Energy correlators at finite coupling

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Acknowledgments

- Energy correlators from correlation functions

  [work with Belitsky,Chen,Chicherin,Henn,Hohenegger,Karlsson,Korchemsky,Sokatchev,Yan '13-...]

- OPE methods for energy correlators

  [work with Chang,Kologlu,Kravchuk,Simmons-Duffin '19-20]

- Conformal collider bootstrap

  [work with Dempsey,Karlsson,Pufu,Zahraee '25-wip]

- Positivity bootstrap

  [work with Belin,Borak,Henriksson,Piron wip]

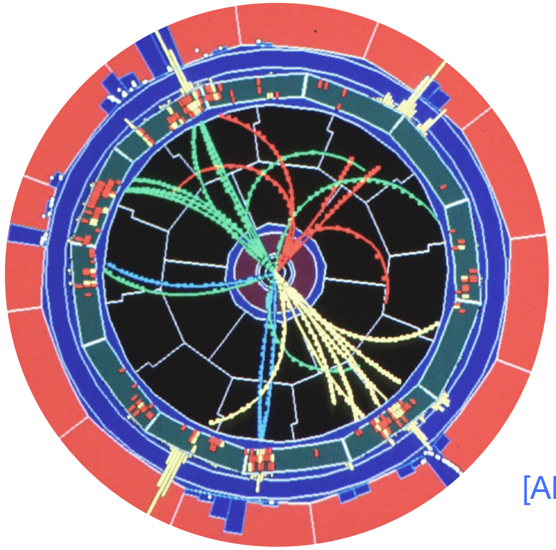
Energy correlators

Event shapes characterize the energy-momentum flow in a collision event.

$$\mathrm{EEC}(z) = \frac{1}{\sigma_{\mathrm{tot}}}\frac{d\Sigma}{dz} = \left\langle \sum_{i,j} \frac{E_i E_j}{Q^2}\,\delta(z - z_{ij}) \right\rangle_{\mathrm{events}}$$

[Basham,Brown,Ellis,Love '78]

- average over final states (events)
- weighted by normalized energy
- function of angular separation $z = \dfrac{1-\cos\theta}{2}$
- infrared and collinear safe
- perturbatively computable (at high energies)

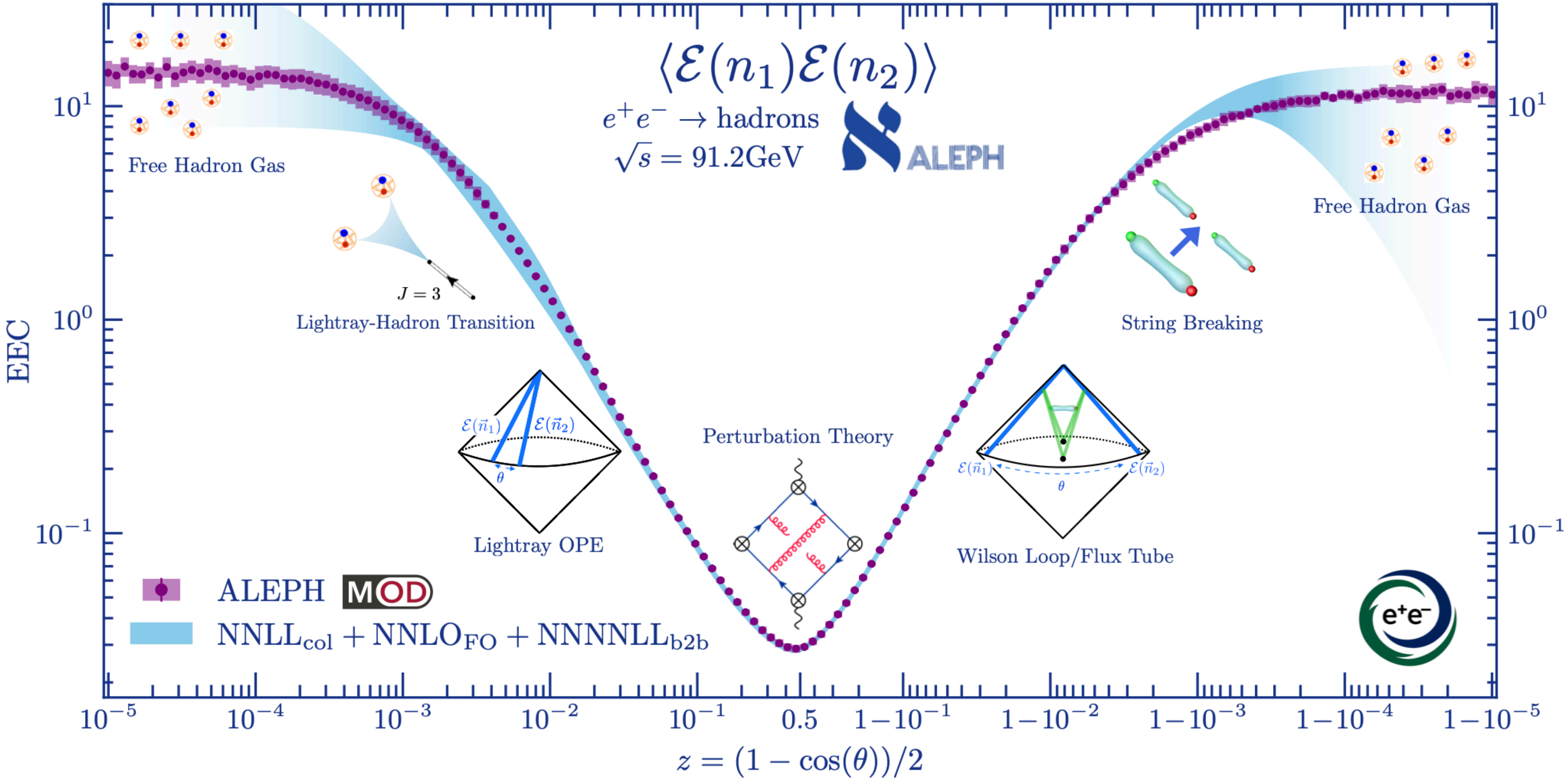


[ALEPH]

This talk: energy correlators at finite coupling (N=4 SYM), when perturbation theory fails.

QCD across angles in $e^+e^- \to$ hadrons

The angular transition between perturbative and hadronic physics

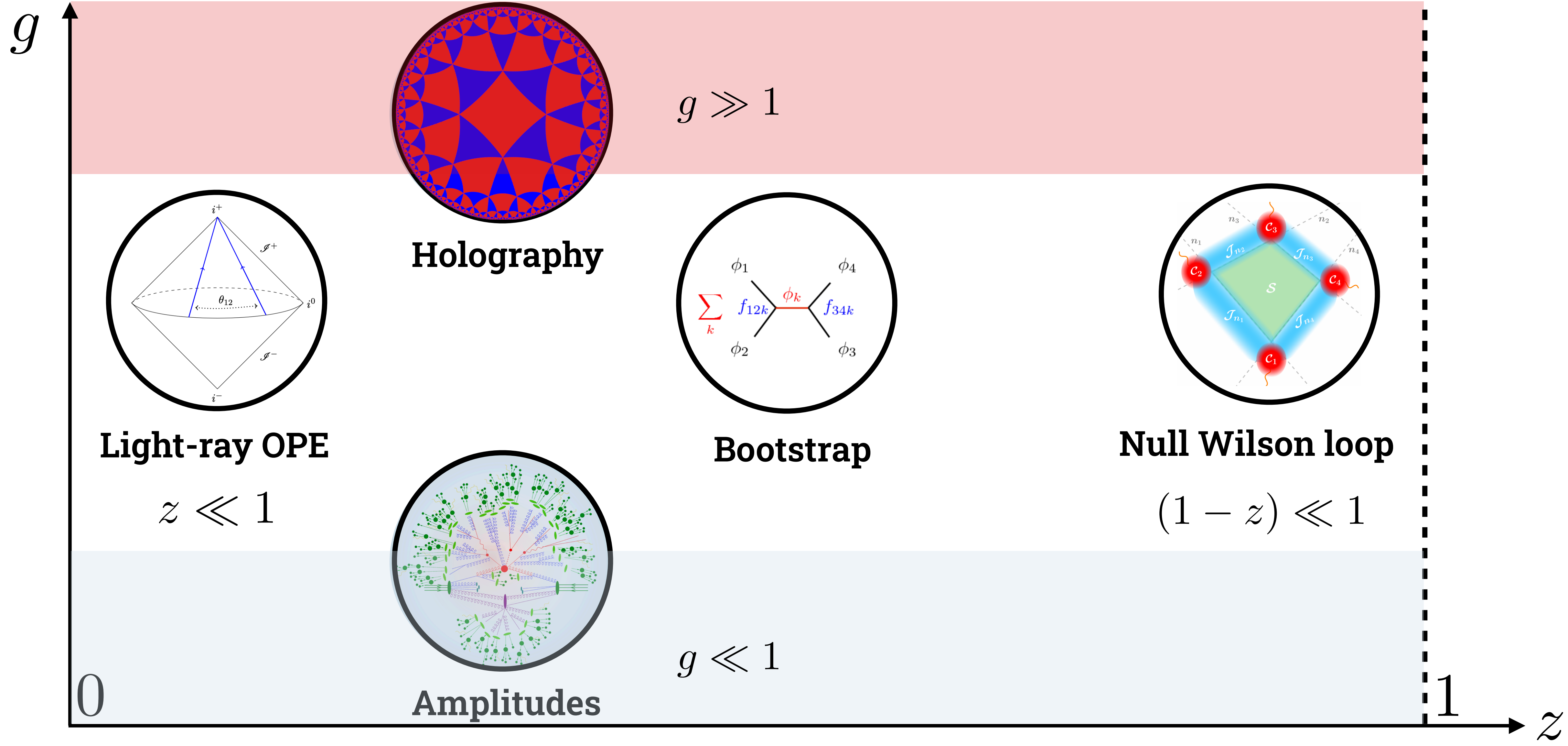


["Energy Correlators: A Journey From Theory to Experiment" Moult, Zhu '25]

[Bossi et al. '25]
[Jaarsma et al. '25]

