COSMOLOGICAL GRASSMANNIAN

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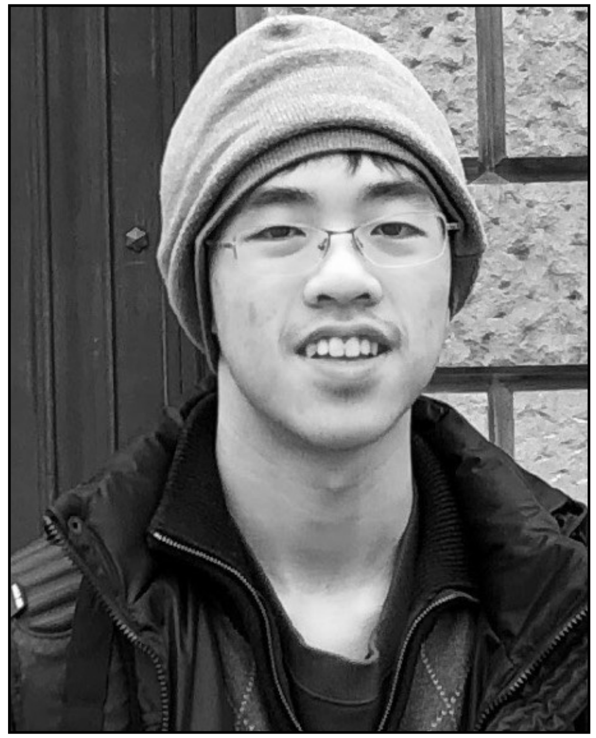
Based on work with

Mattia Arundine



Gordon Lee

Guilherme Pimentel

Facundo Rost

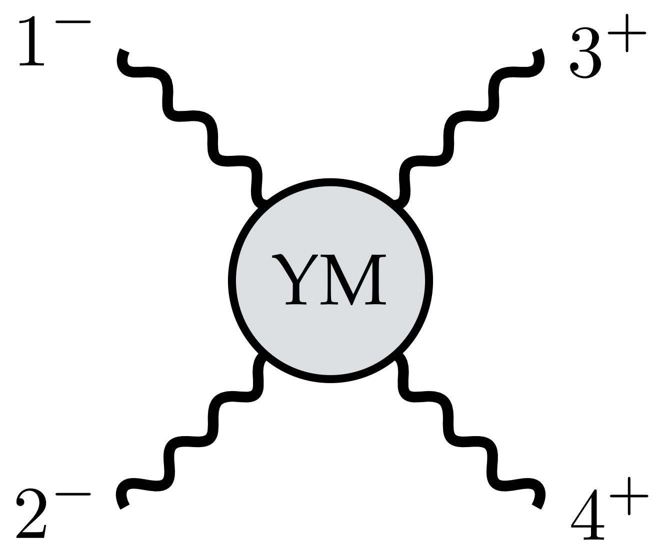
"The Cosmological Grassmannian" [2602.07117]

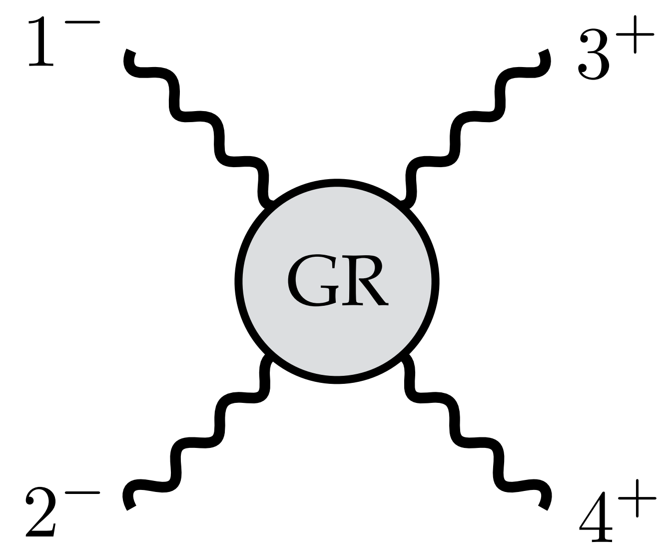
See also related work by:

Carrillo Gonzales and Keseman [2606.13770]
Arundine and Pimentel [2605.21581]
Bala, Jain and Singh [2606.25032]
Bala, Jain and Dhruva [2605.06811]
Bala, Jain, Dhruva and Rao [2604.07446]
Huang, Kuo, Liu and Mei [2604.08512]
De and Lee [2603.24656]
Ansari, Jain and Dhruva [2512.04172]
Carrillo Gonzales and Keseman [2510.00096]
DB, Mathys, Pimentel and Rost [2408.02727]

Challenge

Massless theories have **beautiful structure in flat space**:





Parke-Taylor

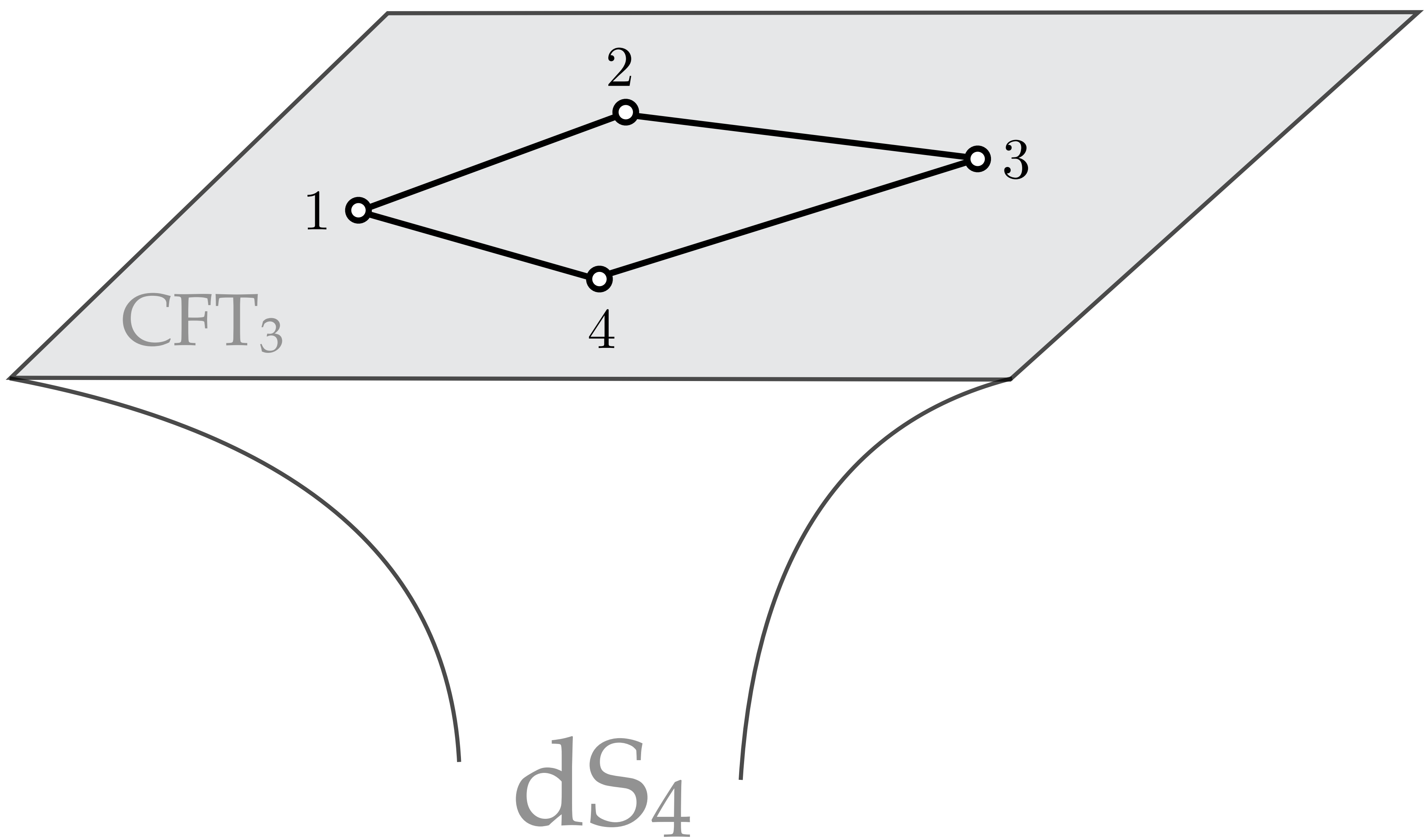
$$\mathcal{M} = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle (n-1) n \rangle}$$

De Witt

$$\mathcal{M} = \frac{(\langle 12 \rangle [34])^4}{stu}$$

Challenge

Is this also true in **curved spacetime**?



YES. If using the right kinematics !!

Main Result

The right variables are **twistors**:

$$\Psi_n(Z_a) = \int \mathrm{d}c_{ab}\, e^{i c_{ab} Z_a \cdot Z_b}\, A_n(c_{ab})$$

half-FT

Orthogonal Grassmannian

Beautiful

Momentum space:

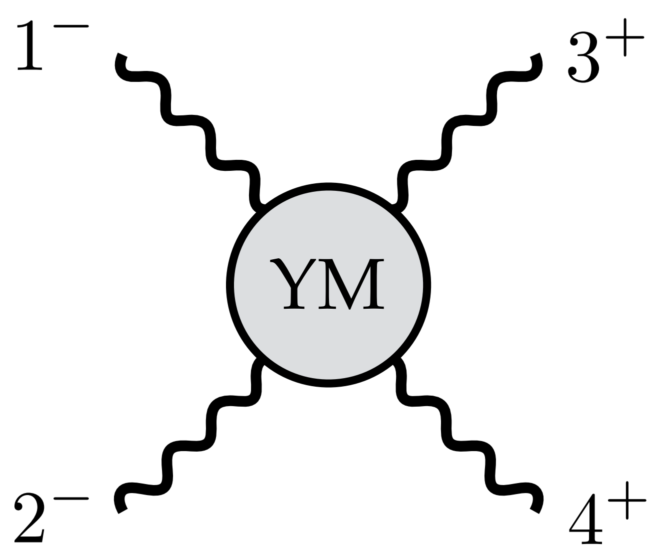
$$\Psi_n(\Lambda) = \int \mathrm{d}C\, \delta(C \cdot \Lambda)\, A_n(C)$$

