Understanding Misaligned Condensates with Yukawa Couplings

Based on

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Outline

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4. Asymptotic Stationary States & Other Implications
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1. Condensates and the Effective Potential

Condensates of Quantum Fields

- **Coherent Field States:** Special quantum mechanical state that behaves as "classically" as possible. In bosonic systems, it is an eigenstate of the annihilation operator.
- **Field Condensates:** Non-zero expectation value of a quantum field operator. Indicates the system is "filled" with macroscopic occupation of some mode.
- **Mean Field:** An approximation where a quantum field operator is decomposed into a classical (c-number) field plus a fluctuation (field operator)

- Condensates are ubiquitous in modern particle physics (e.g. Inflation, Axions/ALPs, Higgs)

- How do we consistently describe the time-evolution of the classical object together with its quantum fluctuations?

The Effective Potential (Zero-Temp)

- The effective potential origins: how do radiative corrections modify spontaneous symmetry breaking?
- Defined as the generating functional of single particle irreducible Green's functions at zero momentum transfer.
- Useful in understanding phase transitions in quantum field theories.
- While originally computed using Feynman diagrams or functional methods, Symanzik (1970) gave a more expedient and intuitive Hamiltonian derivation (for zero temperature):

$$V_{eff}(\varphi) = \frac{1}{\mathcal{V}} \langle \Phi | H | \Phi \rangle$$

Volume

Coherent State

The Model: Scalar Condensate with Yukawa Couplings

- Consider a real scalar coupled to N_f identical massless fermions in Minkowski spacetime:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \sum_{i=1}^{N_f}\overline{\psi}_i\Big(i\partial\!\!\!/ - \frac{y_0}{\sqrt{N_f}}\,\phi\Big)\psi_i \qquad V(\hat{\phi}) = V_0 + \frac{1}{2}\,m_0^2\hat{\phi}^2 + \frac{\lambda_0}{4N_f}\,\hat{\phi}^4$$

- Which has the associated Hamiltonian:

$$H = \int d^3x\left\{\frac{\hat{\pi}^2}{2} + \frac{(\nabla\hat{\phi})^2}{2} + V(\hat{\phi}) + \sum_{i=1}^{N_f}\psi_i^\dagger\Big(-i\vec{\alpha}\cdot\vec{\nabla} + \gamma^0\,\frac{y_0}{\sqrt{N_f}}\,\hat{\phi}\Big)\psi_i\right\}$$

The Dynamical Case

- Increasingly, phenomenologists have used the effective potential to describe the time evolution of the expectation values of homogeneous fields.
- The idea is to use the equation of motion:

$$\ddot{\varphi}(t) + \frac{d}{d\varphi} V_{eff}(\varphi(t)) = 0$$

- But is this ultimately justified?
- Consider the Hamiltonian but with conditions: $\langle \Phi; 0_F | \hat{\phi}(\vec{x}, t) | \Phi; 0_F \rangle = \sqrt{N_f}\, \varphi(t) \;\; ; \;\; \langle \Phi; 0_F | \hat{\pi}(\vec{x}, t) | \Phi; 0_F \rangle = \sqrt{N_f}\, \dot{\varphi}(t)$

$$\hat{\phi}(\vec{x}, t) \equiv \sqrt{N_f}\, \underline{\varphi(t)} + \underline{\hat{\delta}(\vec{x}, t)} \quad ; \quad \hat{\pi}(\vec{x}, t) = \sqrt{N_f}\, \dot{\varphi}(t) + \hat{\pi}_\delta(\vec{x}, t)$$

Time-varying mean field

Quantum Fluctuation

The Dynamical Case

- The fluctuation is subject to the constraints: $\langle \Phi; 0_F | \hat{\delta}(\vec{x}, t) | \Phi; 0_F \rangle = 0 \;\; ; \;\; \langle \Phi; 0_F | \hat{\pi}_\delta(\vec{x}, t) | \Phi; 0_F \rangle = 0$
- Obtain Heisenberg equations of motion up to order $\hat{\delta}^3$:

$$\sqrt{N_f}\Big[\ddot{\varphi}(t) + v'(\varphi(t)) + y_0\, \overline{\psi}\psi\Big] + \ddot{\hat{\delta}}(\vec{x}, t) - \vec{\nabla}^2\, \hat{\delta}(\vec{x}, t) + \mathcal{M}^2(\varphi(t))\, \hat{\delta}(\vec{x}, t) + \frac{v'''(\varphi(t))}{2\sqrt{N_f}}\, \hat{\delta}^2(\vec{x}, t) + \cdots = 0$$