EEC in a conformal collider

Different limits expose different theoretical descriptions



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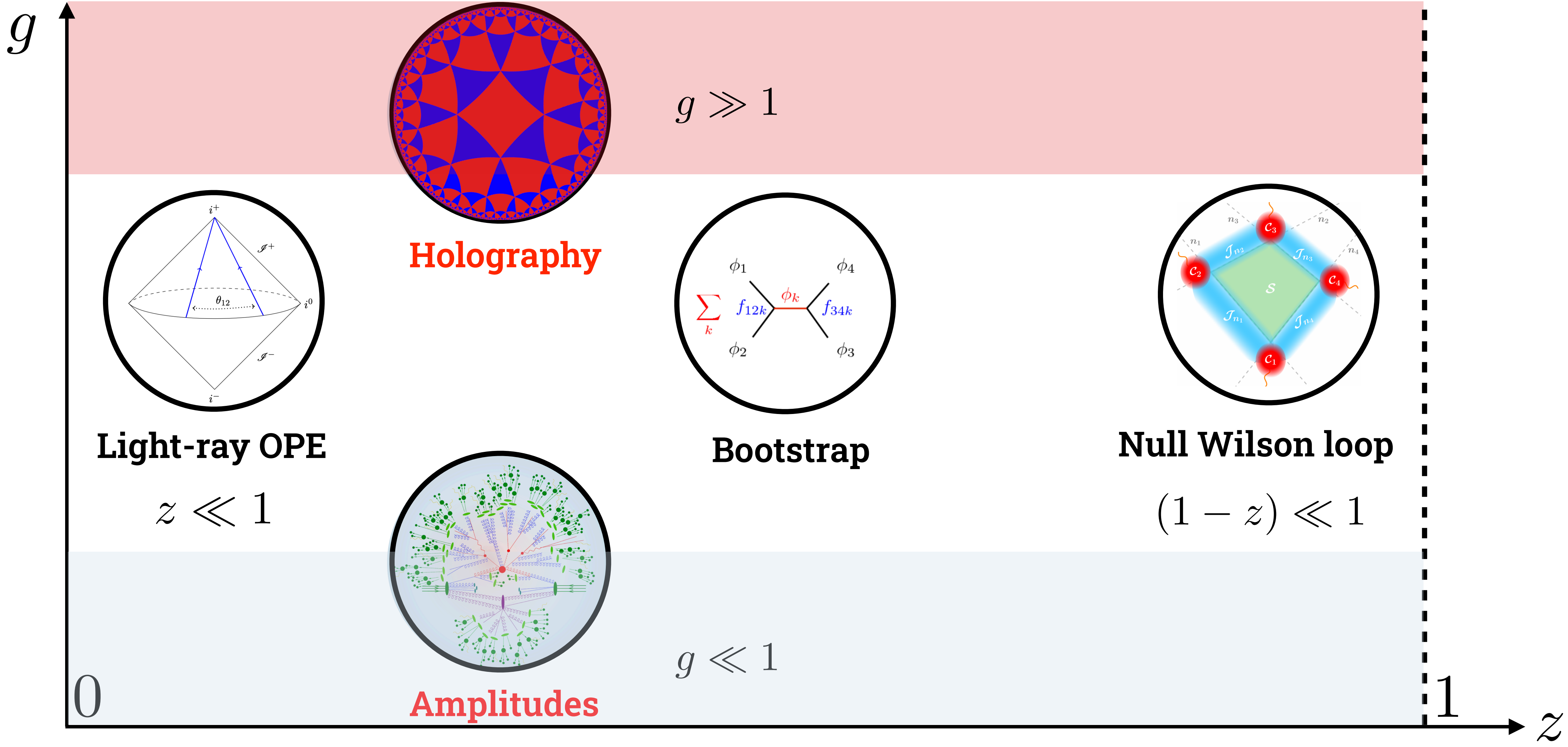
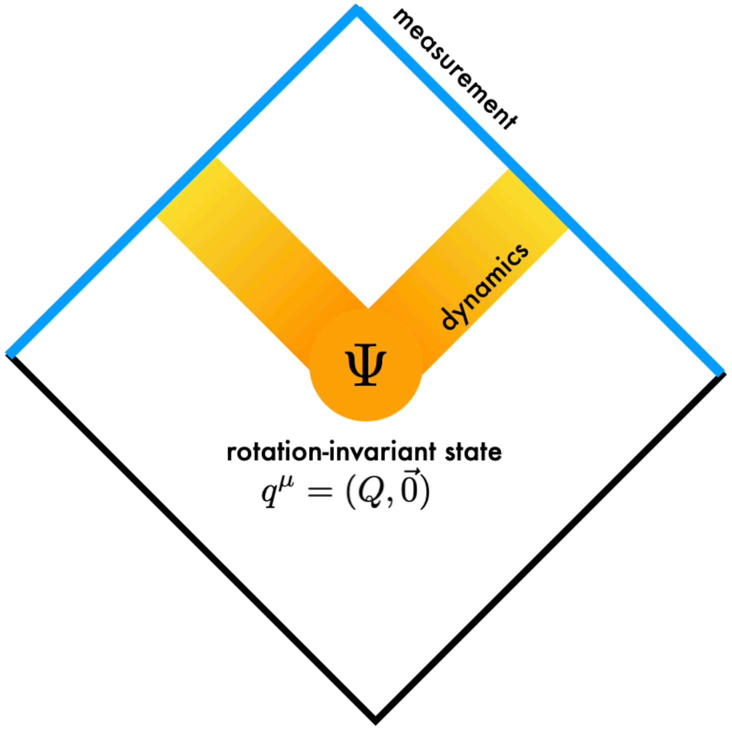


Energy flux operator

[Sveshnikov,Tkachov,Korchemsky,Sterman,Hofman,Maldacena]

Energy correlators are matrix elements of **the energy flux operator**



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{t_0}^{\infty} dt\; n^i T_{0i}(t, r\vec{n})$$

[light transform]

$$\langle \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_k) \rangle \equiv \langle \psi | \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_k) | \psi \rangle$$

$$|\psi\rangle = \mathcal{O}(q)|0\rangle, \quad q^\mu = (Q, \vec{0})$$

The correlator approach is particularly useful in CFTs (trivial Q-dependence).

$\mathcal{N} = 4$ SYM: A_μ, 4λ, 6ϕ

- EEC is a multi-fold integral of the **Wightman** four-point function

$$\mathrm{EEC}(z) \sim \underbrace{\int d^4x\, e^{iqx}}_{\text{Fourier}} \underbrace{\int_0^\infty dt_1\, dt_2 \lim_{r_i \to \infty} r_1^2 r_2^2}_{\text{Detector limit}} \underbrace{\langle 0 | J^\dagger(x)\, T_{0\vec{n}_1}(x_1)\, T_{0\vec{n}_2}(x_2)\, J(0) | 0 \rangle}_{\text{Wightman 4-pt. function}} \Big|_{x_i = (t_i, r_i \vec{n}_i)}$$

$\mathcal{N} = 4$ SYM: A_μ, 4λ, 6ϕ

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- The 4-pt correlator of $J = \left(\mathcal{O}_{20'}, J_{\mu,R}, T_{\mu\nu}\right)$ by a single scalar function $\Phi(u,v)$

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle = \frac{\Phi\left(u, v, \tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\mathrm{YM}}^2}, N_c\right)}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2}$$

$$\langle J(x_1)T(x_2)T(x_3)J(x_4)\rangle \sim \underbrace{P(\partial_u, \partial_v)}_{\text{diff. operator}}\, \Phi(u, v, \tau, N_c)$$

SUSY

- Cross-ratios $u = (x_{12}^2 x_{34}^2)/(x_{13}^2 x_{24}^2)$, $v = (x_{14}^2 x_{23}^2)/(x_{13}^2 x_{24}^2)$

EEC at weak coupling $g \ll 1$

$$\mathrm{EEC}_{\mathrm{QCD}}(z) = \underbrace{\alpha_s(Q)\,A(z)}_{\text{Basham et al. '78}} + \underbrace{\alpha_s^2(Q)\,B(z)}_{\text{Dixon et al. '18}} + \underbrace{\alpha_s^3(Q)\,C(z)}_{\text{NNLO numeric}} + O(\alpha_s^4) \qquad g^2 \equiv \frac{g_{\mathrm{YM}}^2 N_c}{16\pi^2}$$

$$\mathrm{EEC}_{\mathcal{N}=4}(z) = \frac{1}{4z^2(1-z)}\Big(\underbrace{g^2\,F_1(z)}_{\text{Engelund–Roiban '12}} + \underbrace{g^4\,F_2(z)}_{\text{Belitsky et al. '13}} + \underbrace{g^6\,F_3(z)}_{\text{Henn et al. '19}} + \underbrace{O(g^8)}_{\text{non-planar + 1-instanton}} \Big)$$

- The calculations are conveniently done in Mellin space
- $F_1(z) = -\log(1-z)$
- $$\begin{aligned} F_2(z) = {} & (1-z)\left(4\sqrt{z}\left[\mathrm{Li}_2(-\sqrt{z}) - \mathrm{Li}_2(\sqrt{z}) + \frac{1}{2}\ln z\,\ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] + (1+z)\left[2\mathrm{Li}_2(z) + \ln^2(1-z)\right] + 2\ln(1-z)\ln\left(\frac{z}{1-z}\right) + \frac{\pi^2}{3}z\right) \\ & + (1-z)(1+2z)\left[\ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\ln\left(\frac{1-z}{z}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right)\right] - 4(z-4)\mathrm{Li}_3(z) + 6(3+3z-4z^2)\mathrm{Li}_3\left(\frac{z}{z-1}\right) \\ & - 2z(1+4z)\zeta_3 + 2\left[(3-4z)z\ln z + 2(2z^2-z-2)\ln(1-z)\right]\mathrm{Li}_2(z) + \frac{1}{3}\ln^2(1-z)\left[4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z\right] \\ & + \frac{\pi^2}{3}\left[2z^2\ln z - (2z^2+z-2)\ln(1-z)\right]. \end{aligned}$$
- $F_3(z) = \text{harmonic polylogarithms} + \text{two-fold elliptic integral}$
- Large corrections for $z \to 0$ (small angles), and $z \to 1$ (back-to-back)

EEC at strong coupling $g \gg 1$

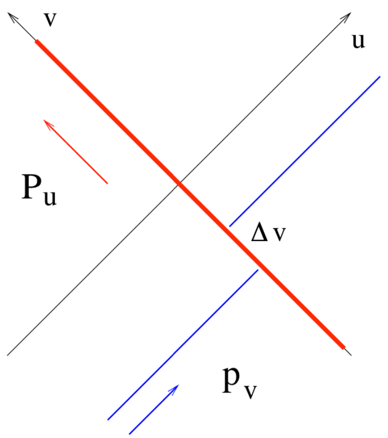
Energy correlators are sensitive to **high-energy gravitational scattering** in the bulk.

$$T_{\mu\nu} \longleftrightarrow g_{\mu\nu}^{\mathrm{AdS}} + \delta h_{\mu\nu}$$

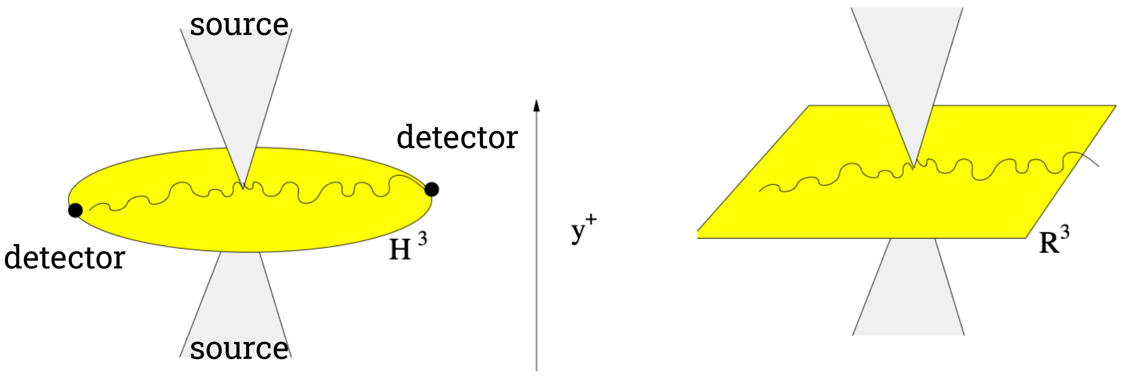
$$\mathcal{E}(\vec{n}) \sim \int dy^- T_{--}(y^+ = 0, y^-) \longleftrightarrow \text{high-energy graviton}$$

$$\text{single-trace source} \longleftrightarrow \text{one-particle state in AdS}$$