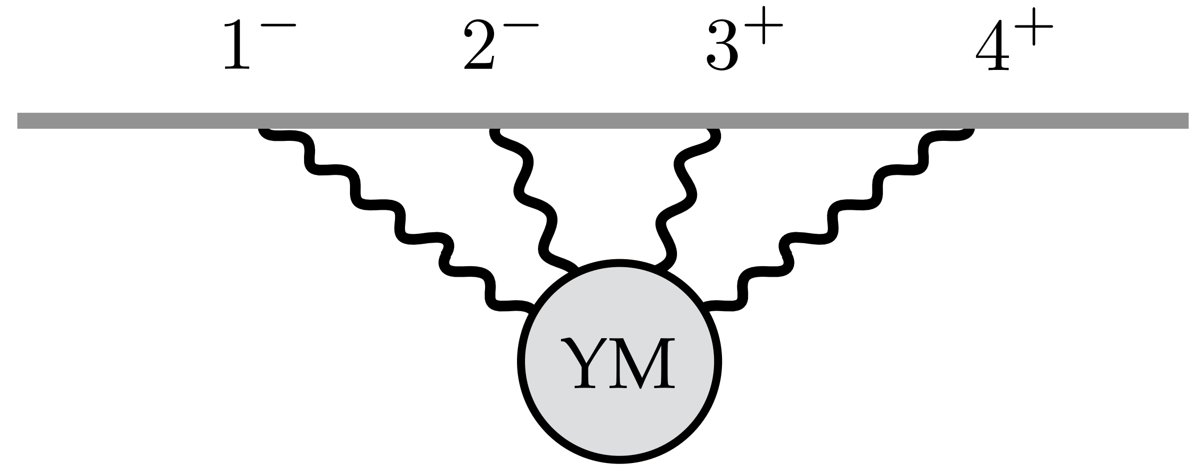
DB, Mathys, Pimentel and Rost [2025]

Arundine, DB, Lee, Pimentel and Rost [2026]

Main Result

Correlators drastically simplify in **Grassmannian space**:





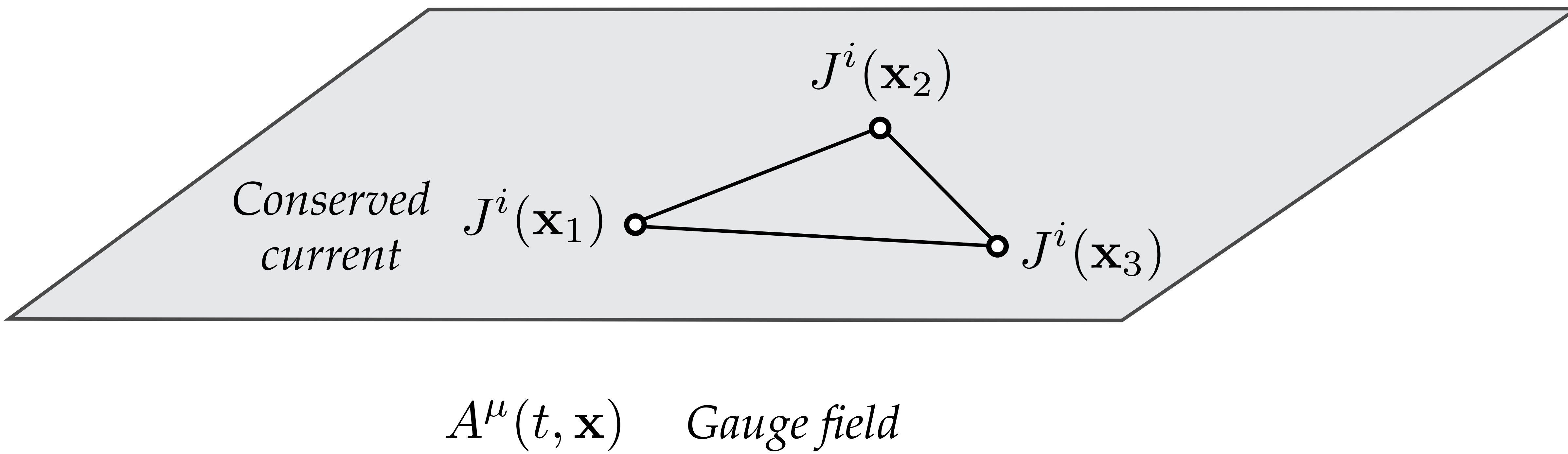
$$\mathcal{M}_4 = \frac{(\langle 12\rangle[34])^2}{st} \qquad A_4 = \frac{(12\bar{3}\bar{4})^2}{ST}\left[\frac{1}{S+T+U}+\frac{1}{S+T-U}\right]$$

Similar to amplitudes in flat space.

