Analytic superspace applied to non SUSY CFTs

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based on: 2512.14618 and WIP with Hector Puerta Ramisa

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Motivation

- CFT correlators with spin
- Dual to amplitudes: (A)dS/CFT, cosmology
- Relatively unexplored territory (compared to scalar case)
- Recent related work: correlators in twistor space / Grassmannian approach Baumann, Mathys, Pimentel, Rost (2024); Bala, Jain, Dhruva (2026);Carrillo González, Keseman (2026)
- SCFT bootstrap beyond half-BPS operators
- Even less well explored territory!
- Analytic superspace: powerful formalism
- Recent surprise: analytic superspace techniques simplify non-SUSY case too!
- Recent surprise 2: non-square analytic superspace

- **Warm up: CFT (spinor formalism)**
- **Analytic superspace**
- **Non-square superspace**
- **Conclusions**

CFT data (4D)

- CFT data: spectrum of local operators:

$$\mathcal{O}^{\tau}_{s,\bar{s}} = \mathcal{O}^{\tau}_{\underbrace{(\alpha\beta\ldots\gamma)}_{s},\underbrace{(\dot{\alpha}\dot{\beta}\ldots\dot{\gamma})}_{\bar{s}}} \qquad \tau = \Delta - \tfrac{1}{2}(s+\bar{s})$$

- and three-point tensor structures, eg

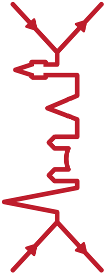
$$\langle \mathcal{O}^{\tau_1}_{\alpha_1\dot{\alpha}_1}(x_1)\mathcal{O}^{\tau_2}_{\alpha_2\dot{\alpha}_2}(x_2)\mathcal{O}^{\tau_3}_{\alpha_3\dot{\alpha}_3}(x_3)\rangle = \sum_i \lambda^{(i)}_{123}\langle\!\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\rangle\!\rangle^{(i)}_{\alpha_1\dot{\alpha}_1\alpha_2\dot{\alpha}_2\alpha_3\dot{\alpha}_3}$$

- Kinematics: classify allowed 3-point ‘tensor structures’
- Dual to 3-point amplitudes in different EFTs

CFT Bootstrap

$$\sum \lambda^{(i)}_{12\mathcal{O}}\lambda^{(i)}_{\mathcal{O}34}$$

$$= \sum \lambda^{(i)}_{13\mathcal{O}}\lambda^{(i)}_{\mathcal{O}24}$$



(With apologies for the abuse of the logo)

Spinor notation

- Conformal transformations simple in spinor formalism

$$\delta x^{\alpha\dot\alpha} = P^{\alpha\dot\alpha} + L^{\alpha}{}_{\beta}\, x^{\beta\dot\alpha} + x^{\alpha\dot\beta}\, \bar L_{\dot\beta}{}^{\dot\alpha} + x^{\alpha\dot\beta}\, K_{\beta\dot\beta}\, x^{\beta\dot\alpha}$$

- Simpler than Lorentz, or even embedding space
- eg Lorentz: $\delta x^{\mu} = 2K.xx^{\mu} - K^{\mu}x^{2}$.

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- Simpler than Lorentz, or even embedding space
- eg Lorentz: $\delta x^\mu = 2K.x x^\mu - K^\mu x^2$.
- Everything is a 2×2 matrix, $x, P, L, \bar L, K$ therefore index free:

$$\delta x = P + L\, x + x\, \bar L + x\, K\, x$$

- Matrix version of Möbius transformation

Operator transformations simple

- Defining $x_{ij} := x_i - x_j$:

$$\delta x_{ij} = L_i\, x_{ij} + x_{ij}\, \bar{L}_j \qquad L_i := L + x_i K, \qquad \bar{L}_j := \bar{L} + K x_j .$$

- conformal $\equiv$ pointwise 'local' Lorentz, $L_i, \bar{L}_i$
- Operators transform according to their indices

$$\delta \mathcal{O}^{\tau}_{s,\bar{s}}(x_i) = -(\det L_i)^{\tau} \mathcal{O}^{\tau}_{s,\bar{s}} - L_i \cdot \mathcal{O}^{\tau}_{s,\bar{s}} - \bar{L}_i \cdot \mathcal{O}^{\tau}_{s,\bar{s}}$$

Solving Ward identities simple

- Building blocks:

$$(x_{ij})^{\alpha\dot{\alpha}}, \quad (I_{ji})_{\dot{\alpha}\alpha} := (x_{ij}^{\alpha\dot{\alpha}})^{-1}, \quad x_{ij}^2$$