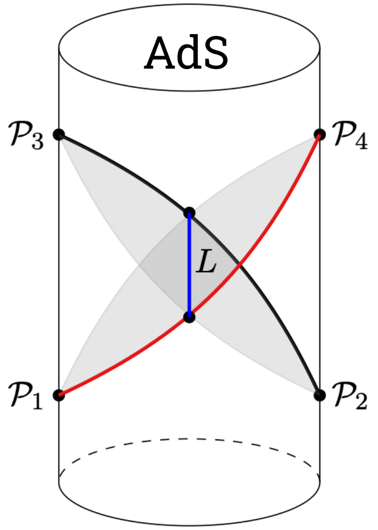
classical theory



string scattering



gravitational scattering



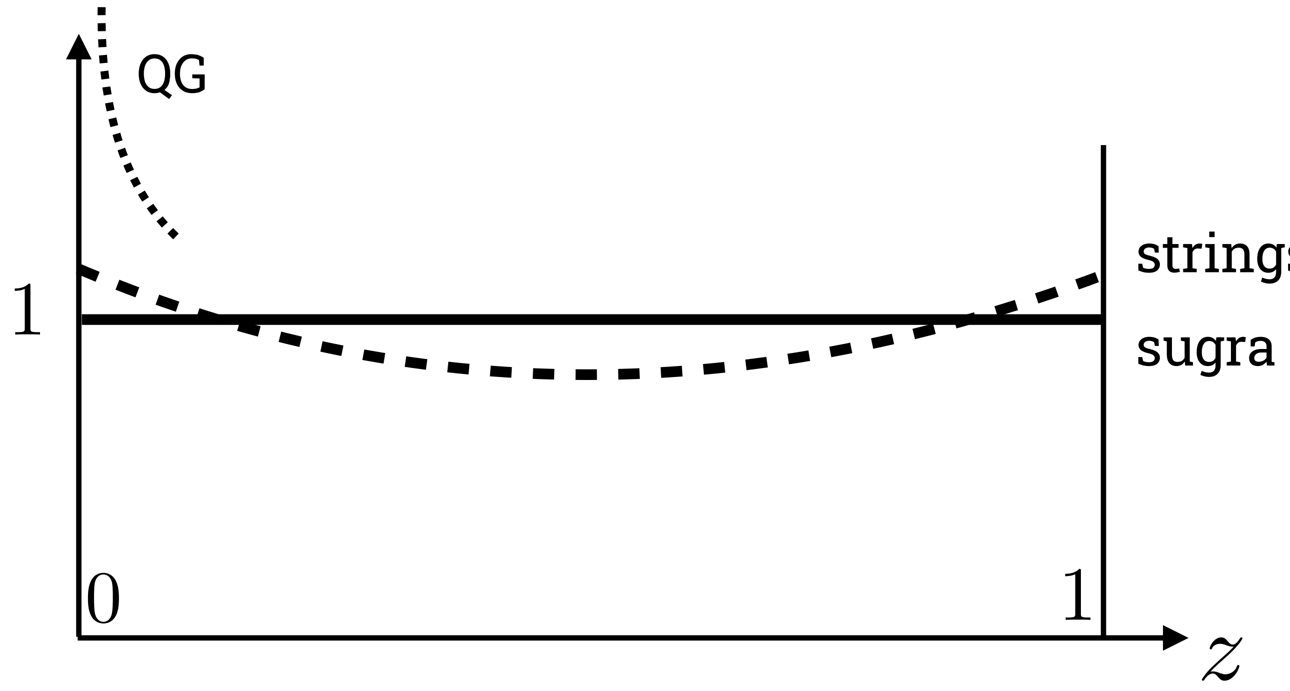
EEC at strong coupling $g \gg 1$

$$g^2 \equiv \frac{g_{\rm YM}^2 N_c}{16\pi^2}$$

$$\mathrm{EEC}(z) = 1 + \frac{1}{4g^2}\left(1 + O(1/g)\right) P_2(1-2z) + \frac{45\zeta_3}{4\pi^3 g^3} P_3(1-2z) + \underbrace{O\left(\frac{\log N_c}{N_c^2}\log z\right)}_{\mathrm{QG}}$$

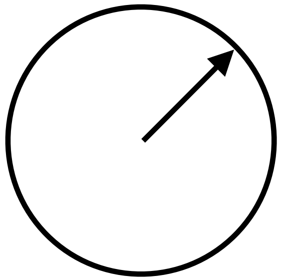
[Legendre polynomial]

- Tree-level supergravity produces homogeneous distribution
- Flat space string scattering
- AdS scattering amplitude
- QG = eikonal in AdS

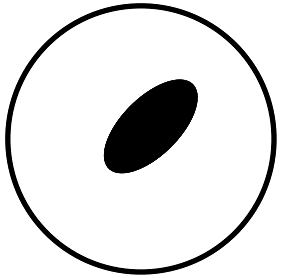


Supergravity: homogeneous distribution

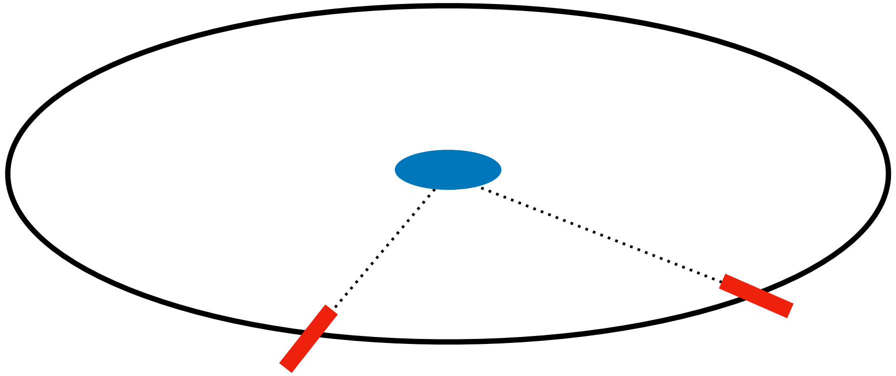
At strong coupling, copious particle production is unsuppressed.
Particles dissolve into **blobs** (=particles in AdS/blobicles).

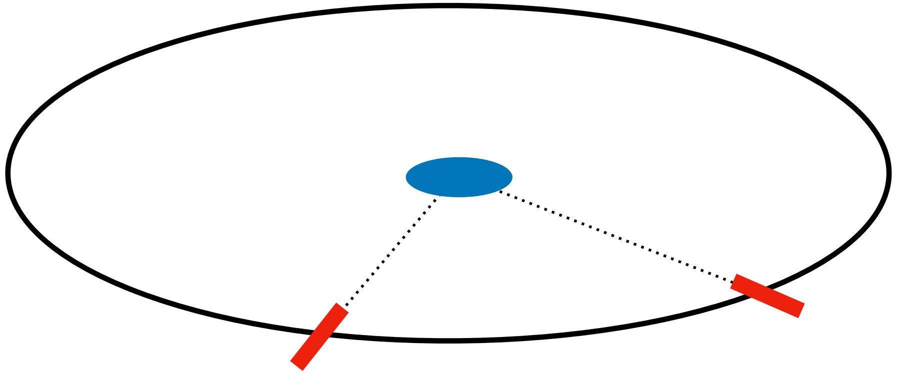


$$\varepsilon_p(\vec{n}) = q^0 \delta(\vec{n} - \frac{\vec{q}}{|\vec{q}|})$$



$$\varepsilon_{\mathrm{AdS}-p}(\vec{n}) = \frac{q^4}{4\pi(q^0 - \vec{q}\cdot\vec{n})^3}$$





A single-trace operator is dual to a **single particle in AdS**

$$\langle \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_k) \rangle_{q,\mathrm{SUGRA}} = \prod_{i=1}^{k} \varepsilon_{\mathrm{AdS}-p}(\vec{n}_i)$$

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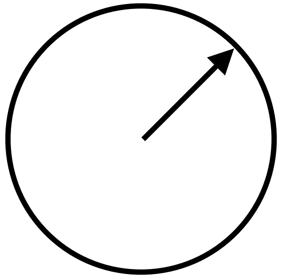

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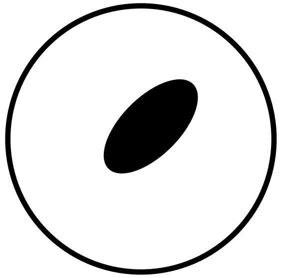

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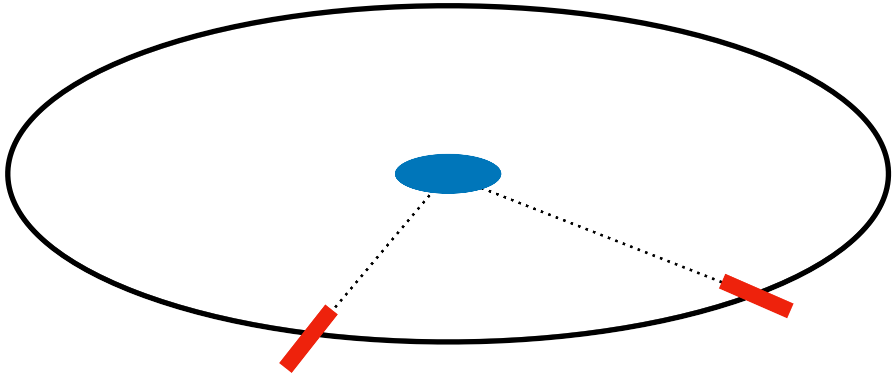

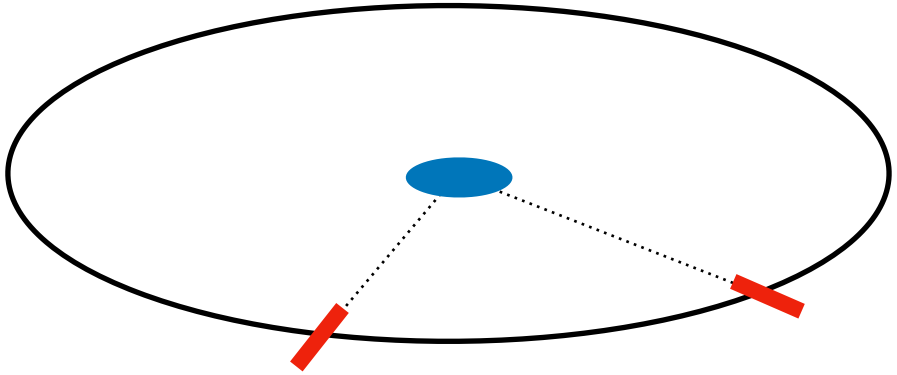

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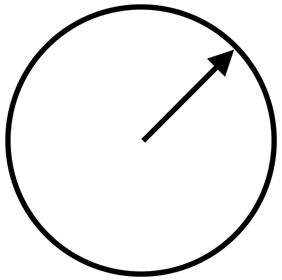

