Perturbative Yang-Mills Theory in de Sitter spacetime

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mostly hep-th 2604.24844

quantum YM but gravity as a classical background

I wonder if someone

had already told you . . .

"S-matrix does not exist"

$m = 0$ $\qquad E, \vec{p} \to 0$

Infrared divergences

zero frequency, ∞ wavelength

That person (agent ?) forgot

that spacetime is curved

It is difficult to develop S-matrix with arbitrary curvature

main problem : no guiding symmetry principles

QFT relies on Poincaré

(all inertial observers are equivalent)

A tractable case study
(and approximation)

constant curvature

maximal symmetry group dimension 10

→ de Sitter

global or Poincaré patch

formalism developed
T.T., Zhu 2025-6
see also:
Marolf, Morrison, Srednicki
(2013)

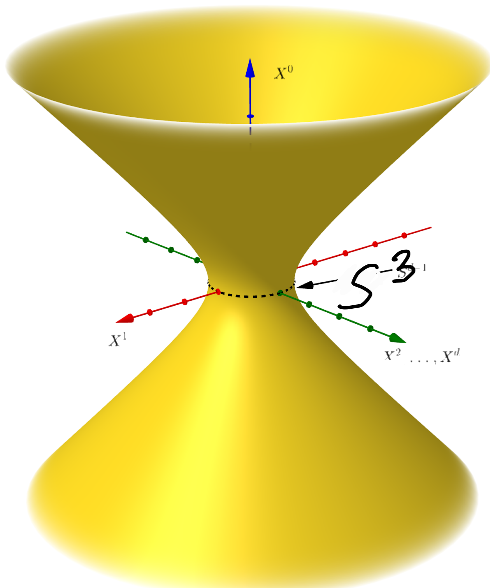
$dS_4 \subset \mathbb{R}^5$

$-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = \ell^2$

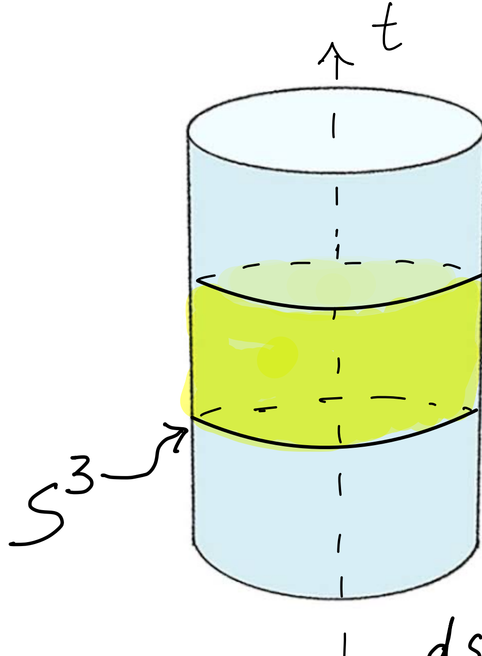
$SO(1,4)$ ≡ Lorentz in 5D

strip $t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ of Einstein Cylinder

$\mathbb{R} \times S^3$



$\underset{\text{conf.}}{\cong}$



$ds^2 = -dt^2 + d\Omega^2$

Quantization : QFT in deS background

Gravity Classical

Hilbert space : UIRs of $SO(1, \underbrace{4}_{})$

$$\mathcal{H} = \bigoplus_{j, j' \in \Gamma} \underbrace{\mathcal{H}_{j,j'}}_{|j\nu\rangle |j'\nu'\rangle} \quad \text{of} \quad SU(2) \times SU(2)'$$

