Cosmological Dressing Rules

Arthur Lipstein

Durham University

Based on:

2312.13803 with C. Chowdhury, J. Mei, I. Sachs, P. Vanhove

2503.10598 with C. Chowdhury, J. Mei, J. Marshall, I. Sachs

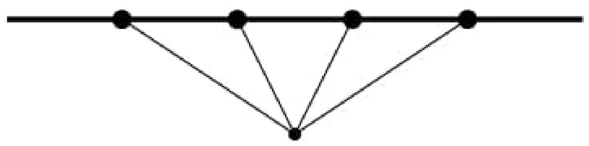
2602.03841 with C. Chowdhury, S. Jazayeri, J. Marshall, J. Mei, I. Sachs

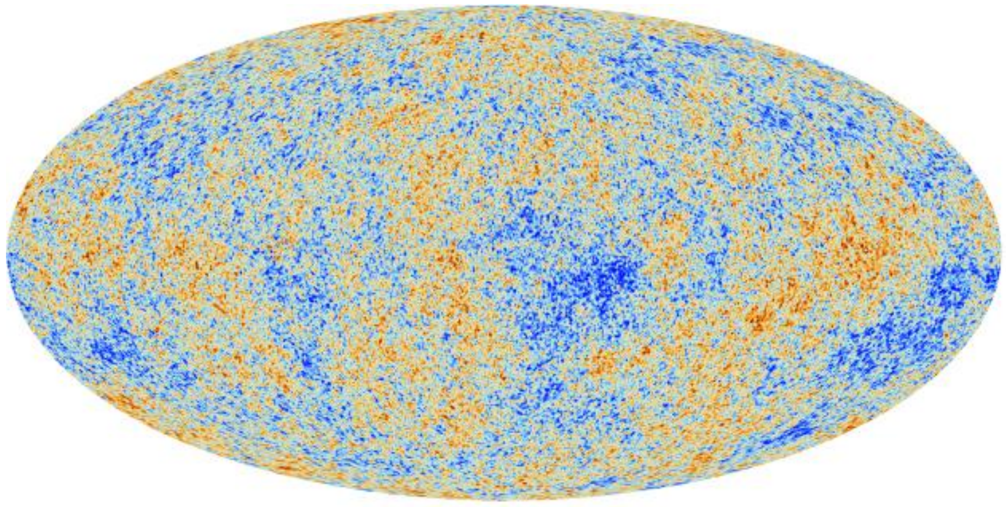
Cosmological Observables

- Inflation: early Universe approximately described by dS_4.
  CMB comes from correlations on future boundary:

$$ds^2 = \frac{-d\eta^2 + (dx^i)^2}{\eta^2}, \qquad -\infty < \eta < 0$$

η=0





Cosmological Correlators

- In-in correlators (Maldacena/Weinberg):

$$\left\langle \phi(\vec{k}_1)\ldots\phi(\vec{k}_n) \right\rangle = \frac{\int \mathcal{D}\phi \, \phi(\vec{k}_1)\ldots\phi(\vec{k}_n) \left|\Psi\left[\phi\right]\right|^2}{\int \mathcal{D}\phi \left|\Psi\left[\phi\right]\right|^2}$$

- Wavefunction:

$$\ln \Psi\left[\phi\right] = -\sum_{n=2}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \psi_n \left(\vec{k}_1, \ldots \vec{k}_n\right) \phi(\vec{k}_1)\ldots\phi(\vec{k}_n)$$

- Can be computed from Feynman diagrams in EAdS (Maldacena,Pimentel/McFadden,Skenderis)

Punchline:

- In-in correlators exhibit surprising mathematical simplicity
- They can be directly uplifted from flat space Feynman diagrams
- These properties are often obscured by traditional approaches
- For related work see Donath,Pajer/Arkani-Hamed,Glew,Vazão/Das,Karan,Khatun,Kundu/Ansari,Jain,Mazumdar,Thakkar