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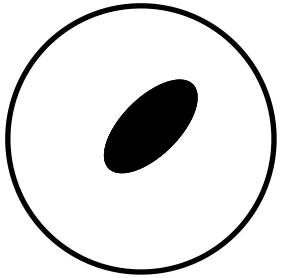

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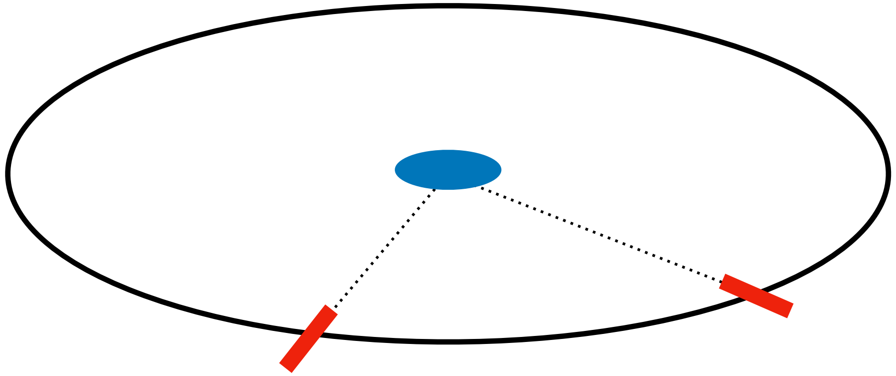

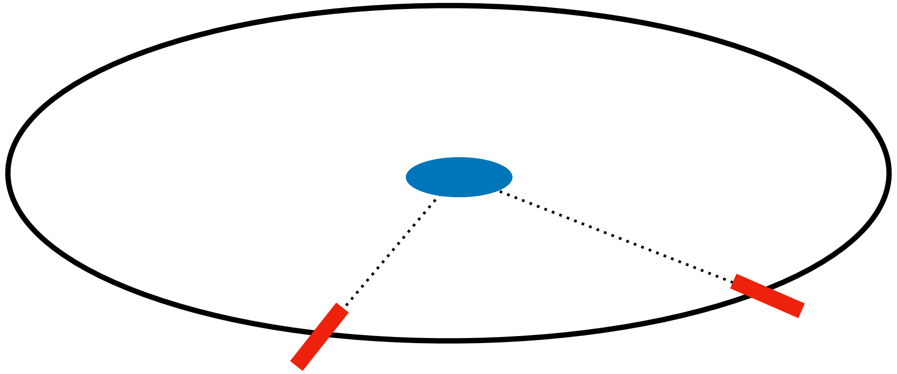

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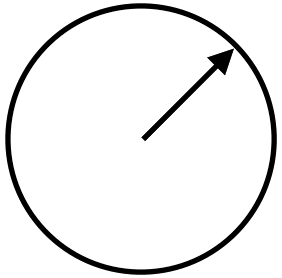

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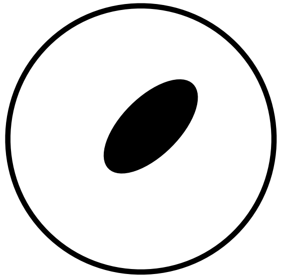

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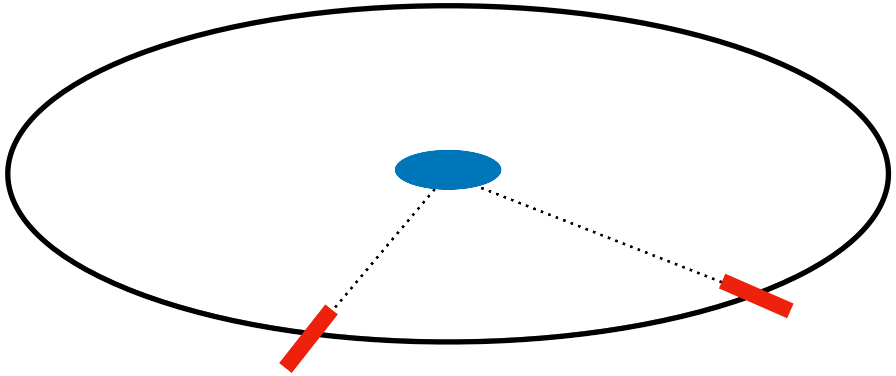

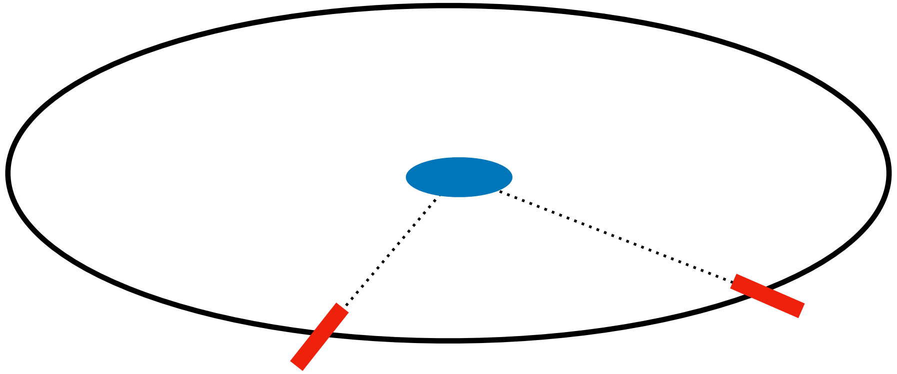

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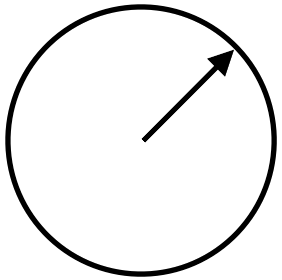

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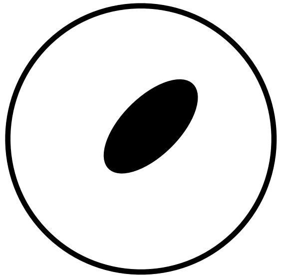

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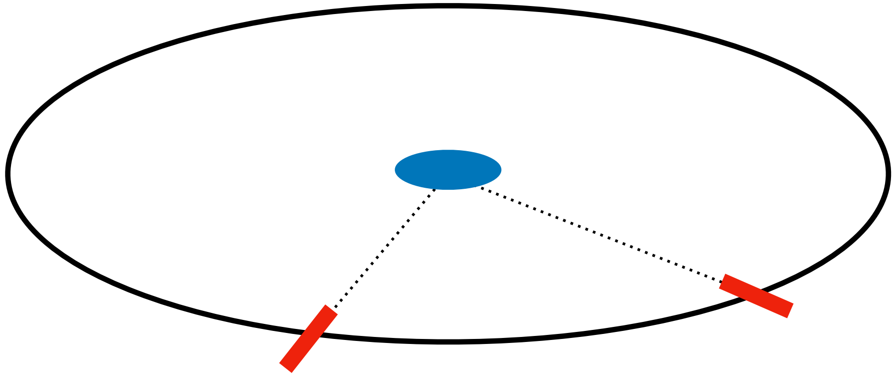

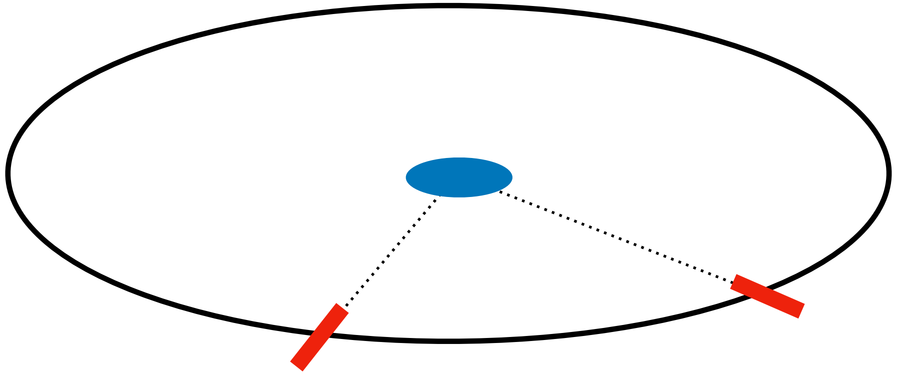

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Diagonalizing the calorimeters

[Hofman,Maldacena ’08]
[Belin,Borak,Henriksson,Piron,AZ, wip]

$$\frac{\langle \mathcal{O}^\dagger \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)\mathcal{O}\rangle}{\langle \mathcal{O}^\dagger\mathcal{O}\rangle}$$

matrix element of **energy flux operator**

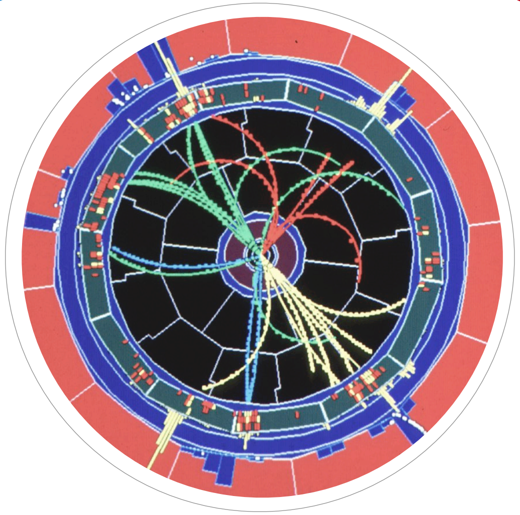
positivity

$$\mathcal{E}(\vec{n}) \geq 0$$

infinite hierarchy

$$\langle \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)\rangle = \int \sigma(d\varepsilon)\prod_{i=1}^{k}\varepsilon(\vec{n}_i)$$

moments of the measure on the space of energy-flow measures



theories with particles

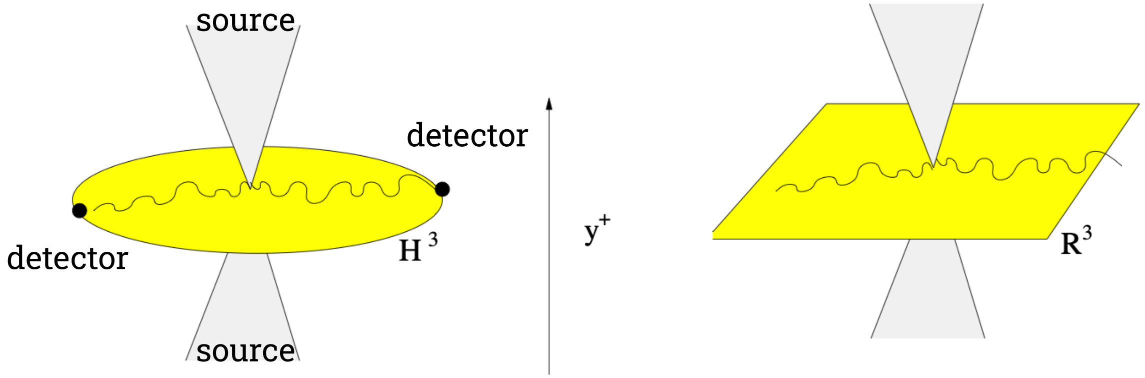
cross-section representation

$$\sigma(d\varepsilon) = \frac{1}{\sigma_{\text{tot}}}\sum_X d\sigma_X$$

collider event = atomic energy-flow measure

$$\varepsilon_X(\vec{n}) = \sum_{a\in X} E_a \delta^{(d-2)}\left(\vec{n} - \frac{\vec{p}_a}{|\vec{p}_a|}\right)$$

