Holographic correlators and AdS amplitudes

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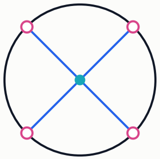
THE HOLOGRAPHIC AMPLITUDE PROBLEM



bulk physics in an AdS box

- In flat space, amplitudes are defined by asymptotic in/out states.
- In AdS, excitations reach the boundary and return: the natural gauge-invariant observables are boundary correlators.
- The amplitude question becomes: which CFT data reorganises itself as local bulk scattering?

Study correlators to understand AdS amplitudes.

Maldacena; Gubser–Klebanov–Polyakov; Witten

THE STRATEGY OF THIS TALK

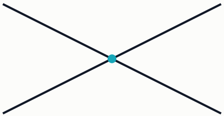
- Start from the AdS/CFT dictionary: fields, masses and couplings become CFT data.
- Use boundary consistency—symmetries, OPE, crossing, analyticity and unitarity—as the computational input.
- Choose the representation adapted to the question: position space, Mellin space, dispersive sum rules or composite correlators.

The goal is not to avoid the bulk, but to find coordinates in which the bulk answer is simple.

WHY ARE AdS AMPLITUDES DIFFERENT?

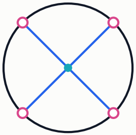
FLAT SPACE

- Asymptotic in/out states at null infinity.
- Momentum conservation and an ordinary S-matrix.
- Factorisation poles and unitarity cuts.

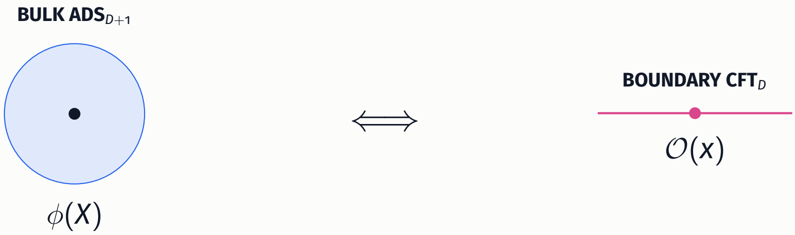


ANTI-DE SITTER SPACE

- No standard asymptotic region: excitations reach the boundary and return.
- The natural observables are boundary correlation functions.
- Flat-space scattering is recovered only in a limit.



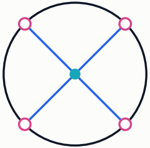
THE AdS/CFT DICTIONARY



$m^2R^2 = \Delta(\Delta - d)$	$\leftrightarrow$	operator dimension Δ
bulk interaction	$\leftrightarrow$	OPE coefficient λ_{ijk}
G_N/R^{d-1}	$\leftrightarrow$	inverse central charge

Maldacena; Gubser–Klebanov–Polyakov; Witten

CORRELATORS ARE THE AdS SCATTERING OBSERVABLES



bulk process in a finite "box"

$$\Longleftrightarrow$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$

boundary four-point function

Position space Mellin space flat-space / bulk-point limit

THE CFT INPUT

SPECTRUM

$$\{\Delta_i, \ell_i, \mathcal{R}_i\}$$

- which primary operators exist;
- their dimensions, spin and global charges.

$$+$$

OPE COEFFICIENTS

$$\{\lambda_{ijk}\}$$

- the three-point couplings;
- the boundary image of bulk interactions.

In principle, spectrum $+$ OPE coefficients determine every correlator.

FOUR-POINT FUNCTIONS ORGANISE THE SPECTRUM

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \mathcal{K}(x_i)\,\mathcal{G}(u,v)$$

$$\mathcal{G}(u,v) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}\, G_{\Delta,\ell}(u,v)$$

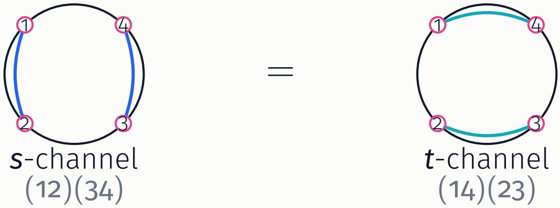
CROSS-RATIOS

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

CONFORMAL BLOCKS

- fixed by symmetry;
- each block isolates one primary and its descendants.