$$i\frac{\partial}{\partial t}\psi_i(\vec{x}, t) = \Big(- i\vec{\alpha}\cdot\vec{\nabla} + \gamma^0\, m_F(\varphi(t)) \Big)\psi_i(\vec{x}, t) + \cdots \qquad \overline{\psi}\psi \equiv \frac{1}{N_f}\sum_{i=1}^{N_f} \overline{\psi}_i(\vec{x}, t)\psi_i(\vec{x}, t)$$

- Spatial homogeneity → Express $\hat{\delta}$ and $\hat{\psi}$ in terms of Fourier modes.

The Dynamical Case

- Inserting into the Hamiltonian one obtains the energy density:

$$\mathcal{E} = \frac{1}{\mathcal{V}} \langle \Phi; 0_F | H | \Phi; 0_F \rangle = N_f \left\{ \frac{\dot{\varphi}^2}{2} + v(\varphi) + \frac{1}{\mathcal{V}} \sum_{\vec{k},s} V^\dagger_{-\vec{k},s}(t) \left[\vec{\alpha}\cdot\vec{k} + \gamma^0 m_F(\varphi(t)) \right] V_{-\vec{k},s}(t) + \frac{1}{2N_f\,\mathcal{V}} \sum_{\vec{k}} \left[|\dot{g}_k(t)|^2 + \omega_k^2(t) |g_k(t)|^2 \right] \right\}$$

- Kinetic
- Classical Potential
- Scalar Fluctuation
- Fermions

Recall: In Minkowski Spacetime manifest energy conservation. $\dot{\mathcal{E}} = 0$

We can deduce the concomitant equations of motion from this condition:

$$\dot{\mathcal{E}} = \left(\sqrt{N_f}\, \dot{\varphi} \right) \sqrt{N_f} \left[\ddot{\varphi}(t) + v'(\varphi(t)) + y_0 \frac{1}{\mathcal{V}} \sum_{\vec{k},s} V^\dagger_{-\vec{k},s}(t) \gamma^0 V_{-\vec{k},s}(t) + \frac{v'''(\varphi(t))}{2\,N_f} \frac{1}{\mathcal{V}} \sum_{\vec{k}} |g_k(t)|^2 \right]$$

- Large N limit can suppress scalar fluctuation $(N_f \to \infty)$

The Dynamical Case: Large N Limit

- Inserting into the Hamiltonian one obtains the energy density:

$$\mathcal{E} = \frac{1}{\mathcal{V}}\langle\Phi;0_F|H|\Phi;0_F\rangle = N_f\left\{\frac{\dot{\varphi}^2}{2} + v(\varphi) + \frac{1}{\mathcal{V}}\sum_{\vec{k},s} V^{\dagger}_{-\vec{k},s}(t)\left[\vec{\alpha}\cdot\vec{k} + \gamma^0 m_F(\varphi(t))\right]V_{-\vec{k},s}(t) + \frac{1}{2N_f\,\mathcal{V}}\sum_{\vec{k}}\left[|\dot{g}_k(t)|^2 + \omega_k^2(t)|g_k(t)|^2\right]\right\}$$

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- Large N limit can suppress scalar fluctuation $(N_f \to \infty)$

2. Fermion Effects: Dynamical Dressing & Particle Production

$$\ddot{\varphi}(t) + v'(\varphi(t)) + y_0 \sum_s \int V^\dagger_{-\vec{k},s}(t)\gamma^0 V_{-\vec{k},s}(t) \, \frac{d^3k}{(2\pi)^3} = 0$$

$$\left[i\,\gamma^0\,\partial_t - \vec{\gamma}\cdot\vec{k} - m_F(t) \right] U_{\vec{k},s}(t) = 0$$

$$\left[i\,\gamma^0\,\partial_t - \vec{\gamma}\cdot\vec{k} - m_F(t) \right] V_{-\vec{k},s}(t) = 0$$

Quasi-Static/Adiabatic Approximation of Dirac Eqn.