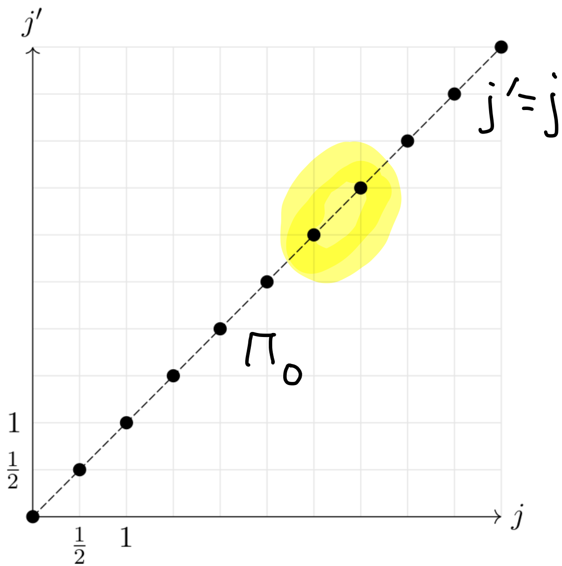
basis

$\Gamma \longleftrightarrow$ UIR

J. Dixmier (1961)

Audrey, Lindsay, T (2026)

Physical UIRs

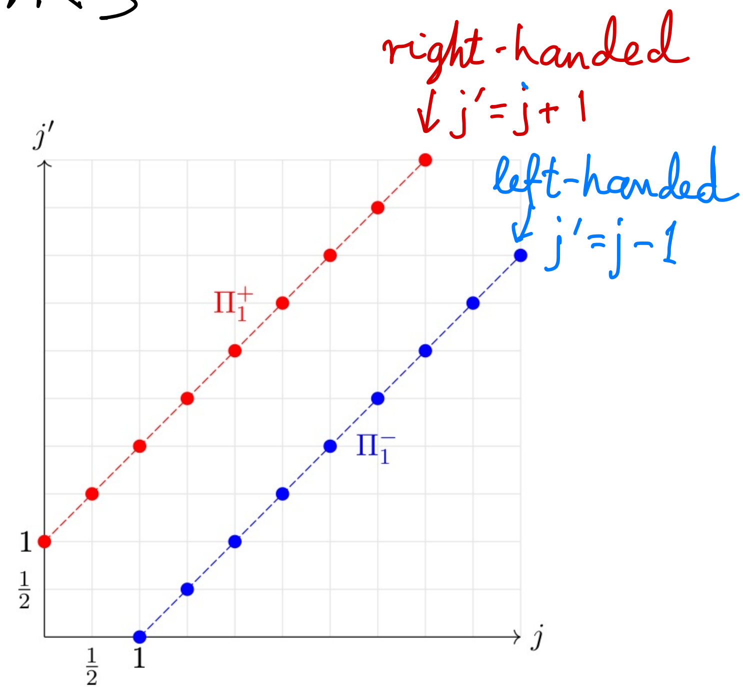


Spin 0 principal series representation.

Principal Series

scalars $m \geq 0$

Π_0



Spin 1 representations for polarized gauge bosons.