- Ward identities $\equiv$ “match up indices pointwise”
- Two-point Ward identities admit a unique solution:

$$\langle \mathcal{O}^{\tau}_{s,\bar{s}}(x_1)\mathcal{O}^{\tau}_{\bar{s},s}(x_2)\rangle = N\frac{I_{12}^{s}I_{21}^{\bar{s}}}{|x_{12}|^{2\tau}}$$

- Three-point functions allow new covariant structures

$$(Y_{i,jk})_{\dot{\alpha}\alpha} = (I_{ij}x_{jk}I_{ki})_{\dot{\alpha}\alpha} = (x_{ji}^{-1} - x_{ki}^{-1})_{\dot{\alpha}\alpha}$$

Three-point solutions

- eg
$$\langle\!\langle \mathcal{O}^{\tau_1}_{\alpha_1\dot\alpha_1}\mathcal{O}^{\tau_2}_{\alpha_2\dot\alpha_2}\mathcal{O}^{\tau_3}\rangle\!\rangle^{(1)} = P(Y_1)_{\dot\alpha_1\alpha_1}(Y_2)_{\dot\alpha_2\alpha_2}$$
$$\langle\!\langle \mathcal{O}^{\tau_1}_{\alpha_1\dot\alpha_1}\mathcal{O}^{\tau_2}_{\alpha_2\dot\alpha_2}\mathcal{O}^{\tau_3}\rangle\!\rangle^{(2)} = P(I_{12})_{\dot\alpha_1\alpha_2}(I_{21})_{\dot\alpha_2\alpha_1}$$
- General tensor structures for balanced ($\sum s = \sum \bar{s}$) unconstrained operators:

$$\boxed{\langle\!\langle \mathcal{O}^{\tau_1}_{s_1,\bar{s}_1}\mathcal{O}^{\tau_2}_{s_2,\bar{s}_2}\mathcal{O}^{\tau_3}_{s_3,\bar{s}_3}\rangle\!\rangle = P\prod_{i,j=1}^{3} V_{ij}^{\,b_{ij}}}$$

$$\sum_i b_{ij} = s_j, \sum_j b_{ij} = \bar{s}_i$$

$$P = \frac{1}{(x_{12}^2)^{\frac{\tau_1+\tau_2-\tau_3}{2}}(x_{23}^2)^{\frac{\tau_2+\tau_3-\tau_1}{2}}(x_{31}^2)^{\frac{\tau_3+\tau_1-\tau_2}{2}}},$$

$$V_{ij} = \begin{pmatrix} Y_1 & I_{12} & I_{13} \\ I_{21} & Y_2 & I_{23} \\ I_{31} & I_{32} & Y_3 \end{pmatrix}$$

Constraints?

- Number of tensor structures reduced by conservation constraints (irreps of conformal group) eg $\partial.j = 0$
- Complicated differential equations to solve
- Solutions $\rightarrow$ arbitrary basis

See Borovik, de Korte, Meurrens, Pavlov: for a recent algebraic geometry approach along those lines

- Warm up: CFT (spinor formalism)
- **Analytic superspace**
- Non-square superspace
- Conclusions

History of analytic superspace

- Harmonic superspace. Offshell $\mathcal{N}=2$ supergraphs:
  Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev (1984)
- Analytic superspace, onshell, applied to $\mathcal{N}=4$, Half BPS:
  Howe, Hartwell (1995)
- Anlaytic superspace for any multiplet via superindices:
  Howe, West (2001); PH (2001); Howe, PH (2001-2003)
- Applications:
  - Supercorrelator $\to$ super WL; hidden symmetry, (4 point integrands, $O(g^{12})$) Eden, Korchemsky, Sokatchev PH (2011)
  - Bootstrap higher point correlators Chcherin, Doobary, Eden, Korchemsky, Sokatchev, PH (2015); Fernandes, Goncalves, Huang, Tang, Vilas Boas, Ye Yuan (2025); Bargheer,Dian,Siebrecht (WIP)
  - Half BPS superblocks Doobary,PH (2015);Aprile, Drummond, Paul, Santagata (2017-); Aprile, Heslop (2021);
  - General (beyond HBPS) correlators: Howe,PH (2003);Hansen, Puerta Ramisa, PH (2025)

Analytic superspace: basic idea

- Basic idea:
  Lift 2×2 **matrices** $\rightarrow (m|n) \times (m|n)$ **supermatrices**
- Superconformal group
  $SU(2,2) \rightarrow SU(m,m|2n) =_{\mathbb{C}} SL(m|2n|m)$
- Additional compact internal coordinates y
- Super Grassmannian $Gr(2,4) \rightarrow Gr(m|n,\ m|2n|m)$
- Underlying rep theory SUSY version of Borel-Weil theorem