- Express spinors in terms of mode functions and time-independent spinors

$$U_{\vec{k},s}(t) = \left[i\,\gamma^0\,\partial_t - \vec{\gamma}\cdot\vec{k} + m_F(t)\right] f_k(t)\,u_s$$
$$V_{-\vec{k},s}(t) = \left[i\,\gamma^0\,\partial_t - \vec{\gamma}\cdot\vec{k} + m_F(t)\right] h_k(t)\,v_s$$

→

$$\left[\frac{d^2}{dt^2} + E_k^2(t) - i\,\dot{m}_F(t)\right] f_k(t) = 0$$
$$\left[\frac{d^2}{dt^2} + E_k^2(t) + i\,\dot{m}_F(t)\right] h_k(t) = 0$$

$$E_k(t) = \sqrt{k^2 + m_F^2(t)}$$

- WKB Ansatz: $f_k(t) = e^{-i\int_0^t \Omega_k(t')dt'}$ $\quad \Omega_k(t) = \Omega_{Rk}(t) + i\,\Omega_{Ik}(t)$

- Adiabatic Expansion:

$$\Omega_{Rk}(t) = E_k(t) - \frac{1}{4}\left(\frac{\ddot{E}_k(t)}{E_k^2(t)} - \frac{3}{2}\,\frac{\dot{E}_k^2(t)}{E_k^2(t)}\right) + \frac{1}{2}\left(\frac{\dot{m}_F^2(t)}{4E_k^3(t)} + \frac{\dot{m}_F^2(t)\,m_F(t)}{E_k^4(t)}\right) - \frac{\ddot{m}_F(t)}{4E_k^2(t)}\cdots$$

$$\Omega_{Ik}(t) = -\frac{\dot{m}_F(t)}{2E_k(t)}\left[1 + \frac{m_F(t)}{E_k(t)}\right] + \cdots$$

Quasi-Static/Adiabatic Approximation of Dirac Eqn.

- Anticipating particle production, we change basis to the *zeroth order adiabatic spinors* $(\mathcal{U}, \mathcal{V})$:

$$
\begin{aligned}
U_{\vec{k},s}(t) &= A_{k,s}(t)\,\mathcal{U}_{\vec{k},s}(t) + B_{k,s}(t)\,\mathcal{V}_{-\vec{k},s}(t) \\
V_{-\vec{k},s}(t) &= C_{k,s}(t)\,\mathcal{V}_{-\vec{k},s}(t) + D_{k,s}(t)\,\mathcal{U}_{\vec{k},s}(t)
\end{aligned}
$$

- The price we pay for using this basis are time-dependent Bogoliubov coefficients.

Energy Conservation and Equation of Motion

- The time-dependent fermion particle number: $n_k(t) = \overline{n}_k(t) = |B_{k,s}(t)|^2 = \dfrac{N_k^2\, k^2 \left| i\dot{f}_k(t) - E_k(t) f_k(t) \right|^2}{\left[2E_k(t)\Big(E_k(t) + m_F(t)\Big)\right]}$

- Thus the condensate's energy density takes the following form:

$$\mathcal{E} = N_f \left\{ \frac{\dot{\varphi}^2}{2} + \underbrace{v(\varphi) - 2\int E_k(t) \frac{d^3k}{(2\pi)^3}}_{v_{eff}(\varphi)} + 2\int E_k(t) \underbrace{\Big[n_k(t) + \overline{n}_k(t) \Big]}_{2\,|B_{k,s}(t)|^2} \frac{d^3k}{(2\pi)^3} \right\}$$

- Energy conservation entails the corresponding equations of motion $\dot{\mathcal{E}} = 0$

$$\ddot{\varphi} + \frac{d}{d\varphi} v_{eff}(\varphi) + 4\, y_0 \int k^2 \left[N_k^2 \Big| f_k(t) \Big|^2 - \frac{1}{2E_k(t)\Big(E_k(t) + m_F(t)\Big)} \right] \frac{d^3k}{(2\pi)^3} = 0$$

Particle and Entropy Production

- By constructing the system's density matrix in the adiabatic basis, evolving in time one obtains:

$$\rho_S(t) = \Pi_{\vec{k},s}\Pi_{\vec{p},s'} \sum_{n_{\vec{k},s}=0}^{1} \sum_{m_{\vec{p},s'}=0}^{1} \mathcal{C}^*_{m_{\vec{p},s'}}(p)\,\mathcal{C}_{n_{\vec{k},s}}(k)\,|n_{\vec{k},s};\overline{n}_{-\vec{k},s}\rangle\langle m_{\vec{p},s'};\overline{m}_{-\vec{p},s'}|\, e^{2i\left(m_{\vec{p},s'}\,E_p(\infty)-n_{\vec{k},s}\,E_k(\infty)\right)(t-t^*)}$$

