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# Anomalies from three-point on-shell bootstrap

based on work with Hiren Kakkad & Rémy Larue
arXiv:2606.28193 [hep-th]


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# Introduction

Quantum anomalies: classical symmetries may break upon quantization

- Chiral anomaly $\Rightarrow$ $\pi^0 \to \gamma\gamma$, gauge-anomaly cancellation, index theorem

  Adler '69; Bell, Jackiw '69; Bouchiat, Iliopoulos, Meyer '72; Gross, Jackiw '72; Atiyah, Singer '68
- Weyl anomaly $\Rightarrow$ Hawking radiation, Casimir effect, holography

  Capper, Duff '74; Christensen, Fulling '77; Setare, Rezaeian '00; Polyakov '81; Henningson, Skenderis '98
- constrained by Wess–Zumino consistency conditions / cohomology

  Wess, Zumino '71; Becchi, Rouet, Stora '75; Bonora, Cotta-Ramusino, Reina '83, Dubois-Violette, Talon, Viallet '85
- gauge anomaly indirectly via 4pt and 5pt amplitudes:
  tension between locality & unitarity/collinear factorization

  Huang, McGady '13, + Chen '14; Accettulli Huber, De Angelis '21; Alviani, Falkowski '25
- no direct on-shell characterization

**This talk:** bootstrap anomalies directly on shell at 3 pts,
derive chiral/gauge, diffeomorphism, Lorentz, & Weyl/trace anomalies
(no explicit Lagrangian, cohomological complex, or regularization)

# Anomalies “from almost nothing”

cf Grant Remmen's talk

Assumptions:

(SYM) bootstrap only anomalies of already-classical symmetries

(EA) effective action $\Gamma$ fully encodes quantum dynamics

(CPT) $\mathcal{CPT}$ and (global) Lorentz invariance of $\Gamma$

(CP) $\mathcal{C}$, $\mathcal{P}$ invariance unless classically broken

(COV) anomalies expressible in covariant form (vs “consistent anomalies”)

(POL) nontrivial on-shell anomalies are polynomial in helicity spinors

**Results:** chiral/gauge, diffeomorphism, Lorentz, Weyl/trace anomalies
(up to real prefactors such as $\frac{1}{16\pi^2}$)

# Outline

0. Introduction

1. Chiral/gauge anomaly
2. Diffeomorphism anomaly
3. Lorentz anomaly
4. Weyl/trace anomaly

5. Summary & outlook

# Chiral/gauge anomaly

# 3-point on-shell bootstrap

2 complex branches:

$$\text{MHV} : |1] \propto |2] \propto |3] \qquad \overline{\text{MHV}} : |1\rangle \propto |2\rangle \propto |3\rangle$$

2 possible expressions for fixed helicities:

Benincasa, Cachazo '07

$$\begin{aligned} \text{MHV}: &\quad \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1} \\ \overline{\text{MHV}}: &\quad [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2} \end{aligned}$$

Finiteness on real MHV $\cap$ $\overline{\text{MHV}}$ kin. + (POL)
$\Rightarrow$ pick polynomial branch, $h_1 + h_2 + h_3 = 0$ ruled out

McGady, Rodina '13

# Gauge-anomaly definitions

Linearized gauge transformation + variation:

$$\delta_\xi^{\mathsf{g}} A_\mu^a = \partial_\mu \xi^a - g f^{abc} \xi^b A_\mu^c =: D_\mu^{\mathsf{adj}} \xi^a, \qquad \delta_\xi^{\mathsf{g}} \varphi = ig\xi^a t_r^a \varphi$$

$$\delta_\xi^{\mathsf{g}} \Gamma[A, \varphi] = \int \mathrm{d}^4 x \left[ \frac{\delta \Gamma}{\delta A_\mu^a} D_\mu^{\mathsf{adj}} \xi^a + ig\xi^a \frac{\delta \Gamma}{\delta \varphi} t_r^a \varphi \right]$$

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Gauge-anomaly functional:

$$\mathcal{A}^c(z)[A, \varphi] \coloneqq - \frac{\delta(\delta^{\mathsf{g}}_{\xi} \Gamma[A, \varphi])}{\delta \xi^c(z)} = D^{\mathsf{adj}}_\mu \frac{\delta \Gamma}{\delta A^c_\mu} - i g \frac{\delta \Gamma}{\delta \varphi} t^c_r \varphi$$

$$\mathcal{A}^{abc}_{\lambda\mu}(x, y, z) \coloneqq \left. \frac{\delta^2 \mathcal{A}^c(z)[A, \varphi]}{\delta A^{a\lambda}(x) \delta A^{b\mu}(y)} \right|_{A=\varphi=0} = \frac{\partial}{\partial z^\nu} \left. \frac{\delta^3 \Gamma[A, 0]}{\delta A^{a\lambda}(x) \delta A^{b\mu}(y) \delta A^c_\nu(z)} \right|_{A=0}$$

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3pt gauge-anomaly amplitude:

$$\delta^4(p_1+p_2+p_3)\, \mathcal{A}^{abc}(1^{h_1}, 2^{h_2}, 3) \coloneqq -i \varepsilon^\lambda_1 \varepsilon^\mu_2 p^\nu_3 \left. \frac{\delta^3 \Gamma[A,0]}{\delta A^{a\lambda}(p_1) \delta A^{b\mu}(p_2) \delta A^{c\nu}(p_3)} \right|_{A=0,\, p_i^2=0}$$