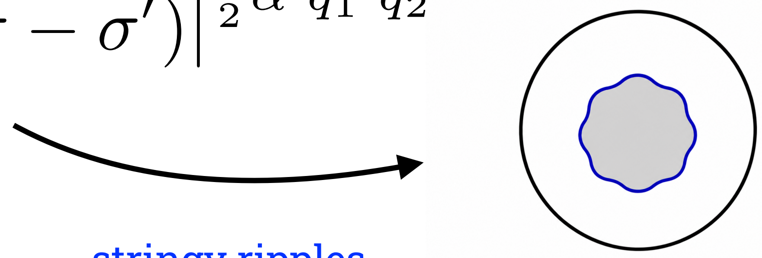
Stringy ripples from the AdS Virasoro-Shapiro



[strings on a shockwave are solvable]

$$\langle 0_{\rm osc}| e^{i\vec{q}_1\cdot\vec{X}_{\rm osc}(\sigma,0)} e^{i\vec{q}_2\cdot\vec{X}_{\rm osc}(\sigma',0)} |0_{\rm osc}\rangle$$

$$= |2\sin(\sigma-\sigma')|^{\frac{1}{2}\alpha'\vec{q}_1\cdot\vec{q}_2}$$



stringy ripples

$\delta\varepsilon \sim 1/g$

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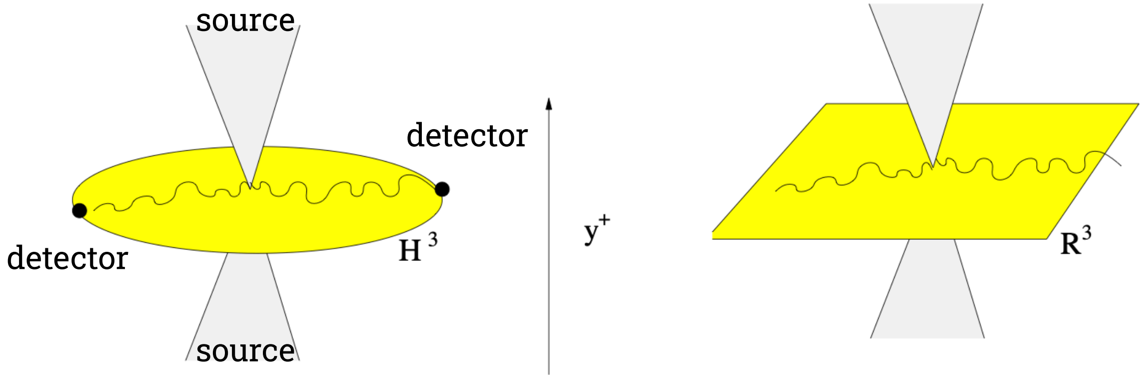

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$$\langle 0_{\text{osc}} | e^{i\vec{q}_1 \cdot \vec{X}_{\text{osc}}(\sigma,0)} e^{i\vec{q}_2 \cdot \vec{X}_{\text{osc}}(\sigma',0)} | 0_{\text{osc}} \rangle$$

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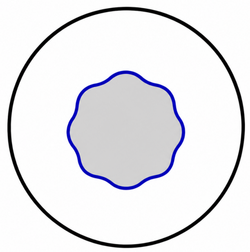

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$\delta\varepsilon \sim 1/g$

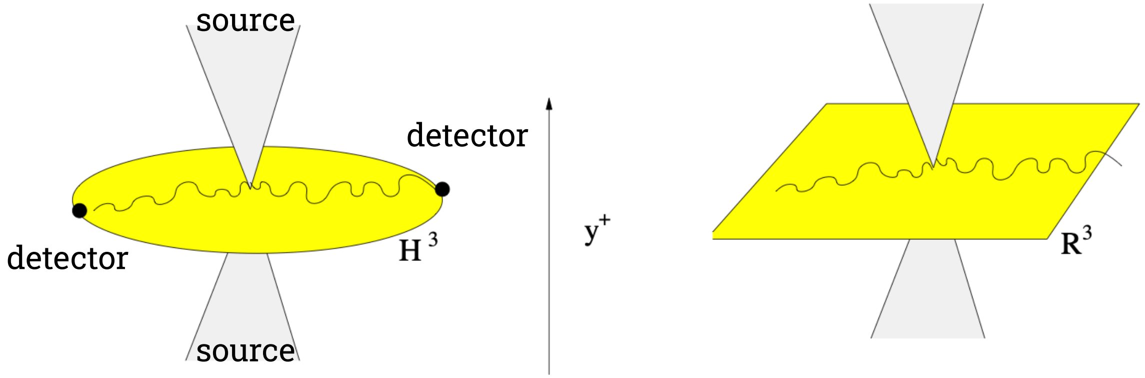
- Leading correction is controlled by **flat-space string shockwave scattering**

$$\text{EEC}(z) = 1 + \sum_{J=2}^{\infty} \frac{\tilde{d}_J}{g^J} P_J(1-2z)$$

$\tilde{d}_J$ known explicitly.

[Hofman,Maldacena '08; Dempsey et al. '25]

Stringy ripples from the AdS Virasoro-Shapiro



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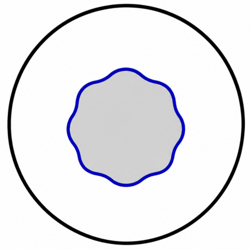


$\delta\varepsilon \sim 1/g$

- Leading correction is controlled by **flat-space string shockwave scattering**

$$\text{EEC}(z) = 1 + \sum_{J=2}^{\infty} \frac{\tilde{d}_J}{g^J} P_J(1-2z)$$

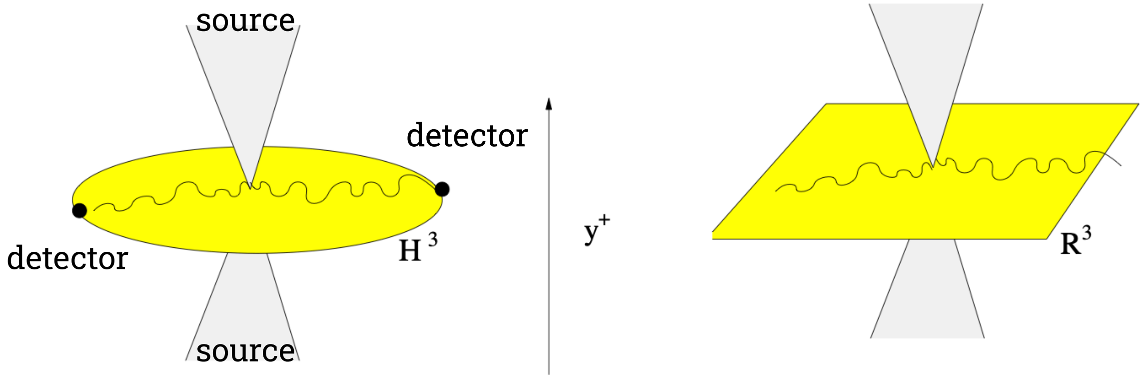
$\tilde{d}_J$ known explicitly.

[Hofman,Maldacena '08; Dempsey et al. '25]

- Sub-leading corrections can be calculated using the OPE and corrections to the AdS Virasoro-Shapiro amplitude [two paths, two different answers]

[Alday,Hansen,Silva '22; Dempsey et al. '25; Ren,Wang,Wen '26]

Stringy ripples from the AdS Virasoro-Shapiro

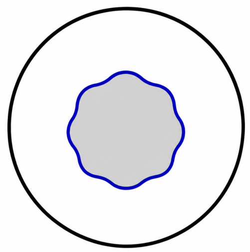


[strings on a shockwave are solvable]

$$\langle 0_{\mathrm{osc}}| e^{i\vec{q}_1\cdot \vec{X}_{\mathrm{osc}}(\sigma,0)} e^{i\vec{q}_2\cdot \vec{X}_{\mathrm{osc}}(\sigma',0)} |0_{\mathrm{osc}}\rangle$$

$$= \left|2\sin(\sigma-\sigma')\right|^{\frac{1}{2}\alpha' \vec{q}_1\cdot\vec{q}_2}$$

stringy ripples



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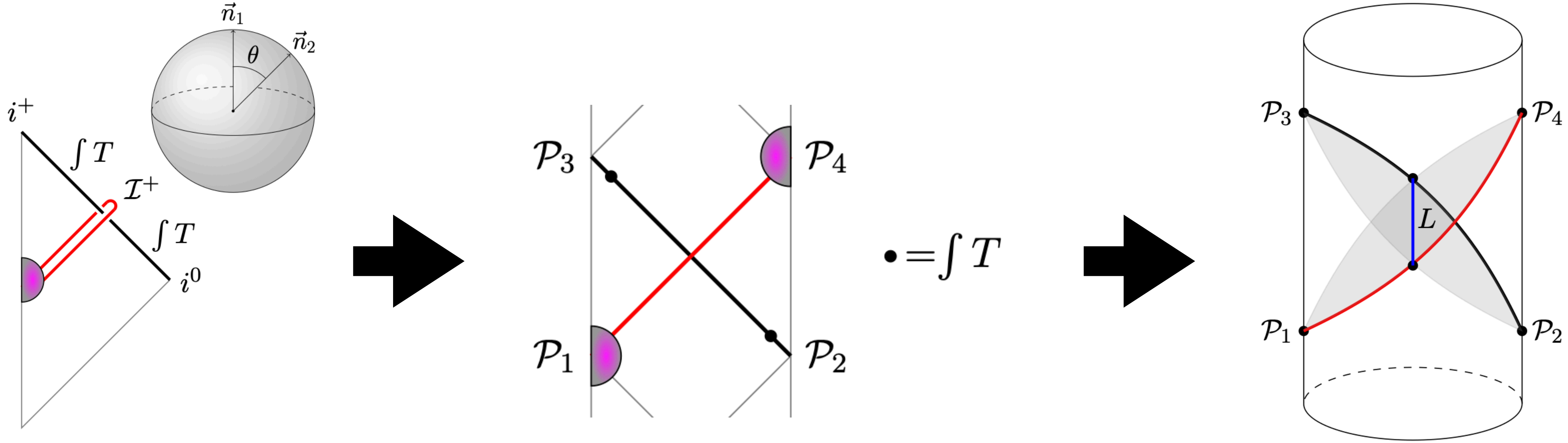
[Alday,Hansen,Silva '22; Dempsey et al. '25; Ren,Wang,Wen '26]

- **Lesson**: light transform and strong coupling $1/g$ expansion do not commute.

