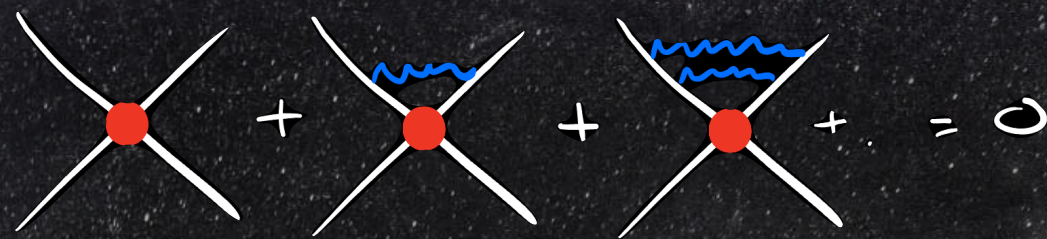


# Positivity Bounds without Vanishing Couplings

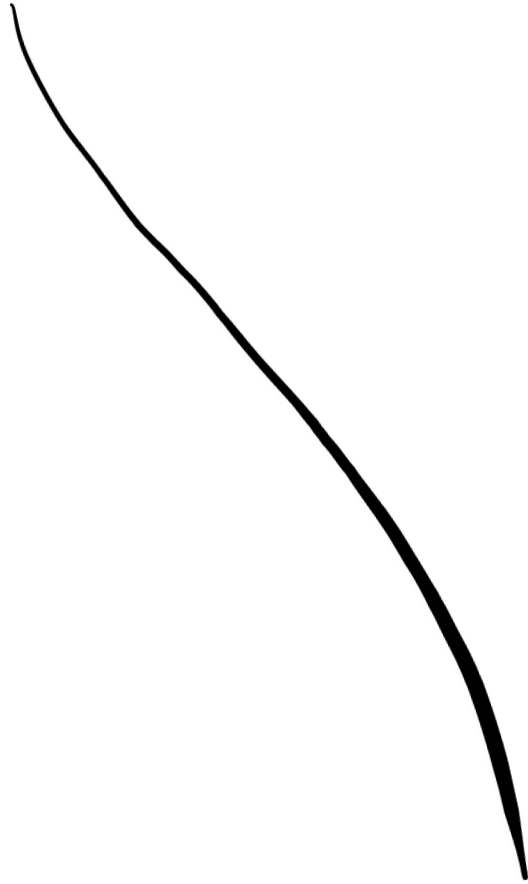


The diagram shows a sum of three Feynman diagrams, each with a red vertex, followed by an equals sign and a zero. The first diagram is a simple four-point vertex. The second diagram has a blue wavy line connecting the top two external lines. The third diagram has a blue wavy line connecting the top two external lines and another blue wavy line connecting the bottom two external lines. The diagrams are separated by plus signs, and the entire sum is set equal to zero.

$$\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} = 0$$

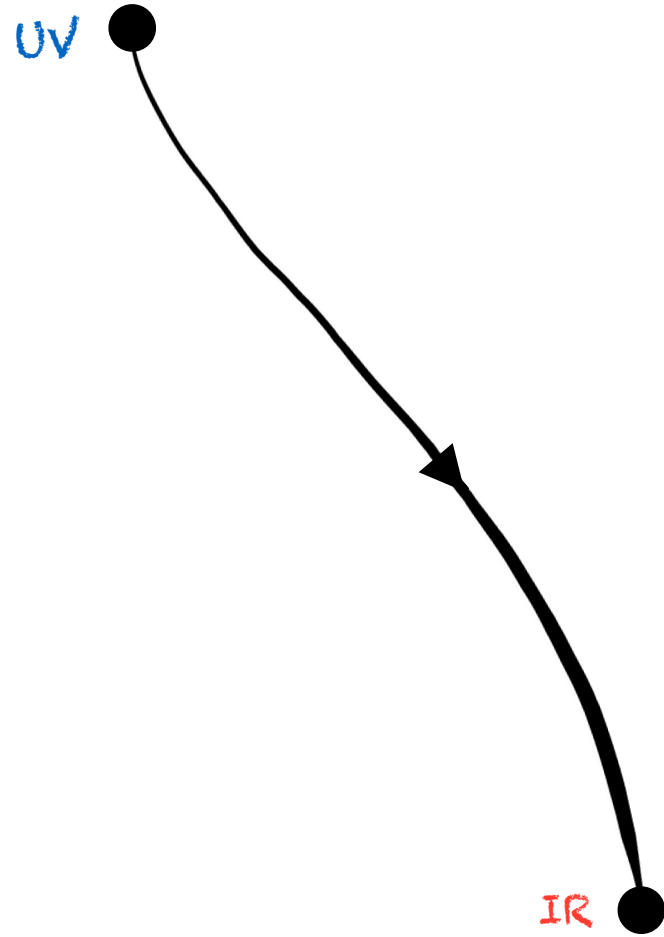
Francesco Riva  
(Geneva University)

UV → IR



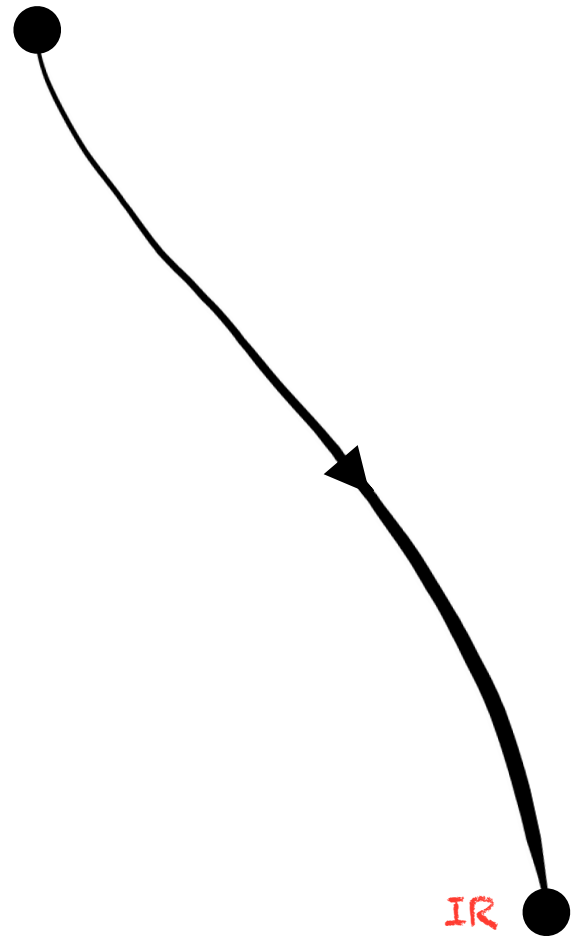


UV → IR



UV → IR

UV



IR

Experiments

EFT: Particles+Symmetries





UV

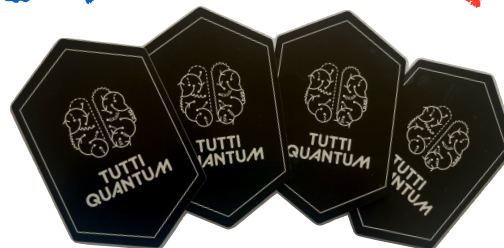
Hypotheses

QFT: Causal+Unitary

UV



IR



IR

Experiments

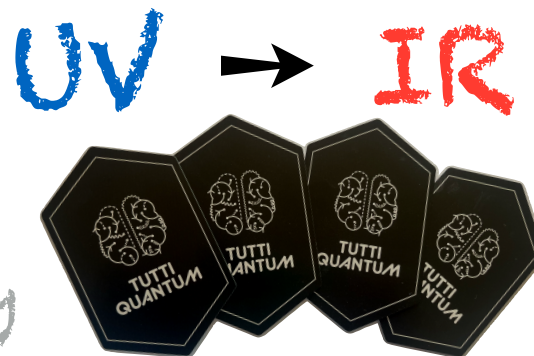
EFT: Particles+Symmetries



UV

Hypotheses

QFT: Causal+Unitary



IR

Experiments

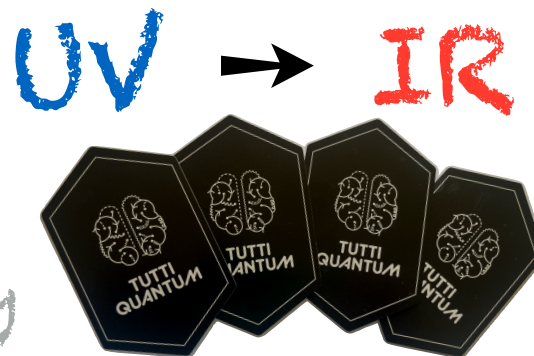
EFT: Particles+Symmetries



UV

Hypotheses

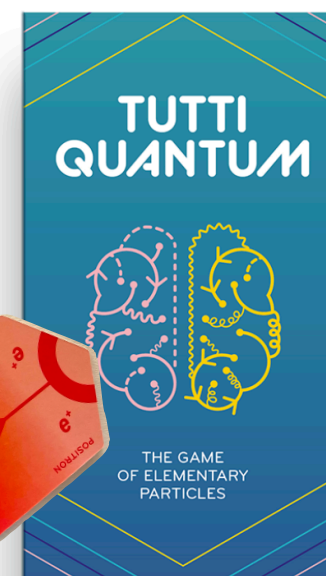
QFT: Causal+Unitary



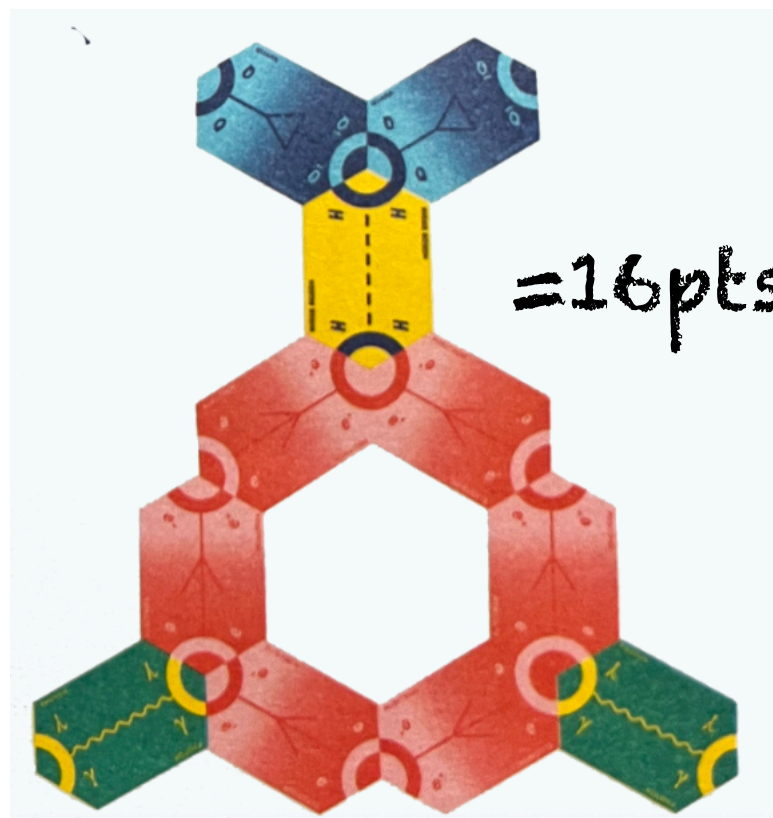
IR

Experiments

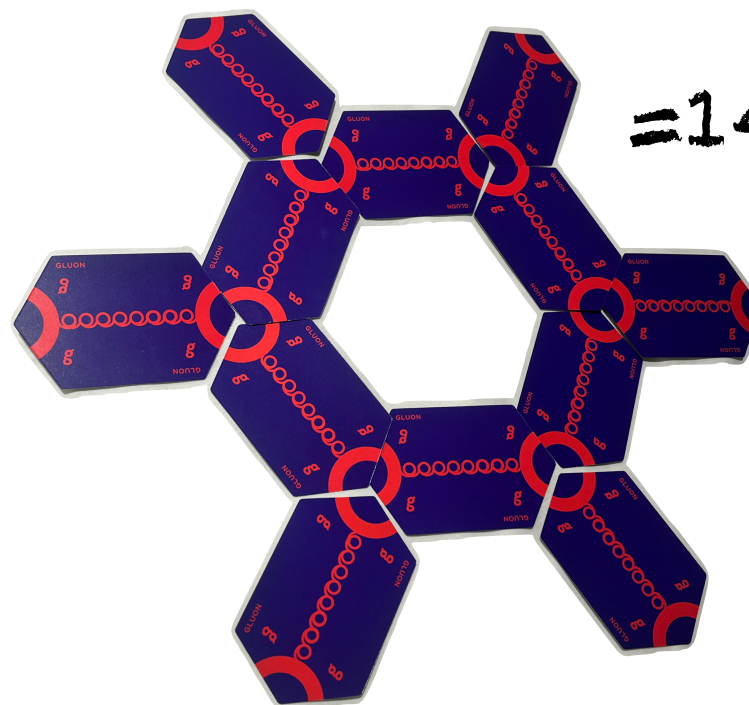
EFT: Particles+Symmetries



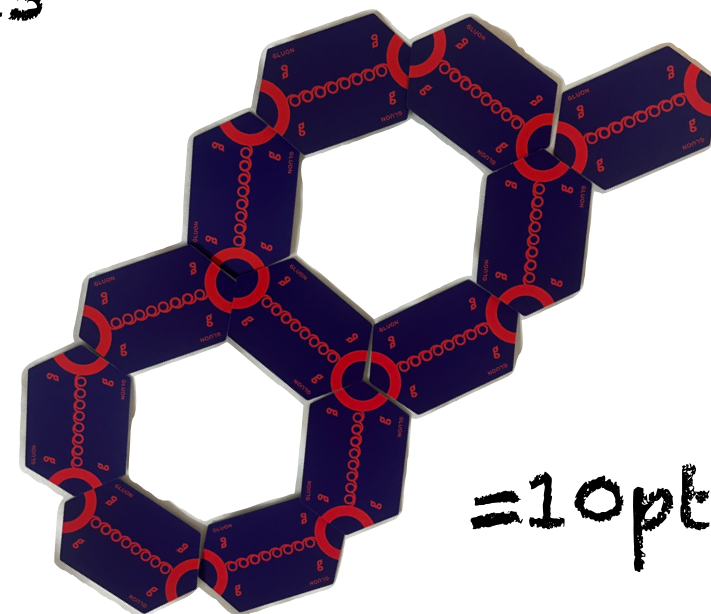
=16pts



=14pts



=10pts

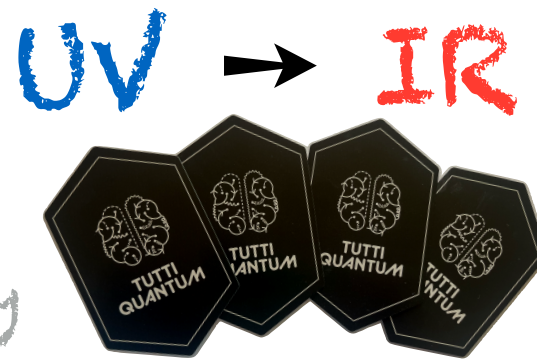




UV

Hypotheses

QFT: Causal+Unitary



IR

Experiments

EFT: Particles+Symmetries



UV

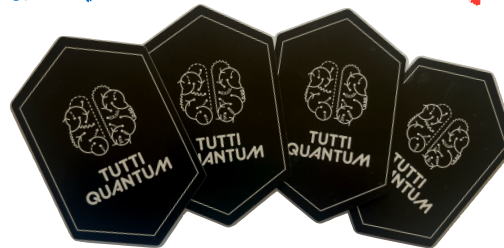
Hypotheses

QFT: Causal+Unitary

UV



IR



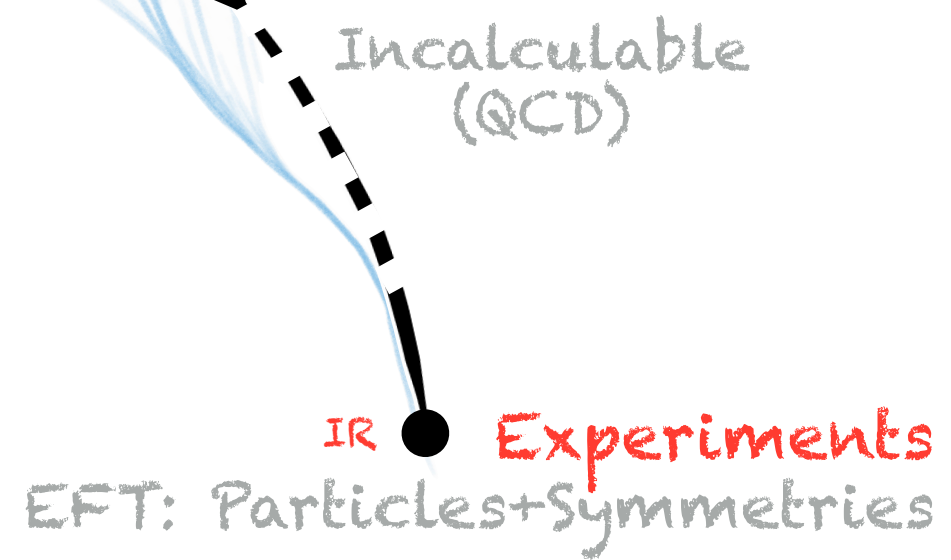
Incalculable  
(QCD)

IR

Experiments

EFT: Particles+Symmetries



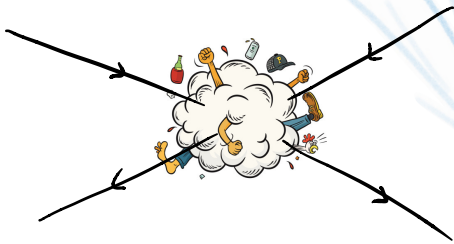






UV

Hypotheses  
QFT: Causal+Unitary



Incalculable  
(QCD)

IR

Experiments  
EFT: Particles+Symmetries



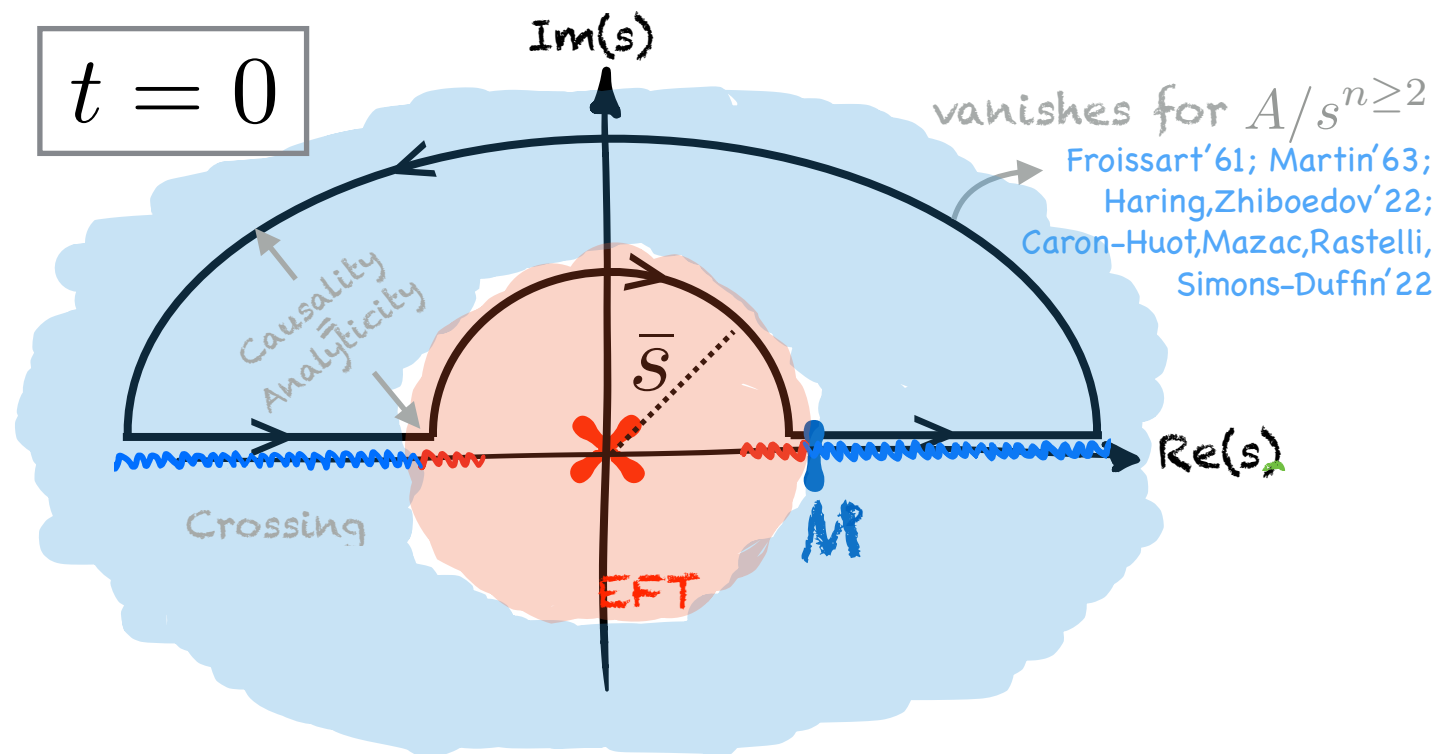
UV  $\rightarrow$  IR

UV  
**Hypotheses**  
 QFT: Causal+Unitary

$$A_{2 \rightarrow 2}(s, t)$$

Incalculable  
 (QCD)

IR  
**Experiments**  
 EFT: Particles+Symmetries



UV  $\rightarrow$  IR

UV

Hypotheses

QFT: Causal+Unitary

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR

Experiments

EFT: Particles+Symmetries

$$\mathcal{A}_n(\bar{s}) \equiv \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\text{Im} A(s)}{s^{n+1}}$$

$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$

$t = 0$

Im(s)

vanishes for  $A/s^{n \geq 2}$

Froissart'61; Martin'63;  
Haring,Zhiboedov'22;  
Caron-Huot,Mazac,Rastelli,  
Simons-Duffin'22

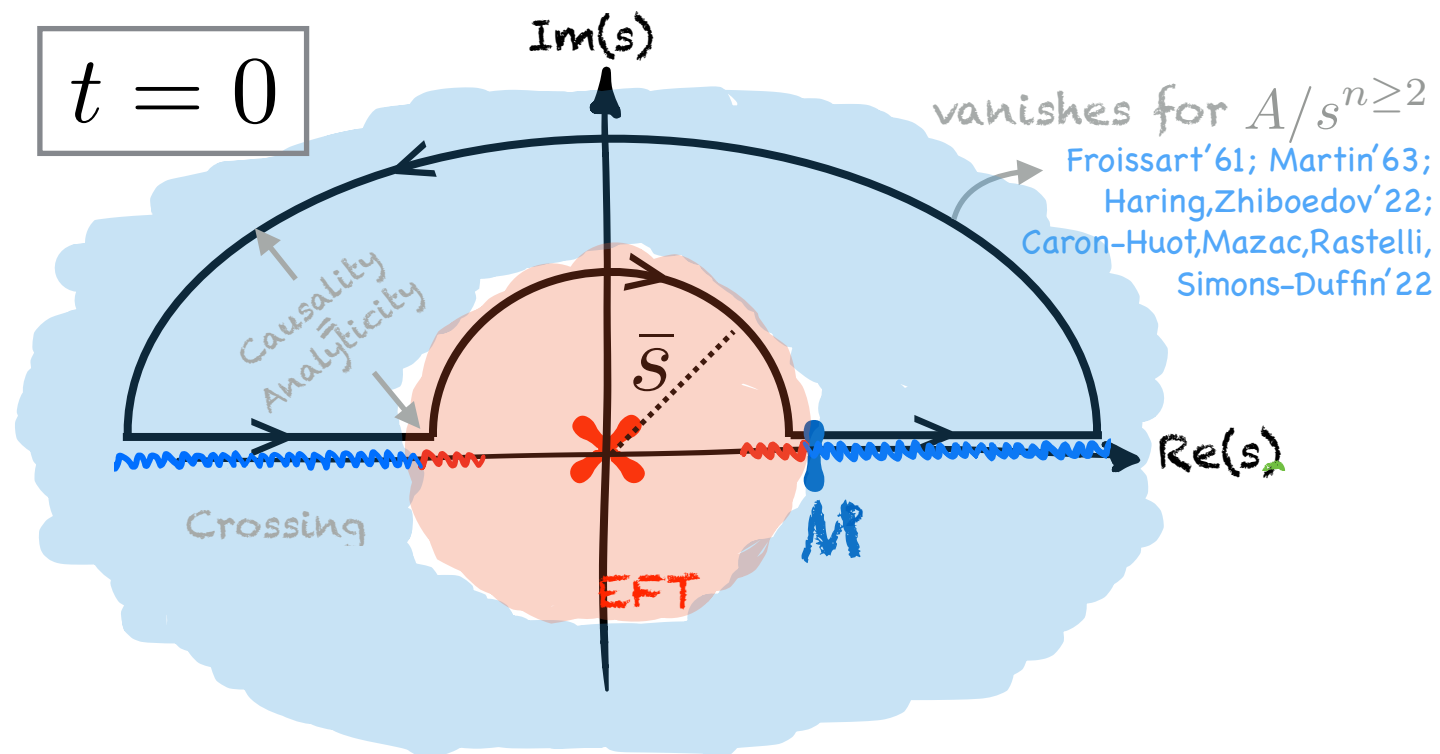
Causality  
Analyticity

Crossing

EFT

M

Re(s)



UV  $\rightarrow$  IR

Hypotheses

QFT: Causal+Unitary

$$\mathcal{A}_n(\bar{s}) \equiv \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\text{Positive Im}A(s)}{s^{n+1}}$$

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR

Experiments

EFT: Particles+Symmetries

$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$

$t = 0$

Im(s)

vanishes for  $A/s^{n \geq 2}$

Froissart'61; Martin'63;

Haring,Zhiboedov'22;

Caron-Huot,Mazac,Rastelli,

Simons-Duffin'22

Causality  
Analyticity

Crossing

EFT

$M$

Unitarity  
Positivity

Re(s)

$\bar{s}$



UV  $\rightarrow$  IR

UV

Hypotheses

QFT: Causal+Unitary

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR

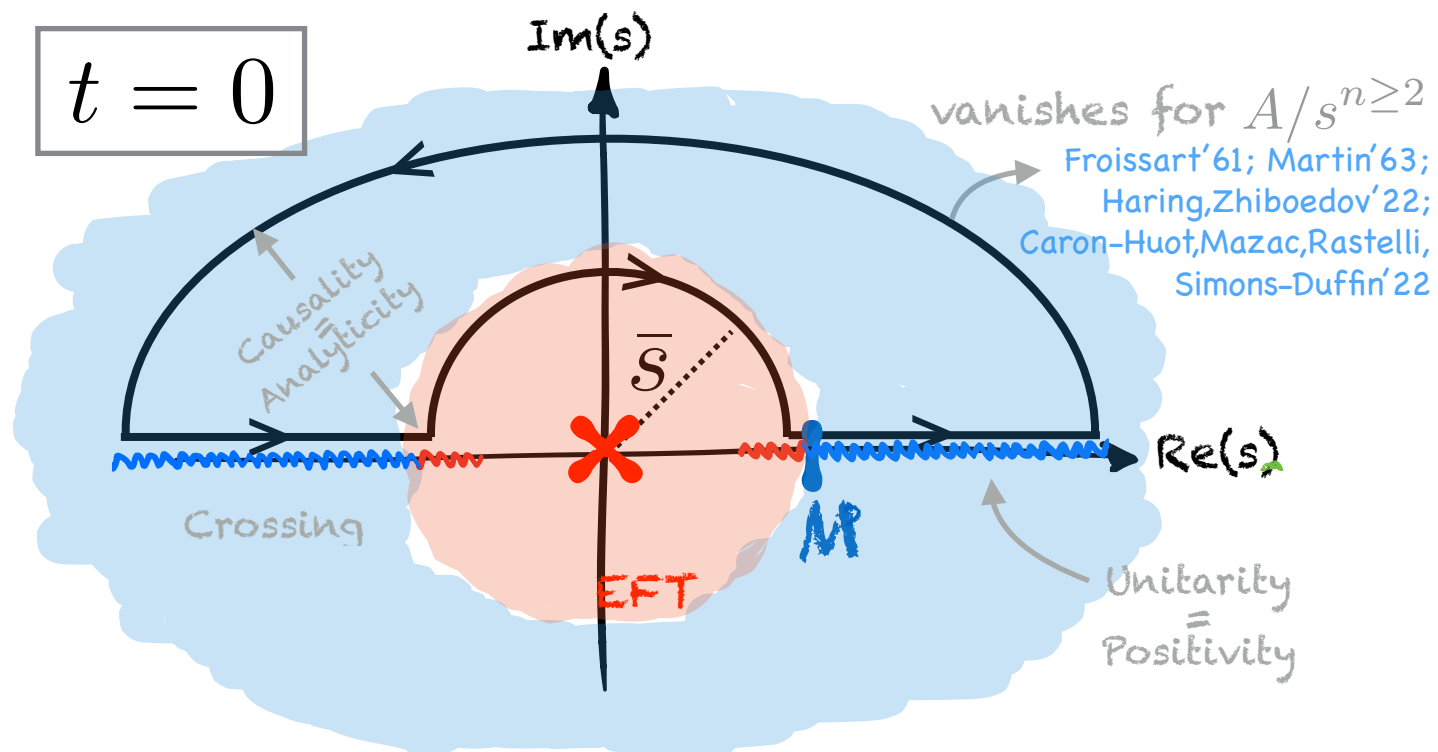
Experiments

EFT: Particles+Symmetries

$$A_n(\bar{s}) \equiv$$

$$\int_0^1 d\mu(x) x^n$$

Positive Moments



UV  $\rightarrow$  IR

uv ● Hypotheses  
QFT: Causal+Unitary

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR ● Experiments  
EFT: Particles+Symmetries

$$A_n(\bar{s}) \equiv \int_0^1 \text{Positive Moments } d\mu(x) x^n$$

Positive polynomials in  $[0,1]$   
 $\nabla$   
 Positivity relations for arcs

$t = 0$

Im(s)

vanishes for  $A/s^{n \geq 2}$

Froissart'61; Martin'63;  
Haring,Zhiboedov'22;  
Caron-Huot,Mazac,Rastelli,  
Simons-Duffin'22

Causality  
Analyticity

Crossing

EFT

M

Unitarity  
Positivity

Re(s)

$\bar{s}$

UV  $\rightarrow$  IR

UV  
Hypotheses  
QFT: Causal+Unitary

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR  
Experiments  
EFT: Particles+Symmetries

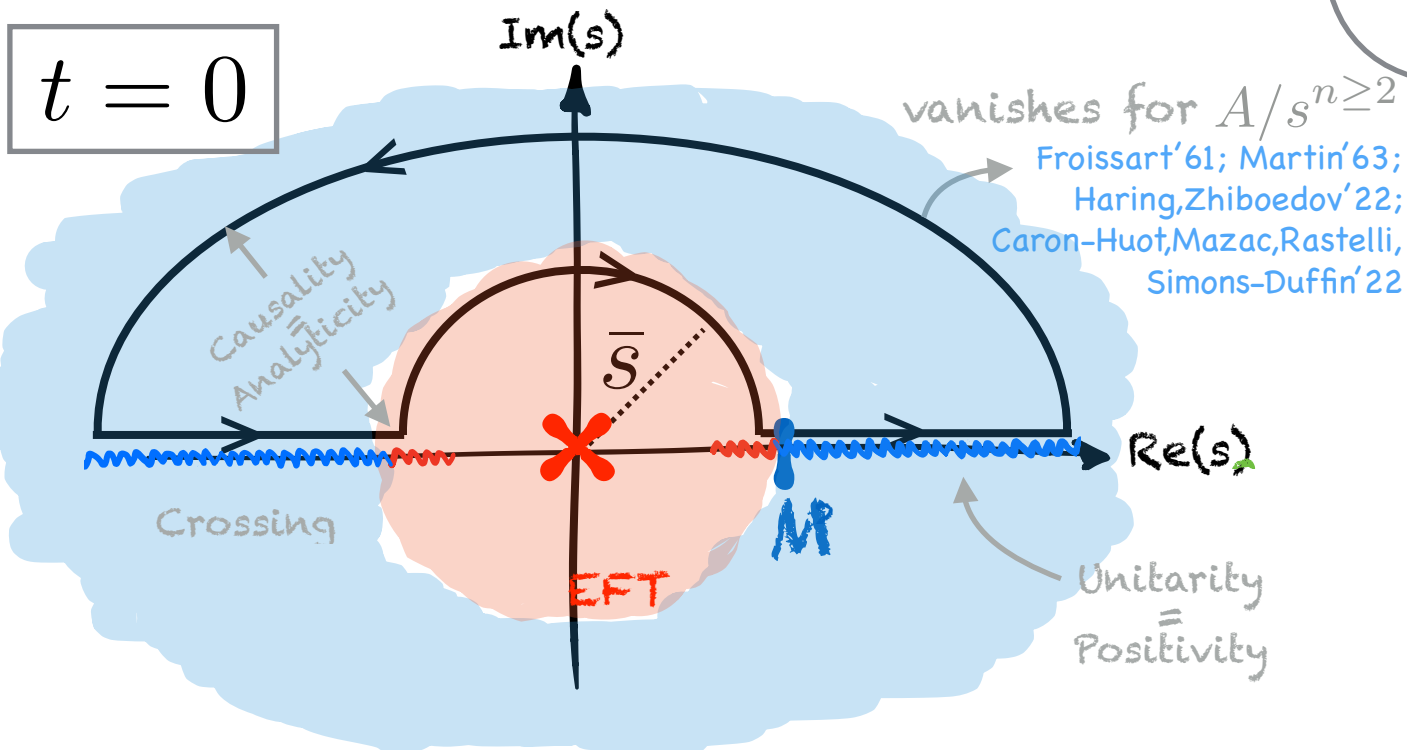
$$A_n(\bar{s}) \equiv \int_0^1 d\mu(x) x^n$$

Positive Moments

Positive polynomials in  $[0,1]$

Positivity relations for arcs

$$0 < \int_0^1 d\mu(x) \left( \sum \alpha_i x^i \right)^2 = \sum_{i,j=0}^{N/2} \alpha_i A_{i+j} \alpha_j$$



UV  $\rightarrow$  IR

UV  
Hypotheses  
QFT: Causal+Unitary

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR  
Experiments  
EFT: Particles+Symmetries

$$A_n(\bar{s}) \equiv \int_0^1 d\mu(x) x^n$$

Positive Moments

Positive polynomials in  $[0,1]$

Positivity relations for arcs

$$0 < \int_0^1 d\mu(x) \left( \sum \alpha_i x^i \right)^2 = \sum_{i,j=0}^{N/2} \alpha_i A_{i+j} \alpha_j$$

Hankel: 
$$\begin{pmatrix} A_0 & A_1 & A_2 \\ A_1 & A_2 & A_3 \\ A_2 & A_3 & A_4 \\ \vdots & \vdots & \vdots \end{pmatrix} \succ 0$$

$t = 0$

Im(s)

vanishes for  $A/s^n \geq 2$

Froissart'61; Martin'63;  
Haring,Zhiboedov'22;  
Caron-Huot,Mazac,Rastelli,  
Simons-Duffin'22

Causality  
Analyticity

Crossing

EFT

Unitarity  
Positivity

Re(s)

M



UV  $\rightarrow$  IR

Hypotheses

QFT: Causal+Unitary

UV

$A_{2 \rightarrow 2}(s, t)$

Incalculable  
(QCD)

IR

Experiments

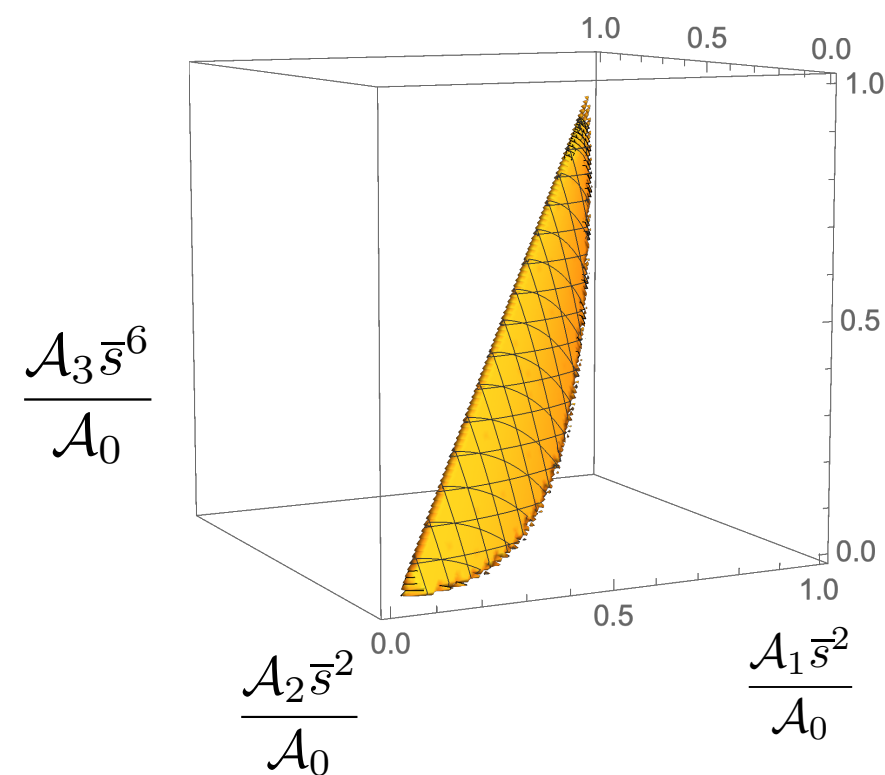
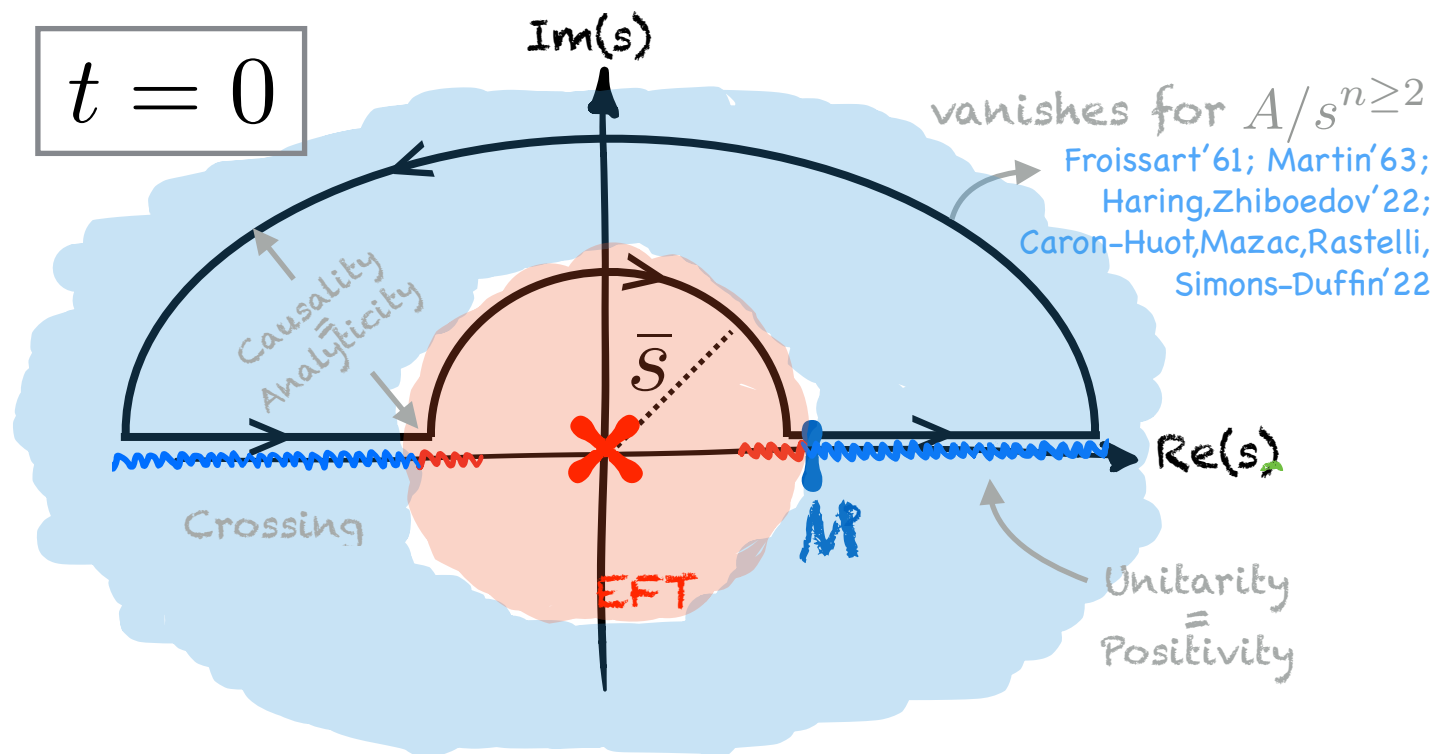
EFT: Particles+Symmetries

$$A_n(\bar{s}) \equiv \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\text{Im} A(s)}{s^{n+1}}$$

Positive Moments

$$A_2 \geq \bar{s}^2 A_4 \geq \bar{s}^4 A_6 \geq \dots \geq 0$$

$$A_n(\bar{s}) \equiv \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$



UV  $\rightarrow$  IR

Hypotheses  
QFT: Causal+Unitary

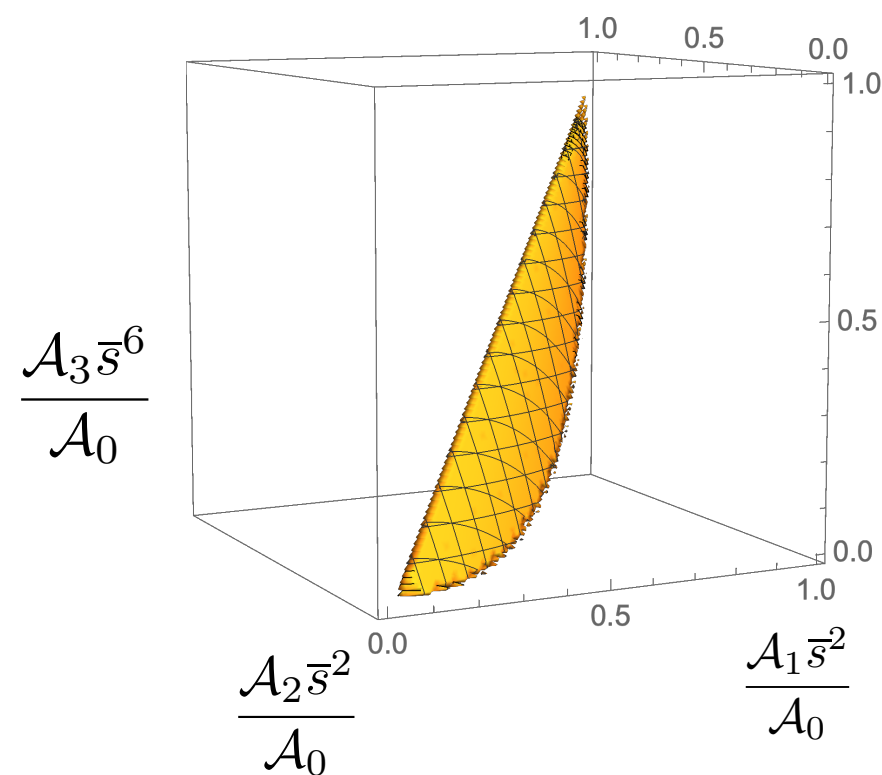
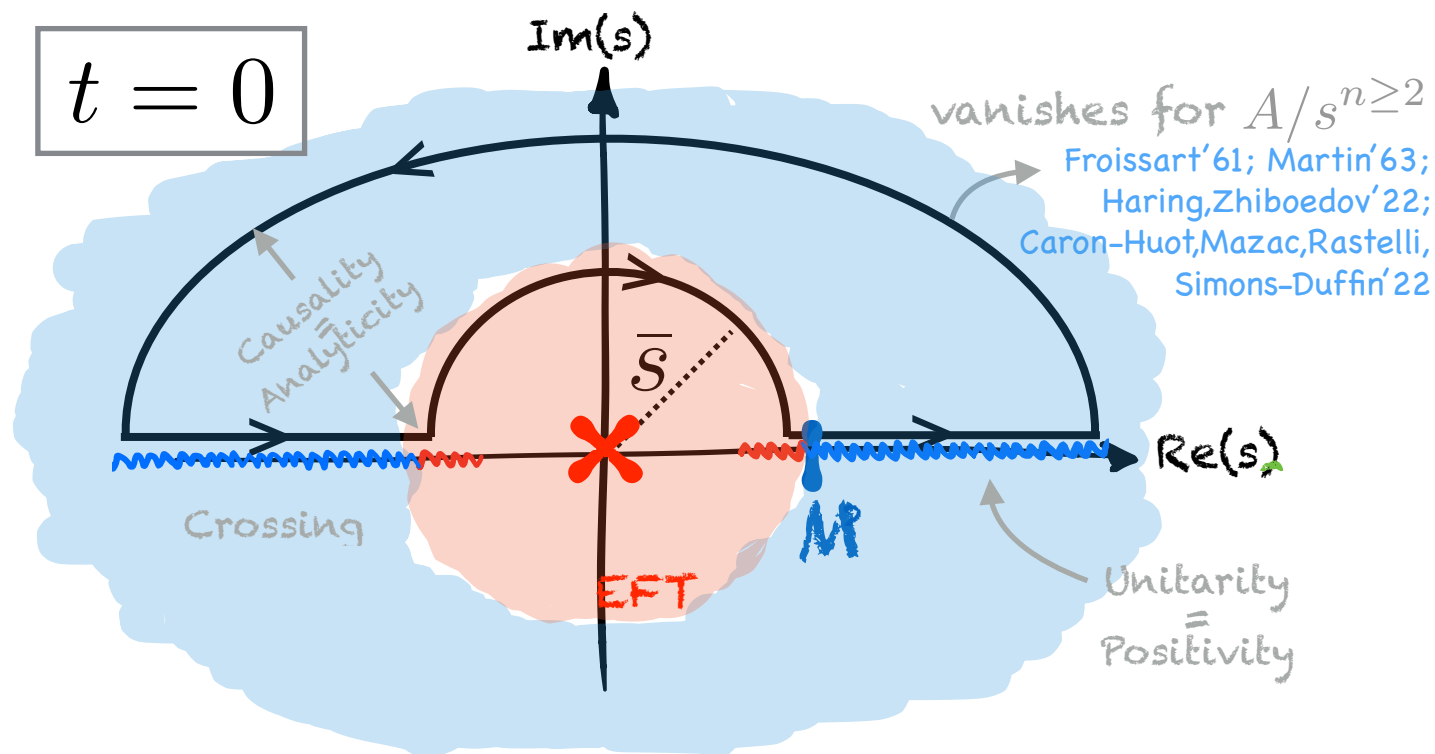
$$A_n(\bar{s}) \equiv \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\text{Im}A(s)}{s^{n+1}}$$