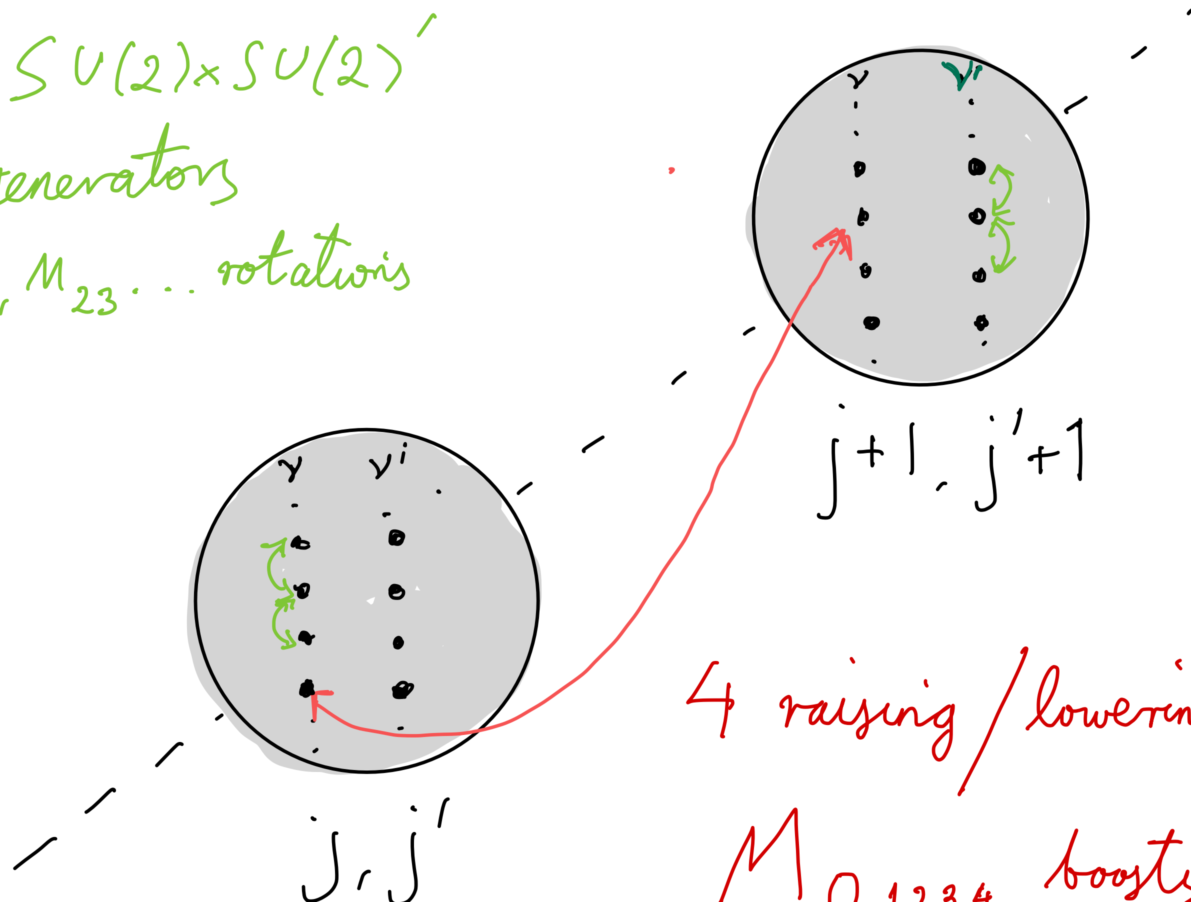
Gauge bosons

"GLUONS" $m=0$

$\Pi_1^{\pm}$

6 $SU(2) \times SU(2)'$ generators

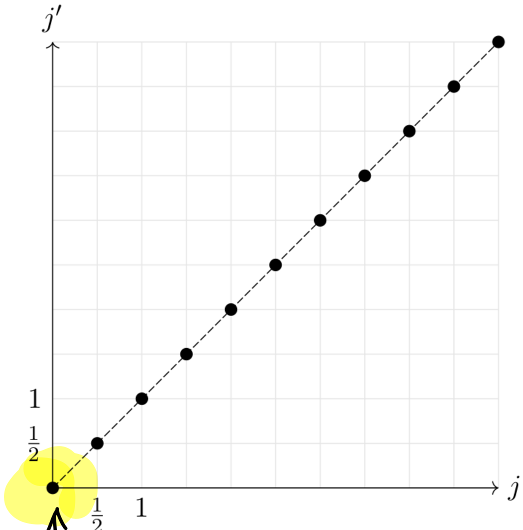
$M_{12}, M_{23} \ldots$ rotations



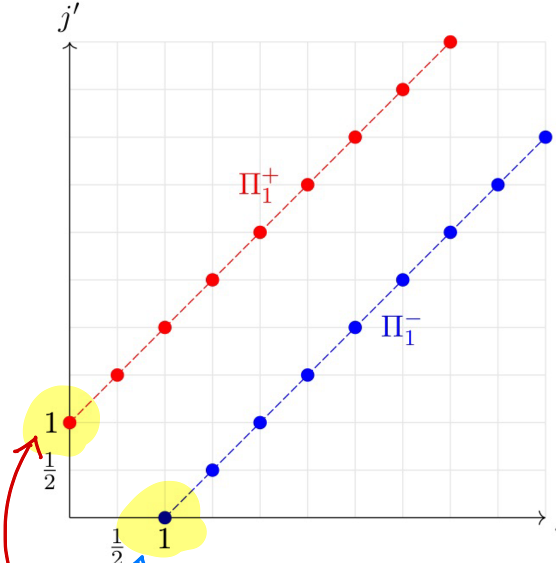
4 raising/lowering

$M_{0\,\underline{1,2,3,4}}$ boosts

Frequency ~ energy ~ $j + j'$ ~ ω



becomes singlet
as $m = 0$

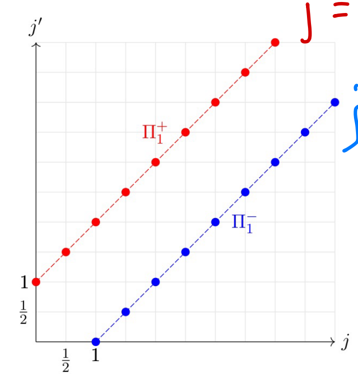


$\omega \geq 1$

A. Lindsay, T (2026)
2512.13781

no infrared "soft" particles in de S

Yang - Mills theory



$j' = j+1$

$j' = j-1$

$$A = A_\mu \, dx^\mu$$

Lie algebra valued

Rios-Fukelman, Sempé, Silva (2024)
see also: Higuchi, ...

$$F = dA + A \wedge A$$

action
$$S = \frac{1}{2g^2} \int \mathrm{Tr}\,(F \wedge * F)$$

$dS \cong \mathbb{R}_t \times S^3$

