lambda^2} + \cdots$$

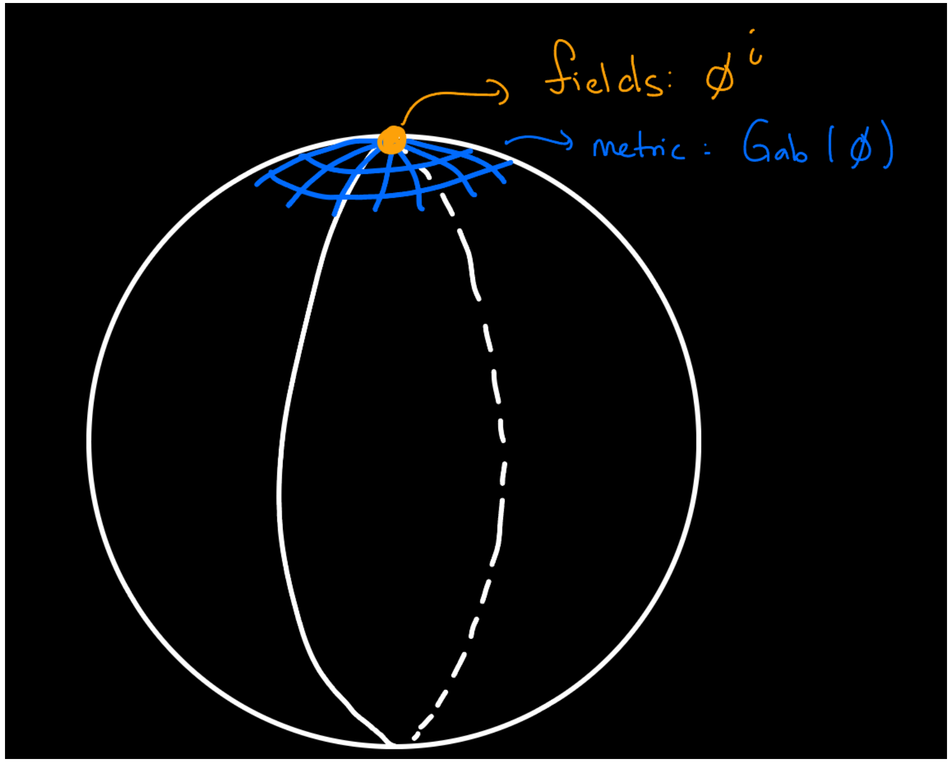
Warsaw Example: $A(W^+W^- \to hh)$



$$\mathscr{L} = \partial_\mu \phi^a G_{ab} \partial^\mu \phi^b + V(\phi)$$

Field Redefinition

$$\phi^i \to F^{ij}(\varphi)\varphi^j \quad \text{where } F^{ij}(0) = \delta^{ij}$$



[A. Manohar:hep-ph/1804.05863]
[Alvarez-Gaume, Freedman, Mukhi (1981)]
[Vilkovisky (1984), DeWitt (1985)]

Field Redefinition

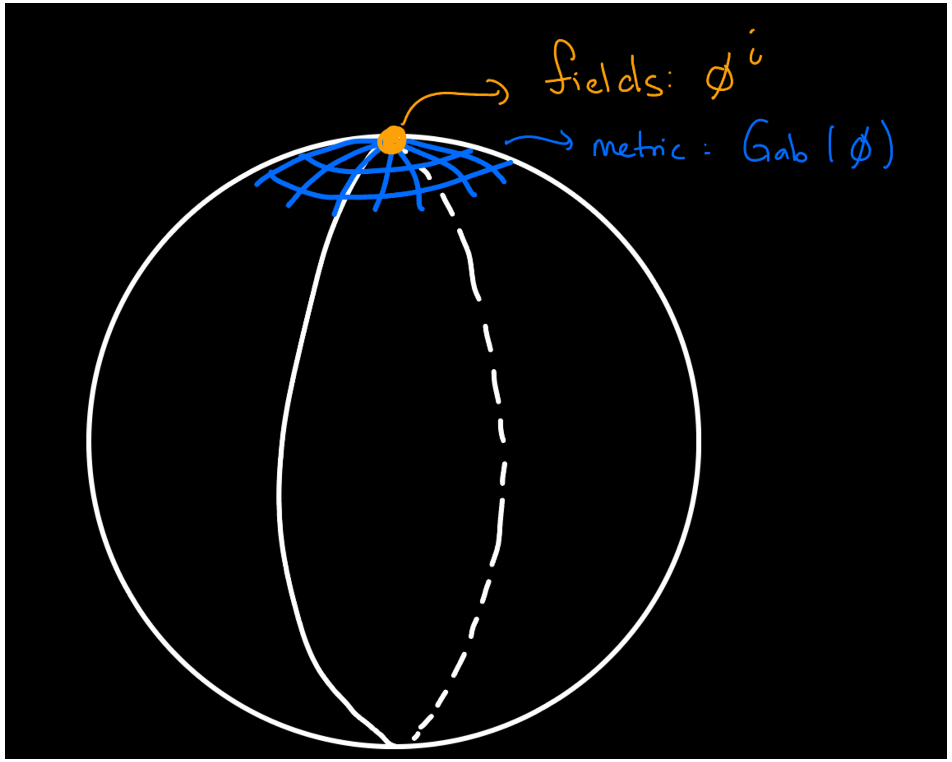
$$\phi^i \rightarrow F^{ij}(\varphi)\varphi^j \quad \text{where } F^{ij}(0) = \delta^{ij}$$

$$\mathscr{L} \ni \partial_\mu\phi^a G_{ab}(\phi)\partial^\mu\phi^b$$

$$\ni \partial_\mu\varphi^c\left(\frac{\partial\phi^a}{\partial\varphi^c}\right)G_{ab}(\phi(\varphi))\left(\frac{\partial\phi^b}{\partial\varphi^d}\right)\partial^\mu\varphi^d$$

$$\ni \partial_\mu\varphi^c\tilde{G}_{cd}\partial^\mu\varphi^d$$

Forms a metric!