CROSSING IS BULK CONSISTENCY



$$\sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} G^{(s)}_{\mathcal{O}}(u,v) = \sum_{\mathcal{O}} \lambda_{14\mathcal{O}}\lambda_{23\mathcal{O}} G^{(t)}_{\mathcal{O}}(v,u)$$

Associativity of the OPE and crossing symmetry.

WHEN DOES A CFT HAVE A LOCAL BULK DUAL?

$C_T \gg 1$

large central charge

- factorisation of correlators;
- expansion in $1/C_T$;
- bulk loop expansion.

$+$

$\Delta_{\text{gap}} \gg 1$

large higher-spin gap

- finitely many light bulk fields;
- derivative expansion;
- approximate bulk locality.

$1/C_T$ counts loops

$1/\Delta_{\text{gap}}$ counts higher derivatives

A CONCRETE LABORATORY: $\mathcal{N} = 4$ SYM

CFT

four-dimensional $\mathcal{N} = 4$ SYM
gauge group $SU(N)$

- central charge $c \sim N^2$;
- 't Hooft coupling $\lambda = g^2_{\mathrm{YM}} N$;
- exact superconformal symmetry.

$\Longleftrightarrow$

BULK

type IIB string theory
on $AdS_5 \times S^5$

- $R^4/\alpha'^2 \sim \lambda$;
- $g_s \sim \lambda/N$;
- supergravity for $N, \lambda \gg 1$.

This is the best-controlled arena for connecting exact CFT data, AdS amplitudes and the ten-dimensional type-IIB S-matrix.

THE OPERATORS WE USE

HALF-BPS SINGLE-TRACE OPERATORS

$$\mathcal{O}_p(x,y) \sim y^{I_1} \cdots y^{I_p}\, \mathrm{Tr}(\phi_{I_1} \cdots \phi_{I_p}) + \cdots \qquad \Longleftrightarrow$$

$$\Delta = p, \qquad [0,p,0]_{SU(4)_R}$$

KALUZA–KLEIN SCALARS

$$m^2 R^2 = p(p-4)$$

- $p = 2$: stress-tensor / graviton multiplet;
- $p > 2$: higher spherical harmonics on S^5.

SUPERCONFORMAL SYMMETRY ISOLATES THE DYNAMICS

$$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle = \mathcal{K}(x_i, y_i)\, \mathcal{G}(u, v; \sigma, \tau)$$

$$\boxed{\mathcal{G} = \mathcal{G}_{\text{protected}} + \mathcal{R}(u, v; \sigma, \tau)\, \mathcal{H}(u, v; \sigma, \tau)}$$

$\mathcal{G}_{\text{protected}}$ fixed by short multiplets

$\mathcal{H}$ contains the long-multiplet / bulk dynamics

Dolan–Osborn; superconformal Ward identities

PROTECTED AND EXACT DATA FEED THE DYNAMICS

PROTECTED SECTOR

- short/BPS multiplets;
- Ward identities;
- protected OPE data.

EXACT INPUT

- integrable spectrum;
- localisation constraints;
- Regge/string trajectories.

DYNAMICAL OUTPUT

- long-multiplet data;
- anomalous dimensions;
- OPE coefficients.

$$\text{protected data} \quad + \quad \text{integrability / localisation} \quad + \quad \text{crossing} \quad \Longrightarrow \quad \mathcal{H}_{\text{dyn}}$$

In supersymmetric holography, symmetry is not just a simplification: it supplies exact data for the dynamical correlator.

Beem–Rastelli–van Rees; Beem–Lemos–Rastelli–van Rees; Caron-Huot–Coronado–Trinh–Zahraee

TWO PERTURBATIVE EXPANSIONS

$$\mathcal{H} = \mathcal{H}^{(0)} + \frac{1}{c}\mathcal{H}^{(1)} + \frac{1}{c^2}\mathcal{H}^{(2)} + \cdots$$

disconnected tree-level AdS one-loop AdS

At large N, the key reconstructed data are usually double-trace data: $[\mathcal{O}\mathcal{O}]_{n,\ell}$, with $\Delta = 2\Delta_{\mathcal{O}} + 2n + \ell + \gamma_{n,\ell}$.

$$\mathcal{H}^{(1)} = \mathcal{H}_{\rm sugra} + \lambda^{-3/2}\mathcal{H}_{R^4} + \lambda^{-5/2}\mathcal{H}_{D^4R^4} + \lambda^{-3}\mathcal{H}_{D^6R^4} + \cdots$$

loops are counted by $1/c$; string corrections by $1/\sqrt{\lambda}$.