Classical $\rightarrow$ Quantum

positive frequency solutions $\rightarrow$ wave functions

In Coulomb gauge $A_t = 0$:

$$A = e^{-i\omega t} E$$

$\hookrightarrow$ one form in $T^* S^3$

Maxwell (Hodge Laplace) Equations

$$(\delta d + d\delta) E = k^2 E$$

$$k = \omega$$

in radiation gauge $\quad \partial^i A_i = 0$

$$\delta E = 0$$

"transverse solutions"

E : transverse covector harmonics

- integer $k \geqslant 2$ $\quad (\omega \geqslant 2$, no IR ✓$)$

- two classes: right- and left-handed $\pm$

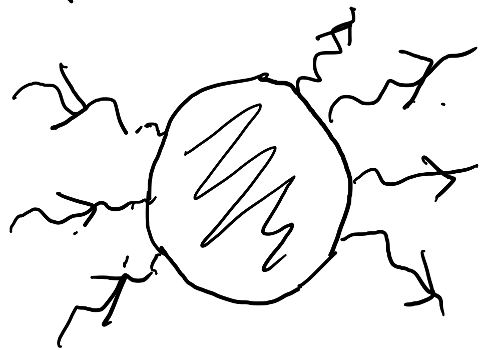
$$*dE^{\pm} = \pm k E^{\pm}$$

- Each level k degenerate

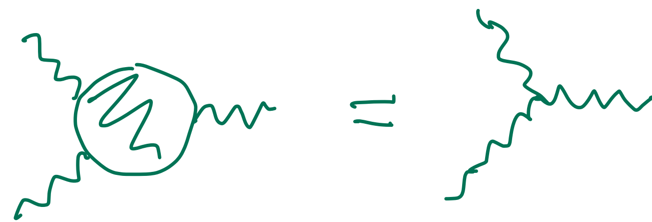
$$\left.E^{\pm}_{(j\nu)(j'\nu')} \longleftrightarrow |j\,\nu\rangle |j'\,\nu'\rangle \in \Pi^{\pm}_{1}\right\} k = j + j' + 1$$