Shadow Formalism

- Scalar in-in correlators can be computed from Feynman diagrams in EAdS using the following Lagrangian:

$$
\begin{aligned}
iS_c = \int_0^\infty \frac{\mathrm{d}z\mathrm{d}^d x}{z^{d+1}} \Bigg[&\sin\left(\pi(\Delta_+ - \frac{d}{2})\right)\left((\partial\phi_+)^2 - m^2{\phi_+}^2\right) \\
&+ \sin\left(\pi(\Delta_- - \frac{d}{2})\right)\left((\partial\phi_-)^2 - m^2{\phi_-}^2\right) \\
&+ e^{i\pi\frac{d-1}{2}} V\left(e^{-i\frac{\pi}{2}\Delta_+}\phi_+ + e^{-i\frac{\pi}{2}\Delta_-}\phi_-\right) + e^{-i\pi\frac{d-1}{2}} V\left(e^{i\frac{\pi}{2}\Delta_+}\phi_+ + e^{i\frac{\pi}{2}\Delta_-}\phi_-\right) \Bigg]
\end{aligned}
$$

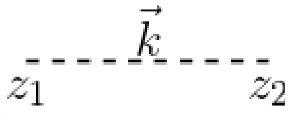
(Sleight,Taronna/Di Pietro,Gorbenko,Komatsu)

- Only ϕ_- appears on external lines

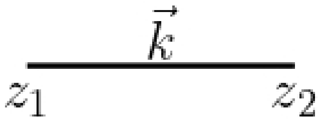
Conformally coupled φ^4

- Choose d=3 and $\Delta_-=d-\Delta_+=1$: $V(\phi_+,\phi_-) = \frac{1}{2}\frac{\lambda}{4!}\left(\phi_+^4 - 6\phi_+^2\phi_-^2 + \phi_-^4\right)$

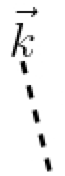
- Feynman rules: (ϕ_- dotted, ϕ_+ solid)



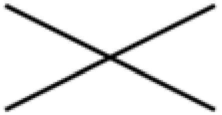
$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{dp}{p^2+\vec{k}^2}\cos(pz_1)\cos(pz_2)$$



$$= -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{dp}{p^2+\vec{k}^2}\sin(pz_1)\sin(pz_2)$$



$$= \quad e^{-kz}$$

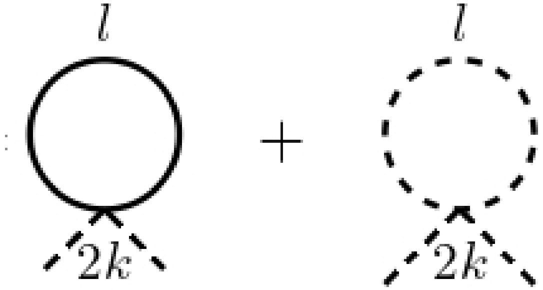


$$= \frac{\lambda}{2},$$

$$= -\frac{\lambda}{2},$$

$$= \frac{\lambda}{2}$$

1-loop 2-point

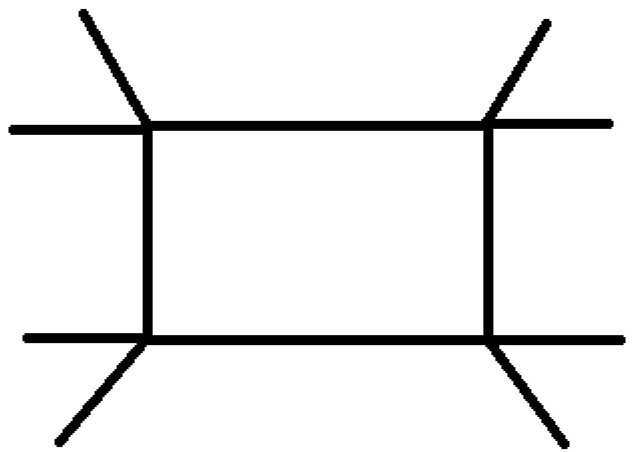


$$= \lambda \int \frac{d^3 l dp}{(2\pi)^4} \frac{1}{p^2 + l^2} \int_0^\infty dz e^{-2kz} \left(\sin^2(pz) + \cos^2(pz) \right)$$