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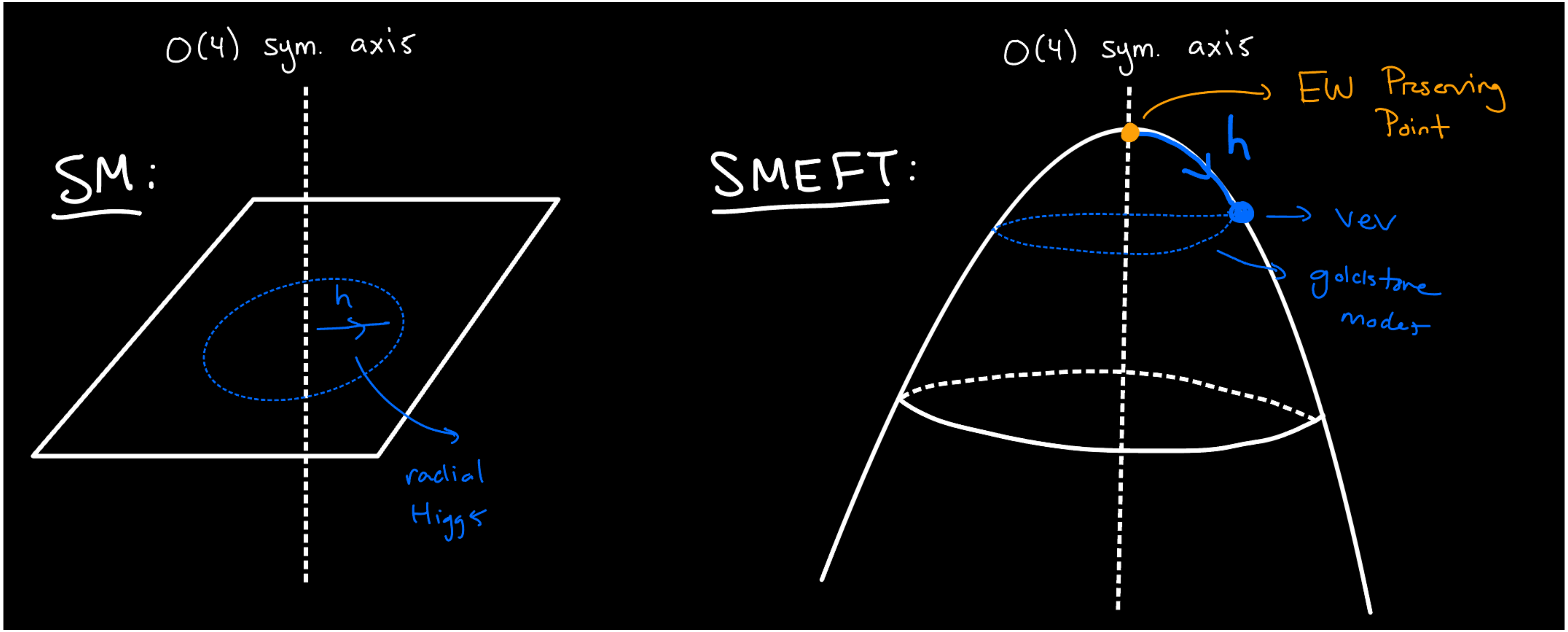
Geometry of EFTs



[T. Cohen, N. Craig, X. Lu, D. Sutherland:2108.03240]

[R. Alonso, E.Jenkins, A.Manohar:1602.00706]

Riemann Normal Coordinates (RNC)

$$\tilde{G}_{ab}(\phi) = \delta_{ab} + \frac{1}{3}\overline{R}_{aijb}\phi^i\phi^j + \frac{1}{6}\overline{\nabla_i R}_{ajkb}\phi^i\phi^j\phi^k + \frac{1}{4!}\frac{6}{5}\left(\overline{\nabla_i \nabla_j R}_{aklb} + \frac{8}{9}\overline{R}_{aijc}\overline{R}^c_{klb}\right)\phi^i\phi^j\phi^k\phi^l + \cdots$$

[A.Hatzinikitas:hep-ph/00010778]

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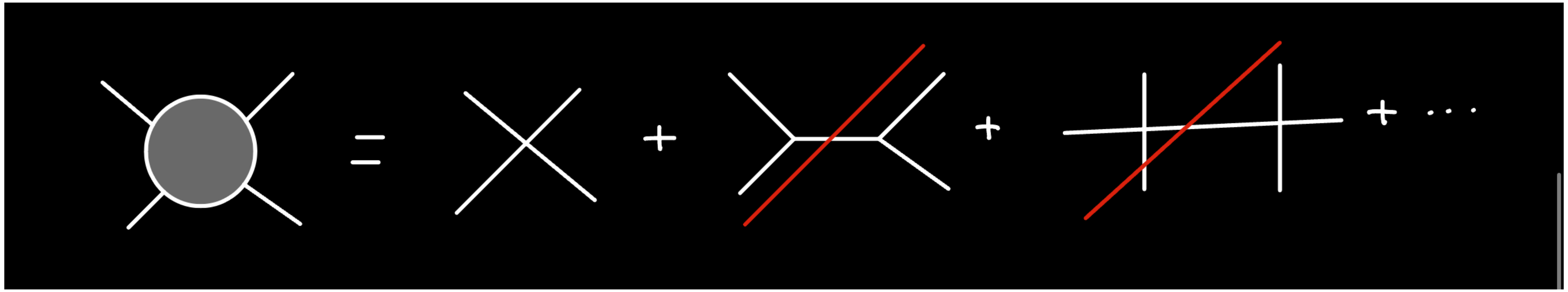
[A.Hatzinikitas:hep-ph/00010778]

This structure pulls out distinct Kinematic Poles:

$$A_4 \sim R \quad A_5 \sim \nabla R \quad A_6 \sim \nabla^2 R \quad A_n \sim \nabla^n R$$



[C. Cheung, A. Helset & J. Parra-Martinez:2111.03045]

The Symmetric Point

Use Isotropy and Riemann Tensor Properties to build Rules to simplify the RNC Metric:

$$\tilde{G}_{ab} = \delta_{ab} + \frac{c_1}{\Lambda^2}\tilde{R}_{a(ij)b}\phi^i\phi^j + \frac{c_2}{\Lambda^4}\delta_{(ij}\tilde{R}_{|a|kl)b}\phi^i\phi^j\phi^k\phi^l + \mathcal{O}\left(\frac{\phi^6}{\Lambda^6}\right)$$