S-matrix

$$\langle \text{in} \mid \text{out} \rangle$$

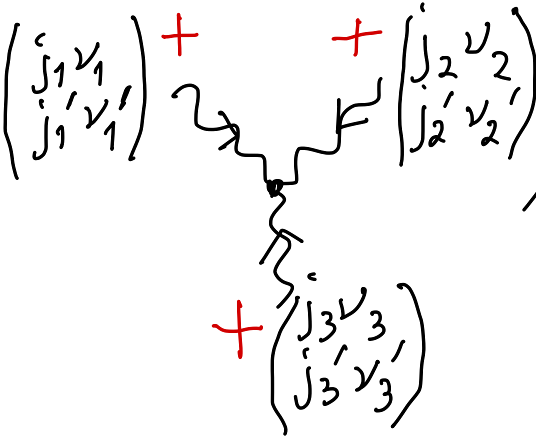


In flat spacetime,
all tree amplitudes can be
constructed by BCFW recursion
from three-gluon amplitudes



deSitter S-matrix
constructed in T.T & Bin Zhu
2411.02504, 2509.25407
see also Marolf, Morrison, Srednicki (2013)

3 gluons in de S



"overlap of wavefunctions"

$$ig \sum_{\text{Perms}(1,2,3)} \mathrm{Tr} \int_{\mathbb{R}_t \times S^3} A_{a_1} \wedge A_{a_2} \wedge * \underbrace{dA_{a_3}}_{= k_3 A_{a_3}}$$

$$\mathcal{M}(1^+_{in} 2^+_{in} 3^+_{in}) = -ig f_{a_1 a_2 a_3} (k_1 + k_2 + k_3) \int e^{-it(\omega_1+\omega_2+\omega_3)} dt \wedge E_1^+ \wedge E_2^+ \wedge E_3^+$$

$$= -2ig f_{a_1 a_2 a_3} \frac{\sin[(\omega_1+\omega_2+\omega_3)\frac{\pi}{2}]}{\omega_1+\omega_2+\omega_3} (k_1+k_2+k_3)\, I_3\,,$$

$$I_3 = \int_{S^3} E^+_{(j_1,\nu_1)(j_1',\nu_1')} \wedge E^+_{(j_2,\nu_2)(j_2',\nu_2')} \wedge E^+_{(j_3,\nu_3)(j_3',\nu_3')}\,.$$

Intertwiner

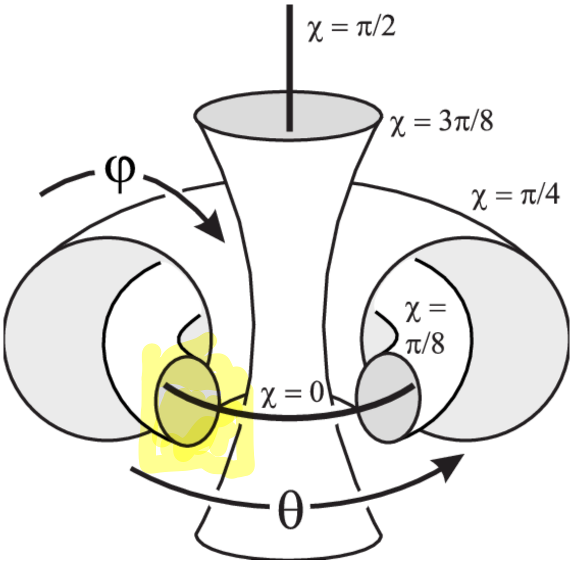
$$I_3 = \int_{S^3} E^+_{(j_1,\nu_1)(j'_1,\nu'_1)} \wedge E^+_{(j_2,\nu_2)(j'_2,\nu'_2)} \wedge E^+_{(j_3,\nu_3)(j'_3,\nu'_3)} \, .$$

$$T^* S^3 \longrightarrow \text{Hilbert}$$

$SU(2) \times SU(2)'$ singlet

important in representation theory, Langlands ..