Kinematics

Boundary Correlators

We are interested in correlations on the **boundary of de Sitter space**:



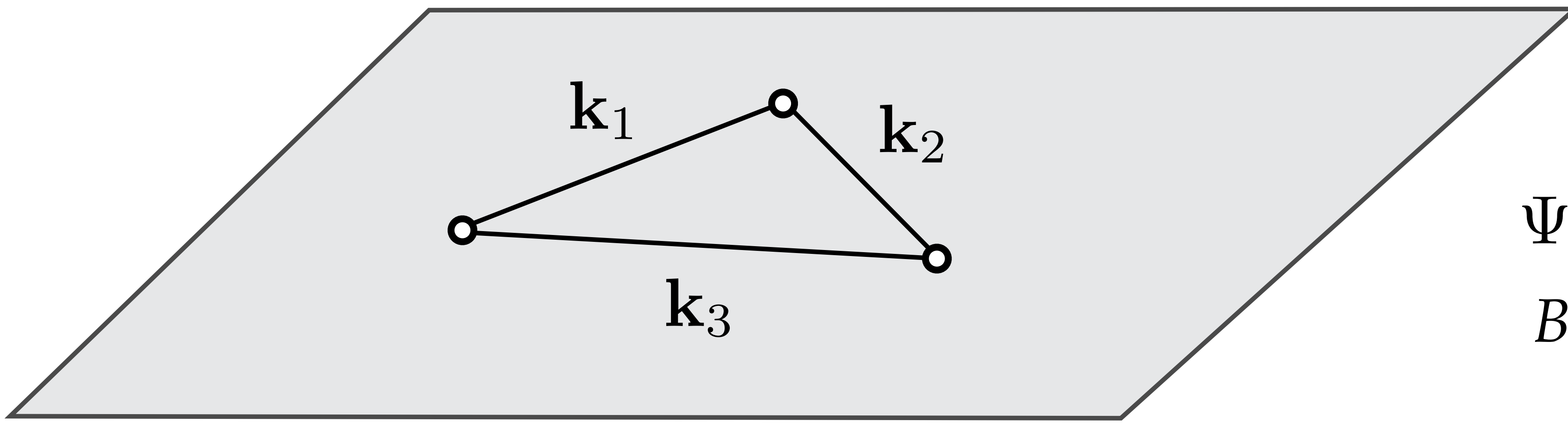
Like in AdS/CFT, bulk gauge fields are dual to **conserved currents**.

We wish to compute: $$\boxed{\Psi_n \equiv \langle J_1 J_2 \cdots J_n \rangle'}$$

These correlators are constrained by **conformal symmetry**.

Fourier Space

Cosmologists like to work in **Fourier space**.



$$\Psi_n \equiv \langle J_1 J_2 \cdots J_n \rangle'$$

Boundary correlators

However, **conformal symmetry** becomes a nontrivial constraint:

$$\boxed{\sum_{a=1}^{n} \left(\frac{k_a^i}{2} \frac{\partial^2}{\partial k_a^2} - k_a^j \frac{\partial^2}{\partial k_a^j \partial k_{a,i}} - 2 \frac{\partial}{\partial k_{a,i}} \right) \Psi_n = 0}$$

Are there better kinematic variables?

Spinor Helicity Variables

Express momentum vectors in terms of **spinors**:

$$\mathbf{k}, \boldsymbol{\epsilon}^{\pm} \longrightarrow k_i(\sigma^i)_{\alpha\beta} \equiv \lambda_{(\alpha}\bar{\lambda}_{\beta)}$$

3-momentum *spinors*

It is convenient to define

$$\Lambda \equiv \begin{pmatrix} \lambda_1^1 & \lambda_1^2 \\ \vdots & \vdots \\ \lambda_n^1 & \lambda_n^2 \\ \bar{\lambda}_1^1 & \bar{\lambda}_1^2 \\ \vdots & \vdots \\ \bar{\lambda}_n^1 & \bar{\lambda}_n^2 \end{pmatrix}$$

We then have

$$\boxed{\left(\partial_\Lambda^T \cdot Q \cdot \partial_\Lambda\right)\Psi_n = 0 \qquad \textit{Conformal symmetry}}$$

$$\boxed{\Psi_n \propto \delta(\Lambda^T \cdot Q \cdot \Lambda) \qquad \textit{Momentum conservation}}$$

$$Q \equiv \begin{pmatrix} 0 & 1_{n\times n} \\ 1_{n\times n} & 0 \end{pmatrix}$$

Can these constraints be solved simultaneously?

Orthogonal Grassmannian

Conformal Ward identity is a Laplace-like equation

$$\frac{\partial}{\partial \Lambda^T} \cdot Q \cdot \frac{\partial}{\partial \Lambda} \Psi_n = 0$$

Recall: 2D Laplace is solved by $\{\psi(x+iy)\,,\,\psi(x-iy)\} \equiv \psi(\vec{c}\cdot\vec{x})$