▸ Underlying (super)-group theory

Specific cases

- Unified treatment for arbitrary (m, n)
- $m = 2 \;\Rightarrow\; 4d, \mathcal{N} = 2n$
- $m = 2, n = 0 \;\Rightarrow\; 4D$ non-SUSY CFT
- $m = 1 \;\Rightarrow\;$ 1D SCFT (or chiral half of 2D)
- $m = 0 \;\Rightarrow\; SU(2n)$ theory pure finite rep theory

From Minkowski space to analytic superspace

- Spinor indices $\to$ superindices $(m|n)$

$$\alpha,\, \dot\alpha \in \{1,2\} \qquad \longrightarrow \qquad \mathfrak{a},\, \dot{\mathfrak{a}} \in \{1,\dots,m \mid m{+}1,\dots,m{+}n\}$$

- Coordinates: $x^{\alpha\dot\alpha} \to \mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}} = \begin{pmatrix} x^{\alpha\dot\alpha} & \theta^{\alpha\dot a} \\ \bar\theta^{a\dot\alpha} & y^{a\dot a} \end{pmatrix}$,

$\mathfrak{a} = (\alpha \mid a)$, $\dot{\mathfrak{a}} = (\dot\alpha \mid \dot a)$, $\alpha, \dot\alpha \le m$, $\quad a, \dot a > m$

- Superconformal transformation = super-matrix Möbius:

$$\delta\mathbf{x} = \mathbf{P} + \mathbf{L}\,\mathbf{x} + \mathbf{x}\,\bar{\mathbf{L}} + \mathbf{x}\,\mathbf{K}\,\mathbf{x}$$

- Operators have superindices $\mathcal{O}^\tau_{\alpha_1 \cdot\, \dot\alpha_1 \cdot}(x_i) \to \mathcal{O}^\gamma_{\mathfrak{a}_1 \cdot\, \dot{\mathfrak{a}}_1 \cdot}(\mathbf{x_i})$
- Operator transformations / Ward identities ($\det \to \mathrm{sdet}$)

$$\delta\mathcal{O}^\tau_{s,\bar s}(\mathbf{x}_i) = -(\mathrm{sdet}\,\mathbf{L}_i)^\tau \mathcal{O}^\tau_{s,\bar s} - \mathbf{L}_i \cdot \mathcal{O}^\tau_{s,\bar s} - \bar{\mathbf{L}}_i \cdot \mathcal{O}^\tau_{s,\bar s}$$

Superindices? Superdeterminant?

- Index parity (internal indices 'fermionic')

$$|\mathfrak{a}| = \begin{cases} 0, & \mathfrak{a} = \alpha \leq m, \\ 1, & \mathfrak{a} = a > m, \end{cases} \qquad |\dot{\mathfrak{a}}| = \begin{cases} 0, & \dot{\mathfrak{a}} = \dot{\alpha} \leq m, \\ 1, & \dot{\mathfrak{a}} = \dot{a} > m. \end{cases}$$

- Symmetrisation of superindices: antisymmetrised internal indices = symmetrised superindex!

$$T_{(\mathfrak{a}\mathfrak{b})} := \frac{1}{2}\Big(T_{\mathfrak{a}\mathfrak{b}} + (-1)^{|\mathfrak{a}||\mathfrak{b}|} T_{\mathfrak{b}\mathfrak{a}}\Big) \quad T_{[\mathfrak{a}\mathfrak{b}]} := \frac{1}{2}\Big(T_{\mathfrak{a}\mathfrak{b}} - (-1)^{|\mathfrak{a}||\mathfrak{b}|} T_{\mathfrak{b}\mathfrak{a}}\Big)$$

- $\operatorname{sdet} \left(\begin{array}{c|c} a & \beta \\ \hline \gamma & d \end{array}\right) := \frac{\det a + O(\beta,\gamma)}{\det d}$

Solving superconformal Ward identities

Also same form as for CFT with $|x_{ij}|^2 = \det x_{ij} \to \operatorname{sdet} \mathbf{x}_{ij}$

Recall CFT solution:

- Two-point Ward identities admit a unique solution

$$\langle \mathcal{O}^{\tau}_{s,\bar{s}}(x_1)\mathcal{O}^{\tau}_{\bar{s},s}(x_2)\rangle = N\frac{I_{12}^{s}I_{21}^{\bar{s}}}{|x_{12}|^{2\tau}} \qquad (I_{ji})_{\dot{\alpha}\alpha} := (x_{ij}^{\alpha\dot{\alpha}})^{-1}$$