but need explicit expression ☺



I_3 can be computed (with little help of Akshay Venkatesh, X. Griffin Wang)

in toroidal ≡ Hopf coordinates

$\theta, \varphi \in (0, 2\pi)\,, \quad \chi \in (0\ \frac{\pi}{2})$

← maximal torus $SU(2) \times SU(2)'$

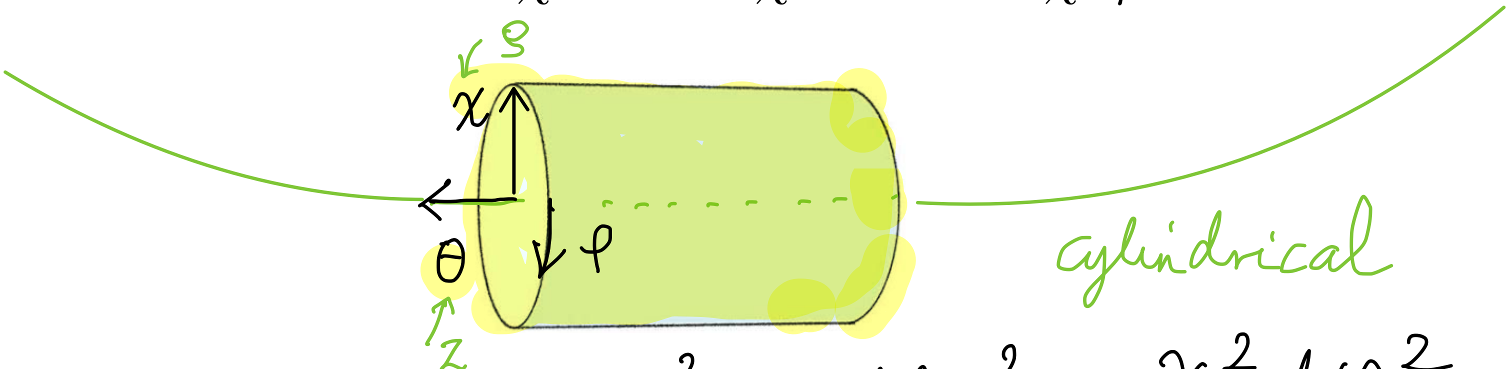
$$d\Omega^2 = d\chi^2 + \cos^2\chi d\theta^2 + \sin^2\chi d\varphi^2.$$

Spherical harmonics → covector harmonics from scalar harmonics

Lehoucq, Uzan, Weeks (2003) → Ben Achour et al (2016)

Comment: Flat limit

$$d\Omega^2 = d\chi^2 + \cos^2\chi\, d\theta^2 + \sin^2\chi\, d\varphi^2.$$



$$d\Omega^2 \approx d\theta^2 + d\chi^2 + \chi^2 d\varphi^2$$

In (angular) momentum space:

$k \approx p_\rho^2 \quad l \approx p_z^2 \quad \ldots \longrightarrow$ flat $\equiv$ large ang. momentum (Jacobi-Anger)

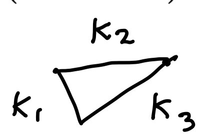
$$I_3 = \int_{S^3} E^+_{(j_1,\nu_1)(j'_1,\nu'_1)} \wedge E^+_{(j_2,\nu_2)(j'_2,\nu'_2)} \wedge E^+_{(j_3,\nu_3)(j'_3,\nu'_3)} = \underbrace{?}_{SU(2)\times SU(2)' \text{ singlet}}$$

Unique 3j symbol

Wigner 3j: $$\begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} = \frac{(-1)^{j_1-j_2-n_3}}{\sqrt{2j_3+1}} \underbrace{\langle j_1 n_1, j_2 n_2 | j_3 \, -n_3 \rangle}_{\text{Clebsch-Gordan}} \, .$$

$$I_3 = \frac{i\mathsf{S}}{\pi}\sqrt{s^2-1} \begin{pmatrix} j_1 & j_2 & j_3 \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \begin{pmatrix} j'_1 & j'_2 & j'_3 \\ \nu'_1 & \nu'_2 & \nu'_3 \end{pmatrix} ,$$