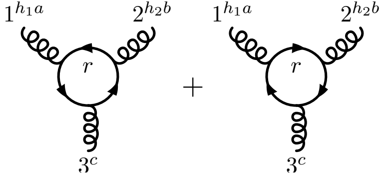
mass dimension 2, spins $(1,1,0)$

# Bootstrapping color + kinematics

Polynomials only for $(\mp 1, \mp 1, 0)$: unique $\langle 12\rangle^2$ & $[12]^2$

$$\mathcal{A}^{abc}(1^{h_1}, 2^{h_2}, 3) = g^3 d_r^{abc} \times \begin{cases} C^- \langle 12\rangle^2, & h_1 = h_2 = -1, \\ C^+ [12]^2, & h_1 = h_2 = +1, \\ 0, & \text{otherwise} \end{cases}$$

- Bose symmetry $1 \leftrightarrow 2 \quad \Rightarrow \quad$ color factor $\propto d_r^{abc}$ (symmetric), not $f^{abc}$
- $d_r^{abc} \neq 0 \quad \Leftarrow \quad$ complex representation $r$
  $\Rightarrow$ matter (not gauge bosons) in the loop




- $\mathcal{C}$ invariance: purely vectorial case is forbidden (generalized Furry's theorem)

# Chiral fermions & L/R gauging

- spin-0, spin-1 matter cannot produce this anomaly; need fermions for $\mathcal{C}$ invariance
- vector + axial gauging via $\gamma_5 \Rightarrow$ independent "left" & "right" currents

$$D^\mu \Psi = [\partial^\mu - igV^\mu - ig'A^\mu \gamma_5]\binom{\chi_\alpha}{\tilde{\eta}^{\dot{\alpha}}} \quad \Leftrightarrow \quad \begin{cases} D^\mu \chi = [\partial^\mu - ig_{\mathsf{L}} L^{a\mu} t^a_r]\chi \\ D^\mu \tilde{\eta} = [\partial^\mu - ig_{\mathsf{R}} R^{a\mu} t^a_r]\tilde{\eta} \end{cases}$$

- mass term obstructs double gauging $\Rightarrow$ only *massless chiral* fermions contribute
- mixed-chirality anomalies vanish e.g. $\mathcal{A}^{abc}_{\mathsf{LLR}} = 0$
- vectorlike Dirac case: $0 = C^{\pm}_{\mathsf{VVV}} = C^{\pm}_{\mathsf{LLL}} + C^{\pm}_{\mathsf{RRR}}$

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| | $V^\mu_{i\bar\jmath}$ | $A^\mu_{i\bar\jmath}$ | $L^\mu_{i\bar\jmath}$ | $R^\mu_{i\bar\jmath}$ | $\partial_\mu$ | $\gamma_5$ |
|---|---|---|---|---|---|---|
| $\mathcal{C}$ | $-V^\mu_{j\bar\imath}$ | $+A^\mu_{j\bar\imath}$ | $-\frac{g_R}{g_L} R^\mu_{j\bar\imath}$ | $-\frac{g_L}{g_R} L^\mu_{j\bar\imath}$ | $+\partial_\mu$ | $-\gamma_5^\top$ |
| $\mathcal{P}$ | $+V_{\mu i\bar\jmath}$ | $-A_{\mu i\bar\jmath}$ | $+\frac{g_R}{g_L} R_{\mu i\bar\jmath}$ | $+\frac{g_L}{g_R} L_{\mu i\bar\jmath}$ | $+\partial^\mu$ | $-\gamma_5$ |
| $\mathcal{T}$ | $+V_{\mu i\bar\jmath}$ | $+A_{\mu i\bar\jmath}$ | $+L_{\mu i\bar\jmath}$ | $+R_{\mu i\bar\jmath}$ | $-\partial^\mu$ | $+\gamma_5$ |
| $\mathcal{CPT}$ | $-V^\mu_{j\bar\imath}$ | $-A^\mu_{j\bar\imath}$ | $-L^\mu_{j\bar\imath}$ | $-R^\mu_{j\bar\imath}$ | $-\partial_\mu$ | $+\gamma_5^\top$ |

| | $\mathcal{C}$ (unitary) | $\mathcal{P}$ (unitary) | $\mathcal{T}$ (antiunitary) | $\mathcal{CPT}$ |
|---|---|---|---|---|
| $t^a_{i\bar\jmath}$ | $t^a_{j\bar\imath}$ | $t^a_{i\bar\jmath}$ (inv.) | $t^a_{i\bar\jmath}$ (inv.) | $t^a_{j\bar\imath}$ |
| $\|p\rangle_\alpha$ | $\|p\rangle_\alpha$ (inv.) | $\|p]^{\dot\alpha}$ | $\langle p\|^\alpha$ | $-[p\|_{\dot\alpha}$ |
| $\|p]^{\dot\alpha}$ | $\|p]^{\dot\alpha}$ (inv.) | $-\|p\rangle_\alpha$ | $-[p\|_{\dot\alpha}$ | $-\langle p\|^\alpha$ |
| $p^\mu$ | $p^\mu$ (inv.) | $p_\mu$ | $p_\mu$ | $p^\mu$ (inv.) |
| $\varepsilon^\mu_{p,\pm}$ | $\varepsilon^\mu_{p,\pm}$ (inv.) | $\varepsilon^\mp_{p\mu}$ | $\varepsilon^\pm_{p\mu}$ | $\varepsilon^\mu_{p,\mp}$ |
| $i$ | $i$ (inv.) | $i$ (inv.) | $-i$ | $-i$ |