Where $\tilde{R}_{aijb} = \delta_{aj}\delta_{ib} - \delta_{ab}\delta_{ij}$

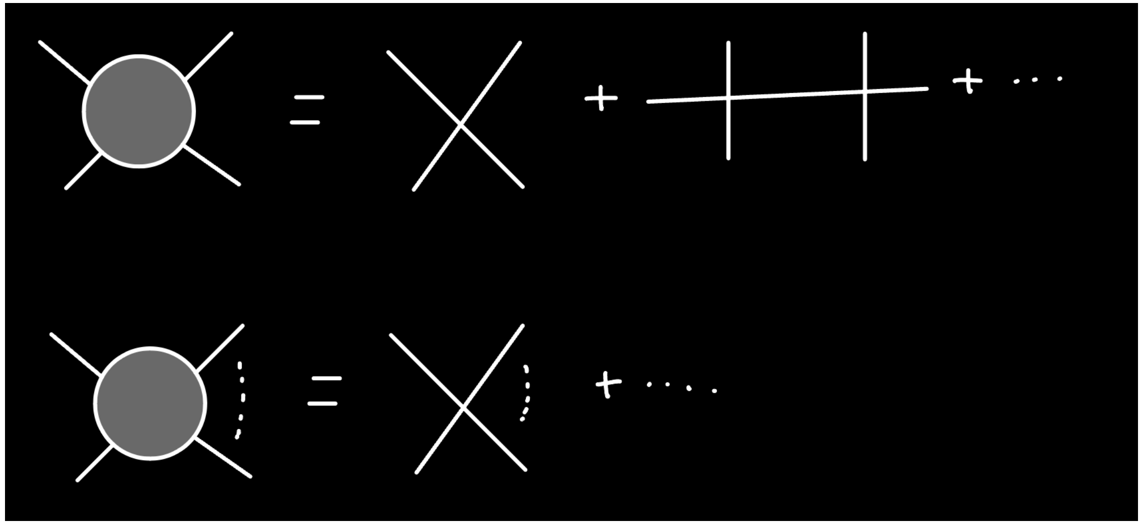
The Symmetric Point

R-Rules around the symmetric point:

(1) $\tilde{R}^n\tilde{R} = (-1)^n\delta\tilde{R}$

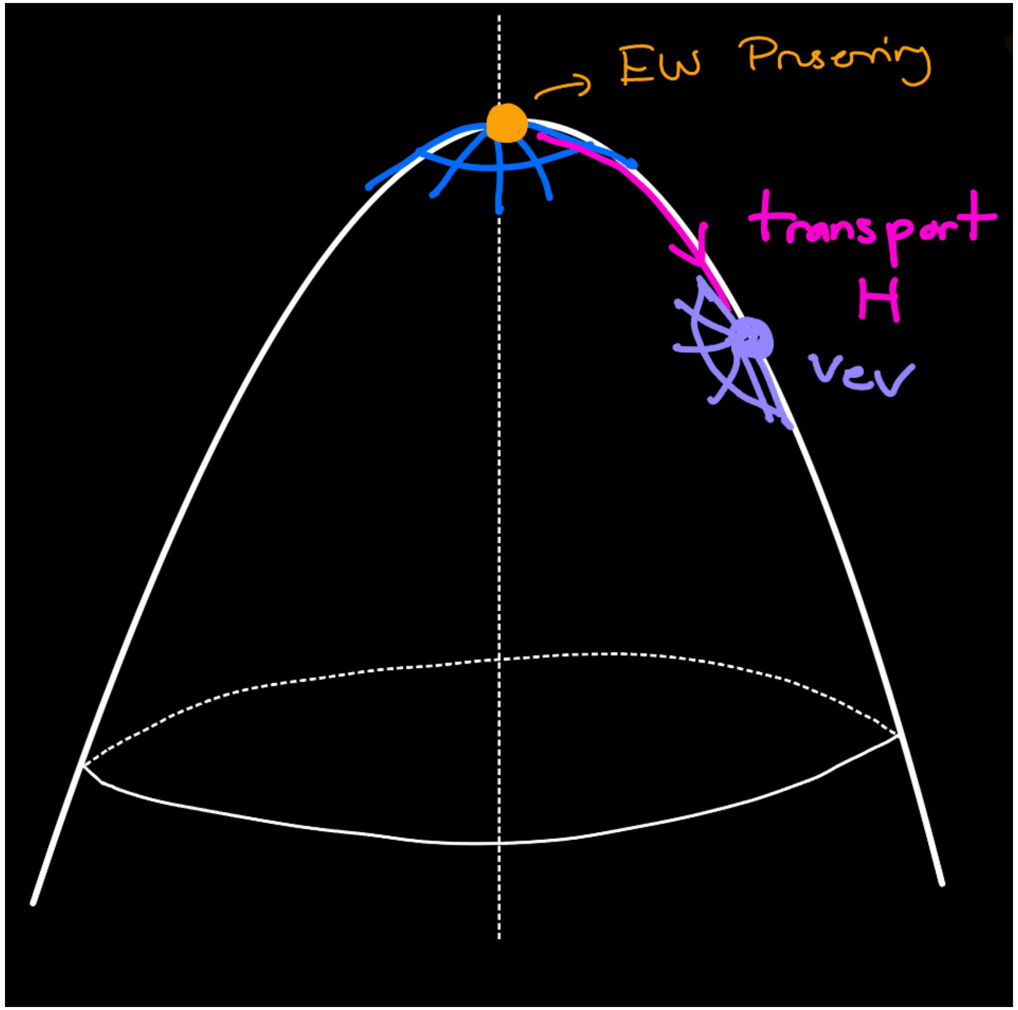
(2) $\nabla^n R = \begin{cases} \delta^{n-1}\tilde{R} & \text{for even } n \\ 0 & \text{for odd } n \end{cases}$