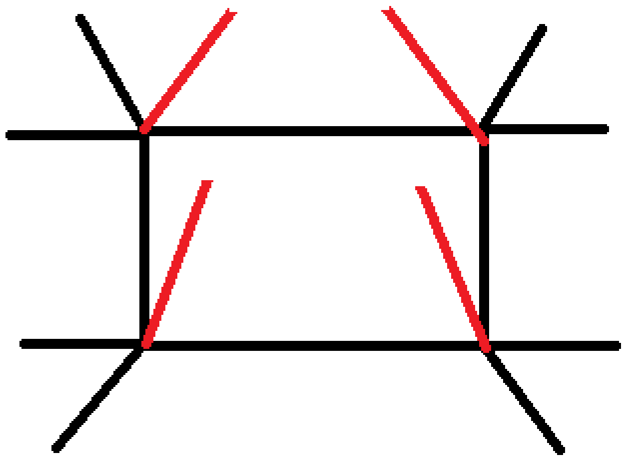
$$= \frac{\lambda}{k} \int \frac{d^4 L}{(2\pi)^4} \frac{1}{L^2} \,, \quad L^\mu = \left(p, \vec{l} \right)$$

- 4d Lorentz invariance is restored
- Doesn’t happen for wavefunction

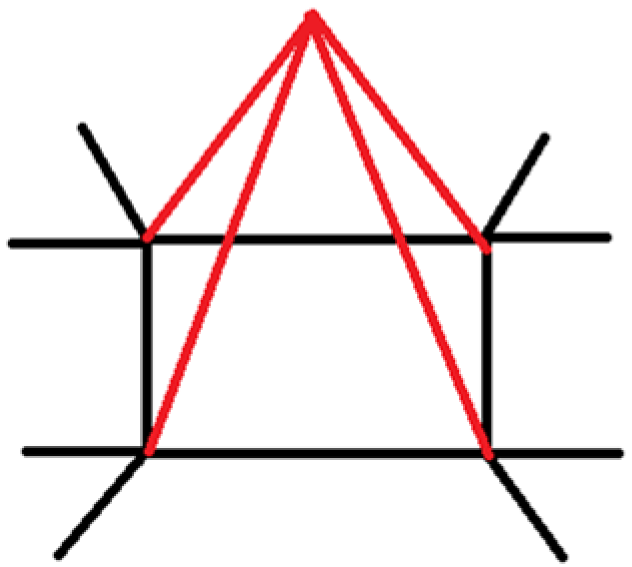
- How do we make simplicity manifest?
- Start with a flat space Feynman diagram



- How do we make simplicity manifest?
- Start with a flat space Feynman diagram
- Add an auxiliary propagator at each vertex

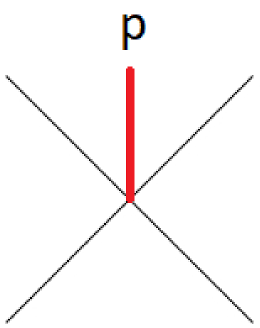


- How do we make simplicity manifest?
- Start with a flat space Feynman diagram
- Add auxiliary propagator at each vertex
- Connect auxiliary propagators to a common point
- Impose internal energy conservation at each vertex
- Integrate over auxiliary energies



Dressing Rules

- For conformally coupled ϕ^4, the auxiliary propagator is:

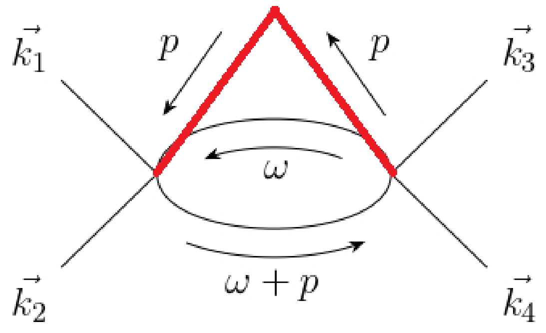


$$\frac{k_{ext}}{p^2 + k_{ext}^2}$$

- k_{ext} = total energy of external legs attached to vertex

- p = total energy of internal legs attached to vertex

Example:



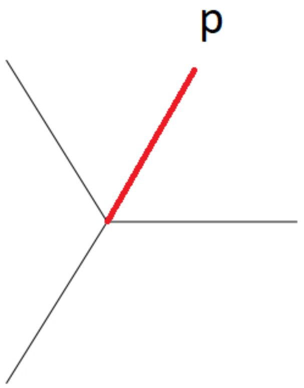
$$= \left(\frac{\lambda}{2}\right)^2 \int \frac{1}{(2\pi)^5} dp\, d\omega\, d^3 l \left(\frac{k_{12}}{p^2 + k_{12}^2}\right) \left(\frac{k_{34}}{p^2 + k_{34}^2}\right) \left(\frac{1}{\omega^2 + \vec{l}^2}\right) \left(\frac{1}{(\omega + p)^2 + \left(\vec{l} + \vec{k}_{12}\right)^2}\right)$$

$$= \left(\frac{\lambda}{2}\right)^2 k_{12} k_{34} \int \frac{dp}{(2\pi)^5} \frac{1}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \int \frac{d^4 L}{L^2 (L - P)^2},$$