- $\mathcal{CPT}\,\mathcal{A}^{abc}_{\mathsf{LLL}}(1^\pm, 2^\pm, 3) = \mathcal{A}^{abc}_{\mathsf{LLL}}(1^\mp, 2^\mp, 3) \quad \Rightarrow \quad (C^{\pm}_{\mathsf{LLL}})^* = C^{\mp}_{\mathsf{LLL}},\ (C^{\pm}_{\mathsf{RRR}})^* = C^{\mp}_{\mathsf{RRR}}$
- $\mathcal{P}\,\mathcal{A}^{abc}_{\mathsf{LLL}}(1^\pm, 2^\pm, 3) = \frac{g_{\mathsf{L}}^3}{g_{\mathsf{R}}^3} \mathcal{A}^{abc}_{\mathsf{RRR}}(1^\mp, 2^\mp, 3) \quad \Rightarrow \quad C^{\pm}_{\mathsf{LLL}} = C^{\mp}_{\mathsf{RRR}} \qquad =: \pm iC,\ C \in \mathbb{R}$

# Final gauge anomaly

$$\mathcal{A}^{abc}_{\mathsf{LLL}}(1^{h_1},2^{h_2},3) = iC \sum_{\substack{\text{left Weyl}\\ \text{fermions } \chi}} g_\chi^3 d^{abc}_{r_\chi} \times \begin{cases} \langle 12\rangle^2, & h_1{=}h_2{=}{-1},\\ -[12]^2, & h_1{=}h_2{=}{+1},\\ 0, & \text{otherwise} \end{cases}$$

$$\mathcal{A}^{abc}_{\mathsf{RRR}}(1^{h_1},2^{h_2},3) = iC \sum_{\substack{\text{right Weyl}\\ \text{fermions } \tilde\eta}} g_{\tilde\eta}^3 d^{abc}_{r_{\tilde\eta}} \times \begin{cases} -\langle 12\rangle^2, & h_1{=}h_2{=}{-1},\\ [12]^2, & h_1{=}h_2{=}{+1},\\ 0, & \text{otherwise} \end{cases}$$

Uplifts to the familiar covariant form

$$\delta^{\mathsf{g}}_{\xi}\Gamma = -\frac{C}{2}\int \mathrm{d}^4x\sqrt{|g|}\Big[\sum_{\chi\in\text{left}} g_\chi^3\,\mathrm{tr}_{r_\chi}(\xi_{\mathsf{L}}F^{\mathsf{L}}_{\mu\nu}\widetilde{F}^{\mu\nu}_{\mathsf{L}}) - \sum_{\tilde\eta\in\text{right}} g_{\tilde\eta}^3\,\mathrm{tr}_{r_{\tilde\eta}}(\xi_{\mathsf{R}}F^{\mathsf{R}}_{\mu\nu}\widetilde{F}^{\mu\nu}_{\mathsf{R}})\Big]$$

- relative sign between helicities encodes $F\cdot F$ vs $F\cdot\widetilde{F}$; matches $C=\dfrac{1}{8\pi^2}$ Fujikawa, Suzuki '04
- make $\xi_{\mathsf{L/R}}$ constant $\Rightarrow$ global/singlet/abelian chiral anomaly

# Mixed gauge-gravitational anomaly

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{bc}_{gA}(1^{h_1},2^{h_2},3) := -i\kappa\,\varepsilon_1^{\mu\nu}\varepsilon_2^{\rho}\,p_3^{\lambda}\,\frac{\delta^3\Gamma[g,A,0]}{\delta g^{\mu\nu}(p_1)\delta A^{b\rho}(p_2)\delta A^{c\lambda}(p_3)}\bigg|_{g=\eta,\,A=0,\,p_i^2=0}$$

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{c}_{gg}(1^{h_1},2^{h_2},3) := -i\kappa^2\varepsilon_1^{\mu\nu}\varepsilon_2^{\rho\sigma}p_3^{\lambda}\,\frac{\delta^3\Gamma[g,A,0]}{\delta g^{\mu\nu}(p_1)\delta g^{\rho\sigma}(p_2)\delta A^{c\lambda}(p_3)}\bigg|_{g=\eta,\,A=0,\,p_i^2=0}$$

mass dimension 2, spins $(2,1$ or $2,0)$

Pure-gravity sector: double copy of pure-gauge $\Rightarrow$ $\mathcal{A}^c_{gg} \propto \langle 12\rangle^4$ & $[12]^4$

Single colored leg $\Rightarrow$ abelian only; same $\mathcal{C}$, $\mathcal{P}$, $\mathcal{CPT}$ logic gives

$$\mathcal{A}_{gg\mathsf{L}}(1^{h_1}; 2^{h_2}; 3) = iC\kappa^2\sum_{\chi\,\in\,\text{left}} Q_\chi \times \begin{cases} \langle 12\rangle^4, & h_1{=}h_2{=}-2,\\ -[12]^4, & h_1{=}h_2{=}+2,\\ 0, & \text{otherwise}\end{cases}$$

$$\mathcal{A}_{gg\mathsf{R}}(1^{h_1}; 2^{h_2}; 3) = iC\kappa^2\sum_{\tilde\eta\,\in\,\text{right}} Q_{\tilde\eta} \times \begin{cases} -\langle 12\rangle^4, & h_1{=}h_2{=}-2,\\ [12]^4, & h_1{=}h_2{=}+2,\\ 0, & \text{otherwise}\end{cases}$$

Covariantly: $$\delta'^{\mathsf{g}}_{\xi}\Gamma = C\int \mathrm{d}^4x\sqrt{|g|}R_{\lambda\mu\nu\rho}\widetilde{R}^{\lambda\mu\nu\rho}\Big[\xi_{\mathsf{L}}\sum_{\chi\,\in\,\text{left}}Q_\chi - \xi_{\mathsf{R}}\sum_{\tilde\eta\,\in\,\text{right}}Q_{\tilde\eta}\Big]$$