- Three-point functions allow new covariant structures

$$(Y_{i,jk})_{\dot{\alpha}\alpha} = (I_{ij}x_{jk}I_{ki})_{\dot{\alpha}\alpha} = (x_{ji}^{-1} - x_{ki}^{-1})_{\dot{\alpha}\alpha}$$

Solving superconformal Ward identities

Also same form as for CFT with $|x_{ij}|^2 = \det x_{ij} \to \operatorname{sdet} \mathbf{x}_{ij}$

$\to$ SCFT solution:

- Two-point Ward identities admit a unique solution

$$\langle \mathcal{O}^{\tau}_{s,\bar{s}}(\mathbf{x}_1) \mathcal{O}^{\tau}_{\bar{s},s}(\mathbf{x}_2) \rangle = N \frac{I_{12}^{s} I_{21}^{\bar{s}}}{\operatorname{sdet}(\mathbf{x}_{12})^{\tau}} \qquad (I_{ji})_{\dot{\mathfrak{a}}\mathfrak{a}} := (\mathbf{x}_{ij}^{\mathfrak{a}\dot{\mathfrak{a}}})^{-1}$$

- Three-point functions allow new covariant structures

$$(Y_{i,jk})_{\dot{\mathfrak{a}}\mathfrak{a}} = (I_{ij} \mathbf{x}_{jk} I_{ki})_{\dot{\mathfrak{a}}\mathfrak{a}} = (\mathbf{x}_{ji}^{-1} - \mathbf{x}_{ki}^{-1})_{\dot{\mathfrak{a}}\mathfrak{a}}$$

Three-point tensor structures

- Same general solution as for CFT for unconstrained multiplets:

$$\boxed{\langle\!\langle \mathcal{O}^{\tau_1}_{s_1,\bar{s}_1}\mathcal{O}^{\tau_2}_{s_2,\bar{s}_2}\mathcal{O}^{\tau_3}_{s_3,\bar{s}_3}\rangle\!\rangle = P\prod_{i,j=1}^{3} V_{ij}^{\,b_{ij}}}$$

$$P = \frac{1}{\operatorname{sdet}(\mathbf{x}_{12})^{\frac{\tau_1+\tau_2-\tau_3}{2}}\operatorname{sdet}(\mathbf{x}_{23})^{\frac{\tau_2+\tau_3-\tau_1}{2}}\operatorname{sdet}(\mathbf{x}_{31})^{\frac{\tau_3+\tau_1-\tau_2}{2}}}$$

$$V_{ij} = \begin{pmatrix} Y_1 & I_{12} & I_{13} \\ -I_{21} & Y_2 & I_{23} \\ -I_{31} & -I_{32} & Y_3 \end{pmatrix}$$

Extra bonus: harmonic analyticity

- Superfields analytic on compact internal space (y variables, coset of $SU(2n)$)
- Analyticity $\Rightarrow$ irreducible reps: differential constraints automatic PH (2002)
- In practice: harmonic analyticity, no y poles

Example: supercurrents $\gamma_i = 2$

- Building blocks I_{ij}, Y_i made from inverse (super-)matrices $\Rightarrow y^{-1}$ poles

The scalar prefactor is (superdeterminant / Berezinian of supermatrix)

$$\frac{1}{\operatorname{sdet}\mathbf{x}_{12}\ \operatorname{sdet}\mathbf{x}_{23}\ \operatorname{sdet}\mathbf{x}_{31}} \sim \frac{\det(y_{12})\ \det(y_{23})\ \det(y_{31})}{\det(x_{12})\ \det(x_{23})\ \det(x_{31})}.$$

- Single $\det(y_{ij})$ can cancel poles....
- ....$\Rightarrow$ tensor y_{ij}^{-1} indices fully antisymmetrised

Simplest example: $\langle J J \phi^2 \rangle$

- Before imposing analyticity: $\langle J_{\mathfrak{a}\dot{\mathfrak{a}}}(\mathbf{x}_1)\, J_{\mathfrak{b}\dot{\mathfrak{b}}}(\mathbf{x}_2)\, \phi^2(\mathbf{x}_3)\rangle =$

$$\frac{\det(y_{12})\ \det(y_{23})\ \det(y_{31})}{\det(x_{12})\ \det(x_{23})\ \det(x_{31})}\Big(c_1\, I_{12\,\dot{\mathfrak{a}}\mathfrak{b}}\, I_{21\,\dot{\mathfrak{b}}\mathfrak{a}} + c_2\, Y_{1\,\dot{\mathfrak{a}}\mathfrak{a}}\, Y_{2\,\dot{\mathfrak{b}}\mathfrak{b}}\Big).$$