(3) $\tilde{R}_{aijk}\phi^i\phi^j\phi^k = 0$

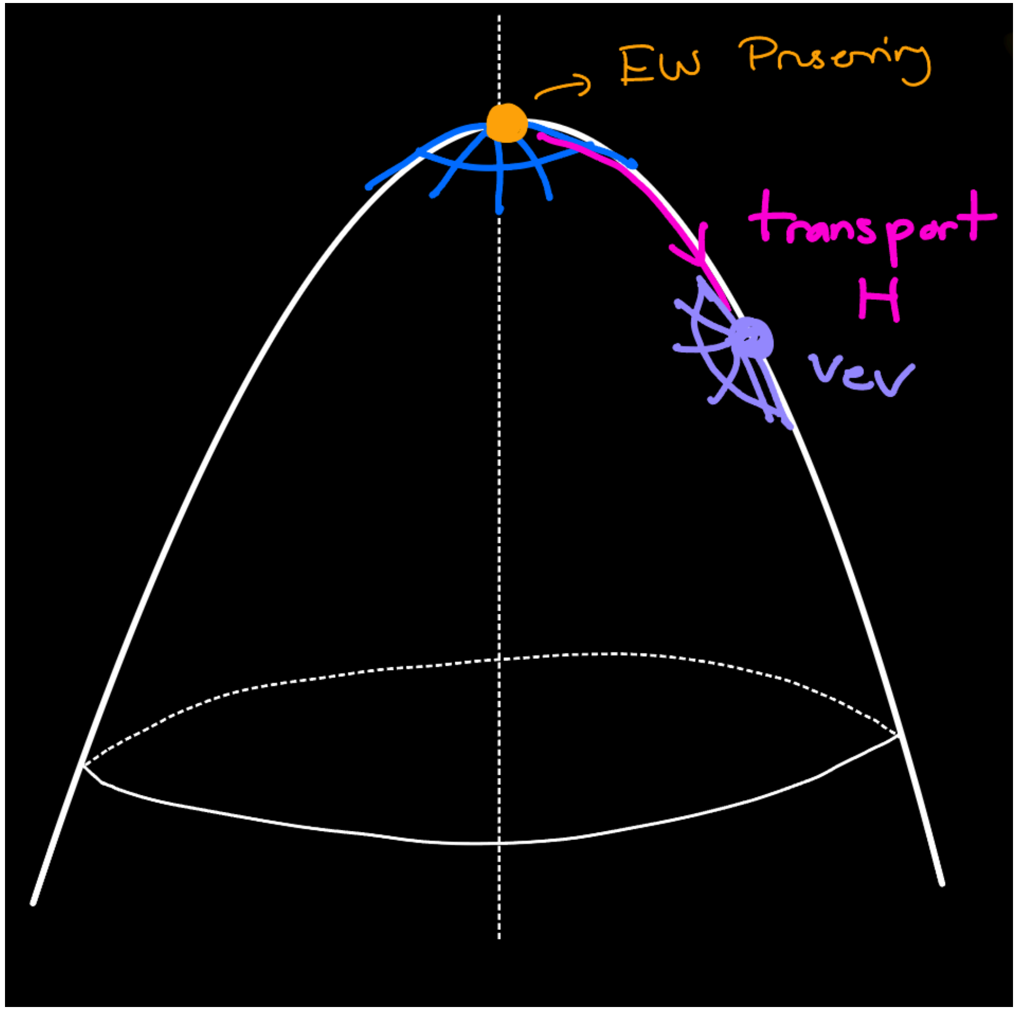


$A_4 \sim \tilde{R}$ $A_6 \sim \delta\tilde{R}$ $A_n \sim \delta^n\tilde{R}$

Expanding around the vacuum



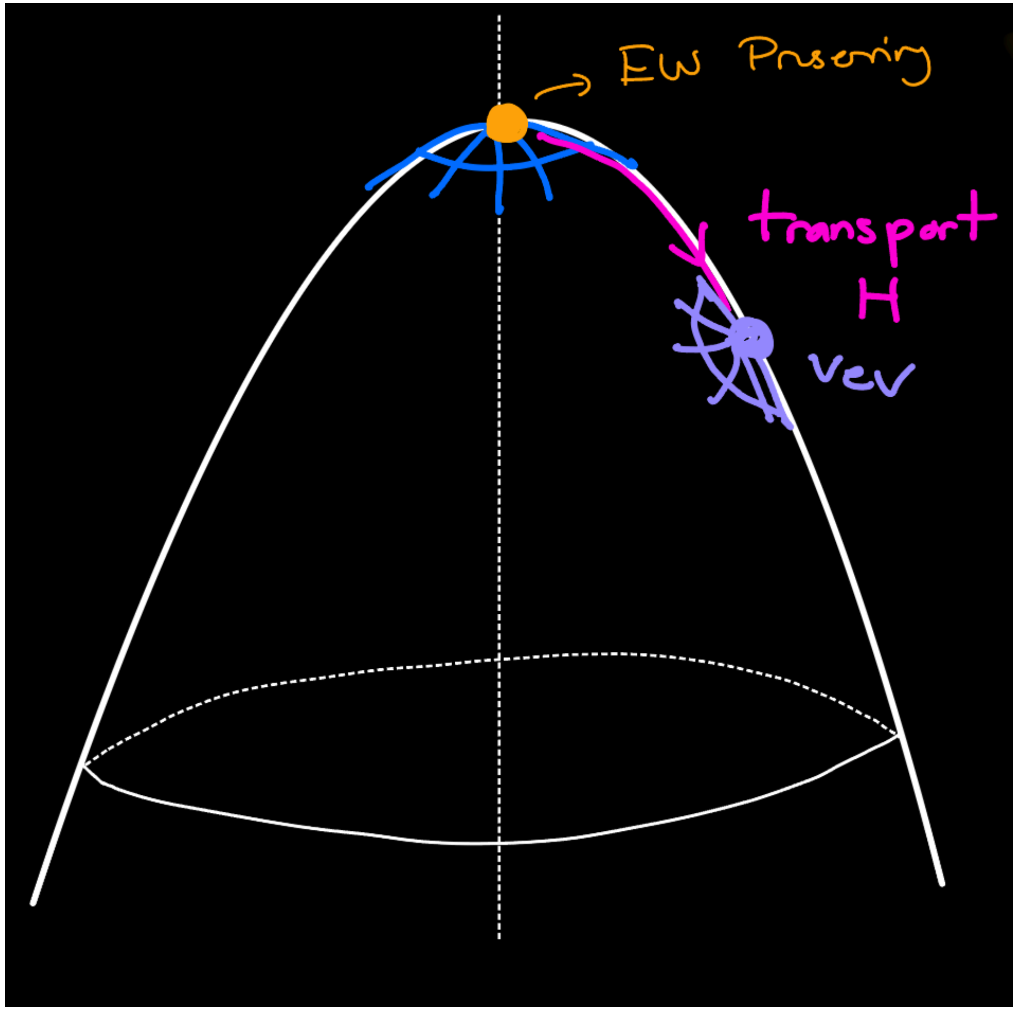
Expanding around the vacuum



Simple expression for the vertex:

$$\mathscr{M}_4 = -\frac{c_1}{\Lambda^2}\left((h\partial\eta_i - \eta_i\partial h)^2 + \eta_i^2\partial\eta_i^2 - \eta_i\partial\eta_i\eta_j\partial\eta_j\right) : i \neq j$$

Expanding around the vacuum



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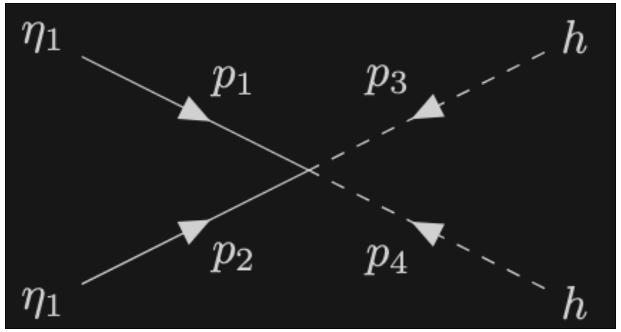


$$= -i\frac{c_1}{\Lambda^2}\left(-p_1\cdot p_2 - p_3\cdot p_4 + 2(p_1\cdot p_3 + p_1\cdot p_4 + p_2\cdot p_3 + p_2\cdot p_4)\right)$$

$$A(W^+W^- \to hh) = \frac{36c_1^2}{\Lambda^4}s_{12}^2$$

Takeaways

"SMEFT in RNC manifests observables to improve how we do physics."

- Cleaner calculations. Removing spurious poles.
- Easier to map Observables from Lagrangian
- Future collider physics to probe the nature of EWSB.

Back up Slides

SMEFT and HEFT

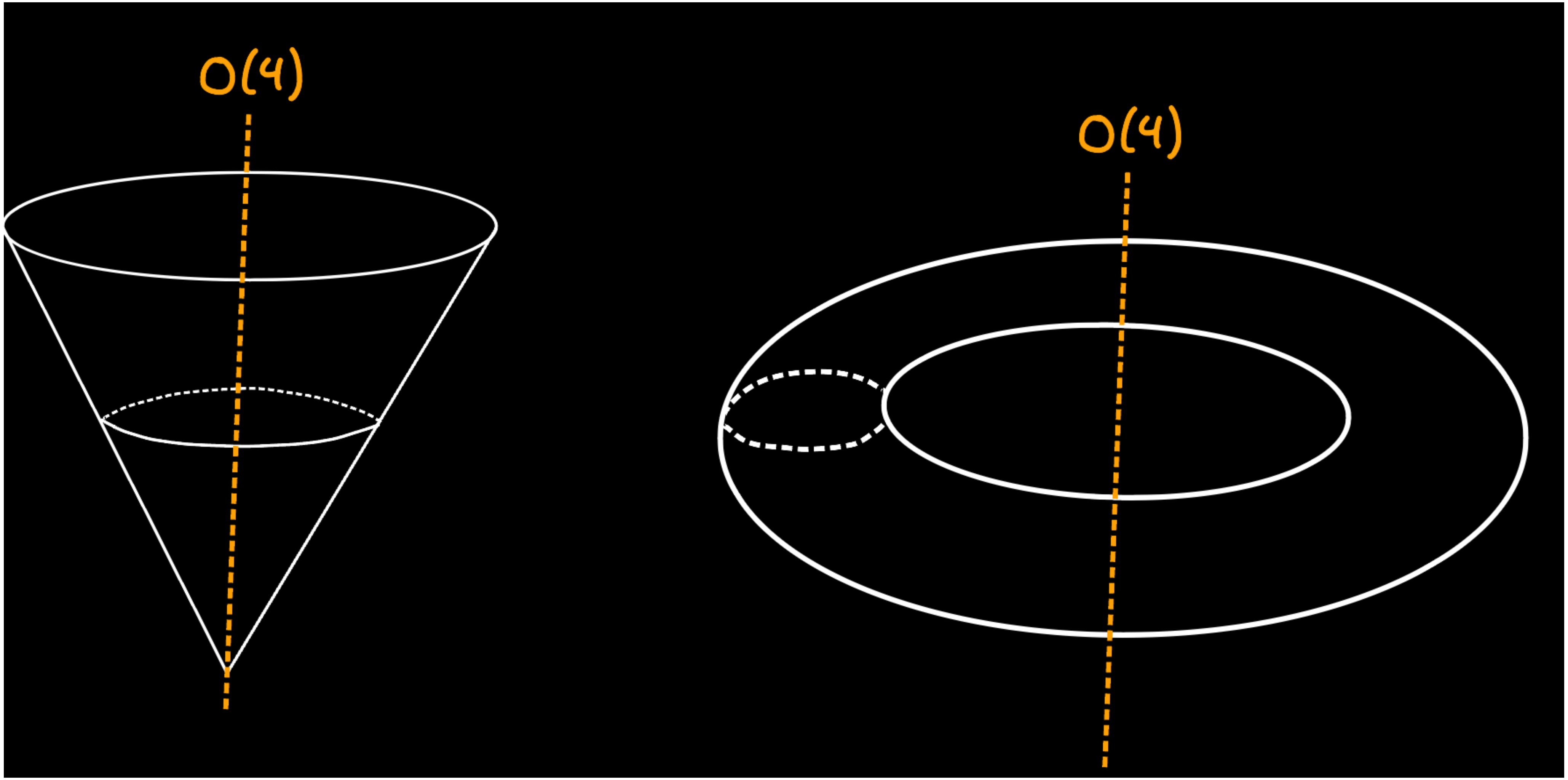
Two possibilities to use as the EFT:

(1) SMEFT: linear realization . Created around the EW preserving point.