Positive Moments

$$A_2 \geq \bar{s}^2 A_4 \geq \bar{s}^4 A_6 \geq \dots \geq 0$$

IR Experiments  
EFT: Particles+Symmetries

$$A_n(\bar{s}) \equiv \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$



UV  $\rightarrow$  IR

Hypotheses  
QFT: Causal+Unitary

$$\mathcal{A}_n(\bar{s}) \equiv \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\text{Im} A(s)}{s^{n+1}}$$

Positive Moments

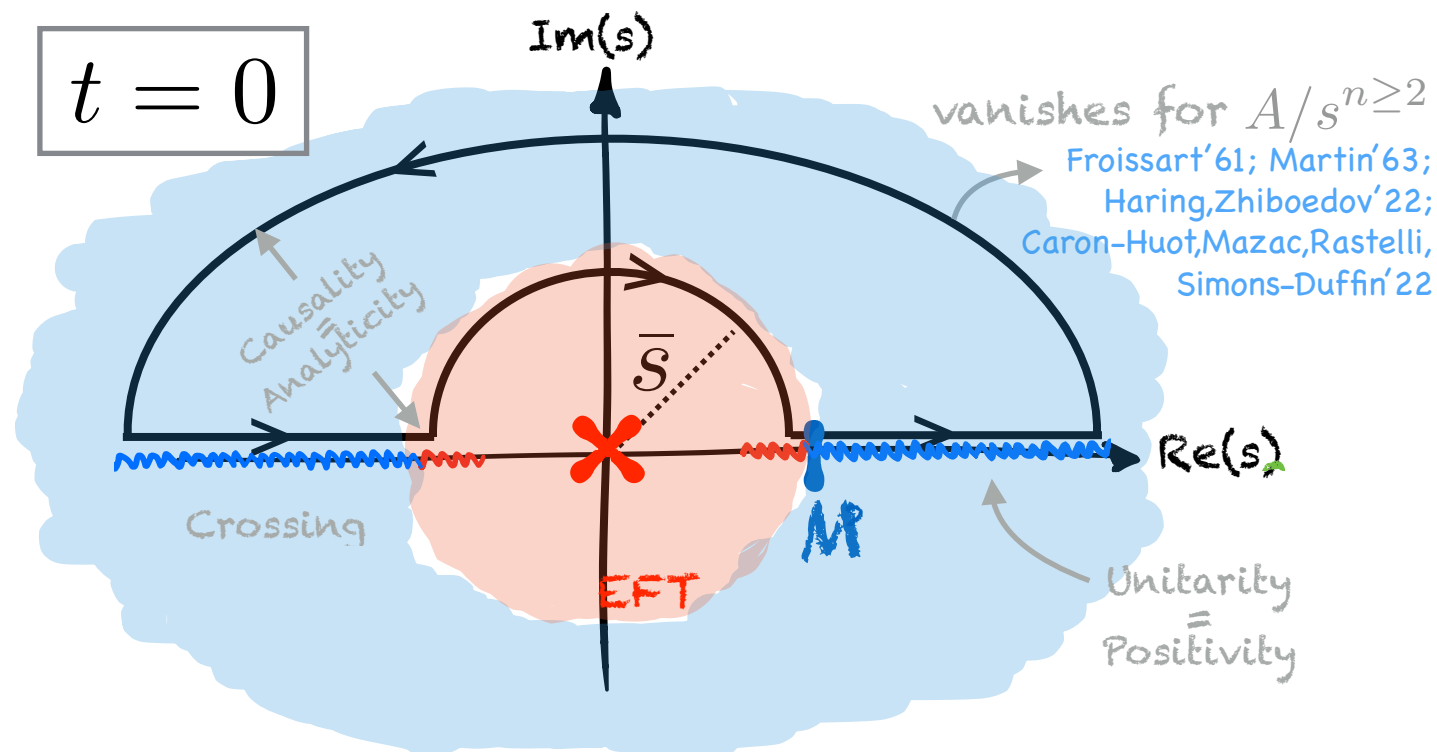
Double-Sided bounds

$$A_2 \geq \bar{s}^2 A_4 \geq \bar{s}^4 A_6 \geq \dots \geq 0$$

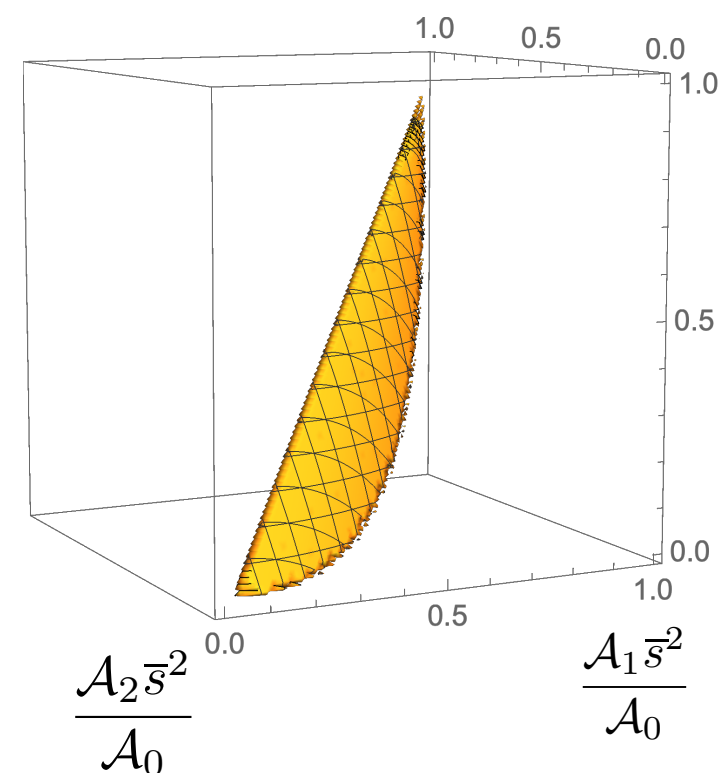
Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi'06  
Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20  
Arkani-Hamed, Huang, Huang'20

IR Experiments  
EFT: Particles+Symmetries

$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$



$$\frac{A_3 \bar{s}^6}{A_0}$$



UV  $\rightarrow$  IR implications at weakest coupling

UV  $\rightarrow$  IR with finite coupling effects

# UV $\rightarrow$ IR implications

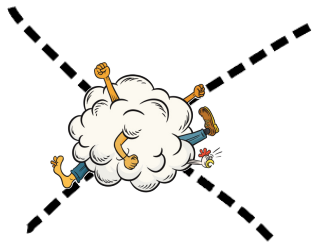
CFT UV  $a_{UV}$  Weyl-anomaly

~~CFT~~

A-Theorem:

$$a_{UV} > a_{IR}$$

Cardy'88

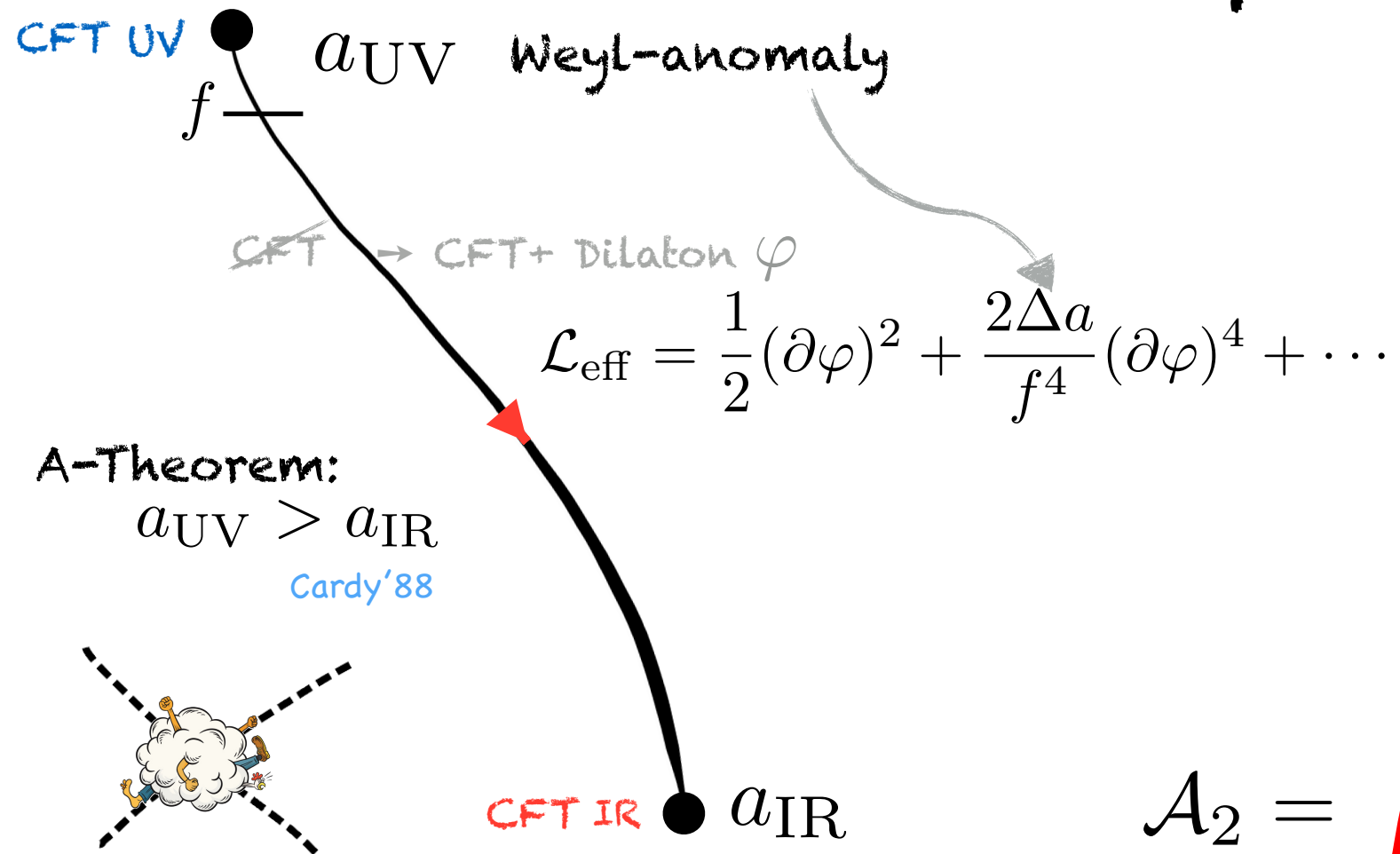


CFT IR  $a_{IR}$

$$\mathcal{A}_2 = \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^3}$$



# UV $\rightarrow$ IR implications



$$\mathcal{A}_2 = \int_{\cap_{\bar{s}}} \frac{ds}{\pi i} \frac{A(s)}{s^3}$$

# UV $\rightarrow$ IR implications

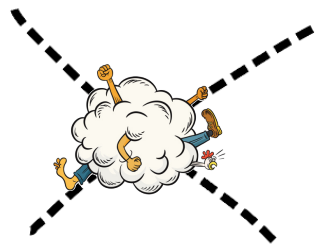
CFT UV  $\bullet$   $a_{UV}$  Weyl-anomaly

~~CFT~~  $\rightarrow$  CFT + Dilaton  $\varphi$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\varphi)^2 + \frac{2\Delta a}{f^4}(\partial\varphi)^4 + \dots$$

▶  $A(s) = \frac{16\Delta a}{f^4}s^2$

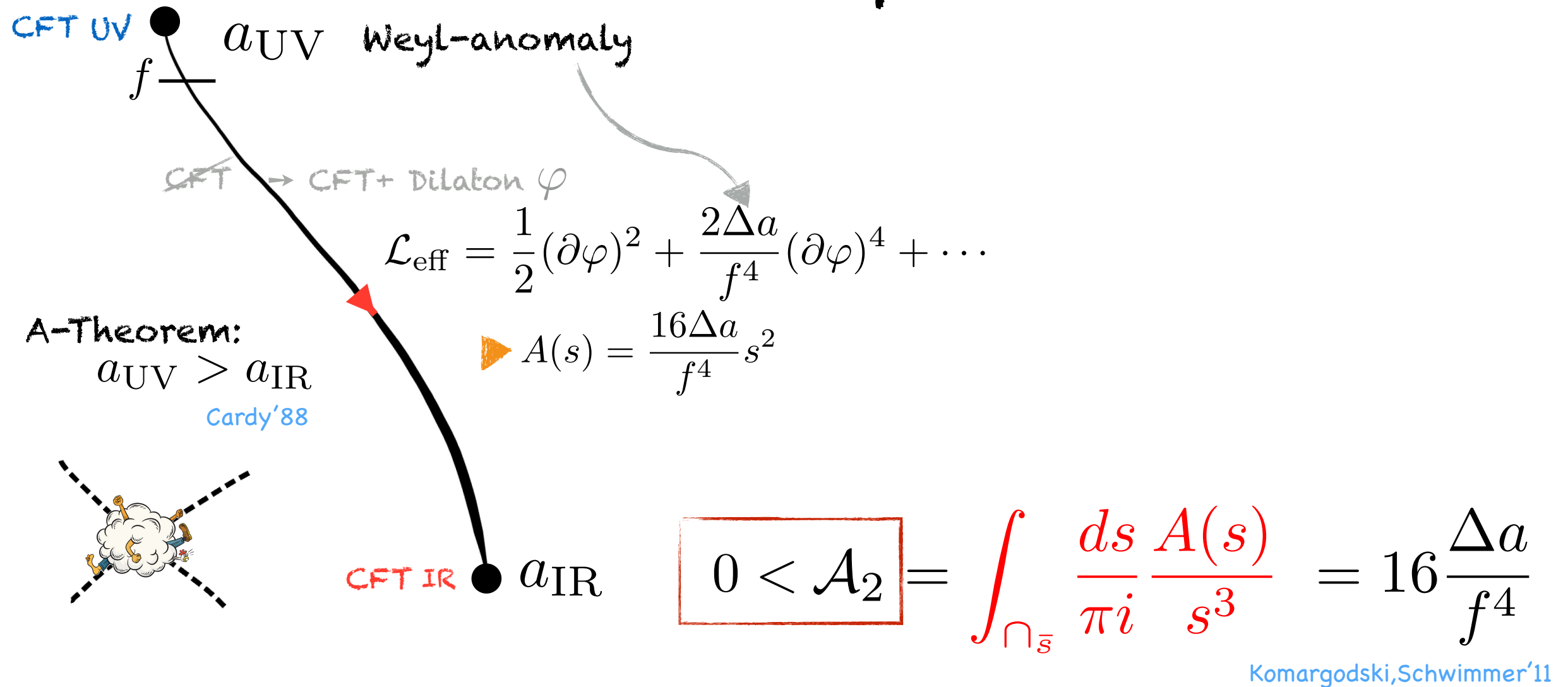
A-Theorem:  
 $a_{UV} > a_{IR}$   
Cardy'88



CFT IR  $\bullet$   $a_{IR}$

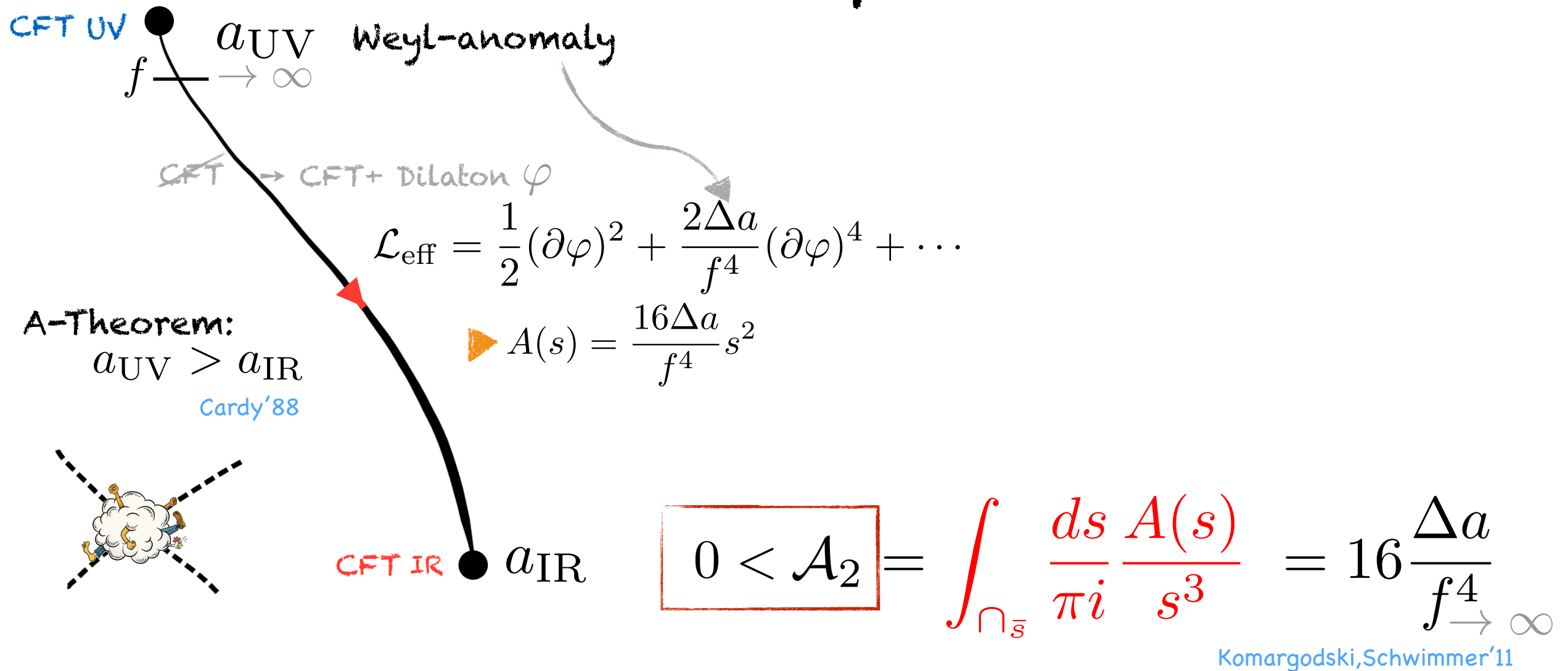
$$\mathcal{A}_2 = \int_{\cap \bar{s}} \frac{ds}{\pi i} \frac{A(s)}{s^3} = 16 \frac{\Delta a}{f^4}$$

# UV $\rightarrow$ IR implications



$\triangleright$  Flow in theory space must follow  $\Delta a > 0$  direction

# UV $\rightarrow$ IR implications



$\blacktriangleright$  Flow in theory space must follow  $\Delta a > 0$  direction

UV  $\rightarrow$  IR implications: symmetries

UV ?



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$

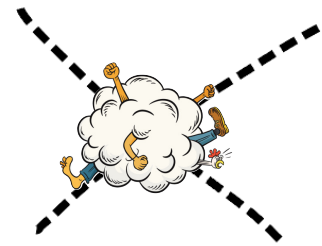


$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$



UV  $\rightarrow$  IR implications: symmetries

UV ?



EFT:  $\blacklozenge$  spin 0  
 $\blacklozenge$   $m=0$

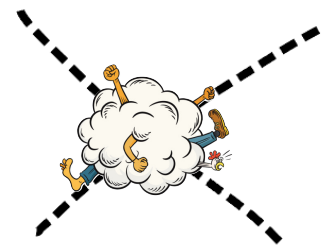


$$A_{2 \rightarrow 2}^{Tree} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$

# UV $\rightarrow$ IR implications: symmetries

UV ?



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$



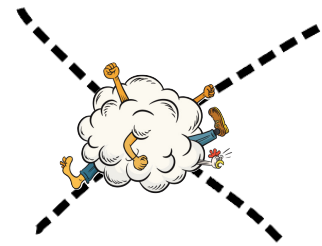
$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$

$$A_{2 \rightarrow 2}^{Tree} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

# UV $\rightarrow$ IR implications: symmetries

UV ?



EFT: • spin 0

•  $m=0$



$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$

$$A_{2 \rightarrow 2}^{Tree} = \underbrace{g_0}_{\ll} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\ll} - \underbrace{g_3 stu}_{\ll} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\ll} + \dots$$

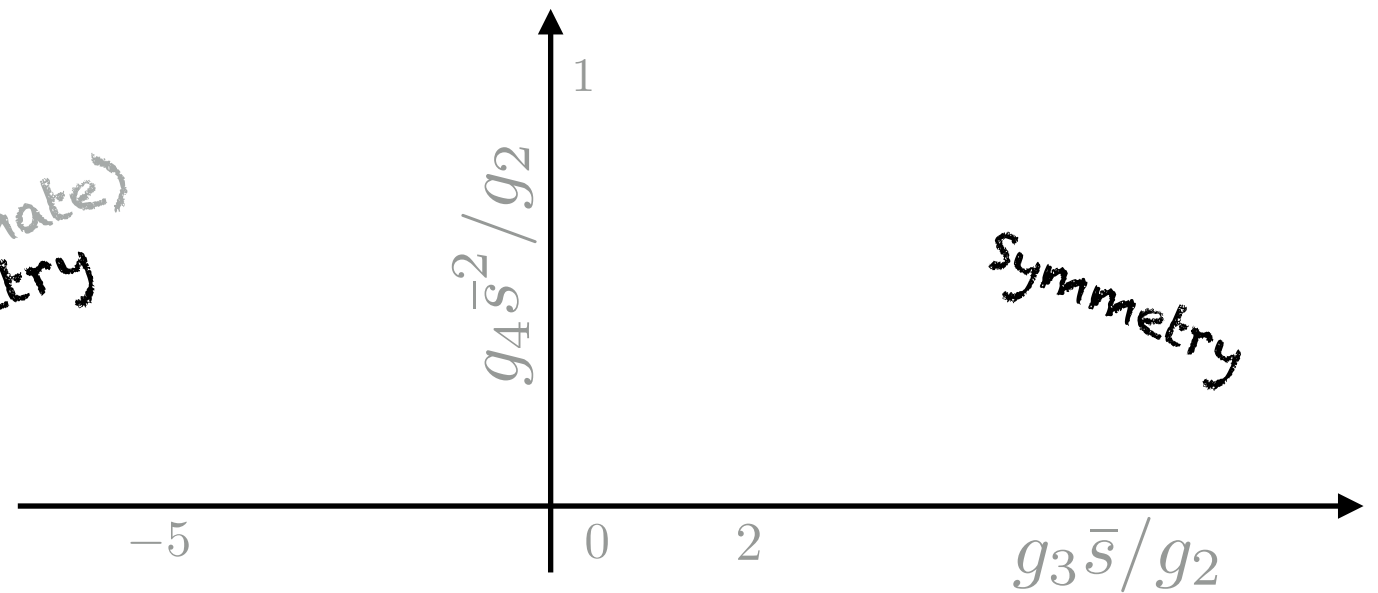
Pions      Galileons      Galileon-like

Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

# UV $\rightarrow$ IR implications: symmetries

UV ?

(approximate)  
symmetry



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$



$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}}$$

$$A_{2 \rightarrow 2}^{Tree} = \underbrace{g_0}_{\ll} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\ll} - \underbrace{g_3 stu}_{\ll} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\ll} + \dots$$

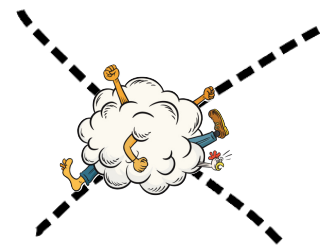
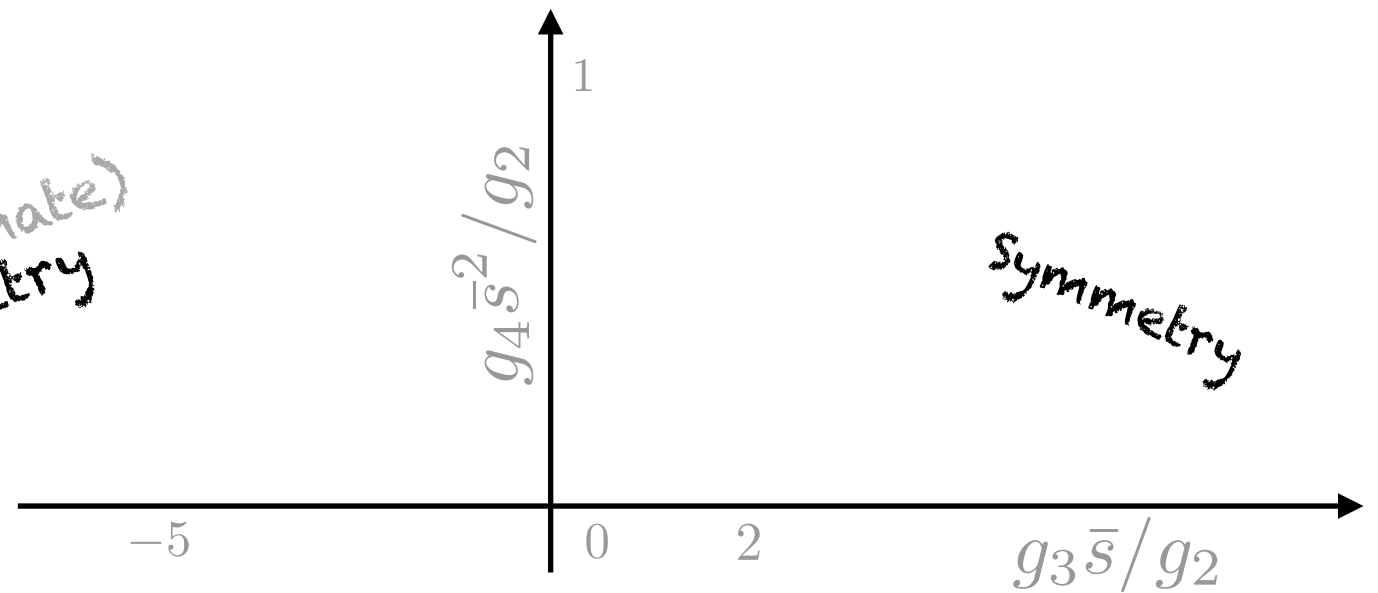
Pions      Galileons      Galileon-like

Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

# UV $\rightarrow$ IR implications: symmetries

UV ?

(approximate)  
symmetry



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$



$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = g_n$$

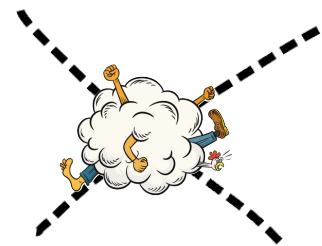
$$A_{2 \rightarrow 2}^{Tree} = \underbrace{g_0}_{\ll} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\ll} - \underbrace{g_3 stu}_{\ll} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\ll} + \dots$$

Pions      Galileons      Galileon-like

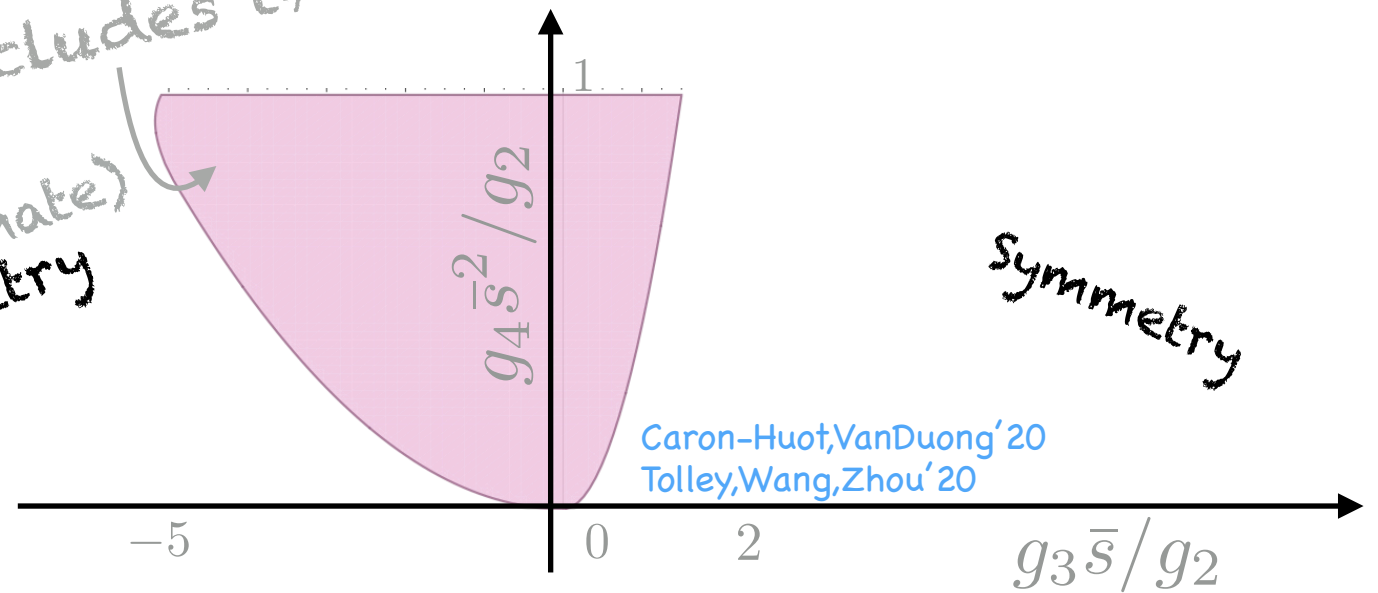
Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

# UV $\rightarrow$ IR implications: symmetries

UV ?



Includes  $t \neq 0$   
(approximate) symmetry



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$



$$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = g_n$$

$$g_2 \geq \bar{s}^2 g_4 \geq \bar{s}^4 g_6 \geq \dots \geq 0$$

$$A_{2 \rightarrow 2}^{Tree} = \underbrace{g_0}_{\ll \text{Pions}} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\ll \text{Galileons}} - \underbrace{g_3 stu}_{\ll \text{Galileon-like}} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\ll \dots} + \dots$$

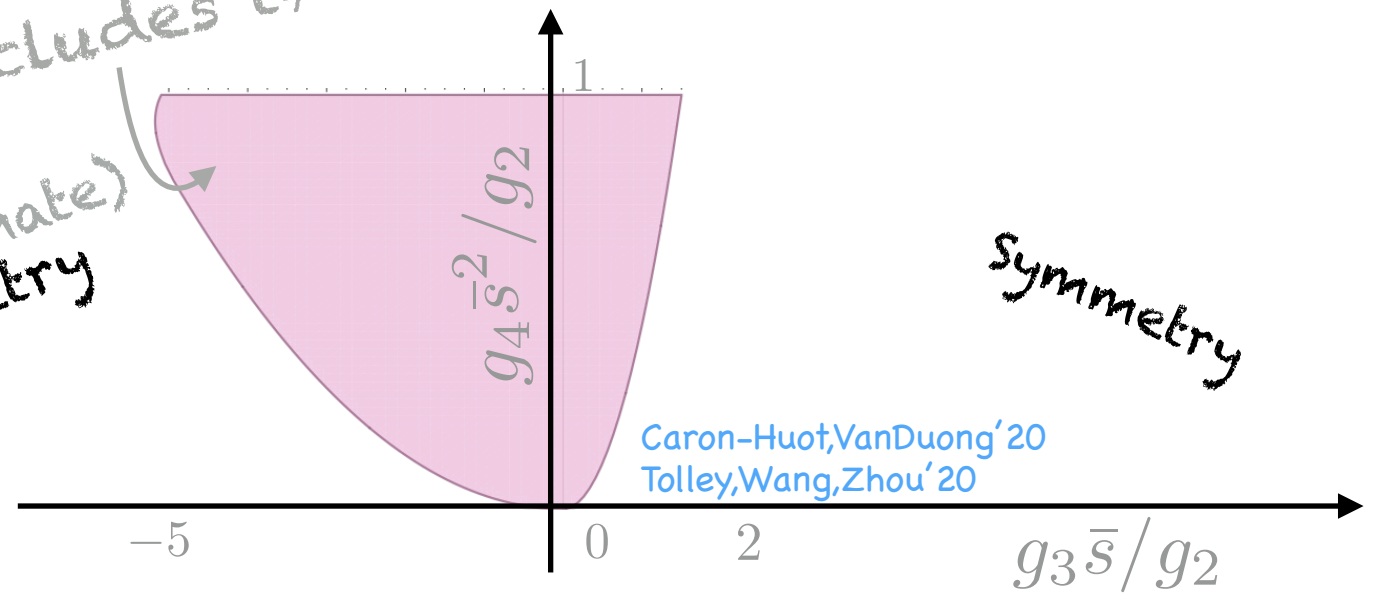
Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$



# UV $\rightarrow$ IR implications: symmetries

UV ?

Includes  $t \neq 0$   
(approximate) symmetry



EFT: • spin 0

•  $m=0$



$$A_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = g_n$$

$$g_2 \geq \bar{s}^2 g_4 \geq \bar{s}^4 g_6 \geq \dots \geq 0$$

$$A_{2 \rightarrow 2}^{Tree} = \underbrace{g_0}_{\text{Pions}} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\text{Galileons}} + \underbrace{g_3 stu}_{\text{Galileon-like}} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\text{Galileon-like}} + \dots$$

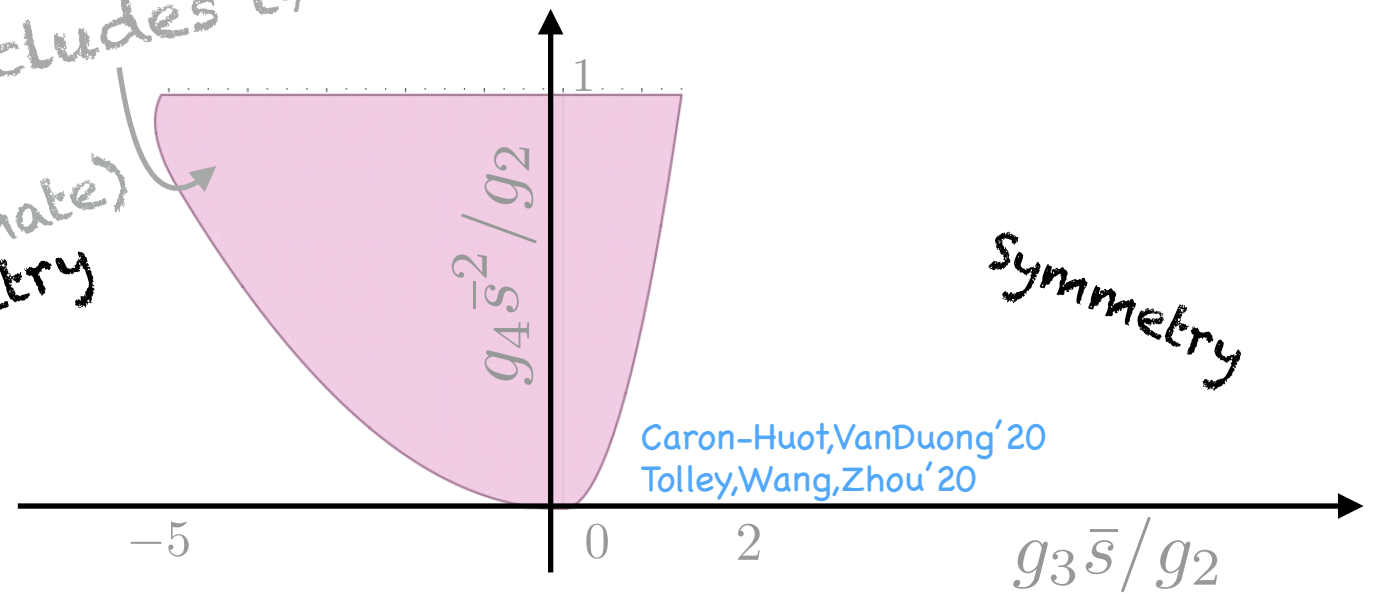
Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

► Amplitude growth faster than  $\sim s^2$  is forbidden  
(large high-derivative interactions)

# UV $\rightarrow$ IR implications: symmetries

UV ?

Includes  $t \neq 0$   
(approximate) symmetry



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$



$$A_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = g_n$$

$$g_2 \geq \bar{s}^2 g_4 \geq \bar{s}^4 g_6 \geq \dots \geq 0$$

$$A_{2 \rightarrow 2}^{\text{Tree}} = \underbrace{g_0}_{\text{Pions}} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\text{Galileons}} + \underbrace{g_3 stu}_{\text{Galileon-like}} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\text{Galileon-like}} + \dots$$

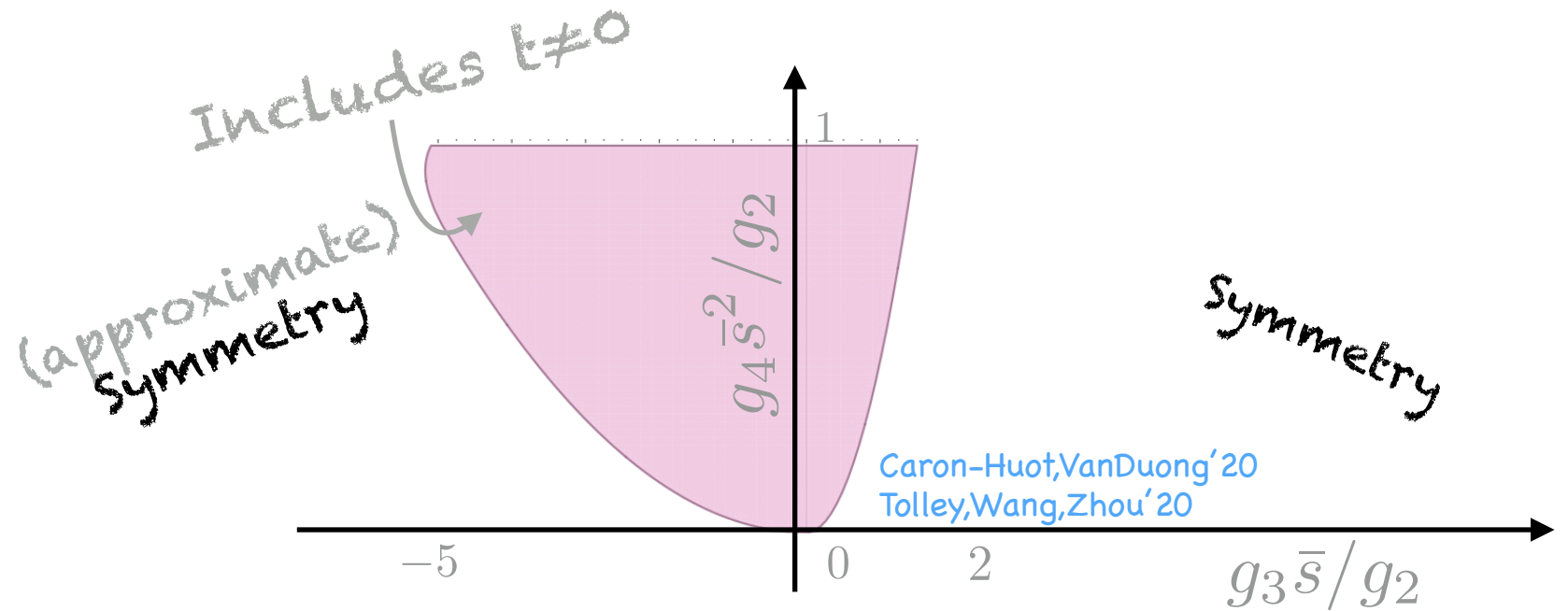
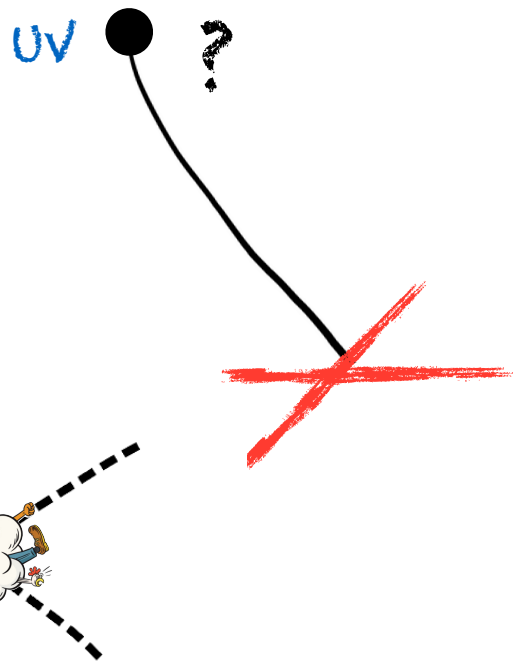
Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

► Amplitude growth faster than  $\sim s^2$  is forbidden  
(large high-derivative interactions)

► Theories with galilean-like symmetries...

...don't have UV completions!