- As $y_{12} \to 0$, $I_{12}, I_{21}, Y_1, Y_2 \to y_{12}^{-1}$.
- Therefore double poles in the parentheses.... unless indices antisymmetrised:

$$(y_{12}^{-1})_{[a[a'}\,(y_{12}^{-1})_{b]b'} \ \propto\ \frac{1}{\det y_{12}}.$$

- BUT RECALL: Antisymmetrisation of internal indices $\iff$ symmetrisation of superindices:

$$\sim (I_{12})_{(\mathfrak{a}(\dot{\mathfrak{a}}}\,(I_{12})_{\mathfrak{b})\dot{\mathfrak{b}})}. \qquad \Rightarrow \qquad c_1 = c_2$$

Coincidence limit constraint

- More generally: for any three higher spin supercurrents (twist 2 multiplets) harmonic analyticity implies:

$$\langle\!\langle J_{s_1,\bar{s}_1}(\mathbf{x}_1) J_{s_2,\bar{s}_2}(\mathbf{x}_2) J_{s_3,\bar{s}_3}(\mathbf{x}_3) \rangle\!\rangle \overset{x_{ij}\to 0}{\sim} \frac{(I_{ij})_{(\dot{\mathfrak{a}}_1(\mathfrak{a}_1} \cdots (I_{ij})_{\dot{\mathfrak{a}}_p)\dot{\mathfrak{a}}_p)}}{\mathrm{sdet}(x_{ij})} T(x_j, x_k)$$

- This consequence is a description purely in CFT (no need for superspace)

General solution of these constraints

- Form of constraints simple, general solution simple

$$\langle\!\langle \mathcal{O}^2_{[s_1],[\bar{s}_1]}\, \mathcal{O}^2_{[s_2],[\bar{s}_2]}\, \mathcal{O}^2_{[s_3],[\bar{s}_3]} \rangle\!\rangle^{(a)} = \frac{1}{\operatorname{sdet} \mathbf{x}_{12}\ \operatorname{sdet} \mathbf{x}_{23}\ \operatorname{sdet} \mathbf{x}_{31}} \sum_{S_a} \prod_{i,j=1}^{3} \frac{V_{ij}^{\,b_{ij}}}{b_{ij}!}$$

- With the constraints
  $\sum_{i=1}^3 b_{ij} = s_j,\ \sum_{j=1}^3 b_{ij} = \bar{s}_i,\ b_{12} - b_{21} = a.$
- Same formula for all m, n, including the non SUSY case $m = 2, n = 0$, even though no harmonic analyticity!!
- Conservation constraints automatically solved
- General counting formula:
  $\max(-s_1, -\bar{s}_2, \bar{s}_1 - s_1 - s_3) \leq a \leq \min(\bar{s}_1, s_2, \bar{s}_1 + \bar{s}_3 - s_1)$

$\Rightarrow$ **# tensor structures** $= \min_{i\neq j}(s_i + \bar{s}_i, s_i + s_j, \bar{s}_i + \bar{s}_j) + 1$

Summary: lift to SUSY (analytic superspace) and reduce back

$$\begin{tikzcd}
\boxed{\begin{array}{c}\textbf{SCFT tensor structures}\\ \text{for supermultiplets } (m,n)\end{array}} \arrow[r, "\text{harmonic analyticity}"] & \boxed{\begin{array}{c}\textbf{SCFT tensor structures}\\ \text{constraints solved } (m,n)\end{array}} \arrow[d, "\text{reduce to } n=0"] \\
\boxed{\begin{array}{c}\textbf{CFT tensor structures}\\ \text{without constraints } (2,0)\end{array}} \arrow[u, "\text{lift to SUSY } (n\neq 0)"] \arrow[r, dashed, "\text{difficult}"'] & \boxed{\begin{array}{c}\textbf{CFT tensor structures}\\ \text{constraints solved } (2,0)\end{array}}
\end{tikzcd}$$

- Same strategy applies when only *some* operators are conserved

Pure internal space

- Special case $(m, n) = (0, n)$: compact internal space only; $SU(2n)$ irreps
- Counting of tensor structures reduces to group theory
- $SU(2n)$ Clebsch–Gordan and Littlewood–Richardson coefficients
- Alternative formulation:
  - Counting tensor structures
  - Proofs