Try: $\Psi_n(C \cdot \Lambda)$

$n \times 2n$ matrix

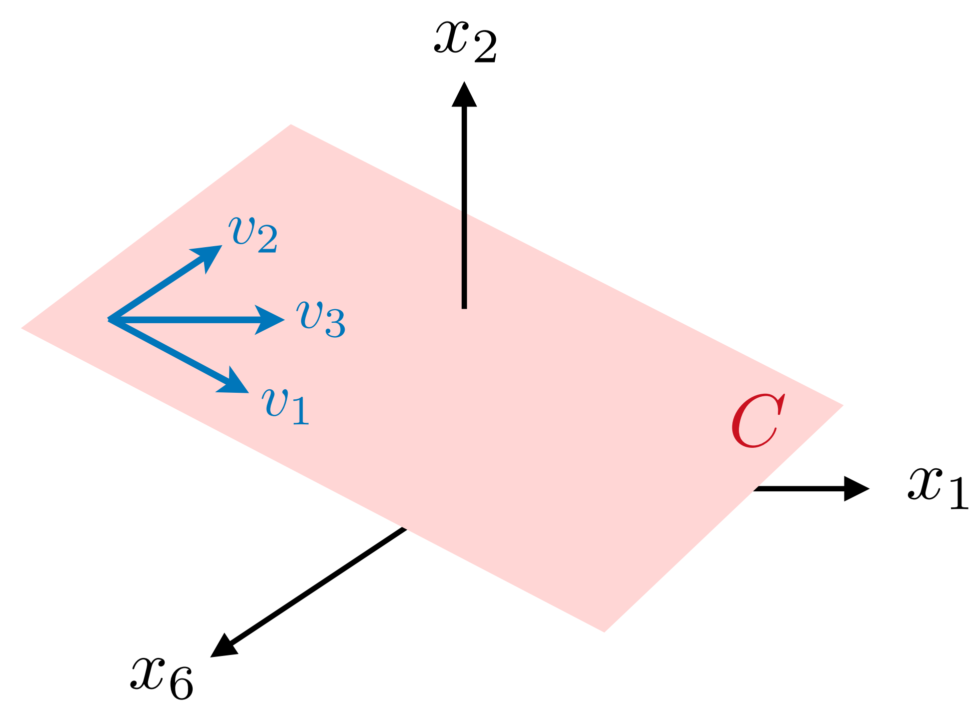
This is a solution if $\boxed{C \cdot Q \cdot C^T = 0}$ *Orthogonal Grassmannian* $\mathrm{OGr}(n, 2n)$

Orthogonal Grassmannian

Geometrically: $\mathrm{OGr}(n, 2n) \longrightarrow$ Null n-planes in $\mathbb{R}^{2n}$

Example: $\mathrm{OGr}(3,6)$

$$
\begin{array}{cc}
\begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} &
\begin{bmatrix}
1 & 0 & 0 & 0 & c_{12} & c_{13} \\
0 & 1 & 0 & -c_{12} & 0 & c_{23} \\
0 & 0 & 1 & -c_{13} & -c_{23} & 0
\end{bmatrix} \\
 & \begin{array}{cccccc} \bar{1} & \bar{2} & \bar{3} & 1 & 2 & 3 \end{array}
\end{array}
$$



Each matrix is defined up to an GL(n) transformation.

Integrating over the Grassmannian gives

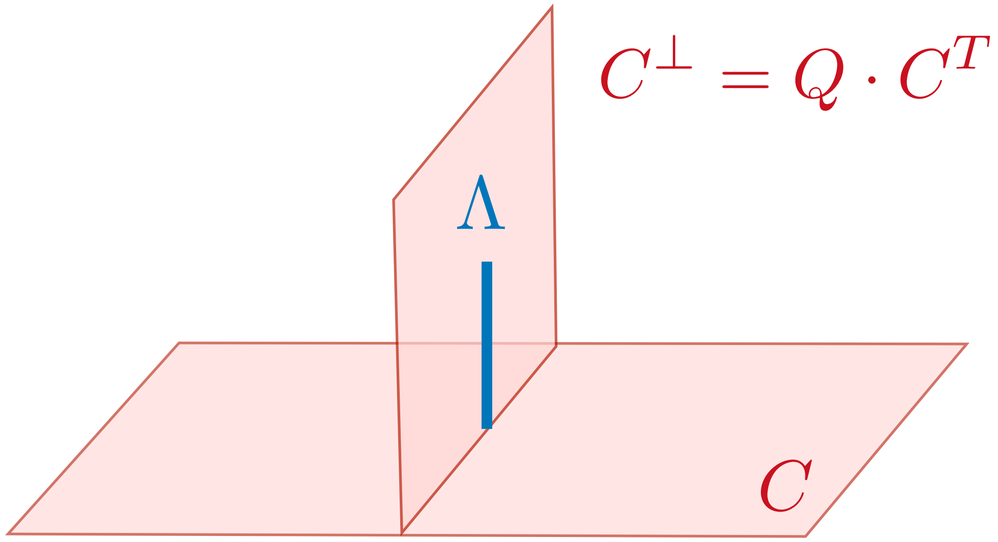
$$
\boxed{\Psi_n(\Lambda) = \int \frac{\mathrm{d}C}{\mathrm{GL}(n)}\, \delta(C \cdot Q \cdot C^T)\, \Psi_n(C \cdot \Lambda)}
$$

Momentum Conservation

What about momentum conservation? $\Psi_n \propto \delta(\Lambda^T \cdot Q \cdot \Lambda)$

Let: $\Psi_n(C \cdot \Lambda) = \delta(C \cdot \Lambda) A_n(C)$

Orthogonal 2-planes



$$\Lambda \equiv \begin{array}{c c} \begin{array}{cc} v_1 & v_2 \end{array} & \\ \begin{bmatrix} \lambda_1^1 & \lambda_1^2 \\ \vdots & \vdots \\ \lambda_n^1 & \lambda_n^2 \\ \bar{\lambda}_1^1 & \bar{\lambda}_1^2 \\ \vdots & \vdots \\ \bar{\lambda}_n^1 & \bar{\lambda}_n^2 \end{bmatrix} & \begin{array}{c} 1 \\ \vdots \\ n \\ \bar{1} \\ \vdots \\ \bar{n} \end{array} \end{array}$$