# UV $\rightarrow$ IR implications: symmetries



EFT:  $\bullet$  spin 0  
 $\bullet$   $m=0$

$$A_n(\bar{s}) \equiv \int_{\cap_{M^2}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = g_n$$

$$g_2 \geq \bar{s}^2 g_4 \geq \bar{s}^4 g_6 \geq \dots \geq 0$$

$$A_{2 \rightarrow 2}^{\text{Tree}} = \underbrace{g_0}_{\text{Pions}} + \underbrace{g_2(s^2 + t^2 + u^2)}_{\text{Galileons}} + \underbrace{g_3 stu}_{\text{Galileon-like}} + \underbrace{g_4(s^2 + t^2 + u^2)^2}_{\text{Galileon-like}} + \dots$$

Symmetry:  $\pi \rightarrow \pi + \alpha x^n \rightarrow A \sim s^N$

► Amplitude growth faster than  $\sim s^2$  is forbidden  
 (large high-derivative interactions)

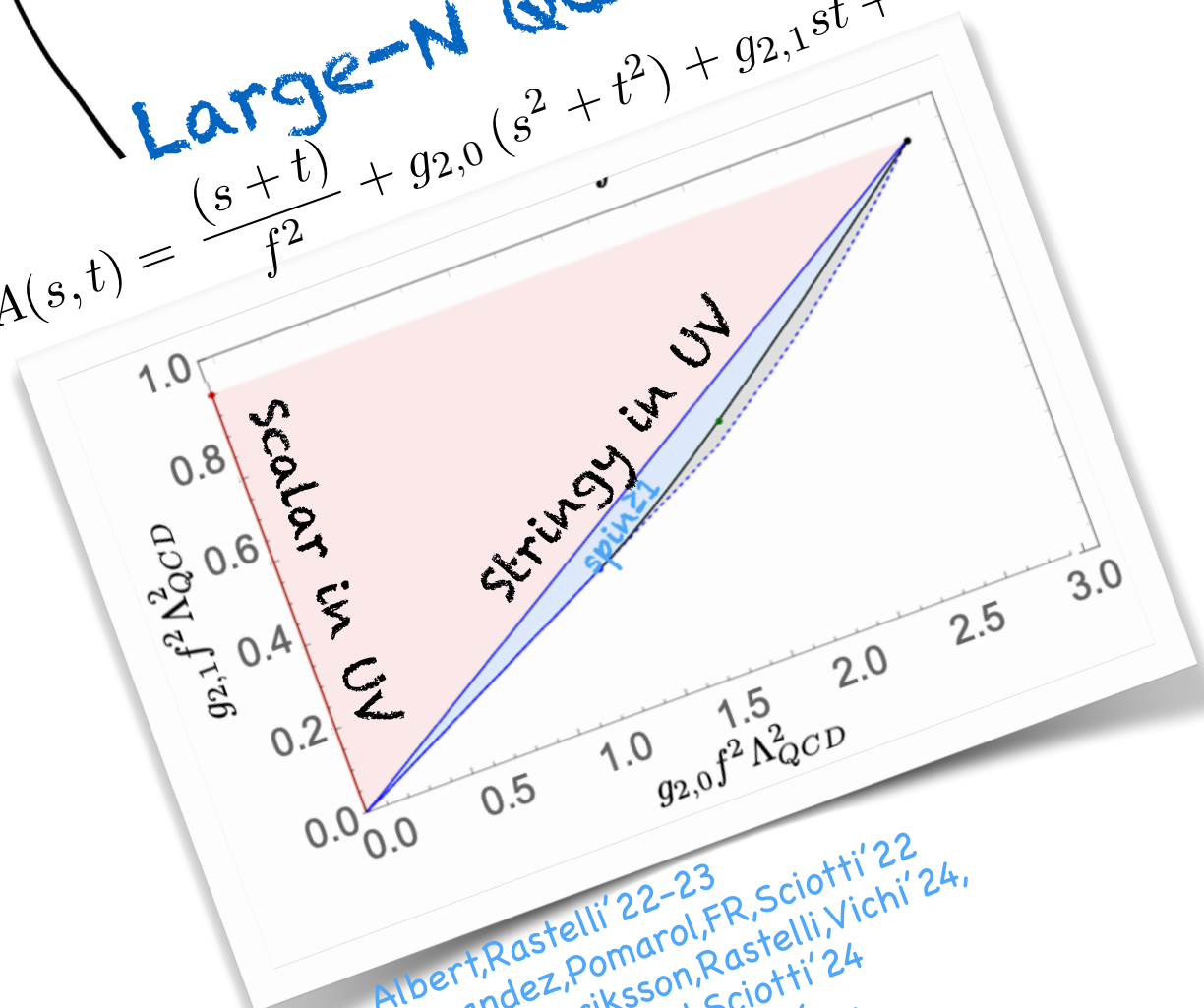
► Theories with galilean-like symmetries...  
 ...don't have UV completions!

# UV $\rightarrow$ IR implications: symmetries

UV ?

Large-N QCD

$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}(s^2 + t^2) + g_{2,1}st + \dots$$



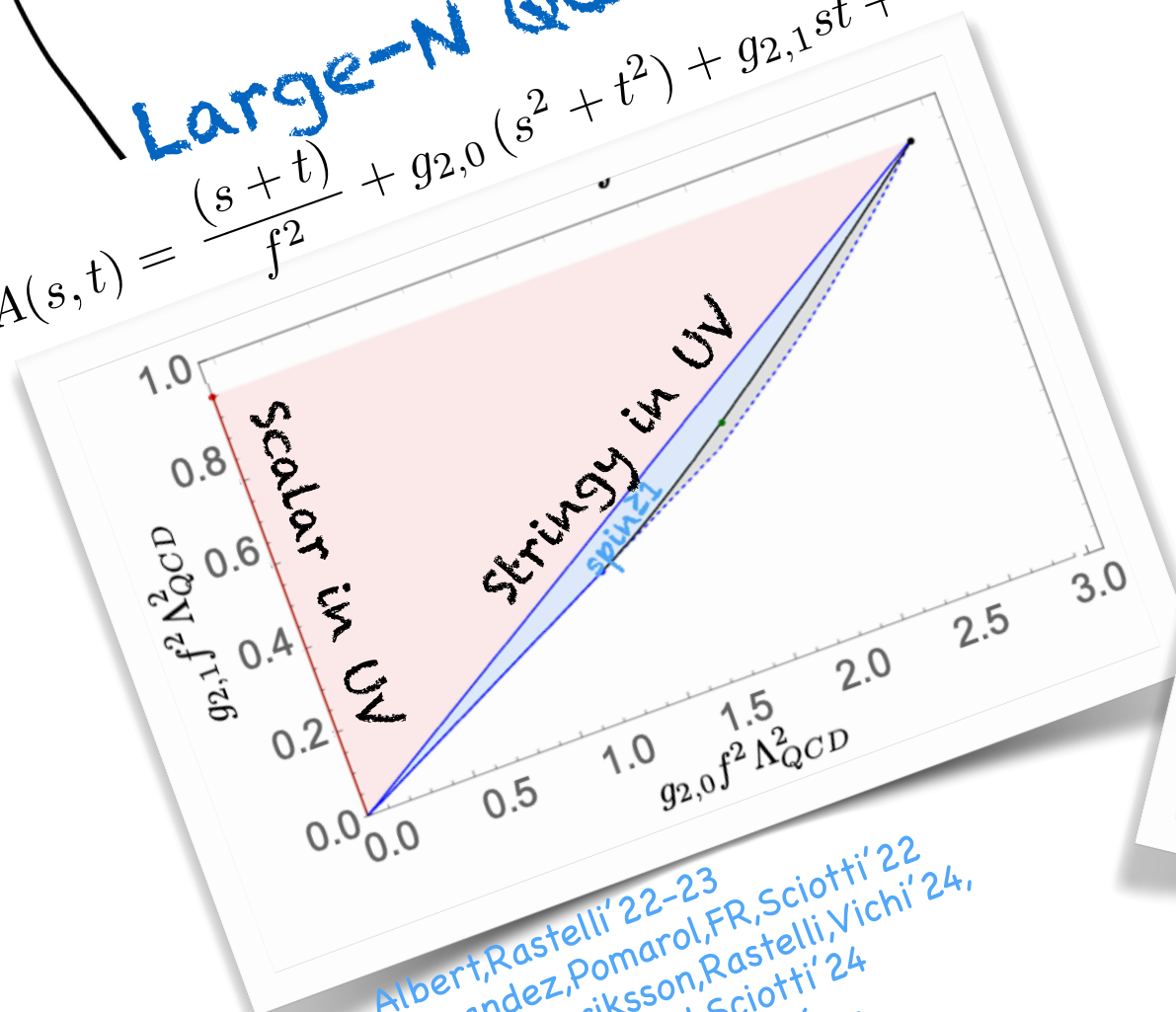
Albert, Rastelli'22-23  
 Fernandez, Pomarol, FR, Sciotti'22  
 Albert, Henriksson, Rastelli, Vichi'24,  
 Dong, Ma, Pomarol, Sciotti'24  
 Albert, Kosva, Rastelli'26 ...

# UV $\rightarrow$ IR implications: symmetries

UV ?

Large-N QCD

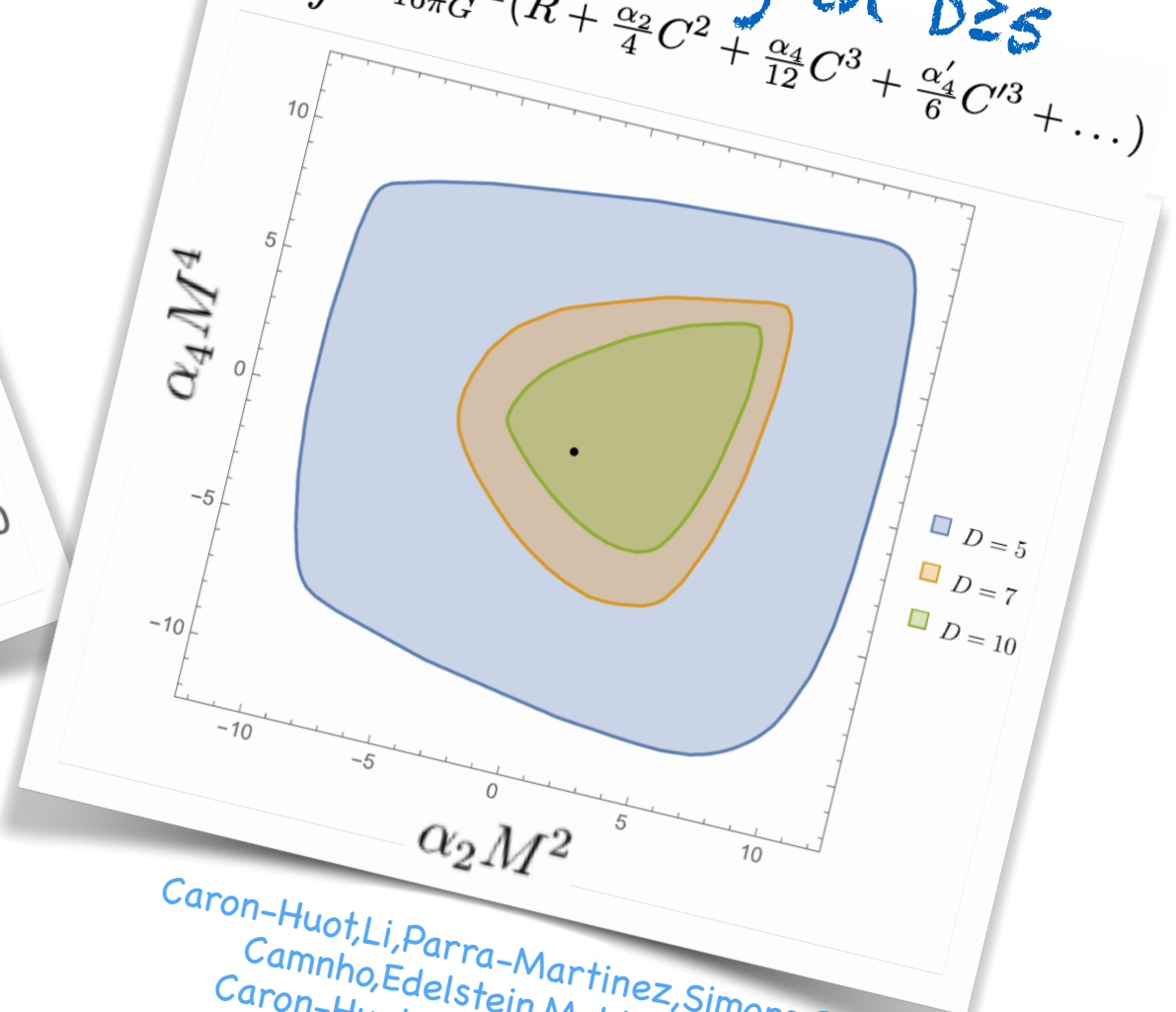
$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}(s^2+t^2) + g_{2,1}st + \dots$$



Albert, Rastelli'22-23  
 Fernandez, Pomarol, FR, Sciotti'22  
 Albert, Henriksson, Rastelli, Vichi'24,  
 Dong, Ma, Pomarol, Sciotti'24  
 Albert, Kosva, Rastelli'26 ...

Gravity in D=5

$$S = \int \frac{d^D x \sqrt{-g}}{16\pi G} (R + \frac{\alpha_2}{4} C^2 + \frac{\alpha_4}{12} C^3 + \frac{\alpha'_4}{6} C'^3 + \dots)$$



Caron-Huot, Li, Parra-Martinez, Simons-Duffin'22  
 Camanho, Edelstein, Maldacena, Zhiboedov'14  
 Caron-Huot, Mazac, Rastelli, Simons-Duffin'21

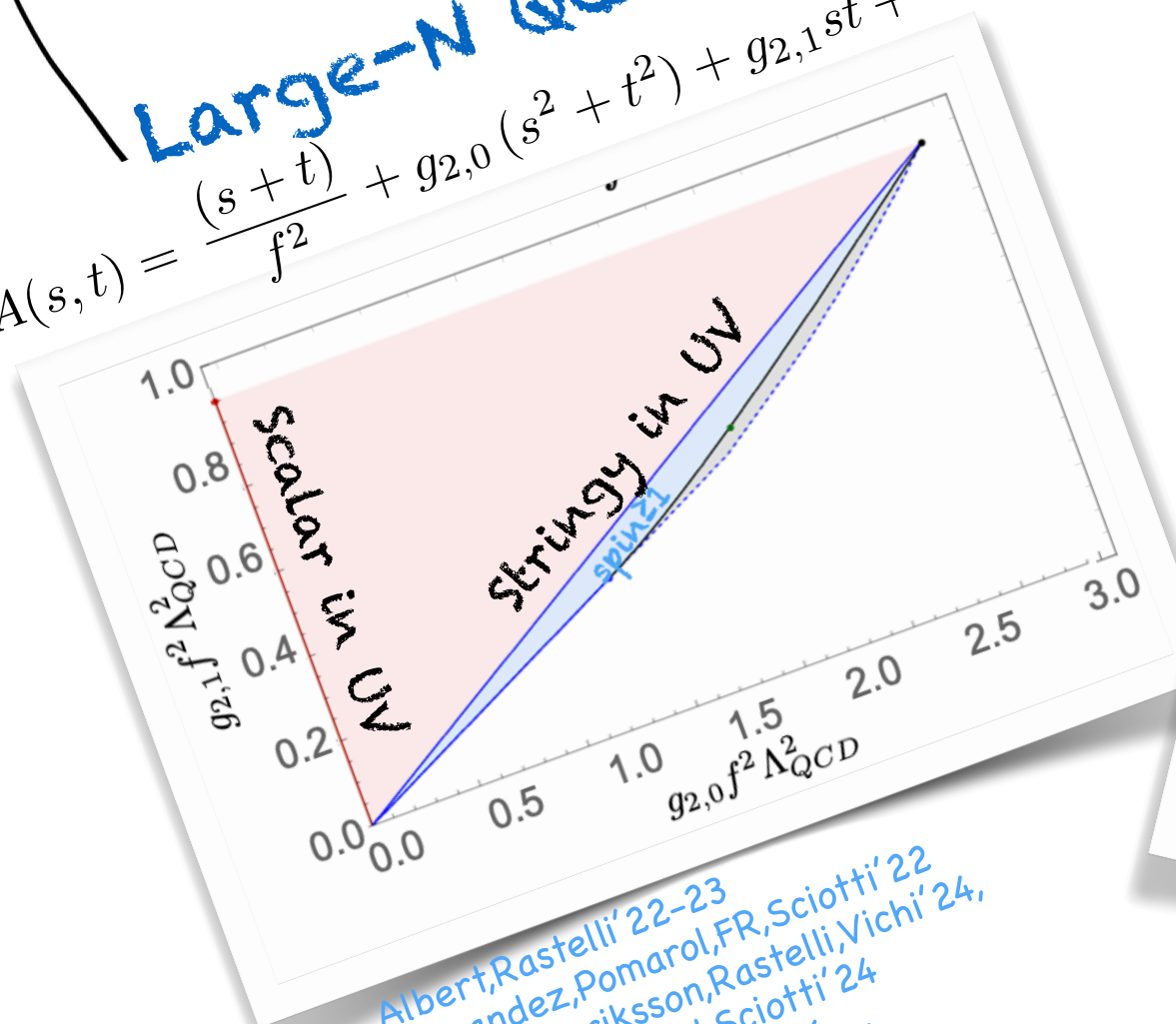


# UV $\rightarrow$ IR implications: symmetries

UV ● ?

Large-N QCD

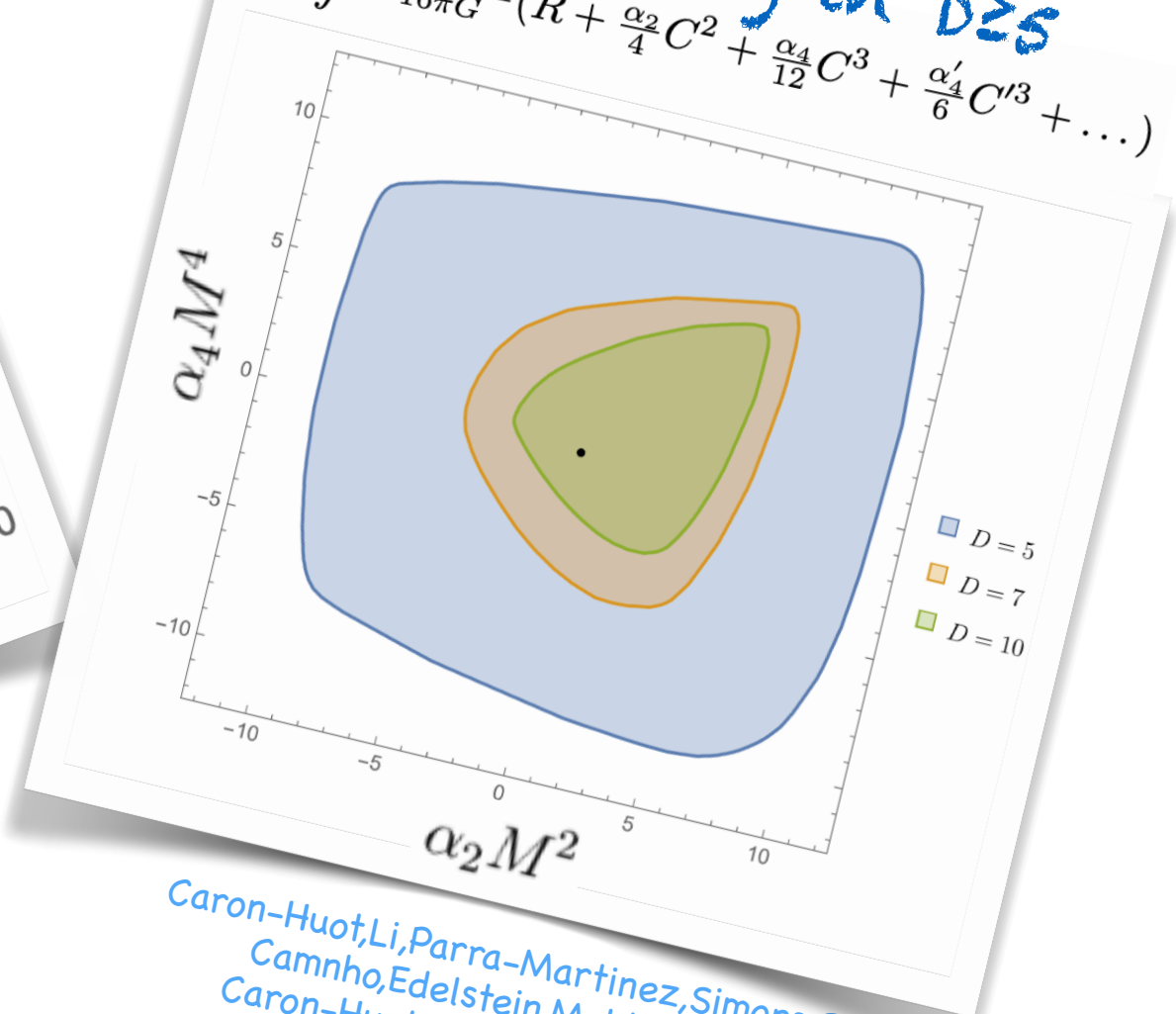
$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}(s^2 + t^2) + g_{2,1}st + \dots$$



Albert, Rastelli'22-23  
 Fernandez, Pomarol, FR, Sciotti'22  
 Albert, Henriksson, Rastelli, Vichi'24,  
 Dong, Ma, Pomarol, Sciotti'24  
 Albert, Kosva, Rastelli'26 ...

Gravity in D=5

$$S = \int \frac{d^D x \sqrt{-g}}{16\pi G} (R + \frac{\alpha_2}{4} C^2 + \frac{\alpha_4}{12} C^3 + \frac{\alpha'_4}{6} C'^3 + \dots)$$



Caron-Huot, Li, Parra-Martinez, Simons-Duffin'22  
 Camanho, Edelstein, Maldacena, Zhiboedov'14  
 Caron-Huot, Mazac, Rastelli, Simons-Duffin'21

more irrelevant couplings  $\leq O(1)$  x more relevant couplings

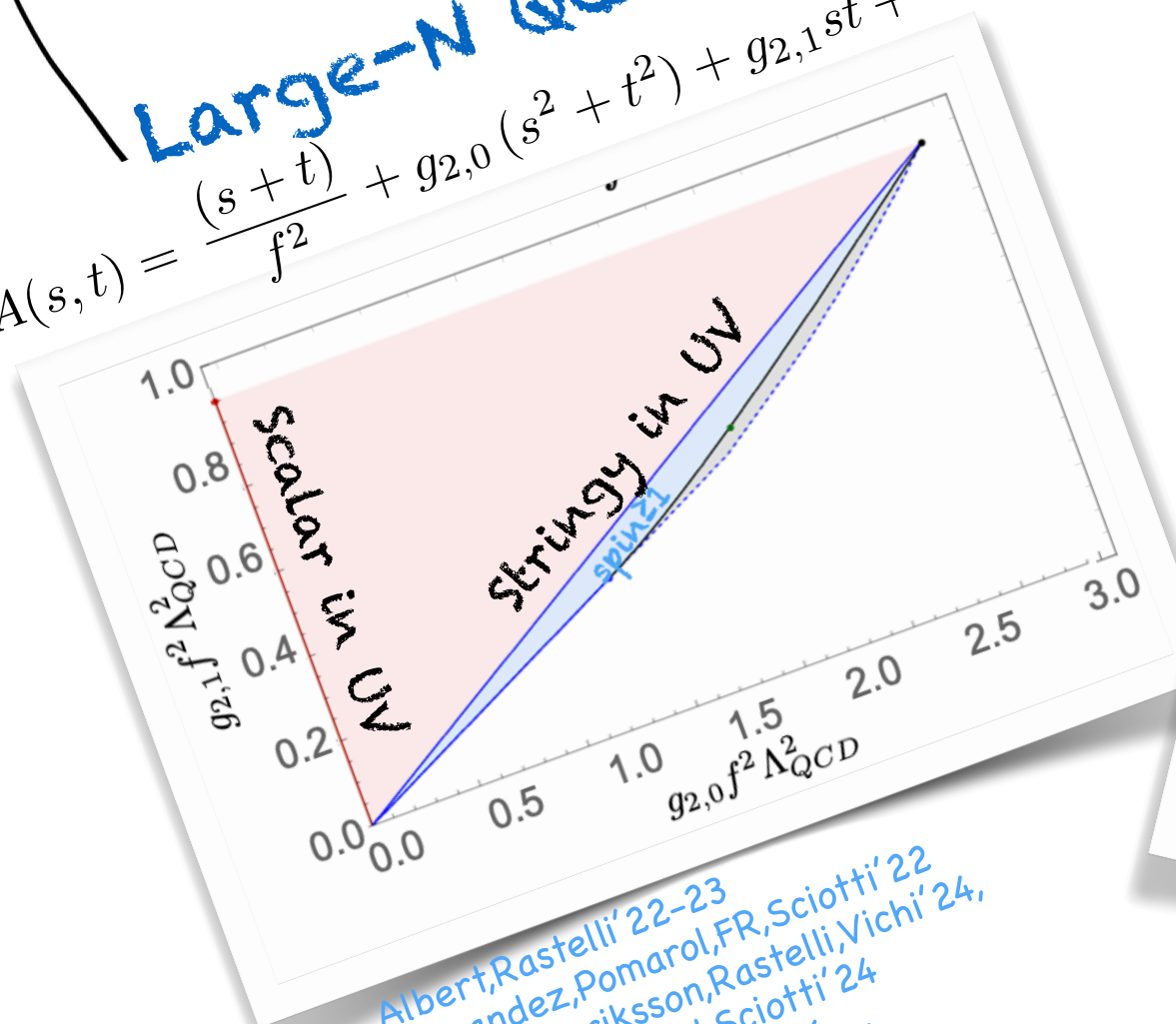


# UV $\rightarrow$ IR implications: symmetries

UV ?

Large-N QCD

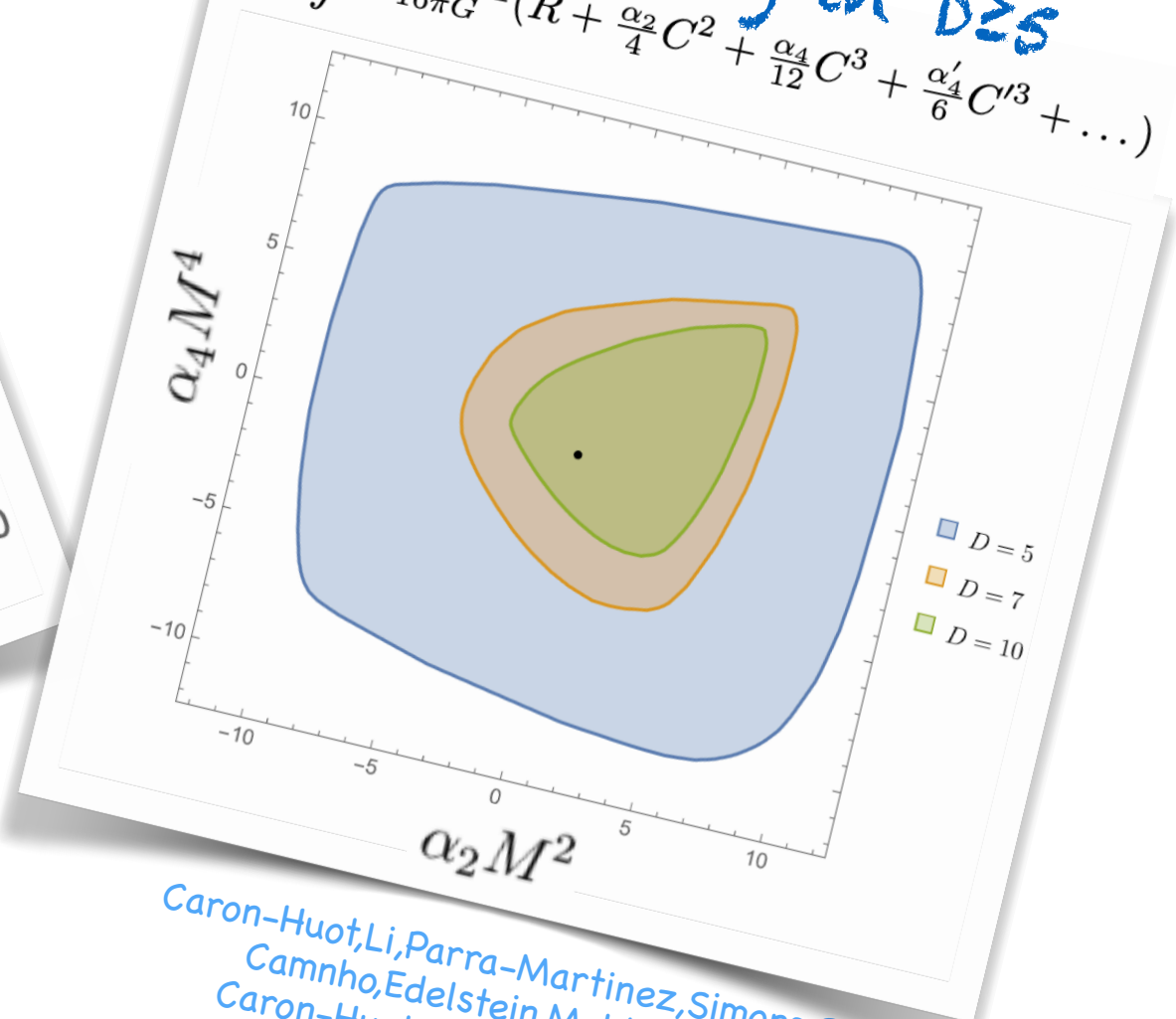
$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}(s^2 + t^2) + g_{2,1}st + \dots$$



Albert, Rastelli'22-23  
 Fernandez, Pomarol, FR, Sciotti'22  
 Albert, Henriksson, Rastelli, Vichi'24,  
 Dong, Ma, Pomarol, Sciotti'24  
 Albert, Kosva, Rastelli'26 ...

Gravity in D=5

$$S = \int \frac{d^D x \sqrt{-g}}{16\pi G} (R + \frac{\alpha_2}{4} C^2 + \frac{\alpha_4}{12} C^3 + \frac{\alpha'_4}{6} C'^3 + \dots)$$



Caron-Huot, Li, Parra-Martinez, Simons-Duffin'22  
 Camanho, Edelstein, Maldacena, Zhiboedov'14  
 Caron-Huot, Mazac, Rastelli, Simons-Duffin'21

more irrelevant couplings  $\leq O(1)$  x more relevant couplings

(e.g. in Black Hole inspiral experiments: no UV completion can induce effects larger than Einstein gravity, at any scale)

Caron-Huot, Li, Parra-Martinez, Simons-Duffin'22  
 Serra, Serra, Trincherini, Trombetta'22

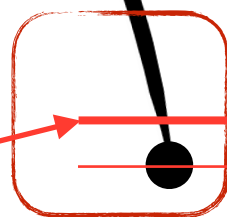
Only experiments with  $\ll O(1)$  precision detect New Physics

# UV $\rightarrow$ IR implications: spectra

1



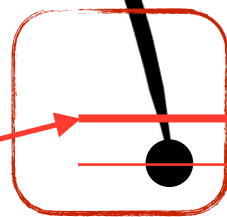
EFT:  $\diamond$  spin 2  
 $\diamond$   $m \neq 0$



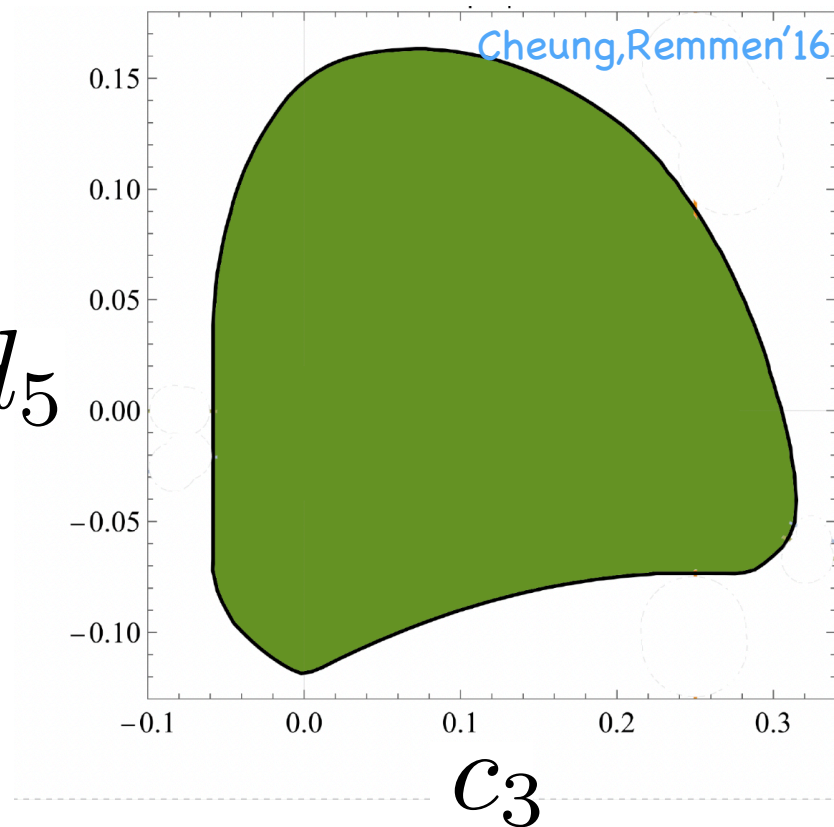
# UV $\rightarrow$ IR implications: spectra



EFT:  $\blacklozenge$  spin 2  
 $\blacklozenge$   $m \neq 0$



$d_5$



1

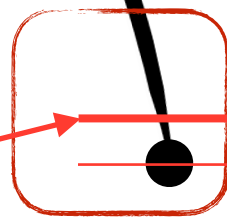
Massive Gravity:  $A(hh \rightarrow hh) \sim \frac{(c_3, d_5)}{\Lambda_3^6} [m_g^2 s^2 + s^2 t]$

Fierz, Pauli '1930s  
 Arkani-Hamed, Georgi, Schwartz '02  
 deRham, Gabadatz, Tolley '10

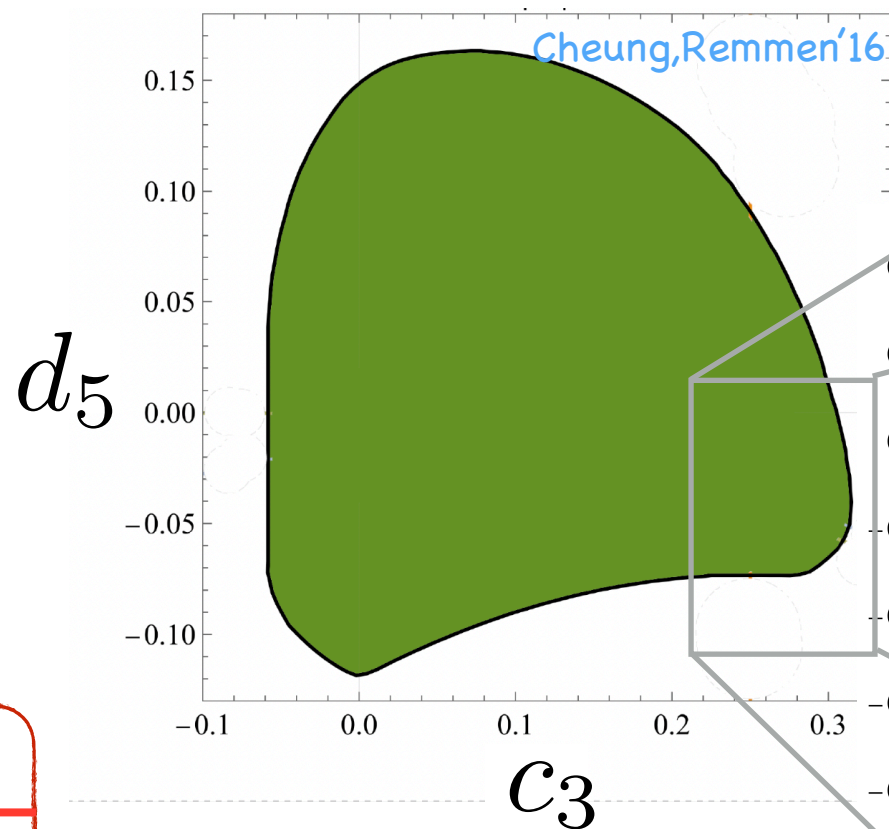
# UV $\rightarrow$ IR implications: spectra



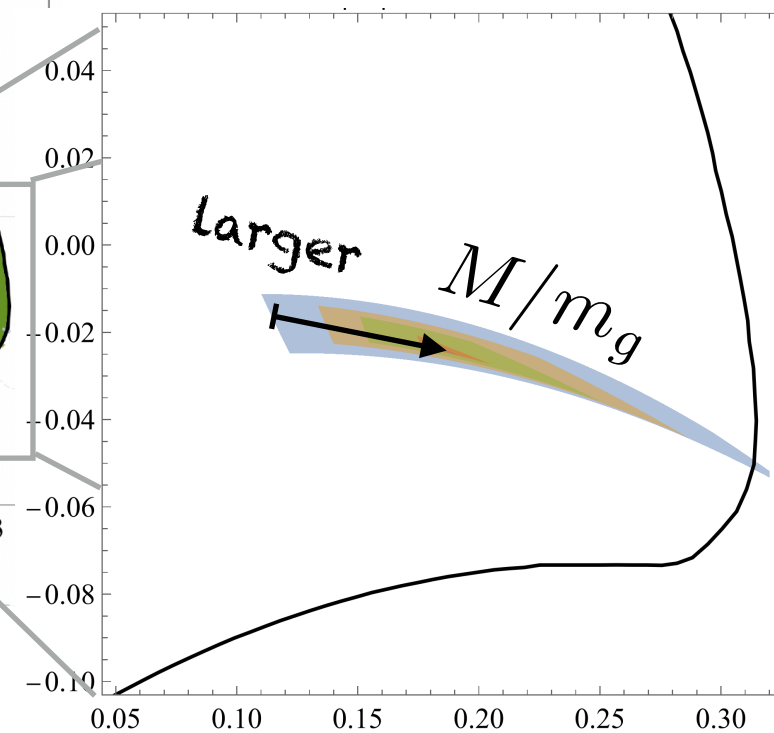
EFT:  $\blacklozenge$  spin 2  
 $\blacklozenge$   $m \neq 0$



Massive Gravity:  $A(hh \rightarrow hh) \sim \frac{(c_3, d_5)}{\Lambda_3^6} [m_g^2 s^2 + s^2 t]$   
 Fierz, Pauli '1930s  
 Arkani-Hamed, Georgi, Schwartz '02  
 deRham, Gabadatzte, Tolley '10



1

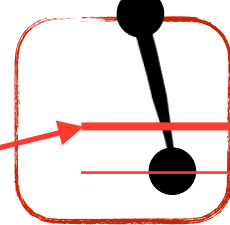


Bellazzini, Isabella, Ricossa, FR '23

# UV $\rightarrow$ IR implications: spectra

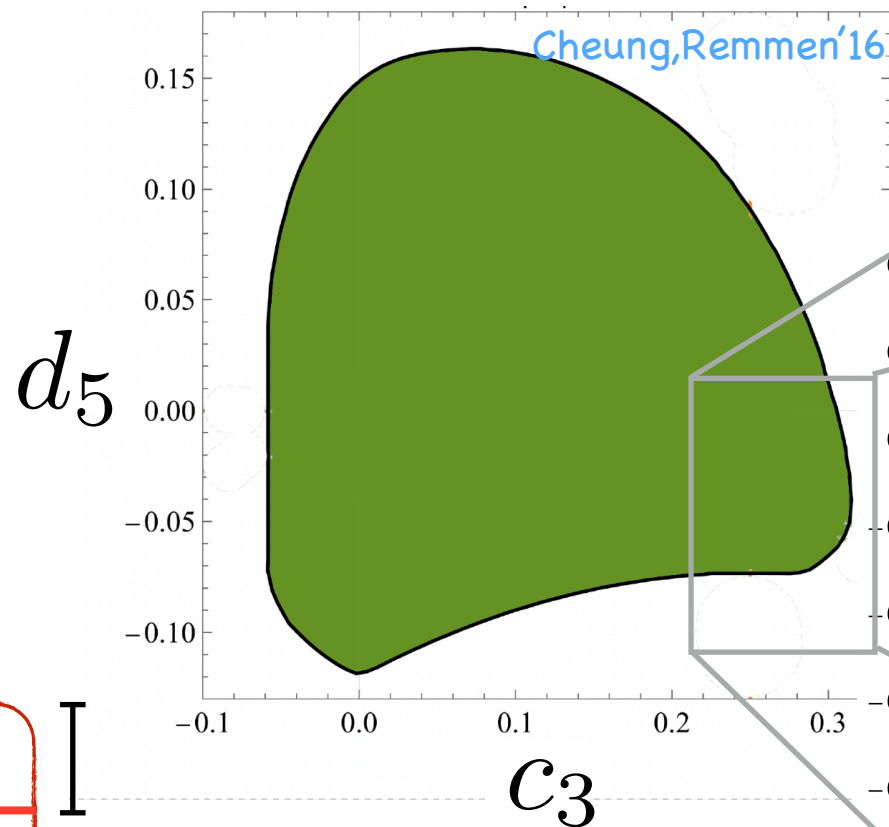


UV

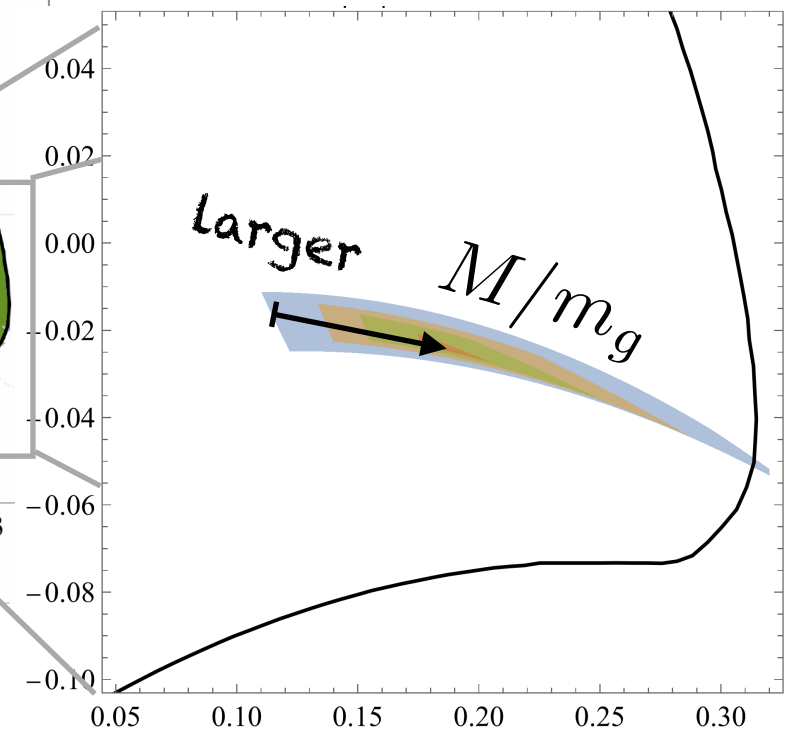


EFT:  $\bullet$  spin 2  
 $\bullet$   $m \neq 0$

Massive Gravity:  $A(hh \rightarrow hh) \sim \frac{(c_3, d_5)}{\Lambda_3^6} [m_g^2 s^2 + s^2 t]$   
 Fierz, Pauli '1930s  
 Arkani-Hamed, Georgi, Schwartz '02  
 deRham, Gabadatz, Tolley '10



1



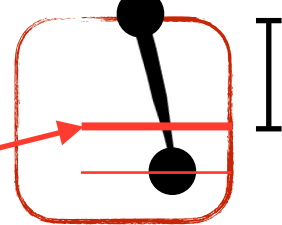
► Massive gravity only consistent over range  $M \lesssim 30m_g$



# UV $\rightarrow$ IR implications: spectra



UV



EFT: • spin 3/2

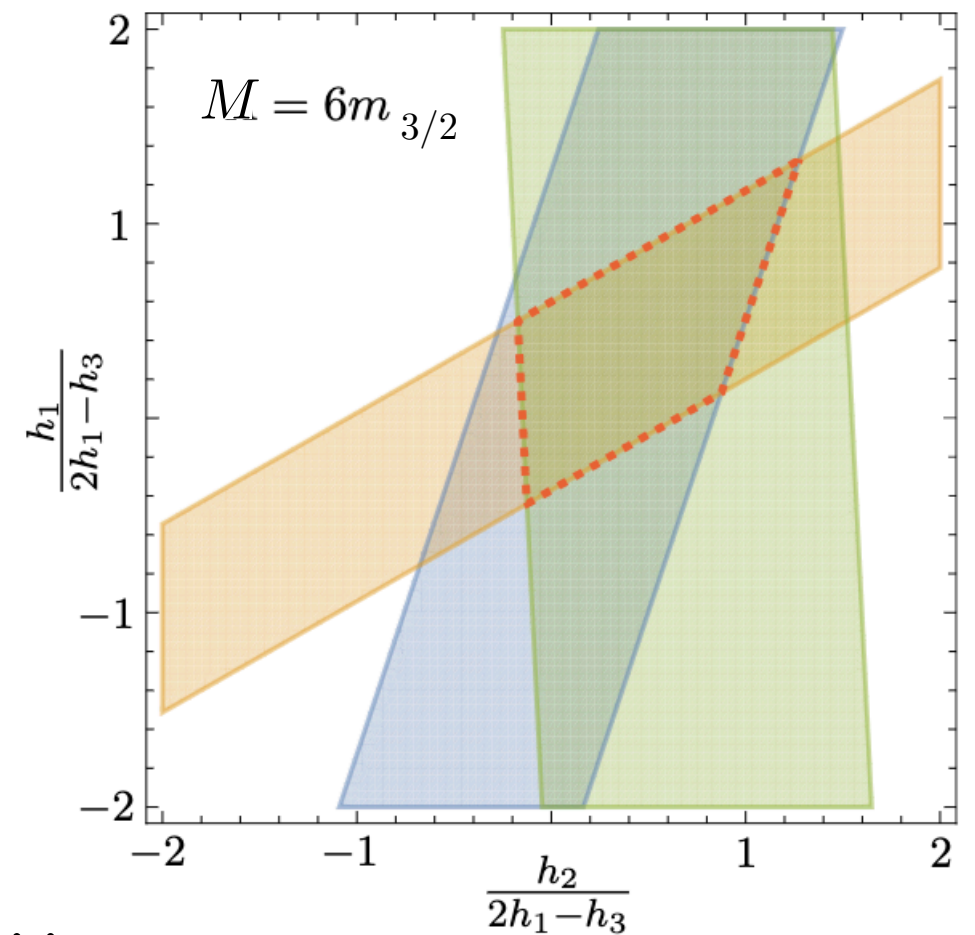
•  $m \neq 0$

Massive spin-3/2:

$$A^{--} = -\frac{2h_2s^3 + h_3s[s^2 + 2t(s+t)]}{18M^6} + \dots$$

$$A^{+-} = -\frac{h_1st(s+t)}{3M^6} + \dots$$

Bellazzini, Pomarol, Romano, Sciotti '25



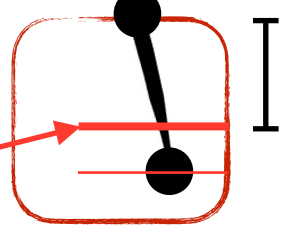
► Massive gravity only consistent over range  $M \lesssim 30m_g$