Delbourgo, Salam '72; Eguchi, Freund '76

# Diffeomorphism anomaly

# Diffeo-anomaly setup

Linearized diffeomorphism + variation:

$$\delta^{\mathsf{d}}_{\xi} g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \qquad \delta^{\mathsf{d}}_{\xi} A^a_\mu = \xi^\nu \nabla_\nu A^a_\mu + A^a_\nu \nabla_\mu \xi^\nu \qquad \sqrt{|g|}\langle T_{\mu\nu}\rangle := 2\frac{\delta\Gamma}{\delta g^{\mu\nu}}$$

$$\delta^{\mathsf{d}}_{\xi}\Gamma = \int \mathrm{d}^4x \left\{ \xi^\mu \left[ \sqrt{|g|}\nabla^\lambda \langle T_{\lambda\mu}\rangle - \sqrt{|g|} A^c_\mu \mathcal{A}^c + F^c_{\mu\nu}\frac{\delta\Gamma}{\delta A^c_\nu} \right] + \left[\delta^{\mathsf{d}}_{\xi}\varphi - (\xi\cdot A^c) i g t^c_r \varphi\right] \frac{\delta\Gamma}{\delta\varphi} \right\}$$

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Diffeo-anomaly functional:

$$\mathcal{A}^\mu(z)[g,A,\varphi] := -\frac{\delta(\delta^{\mathsf{d}}_\xi\Gamma[g,A,\varphi])}{\delta\xi_\mu(z)} = 2\nabla_\lambda \frac{\delta\Gamma}{\delta g_{\lambda\mu}(z)} + \cdots =: -\sqrt{|g|}\nabla_\lambda\langle T^{\lambda\mu}\rangle(z) + \ldots$$

3pt diffeo-anomaly on-shell currents: mass dimension 3, spins $(1 \text{ or } 2, 1 \text{ or } 2, 0)$, vector

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{ab\mu}_{AA}(1^{h_1},2^{h_2},3) := -2i\varepsilon_{1\nu}\varepsilon_{2\rho}p_{3\lambda}\frac{\delta\Gamma[g,A,0]}{\delta A^a_\nu(p_1)\delta A^b_\rho(p_2)\delta g_{\lambda\mu}(p_3)}\Bigg|_{g=\eta,\,A=0,\,p_i^2=0}$$

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{b\mu}_{gA}(1^{h_1},2^{h_2},3) := -2i\kappa\,\varepsilon_{1\nu\rho}\varepsilon_{2\sigma}p_{3\lambda}\frac{\delta\Gamma[g,A,0]}{\delta g_{\nu\rho}(p_1)\delta A^b_\sigma(p_2)\delta g_{\lambda\mu}(p_3)}\Bigg|_{g=\eta,\,A=0,\,p_i^2=0}$$

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{\mu}_{gg}(1^{h_1},2^{h_2},3) := -2i\kappa^2\varepsilon_{1\nu\rho}\varepsilon_{2\sigma\tau}p_{3\lambda}\frac{\delta\Gamma[g,0,0]}{\delta g_{\nu\rho}(p_1)\delta g_{\sigma\tau}(p_2)\delta g_{\lambda\mu}(p_3)}\Bigg|_{g=\eta,\,p_i^2=0}$$

# Pure-gauge & -gravity sectors vanish

Vector ansatz in terms of available vectors:

$$\mathcal{A}^{ab\mu}_{AA}(1^-,2^-,3) = \delta^{ab}(C_1 p_1^\mu + C_2 p_2^\mu)\langle 12\rangle^2$$

$$\vdots$$

$$\mathcal{A}^{\mu}_{gg}(1^+,2^+,3) = \kappa^2(C_1 p_1^\mu + C_2 p_2^\mu)[12]^4$$

- Bose symmetry $\Rightarrow\ C_1 = C_2\ \Rightarrow\ \mathcal{A}^{ab\mu}_{AA} \propto p_1^\mu + p_2^\mu = -p_3^\mu$
- graviton transversality $h_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}/\kappa$ forces $p_3 \cdot \xi(p_3) = 0$ on shell, so

$$\xi_\mu \mathcal{A}^{ab\mu}_{AA} = \xi_\mu \mathcal{A}^{\mu}_{gg} = 0$$

No nontrivial pure-gauge or pure-gravity diffeomorphism anomaly

Alvarez-Gaume, Witten '83

# Mixed sector: nontrivial result

May bootstrap $\mathcal{A}^\mu$ as bispinor $\mathcal{A}^{\dot\gamma\gamma} := \mathcal{A}^\mu \bar\sigma^{\dot\gamma\gamma}$

$$\begin{aligned}
\mathcal{A}^{\dot\gamma\gamma} &= \mathcal{M}_1 |1]^{\dot\gamma} \langle 1|^\gamma + \mathcal{M}_2 |2]^{\dot\gamma}\langle 2|^\gamma \\
&= \mathcal{M}_{12} |1]^{\dot\gamma}\langle 2|^\gamma + \mathcal{M}_{21}|2]^{\dot\gamma}\langle 1|^\gamma \\
\mathcal{A}^{b\mu}_{gA}(1^{-2}, 2^{-1}, 3) &= \kappa \, \operatorname{tr} t^b_r \, C_- \, \langle 1|\sigma^\mu|2] \, \langle 12\rangle^3 \\
\mathcal{A}^{b\mu}_{gA}(1^{+2}, 2^{+1}, 3) &= \kappa \, \operatorname{tr} t^b_r \, C_+ \, [1|\bar\sigma^\mu|2\rangle \, [12]^3 \quad \text{easy to miss!}
\end{aligned}$$