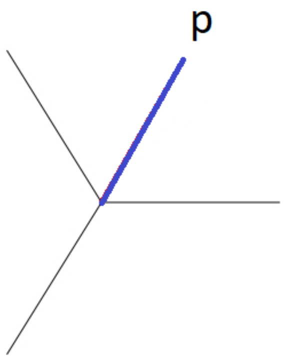
- This gives a log. In contrast, wavefunction is a dilog (Albayrak,Chowdhury,Kharel)

Dressing rules for cc φ³

- Now there are two types of auxiliary props:



$$\int_0^\infty ds \, \frac{p}{p^2 + (s + k_{\text{ext}})^2}$$

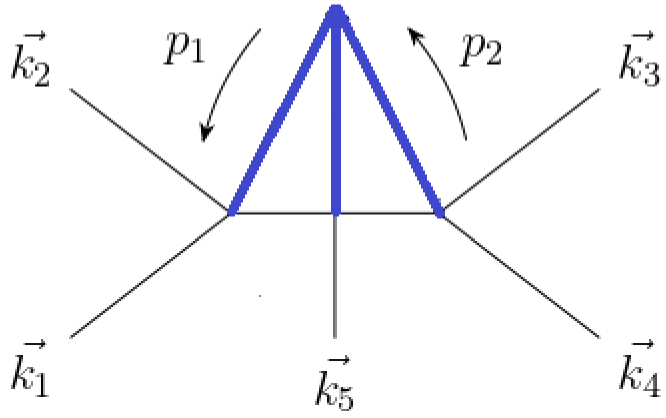


$$-\pi$$

- Need an even number of red ones

Example: 5-point tree

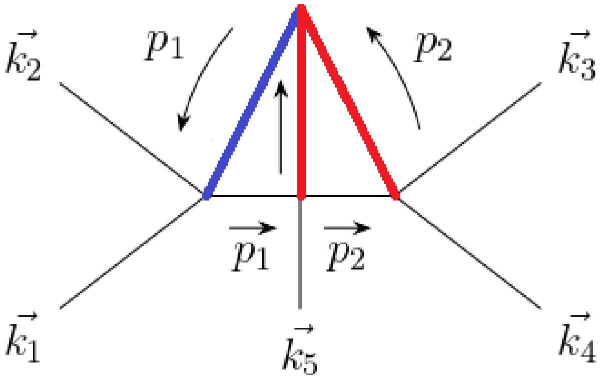
- There are two types of dressed diagrams:



$$= -\lambda^3 \pi \int_{-\infty}^{\infty} dp_1\, dp_2 \frac{1}{(p_1^2 + y_{12}^2)(p_2^2 + y_{34}^2)} = -\frac{\lambda^3 \pi^3}{y_{12} y_{34}}$$

where $\vec{y}_{ij} = \vec{k}_i + \vec{k}_j \,, \;\; y = |\vec{y}_{ij}|$

- Two red and one blue:



$$= \frac{\lambda^3}{\pi} \int_0^\infty ds_2\, ds_3 \int_{-\infty}^\infty dp_1\, dp_2 \frac{p_1 - p_2}{(s_2 + k_5)^2 + (p_1 - p_2)^2} \frac{p_2}{(s_3 + k_{34})^2 + p_2^2} \frac{1}{p_1^2 + y_{12}^2} \frac{1}{p_2^2 + y_{34}^2}$$

- This gives a dilog. In contrast, the wavefunction is a trilog (Hillman)

- Transcendentality = number of red props

Correlator Discontinuities

- After Wick-rotating to Lorentzian signature, a generic dressed diagram in cc ϕ^4 takes the form

$$C_n(\{x\},\{\vec{y}\}) = \int_{-\infty}^{\infty} \prod_{i=1}^{v} \frac{dp_i \delta(\sum_j p_j)}{p_i^2 - x_i^2 + i\epsilon} A_n(\{p\};\{\vec{y}\})$$

  where A_n is a flat space Feynman diagram with v vertices and we strip off a factor of $x_1 \ldots x_v$