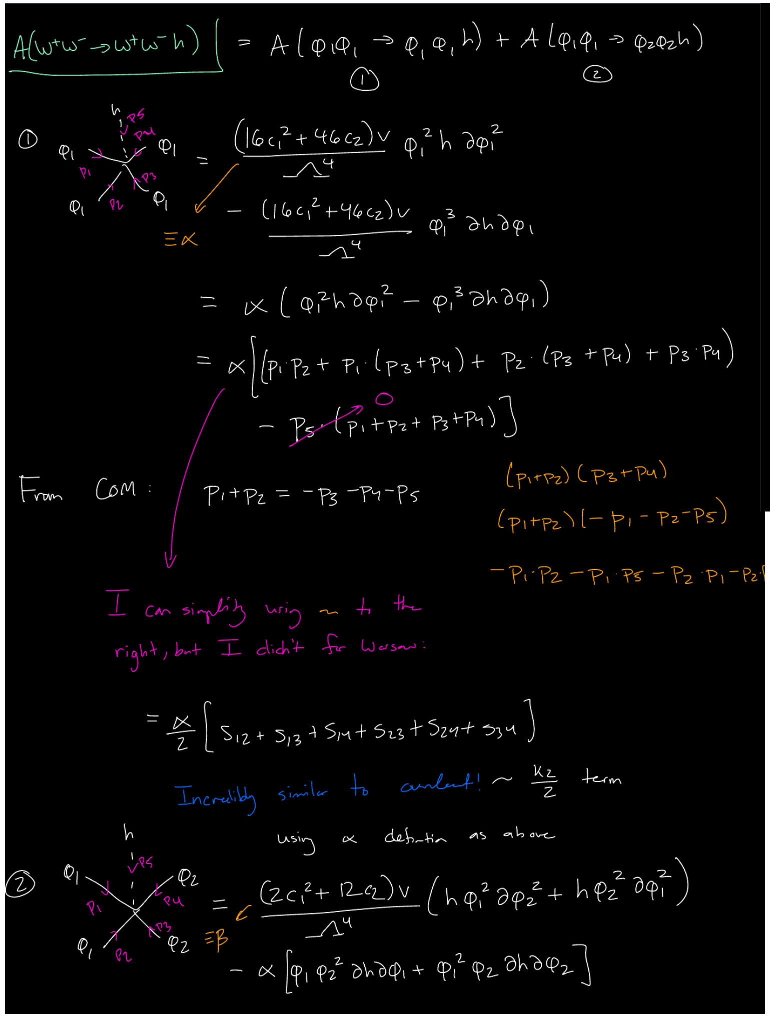
(2) HEFT: nonlinear realization. Created around the vacuum.

Ordering: $SM \subset SMEFT \subset HEFT$

EHEFT-like Manifolds



Examples of amplitudes (RNC)



$$A(w^+w^- \to w^+w^- h) = \underset{①}{A(\varphi_1\varphi_1 \to \varphi_1\varphi_1 h)} + \underset{②}{A(\varphi_1\varphi_1 \to \varphi_2\varphi_2 h)}$$

① $= \frac{(16c_1^2+46c_2)v}{\Lambda^4}\varphi_1^2 h\,\partial\varphi_1^2 - \frac{(16c_1^2+46c_2)v}{\Lambda^4}\varphi_1^3\,\partial h\,\partial\varphi_1$

$\equiv \alpha$

$$= \alpha\left(\varphi_1^2 h\,\partial\varphi_1^2 - \varphi_1^3\,\partial h\,\partial\varphi_1\right)$$

$$= \alpha\Big[\big(p_1\cdot p_2 + p_1\cdot(p_3+p_4) + p_2\cdot(p_3+p_4) + p_3\cdot p_4\big) - \cancelto{0}{p_5\cdot(p_1+p_2+p_3+p_4)}\Big]$$

From CoM: $p_1+p_2 = -p_3-p_4-p_5$

$(p_1+p_2)(p_3+p_4)$

$(p_1+p_2)(-p_1-p_2-p_5)$

$-p_1\cdot p_2 - p_1\cdot p_5 - p_2\cdot p_1 - p_2\cdot$

I can simplify using ~ to the right, but I didn't for Warsaw:

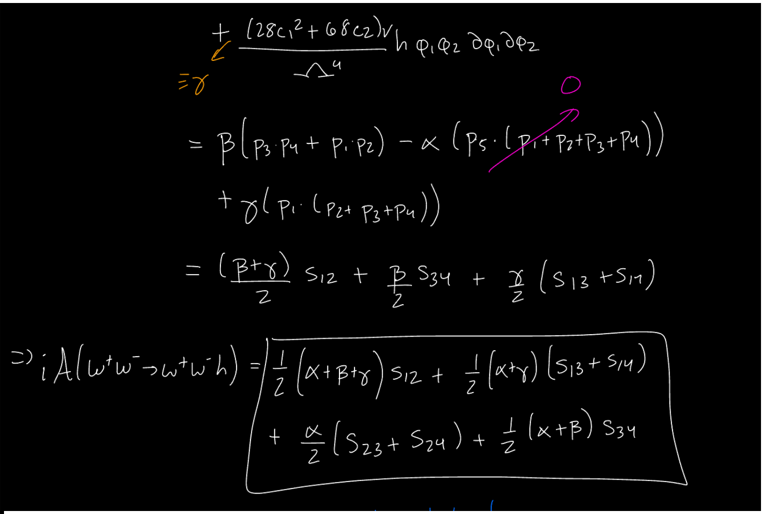
$$= \frac{\alpha}{2}\left[s_{12}+s_{13}+s_{14}+s_{23}+s_{24}+s_{34}\right]$$