- $\operatorname{tr} t^b_r \neq 0$ + (CP) $\Rightarrow$ abelian *axial* gauge field
- $\mathcal{CPT}\mathcal{A}^\mu_{g\mathrm{A}} = \mathcal{A}^\mu_{g\mathrm{A}} \;\Rightarrow\; C_+ = -C_-^* =: iQ'C$
- on-shell parity $\mathcal{P}\mathcal{A}^\mu_{g\mathrm{A}} = -\mathcal{A}_{(g\mathrm{A})\mu}$ (for Dirac case) forces $C \in \mathbb{R}$

Uplifts to

$$\delta^{\mathrm{d}}_\xi \Gamma = -4\sqrt{2}\, C \int \mathrm{d}^4x \sqrt{|g|}\, (\nabla^\lambda \xi^\mu) R_{\lambda\mu\nu\rho} \widetilde{F}^{\nu\rho}_{\mathrm{A}} \sum_{\text{matter } \varphi} Q'_\varphi$$

Nieh '84; Larue, Quevillon, Zwicky '23

# Lorentz anomaly

# Lorentz-anomaly setup

Linearized (local) Lorentz transformation + variation:

$$\delta^{\mathsf{L}}_{\xi} e^{\hat{\imath}}_{\mu} = \xi^{\hat{\imath}}{}_{\hat{\jmath}} e^{\hat{\jmath}}_{\mu}, \qquad \xi_{\hat{\imath}\hat{\jmath}} = -\xi_{\hat{\jmath}\hat{\imath}} \qquad \sqrt{|g|}\langle T^{\hat{\imath}\hat{\jmath}}\rangle := e^{\hat{\imath}\mu} \frac{\delta\Gamma}{\delta e^{\mu}_{\hat{\jmath}}}$$

$$\delta^{\mathsf{L}}_{\xi}\Gamma[e, A, \varphi] = \int \mathrm{d}^4x \left[ -\sqrt{|g|}\, \xi_{\hat{\imath}\hat{\jmath}} \langle T^{\hat{\imath}\hat{\jmath}}\rangle + \frac{\delta\Gamma}{\delta\varphi} \delta^{\mathsf{L}}_{\xi}\varphi \right]$$

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$$\delta^{\mathsf{L}}_\xi \Gamma[e, A, \varphi] = \int \mathrm{d}^4 x \left[ -\sqrt{|g|}\,\xi_{\hat\imath\hat\jmath}\langle T^{\hat\imath\hat\jmath}\rangle + \frac{\delta\Gamma}{\delta\varphi}\delta^{\mathsf{L}}_\xi\varphi \right]$$

Lorentz-anomaly functional:

$$\mathcal{A}^{[\hat\imath\hat\jmath]}(z) := -\frac{\delta(\delta^{\mathsf{L}}_\xi\Gamma[e,A,\varphi])}{\delta\xi_{\hat\imath\hat\jmath}(z)} = e^{\lambda[\hat\imath}\frac{\delta\Gamma}{\delta e^\lambda_{\hat\jmath]}(z)} + \cdots =: -\sqrt{|g|}\langle T^{[\hat\imath\hat\jmath]}\rangle(z) + \ldots$$

3pt on-shell Lorentz anomalies:

mass dimension 2, spins $(1 \text{ or } 2, 1 \text{ or } 2, 0)$, 2-form

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{ab[\hat\imath\hat\jmath]}_{AA}(1^{h_1},2^{h_2},3) := \varepsilon_{1\mu}\varepsilon_{2\nu}e_0^{\lambda[\hat\imath}(p_3)\frac{\delta\Gamma[e,A,0]}{\delta A^a_\mu(p_1)\delta A^b_\nu(p_2)\delta e^\lambda_{\hat\jmath]}(p_3)}\bigg|_{e=e_0,\,A=0,\,p_i^2=0}$$

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{b[\hat\imath\hat\jmath]}_{gA}(1^{h_1},2^{h_2},3) := \kappa\,\varepsilon_{1\mu\nu}\varepsilon_{2\rho}e_0^{\lambda[\hat\imath}(p_3)\frac{\delta\Gamma[e,A,0]}{\delta g_{\mu\nu}(p_1)\delta A^b_\rho(p_2)\delta e^\lambda_{\hat\jmath]}(p_3)}\bigg|_{e=e_0,\,A=0,\,p_i^2=0}$$

$$\delta^4(p_1+p_2+p_3)\,\mathcal{A}^{[\hat\imath\hat\jmath]}_{gg}(1^{h_1},2^{h_2},3) := \kappa^2\varepsilon_{1\mu\nu}\varepsilon_{2\rho\sigma}e_0^{\lambda[\hat\imath}(p_3)\frac{\delta\Gamma[e,0,0]}{\delta g_{\mu\nu}(p_1)\delta g_{\rho\sigma}(p_2)\delta e^\lambda_{\hat\jmath]}(p_3)}\bigg|_{e=e_0,\,p_i^2=0}$$

# Bispinor decomposition & pure sectors

Note: unlike diffeo, $\xi_{\hat\imath\hat\jmath}$ need *not* be transverse on shell $\Rightarrow$ no free cancellation
Antisymmetry in flat Minkowski indices $\hat\imath, \hat\jmath \;\; \Rightarrow \;\;$ decompose into left/right bispinors:

$$\bar\sigma_{\hat\imath}^{\dot\gamma\gamma}\bar\sigma_{\hat\jmath}^{\dot\delta\delta}\mathcal{A}^{[\hat\imath\hat\jmath]} = \mathcal{A}^{(\gamma\delta)}\epsilon^{\dot\gamma\dot\delta} + \mathcal{A}^{(\dot\gamma\dot\delta)}\epsilon^{\gamma\delta}$$