Finite N_c: energy correlators probe Planckian scattering

[Chen,Karlsson,AZ '24]

Light transform does not commute with the $1/N_c$ expansion



Finite N_c: energy correlators probe Planckian scattering

[Chen,Karlsson,AZ '24]

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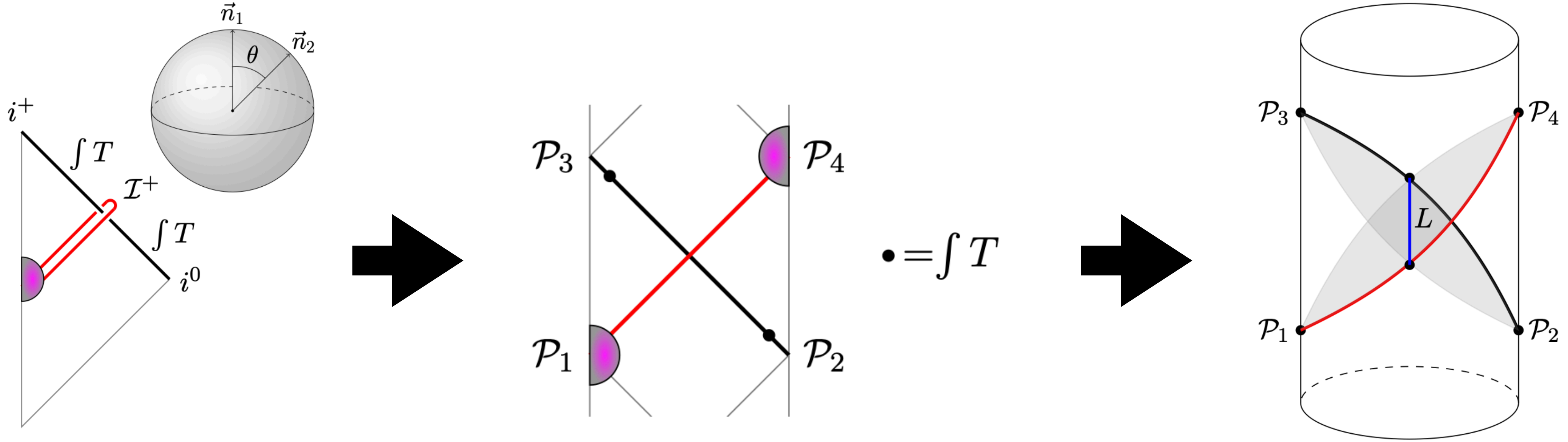


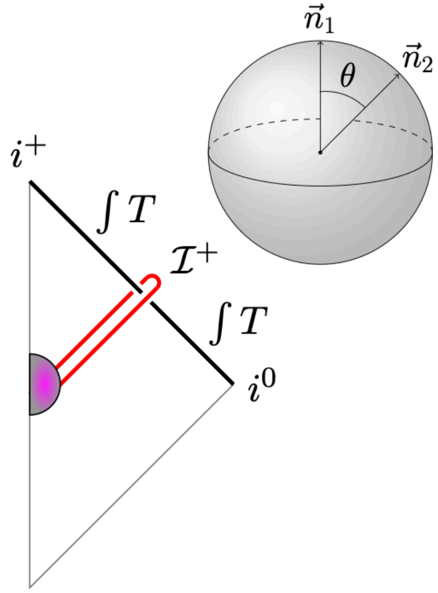
$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \int^{\infty} \frac{dS}{S^3}\int_0^{\infty} dL K(L,z) \mathrm{Re}\left(1 - e^{i\delta(S,L)}\right)$$

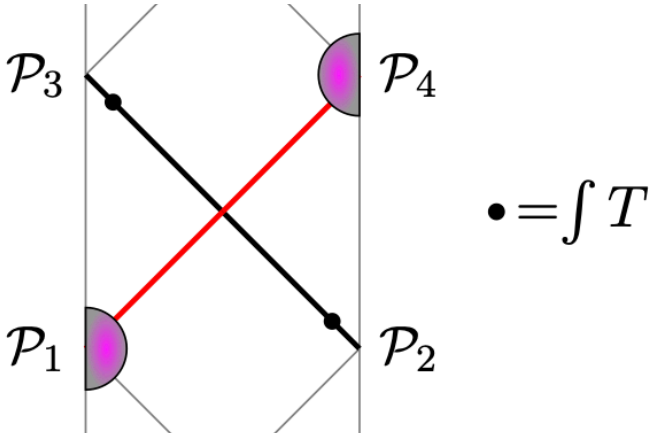
$$\delta^{\mathrm{tree}}(S,L) \sim S$$

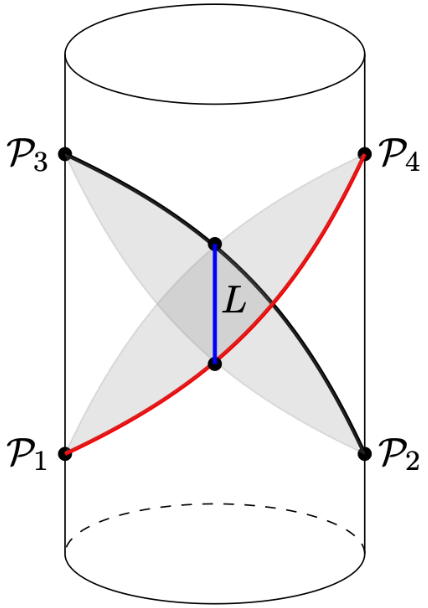
Finite N_c: energy correlators probe Planckian scattering

[Chen,Karlsson,AZ '24]

Light transform does not commute with the $1/N_c$ expansion





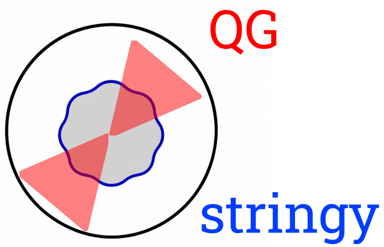


$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \int^{\infty} \frac{dS}{S^3} \int_0^{\infty} dL K(L,z) \mathrm{Re}\left(1 - e^{i\delta(S,L)}\right)$$

$$\delta^{\text{tree}}(S,L) \sim S$$

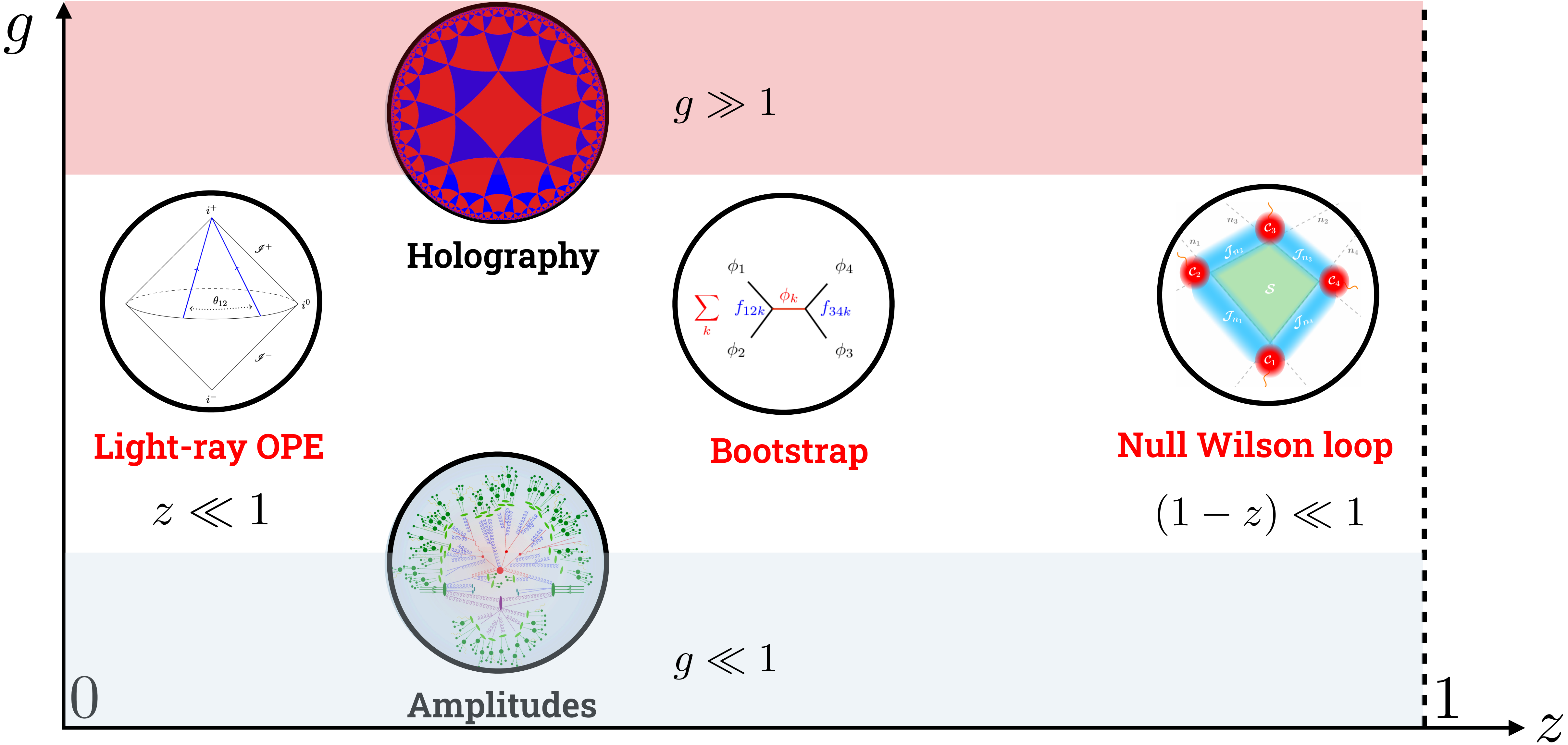
$$\langle \mathcal{E}(\theta)\mathcal{E}(0)\rangle_{\text{QG}} = \gamma_d \left(\frac{p^0}{\Omega_{d-1}}\right)^2 \frac{\log c_T}{c_T} \log\frac{1}{\theta^2}, \quad \theta \to 0$$

- gravitational eikonal
- singular at small angles (rare fluctuations)