POSITION SPACE: THE LORENTZIAN INVERSION FORMULA

$$c(\Delta, \ell) \sim \int_0^1 dz\, d\bar{z}\; \mu(z, \bar{z})\; \mathrm{dDisc}[\mathcal{G}(z, \bar{z})]$$

INPUT	**KERNEL**	**OUTPUT**
crossed-channel singularities	analyticity in spin	poles $\Delta_{\mathcal{O}}$ and residues $a_{\mathcal{O}}$

At loop level, the double discontinuity isolates logarithmic terms fixed by lower-order OPE data.

Caron-Huot; Simmons-Duffin–Stanford–Witten

HOW TO USE THE INVERSION FORMULA

1. FIND DDISC

$\mathrm{dDisc}\,\mathcal{G}$

crossed-channel logs and singularities

2. INVERT

$c(\Delta, \ell)$

analytic function of spin and dimension

3. READ POLES

$$c(\Delta, \ell) \sim \frac{a_{\mathcal{O}}}{\Delta - \Delta_{\mathcal{O}}}$$

dimensions and OPE coefficients

4. RECONSTRUCT

$\mathcal{G}$

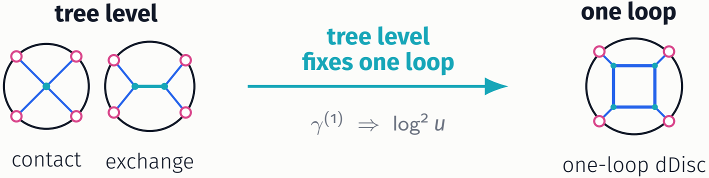
crossing plus contact-term ambiguities

tree data $\gamma^{(1)}$ $\Longrightarrow$ one-loop $\log^2 u$ $\Longrightarrow$ one-loop OPE data

In practice, the formula turns lower-order discontinuities into higher-order CFT data.

WHY THE ONE-LOOP dDisc IS FIXED

$$u^{\frac{1}{2}(\Delta^{(0)}+c^{-1}\gamma^{(1)}+\cdots)} = u^{\Delta^{(0)}/2}\left[1+\frac{\gamma^{(1)}}{2c}\log u+\frac{(\gamma^{(1)})^2}{8c^2}\log^2 u+\cdots\right]$$



The one-loop double discontinuity is fixed by tree-level OPE data; only contact-term ambiguities remain.

Alday–Bissi; Aprile–Drummond–Heslop–Paul

WHY MELLIN SPACE?

POSITION SPACE KNOWS

- OPE limits $u \to 0$, $v \to 1$;
- crossing as equality of functions;
- logarithms from anomalous dimensions.

MELLIN SPACE EXPOSES

- poles from exchanged AdS states;
- residues that factorise into OPE data;
- polynomial ambiguities as local contact terms.

OPE singularities $\longleftrightarrow$ Mellin poles $\longleftrightarrow$ bulk factorisation

Mellin space is the representation where AdS locality looks most like amplitudes.

Mack; Penedones

MELLIN / MANDELSTAM VARIABLES AND CFT DATA

MELLIN REPRESENTATION

$$\mathcal{G}(u,v) = \int \frac{ds\,dt}{(4\pi i)^2}\, u^{s/2} v^{t/2}$$
$$\times \Gamma_{p_i}(s,t)\,\mathcal{M}(s,t)$$

- Γ_{p_i}: universal kinematics;
- $\mathcal{M}$: dynamical AdS amplitude;
- $s + t + \tilde{u} = \sum_i \Delta_i$.

CFT MEANING OF MANDELSTAM CHANNELS

s	twist flowing in the $12 \to 34$ OPE channel
t	twist in a crossed channel
poles	$s = \Delta - \ell + 2m$ for primaries and descendants
residues	$\lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}\, Q_{\ell,m}(t)$

Mellin variables are finite-AdS Mandelstam invariants: their poles are fixed by exchanged CFT twists.