MHV basis: $\langle 1|^\gamma\langle 1|^\delta$, $\langle 2|^\gamma\langle 2|^\delta$, $\langle 1|^{(\gamma}\langle 2|^{\delta)}$; pure-gauge sector gives

$$\mathcal{A}_{AA}^{ab(\gamma\delta)}(1^-,2^-,3) = \delta^{ab}\Big\{C_1\frac{\langle 12\rangle\langle 23\rangle}{\langle 13\rangle}\langle 1|^\gamma\langle 1|^\delta + C_2\frac{\langle 21\rangle\langle 13\rangle}{\langle 23\rangle}\langle 2|^\gamma\langle 2|^\delta + C_3\langle 12\rangle\langle 1|^{(\gamma}\langle 2|^{\delta)}\Big\}$$

$$\mathcal{A}_{AA}^{ab(\dot\gamma\dot\delta)}(1^-,2^-,3) = C_4\,\delta^{ab}\,\langle 12\rangle^3|1]^{(\dot\gamma}|2]^{\dot\delta)}$$

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$$\mathcal{A}_{AA}^{ab(\dot{\gamma}\dot{\delta})}(1^-,2^-,3) = C_4\,\delta^{ab}\,\langle 12\rangle^3|1]^{(\dot{\gamma}}|2]^{\dot{\delta})}$$

- all other helicity conf. $\ni$ denominators on all basis bispinors $\Rightarrow$ (POL) kills them
- (POL) also kills $C_1 = C_2 = 0$ (denominators on $\langle 1|\langle 1|$ & $\langle 2|\langle 2|$ terms)
- Bose symmetry $1 \leftrightarrow 2$ forces $C_3 = C_4 = 0$

Pure-gauge & pure-gravity Lorentz anomalies are trivial (may be renormalized away)

Bertlmann '96

# Mixed sector: what survives

Full MHV ansatz in the cross-sector (spins $(-2,-1,0)$ with $[\xi_{\hat{\imath}\hat{\jmath}}]=0$):

$$\mathcal{A}_{gA}^{b(\gamma\delta)}(1^{-2},2^{-1},3)=\kappa\ \mathrm{tr}\,t_r^b\Big\{C_1\langle 12\rangle^2\langle 1|^\gamma\langle 1|^\delta+C_2\frac{\langle 12\rangle^2\langle 31\rangle^2}{\langle 23\rangle^2}\langle 2|^\gamma\langle 2|^\delta+C_3\frac{\langle 12\rangle^2\langle 31\rangle}{\langle 23\rangle}\langle 1|^{(\gamma}\langle 2|^{\delta)}\Big\}$$

$$\mathcal{A}_{gA}^{b(\dot\gamma\dot\delta)}(1^{-2},2^{-1},3)=\kappa\,C_4\ \mathrm{tr}\,t_r^b\,\langle 12\rangle^4|2]^{\dot\gamma}|2]^{\dot\delta}$$

- (POL) kills $C_2=C_3=0$; $\overline{\text{MHV}}$ branch is analogous
- $\mathrm{tr}\,t_r^b\neq 0$ + (CP) $\Rightarrow$ abelian *axial* gauge field (as for diffeo)
- $C_4$ term: requires $[C_4]=-2\Rightarrow$ higher-dim.; explicit local counterterm $\int \mathrm{d}^4x\,\omega^\lambda{}_{\mu\nu}R^\mu{}_{\lambda\rho\sigma}\big(c_3\nabla^\rho F^{\nu\sigma}+\tilde c_3\nabla^\rho\widetilde{F}^{\nu\sigma}\big)\Rightarrow$ *trivial* anomaly Bertlmann '96

Only dim.-2 terms survive, one per branch:

$$\mathcal{A}_{gA}^{b(\gamma\delta)}(1^{-2},2^{-1},3)=\kappa Q'C_-\,\langle 12\rangle^2\langle 1|^\gamma\langle 1|^\delta$$

$$\mathcal{A}_{gA}^{b(\dot\gamma\dot\delta)}(1^{+2},2^{+1},3)=\kappa Q'C_+\,[12]^2|1]^{\dot\gamma}|1]^{\dot\delta}$$

# Final Lorentz anomaly

$\mathcal{CPT}$ & $\mathcal{P}$ act on the bispinor anomaly as (using spinor parity rules from CPT table):

$$\mathcal{CPT}\,\mathcal{A}_{g\mathsf{A}}^{(\gamma\delta)} = -\mathcal{A}_{g\mathsf{A}}^{(\dot\gamma\dot\delta)}, \qquad \mathcal{P}\mathcal{A}_{g\mathsf{A}}^{(\gamma\delta)} = -\epsilon_{\dot\gamma\dot\varepsilon}\epsilon_{\dot\delta\dot\varphi}\,\mathcal{A}_{g\mathsf{A}}^{(\dot\varepsilon\dot\varphi)}$$

- $\mathcal{CPT}$: $\kappa Q' C_-\langle 12\rangle^2\langle 1|\langle 1| \xrightarrow{\mathcal{CPT}} \kappa Q' C_-^*[12]^2|1]|1] \quad \Rightarrow \quad C_-^* = -C_+$
- $\mathcal{P}$: $\kappa Q' C_-\langle 12\rangle^2\langle 1|\langle 1| \xrightarrow{\mathcal{P}} \kappa Q' C_-[12]^2[1|[1| \quad \Rightarrow \quad C_- = -C_+$
- together: $\pm C_\pm =: C \in \mathbb{R}$