Grassmannian Correlators

We have arrived at an integral representation of conformal correlators:

$$\Psi_n(\Lambda) = \int \mathrm{d}C \; \delta(C \cdot Q \cdot C^T)\, \delta(C \cdot \Lambda)\, A_n(C)$$

Conformal symmetry

Momentum conservation

Grassmannian Correlators

We have arrived at an integral representation of conformal correlators:

$$\Psi_n(\Lambda) = \int \mathrm{d}C \; \delta(C \cdot Q \cdot C^T)\, \delta(C \cdot \Lambda)\, A_n(C)$$

Momentum space

Ugly

Grassmannian space

Beautiful

Grassmannian Correlators

We have arrived at an integral representation of conformal correlators:

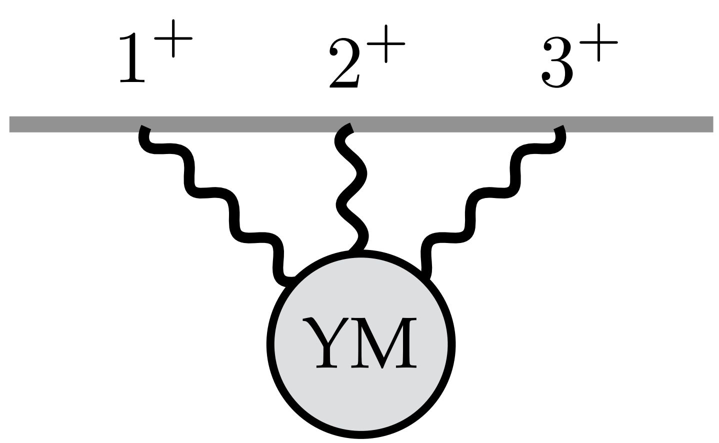
$$\Psi_n(\Lambda) = \int \mathrm{d}C \; \delta(C \cdot Q \cdot C^T)\, \delta(C \cdot \Lambda)\, A_n(C)$$

Momentum space

Grassmannian space

Beautiful

Three-point functions are fixed by kinematics



$$= \frac{1}{c_{12}c_{23}c_{31}}$$

For higher-point functions, we need extra input from dynamics.

Dynamics

Mandelstam Variables

At four points, the correlator depends on Mandelstam-like variables:

$$\begin{aligned} S &\equiv c_{14}c_{23} - c_{13}c_{24} = (1\bar{1}2\bar{2}) \\ T &\equiv c_{12}c_{34} - c_{13}c_{24} = (1\bar{1}4\bar{4}) \\ U &\equiv c_{12}c_{34} + c_{14}c_{23} = (1\bar{1}3\bar{3}) \end{aligned}$$

Minors

$$C = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & c_{12} & c_{13} & c_{14} \\ 0 & 1 & 0 & 0 & -c_{12} & 0 & c_{23} & c_{24} \\ 0 & 0 & 1 & 0 & -c_{13} & -c_{23} & 0 & c_{34} \\ 0 & 0 & 0 & 1 & -c_{14} & -c_{24} & -c_{34} & 0 \end{bmatrix} \\ \begin{array}{cccccccc} \bar{1} & \bar{2} & \bar{3} & \bar{4} & 1 & 2 & 3 & 4 \end{array} \end{array} \in \mathrm{OGr}(4,8)$$