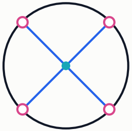
Mack; Penedones; Fitzpatrick–Kaplan–Penedones–Raju–van Rees

TREE-LEVEL MELLIN AMPLITUDES BECOME SIMPLE

$$\mathcal{M}_{p_1 p_2 p_3 p_4} = \sum_{i,j} \sigma^i \tau^j \, \mathcal{M}_{ij}(s,t),$$

$$\mathcal{M}_{ij}(s,t) = \sum_{m,n} \frac{R_{ijmn}}{(s-s_m)(t-t_n)} + \text{crossed.}$$

- compact formula for arbitrary external weights p_i;
- poles fixed by the exchanged KK spectrum;
- no need to reconstruct the complete five-dimensional effective action.



exchange poles + contact terms

Rastelli–Zhou

SUPERSYMMETRY CAN BE IMPOSED DIRECTLY IN MELLIN SPACE

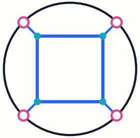
- Superconformal Ward identities become difference constraints on $\mathcal{M}(s,t)$.
- The construction is universal for SCFTs in $3 \leq d \leq 6$.
- It gives a complementary route to holographic correlators in $AdS_4 \times S^7$, $AdS_5 \times S^5$ and $AdS_7 \times S^4$.

Symmetry acts before the inverse Mellin transform, where factorisation is manifest.

Xinan Zhou

LOOP AMPLITUDES FROM TREE-LEVEL DATA

- Tree-level anomalous dimensions fix the one-loop double discontinuity.
- The full loop correlator is reconstructed by OPE/crossing, up to local contact ambiguities.
- Mixing requires the relevant tree-level KK family, not only one correlator.
- In Mellin space this becomes a compact one-loop amplitude with simultaneous poles.



one-loop box in AdS

$$\mathcal{M}^{1-\text{loop}}(s,t) \sim \sum_{m,n} \frac{c_{mn}}{(s-s_m)(t-t_n)}$$

tree OPE data $\longrightarrow$ dDisc / unitarity $\longrightarrow$ one-loop AdS amplitude

Alday–Bissi; Aprile–Drummond–Heslop–Paul; Alday–Zhou

DISPERSIVE SUM RULES: WHAT THEY DO

$$\text{crossing equation} \xrightarrow{\text{linear functional } \omega} \textstyle\sum_{\mathcal{O}} a_{\mathcal{O}}\, \omega_{\Delta,\ell} = 0$$

WHAT ARE THEY?

Exact projections of crossing. A functional ω acts on the conformal-block expansion and gives a weighted identity for CFT spectral data.

WHY USEFUL?

They isolate one target coefficient. Zeros of ω remove known generalized-free terms; the dDisc gives positive/absorptive input from heavy states.

HOW TO USE THEM?

Insert the known CFT data explicitly. Choose kernels that suppress or sign-control the unknown spectrum. Then obtain bounds, or with enough input reconstruct the target AdS/string coefficient.

Main advantage: replace a hard bulk computation by a controlled boundary spectral sum.

Caron-Huot–Mazáč–Rastelli–Simmons-Duffin

SUM RULES RECONSTRUCT THE AdS VIRASORO–SHAPIRO AMPLITUDE

- Keep the leading order in $1/c$, but include all orders in $1/\lambda$.
- Mellin dispersion relations produce an infinite family of sum rules.
- Low-energy coefficients are determined by CFT data of heavy string operators.
- Integrability and localisation provide the first non-trivial data and select a unique solution.

$$\mathcal{M}^{\mathrm{AdS}}_{\mathrm{string}} = \mathcal{M}_{\mathrm{sugra}} + \lambda^{-3/2}\mathcal{M}_{R^4} + \lambda^{-5/2}\mathcal{M}_{D^4R^4} + \cdots$$

Alday–Hansen–Silva; Alday–Hansen

STRING AMPLITUDES FROM LOCALISATION AND MODULARITY

INTEGRATED CFT CONSTRAINTS

- derivatives of the mass-deformed S^4 partition function;
- exact dependence on the Yang–Mills coupling;
- constraints on separated-point correlators.

BULK INTERPRETATION

- R^4 and D^4R^4 string corrections;
- Eisenstein series and $SL(2,\mathbb{Z})$ invariance;
- flat-space limit matches the type-IIB S-matrix.

Punch line: exact CFT data fixes finite-AdS string corrections; the ten-dimensional S-matrix is recovered as a limit.