$$\mathsf{S} = \sqrt{s(s-k_1)(s-k_2)(s-k_3)} \, , \qquad s = \frac{k_1+k_2+k_3}{2}$$

area of 

$$k = j + j' + 1$$

All three-gluon amplitudes

$$\begin{array}{llcll} \lambda = +1 & \text{helicity} + & \qquad & \epsilon = +1 & \text{incoming} \\ \lambda = -1 & \text{helicity} - & & \epsilon = -1 & \text{outgoing} \end{array}$$

$$\lambda_k = \frac{\lambda_1 k_1 + \lambda_2 k_2 + \lambda_3 k_3}{2}\,, \qquad \epsilon_k = \frac{\epsilon_1 k_1 + \epsilon_2 k_2 + \epsilon_3 k_3}{2}$$

$$\mathcal{M}(1^{\lambda_1}_{\epsilon_1}\, 2^{\lambda_2}_{\epsilon_2}\, 3^{\lambda_3}_{\epsilon_3}) = 2g f_{a_1 a_2 a_3}\, \lambda_k \sqrt{\lambda_k^2 - 1}\, \mathsf{S} \begin{pmatrix} j_1 & j_2 & j_3 \\ \epsilon_1 \nu_1 & \epsilon_2 \nu_2 & \epsilon_3 \nu_3 \end{pmatrix} \begin{pmatrix} j_1' & j_2' & j_3' \\ \epsilon_1 \nu_1' & \epsilon_2 \nu_2' & \epsilon_3 \nu_3' \end{pmatrix} \frac{\sin \epsilon_k \pi}{\epsilon_k \pi}\,.$$

selection rules $\rightarrow \epsilon_k \in \mathbb{N}$

$$\Rightarrow \mathcal{M} = 0$$

but in an interesting way ☺

Recall

$$\cdots \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle} \delta^{(4)}(p_1 + p_2 + p_3) \; ? \qquad = 0$$

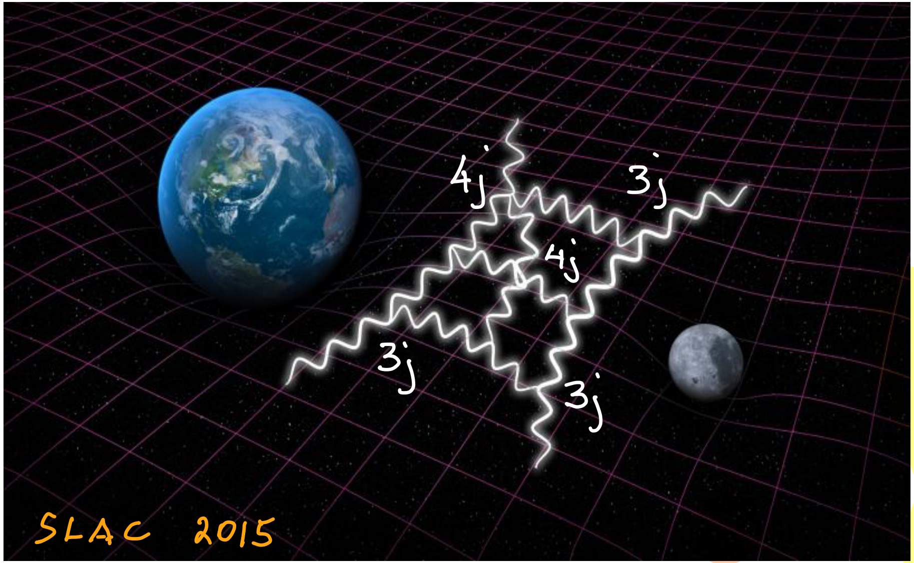
Outlook: Multigluon amplitudes

(will be able to do
flat limit
helicity formalism)

Feynman rules $\longleftrightarrow$ graphical theory of angular momentum

$\downarrow$

3j, 6j (c.f. Venkatesh)



new insights into IR
modifications of soft thms
spin foam of LQG ???

Thank you!

Dedicated to Jurek Lewandowski 1959 - 2024