# UV $\rightarrow$ IR implications: spectra



UV



EFT: • spin 3/2

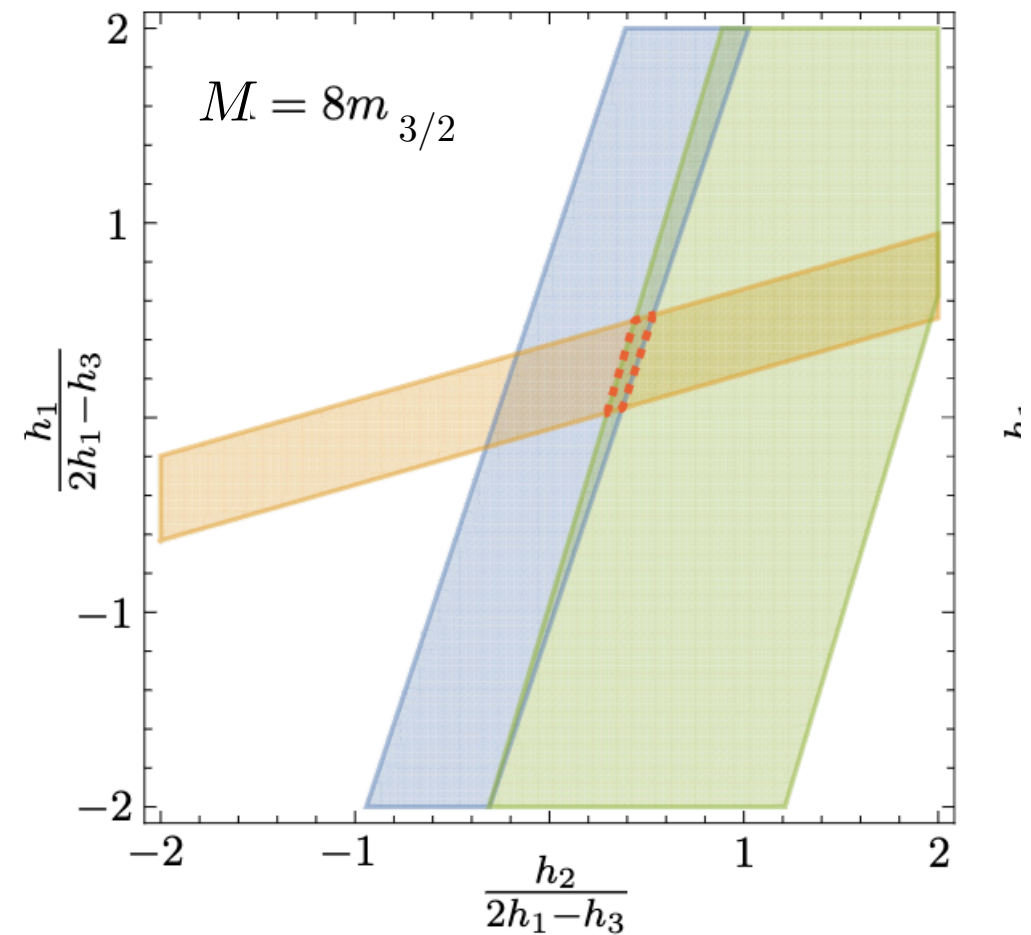
•  $m \neq 0$

Massive spin-3/2:

$$A^{--} = -\frac{2h_2s^3 + h_3s[s^2 + 2t(s+t)]}{18M^6} + \dots$$

$$A^{+-} = -\frac{h_1st(s+t)}{3M^6} + \dots$$

Bellazzini, Pomarol, Romano, Sciotti '25



► Massive gravity only consistent over range  $M \lesssim 30m_g$

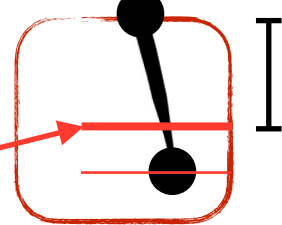
# UV $\rightarrow$ IR implications: spectra



UV

EFT:  $\bullet$  spin 3/2

$\bullet m \neq 0$

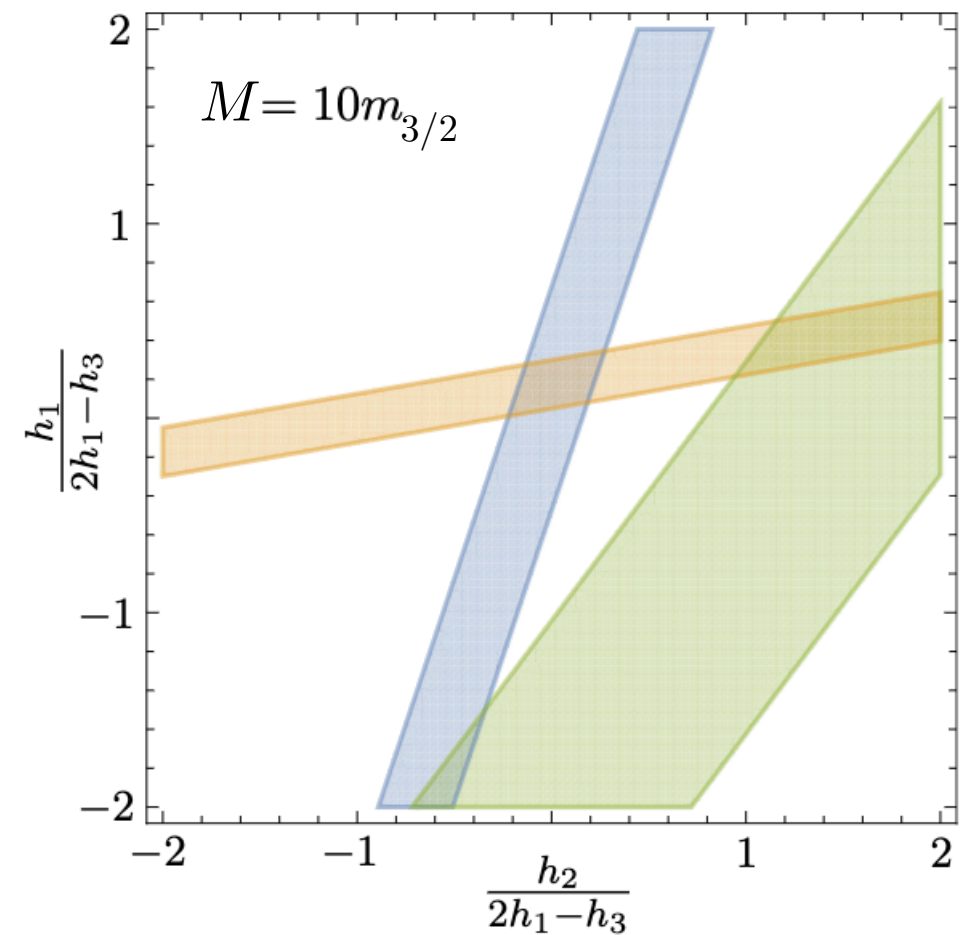


Massive spin-3/2:

$$A^{--} = -\frac{2h_2s^3 + h_3s[s^2 + 2t(s+t)]}{18M^6} + \dots$$

$$A^{+-} = -\frac{h_1st(s+t)}{3M^6} + \dots$$

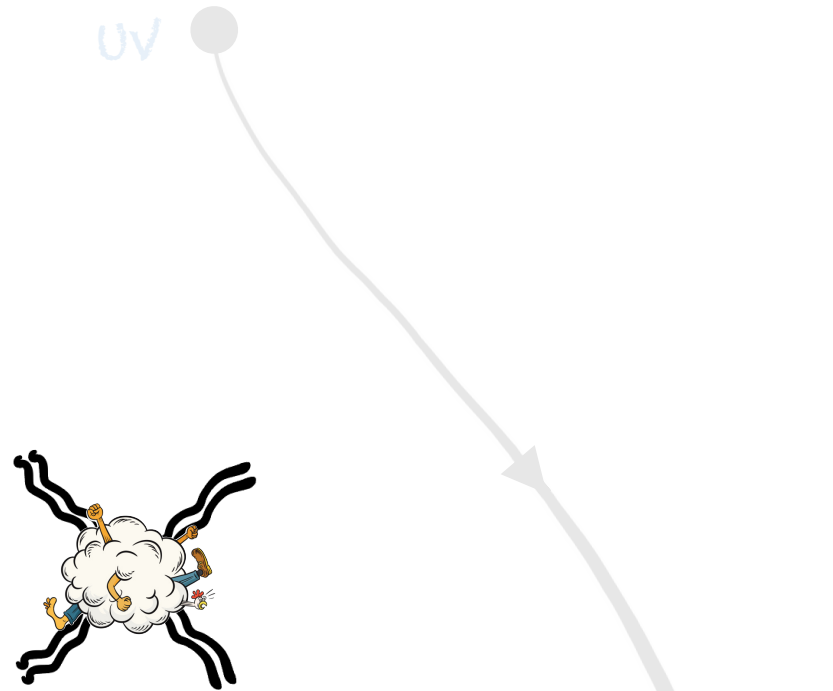
Bellazzini, Pomarol, Romano, Sciotti '25



► Massive gravity only consistent over range  $M \lesssim 30m_g$

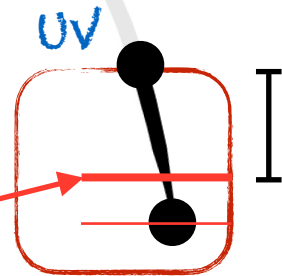
# UV $\rightarrow$ IR implications: spectra

Bellazzini, Pomarol, Romano, Sciotti '25



EFT: • spin 3/2

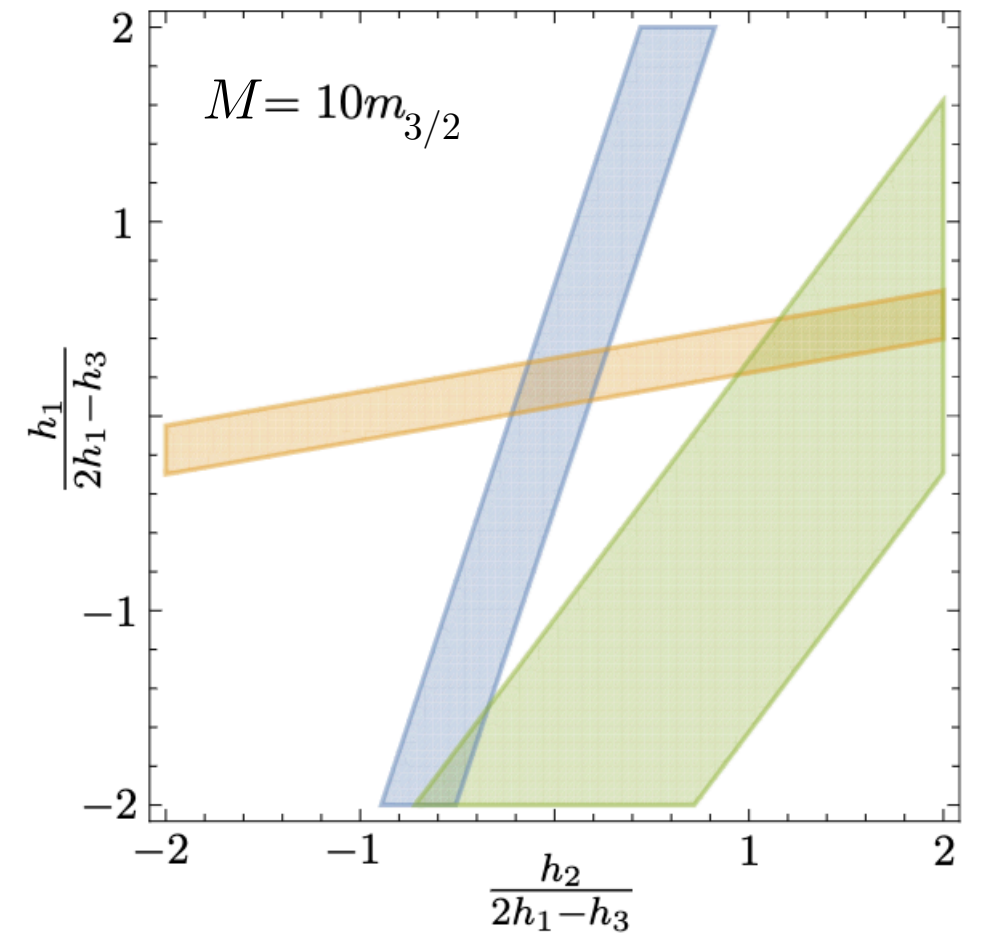
•  $m \neq 0$



Massive spin-3/2:

$$A^{--} = -\frac{2h_2s^3 + h_3s[s^2 + 2t(s+t)]}{18M^6} + \dots$$

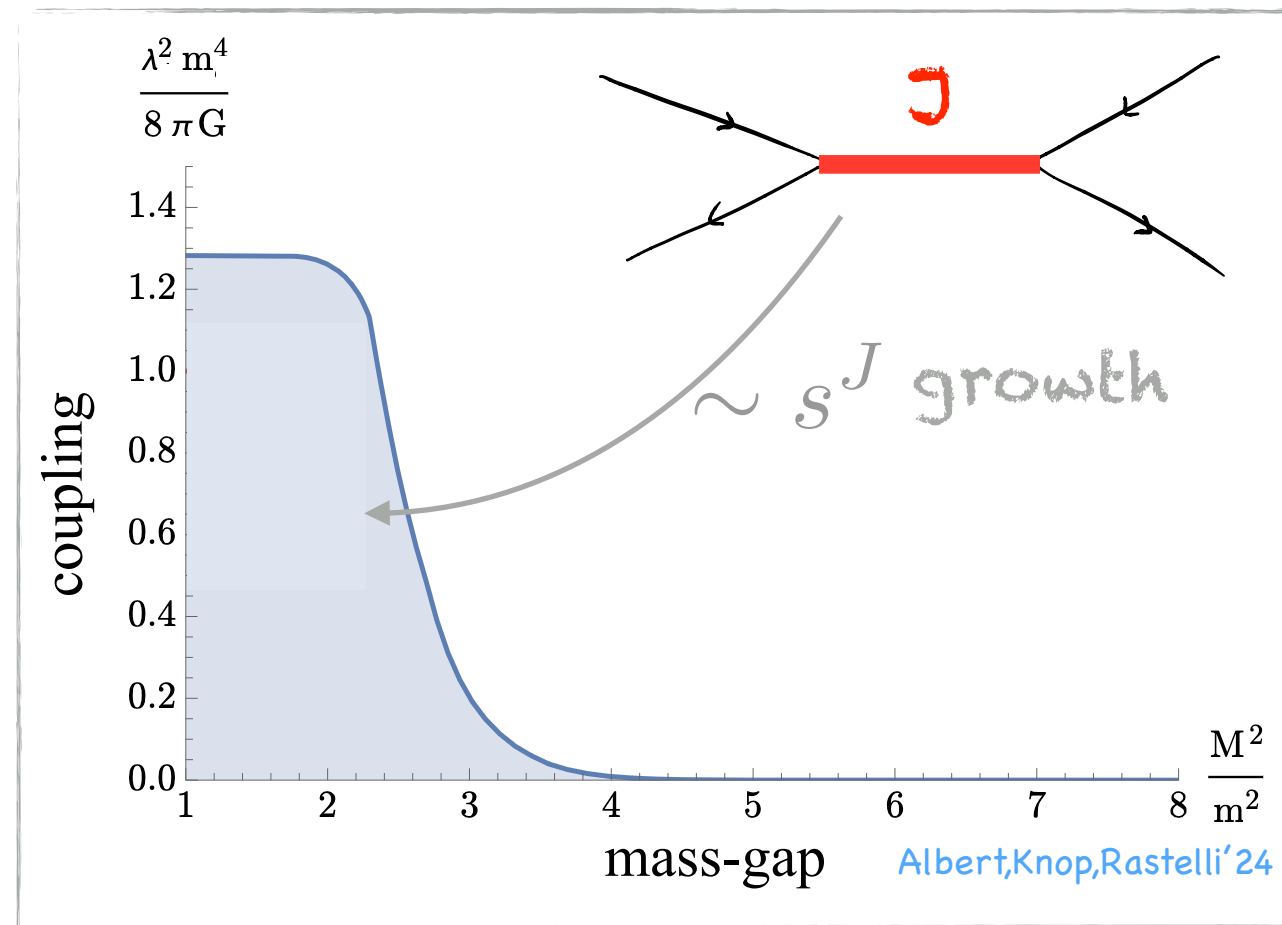
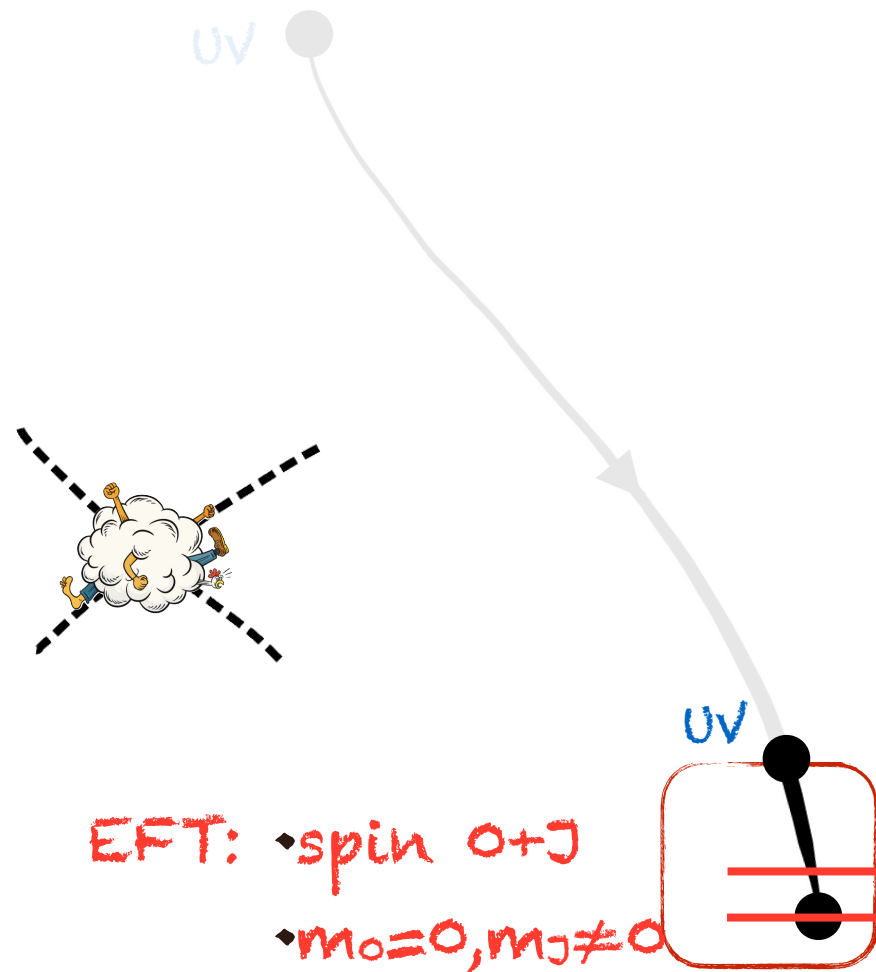
$$A^{+-} = -\frac{h_1st(s+t)}{3M^6} + \dots$$



► Massive gravity only consistent over range  $M \lesssim 30m_g$

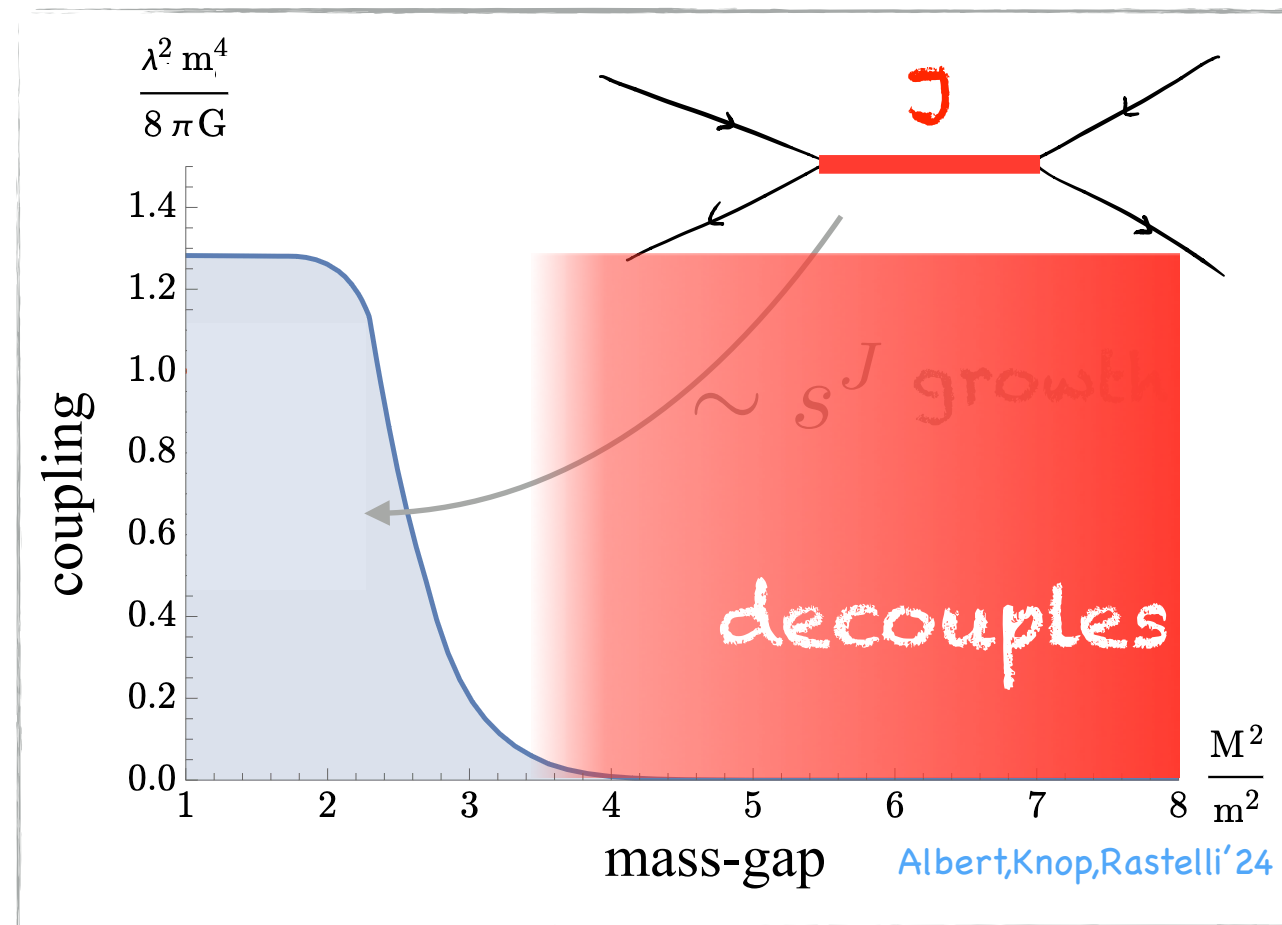
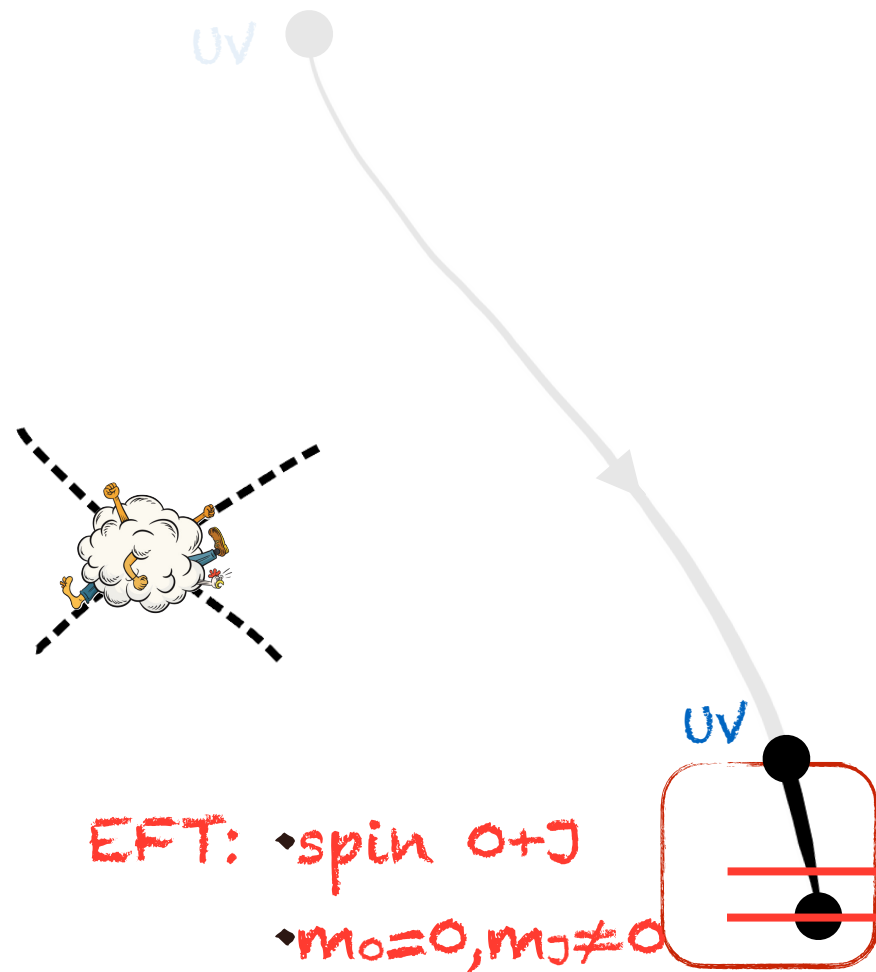
► Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
 ( $M \rightarrow \infty$  with graviton and tuned couplings)

# UV $\rightarrow$ IR implications: spectra



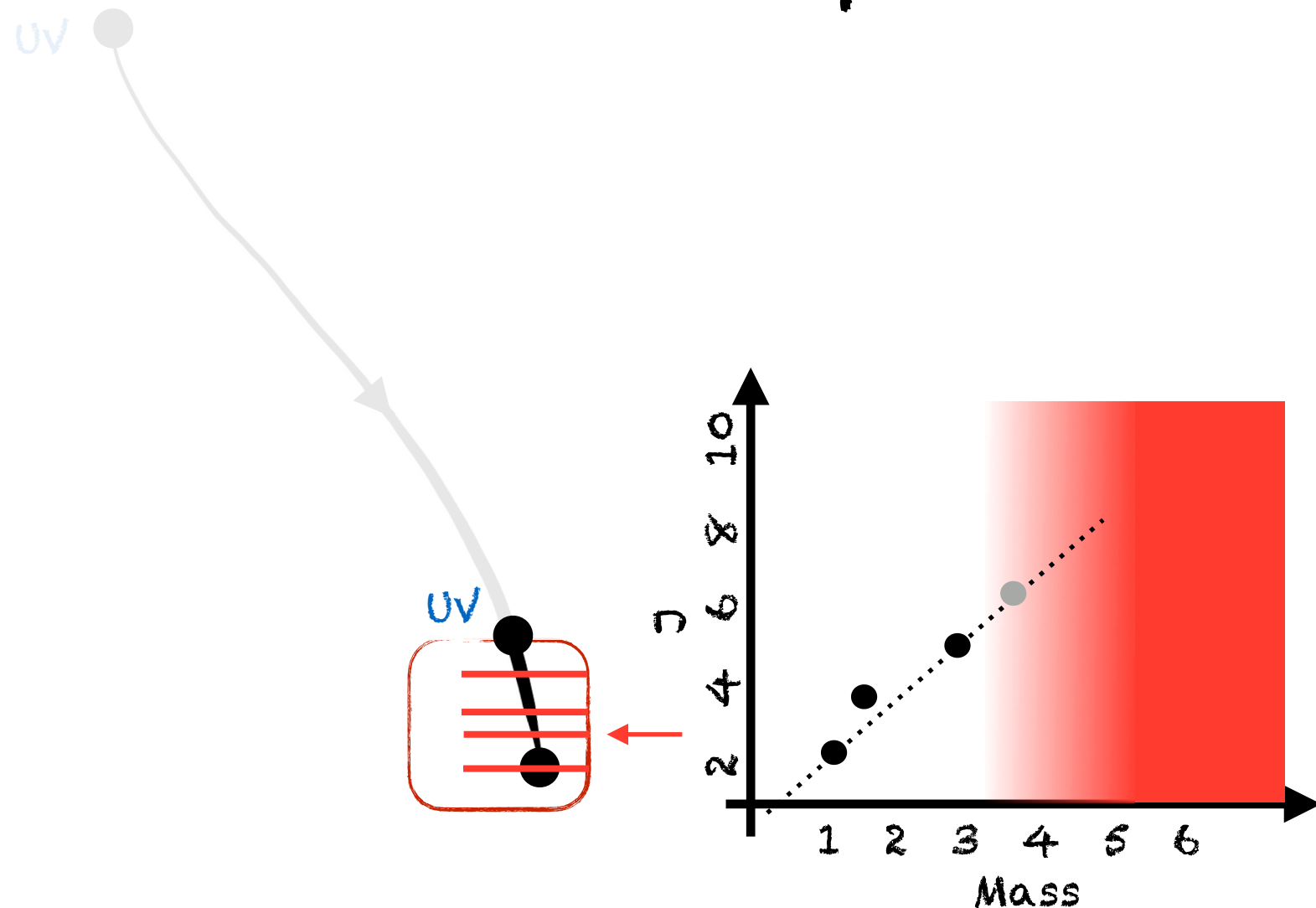
- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
 ( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass

# UV $\rightarrow$ IR implications: spectra



- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
 ( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass

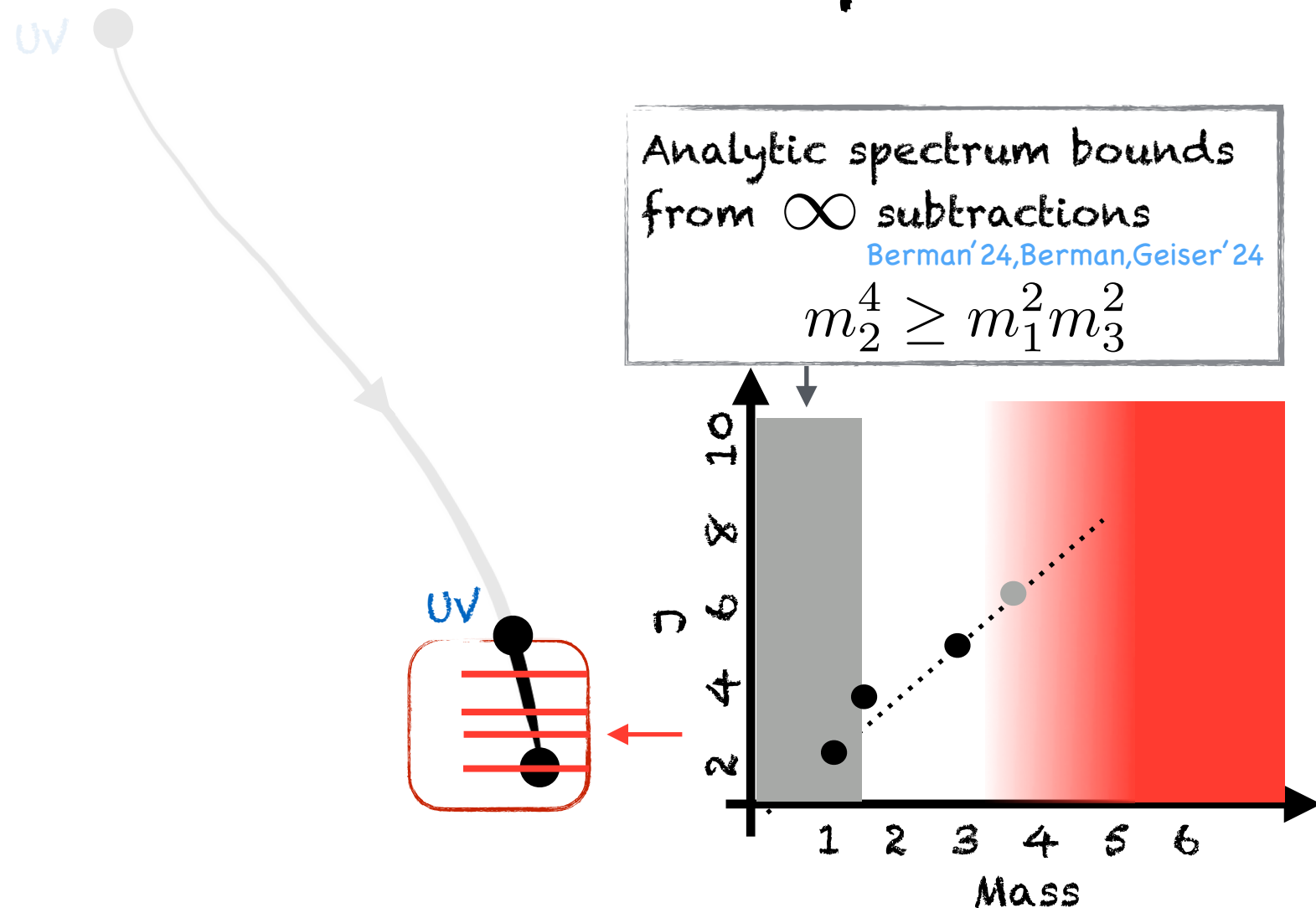
# UV $\rightarrow$ IR implications: spectra



- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass



# UV $\rightarrow$ IR implications: spectra



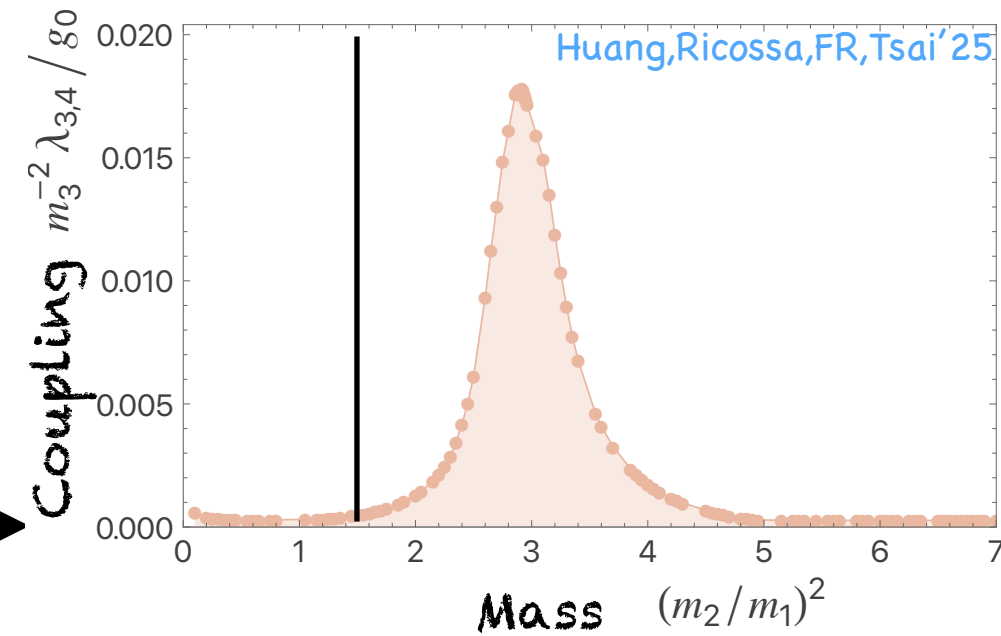
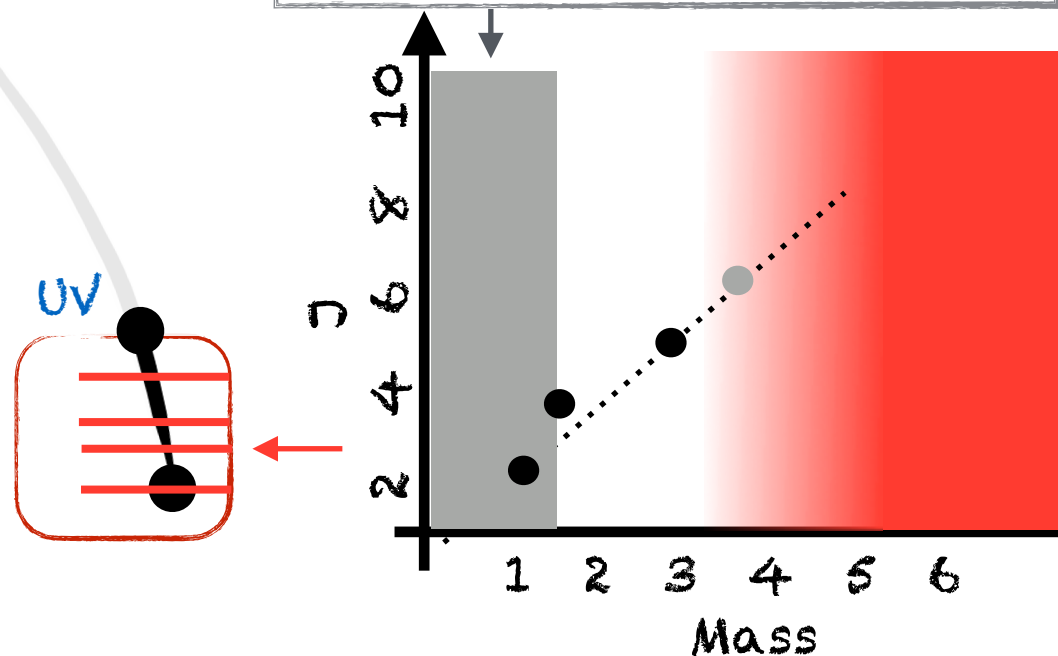
- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass

# UV $\rightarrow$ IR implications: spectra

UV

UV

Analytic spectrum bounds  
from  $\infty$  subtractions  
Berman'24, Berman, Geiser'24  
$$m_2^4 \geq m_1^2 m_3^2$$



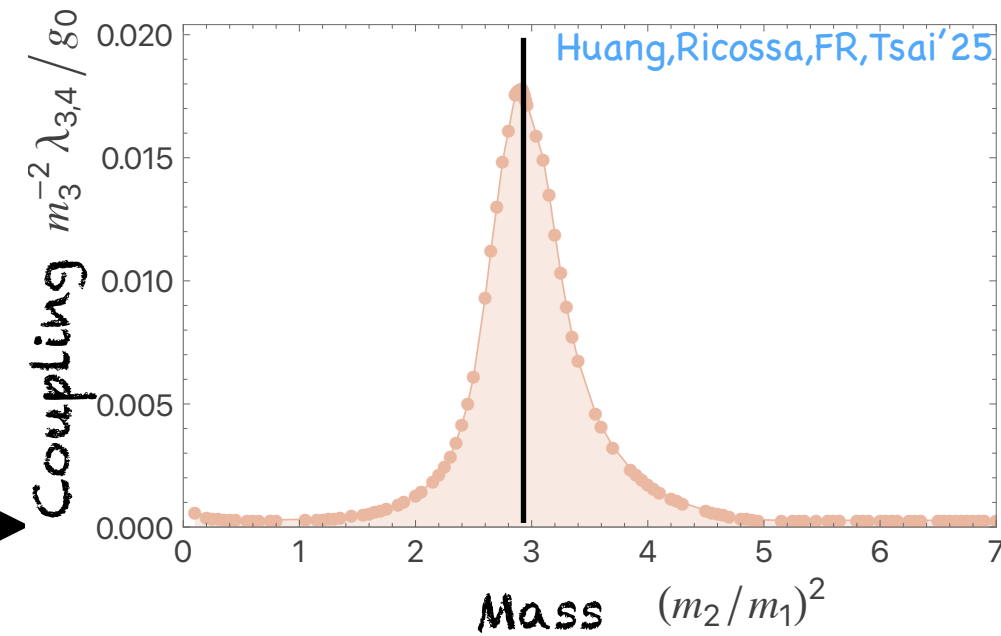
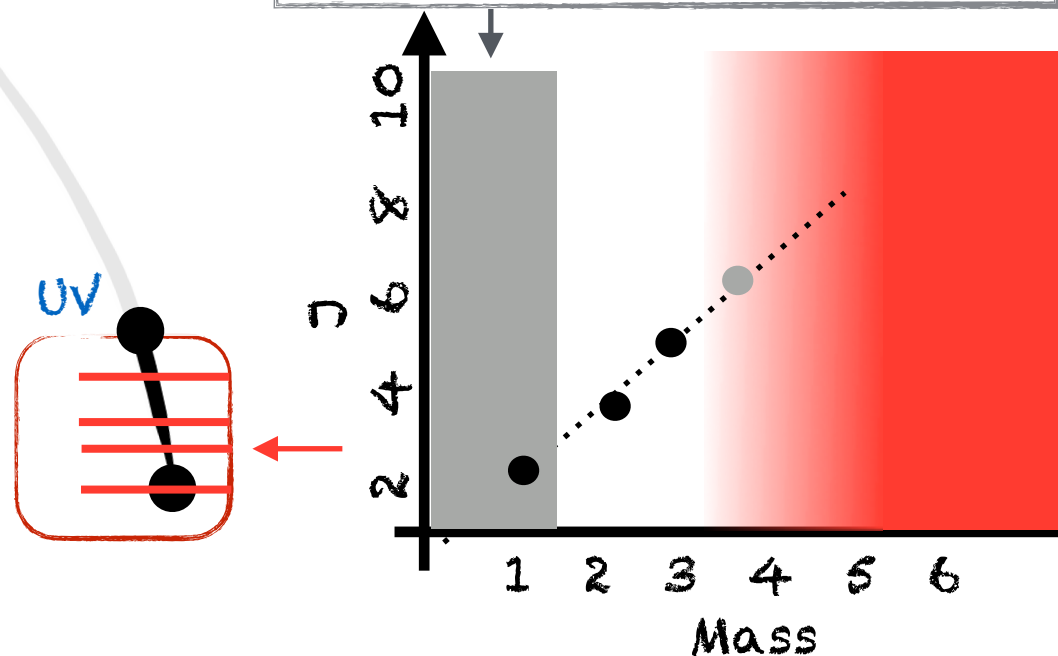
- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass

# UV $\rightarrow$ IR implications: spectra

UV

UV

Analytic spectrum bounds  
from  $\infty$  subtractions  
Berman'24, Berman, Geiser'24  
$$m_2^4 \geq m_1^2 m_3^2$$



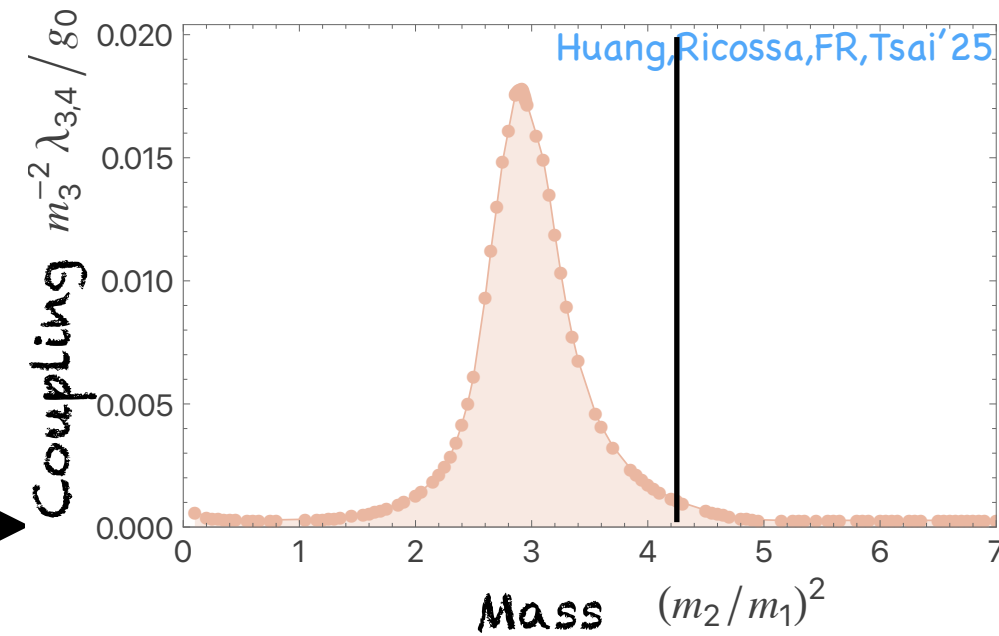
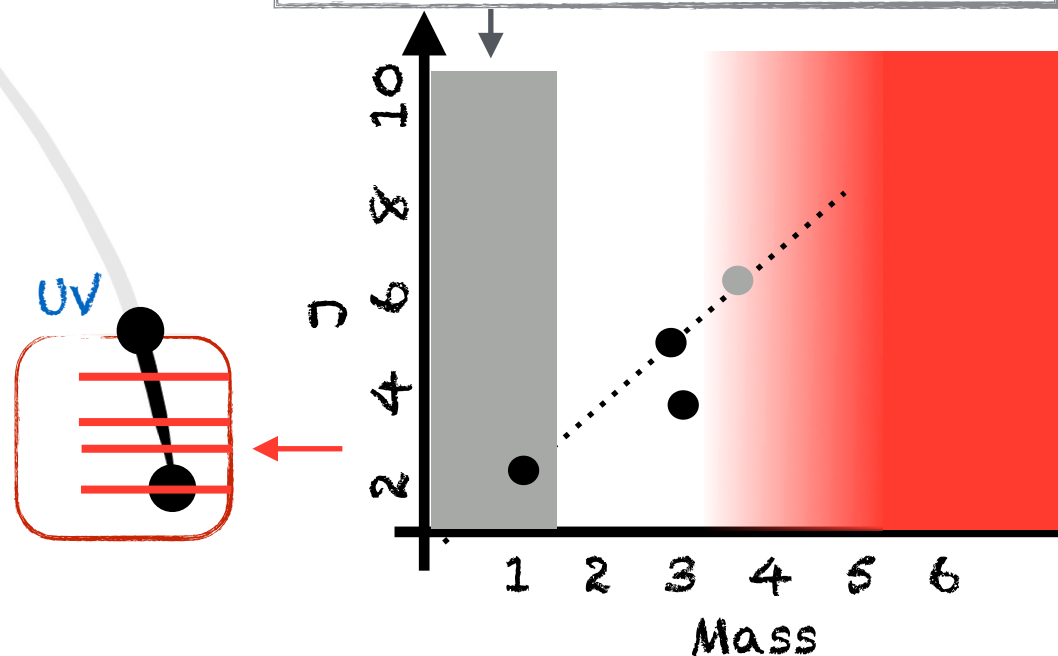
- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass

# UV $\rightarrow$ IR implications: spectra

UV

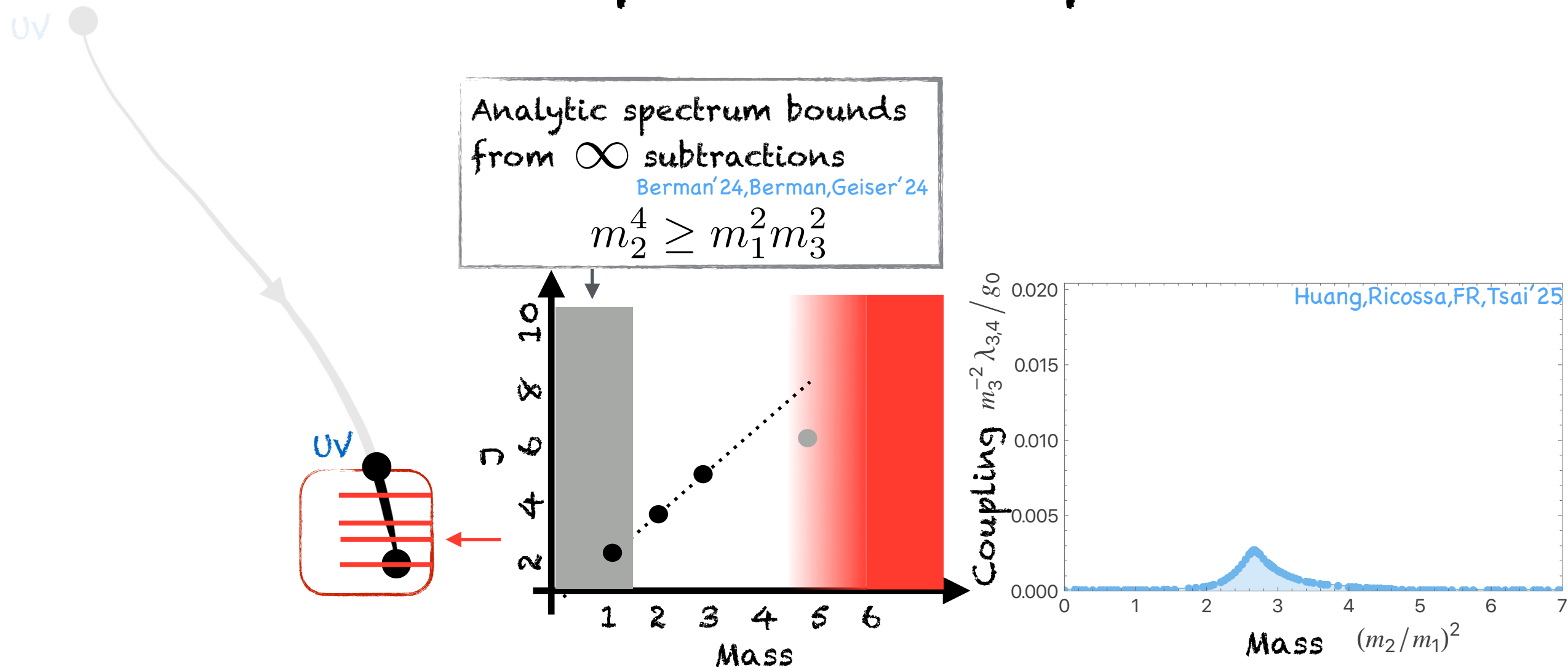
UV

Analytic spectrum bounds  
from  $\infty$  subtractions  
Berman'24, Berman, Geiser'24  
$$m_2^4 \geq m_1^2 m_3^2$$



- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass

# UV $\rightarrow$ IR implications: spectra



- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass
- ▶ Higher-spin spectra prefer to organise in linear trajectories

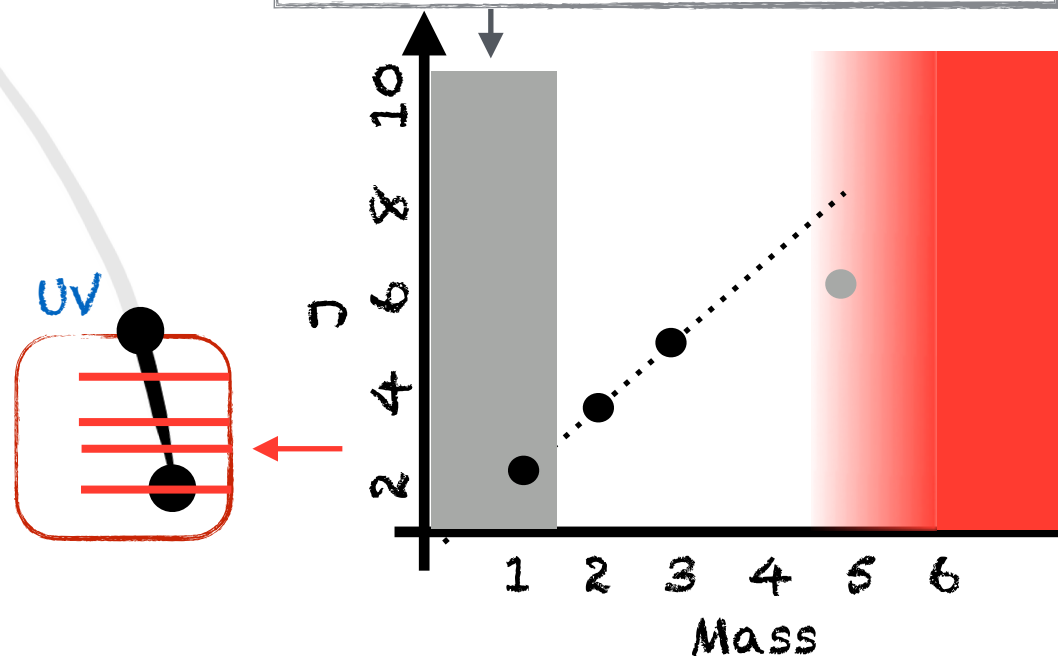
# UV $\rightarrow$ IR implications: spectra

UV

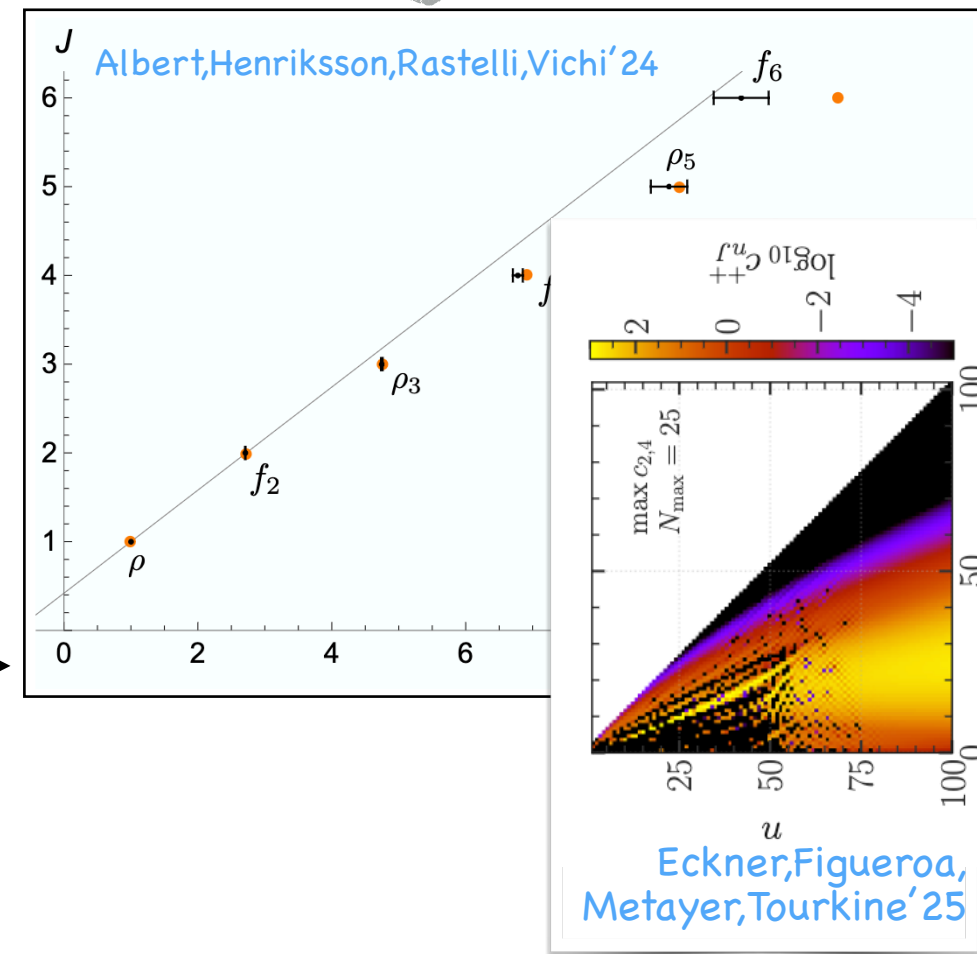
UV

Analytic spectrum bounds  
from  $\infty$  subtractions  
Berman'24, Berman, Geiser'24

$$m_2^4 \geq m_1^2 m_3^2$$



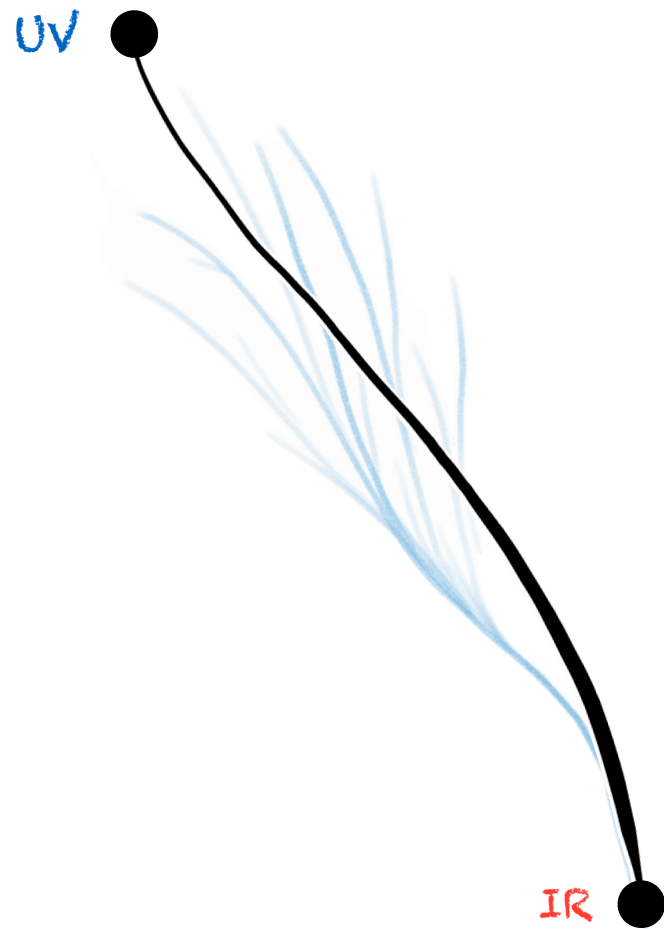
Large-N QCD kink?



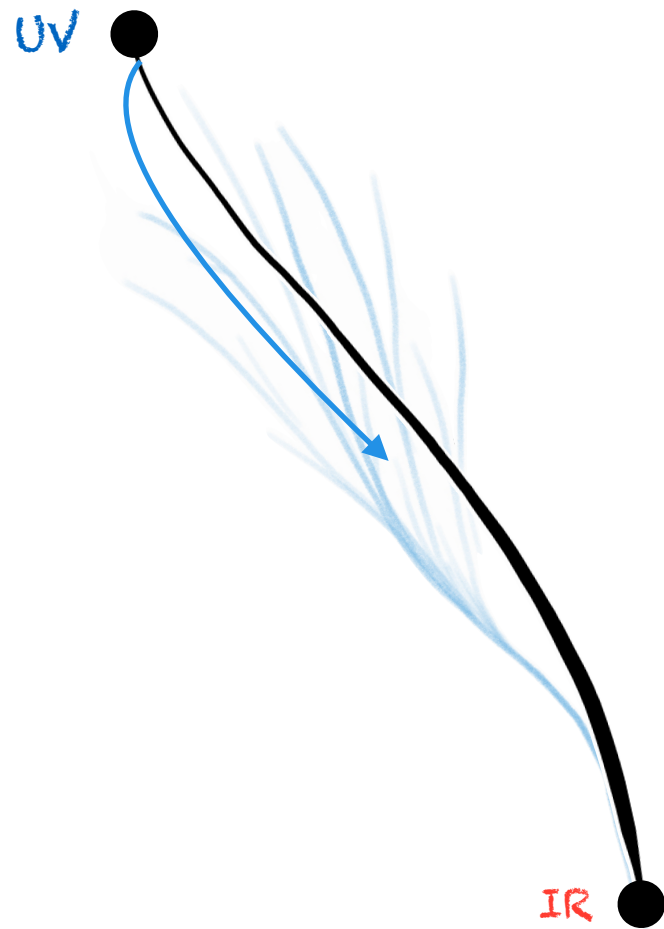
- ▶ Massive gravity only consistent over range  $M \lesssim 30m_g$
- ▶ Massive spin-3/2 only for  $M \lesssim 10m_{3/2}$   
( $M \rightarrow \infty$  with graviton and tuned couplings)
- ▶ Massive spin- $J > 2$  have finite gap fixed by mass
- ▶ Higher-spin spectra prefer to organise in linear trajectories



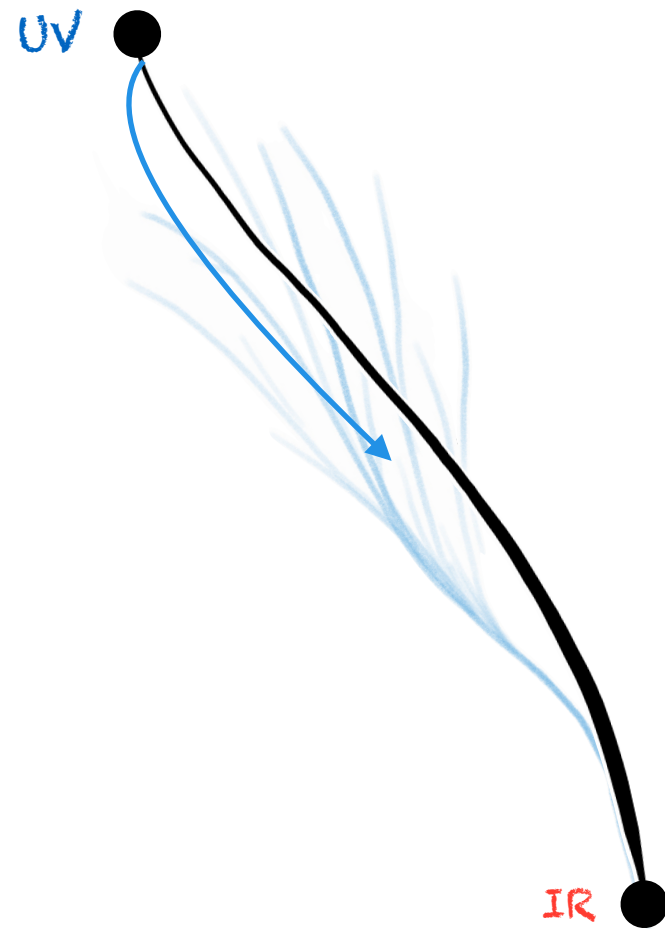
# UV and IR



# UV and IR



# UV and IR

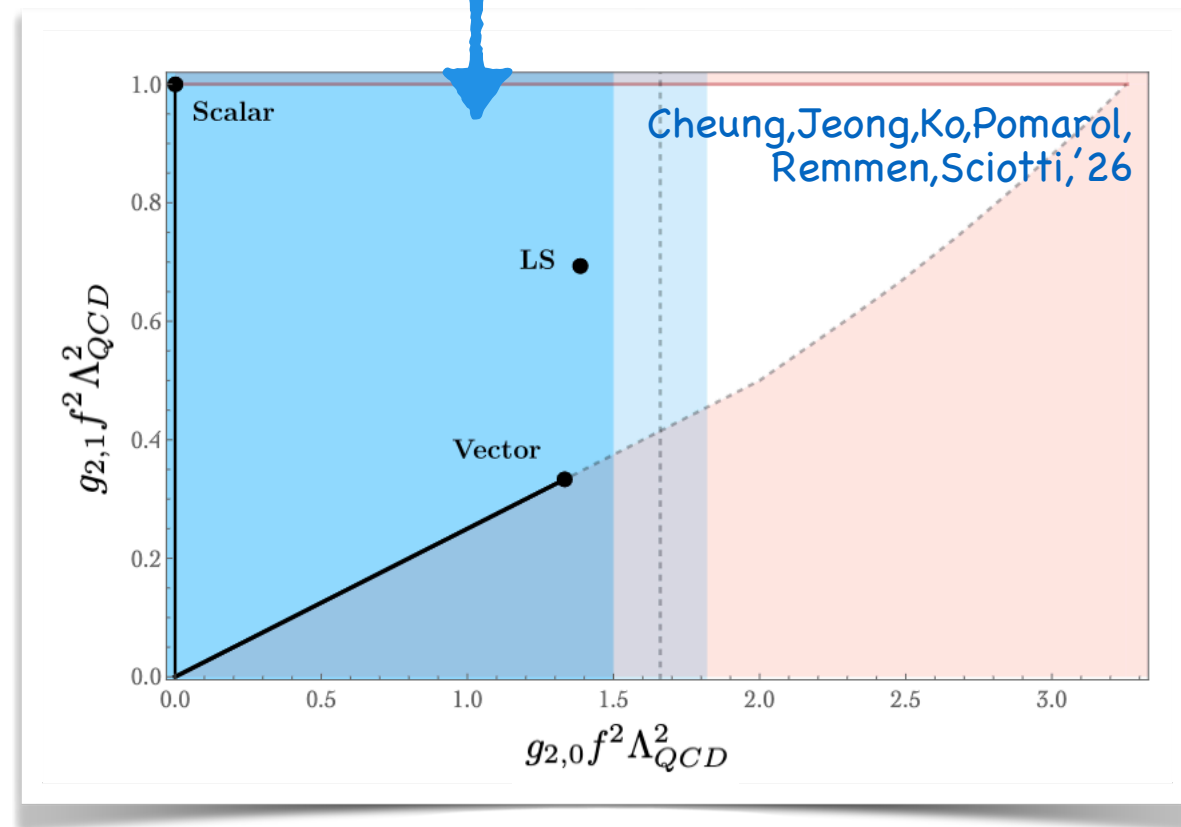


Multipositivity:

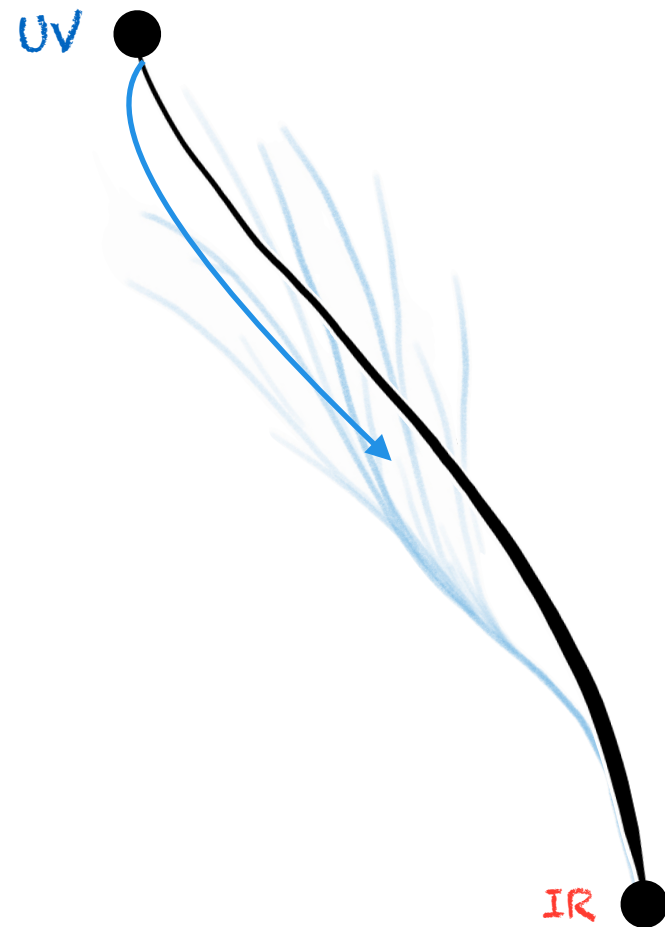
+  
Anomaly

$$\left| \begin{array}{c} \text{---} \text{---} \text{---} \end{array} \right|^2 \leq \left( \begin{array}{c} \text{---} \text{---} \end{array} \right) \left( \begin{array}{c} \text{---} \text{---} \end{array} \right)$$

Cheung, Remmen, '25



# UV and IR

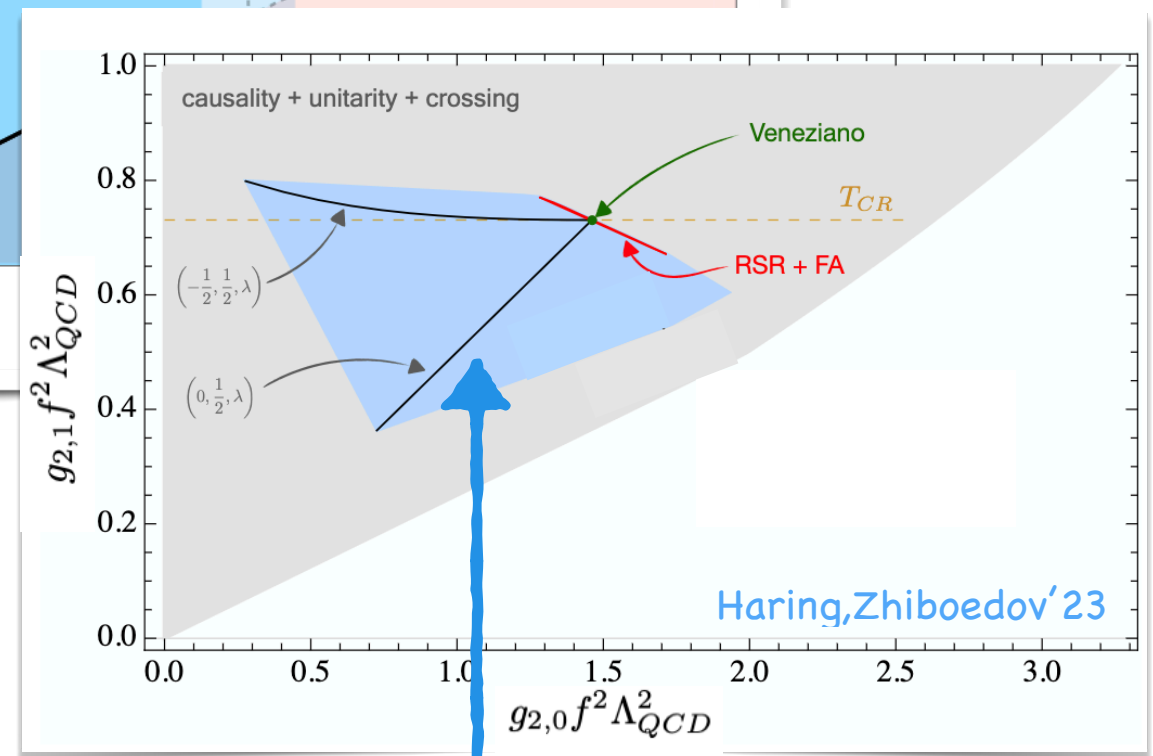
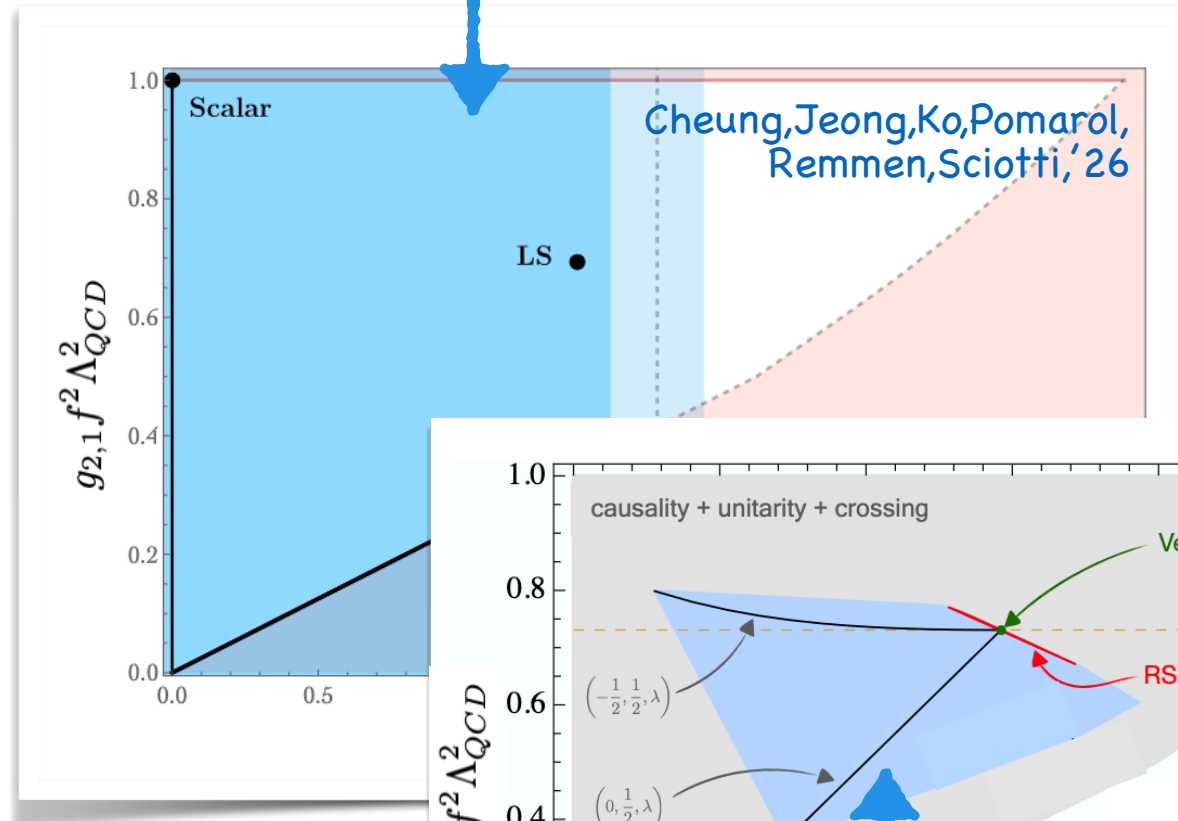


Multipositivity:

+  
Anomaly

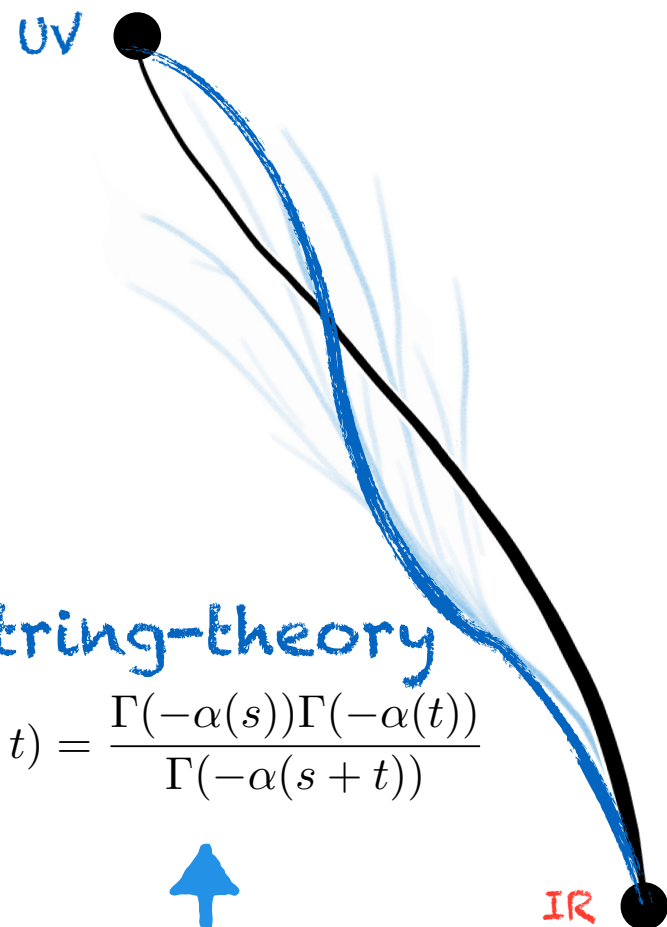
$$\left| \begin{array}{c} \text{---} \text{---} \text{---} \end{array} \right|^2 \leq \left( \begin{array}{c} \text{---} \text{---} \end{array} \right) \left( \begin{array}{c} \text{---} \text{---} \end{array} \right)$$

Cheung, Remmen, '25



Regge:  $A(s, t)_{s \rightarrow \infty} \sim f(t) s^{j(t)}$

# UV and IR



Multipositivity:

+  
Anomaly

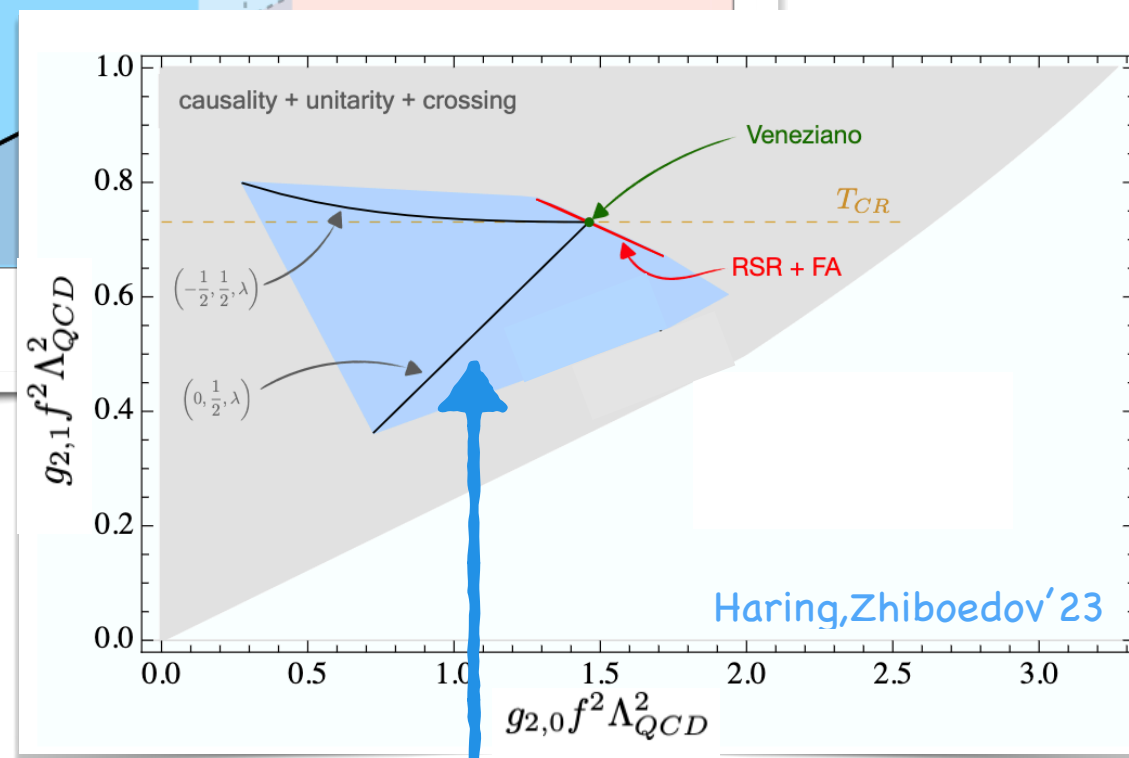
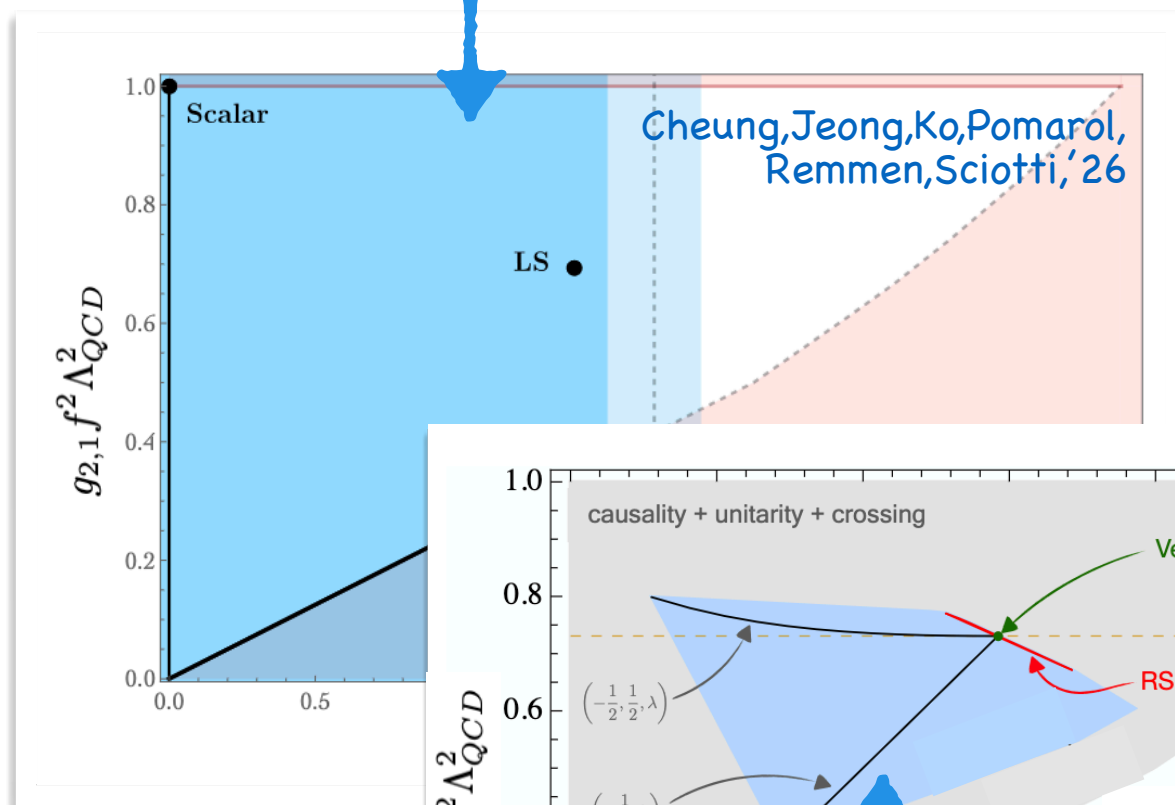
$$\left| \begin{array}{c} \text{---} \text{---} \text{---} \end{array} \right|^2 \leq \left( \begin{array}{c} \text{---} \text{---} \end{array} \right) \left( \begin{array}{c} \text{---} \text{---} \end{array} \right)$$

Cheung, Remmen, '25

String-theory

$$A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s+t))}$$

Causal  
+Unitary  
+?



Haring, Zhiboedov '23

Regge:  $A(s, t)_{s \rightarrow \infty} \sim f(t) s^{j(t)}$

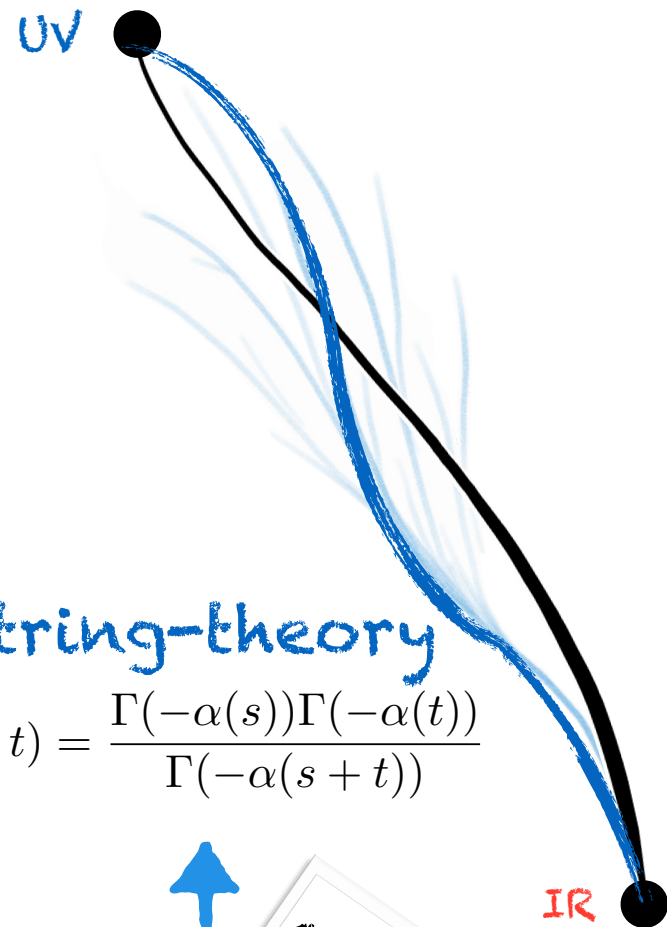
# UV and IR

Multipositivity:

$$\left| \begin{array}{c} \text{---} \bigcirc \text{---} \bigcirc \text{---} \\ \text{---} \bigcirc \text{---} \bigcirc \text{---} \end{array} \right|^2 \leq \left( \begin{array}{c} \text{---} \bigcirc \text{---} \bigcirc \text{---} \\ \text{---} \bigcirc \text{---} \bigcirc \text{---} \end{array} \right) \left( \begin{array}{c} \text{---} \bigcirc \text{---} \bigcirc \text{---} \\ \text{---} \bigcirc \text{---} \bigcirc \text{---} \end{array} \right)$$

Cheung, Remmen, '25

+  
Anomaly



String-theory

$$A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s+t))}$$

Causal  
+Unitary  
+?

See Grant's talk

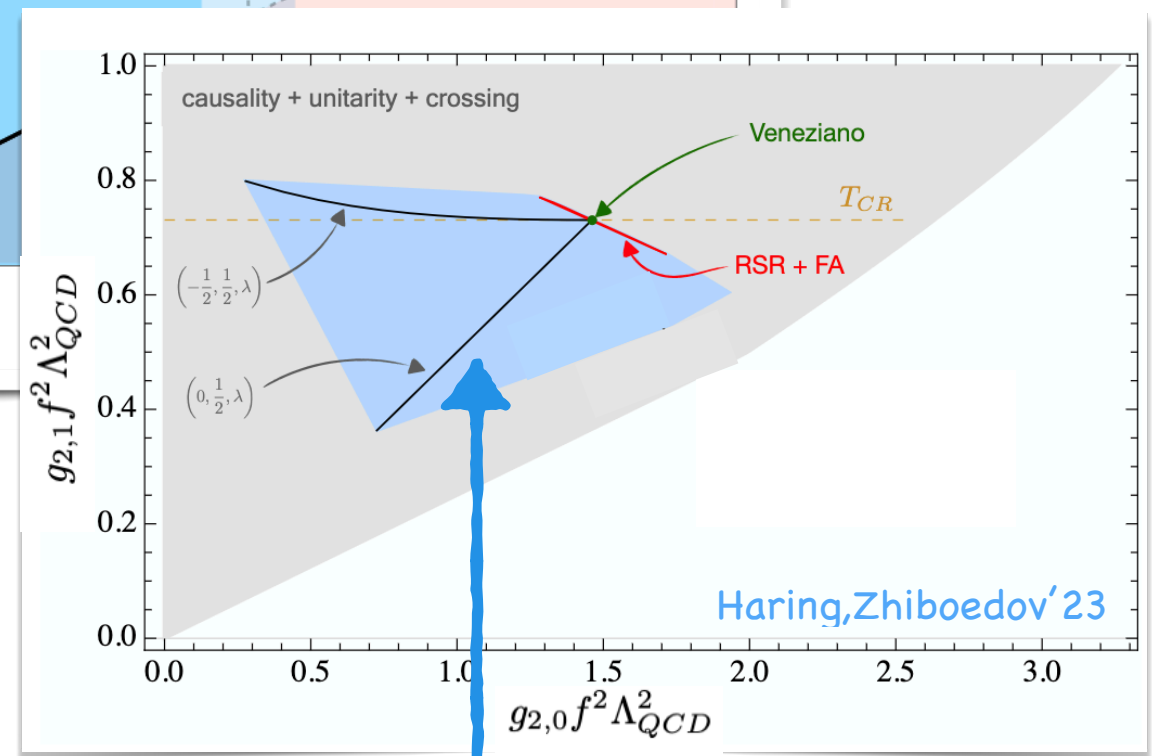
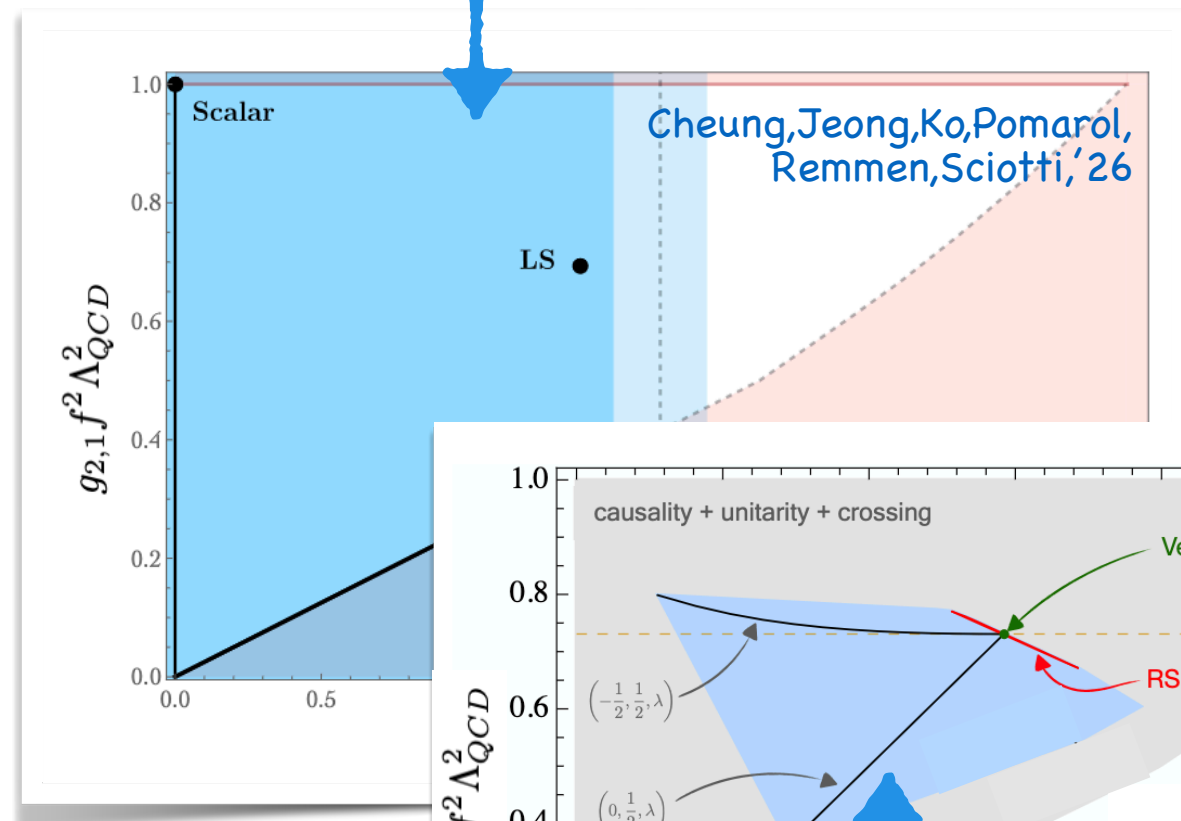
Monodromy

+ SUSY

Ultrasoftness

...