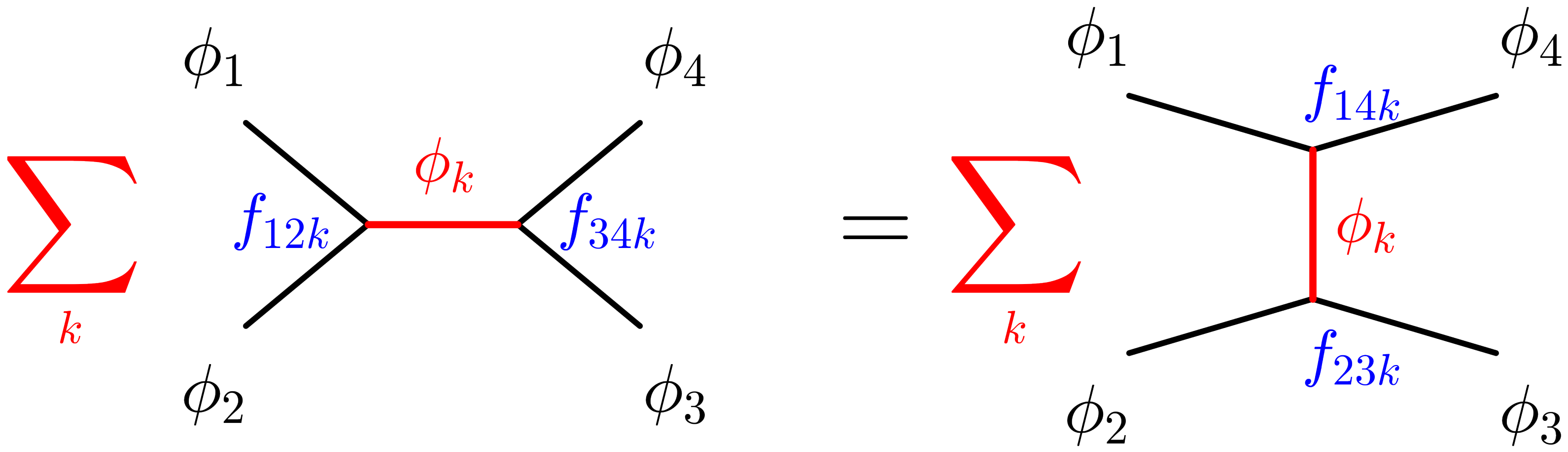
EEC in a conformal collider

Different limits expose different theoretical descriptions



Conformal bootstrap

At finite coupling, CFTs are contrained by the basic axioms



- Local (primary) operators ϕ_k have scaling dimension Δ and spin J
- The OPE coefficients fixed by the three-point functions $\langle \phi_i \phi_j \phi_k \rangle \sim f_{ijk}$
- Crossing symmetry = associativity of the OPE
- +SUSY, integrability, localization, dispersion relations, ...

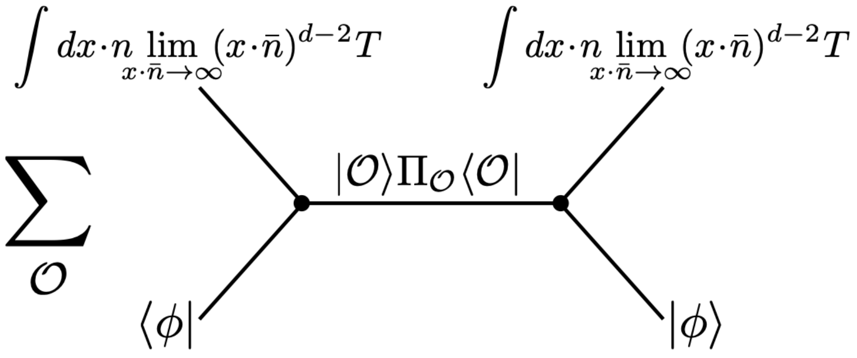
CFT data: (Δ_k, J_k) $\quad f_{ijk}$

Source-detector OPE: from the CFT data to EEC

[Kologlu,Kravchuk,Simmons-Duffin,AZ '19]
[Mecaj,Moult,Walters,Yin '25]

EEC can be expressed through **the CFT data** (dimensions+OPE coefficients).

$$\langle \mathcal{E}\mathcal{E} \rangle = \sum_{\mathcal{O}}$$

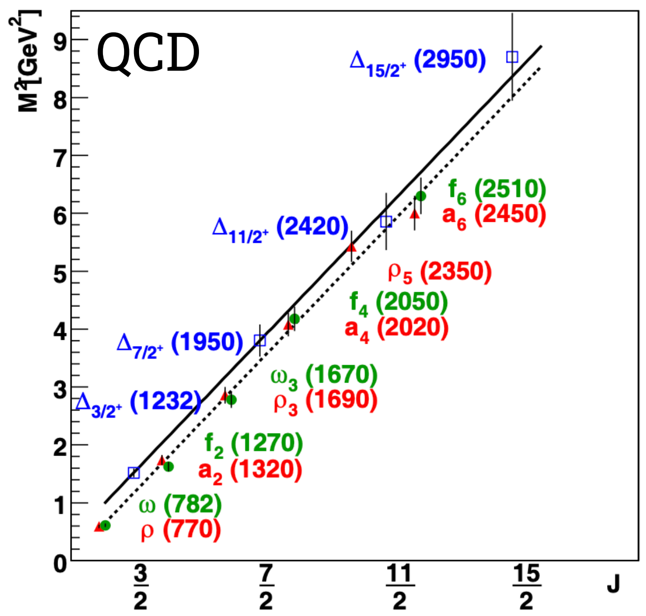


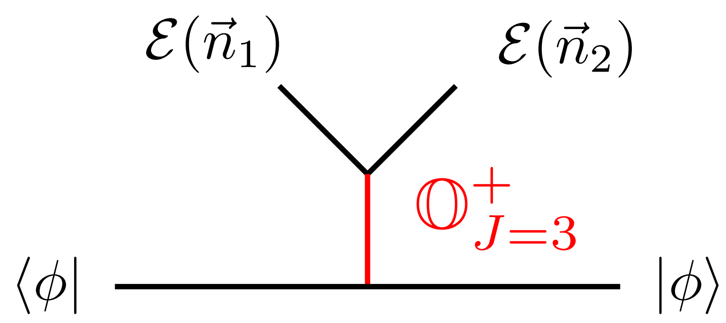
$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \left(\frac{Q}{4\pi}\right)^2 \left(1 + \sum_{J=2}^{\infty} c_J P_J(\cos\theta)\right)$$

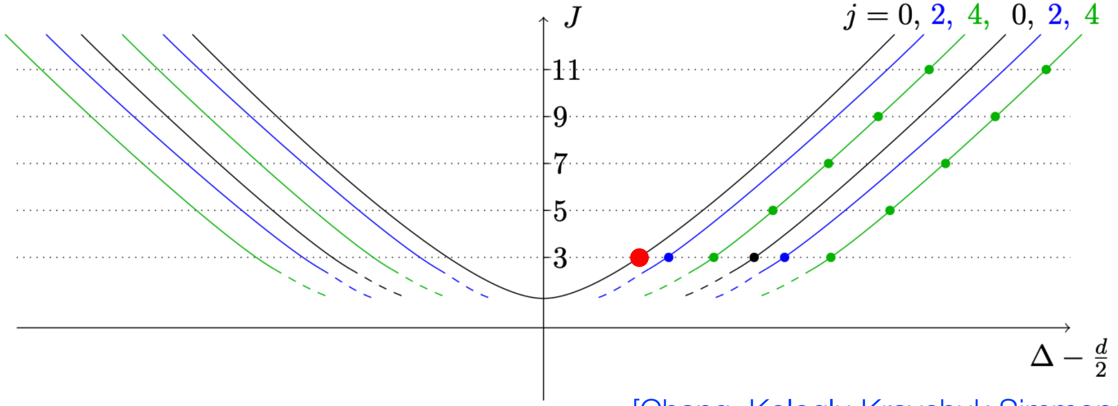
energy multipoles

- Energy multipoles $c_J \geq 0$ (unitarity) are expressed through the CFT data: spin J operator contributes to $0,..,J+2$ multipoles
- Energy/momentum conservation leads to $c_0 = 1,\ c_1 = 0$
- There is a nontrivial interplay between point-wise positivity EEC$(z) \geq 0$ and unitarity $c_J \geq 0$ (applicable to all theories)

The light-ray OPE and the collinear limit







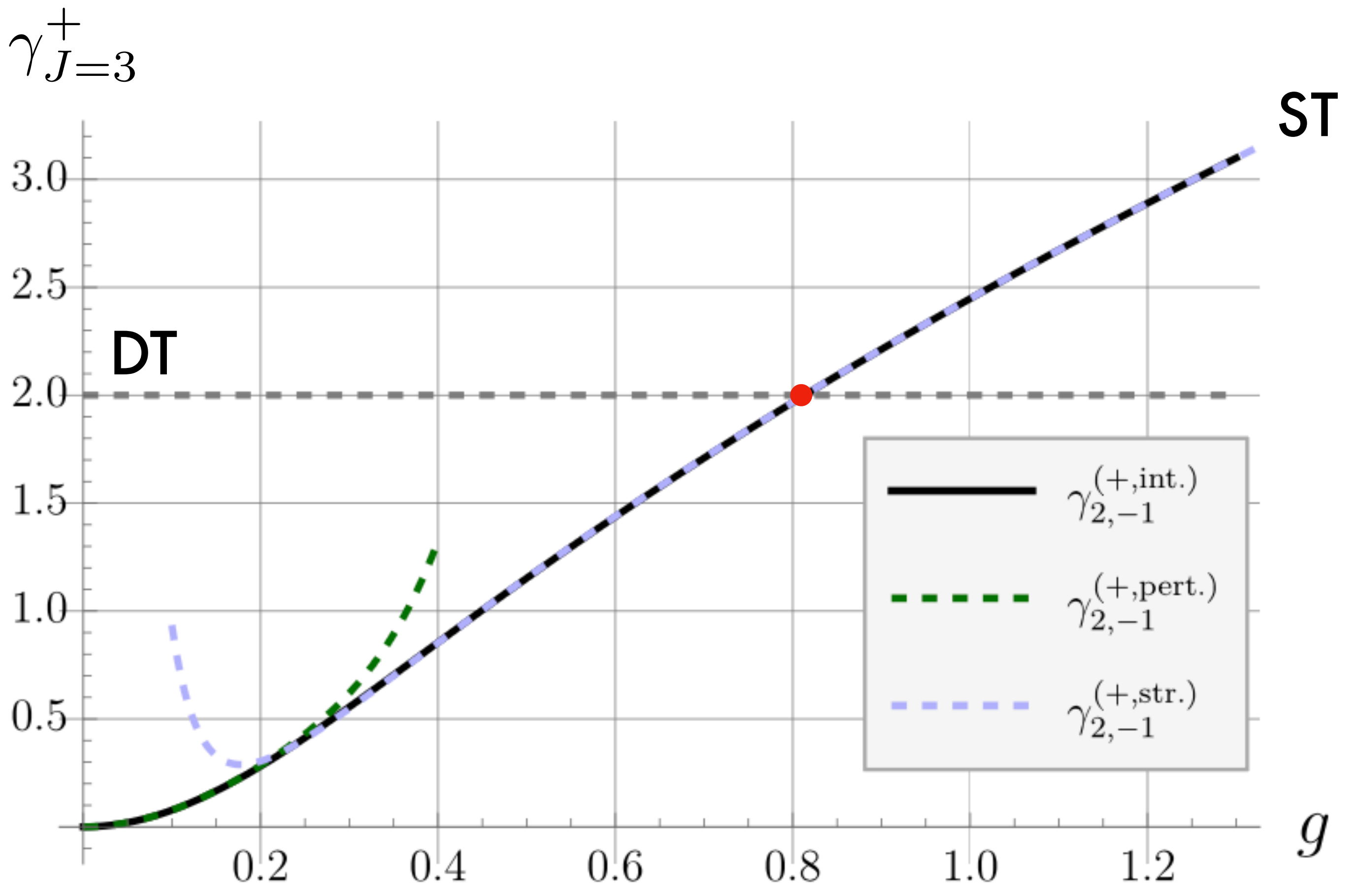
[Chang, Kologlu,Kravchuk,Simmons-Duffin,AZ '19-20]
[Dixon,Moult,Zhu'19][Korchemsky '19]

$$\mathrm{EEC}(z) = \sum_{\text{Regge traj}} p^{+,J=3}_{\Delta_i} z^{\frac{\Delta_i - 7}{2}} {}_2F_1\left(\frac{\Delta_i - 1}{2}, \frac{\Delta_i - 1}{2}, \Delta_i - 1, z\right)$$