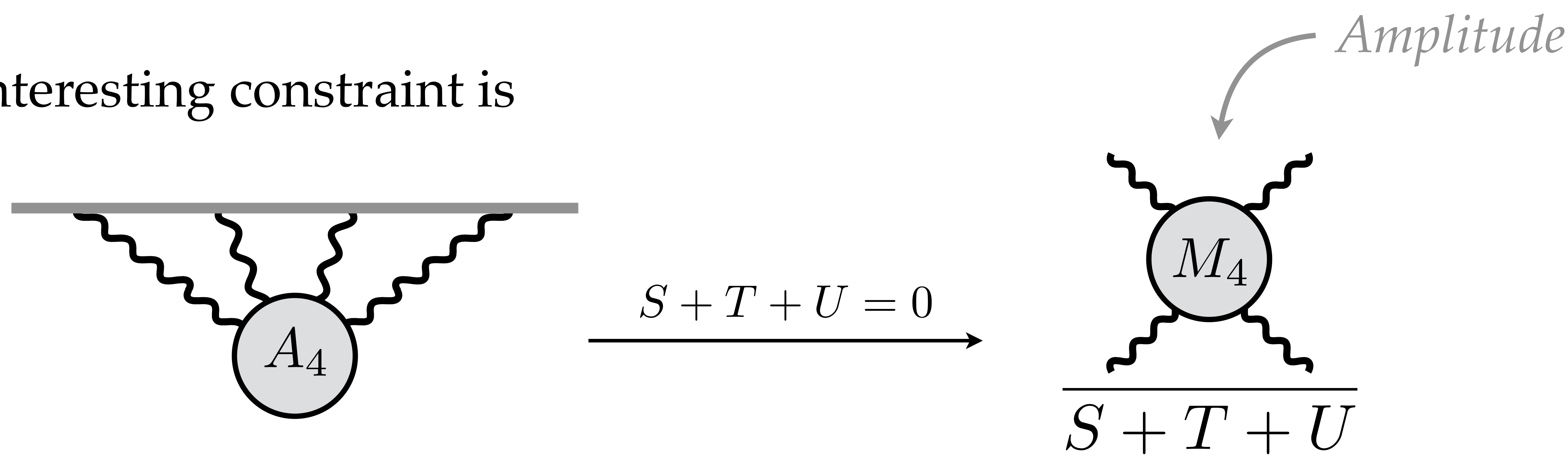
Mandelstam Variables

At four points, the correlator depends on Mandelstam-like variables:

$$
\begin{aligned}
S &\equiv c_{14}c_{23} - c_{13}c_{24} = (1\bar{1}2\bar{2}) \\
T &\equiv c_{12}c_{34} - c_{13}c_{24} = (1\bar{1}4\bar{4}) \\
U &\equiv c_{12}c_{34} + c_{14}c_{23} = (1\bar{1}3\bar{3})
\end{aligned}
$$

Unlike in flat space: $\quad S + T + U \neq 0$

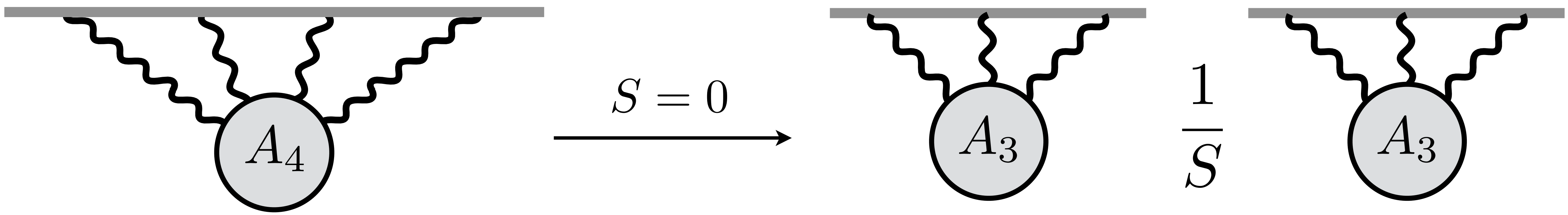
An interesting constraint is



We call this the **flat-space limit**.

Factorization

Further important constraints are the **factorization limits**:



- Identical to the factorization of amplitudes
- Equivalent to unitarity cuts in momentum space

Goodhew, Jazayeri, Lee and Pajer [2021]

We can build four-point functions by **gluing three-point functions**.

Arundine, DB, Lee, Pimentel and Rost [2026]

Yang-Mills

Factorization and flat-space limits together are highly constraining.

For **color-ordered Yang-Mills**, we find

$$\boxed{A_4^{--++} = \frac{(12\bar{3}\bar{4})^2}{ST}\left[\frac{3}{\mathcal{E}} - \frac{1}{\mathcal{E}_S} - \frac{1}{\mathcal{E}_T} + \frac{1}{\mathcal{E}_U}\right]}$$

where $\mathcal{E} \equiv S + T + U\,, \quad \mathcal{E}_S \equiv T + U - S\,,$

$$\mathcal{E}_T \equiv S + U - T\,,$$
$$\mathcal{E}_U \equiv S + T - U\,.$$

Integration produces the correct momentum-space result.*

Arundine, DB, Lee, Pimentel and Rost [2026]

Yang-Mills

It is also useful to write the result as a sum of **reduced correlators**:

$$A_S = \frac{1}{ST}\left(\frac{1}{\mathcal{E}} - \frac{1}{\mathcal{E}_S}\right)$$

$$A_T = \frac{1}{ST}\left(\frac{1}{\mathcal{E}} - \frac{1}{\mathcal{E}_T}\right)$$

$$A_U = \frac{1}{ST}\left(\frac{1}{\mathcal{E}} + \frac{1}{\mathcal{E}_U}\right) = \frac{1}{ST}\left(\frac{1}{S+T+U} + \frac{1}{S+T-U}\right)$$

These are the building blocks of the Yang-Mills correlator.

Arundine, DB, Lee, Pimentel and Rost [2026]

Gravity

Performing the same analysis for gravity, we find

$$\boxed{A_4^{--++} = \frac{(12\bar{3}\bar{4})^4}{STU}\left[\frac{3}{\mathcal{E}^2} - \frac{1}{\mathcal{E}_S^2} - \frac{1}{\mathcal{E}_T^2} - \frac{1}{\mathcal{E}_U^2}\right]}$$

This is radically simpler than the known momentum-space result.