Huang, Liu, Rodina, Wang '20  
Berman, Elvang, Herderschee '23  
Cheung, Hillmann, Remmen, '24  
Cheung, Remmen, Sciotti, Tarquini, '25,  
...



Haring, Zhiboedov '23

Regge:  $A(s, t)_{s \rightarrow \infty} \sim f(t) s^{j(t)}$



UV  $\rightarrow$  IR implications at weakest coupling

UV  $\rightarrow$  IR with finite coupling effects

# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2\rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on tree-level/zero coupling EFT

$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$



# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2\rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on tree-level/zero coupling EFT

$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$

Rest of this talk: finite couplings



# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2\rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on **tree-level/zero coupling EFT**

$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$

Rest of this talk: **finite** couplings

$$A(s, t) = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}(s) P_{\ell}(1 + 2t/s)$$

↓

$$\text{Unitarity: } 0 \leq \text{Im} a_{\ell} \leq 1$$



# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2 \rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on **tree-level/zero coupling EFT**

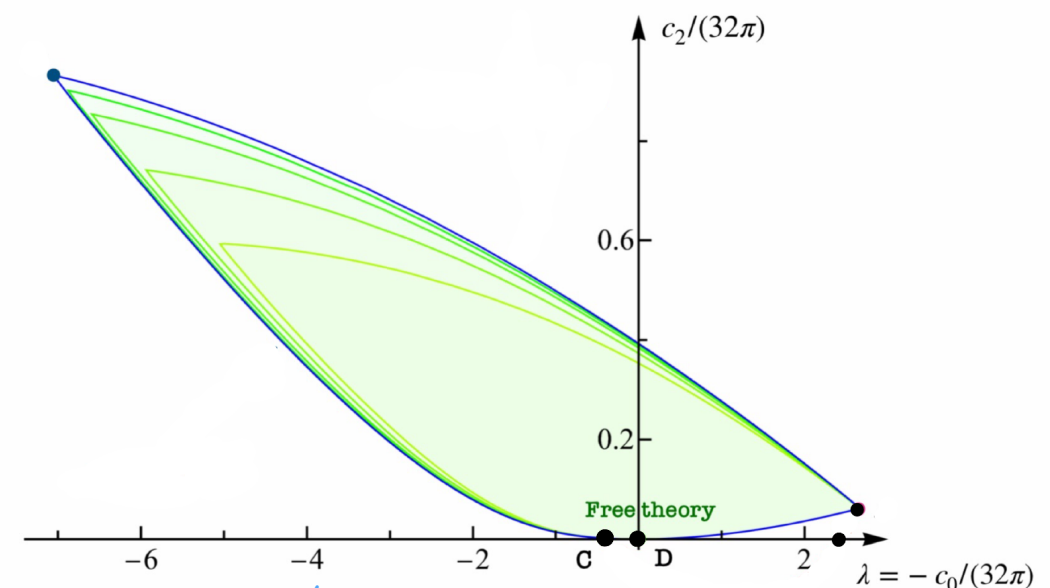
$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$

Rest of this talk: **finite couplings**

$$A(s, t) = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}(s) P_{\ell}(1 + 2t/s)$$

Unitarity:  $0 \leq \text{Im} a_{\ell} \leq 1$   $\rightarrow$  **S-Matrix Bootstrap**

Paulos, Penedones, Toledo, van Rees, Vieira'16, ...



Elias Miro, Guerrieri, Gumus'22  
Acanfora, Guerrieri, Haring, Karateev'23

# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2 \rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on **tree-level/zero coupling EFT**

$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$

Rest of this talk: **finite** couplings

$$A(s, t) = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}(s) P_{\ell}(1 + 2t/s)$$

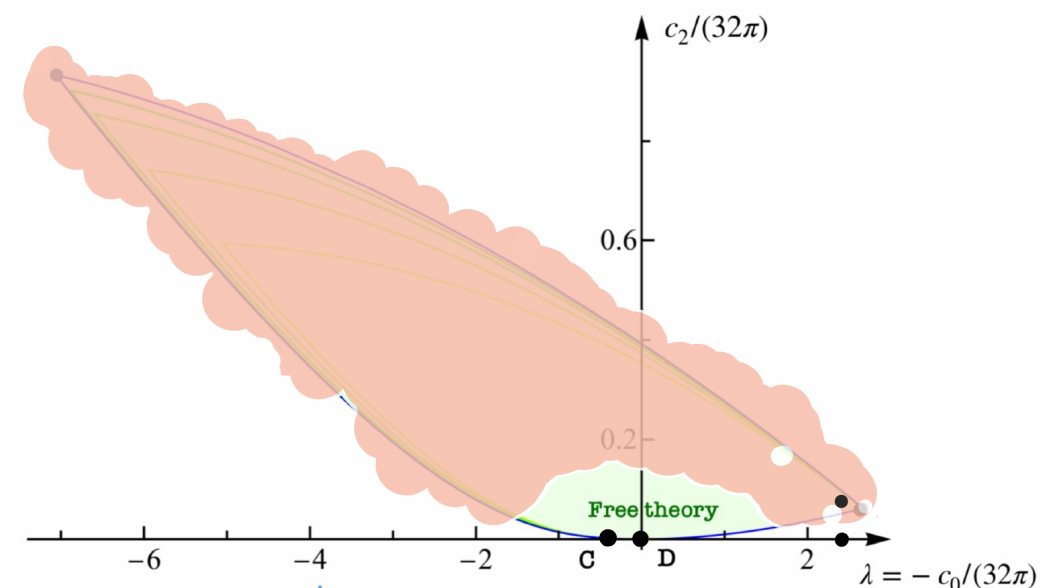
Unitarity:  $0 \leq \text{Im} a_{\ell} \leq 1$   $\blacktriangleright$  **S-Matrix Bootstrap**

Paulos, Penedones, Toledo, van Rees, Vieira'16, ...

Strongly coupled theories  $\blacktriangleleft$

No EFT  $\blacktriangleleft$

Difficult comparison with experiments  $\blacktriangleleft$



Elias Miro, Guerrieri, Gumus'22  
Acanfora, Guerrieri, Haring, Karateev'23



# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2\rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on **tree-level/zero coupling EFT**

$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$

Rest of this talk: **finite** couplings

$$A(s, t) = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}(s) P_{\ell}(1 + 2t/s)$$

Unitarity:  $0 \leq \text{Im} a_{\ell} \leq 1$



Weak but finite-coupling  
effects calculable in the EFT



# UV $\rightarrow$ IR Relations and IR effects

$$\overset{\text{Tree}}{A_{2\rightarrow 2}} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

These results (and most literature):

based on **tree-level/zero coupling** EFT

$$g_n \rightarrow 0 \qquad \frac{g_n}{g_m} = \text{finite}$$

Rest of this talk: **finite** couplings

$$A(s, t) = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}(s) P_{\ell}(1 + 2t/s)$$

Unitarity:  $0 \leq \text{Im} a_{\ell} \leq 1$



Weak but finite-coupling  
effects calculable in the EFT

Aren't these small?

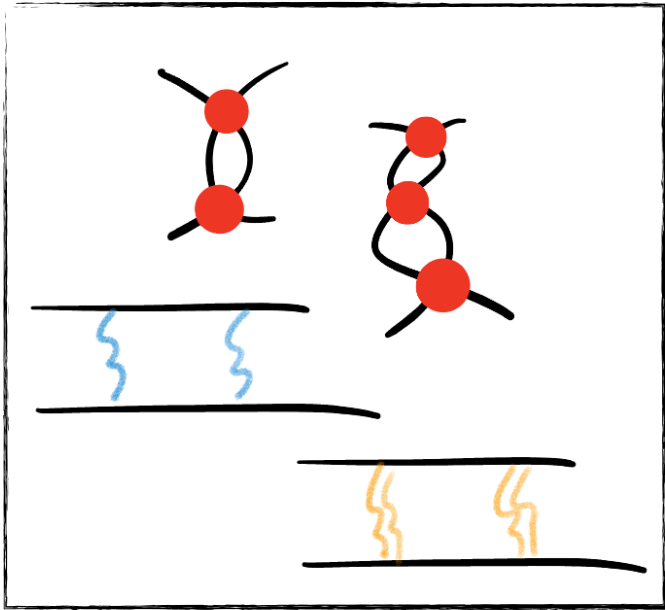


# UV $\rightarrow$ IR Relations and IR effects

Massless particles (Goldstones, Gravitons, Photons,...): enhanced IR

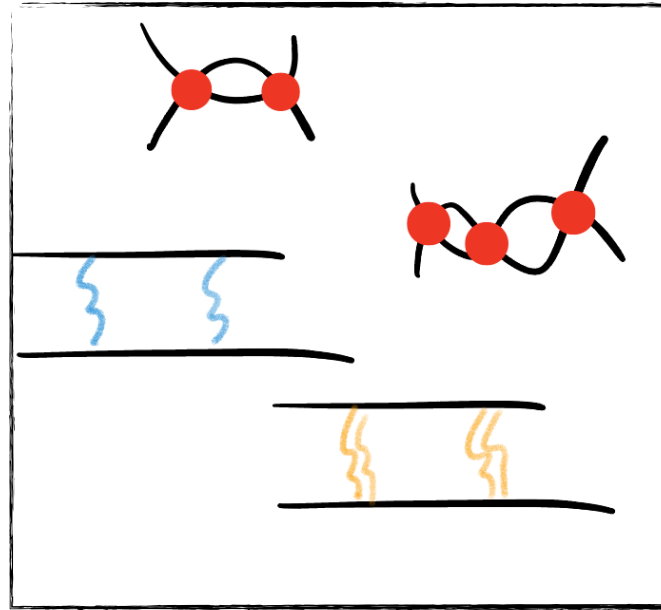
# UV $\rightarrow$ IR Relations and IR effects

Massless particles (Goldstones, Gravitons, Photons,...): enhanced IR



Beadle, Isabella, Perrone, Ricossa, FR, Serra'24-'25  
 Bellazzini, Riembau, FR '22  
 Chang, Parra-Martinez'25

$$A_n \sim \log t \xrightarrow{(t \rightarrow 0)} \infty$$

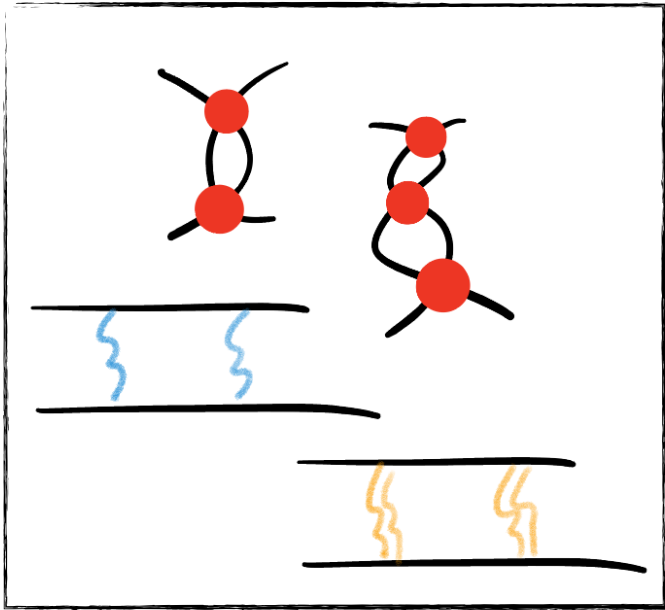


Bellazzini, Elias-Miro, Riembau, Rattazzi, FR'20  
 Beadle, Isabella, Perrone, Ricossa, FR, Serra'24-'25  
 Chang, Parra-Martinez'25  
 Ye, He, Gu'24; Peng, Shao, Tokareva, Xu'25

$$A_n \sim \log s \xrightarrow{(s \rightarrow 0)} \infty$$

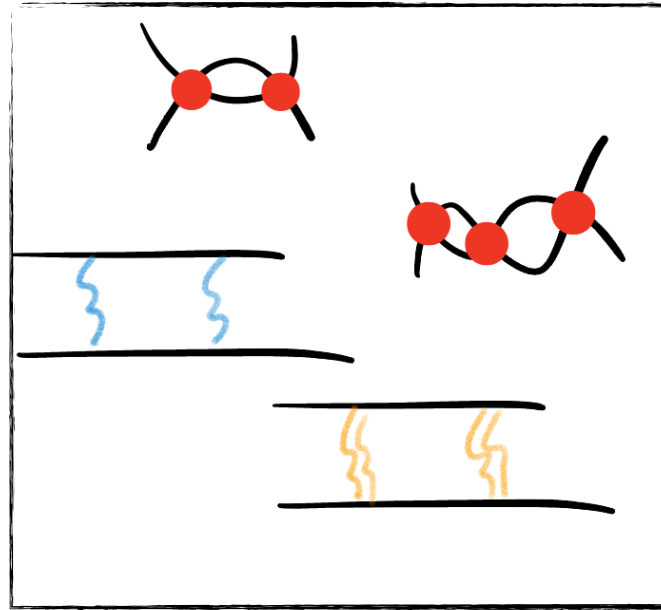
# UV $\rightarrow$ IR Relations and IR effects

Massless particles (Goldstones, Gravitons, Photons,...): enhanced IR



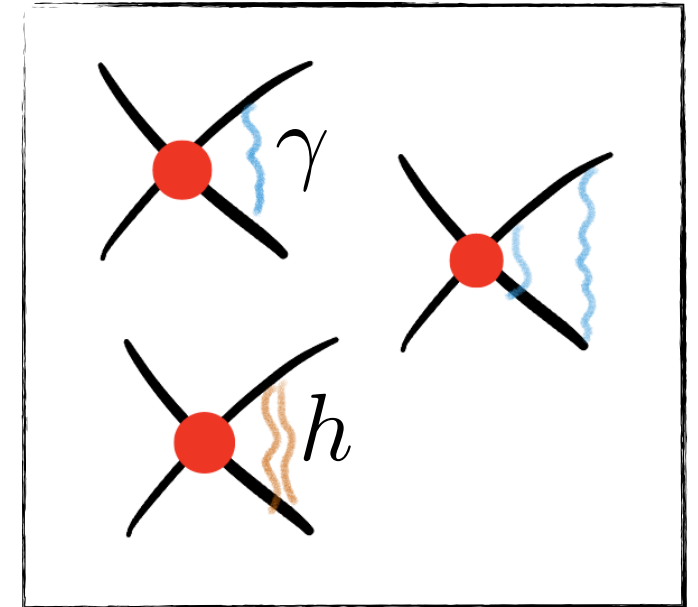
Beadle, Isabella, Perrone, Ricossa, FR, Serra' 24-'25  
Bellazzini, Riembau, FR '22  
Chang, Parra-Martinez' 25

$$A_n \sim \log t \xrightarrow{(t \rightarrow 0)} \infty$$



Bellazzini, Elias-Miro, Riembau, Rattazzi, FR' 20  
Beadle, Isabella, Perrone, Ricossa, FR, Serra' 24-'25  
Chang, Parra-Martinez' 25  
Ye, He, Gu' 24; Peng, Shao, Tokareva, Xu' 25

$$A_n \sim \log s \xrightarrow{(s \rightarrow 0)} \infty$$



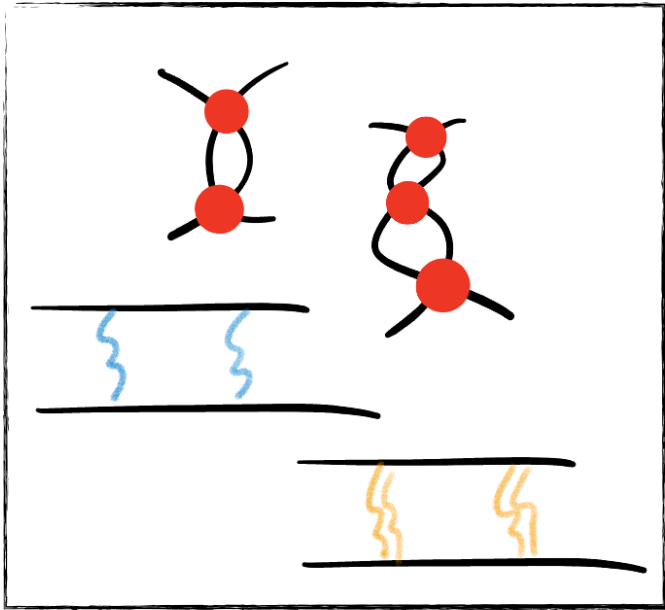
Bellazzini, Berman, Isabella, FR, Romano, Sciotti' 25

$$A(s, t) = \frac{1}{\infty}$$

↑  
long-range  
forces, D=4

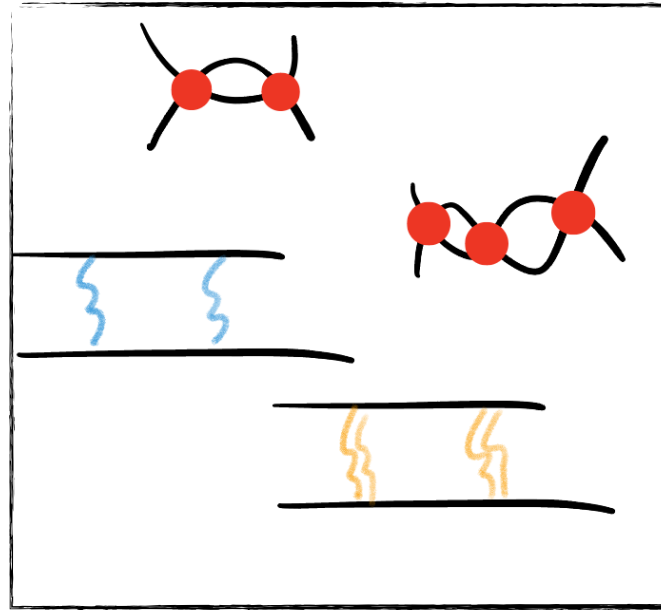
# UV $\rightarrow$ IR Relations and IR effects

Massless particles (Goldstones, Gravitons, Photons,...): enhanced IR



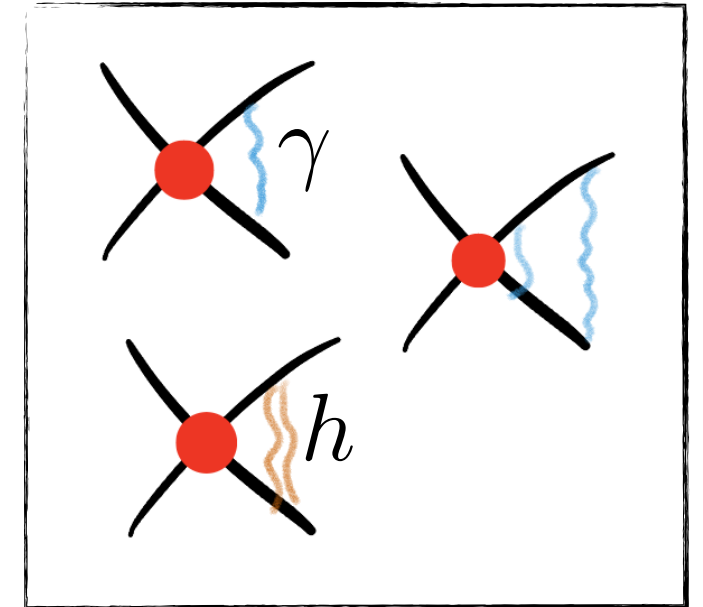
Beadle, Isabella, Perrone, Ricossa, FR, Serra' 24-'25  
Bellazzini, Riembau, FR '22  
Chang, Parra-Martinez' 25

$$A_n \sim \log t \xrightarrow{(t \rightarrow 0)} \infty$$



Bellazzini, Elias-Miro, Riembau, Rattazzi, FR' 20  
Beadle, Isabella, Perrone, Ricossa, FR, Serra' 24-'25  
Chang, Parra-Martinez' 25  
Ye, He, Gu' 24; Peng, Shao, Tokareva, Xu' 25

$$A_n \sim \log s \xrightarrow{(s \rightarrow 0)} \infty$$



Bellazzini, Berman, Isabella, FR, Romano, Sciotti' 25

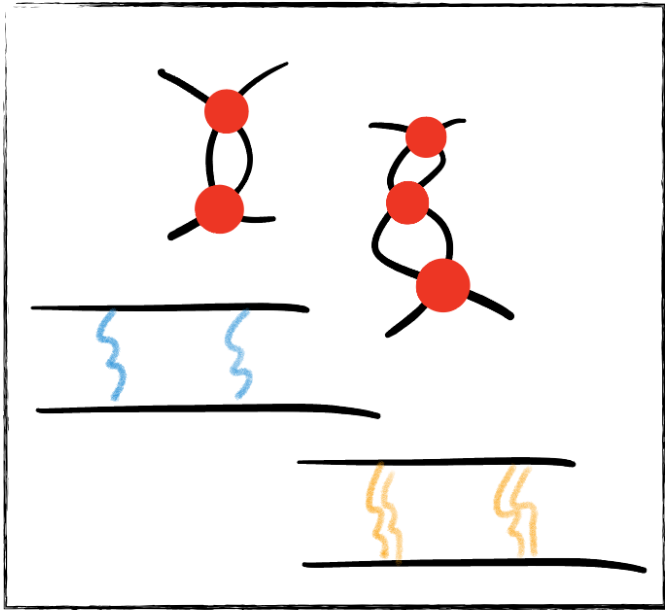
$$A(s, t) \underset{\substack{\uparrow \\ \text{long-range} \\ \text{forces, } D=4}}{=} \frac{1}{\infty}$$

► Possible that bounds are  $O(1)$  modified for  $g_n, \alpha, G_N \neq 0$  ?  
Maybe yes! After all these are UV/IR relations...



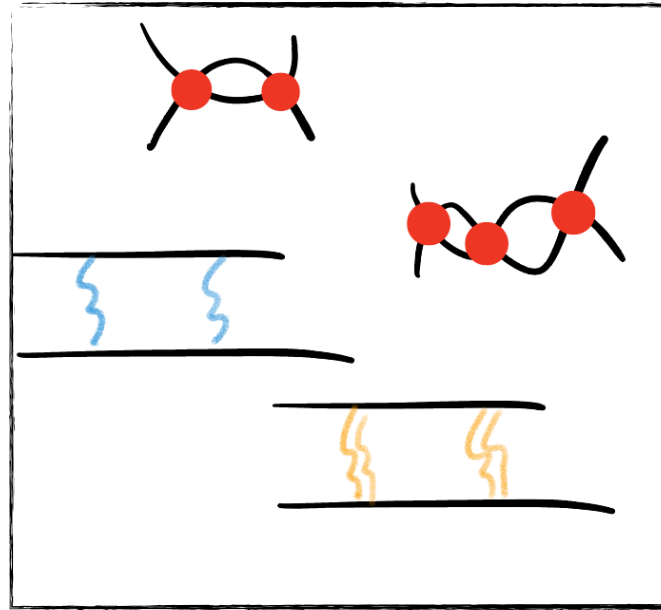
# UV $\rightarrow$ IR Relations and IR effects

Massless particles (Goldstones, Gravitons, Photons,...): enhanced IR



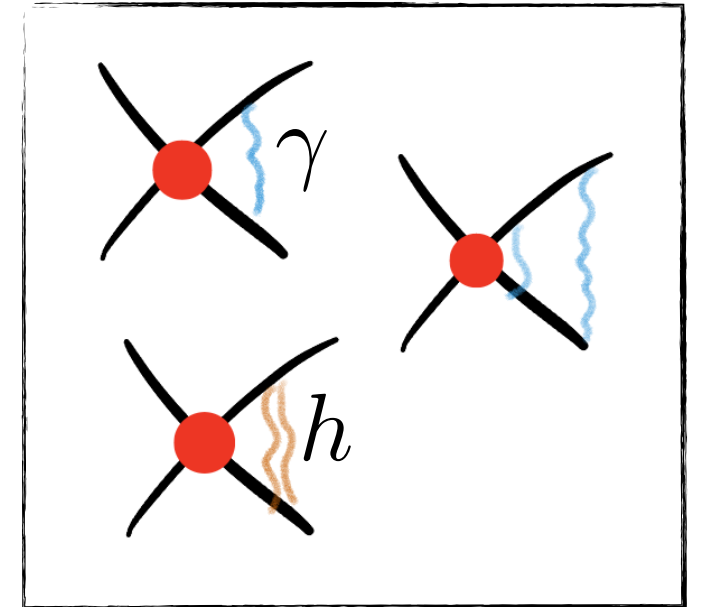
Beadle, Isabella, Perrone, Ricossa, FR, Serra' 24-'25  
Bellazzini, Riembau, FR '22  
Chang, Parra-Martinez' 25

$$A_n \sim \log t \xrightarrow{(t \rightarrow 0)} \infty$$



Bellazzini, Elias-Miro, Riembau, Rattazzi, FR' 20  
Beadle, Isabella, Perrone, Ricossa, FR, Serra' 24-'25  
Chang, Parra-Martinez' 25  
Ye, He, Gu' 24; Peng, Shao, Tokareva, Xu' 25

$$A_n \sim \log s \xrightarrow{(s \rightarrow 0)} \infty$$



Bellazzini, Berman, Isabella, FR, Romano, Sciotti' 25

$$A(s, t) = \frac{1}{\infty}$$

↑  
long-range forces, D=4

► Possible that bounds are  $O(1)$  modified for  $g_n, \alpha, G_N \neq 0$  ?  
Maybe yes! After all these are UV/IR relations...

► Or are there modified dispersion relations for which NLO effects are truly small?

# Negativity

Not all tree-level bounds are affected in the same way...

$$\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$$

Bellazzini, Elias-Miro, Riembau, Rattazzi, FR'20

Bellazzini, Riembau, FR '22

Beadle, Isabella, Perrone, Ricossa, FR, Serra'24-'25

$$A_{2 \rightarrow 2}^{Tree} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

$$0 \leq \mathcal{A}_2 = g_2$$

$$0 \leq \mathcal{A}_4 = g_4$$

$$0 \leq \mathcal{A}_6 = g_6$$

# Negativity

Not all tree-level bounds are affected in the same way...

$$\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$$

Bellazzini, Elias-Miro, Riembau, Rattazzi, FR'20

Bellazzini, Riembau, FR '22

Beadle, Isabella, Perrone, Ricossa, FR, Serra'24-'25

$$A_{2 \rightarrow 2}^{Tree} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

$$0 \leq \mathcal{A}_2 = g_2$$

$$0 \leq \mathcal{A}_4 = g_4$$

$$0 \leq \mathcal{A}_6 = g_6$$

► Each coupling individually bounded, independently of others!

# Negativity

Bellazzini,Elias-Miro,Riembau,Rattazzi,FR'20

Bellazzini,Riembau,FR '22

Beadle,Isabella,Perrone,Ricossa,FR,Serra'24-25

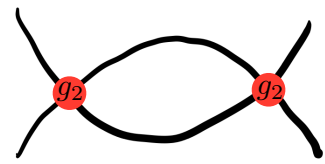
Chang,Parra-Martinez'25

Peng,Saho,Tokareva,Xu'25

Not all tree-level bounds are affected in the same way..

$$\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$$

$$A_{2 \rightarrow 2} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$



$$\beta_4 = \frac{7}{10} \frac{g_2^2}{16\pi^2}$$

exact at order  $\mathcal{O}(s^4)$

$$- \frac{\beta_4}{2} s^2 \left( s^2 - \frac{tu}{21} \right) \log(-s) + (s \leftrightarrow t, u) + \dots$$

$$0 \leq \mathcal{A}_2 = g_2$$

$$0 \leq \mathcal{A}_4 = g_4$$

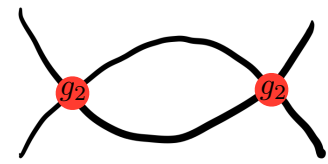
$$0 \leq \mathcal{A}_6 =$$

# Negativity

Not all tree-level bounds are affected in the same way..

$$\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$$

$$A_{2 \rightarrow 2} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$



$$\beta_4 = \frac{7}{10} \frac{g_2^2}{16\pi^2}$$

exact at order  $O(s^4)$

$$- \frac{\beta_4}{2} s^2 \left( s^2 - \frac{tu}{21} \right) \log(-s) + (s \leftrightarrow t, u) + \dots$$

► In amplitude: like running  $g_4(s)$ . In dispersion relation: more..

$$0 \leq \mathcal{A}_2 = g_2 + \frac{\beta_4}{2} s^2 + \dots$$

$$0 \leq \mathcal{A}_4 = g_4(s) + \dots$$

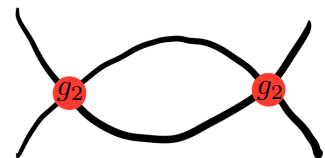
$$0 \leq \mathcal{A}_6 = \frac{\beta_4}{2s^2} + g_6 + \dots$$

# Negativity

Not all tree-level bounds are affected in the same way..

$$\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$$

$$A_{2 \rightarrow 2} = g_0 + g_2(s^2 + t^2 + u^2) - g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$



$$\beta_4 = \frac{7}{10} \frac{g_2^2}{16\pi^2}$$

exact at order  $O(s^4)$

$$- \frac{\beta_4}{2} s^2 \left( s^2 - \frac{tu}{21} \right) \log(-s) + (s \leftrightarrow t, u) + \dots$$

► In amplitude: like running  $g_4(s)$ . In dispersion relation: more..

$$0 \leq \mathcal{A}_2 = g_2 + \frac{\beta_4}{2} s^2 + \dots$$

$$0 \leq \mathcal{A}_4 = g_4(s) + \dots$$

$$0 \leq \mathcal{A}_6 = \frac{\beta_4}{2s^2} + g_6 + \dots$$

► Negativity for irrelevant interactions  
(running vs irrelevant are indistinguishable)



# Long-range Interactions

Exclusive Amplitudes: Unitary and Analytic but...

$$A_{2 \rightarrow 2} = \text{[Diagram: A red circle with two black lines crossing it to form an 'X' shape]} \\ A_{2 \rightarrow 2}^{\text{Tree}}$$

# Long-range Interactions

Exclusive Amplitudes: Unitary and Analytic but...

$$A_{2 \rightarrow 2} = \underbrace{\text{Diagram 1}}_{A_{2 \rightarrow 2}^{Tree}} + \underbrace{\text{Diagram 2}}_{\infty} + \underbrace{\text{Diagram 3}}_{\infty} + \dots = \mathcal{W}_\varepsilon (A_{2 \rightarrow 2}^{Tree} + sub) = 0$$

Diagram 1: A tree-level exchange diagram with a red vertex and two external lines. Labeled  $A_{2 \rightarrow 2}^{Tree}$ .

Diagram 2: A tree-level exchange diagram with a red vertex and two external lines. A blue wavy line labeled  $\Delta E < \varepsilon$  and  $\gamma, h$  is attached to one of the external lines. A red infinity symbol  $\infty$  is below it.

Diagram 3: A tree-level exchange diagram with a red vertex and two external lines. A blue wavy line is attached to one of the external lines. A red infinity symbol  $\infty$  is below it.

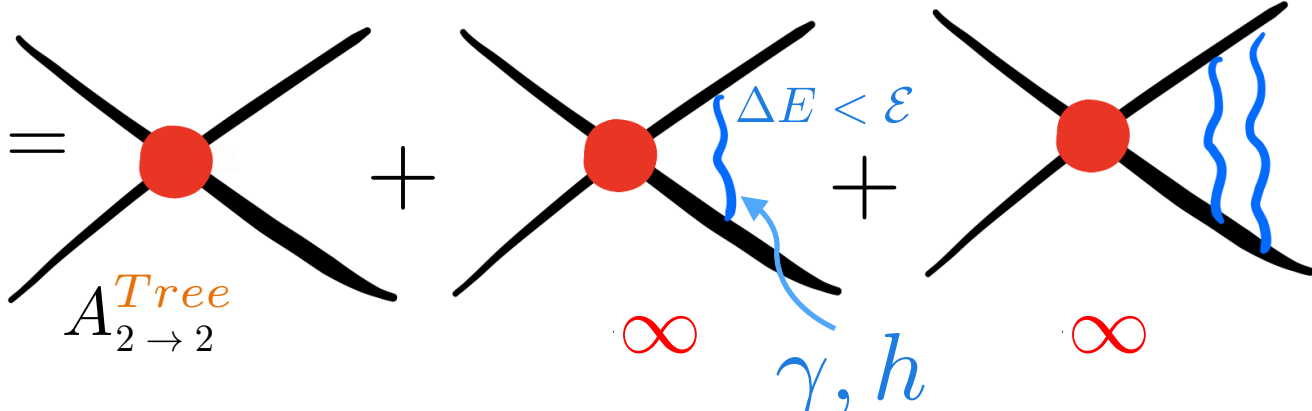
The ellipsis  $\dots$  is followed by a red infinity symbol  $\infty$ .

The operator  $\mathcal{W}_\varepsilon$  is shown with an arrow pointing to it from the label  $e^{-\infty}$ .

# Long-range Interactions

Exclusive Amplitudes: Unitary and Analytic but...

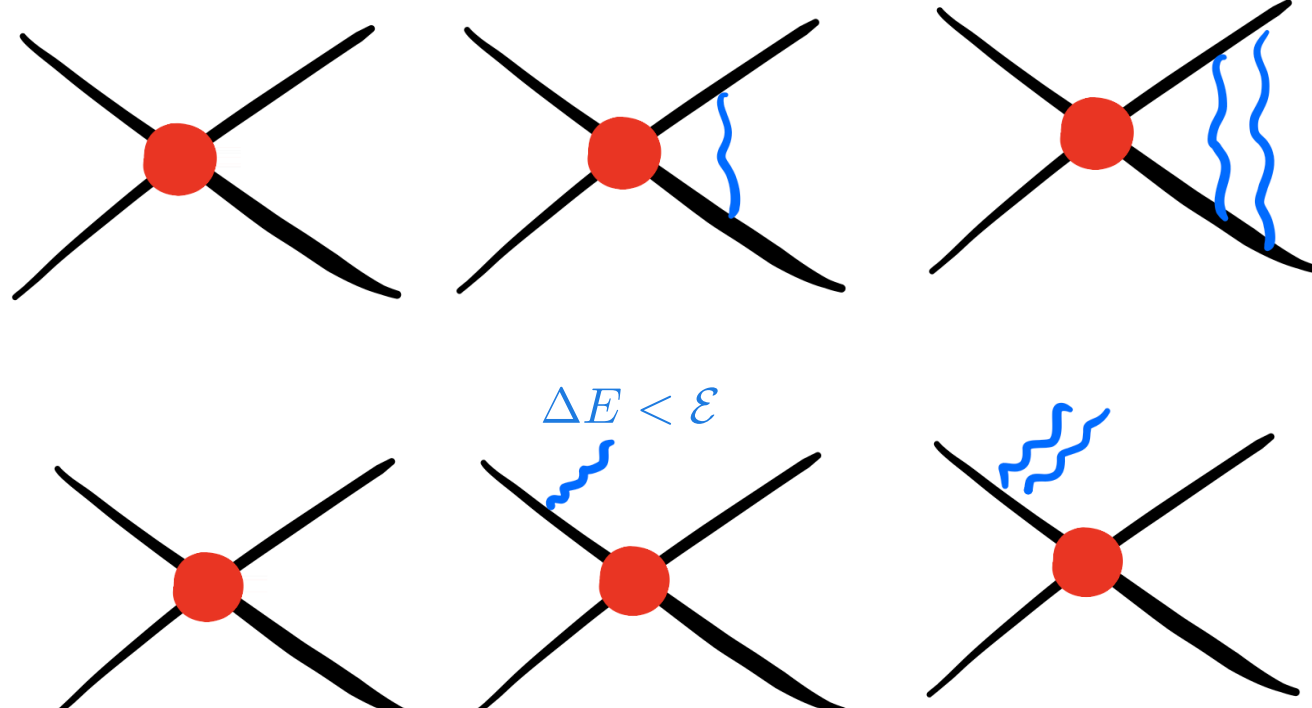
$$A_{2 \rightarrow 2} = \underbrace{\text{Tree}}_{A_{2 \rightarrow 2}^{Tree}} + \underbrace{\text{Loop}}_{\infty}^{\Delta E < \mathcal{E}} + \underbrace{\text{Loop}}_{\infty} + \dots = \mathcal{W}_{\mathcal{E}} \left( A_{2 \rightarrow 2}^{Tree} + sub \right) = 0$$



$e^{-\infty}$

Inclusive Cross-sections: IR finite, but no analyticity

$$\sigma(\Delta E < \mathcal{E}) = \sum^{\infty} |A|^2$$



# Stripped Amplitudes: IR Finite

Inclusive  
Cross-sections

$$\sigma(\Delta E < \mathcal{E})$$

IR safe



Exclusive  
Amplitudes

$$A$$



Unitary/Analytic

# Stripped Amplitudes: IR Finite

Inclusive  
Cross-sections

$$\sigma(\Delta E < \mathcal{E})$$

IR Safe



Exclusive  
Amplitudes

$$A$$



Unitary/Analytic

In this work

Stripped  
Amplitudes

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

IR Safe

Unitary/Analytic

# Stripped Amplitudes: IR Finite

Inclusive  
Cross-sections

$$\sigma(\Delta E < \mathcal{E})$$

IR Safe



Exclusive  
Amplitudes

$$A$$



Unitary/Analytic

In this work

Stripped  
Amplitudes

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

$$\epsilon = D - 4$$

IR Safe

Unitary/Analytic



# Stripped Amplitudes: IR Finite

Inclusive  
Cross-sections

$$\sigma(\Delta E < \mathcal{E})$$

IR Safe



Exclusive  
Amplitudes

$$\sum^{\infty} |A|^2$$

$A$

$$|A_{\mathcal{E}}|^2$$

Unitary/Analytic



In this work

Stripped  
Amplitudes

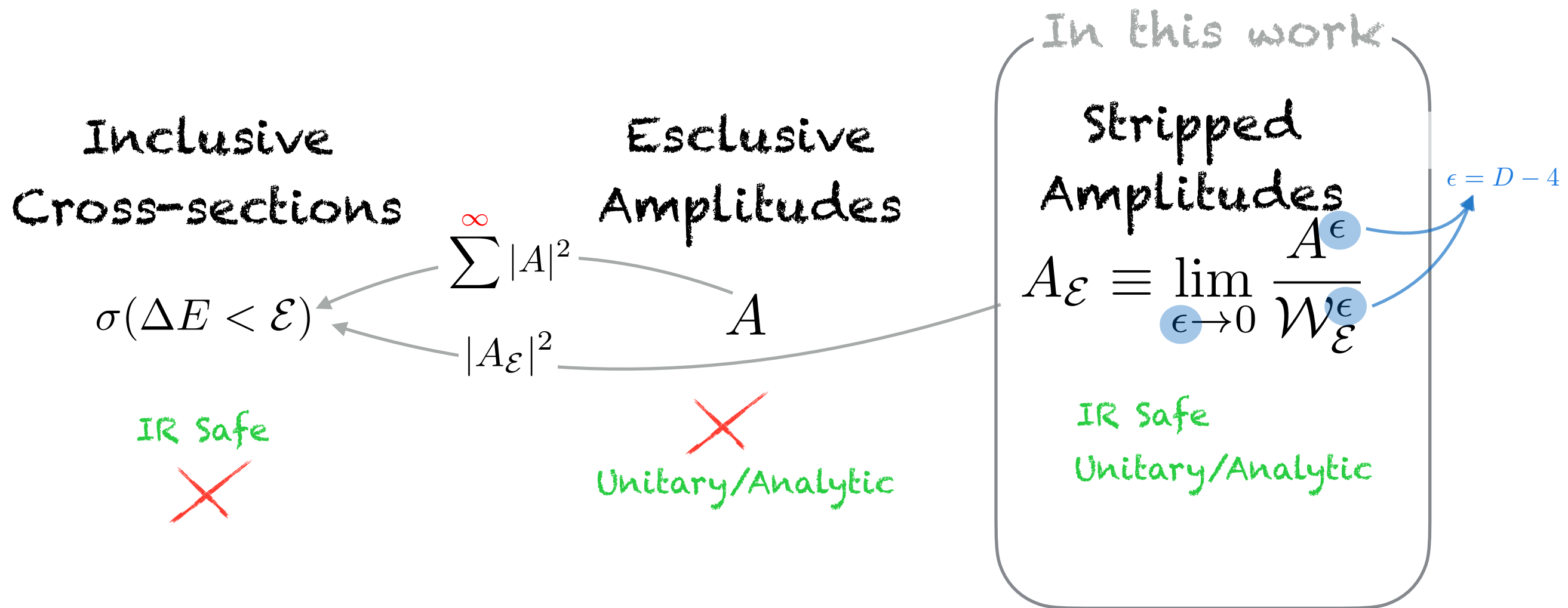
$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

IR Safe

Unitary/Analytic

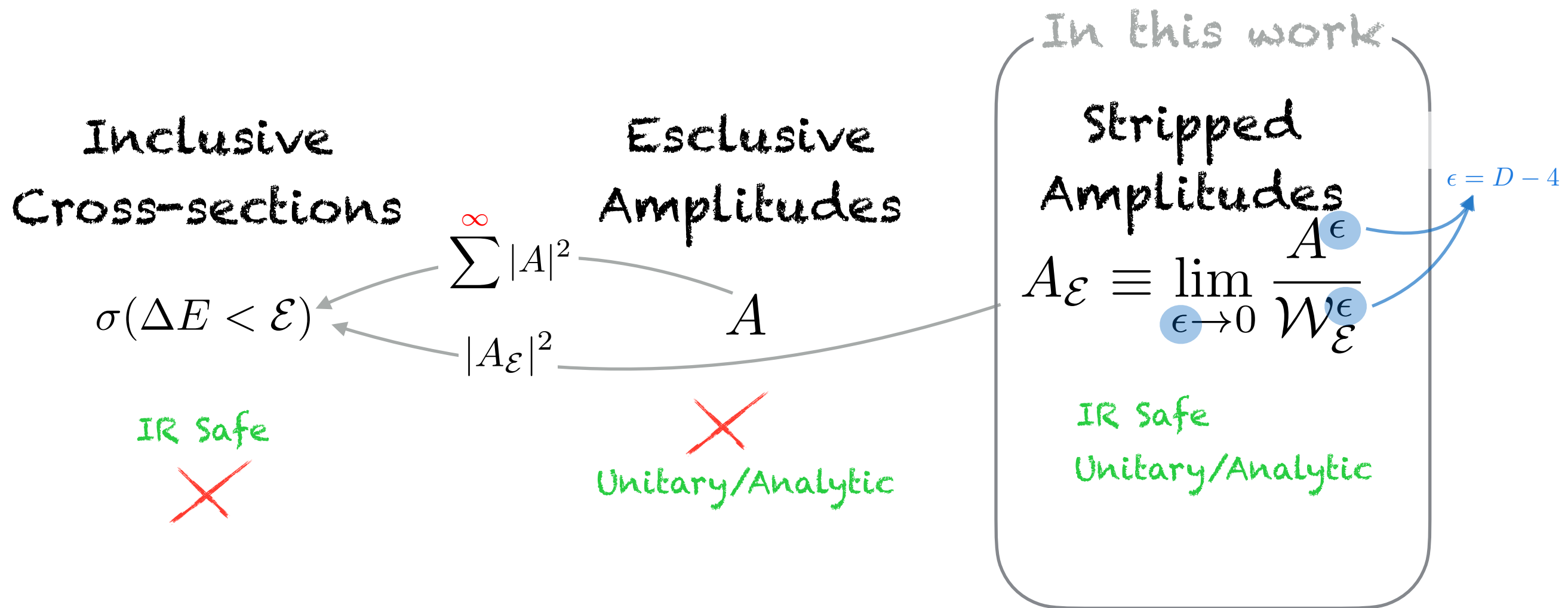
$$\epsilon = D - 4$$

# Stripped Amplitudes: IR Finite



►  $A_{\mathcal{E}}$  are formal amplitudes of finite size  $\mathcal{E}^{-1}$  experiments

# Stripped Amplitudes: IR Finite



- ▶  $A_\mathcal{E}$  are formal amplitudes of finite size  $\mathcal{E}^{-1}$  experiments
- ▶ Regulator  $\epsilon$  disappears, physical  $\mathcal{E}$  remains:  
it specify initial/final states!