Other dimensions

- Conservation constraints exist in any dimension d
- Similar story for 4D,6D: analytic superspace methods
- Supergroup $SU \to OSp$. Grassmannian $\to$ orthogonal Grassmannian
- For three conserved currents 3D,4D,6D all consistent with:

$$\langle\!\langle J_{s_1,\bar{s}_1} J_{s_2,\bar{s}_2} J_{s_3,\bar{s}_3} \rangle\!\rangle_d^{(a)} = \frac{1}{(x_{12}^2 x_{23}^2 x_{31}^2)^{\frac{d-2}{2}}} \sum_{S_a} \prod_{i,j=1}^{3} \frac{V_{ij}^{b_{ij}}}{\left(b_{ij} + \frac{d-4}{2}\delta_{j-i-1}\right)!} .$$

- Harmonic analyticity $\Rightarrow$ leading singular structure $\langle \psi \bar{\psi} \rangle$
- Simplifies and generalises Zhiboedov (2013)

- Warm up: CFT (spinor formalism)
- Analytic superspace
- **Non-square superspace**
- Conclusions

New!! Non-square analytic superspace (WIP) Puerta-Ramisa, PH

- (Square) analytic superspace $Gr(m|n, m|2n|m)$ requires $\mathcal{N} = 2n$ even
- $\mathcal{N} = 4$, $\mathcal{N} = 2$, $\mathcal{N} = 0$ but what about $\mathcal{N} = 1$?
- (Non-square) analytic superspace, eg $Gr(m|n, m|2n-1|m)$?
- Non-square coordinate matrix: $\mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}}$,
  $\mathfrak{a} = (1, \ldots, m|m+1, \ldots, n)$
  $\dot{\mathfrak{a}} = (1, \ldots, m|m+1, \ldots, n-1)$
- Problem: what is $(I_{ij}) = (\mathbf{x})^{-1}$ if $\mathbf{x}$ is non-square????
- Major stumbling bock preventing pursuit of non-square analytic superspace until now...

Inverse of non square matrix?

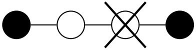
- Solution: extend $\mathbf{x}$ to a square matrix arbitrarily, $\dot{\mathfrak{a}} \to (\dot{\mathfrak{a}}, \hat{a})$
- Then:
  - $(\mathbf{x_{ij}}^{-1})_{\hat{a}\mathfrak{a}}$ is independent of the extension
  - symmetrised combinations $(\mathbf{x_{ij}}^{-1})_{\hat{a}(\mathfrak{a}_1}(\mathbf{x_{ij}}^{-1})_{\dot{\mathfrak{a}}_1\mathfrak{a}_2}\dots(\mathbf{x_{ij}}^{-1})_{\dot{\mathfrak{a}}_{p-1}\mathfrak{a}_p)}$ are also independent of the extension
- Independence of extension $\Rightarrow$ analyticity!

- Byproduct: Interesting relation with maths literature on "super Pluckers". *'wrong minors'* Shemyakova, Voronov (2022); Shemyakova (2023)

▸ Non-square Dynkin diagrams

(anti)chiral superspace = non-square analytic superspace

- Natural framework for $\mathcal{N}=1$ theories: $(m=2, n=1, n'=0)$



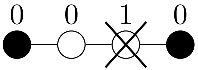
$$\mathfrak{a} = (1,2|3) = (\alpha|3), \quad \dot{\mathfrak{a}} = \dot{\alpha}, \qquad \mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} \\ \bar{\theta}^{\dot{\alpha}} \end{pmatrix}$$

- Chiral multiplets can be described *in antichiral superspace* using superindices!

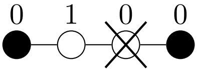
Chiral vs anti-chiral fields

- Chiral multiplets in antichiral superspace?!?

Anti-chiral multiplet, $\Phi(\mathbf{x})$:



Chiral multiplet, $\Phi_{\mathfrak{a}}(\mathbf{x})$:



- $\Phi(\mathbf{x}) = \bar{\phi}(x) + \bar{\theta}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}(x)}$
- $\Phi_{\mathfrak{a}}(\mathbf{x}) = (\Phi_{\alpha}(\mathbf{x}), \Phi_3(\mathbf{x})) = (\psi_{\alpha}(x), \phi(x))$
- On-shell constraints kill higher $\bar{\theta}$ corrections of $\Phi_{\mathfrak{a}}(\mathbf{x})$

$\mathcal{N} = 1$ constraints from reducing non-square analytic superspace

- New formalism for $\mathcal{N} = 1$ SCFT to be explored
- Analogously to CFT case, lift to general non-square superspace
- Analyticity in internal variables $\implies$ constraints solved algebraically
- Reduction to antichiral superspace automatic $\dot{\mathfrak{a}} \to \dot{\alpha}$ etc.
- Exact analogue of the CFT conservation story for $\mathcal{N} = 1$ SUSY