Incredibly similar to current! ~ $\frac{\kappa_2}{2}$ term

using α definition as above

② $= \frac{(2c_1^2+12c_2)v}{\Lambda^4}\left(h\varphi_1^2\,\partial\varphi_2^2 + h\varphi_2^2\,\partial\varphi_1^2\right)$ $\equiv\beta$

$$-\alpha\left[\varphi_1\varphi_2^2\,\partial h\,\partial\varphi_1 + \varphi_1^2\varphi_2\,\partial h\,\partial\varphi_2\right]$$



$$+\frac{(28c_1^2+68c_2)v}{\Lambda^4}h\,\varphi_1\varphi_2\,\partial\varphi_1\partial\varphi_2$$

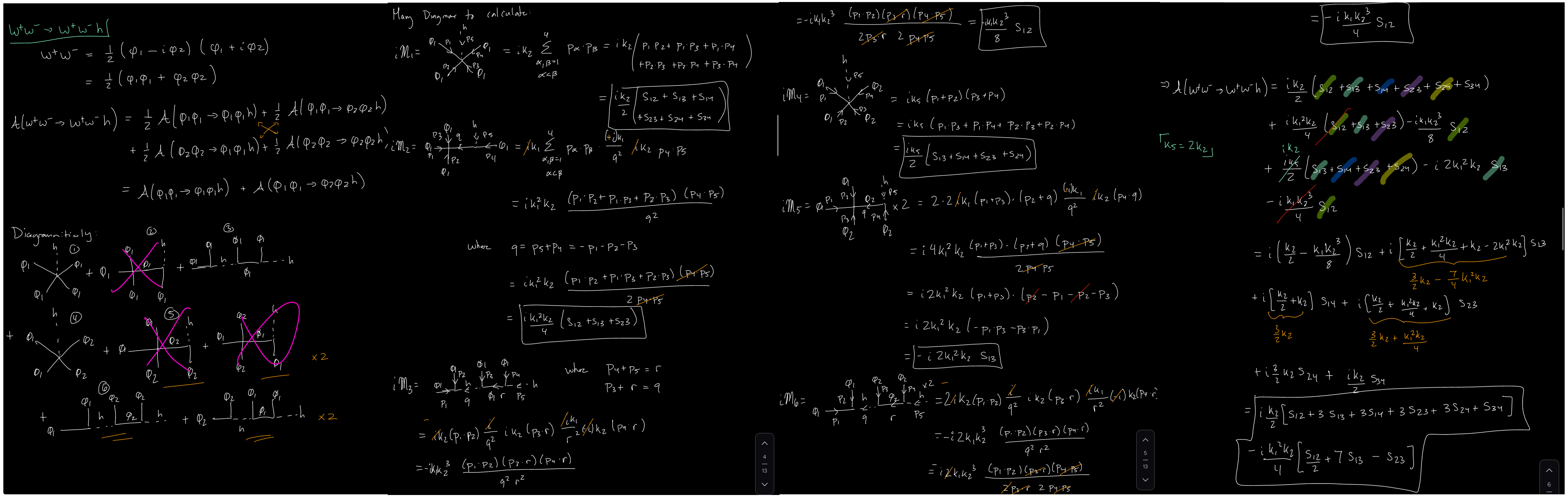
$\equiv\gamma$

$$= \beta(p_3\cdot p_4 + p_1\cdot p_2) - \alpha\left(\cancelto{0}{p_5\cdot(p_1+p_2+p_3+p_4)}\right) + \gamma\left(p_1\cdot(p_2+p_3+p_4)\right)$$

$$= \frac{(\beta+\gamma)}{2}s_{12} + \frac{\beta}{2}s_{34} + \frac{\gamma}{2}(s_{13}+s_{14})$$

$$\Rightarrow iA(w^+w^- \to w^+w^- h) = \boxed{\frac{1}{2}(\alpha+\beta+\gamma)s_{12} + \frac{1}{2}(\alpha+\gamma)(s_{13}+s_{14}) + \frac{\alpha}{2}(s_{23}+s_{24}) + \frac{1}{2}(\alpha+\beta)s_{34}}$$

Examples of amplitudes (Warsaw)



$W^+W^- \to W^+W^- h$

$$W^+W^- = \frac{1}{2}(\varphi_1 - i\varphi_2)(\varphi_1 + i\varphi_2) = \frac{1}{2}(\varphi_1\varphi_1 + \varphi_2\varphi_2)$$

$$A(W^+W^- \to W^+W^- h) = \frac{1}{2}A(\varphi_1\varphi_1 \to \varphi_1\varphi_1 h) + \frac{1}{2}A(\varphi_1\varphi_1 \to \varphi_2\varphi_2 h) + \frac{1}{2}A(\varphi_2\varphi_2 \to \varphi_1\varphi_1 h) + \frac{1}{2}A(\varphi_2\varphi_2 \to \varphi_2\varphi_2 h)$$

$$= A(\varphi_1\varphi_1 \to \varphi_1\varphi_1 h) + A(\varphi_1\varphi_1 \to \varphi_2\varphi_2 h)$$

Diagrammatically:

Many Diagrams to calculate:

$$iM_1 = ik_2 \sum_{\substack{\alpha,\beta=1\\ \alpha<\beta}}^{4} p_\alpha\cdot p_\beta = ik_2\left(p_1\cdot p_2 + p_1\cdot p_3 + p_1\cdot p_4 + p_2\cdot p_3 + p_2\cdot p_4 + p_3\cdot p_4\right)$$

$$= \boxed{\frac{ik_2}{2}\left(s_{12} + s_{13} + s_{14} + s_{23} + s_{24} + s_{34}\right)}$$

$$iM_2 = \cancel{i}k_1 \sum_{\substack{\alpha,\beta=1\\ \alpha<\beta}}^{4} p_\alpha\cdot p_\beta \, \frac{(+i)k_1}{q^2}\, \cancel{i}k_2\, p_4\cdot p_5$$

$$= ik_1^2k_2 \frac{(p_1\cdot p_2 + p_1\cdot p_3 + p_2\cdot p_3)(p_4\cdot p_5)}{q^2}$$

where $q = p_5 + p_4 = -p_1 - p_2 - p_3$

$$= ik_1^2k_2 \frac{(p_1\cdot p_2 + p_1\cdot p_3 + p_2\cdot p_3)\cancel{(p_4\cdot p_5)}}{2\cancel{p_4\cdot p_5}}$$