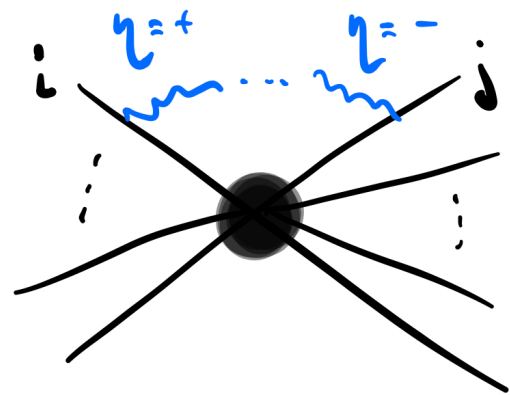
# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

QED:

Weinberg'65

$$\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$$



$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$

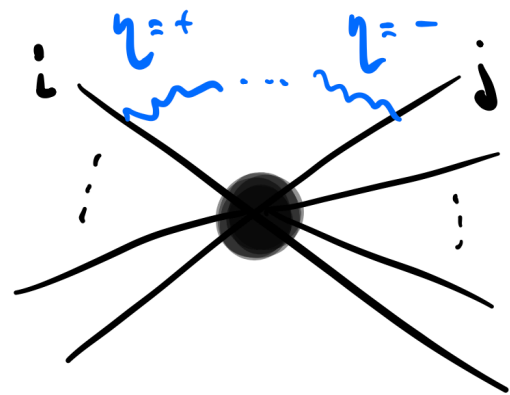
only for real kinematics!

# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

QED:  $\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$

Weinberg'65



$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$

only for real kinematics!

Analytic continuation:

Bellazzini, Berman, Isabella, FR, Romano, Sciotti'25

$$\mathcal{W} = \prod_{i,j} \mathcal{W}_{ij}(s_{ij})$$

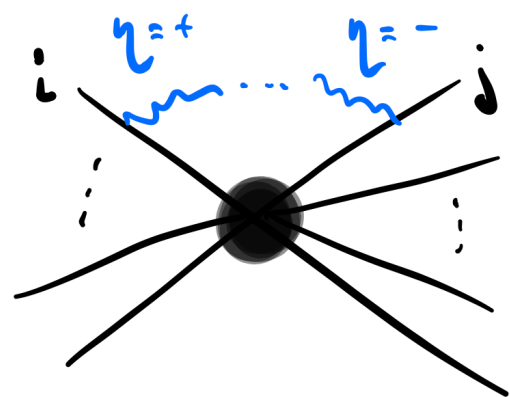
# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

QED:

Weinberg'65

$$\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$$



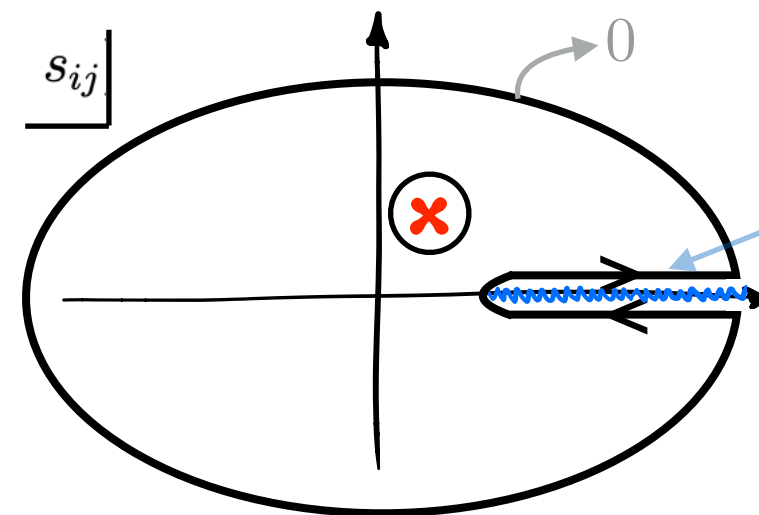
$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$

only for real kinematics!

Analytic continuation:

Bellazzini, Berman, Isabella, FR, Romano, Sciotti'25

$$\mathcal{W} = \prod_{i,j} \mathcal{W}_{ij}(s_{ij})$$





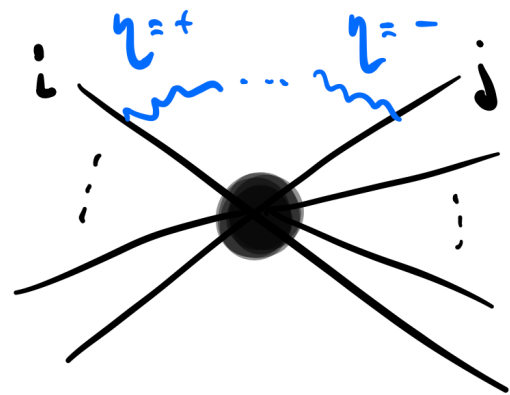
# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

**QED:**  $\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$

Weinberg'65

$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$



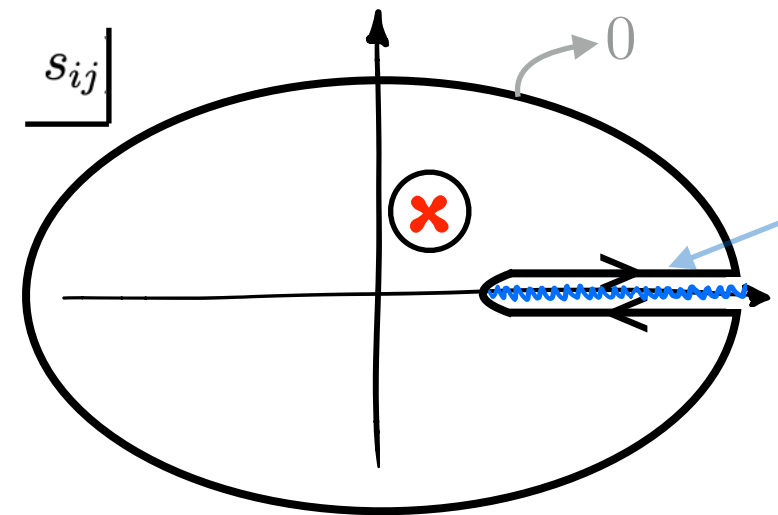
only for real kinematics!

**Analytic continuation:**  $\mathcal{W} = \prod_{i,j} \mathcal{W}_{ij}(s_{ij})$

Bellazzini, Berman, Isabella, FR, Romano, Sciotti'25

$$\mathcal{W}_{ij}(s_{ij}) = \mathcal{W}_{ij}(s_0) + \int_{(m_1+m_2)^2}^{\infty} \frac{d\bar{s}}{2\pi i} \frac{s_{ij} - s_0}{(\bar{s} - s_0)(\bar{s} - s_{ij})} \text{Disc } \mathcal{W}_{ij}(\bar{s}),$$

$$= \exp \left[ -\frac{\alpha q_i q_j}{2\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \frac{s_{ij} - m_i^2 - m_j^2}{\sqrt{(s_{ij} - (m_i - m_j)^2)(-s_{ij} + (m_i + m_j)^2)}} \arctan \frac{\sqrt{s_{ij} - (m_i - m_j)^2}}{\sqrt{-s_{ij} + (m_i + m_j)^2}} \right]$$

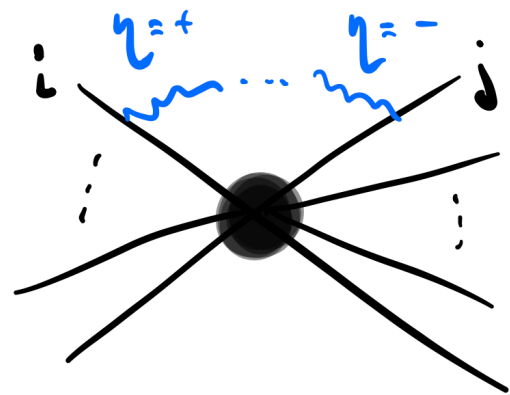


# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

QED:

Weinberg'65



$$\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$$

$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$

only for real kinematics!

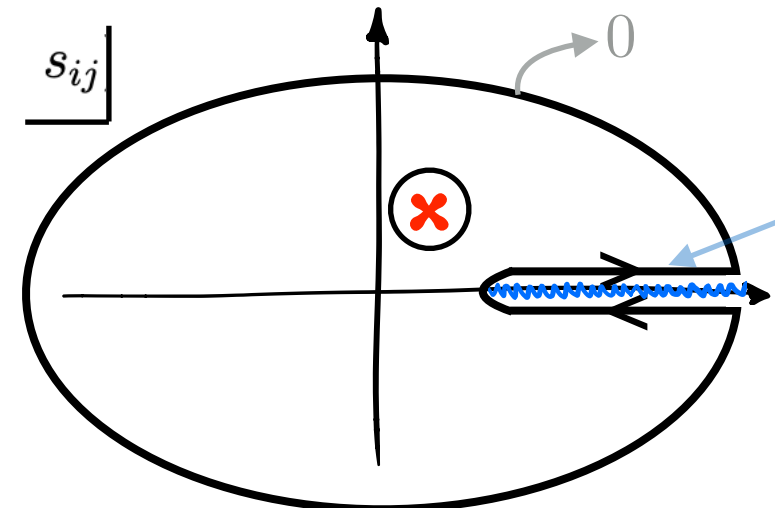
Analytic continuation:

Bellazzini, Berman, Isabella, FR, Romano, Sciotti'25

$$\mathcal{W} = \prod_{i,j} \mathcal{W}_{ij}(s_{ij})$$

$$\mathcal{W}_{ij}(s_{ij}) = \mathcal{W}_{ij}(s_0) + \int_{(m_1+m_2)^2}^{\infty} \frac{d\bar{s}}{2\pi i} \frac{s_{ij} - s_0}{(\bar{s} - s_0)(\bar{s} - s_{ij})} \text{Disc } \mathcal{W}_{ij}(\bar{s}),$$

$$= \exp \left[ -\frac{\alpha q_i q_j}{2\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \frac{s_{ij} - m_i^2 - m_j^2}{\sqrt{(s_{ij} - (m_i - m_j)^2)(-s_{ij} + (m_i + m_j)^2)}} \arctan \frac{\sqrt{s_{ij} - (m_i - m_j)^2}}{\sqrt{-s_{ij} + (m_i + m_j)^2}} \right] \xrightarrow{s \gg m} \exp \left[ \frac{\alpha q_i q_j}{4\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \log \left( -\frac{s_{ij}}{m^2} \right) \right]$$

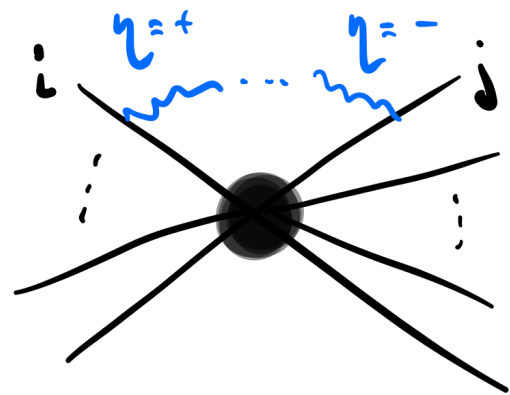


# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

QED:

Weinberg'65



$$\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$$

$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$

only for real kinematics!

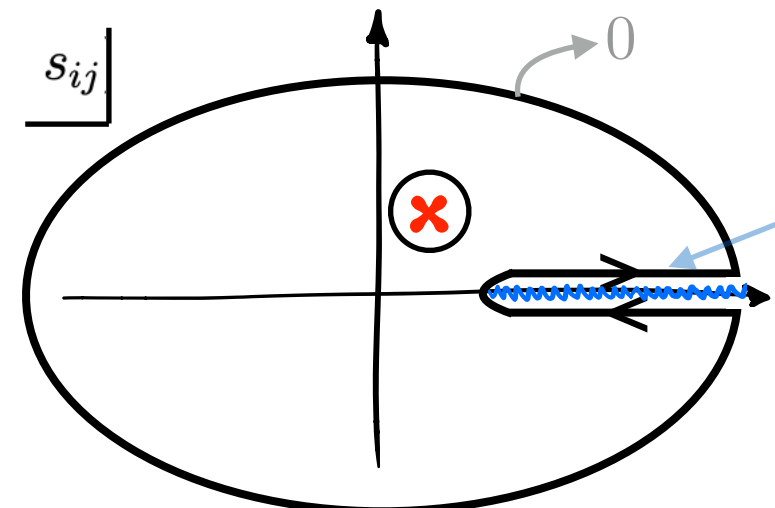
Analytic continuation:

Bellazzini, Berman, Isabella, FR, Romano, Sciotti'25

$$\mathcal{W} = \prod_{i,j} \mathcal{W}_{ij}(s_{ij})$$

$$\mathcal{W}_{ij}(s_{ij}) = \mathcal{W}_{ij}(s_0) + \int_{(m_1+m_2)^2}^{\infty} \frac{d\bar{s}}{2\pi i} \frac{s_{ij} - s_0}{(\bar{s} - s_0)(\bar{s} - s_{ij})} \text{Disc } \mathcal{W}_{ij}(\bar{s}),$$

$$= \exp \left[ -\frac{\alpha q_i q_j}{2\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \frac{s_{ij} - m_i^2 - m_j^2}{\sqrt{(s_{ij} - (m_i - m_j)^2)(-s_{ij} + (m_i + m_j)^2)}} \arctan \frac{\sqrt{s_{ij} - (m_i - m_j)^2}}{\sqrt{-s_{ij} + (m_i + m_j)^2}} \right] \xrightarrow{s \gg m} \exp \left[ \frac{\alpha q_i q_j}{4\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \log \left( -\frac{s_{ij}}{m^2} \right) \right]$$



►  $A_{\mathcal{E}}$  has same analyticity as  $A$  and respects Froissart  $A/s^2 \xrightarrow{s \rightarrow \infty} 0!$

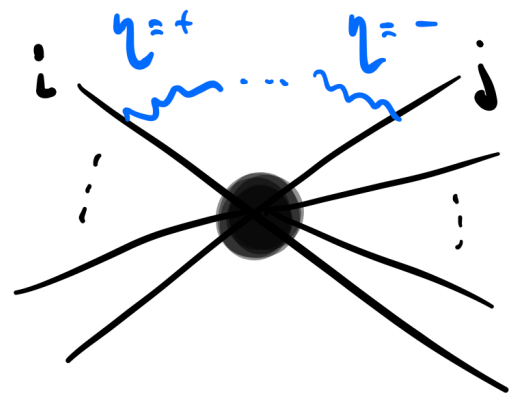
# Stripped Amplitudes: Analytic

$$A_{\mathcal{E}} \equiv \lim_{\epsilon \rightarrow 0} \frac{A^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}}$$

QED:

Weinberg'65

$$\mathcal{W} = \exp \left\{ \frac{\alpha}{8\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \left( \sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} + O(\epsilon) \right) \right\}$$



$$\beta_{ij}^2 = \frac{[s_{ij} - (m_i - m_j)^2][s_{ij} - (m_i + m_j)^2]}{(s_{ij} - m_i^2 - m_j^2)^2}$$

only for real kinematics!

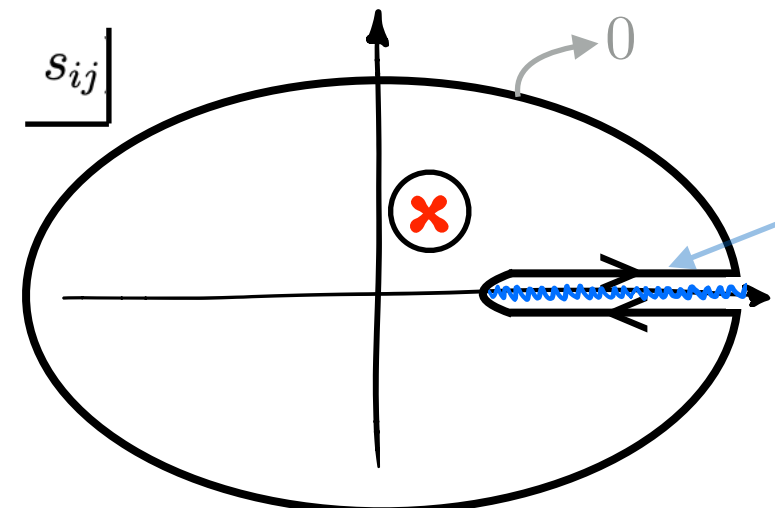
Analytic continuation:

Bellazzini, Berman, Isabella, FR, Romano, Sciotti'25

$$\mathcal{W} = \prod_{i,j} \mathcal{W}_{ij}(s_{ij})$$

$$\mathcal{W}_{ij}(s_{ij}) = \mathcal{W}_{ij}(s_0) + \int_{(m_1+m_2)^2}^{\infty} \frac{d\bar{s}}{2\pi i} \frac{s_{ij} - s_0}{(\bar{s} - s_0)(\bar{s} - s_{ij})} \text{Disc } \mathcal{W}_{ij}(\bar{s}),$$

$$= \exp \left[ -\frac{\alpha q_i q_j}{2\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \frac{s_{ij} - m_i^2 - m_j^2}{\sqrt{(s_{ij} - (m_i - m_j)^2)(-s_{ij} + (m_i + m_j)^2)}} \arctan \frac{\sqrt{s_{ij} - (m_i - m_j)^2}}{\sqrt{-s_{ij} + (m_i + m_j)^2}} \right] \xrightarrow{s \gg m} \exp \left[ \frac{\alpha q_i q_j}{4\pi} \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\epsilon} \log \left( -\frac{s_{ij}}{m^2} \right) \right]$$



►  $A_{\mathcal{E}}$  has same analyticity as  $A$  and respects Froissart  $A/s^2 \xrightarrow{s \rightarrow \infty} 0!$

... and similarly in Gravity!

# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$\alpha$

$$\alpha_{\mathcal{E}} \equiv \alpha \log M/\mathcal{E}$$

Captures finite perturbative effects

Captures leading IR effects

# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$\alpha$

$$\alpha_{\varepsilon} \equiv \alpha \log M/\varepsilon$$




Captures finite perturbative effects

Captures leading IR effects

# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$$\alpha \rightarrow 0$$

$$\mathcal{E} \rightarrow 0$$

$$\alpha_{\mathcal{E}} \equiv \alpha \log M/\mathcal{E} = \text{fixed} \quad \Downarrow$$


Captures finite perturbative effects

Captures leading IR effects



# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$$\alpha \rightarrow 0$$

$$\mathcal{E} \rightarrow 0$$

$$\alpha_{\mathcal{E}} \equiv \alpha \log M/\mathcal{E} = \text{fixed} \quad \Downarrow$$


Captures finite perturbative effects

Captures leading IR effects

In gravity:  $G \rightarrow 0, \mathcal{E} \rightarrow 0, G_{\mathcal{E}} \equiv GM^2 \log M/\mathcal{E} = \text{fixed}$

# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$$\alpha \rightarrow 0$$

$$\mathcal{E} \rightarrow 0$$

$$\alpha_{\mathcal{E}} \equiv \alpha \log M/\mathcal{E} = \text{fixed} \quad \Downarrow$$

Captures finite perturbative effects

Captures leading IR effects

In gravity:  $G \rightarrow 0, \mathcal{E} \rightarrow 0, G_{\mathcal{E}} \equiv GM^2 \log M/\mathcal{E} = \text{fixed}$

$$A_{\mathcal{E}} = A_{\mathcal{E}}^{\text{hard}} + \delta_{\text{sub. soft}}$$

$\alpha$        $\mathcal{E}$

$$E_{\gamma} > \mathcal{E}$$

# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$$\alpha \rightarrow 0$$

$$\mathcal{E} \rightarrow 0$$

$$\alpha_{\mathcal{E}} \equiv \alpha \log M/\mathcal{E} = \text{fixed} \quad \Downarrow$$


Captures finite perturbative effects

Captures leading IR effects

In gravity:  $G \rightarrow 0, \mathcal{E} \rightarrow 0, G_{\mathcal{E}} \equiv GM^2 \log M/\mathcal{E} = \text{fixed}$

$$A_{\mathcal{E}} = A_{\mathcal{E}}^{\text{hard}} + \delta_{\text{sub. soft}}$$


$\alpha \quad \mathcal{E}$

Is unitary (equivalent to  $m_{\gamma} = \mathcal{E}$  theory)  $E_{\gamma} > \mathcal{E}$

# Stripped Amplitudes: Unitary and Positive

Effectively 2 QED couplings:

$$\alpha \rightarrow 0$$

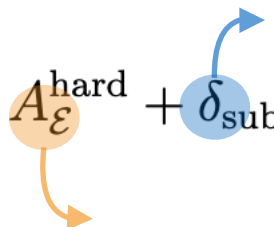
$$\mathcal{E} \rightarrow 0$$

$$\alpha_{\mathcal{E}} \equiv \alpha \log M/\mathcal{E} = \text{fixed} \quad \Downarrow$$


Captures finite perturbative effects

Captures leading IR effects

In gravity:  $G \rightarrow 0, \mathcal{E} \rightarrow 0, G_{\mathcal{E}} \equiv GM^2 \log M/\mathcal{E} = \text{fixed}$

$$A_{\mathcal{E}} = A_{\mathcal{E}}^{\text{hard}} + \delta_{\text{sub. soft}}$$


$\alpha$        $\mathcal{E}$

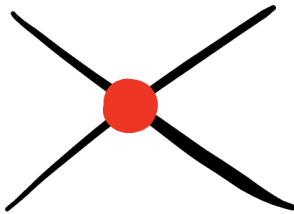
Is unitary (equivalent to  $m_{\gamma} = \mathcal{E}$  theory)  $E_{\gamma} > \mathcal{E}$

► Leads to positivity at **all orders** in  $\alpha_{\mathcal{E}}$  and L.O. in  $\alpha$

$$\mathcal{A}_n(s, t) = \int_{M^2}^{+\infty} \frac{ds}{\pi} \left[ \frac{\overline{\text{Im}} A_{\mathcal{E}}^s(s, t)}{s^{3+n}} + \frac{(-1)^n \overline{\text{Im}} A_{\mathcal{E}}^u(s, t)}{(s + t - 4m^2)^{3+n}} \right] \quad \text{e.g.} \quad \mathcal{A}_n \geq 0 + O(\alpha)$$

# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Tree level: 

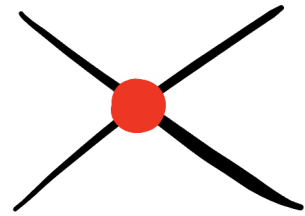
don't enter twice-subtracted relations

$$A_{+0}^{\text{tree}} = c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots$$

# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

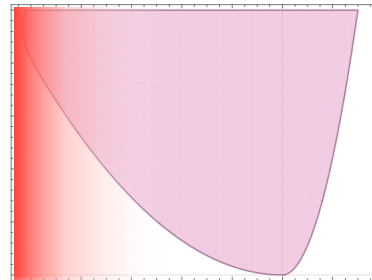
Tree level:



don't enter twice-subtracted relations

$$A_{+0}^{\text{tree}} = c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots$$

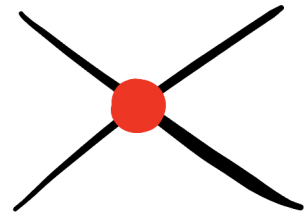
Positivity Bounds:



# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

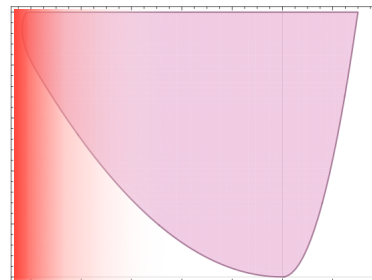
Tree level:



don't enter twice-subtracted relations

$$A_{+0}^{\text{tree}} = c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots$$

Positivity Bounds:



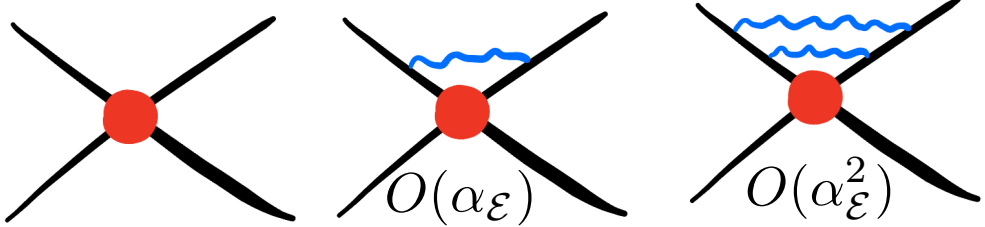
$$c_{20} > 0$$

$$(M^2 - m_{\pm}^2) \frac{c_{31}}{c_{20}} \geq -\frac{3}{2}$$



# Bounds for charged/neutral pions

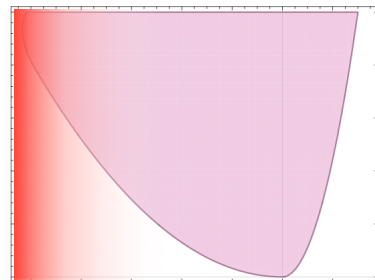
$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Loop level:  + .. = 0 exclusively...

$$A_{\mathcal{E}} = \left[ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right] \times$$

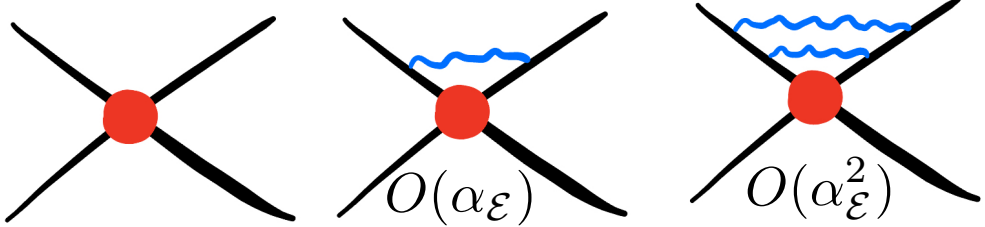
$$\left[ 1 + \frac{\alpha}{2\pi} \log \left( \frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left( \frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

Positivity Bounds:



# Bounds for charged/neutral pions

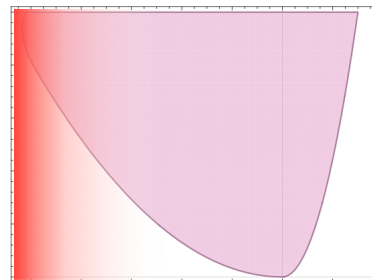
$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Loop level:  + .. = 0 exclusively...

$$A_{\mathcal{E}} = \left[ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right] \times$$

$$\left[ 1 + \frac{\alpha}{2\pi} \log \left( \frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left( \frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

Positivity Bounds:



$$c_{2,0} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

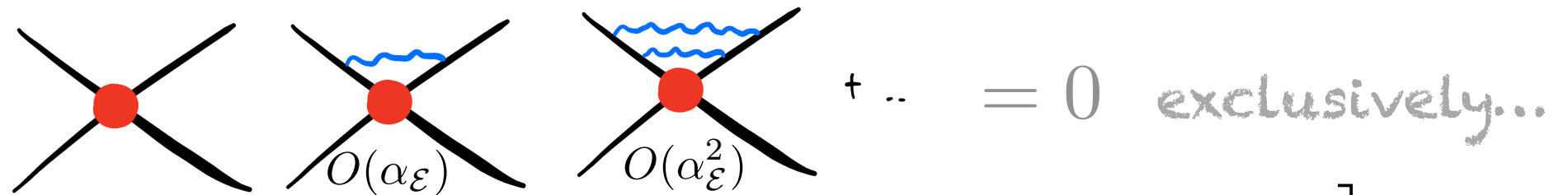


$$c_{2,0} \left[ \frac{3}{2} - \frac{(M^2 - m_{\pm}^2)}{m_{\pm}^2} \frac{\alpha}{3\pi} \log \left( \frac{\mathcal{E}}{m_{\pm}} \right) \right] + (M^2 - m_{\pm}^2)c_{3,1} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Loop level:



$$+ \dots = 0 \text{ exclusively...}$$

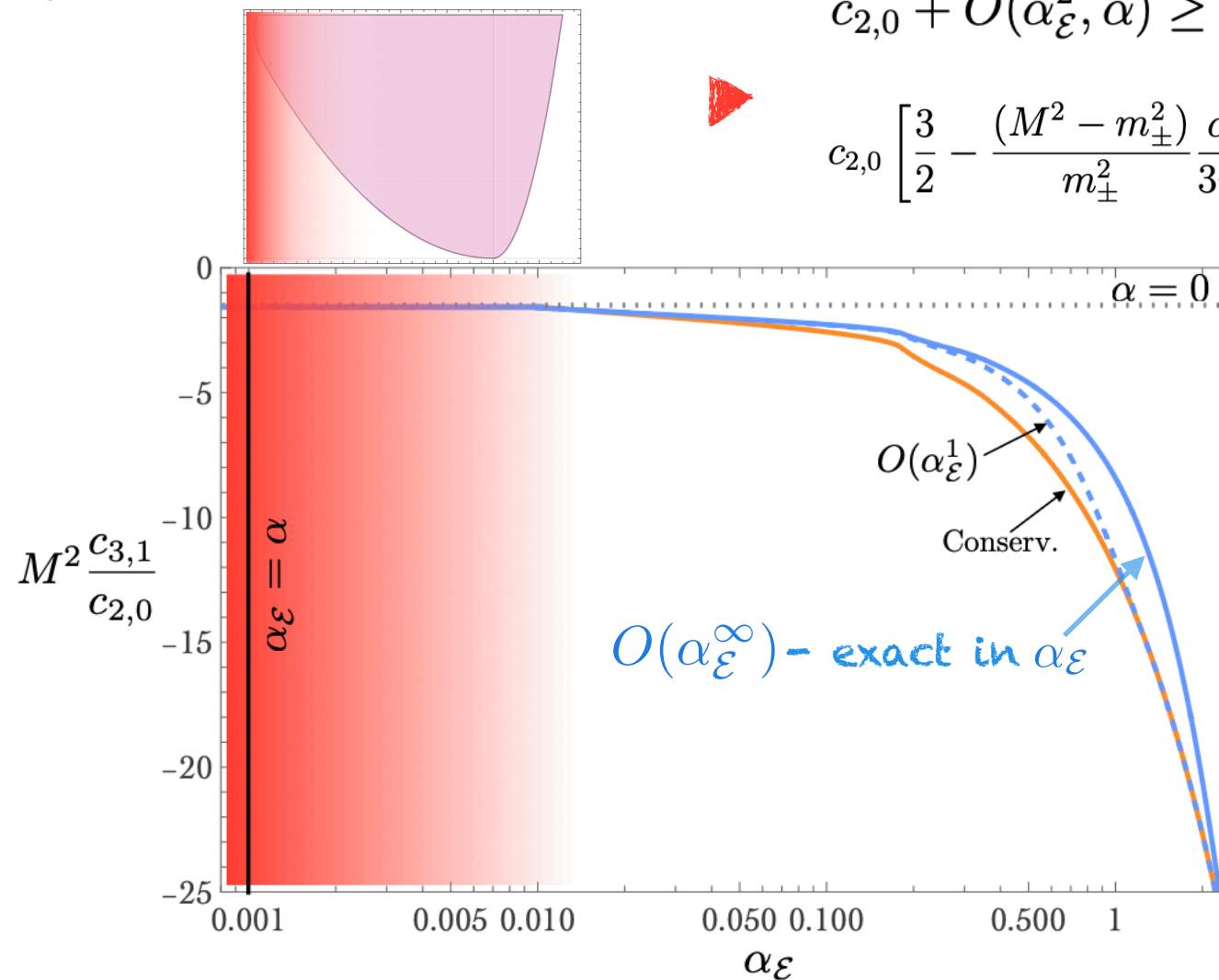
$$A_{\mathcal{E}} = \left[ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right] \times$$

$$\left[ 1 + \frac{\alpha}{2\pi} \log \left( \frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left( \frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

Positivity Bounds:

$$c_{2,0} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

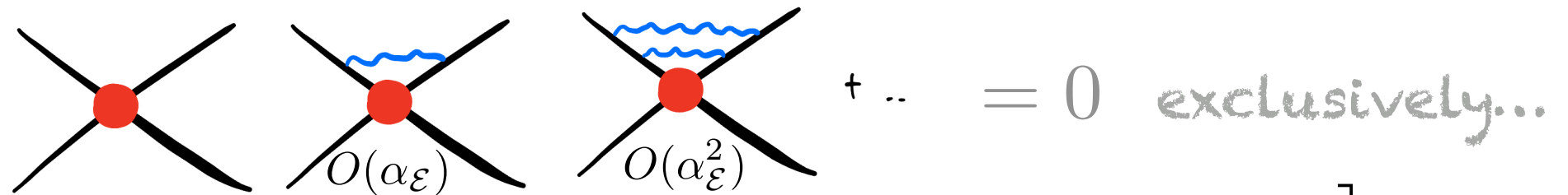
$$c_{2,0} \left[ \frac{3}{2} - \frac{(M^2 - m_{\pm}^2)}{m_{\pm}^2} \frac{\alpha}{3\pi} \log \left( \frac{\mathcal{E}}{m_{\pm}} \right) \right] + (M^2 - m_{\pm}^2)c_{3,1} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$



# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Loop level:



$$A_{\mathcal{E}} = \left[ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right] \times$$

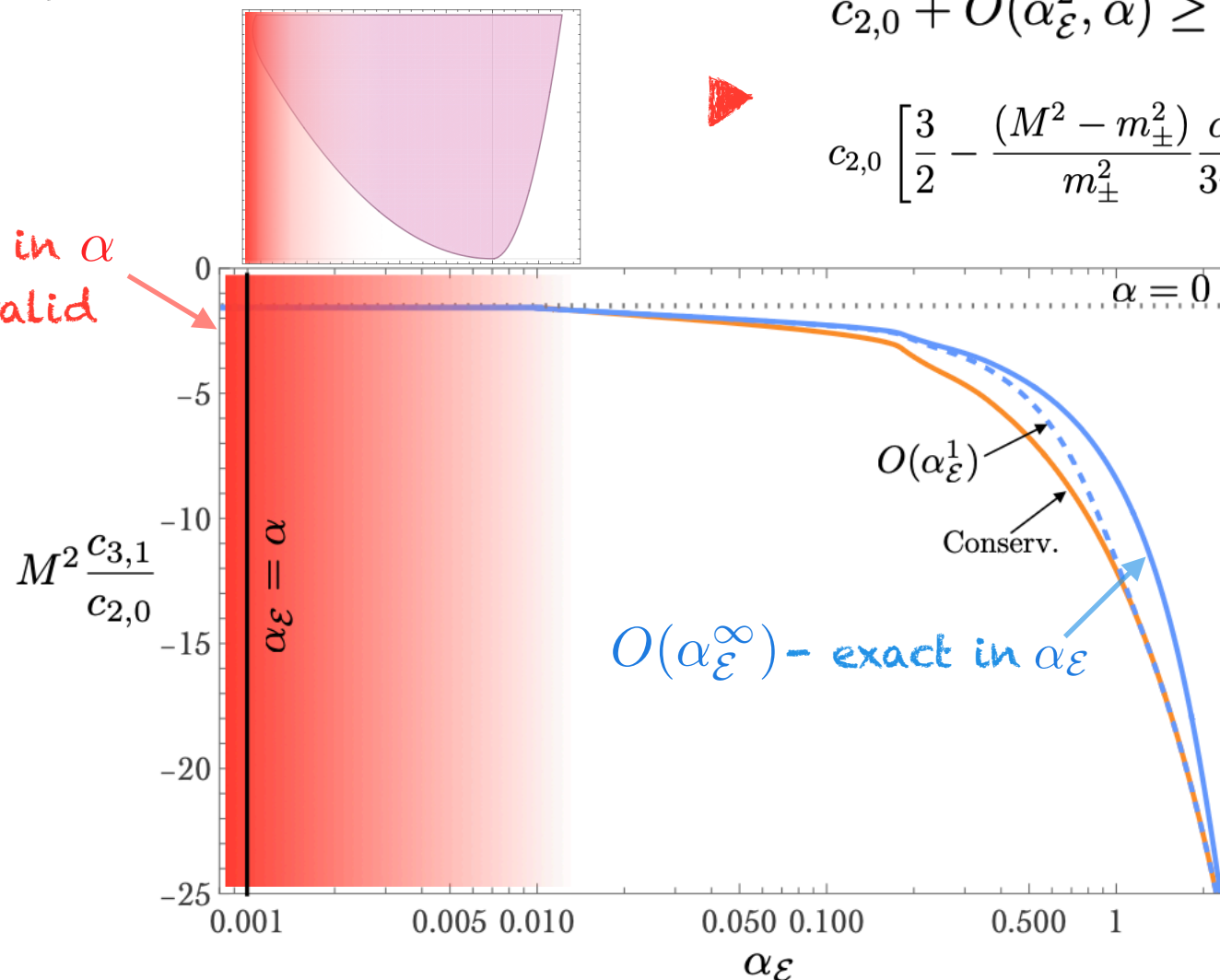
$$\left[ 1 + \frac{\alpha}{2\pi} \log \left( \frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left( \frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

Positivity Bounds:

$$c_{2,0} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

$$c_{2,0} \left[ \frac{3}{2} - \frac{(M^2 - m_{\pm}^2)}{m_{\pm}^2} \frac{\alpha}{3\pi} \log \left( \frac{\mathcal{E}}{m_{\pm}} \right) \right] + (M^2 - m_{\pm}^2)c_{3,1} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

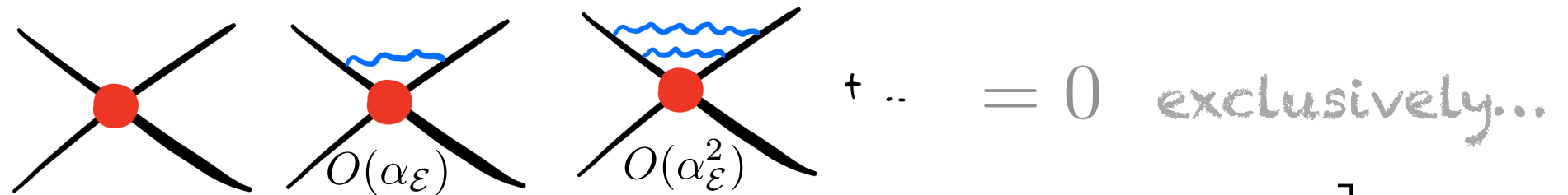
approx in  $\alpha$   
not valid



# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Loop level:



$$A_{\mathcal{E}} = \left[ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right] \times$$

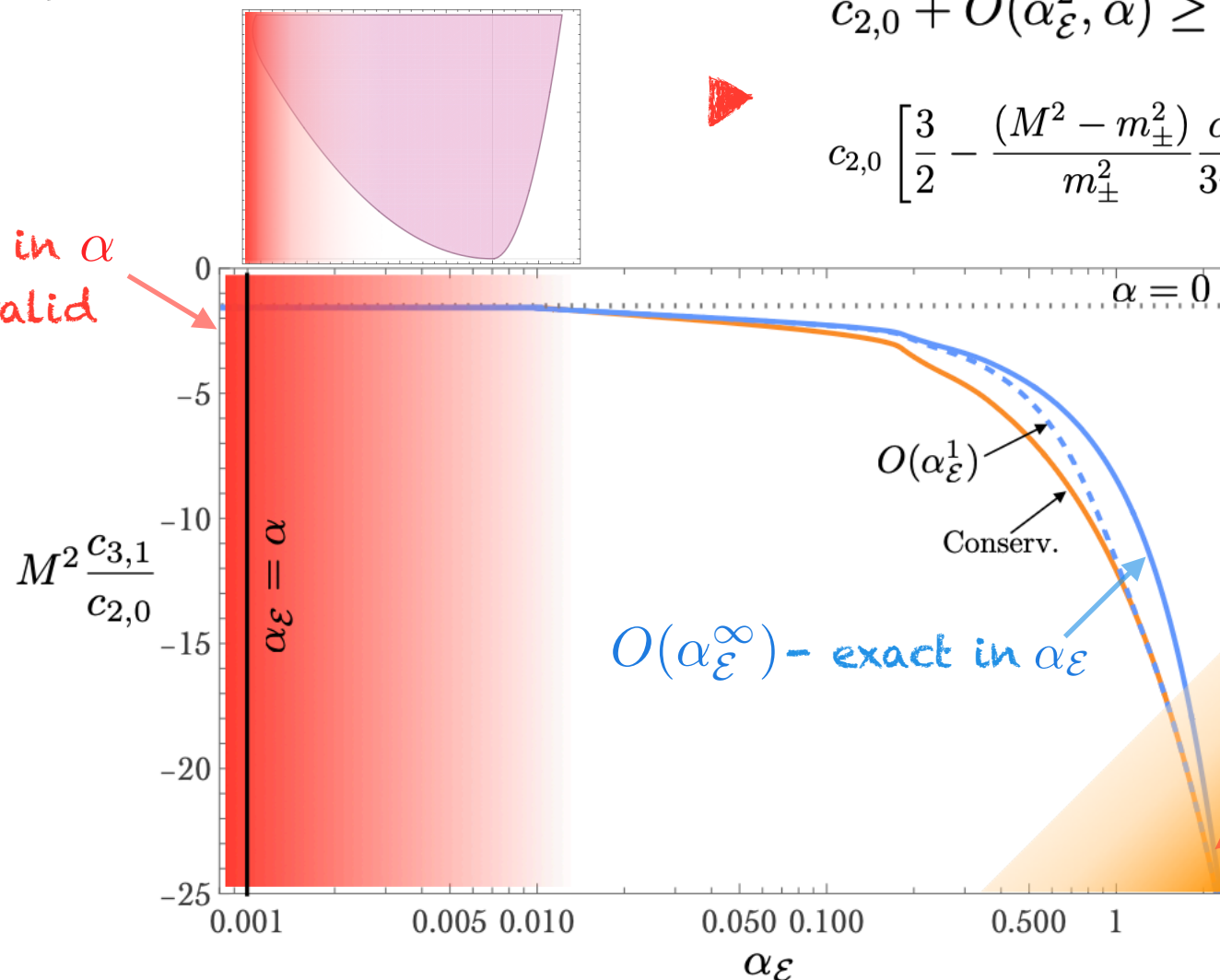
$$\left[ 1 + \frac{\alpha}{2\pi} \log \left( \frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left( \frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

Positivity Bounds:

$$c_{2,0} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

$$c_{2,0} \left[ \frac{3}{2} - \frac{(M^2 - m_{\pm}^2)}{m_{\pm}^2} \frac{\alpha}{3\pi} \log \left( \frac{\mathcal{E}}{m_{\pm}} \right) \right] + (M^2 - m_{\pm}^2)c_{3,1} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

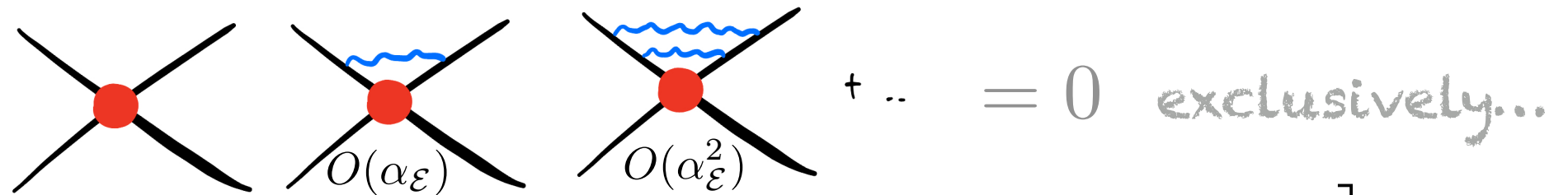
approx in  $\alpha$   
not valid



# Bounds for charged/neutral pions

$$\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$$

Loop level:



$$A_{\mathcal{E}} = \left[ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right] \times$$