Arundine, DB, Lee, Pimentel and Rost [2026]
Bala, Jain and Singh [2026]
Bonifacio, Goodhew, Joyce, Pajer and Stefanyszyn [2022]

Geometry

Based on work of

Mattia Arundine

Veronica Calvo Cortes

Joris Koefler

Facundo Rost

“Positive Geometry of Yang-Mills Correlators” [to appear]

Positive Orthogonal Grassmannian

Write the orthogonal Grassmannian in **Pfaffian coordinates**:

$$(p : p_{12} : p_{13} : p_{23})$$

$$(p : p_{12} : p_{34} : p_{13} : p_{24} : p_{14} : p_{23} : p_{1234})$$

c_{ab} $\qquad$ $p_{1234} \equiv (p_{12}p_{34} - p_{13}p_{24} + p_{14}p_{23})/p$

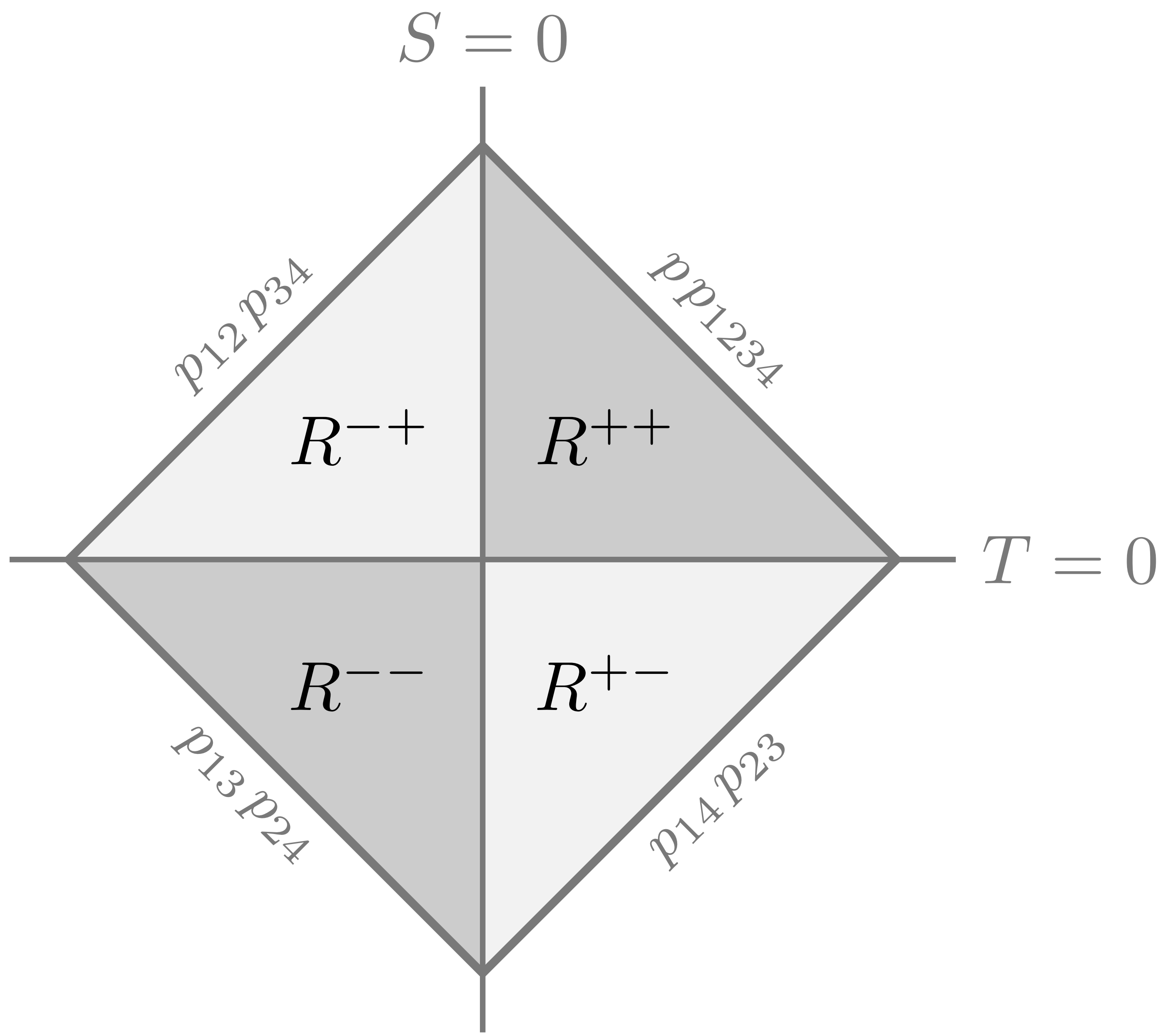
Consider the **positive region**: $p, p_{ab}, p_{1234} > 0$

- At three points, the **canonical form** of this region gives the correlator:

$$\Omega(R) = \frac{1}{p_{12}p_{23}p_{31}} \mathrm{d}^3 p_{ab} \equiv A_3(p_{ab})\, \mathrm{d}^3 p_{ab}$$

Yang-Mills From Geometry

- At four points, the Mandelstams give additional boundaries.



Yang-Mills four-point function arises as a **canonical form**:

$$\Omega(R^{++} \cup R^{--}) = A_U(S,T,U)\,\mathrm{d}^6 p_{ab}$$

Arundine, Calvo Cortes, Koefler and Rost [2026]

Outlook

Future Directions

We have finally found the right kinematic space to describe massless spinning fields in de Sitter space.

Many open problems remain:

- Extension to higher points (recursion)
- Extension to gravity (double copy)
- Connections to positive geometry

Supported by ERC:

Thanks for your attention.