Binder–Chester–Pufu–Wang; Alday–Chester–Hansen

HIGHER-POINT DATA IS BECOMING A GENERATING ENGINE

- AdS$\times S$ Mellin bootstrap gives unified tree-level formulas for five-point half-BPS correlators with arbitrary KK weights.
- Hidden ten-dimensional symmetry and an AdS$\leftrightarrow S$ structure organise whole KK families at once.
- OPE limits of higher-point single-particle correlators generate lower-point data with composite or descendant external states.
- The same method is directly aimed at generic multipoint tree-level computation.

five-point single-particle data $\longrightarrow$ families of four-point amplitudes

Gonçalves–Meneghelli–Pereira–Vilas Boas–Zhou; Fernandes–Gonçalves–Huang–Tang–Vilas Boas–Yuan

NON-SUSY LOOPS: TREE DATA FIXES CUTS

ONE LOOP FROM TREE LEVEL

- no Ward-identity or localisation shortcut;
- input: tree-level OPE data in a generic large-N CFT;
- one-loop double-trace data is completely fixed by tree level;
- Mellin: one-loop polar/cut part reconstructed.

WHAT CHANGES AT LOOPS

- double-trace anomalous dimensions have infinite spin support;
- scalar AdS labs: ϕ^4 and $\phi^3 + \phi^4$;
- higher-loop consecutive cuts are again fixed by tree data;
- flat-space limit: bubble chains and ordinary unitarity cuts.

Tree-level CFT data fixes the one-loop cut and the higher-loop consecutive cuts.

Aharony–Alday–Bissi–Perlmutter; Bissi–Fardelli–Khansari

INTEGRABILITY AS INPUT FOR CORRELATORS

- Planar integrability gives finite-coupling spectral data: dimensions, Regge trajectories and heavy-string states.
- Crossing and dispersive sum rules turn this spectral input into correlator data.
- Conformal-collider bootstrap combines perturbation theory, holography, localisation, integrability and numerical bootstrap for the energy-energy correlator.
- Coulomb-branch scattering uses integrability data as input for a finite-coupling amplitude bootstrap.

integrable spectrum $+$ sum rules / bootstrap $\longrightarrow$ finite-coupling correlators

Caron-Huot–Coronado–Trinh–Zahraee; Dempsey–Karlsson–Pufu–Zahraee–Zhiboedov; Alday–Armanini–Häring–Zhiboedov

MULTI-PARTICLE STATES

SINGLE TRACE

one particle

DOUBLE TRACE

two particles

HIGHER TRACE

multi-particle sector

- OPE limits of higher-point correlators generate composite external states.
- Large-N consistency can fix correlators even when the leading OPE includes higher-particle sectors.
- Composite-operator correlators make the triple-trace sector and multi-particle mixing explicit.
- This is one frontier among several, not the whole recent story.

Aprile–Giusto–Russo–Vilas Boas; Bissi–Fardelli–Manenti; Bissi–Fardelli–Puerta Ramisa

FOUR RECENT FRONTIERS

Higher point

- five-point KK data;
- multipoint bootstrap;
- hidden 10d symmetry.

Spin

- spinning correlators;
- scaffolding;
- AdS gluons / gravitons.

Non-SUSY

- tree $\rightarrow$ loops;
- infinite spin support;
- consecutive cuts.

Integrability

- finite coupling;
- collider correlators;
- Regge/string data.

The field is moving from isolated four-point scalar answers to families of AdS amplitudes.

WHY NOT JUST COMPUTE WITTEN DIAGRAMS?

DIRECT WITTEN-DIAGRAM ROUTE

- requires many KK couplings and exchange diagrams;
- contact terms and field redefinitions obscure uniqueness;
- loop AdS integrals and mixing quickly become hard;
- simple symmetries often appear only after large cancellations.

CORRELATOR / AMPLITUDE ROUTE

- crossing, OPE and symmetries are imposed from the start;
- Mellin poles make factorisation and locality visible;
- sum rules and localisation fix stringy coefficients directly;
- whole families of correlators are solved at once.

Boundary methods are often better coordinates on the same bulk physics.

Penedones; Rastelli–Zhou; Caron-Huot–Mazáč–Rastelli–Simmons-Duffin; Alday–Chester–Hansen

CONCLUSIONS

- The AdS/CFT dictionary makes boundary correlators the natural definition of AdS amplitudes.
- OPE, crossing, analyticity and exact input reconstruct tree and loop data without the full bulk Lagrangian.
- Mellin amplitudes expose AdS factorisation; dispersive sum rules and localisation determine string corrections.

What is the right amplitude basis for AdS correlators? How to systematise the correlator/amplitude description at finite coupling?

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