- Warm up: CFT (spinor formalism)
- Analytic superspace
- Non-square superspace
- **Conclusions**

What we have done

- Analytic superspace as a framework for $\mathcal{N} = 4, 2$ superconformal field theories (half BPS and beyond)
- New: Analytic superspace as a framework for $\mathcal{N} = 0$ conformal field theories - constrained fields
- Application: simple formula for conserved 3-point tensor structures
- Other dimensions: $d = 3, 6$ and analytic continuation to all dimensions
- New non-square analytic superspace (still a Grassmannian): access to $\mathcal{N} = 1, 3$ theories

Future directions

- Analytic results for correlators with just one or two conserved operators
- Extension to non half BPS four-point functions and superconformal blocks (super weight shifting operators)
  Hansen,Puerta Ramisa, PH (2025);Bissi,Fardelli,Puerta Ramisa (WIP)
- $\mathcal{N}=1$ tensor structures and superblocks with non-square analytic superspace (relation to BC Jacobi Aprile,PH (2021))
- Relation to recent twistor $(Z,\bar{Z})$ approaches Baumann, Mathys, Pimentel, Rost (2024);Bala, Jain, Dhruva (2026);Carrillo González, Keseman (2026)
- Grassmannian/ twistor line formalism $(x_{ij})^{\alpha\dot\alpha}=(x_i)^\alpha_A(\bar{x}_j)^{A\dot\alpha}$ v ambitwistor $Z_A\bar{Z}^A$. Solve local $SL(2)$ directly v integrate over $SL(2)$
- Grassmannian of Grassmannians?

Additional slides

Comparing CFT and SCFT operators

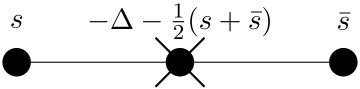
CFT	$\leftrightarrow$	**SCFT**
Scalars $\mathcal{O}^\tau(x)$	$\leftrightarrow$	half-BPS $\mathcal{O}^\gamma(\mathbf{x})$
Spinning ops $\mathcal{O}^\tau(x)_{\alpha_1\dots\alpha_s\,\dot\alpha_1\dots\dot\alpha_{\bar s}}$	$\leftrightarrow$	general mplets $\mathcal{O}^\gamma(\mathbf{x})_{\mathfrak{a}_1\dots\,\dot{\mathfrak{a}}_1\dots}$

- Superindices symmetrised via Young diagrams $\equiv$ finite irrep of $SL(m|n)$
- Theorem: Unitary superconformal reps $\Rightarrow$ finite reps of $SL(m|n)$ PH (2002)
- Read off rep from super Dynkin diagram

- Underlying mathematical machinery supersymmetric Borel-Weil theorem
- Supercoset of the generalised superconformal group $G \Rightarrow$ Superconformal symmetry manifest
- Analytic superspace $= G/H$ with lower block-triangular (parabolic) subgroup H

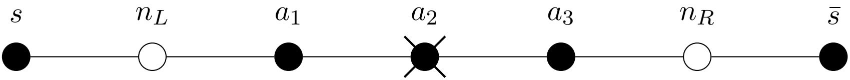
Reps on cosets via Dynkin diagrams

Non-supersymmetric Minkowski space



$$\mathfrak{g} = \mathfrak{su}(2,2) \to \mathfrak{sl}(4;\mathbb{C})$$

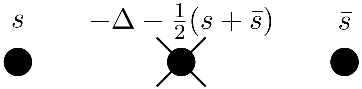
$\mathcal{N}=4$ analytic superspace



$$\mathfrak{g} = \mathfrak{su}(2,2|4) \to \mathfrak{sl}(2|4|2;\mathbb{C})$$

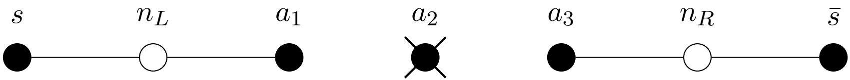
Reps on cosets via Dynkin diagrams

Non-supersymmetric Minkowski space



$$\mathfrak{l} = \mathfrak{sl}(2) \oplus \mathbb{C} \oplus \mathfrak{sl}(2)$$

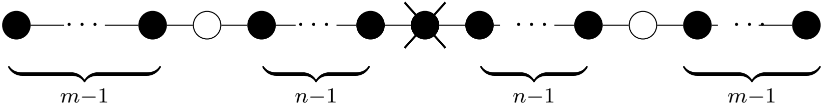
$\mathcal{N} = 4$ analytic superspace



$$\mathfrak{l} = \mathfrak{sl}(2|2) \oplus \mathbb{C} \oplus \mathfrak{sl}(2|2)$$