$$= \boxed{\frac{ik_1^2k_2}{4}\left(s_{12} + s_{13} + s_{23}\right)}$$

$iM_3 =$ where $p_4 + p_5 = r$, $p_3 + r = q$

$$= \cancel{i}k_2(p_1\cdot p_2)\frac{\cancel{i}}{q^2}\, ik_2(p_3\cdot r)\frac{\cancel{i}k_1}{r^2}(\cancel{i})k_2(p_4\cdot r)$$

$$= -ik_1k_2^3 \frac{(p_1\cdot p_2)(p_3\cdot r)(p_4\cdot r)}{q^2 r^2}$$

$$= -ik_1k_2^3 \frac{(p_1\cdot p_2)\cancel{(p_3\cdot r)}(p_4\cdot \cancel{p_5})}{2\cancel{p_3\cdot r}\; 2\cancel{p_4\cdot p_5}} = \boxed{\frac{-ik_1k_2^3}{8} s_{12}}$$

$$iM_4 = ik_5(p_1 + p_2)(p_3 + p_4)$$

$$= ik_5(p_1\cdot p_3 + p_1\cdot p_4 + p_2\cdot p_3 + p_2\cdot p_4)$$

$$= \boxed{\frac{ik_5}{2}\left(s_{13} + s_{14} + s_{23} + s_{24}\right)}$$

$$iM_5 = \times 2 = 2\cdot 2\cancel{i}k_1(p_1 + p_3)\cdot(p_2 + q)\frac{(-i)k_1}{q^2}\cancel{i}k_2(p_4\cdot q)$$

$$= i4k_1^2k_2 \frac{(p_1 + p_3)\cdot(p_2 + q)\cancel{(p_4\cdot p_5)}}{2\cancel{p_4}\cdot p_5}$$

$$= i2k_1^2k_2(p_1 + p_3)\cdot(\cancel{p_2} - p_1 - \cancel{p_2} - p_3)$$

$$= i2k_1^2k_2(-p_1\cdot p_3 - p_3\cdot p_1)$$

$$= \boxed{-i2k_1^2k_2\, s_{13}}$$

$$iM_6 = \times 2 = 2\cancel{i}k_2(p_1\cdot p_2)\frac{\cancel{i}}{q^2}\, ik_2(p_3\cdot r)\frac{\cancel{i}k_1}{r^2}(\cancel{-i})k_2(p_4\cdot r)$$

$$= -i2k_1k_2^3 \frac{(p_1\cdot p_2)(p_3\cdot r)(p_4\cdot r)}{q^2 r^2}$$

$$= -i\cancel{2}k_1k_2^3 \frac{(p_1\cdot p_2)\cancel{(p_3\cdot r)}(p_4\cdot \cancel{p_5})}{\cancel{2}\cancel{p_3\cdot r}\; 2p_4\cdot \cancel{p_5}}$$

$$= \boxed{-\frac{ik_1k_2^3}{4} s_{12}}$$

$$\Rightarrow A(W^+W^- \to W^+W^- h) = \frac{ik_2}{2}\left(s_{12} + s_{13} + s_{14} + s_{23} + s_{24} + s_{34}\right) + \frac{ik_1^2k_2}{4}\left(s_{12} + s_{13} + s_{23}\right) - \frac{ik_1k_2^3}{8}s_{12} + \overset{ik_2}{\cancel{\frac{ik_5}{2}}}\left(s_{13} + s_{14} + s_{23} + s_{24}\right) - i2k_1^2k_2\, s_{13} - \frac{ik_1k_2^3}{4}s_{12}$$

$[k_5 = 2k_2]$

$$= i\left(\frac{k_2}{2} - \frac{k_1k_2^3}{8}\right)s_{12} + i\underbrace{\left[\frac{k_2}{2} + \frac{k_1^2k_2}{4} + k_2 - 2k_1^2k_2\right]}_{\frac{3}{2}k_2 - \frac{7}{4}k_1^2k_2} s_{13} + i\underbrace{\left[\frac{k_2}{2} + k_2\right]}_{\frac{3}{2}k_2} s_{14} + i\underbrace{\left(\frac{k_2}{2} + \frac{k_1^2k_2}{4} + k_2\right)}_{\frac{3}{2}k_2 + \frac{k_1^2k_2}{4}} s_{23} + i\frac{3}{2}k_2 s_{24} + \frac{ik_2}{2}s_{34}$$

$$= \boxed{i\frac{k_2}{2}\left[s_{12} + 3s_{13} + 3s_{14} + 3s_{23} + 3s_{24} + s_{34}\right] - \frac{ik_1^2k_2}{4}\left[\frac{s_{12}}{2} + 7s_{13} - s_{23}\right]}$$

Extra Geometric-EFTs

Supersymmetric Models:

[L. Alvarez-Gaume, D.Z. Freedman & S. Mukhi; Amals.Phys.148(1981)85]

[E. Brazen, T. Cutright, C. Zachos; Nucl.Phys.B 260(1985)630-688]

HEFT:

[R. Alonso, E. Jenkins & A. Manohar:1605.03602]

Geo-SMEFT:

[A. Helset, A. Martin, M. Trott: 2001.01453]

Extra Geometric-EFTs

Derivative Field Redefintions:

Jet bundle Geometry:

[M. Alminawi, I. Brivio & J. Dauighi;2308.00017]

Geometry Kinematics Duality:

[C. Cheung, A. Helset, and J. Parra Martinez:2202.06972]

Functional Geometry:

[T. Cohen, X. Lu & Z. Zhang:2410.21378]

Non-local field redefinitions:

[T. Cohen, M. Forslund, A. Helset:2412.12247]

Extra Geometric-EFTs

Fermions:

Quantum effective action including fermions:

[V. Altus, A. Pilaftsis:2406.13594]

[V. Altus, A. Pilaftsis:2307.01126]

Fermi Normal Coordinates:

[N. Craig, I-K. Lee, Y.T. Lee: 2509.07101]

Two loop Renormalisation of two-fermi SMEFT operators:

[B. Assi, A. Helset, J. Pages, C.H. Shen:2504.18537]