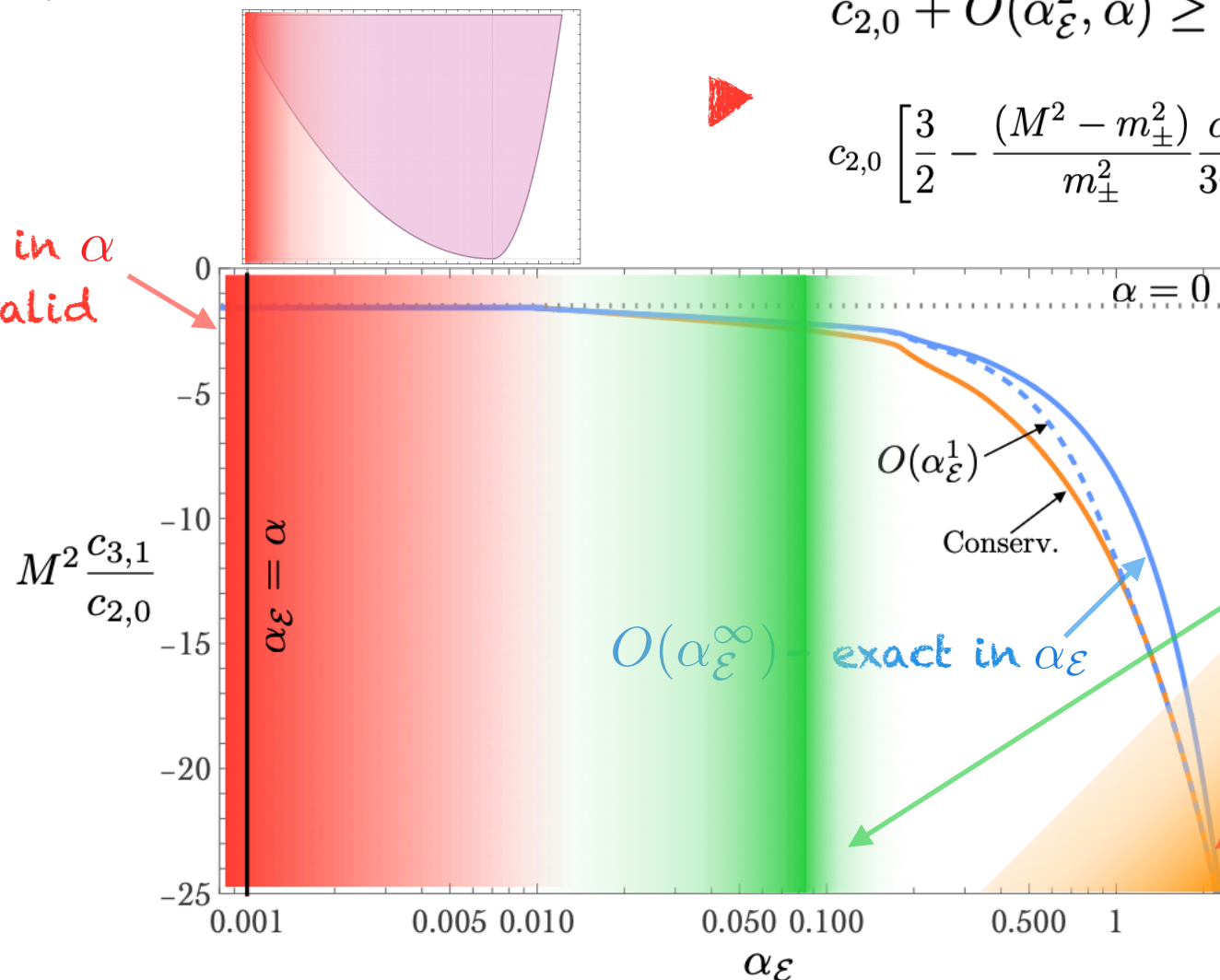
$$\left[ 1 + \frac{\alpha}{2\pi} \log \left( \frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left( \frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

Positivity Bounds:

$$c_{2,0} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

$$c_{2,0} \left[ \frac{3}{2} - \frac{(M^2 - m_{\pm}^2)}{m_{\pm}^2} \frac{\alpha}{3\pi} \log \left( \frac{\mathcal{E}}{m_{\pm}} \right) \right] + (M^2 - m_{\pm}^2)c_{3,1} + O(\alpha_{\mathcal{E}}^2, \alpha) \geq 0$$

approx in  $\alpha$   
not valid



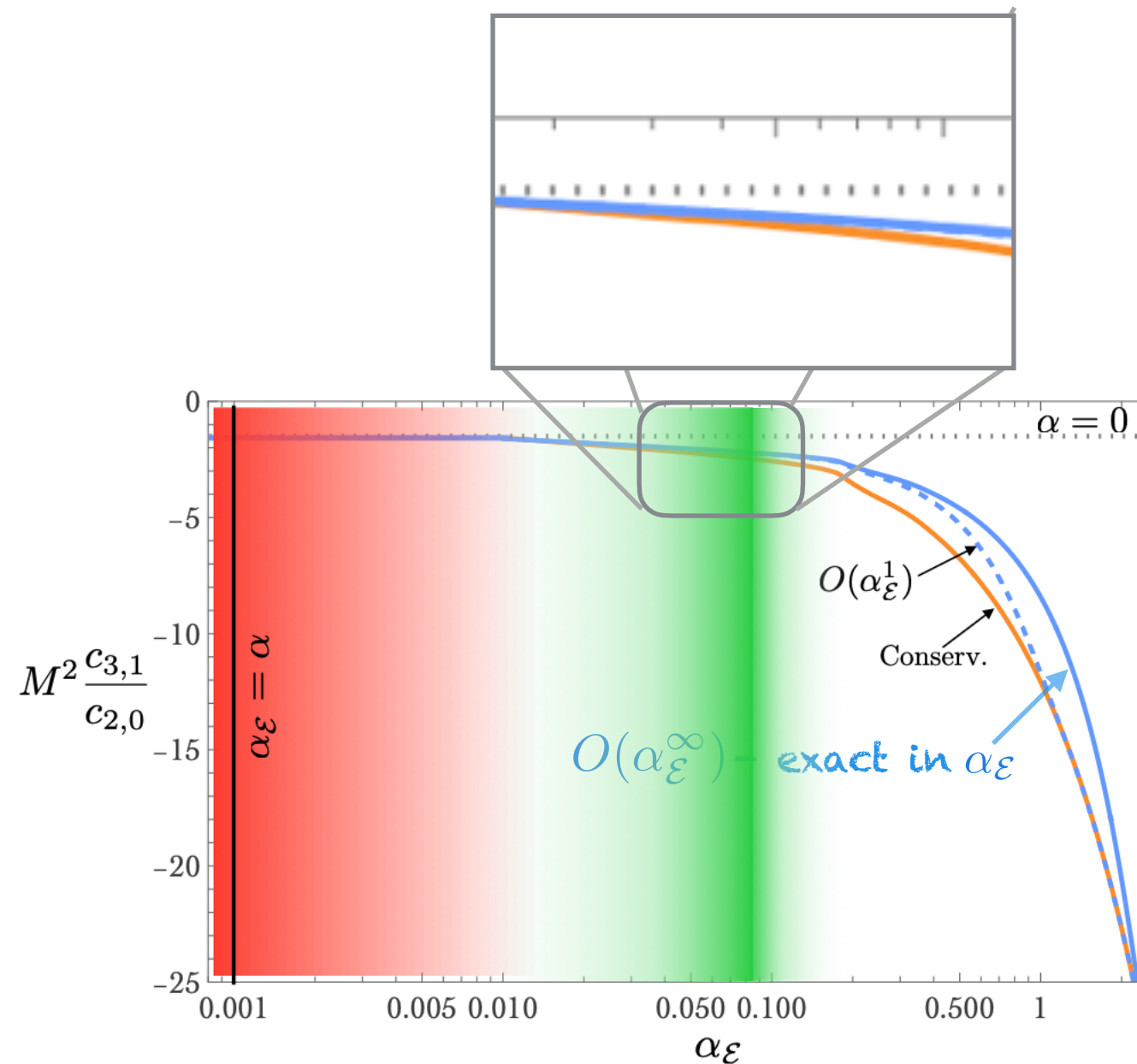


# Bounds for charged/neutral pions

$$\pi^+\pi^0 \rightarrow \pi^+\pi^0$$

Positivity bounds survive to  $\alpha \neq 0$  also in D=4...

QED corrections are perturbative.



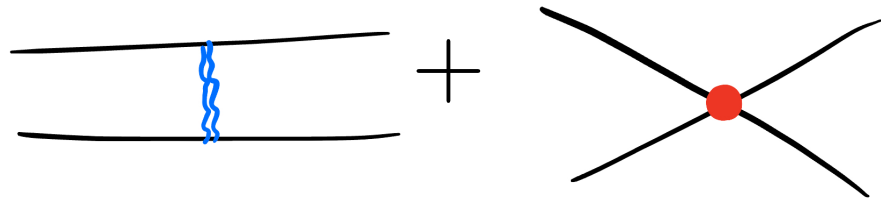


# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

Tree Amplitude:  $G_N \rightarrow 0 \quad \frac{g}{G_N} = \text{finite}$   
 $g_n \rightarrow 0$

# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

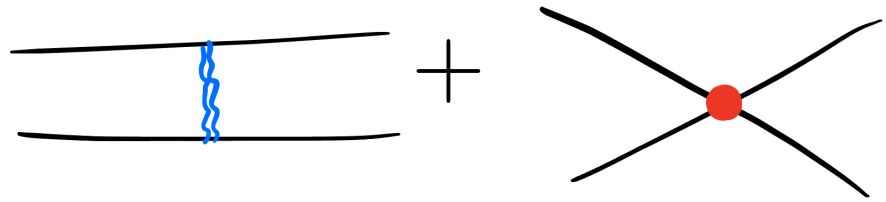
Tree Amplitude:  $G_N \rightarrow 0 \quad \frac{g}{G_N} = \text{finite}$   
 $g_n \rightarrow 0$



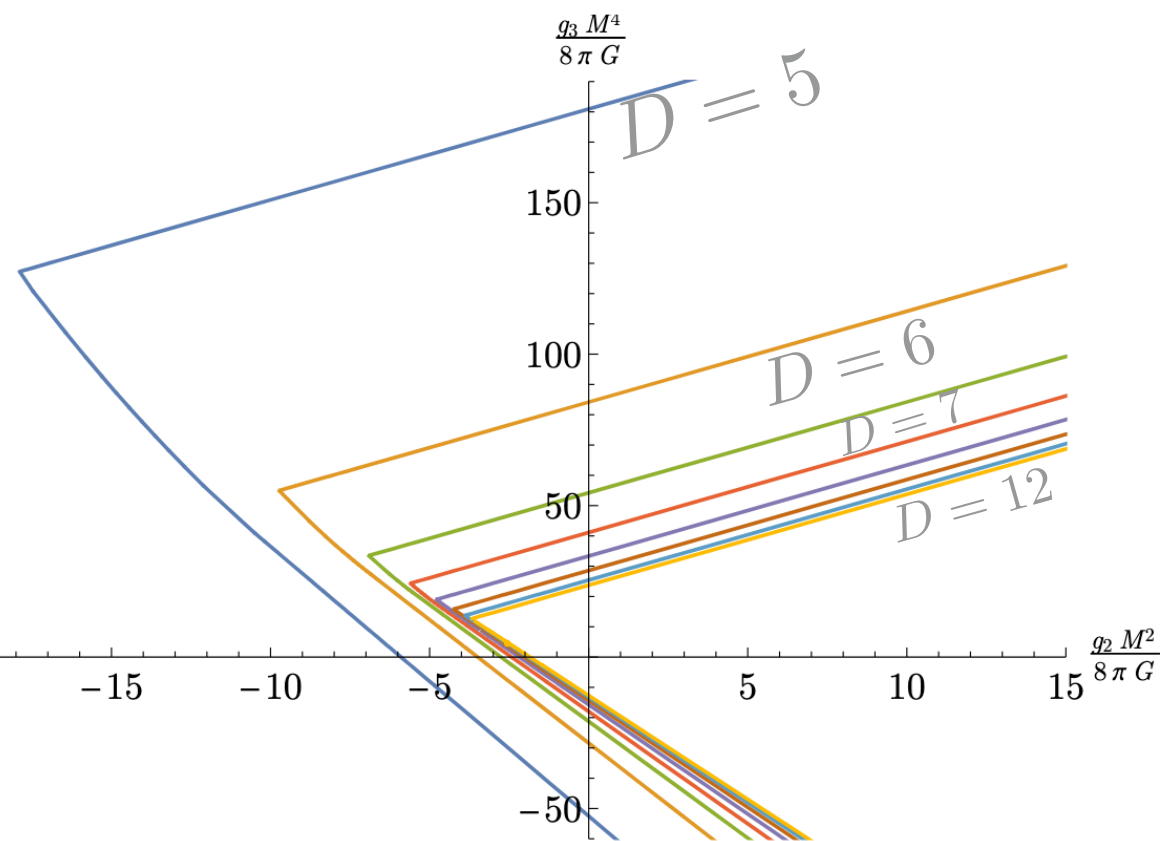
$$A(s, t) = G_N \left( \frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + g_2(s^2 + t^2 + u^2) + g_3 stu + \dots$$

# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

Tree Amplitude:  $G_N \rightarrow 0 \quad \frac{g}{G_N} = \text{finite}$   
 $g_n \rightarrow 0$



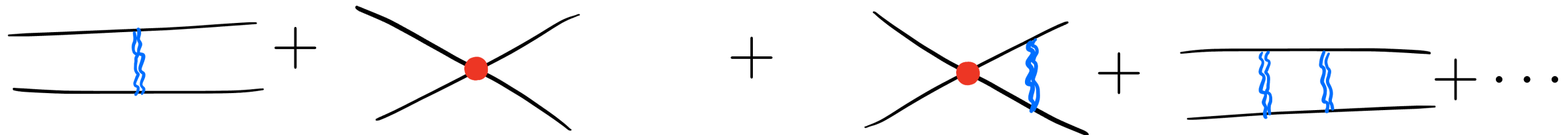
$$A(s, t) = G_N \left( \frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + g_2(s^2 + t^2 + u^2) + g_3 stu + \dots$$



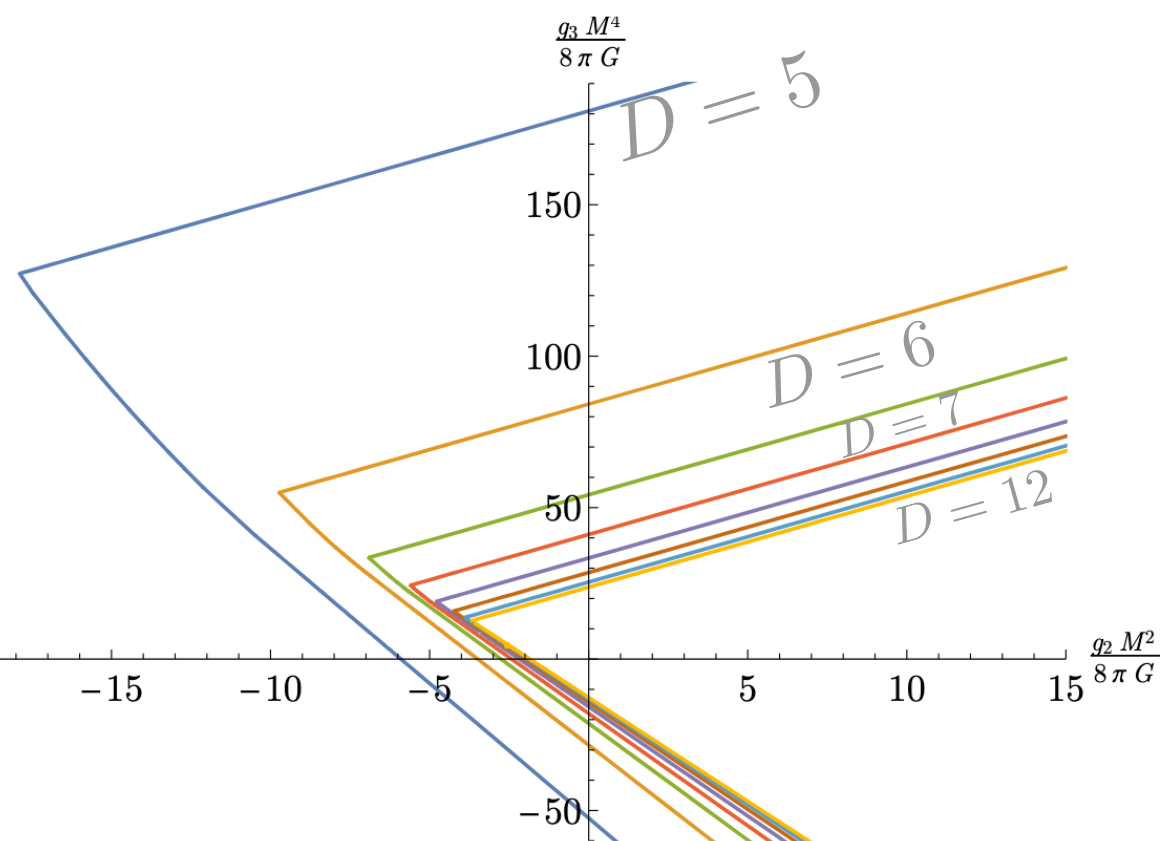
# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

Tree Amplitude:  $G_N \rightarrow 0$   $\frac{g}{G_N} = \text{finite}$

Stripped Amplitude:  $G_N \rightarrow 0$   $\mathcal{E} \rightarrow 0$ ,  
 $G_{\mathcal{E}} \equiv G M^2 \log M / \mathcal{E} = \text{fixed}$

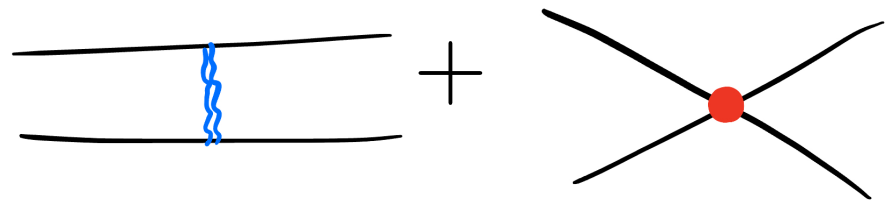


$$A(s, t) = G_N \left( \frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + g_2(s^2 + t^2 + u^2) + g_3 stu + \dots$$



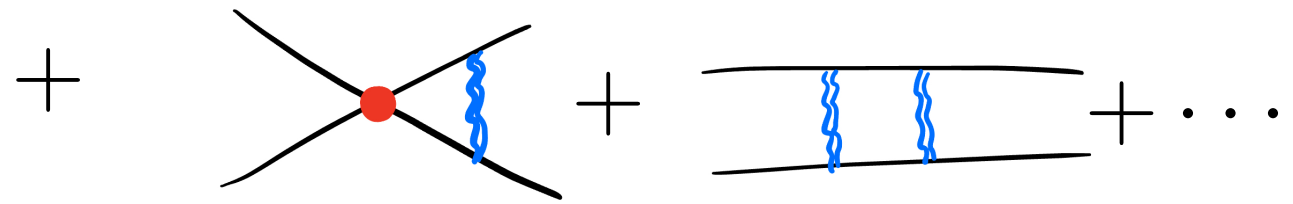
# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

Tree Amplitude:  $G_N \rightarrow 0$   $\frac{g}{G_N} = \text{finite}$



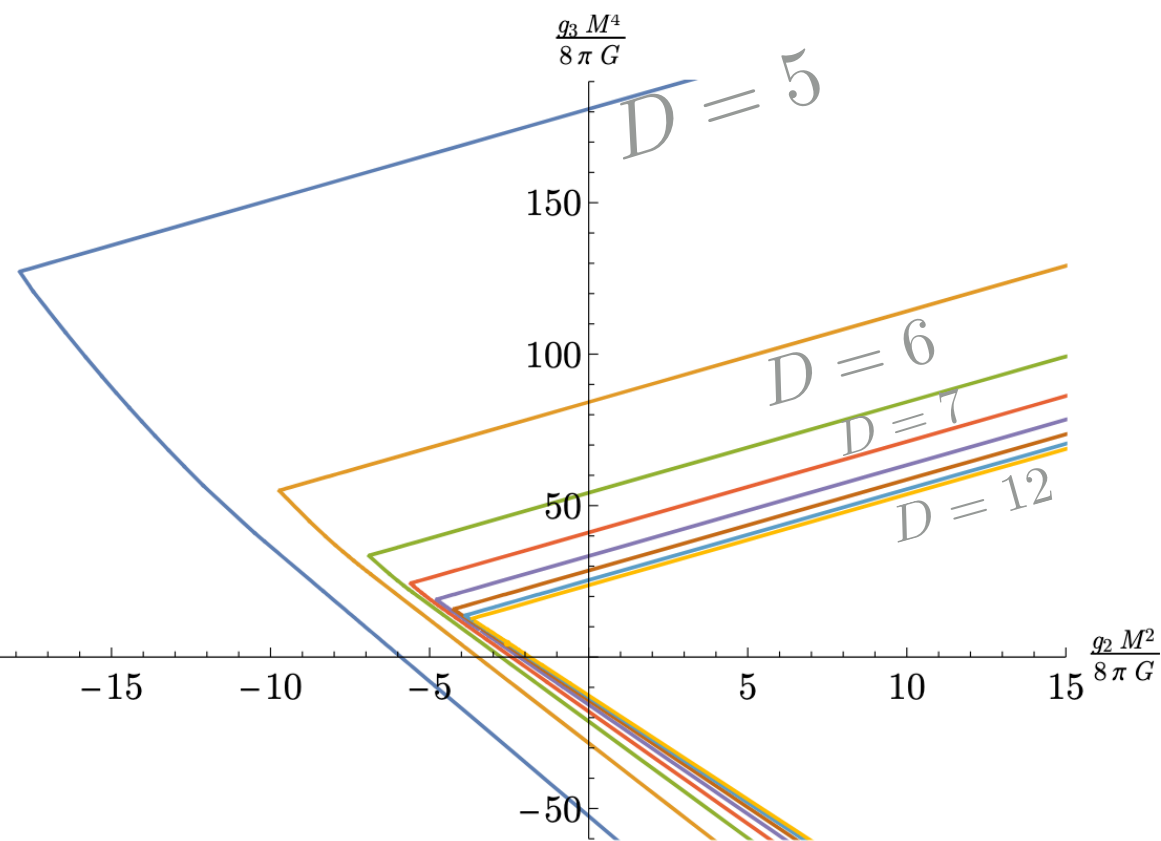
$$A(s, t) = G_N \left( \frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + g_2(s^2 + t^2 + u^2) + g_3 stu + \dots$$

Stripped Amplitude:  $G_N \rightarrow 0$   $\mathcal{E} \rightarrow 0$ ,  
 $G_{\mathcal{E}} \equiv GM^2 \log M/\mathcal{E} = \text{fixed}$



$$A_{\mathcal{E}} = \frac{A}{\mathcal{W}_{GR}} \Big|_{\epsilon \rightarrow 0}$$

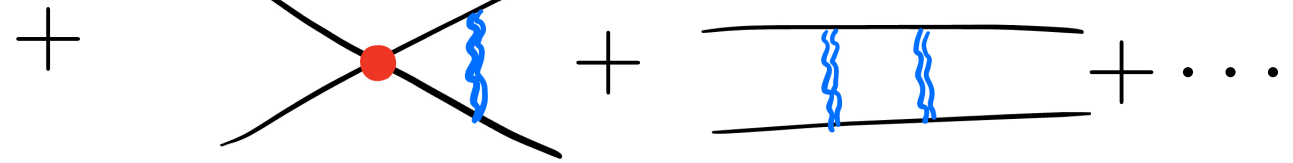
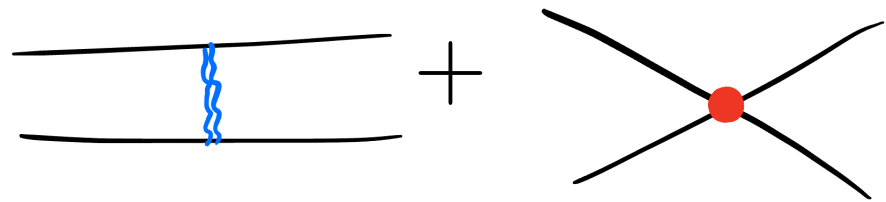
$$\mathcal{W}_{GR}(s, t) = \exp \left[ -G \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\pi\epsilon} (s \log(-s/\mu^2) + s \rightarrow t, u) \right]$$



# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

Tree Amplitude:  $G_N \rightarrow 0$   $\frac{g}{G_N} = \text{finite}$

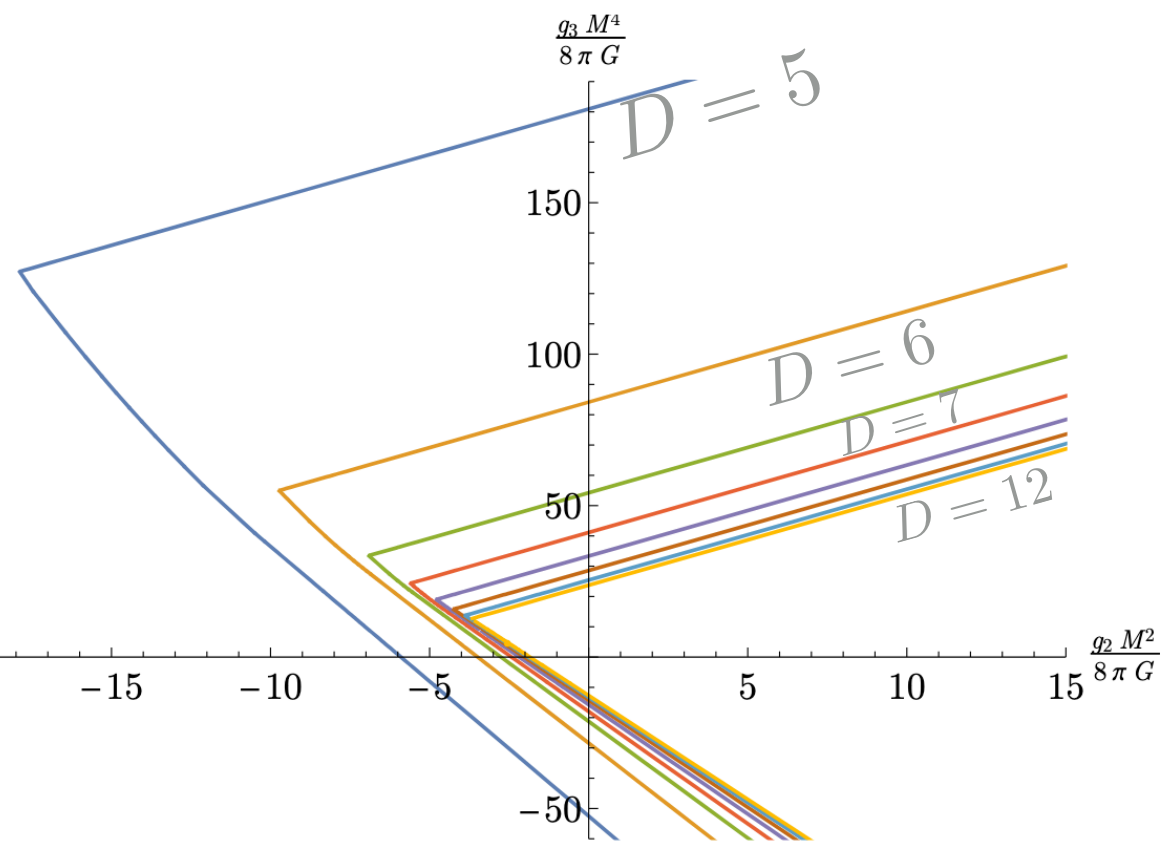
Stripped Amplitude:  $G_N \rightarrow 0$   
 $\mathcal{E} \rightarrow 0$ ,  
 $G_{\mathcal{E}} \equiv G M^2 \log M / \mathcal{E} = \text{fixed}$



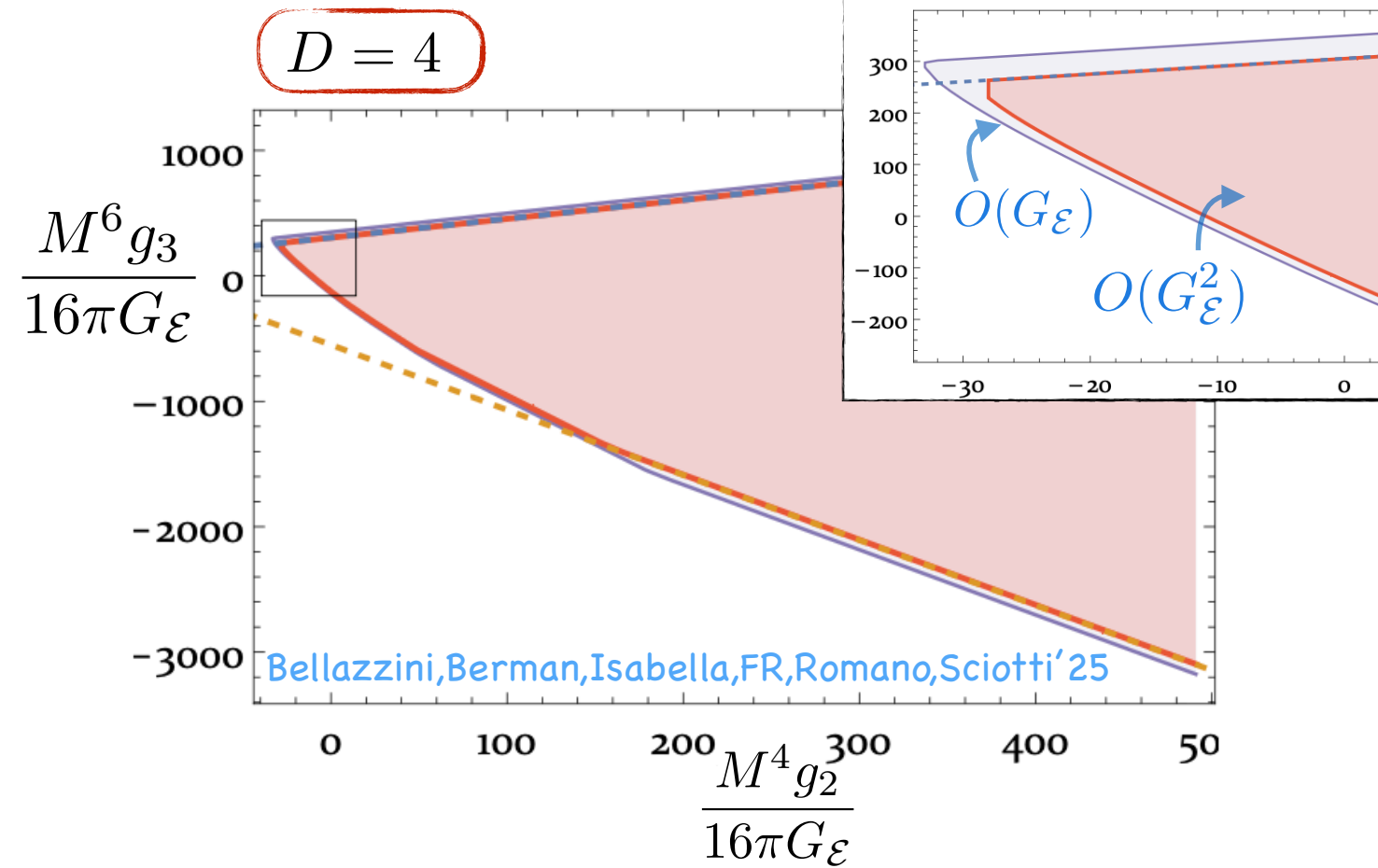
$$A(s, t) = G_N \left( \frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + g_2(s^2 + t^2 + u^2) + g_3 stu + \dots$$

$$A_{\mathcal{E}} = \frac{A}{\mathcal{W}_{GR}} \Big|_{\epsilon \rightarrow 0}$$

$$\mathcal{W}_{GR}(s, t) = \exp \left[ -G \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\pi\epsilon} (s \log(-s/\mu^2) + s \rightarrow t, u) \right]$$



Caron-Huot, Mazac, Rastelli, Simons-Duffin '22

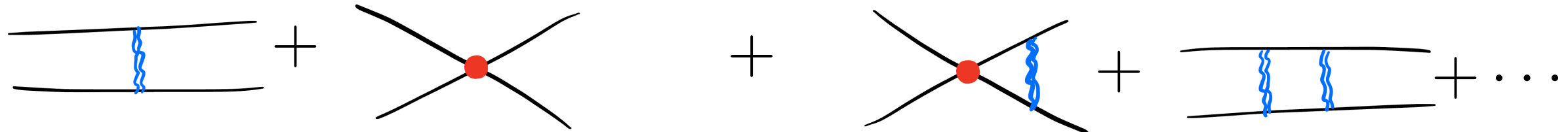


Bellazzini, Berman, Isabella, FR, Romano, Sciotti '25

# Bounds with Gravity: $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$

Tree Amplitude:  $G_N \rightarrow 0$   $\frac{g}{G_N} = \text{finite}$

Stripped Amplitude:  $G_N \rightarrow 0$   $\mathcal{E} \rightarrow 0$ ,  
 $G_{\mathcal{E}} \equiv GM^2 \log M/\mathcal{E} = \text{fixed}$

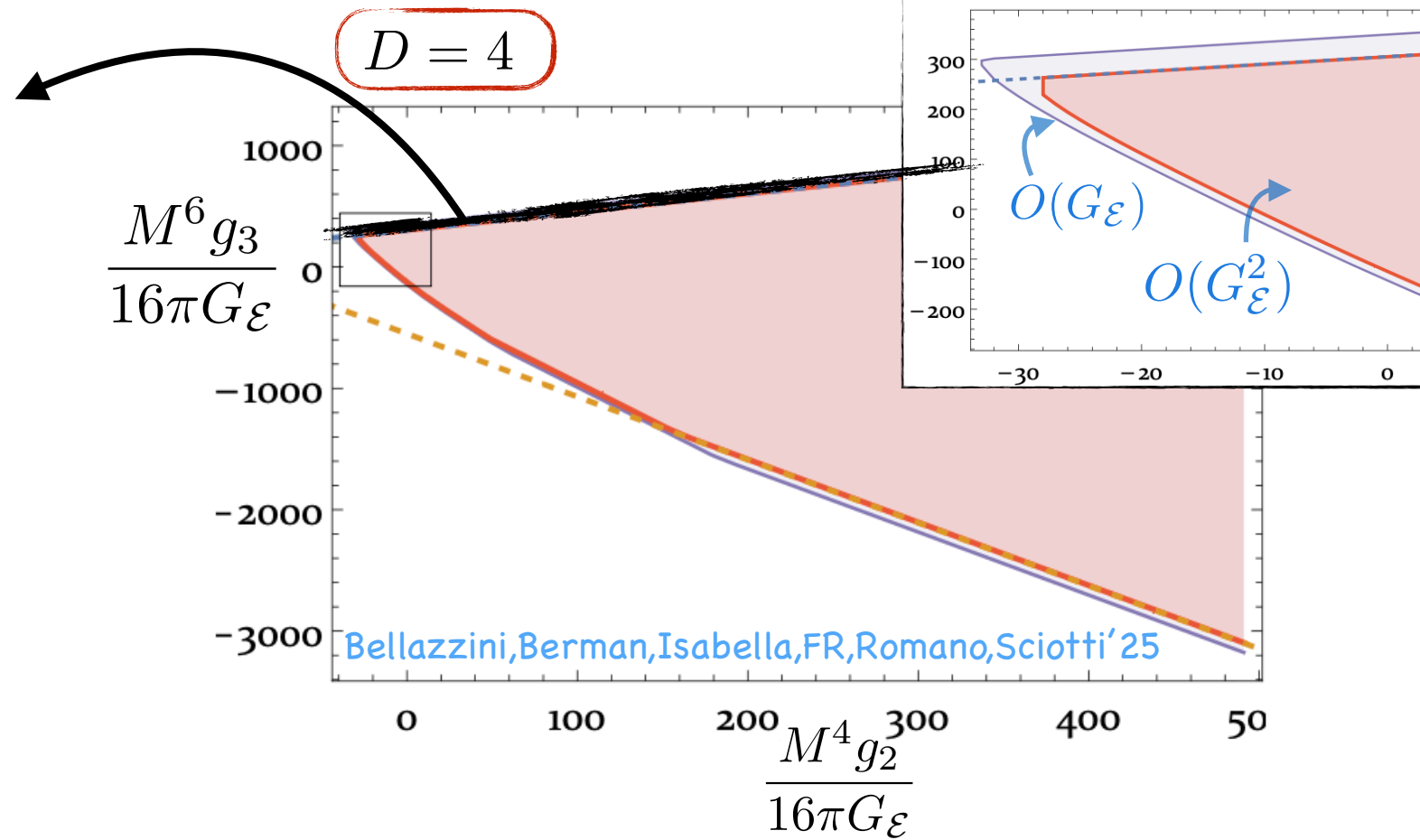
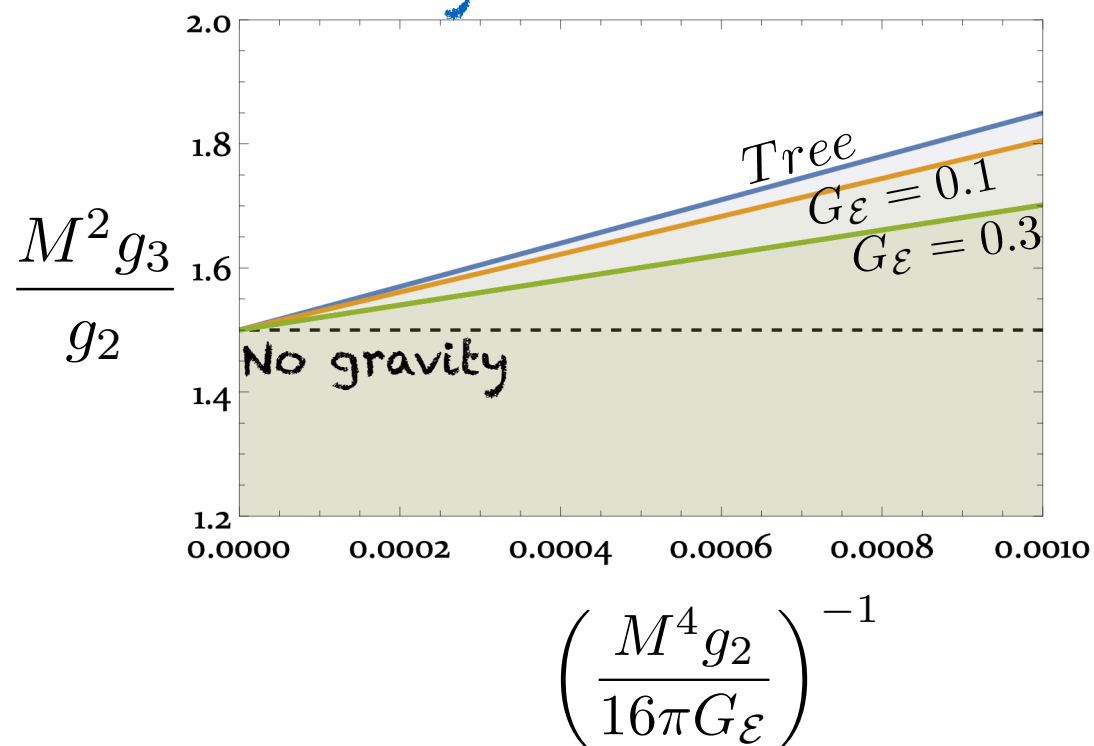


$$A(s, t) = G_N \left( \frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + g_2(s^2 + t^2 + u^2) + g_3 stu + \dots$$

$$A_{\mathcal{E}} = \frac{A}{\mathcal{W}_{GR}} \Big|_{\epsilon \rightarrow 0}$$

$$\mathcal{W}_{GR}(s, t) = \exp \left[ -G \frac{(\mathcal{E}/\mu)^{2\epsilon}}{\pi\epsilon} (s \log(-s/\mu^2) + s \rightarrow t, u) \right]$$

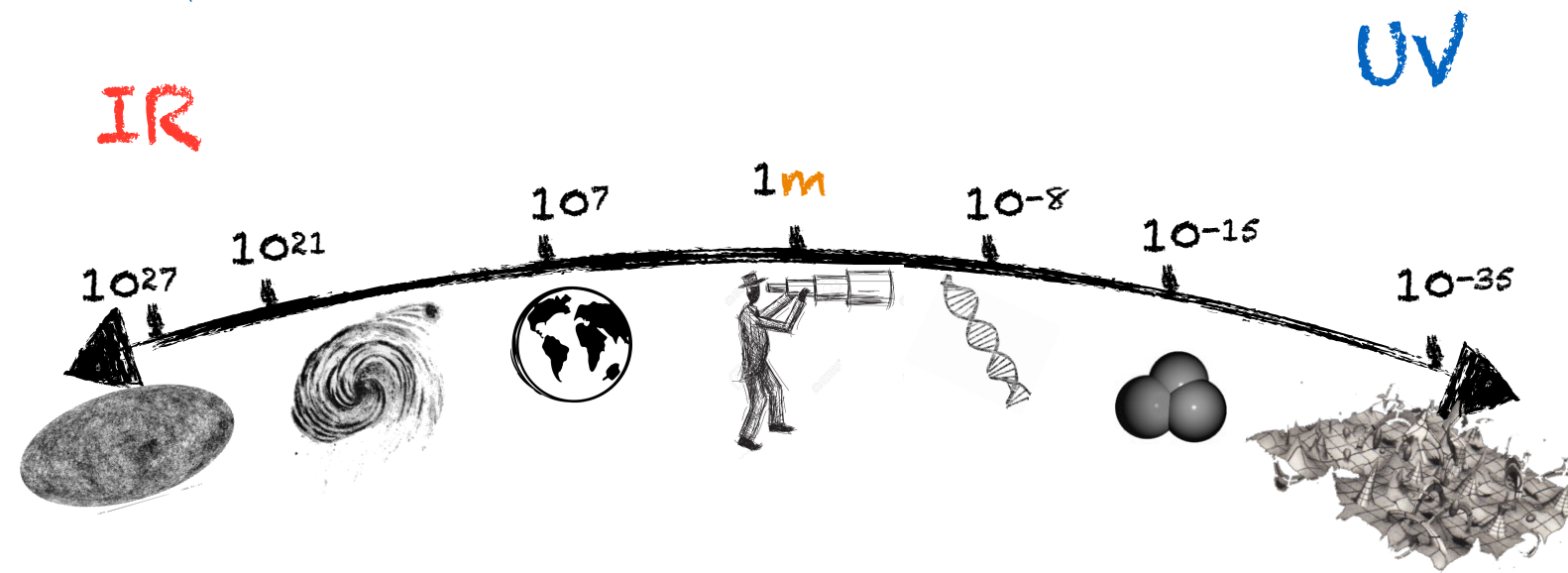
Smoothly connects to  $G=0$



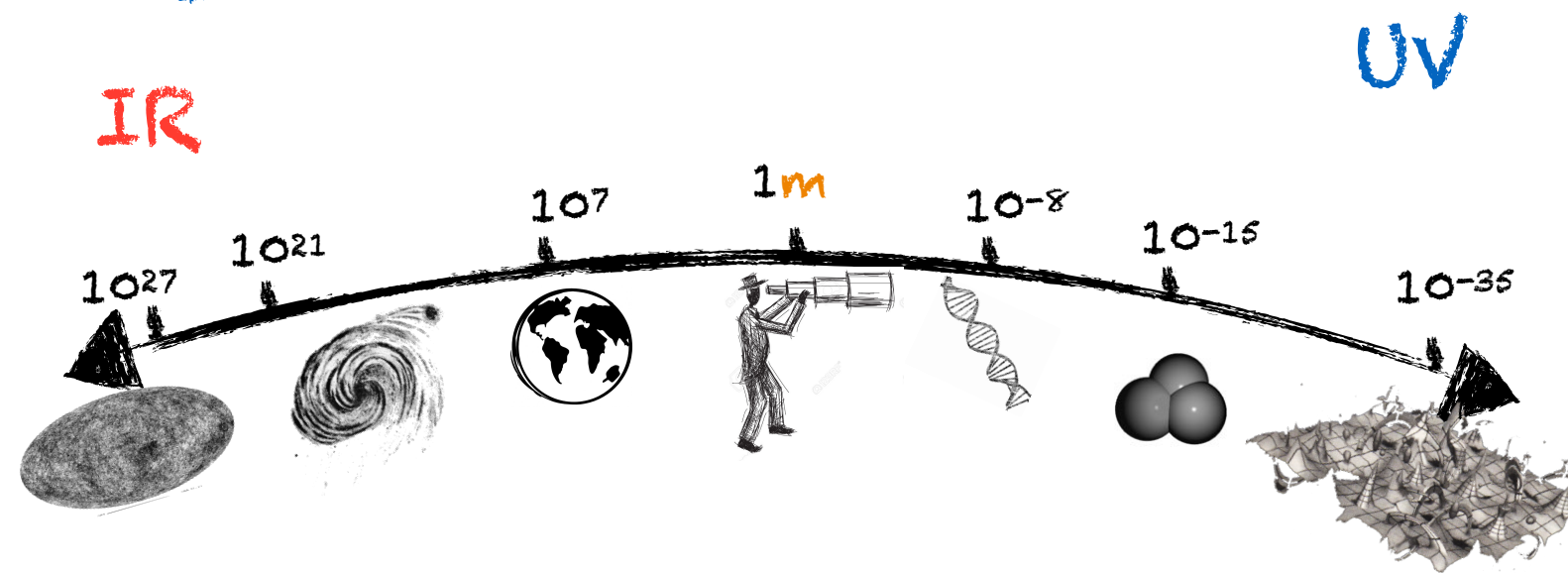




# Summary/Outlook



# Summary/Outlook



- Positivity  $\leftrightarrow$  UV completable EFTs
- Even with  $D=4$  Long-Range interactions bounds smoothly connected to tree level ones: leading IR captured by stripped amplitude
- Physical experiment size enters; bounds on very irrelevant operators washed away
- Zero coupling: (analytic) understanding of infinite spectra amplitudes?
- Finite coupling: progress beyond scaling limit  $\alpha, G, \mathcal{E} \rightarrow 0$  ?