By one-loop additivity & covariant uplift:

$$\delta_\xi^{\mathsf{L}}\Gamma = -\sqrt{2}\,C\int \mathrm{d}^4x\sqrt{|g|}\;\xi_{\hat\imath\hat\jmath}\,R^{\hat\imath\hat\jmath}{}_{\mu\nu}\,\widetilde{F}_{\mathsf{A}}^{\mu\nu}\sum_{\text{matter }\varphi} Q'_\varphi$$

Companion to the diffeomorphism result: same abelian axial coupling

$$\delta_\xi^{\mathsf{d}}\Gamma = -4\sqrt{2}\,C\int \mathrm{d}^4x\sqrt{|g|}\;(\nabla^\lambda\xi^\mu)\,R_{\lambda\mu\nu\rho}\,\widetilde{F}_{\mathsf{A}}^{\nu\rho}\sum_{\text{matter }\varphi} Q'_\varphi$$

Leutwyler '84, '85; Nieh '84; Caneschi, Valtancoli '85; Yajima, Kimura '86; Larue, Quevillon, Zwicky '23

# Weyl/trace anomaly

# Weyl-anomaly setup

Linearized Weyl transformation + variation:

$$\delta^{\mathsf{W}}_{\xi} g_{\mu\nu} = 2\xi g_{\mu\nu}, \qquad \delta^{\mathsf{W}}_{\xi} A^a_\mu = 0, \qquad \delta^{\mathsf{W}}_{\xi} \varphi = \xi \Delta_\varphi \varphi$$

$$\delta^{\mathsf{W}}_{\xi} \Gamma[g, A, \varphi] = \int \mathrm{d}^4 x \, \xi \left[ -\sqrt{|g|} \langle T^{\mu}{}_{\mu} \rangle + \Delta_\varphi \varphi \frac{\delta \Gamma}{\delta \varphi} \right]$$

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Weyl-anomaly functional: (make Weyl local for Noether currents)

$$\mathcal{A}(z)[g, A, \varphi] := -\frac{\delta(\delta_\xi^{\mathsf{W}} \Gamma[g, A, \varphi])}{\delta \xi(z)} = -2 g_{\lambda\mu} \frac{\delta \Gamma}{\delta g_{\lambda\mu}(z)} + \dots =: \sqrt{|g|} \langle T^\mu{}_\mu \rangle(z) + \dots$$

3pt Weyl-anomaly amplitudes:

mass dimension 2, spins $(1 \text{ or } 2, 1 \text{ or } 2, 0)$

$$\delta^4(p_1 + p_2 + p_3)\, \mathcal{A}_{AA}^{ab}(1^{h_1}, 2^{h_2}, 3) := -2 \varepsilon_{1\nu} \varepsilon_{2\rho} \eta_{\lambda\mu} \frac{\delta \Gamma[g, A, 0]}{\delta A_\nu^a(p_1) \delta A_\rho^b(p_2) \delta g_{\lambda\mu}(p_3)} \bigg|_{g=\eta,\, A=0,\, p_i^2=0}$$

$$\delta^4(p_1 + p_2 + p_3)\, \mathcal{A}_{gA}^{b}(1^{h_1}, 2^{h_2}, 3) := -2\kappa\, \varepsilon_{1\nu\rho} \varepsilon_{2\sigma} \eta_{\lambda\mu} \frac{\delta \Gamma[g, A, 0]}{\delta g_{\nu\rho}(p_1) \delta A_\sigma^b(p_2) \delta g_{\lambda\mu}(p_3)} \bigg|_{g=\eta,\, A=0,\, p_i^2=0}$$

$$\delta^4(p_1 + p_2 + p_3)\, \mathcal{A}_{gg}(1^{h_1}, 2^{h_2}, 3) := -2\kappa^2 \varepsilon_{1\nu\rho} \varepsilon_{2\sigma\tau} \eta_{\lambda\mu} \frac{\delta \Gamma[g, 0, 0]}{\delta g_{\nu\rho}(p_1) \delta g_{\sigma\tau}(p_2) \delta g_{\lambda\mu}(p_3)} \bigg|_{g=\eta,\, p_i^2=0}$$

# Weyl anomaly: kinematic bootstrap

$\mathcal{A}_{AA}^{ab}$, $\mathcal{A}_{gA}^{b}$, $\mathcal{A}_{gg}$ are Lorentz *scalars* of dimension 2
$\Rightarrow$ identical kinematic bootstrap to gauge anomaly

- mixed sector $\mathcal{A}_{gA}^{b} = 0$ ruled out by (POL), e.g. $\mathcal{A}_{gA}^{b}(1^{-2}, 2^{-1}, 3) \propto \kappa \frac{\langle 12\rangle^3 \langle 31\rangle}{\langle 23\rangle}$
- pure-gauge sector:

$$\mathcal{A}_{AA}^{ab}(1^{\mp}, 2^{\mp}, 3) = g^2 \delta^{ab} \times \begin{cases} C^{-}\langle 12\rangle^2 & (-,-) \\ C^{+}[12]^2 & (+,+) \end{cases}$$

- pure-gravity sector:

$$\mathcal{A}_{gg}(1^{\mp}, 2^{\mp}, 3) = \kappa^2 \times \begin{cases} C_2^{-}\langle 12\rangle^4 & (-,-) \\ C_2^{+}[12]^4 & (+,+) \end{cases}$$

- $\mathcal{CPT}$ symmetry implies $(C^{\pm})^* = C^{\mp}$, $(C_2^{\pm})^* = C_2^{\mp}$
- $\mathcal{P}$ symmetry implies $C^{-} = C^{+} =: C_1$, $C_2^{-} = C_2^{+} =: C_2 \in \mathbb{R}$
- key difference from gauge anomaly:
  *any* massless particle may contribute; no chirality required

# Parity fixes $C^+ = C^-$ (contrast with gauge anomaly)

Consider Dirac fermion with vector & axial gauging:

- $\mathcal{C}$ invariance: $\mathcal{C}\mathcal{A}^{ab}_{\mathsf{LL}} = \frac{g_{\mathsf{L}}^2}{g_{\mathsf{R}}^2}\mathcal{A}^{ab}_{\mathsf{RR}} \;\Rightarrow\; C^{\pm}_{\mathsf{LL}} = C^{\pm}_{\mathsf{RR}}$ (vs $C^{\pm}_{\mathsf{LLL}} = -C^{\pm}_{\mathsf{RRR}}$ for gauge)
- $\mathcal{P}$ relation:
$$\mathcal{P}\mathcal{A}^{ab}_{\mathsf{LL}}(1^{\pm},2^{\pm},3) = \frac{g_{\mathsf{L}}^2}{g_{\mathsf{R}}^2}\mathcal{A}^{ab}_{\mathsf{RR}}(1^{\mp},2^{\mp},3) \quad\Rightarrow\quad C^{\pm}_{\mathsf{LL}} = C^{\mp}_{\mathsf{RR}} = C_1 \in \mathbb{R}$$

Significance: on-shell field strengths $f^{\mu\nu}_{p,\pm} := -2ip^{[\mu}\varepsilon^{\nu]}_{p,\pm}$ give

$$F\cdot F \;\rightsquigarrow\; \eta_{\lambda\nu}\eta_{\mu\rho}f^{\lambda\mu}_{p_1,\mp}f^{\nu\rho}_{p_2,\mp} = \begin{cases} -\langle 12\rangle^2 & (-,-) \\ -[12]^2 & (+,+) \end{cases}$$

$$F\cdot\widetilde{F} \;\rightsquigarrow\; \tfrac{1}{2}\epsilon_{\lambda\mu\nu\rho}f^{\lambda\mu}_{p_1,\mp}f^{\nu\rho}_{p_2,\mp} = \begin{cases} +i\langle 12\rangle^2 & (-,-) \\ -i[12]^2 & (+,+) \end{cases}$$

same signs $C^+ = C^- \Rightarrow F\cdot F$ vs opp. signs $C^+ = -C^-$ (gauge anomaly) $\Rightarrow F\cdot\widetilde{F}$

# Final Weyl anomaly: no Pontryagin density

By one-loop additivity and covariant uplift:

$$\delta_\xi^{\mathsf{W}}\Gamma = \xi \int \mathrm{d}^4x \sqrt{|g|} \left\{ \frac{C_1}{2} \left[ \sum_\chi g_\chi^2 \operatorname*{tr}_{r_\chi}(F^{\mathsf{L}}_{\mu\nu} F^{\mu\nu}_{\mathsf{L}}) + \sum_{\tilde\eta} g_{\tilde\eta}^2 \operatorname*{tr}_{r_{\tilde\eta}}(F^{\mathsf{R}}_{\mu\nu} F^{\mu\nu}_{\mathsf{R}}) + \sum_\phi g_\phi^2 \operatorname*{tr}_{r_\phi}(F_{\mu\nu} F^{\mu\nu}) \right] \right.$$
$$\left. - \tilde C_2\, R_{\lambda\mu\nu\rho} R^{\lambda\mu\nu\rho} \right\}$$

where $\tilde C_2 = \sum_{\varphi'} C_2$ sums over all massless particles

3pt bootstrap is insensitive to $R^2$, $R_{\mu\nu}R^{\mu\nu}$, $D^2 R$ (vanish on shell)

- Equal-sign constraint $C^+ = C^-$ rules out *both* Pontryagin densities:

$$F_{\mu\nu}\widetilde F^{\mu\nu} \text{ absent} \qquad \text{and} \qquad R_{\lambda\mu\nu\rho}\widetilde R^{\lambda\mu\nu\rho} \text{ absent}$$

- New *model- and regularization-independent* proof, resting only on $\mathcal{P}$ invariance not broken unless classically
- Note: $C^+ = C^-$ derived even *before* invoking $\mathcal{CPT}$, so $\mathcal{P}$ alone is sufficient

Bonora, Giaccari, Lima de Souza '14; Bonora, Duarte Pereira, Lima de Souza'15; Bastianelli, Martelli '16;
Bonora, Cvitan, Dominis Prester, Duarte Pereira, Giaccari, Paulišić, Štemberga '17–'18, Bastianelli, Broccoli '19; Fröb, Zahn '19; Bonora '22;
Abdallah, Franchino-Viñas, Fröb '21,'23; Larue, Quevillon, Zwicky '23,'24; Alvarez, Alvarez-Gaume, Anero, Martin '26

# Summary

- 3pt on-shell bootstrap, using only Lorentz invariance, discrete symmetries, residual gauge freedom, and polynomial ansatz
- reproduces gauge-anomaly cancellation conditions, mixed gauge-gravitational, diffeomorphism, and Lorentz anomalies, each up real constants
- no nontrivial pure-gauge or pure-gravity diffeomorphism/Lorentz anomalies in 4d
- new model- and regularization-independent proof:
  Weyl anomaly contains *no* Pontryagin densities

Anomalies as consequences of on-shell consistency rather than explicit loop calculations

# Outlook

- Understand (POL) $\leftrightarrow$ triviality relationship better
- Derive one-loop exactness directly within bootstrap, w/o regularization
- Systematics of trivial anomalies and their nonpolynomial on-shell realization
- Consider matter on external legs
- 3pt global-anomaly amplitudes as seeds for on-shell recursion to higher pts
- Combine with on-shell methods for Goldstone-boson amplitudes coupling to anomalous current (e.g. axion coupling to Lorentz)

Thank you!