General analytic superspace: $\mathcal{A}_{m|n}$

Generalised superconformal group $\mathfrak{sl}(m|2n|m)$



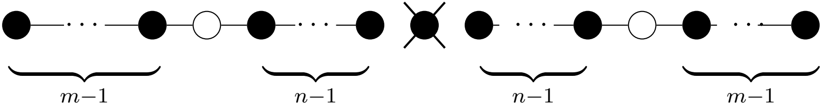
$$\mathcal{A}_{m|n} = \frac{SL(m\,|\,2n\,|\,m)}{\left\{\begin{pmatrix} \boxed{*} & 0 \\ * & \boxed{*} \end{pmatrix}\right\}} \qquad \boxed{*} = \mathfrak{gl}(m|n)$$

Coordinates:
$$\mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} & \theta^{\alpha\dot{a}} \\ \bar{\theta}^{a\dot{\alpha}} & y^{a\dot{a}} \end{pmatrix}, \qquad \begin{array}{l} \mathfrak{a} = (\alpha\mid a),\ \dot{\mathfrak{a}} = (\dot{\alpha}\mid\dot{a}), \\ \alpha, \dot{\alpha} \leq m, \quad a, \dot{a} > m \end{array}$$

Return to main talk

General analytic superspace: $\mathcal{A}_{m|n}$

Corresponding Levi subgroup $\mathfrak{sl}(m|n) \oplus \mathbb{C} \oplus \mathfrak{sl}(m|n)$



$$\mathcal{A}_{m|n} = \frac{SL(m\,|\,2n\,|\,m)}{\left\{\begin{pmatrix} \boxed{*} & 0 \\ * & \boxed{*} \end{pmatrix}\right\}} \qquad \boxed{*} = \mathfrak{gl}(m|n)$$

Coordinates: $$\mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} & \theta^{\alpha\dot{a}} \\ \bar{\theta}^{a\dot{\alpha}} & y^{a\dot{a}} \end{pmatrix}, \qquad \begin{array}{l} \mathfrak{a} = (\alpha\,|\,a),\ \dot{\mathfrak{a}} = (\dot{\alpha}\,|\,\dot{a}), \\ \alpha, \dot{\alpha} \leq m, \quad a, \dot{a} > m \end{array}$$

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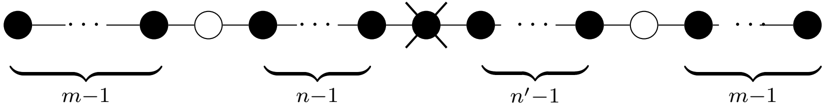
New!! Non-square analytic superspace (WIP) Puerta-Ramisa, PH

Superconformal group $\mathfrak{sl}(m|n+n'|m)$



Non-central crossed note $\Rightarrow$ non-square coordinate matrix $\mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}}$

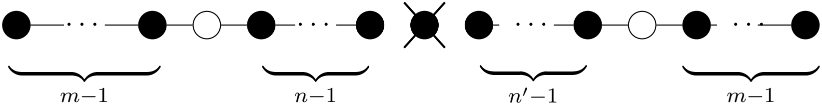
$$s(x) = \begin{pmatrix} 1_{m|n} & \mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}} \\ 0 & 1_{m|n'} \end{pmatrix}$$

$$\mathfrak{a} = (1, \ldots, m|m+1, \ldots, n)$$
$$\dot{\mathfrak{a}} = (1, \ldots, m|m+1, \ldots, n')$$

◂ Return to main talk

New!! Non-square analytic superspace (WIP) Puerta-Ramisa, PH

Corresponding Levi subgroup $\mathfrak{sl}(m|n) \oplus \mathbb{C} \oplus \mathfrak{sl}(m|n')$



Non-central crossed note $\Rightarrow$ non-square coordinate matrix $\mathbf{x}^{\mathfrak{a}\dot{\mathfrak{a}}}$

$$H = \left(\begin{array}{c|c} \mathrm{GL}(m|n) & 0 \\ \hline * & \mathrm{GL}(m|n') \end{array}\right)$$

$$\mathfrak{a} = (1, \ldots, m|m+1, \ldots, n)$$
$$\dot{\mathfrak{a}} = (1, \ldots, m|m+1, \ldots, n')$$

◂ Return to main talk