- Off-diagonal terms average out in long time limit (decoherence via dephasing):

$$\rho_S^{(d)} = \Pi_{\vec{k},s}\left[\cos^2(\theta_k)\right] \sum_{n_{\vec{k},s}=0}^{1} \left(\tan^2(\theta_k)\right)^{n_{\vec{k},s}} |n_{\vec{k},s};\overline{n}_{-\vec{k},s}\rangle\langle n_{\vec{k},s};\overline{n}_{-\vec{k},s}|$$

Particle and Entropy Production

- This produces an effective *mixed state* with concomitant entanglement entropy:

$$S = -\text{Tr}\rho_S^{(d)} \ln \rho_S^{(d)} = -\sum_{\vec{k},s} \sum_{n_{k,s}=0}^{1} p_{n_{k,s}} \ln p_{n_{k,s}}$$

$$S = -2 \sum_{\vec{k}} \left\{ \Big(1 - n_k(\infty)\Big) \ln \Big(1 - n_k(\infty)\Big) + n_k(\infty) \ln n_k(\infty) \right\}$$

- Same form as a thermal fermi gas!
  - However, distribution functions given by asymptotic particle states rather than LTE distributions.
- Why entropy from unitary evolution?
  - Decoherence in density matrix yields mixed state—Halmark of the *eigenstate thermalization hypothesis*
- Using a phenomenologically-inspired effective potential approach misses this "thermalization".

Energy Conservation and Equation of Motion

$$\mathcal{E} = N_f \left\{ \frac{\dot{\varphi}^2}{2} + \underbrace{v(\varphi) - 2\int E_k(t) \frac{d^3k}{(2\pi)^3}}_{v_{eff}(\varphi)} + 2\int E_k(t) \underbrace{\Big[n_k(t) + \overline{n}_k(t) \Big]}_{2\,|B_{k,s}(t)|^2} \frac{d^3k}{(2\pi)^3} \right\}$$

$$\ddot{\varphi} + \frac{d}{d\varphi} v_{eff}(\varphi) + 4\,y_0 \int k^2 \left[N_k^2 \Big| f_k(t) \Big|^2 - \frac{1}{2E_k(t)\Big(E_k(t) + m_F(t)\Big)} \right] \frac{d^3k}{(2\pi)^3} = 0$$

- The zeroth order adiabatic contributions furnish the corrections in the effective potential.
- The higher order terms (2nd and higher) can be interpreted as inducing production of (adiabatic) fermions.
- Particle production contribution to energy density is manifestly positive!
  - Suppose condensate is “rolling down” the effective potential → $\dot{\varphi}$ increases as does particle production!
  - This then “damps” the dynamics of the condensate!

3. Unitary Equations of Motion for Dynamical Condensates

The Dynamical Case: N=1

- Inserting into the Hamiltonian one obtains the energy density:

$$\mathcal{E} = \frac{1}{\mathcal{V}} \langle \Phi; 0_F | H | \Phi; 0_F \rangle = N_f \left\{ \frac{\dot{\varphi}^2}{2} + v(\varphi) + \frac{1}{\mathcal{V}} \sum_{\vec{k},s} V^\dagger_{-\vec{k},s}(t) \left[\vec{\alpha}\cdot\vec{k} + \gamma^0 m_F(\varphi(t)) \right] V_{-\vec{k},s}(t) + \frac{1}{2N_f\,\mathcal{V}} \sum_{\vec{k}} \left[|\dot{g}_k(t)|^2 + \omega_k^2(t) |g_k(t)|^2 \right] \right\}$$

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Recall: In Minkowski Spacetime manifest energy conservation. $\dot{\mathcal{E}} = 0$

We can deduce the concomitant equations of motion from this condition:

$$\dot{\mathcal{E}} = \left(\sqrt{N_f}\,\dot{\varphi} \right) \sqrt{N_f} \left[\ddot{\varphi}(t) + v'(\varphi(t)) + y_0 \frac{1}{\mathcal{V}} \sum_{\vec{k},s} V^\dagger_{-\vec{k},s}(t) \gamma^0 V_{-\vec{k},s}(t) + \frac{v'''(\varphi(t))}{2\,N_f} \frac{1}{\mathcal{V}} \sum_{\vec{k}} |g_k(t)|^2 \right]$$