- Discontinuity of a function f(z) across the real line:

$$\mathrm{disc}_z f(z) \equiv \frac{1}{2\pi i} \lim_{\epsilon \to 0^+} f(z+i\epsilon) - f(z-i\epsilon)$$

- x discontinuities → cuts of auxiliary propagators:

$$\text{disc}_{x_1^2} C_n = \int \prod_{i=2}^{v} \frac{dp_i dp_1 \delta(p_1^2 - x_1^2)}{p_i^2 - x_i^2 + i\epsilon} \delta\left(\sum p_j\right) A_n(\{p\}, \{\vec{y}\}),$$

- y discontinuities → flat space cuts:

$$\text{disc}_{y_1^2} C_n = \left(\int \prod_{i=1}^{v} \frac{dp_i \delta(\sum_j p_j)}{p_i^2 - x_i^2 + i\epsilon} \right) \text{disc}_{y_1^2} A_n(\{p\}, \{\vec{y}\})$$

Note that A_n is a function of Lorentz-invariants of the form p^2-y^2

- **Summary**: correlator discontinuities can be uplifted from flat space!

Sum Rules

- Cutting all auxiliary propagators gives flat space limit:

$$\mathrm{disc}_{x_v^2} \cdots \mathrm{disc}_{x_1^2} C_n \propto \delta(E) A_n(\{x\}; \{\vec{y}\})$$

where E is total energy. For generic kinematics, RHS vanishes.

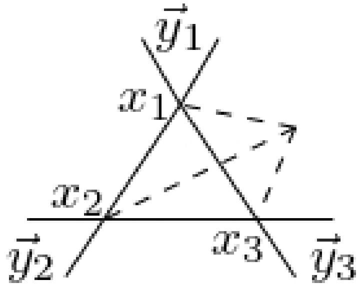
- On the other hand, this equivalent to summing over ±x:

$$\sum_{\{s\}=\pm 1} \Big(\prod_i s_i\Big) C_n(s_1 x_1, s_2 x_2, \cdots, s_v x_v; \{\vec{y}\}) = 0$$

Example: 1-loop triangle

Dressed diagram:

$$C_6^{\Delta} =$$



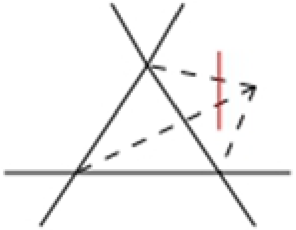
$$= \int_{-\infty}^{\infty} dp_1 dp_2 \frac{1}{(p_1^2 - x_1^2)(p_2^2 - x_2^2)\big((p_1+p_2)^2 - x_3^2\big)} A_6$$

where

$$A_6 = \int d^4 L \frac{1}{L^2 (L+P_1)^2 (L+P_2)^2} \quad , \qquad P_1^{\mu} \ = \ (p_1, \vec{y}_1), \ \ P_2^{\mu} \ = \ (p_2, \vec{y}_2)$$

- Partial sum rule (full sum rule follows trivially from this):

$$\mathrm{disc}_{x_1^2}\mathrm{disc}_{x_2^2} C_6^{\Delta} =$$



$$= \frac{4}{x_1 x_2}\left[\frac{A_6(x_1,x_2)}{x_3^2-(x_1+x_2)^2}+\frac{A_6(x_1,-x_2)}{x_3^2-(x_1-x_2)^2}\right]$$

$$= \sum_{\{s\}=\pm 1} s_1 s_2 C_6^{\Delta}(s_1 x_1, s_2 x_2, x_3; \{\vec{y}\})$$

- Suggests that 1-loop triangle can be written in terms of dilogs, which is consistent with recent computations (Pimentel,Westerdijk)

- In contrast, wavefunction coefficient is an elliptic polylog (Benincasa,Brunello,Mandal,Mastrolia,Vazão)

Future directions

- Differential equations for correlators? (recent progress for wave-functions by Arkani-Hamed,Baumann,Hillman,Joyce,Lee,Pimentel)

- Dressing rules for spinning fields? (recent progress by Chowdhury,Lipstein,Marshall,Zhang)

- All-multiplicity formula for in-in correlators? (recent progress for scalar wavefunctions by Ferro,Lukowski,Ren,Spradlin,Volovich,Weng,Zhang and gluon correlators in AdS_4 by Gomez,Lipinski Jusinskas,Lipstein, Lopez-Arcos)

- de Sitter holography from flat space holography?