Renormalized Equations—Energy Density

- We can now construct a fully renormalized (conserved) energy density:

$$\mathcal{E} = \frac{\dot{\varphi_R}^2}{2} + v_{eff}(\varphi_R) + 4\int E_k(t)\, n_k(t)\, \frac{d^3k}{(2\pi)^3}\Big|_R + \int \omega_k(t)\, \widetilde{\mathcal{N}}_k(t)\, \frac{d^3k}{(2\pi)^3} = v_{eff}(\varphi_R(0))$$

1-Loop Effective Energy

Renormalized Fermion Particle Production

Scalar Particle Production

$$4\int E_k(t)\, n_k(t)\, \frac{d^3k}{(2\pi)^3}\Big|_R = 4\int E_k(t)\, n_k(t)\, \frac{d^3k}{(2\pi)^3} - \frac{1}{2}\,\dot{\varphi}^2(t)\, \frac{y_0^2}{4\pi^2}\,\mathcal{I}[\frac{\Lambda}{\mu};0]$$

Divergent "Counter Term"

4. Asymptotic Stationary States & Other Implications

Asymptotic Stationary States

- In the large N_f limit, the fermion production dominates. Energy is gradually drained from condensate.
  - Possible dynamical fixed point: $\varphi(t \to \infty) = \varphi(\infty); \dot{\varphi}(\infty) = \ddot{\varphi}(\infty) = 0$
  - By energy conservation: $$v_{eff}(\varphi_R(\infty)) + 4 \int E_k(\infty)\, n_k(\infty)\, \frac{d^3k}{(2\pi)^3} = v_{eff}(\varphi_R(0))$$
  - Particle Production Contribution: $$\int E_k(\infty)\, n_k(\infty)\, \frac{d^3k}{(2\pi)^3} = \frac{1}{4}\Big(v_{eff}(\varphi_R(0)) - v_{eff}(\varphi_R(\infty))\Big) = \text{finite} > 0$$

- Interpretation: The condensate produces a "bath" of fermions and settles into fixed location on the (effective) potential landscape.
  - This behavior is not possible in the phenomenologically-inspired equation of motion: $$\ddot{\varphi}(t) + \frac{d}{d\varphi} V_{eff}(\varphi(t)) = 0$$

Anomalous Spontaneous Symmetry Breaking?

- Dissipation by fermion production could "freeze" a condensate $|\varphi_R(\infty) \neq 0|$
  - Stable VEV w/out SSB violating minima in tree-level potential.
- This is a *possible* novel symmetry breaking mechanism which is a confluence of two effects:
  1. Radiatively induced maxima of static effective potential (Coleman-Weinberg) produces instability
  2. Misalignment dynamics with concomitant profuse particle production + conserved energy yields asymptotic stationary state
- Unlike finite temp. effects (where highly excited states restore symmetry),
  - More entanglement entropy → more fermion/anti-fermion pairs → larger $|\varphi_R(\infty)|$
- This is a unique result of dynamical scalar condensates coupled to (large numbers) of fermion fields.

Conclusions

- The usual effective potential **does not** correctly capture the dynamics of a *dynamical mean field*.
- However, the 0^{th} order adiabatic furnishes a natural basis for constructing equations of motion for condensate evolution.
  - Higher order terms encode spontaneous particle production of coupled fermions or scalar fluctuations.
  - Decoherence of asymptotic density matrices suggest emergence of entanglement entropy.
  - This production mechanism efficiently transfers energy from dynamical condensate suggesting asymptotic dynamical fixed points.
  - Though this basis is used as a "pointer" the resulting equations are valid to **all adiabatic orders**.
  - Equations are energy conserving, renormalizable, and unitary.
- Real time dynamics highlights the natural emergence of wave function renormalization through "dynamical dressing" of bare condensate.
- Lingering Questions:
  - Condensate evolution in cosmological spacetimes? Possible interaction with cosmological particle production? Does the classical misalignment mechanism *only* produce "cold dark matter"?
  - Gauge boson behavior?
  - Possible implications for Higgs sector?