At small angles, this predicts $$\mathrm{EEC}(z) \underset{z \to 0}{\sim} \frac{1}{z^{1-\gamma^+_{J=3}/2}}$$

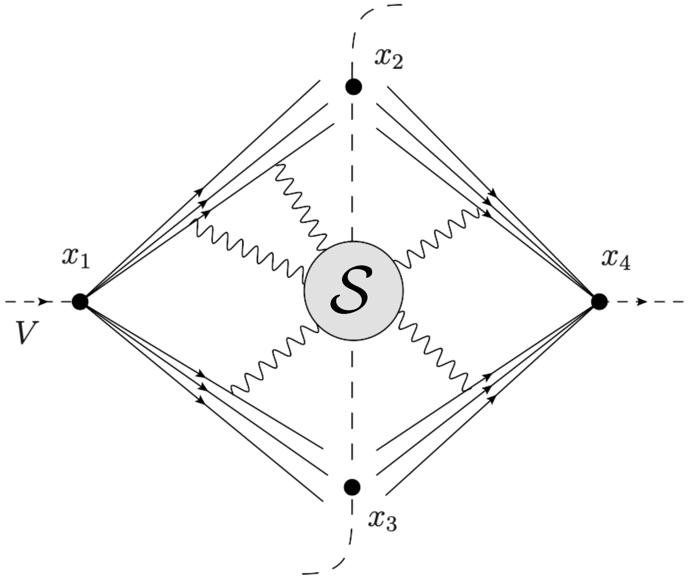
The light-ray OPE: small angles (planar theory)

Collinear limit: $$\mathrm{EEC}(z) \underset{z\to 0}{\sim} \frac{1}{z^{1-\gamma^{+}_{J=3}/2}} + \frac{1}{z^{\gamma_{\mathrm{DT}}/2}}$$



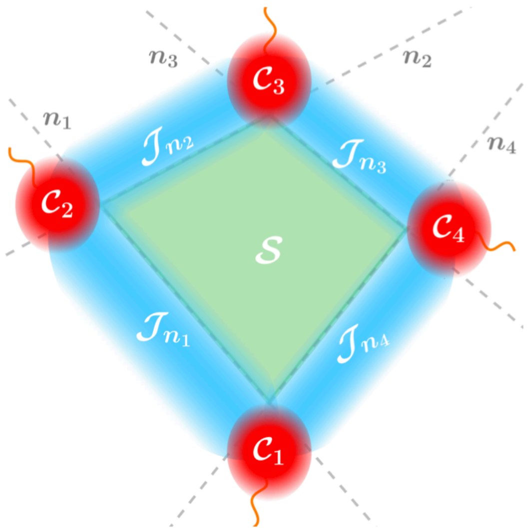
[Gromov,Levkovich-Maslyuk,Sizov,Valatka '14]

Back-to-back limit: soft physics and null Wilson loop



controlled by soft radiation

[Alday,Eden,Korchemsky,Maldacena,Sokatchev '10]
[Chen,Monni,Pang,Vita,Zhu '25]



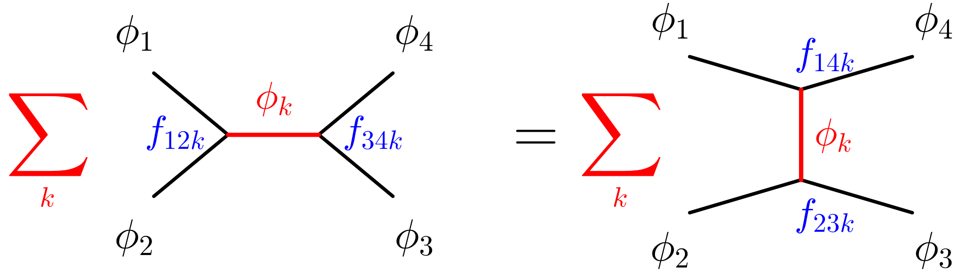
double light-cone limit $u, v \to 0$

$$\mathrm{EEC}(y \equiv 1 - z) = \frac{H(g)}{4y} \int_0^\infty db\, b J_0(b) \exp\left[-\frac{1}{2}\Gamma_{\mathrm{cusp}}(g) \log^2\left(\frac{b^2}{y b_0^2}\right) - \Gamma_{\mathrm{coll}}(g) \log\left(\frac{b^2}{y b_0^2}\right)\right]$$

[Collins,Soper '81; Moult,Zhu '18; Korchemsky '19; Chen,Zhou,Zhu '23]

- twist-2/string flux tube dominates at weak coupling
- twist-4/string breaking is not well understood
- systematic corrections? General CFTs?

Bootstrap problem (numerical)



finite Nc

(+dispersive functionals)

$$
\begin{array}{lll}
\text{min/max} & \sum \beta_{\tau,J}\lambda^2_{\tau,J} & \\
\text{subject to} & \sum \lambda^2_{\tau,J} F_{\tau,J;m,n} + F^{\text{protected}}_{m,n}(N_c) = 0, & m+n \text{ odd}, m<n \\
 & \sum \lambda^2_{\tau,J} I_{k,\tau,J} + (I^{\text{protected}}_{k,N_c} - \mathcal{F}_k(N_c, g_{\text{YM}})) = 0, & k = 2,4 \\
 & \sum \lambda^2_{\tau,J} \sin^2 \frac{\pi\tau}{2} \alpha(\tau, J) f_{J+2,s}(\tau) - \max c_s \leq 0, & s = 2,3,\ldots \\
 & \lambda^2_{\tau,J} \geq 0 & \forall \tau, J
\end{array}
$$

[Bender,Chester,Dempsey,Pufu,Wang,...]

planar

(spectral gap from integrability)

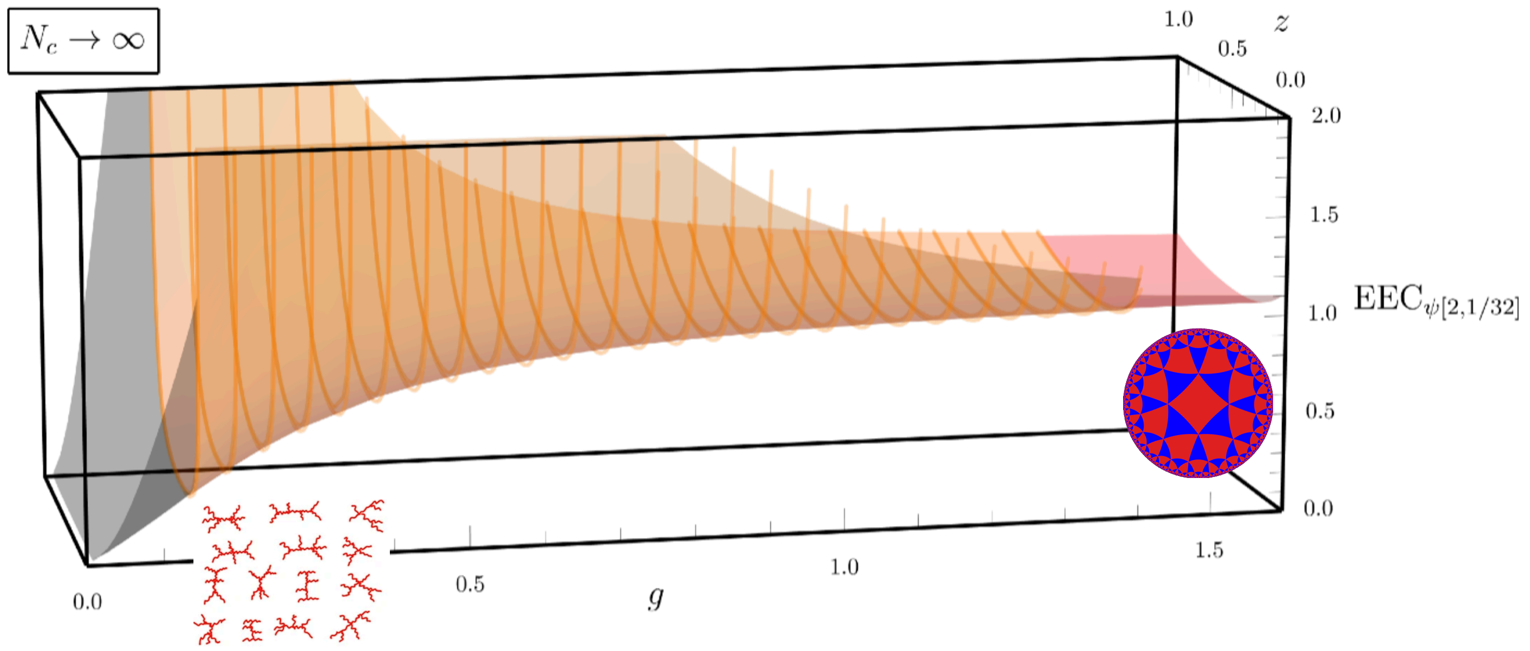
$$
\begin{array}{lll}
\text{min/max} & \sum \beta_{\tau,J}\lambda^2_{\tau,J} & \\
\text{subject to} & \sum \tilde{\lambda}^2_{\tau,J} X_{\tau,J}(u,v) = 0, & (u,v) \text{ Euclidean } (6.14)\,, \\
 & \sum \tilde{\lambda}^2_{\tau,J} B_{\tau,J}(v) + B^{\text{protected}}_v = 0, & v > 0 \text{ real}\,, \\
 & \sum \tilde{\lambda}^2_{\tau,J} \hat{B}_{\tau,J}(t) + \hat{B}^{\text{protected}}_t = 0, & 4 < \text{Re}\, t < 6\,, \\
 & \sum \tilde{\lambda}^2_{\tau,J} \Psi_{\ell;\tau,J} + \Psi^{\text{protected}}_\ell = 0, & \ell = 0,2,\ldots\,, \\
 & \sum \tilde{\lambda}^2_{\tau,J} \Phi_{\ell,\ell+2;\tau,J} = 0, & \ell = 0,2,\ldots\,, \\
 & \sum \tilde{\lambda}^2_{\tau,J} I_{k,\tau,J} + (I^{\text{strong}}_k - I_k(g)) = 0, & k = 2,4\,, \\
 & \tilde{\lambda}^2_{\tau,J} \geq 0 & \forall J = 0,2,\ldots,\ \tau \in \tau(g,J)\,.
\end{array}
$$

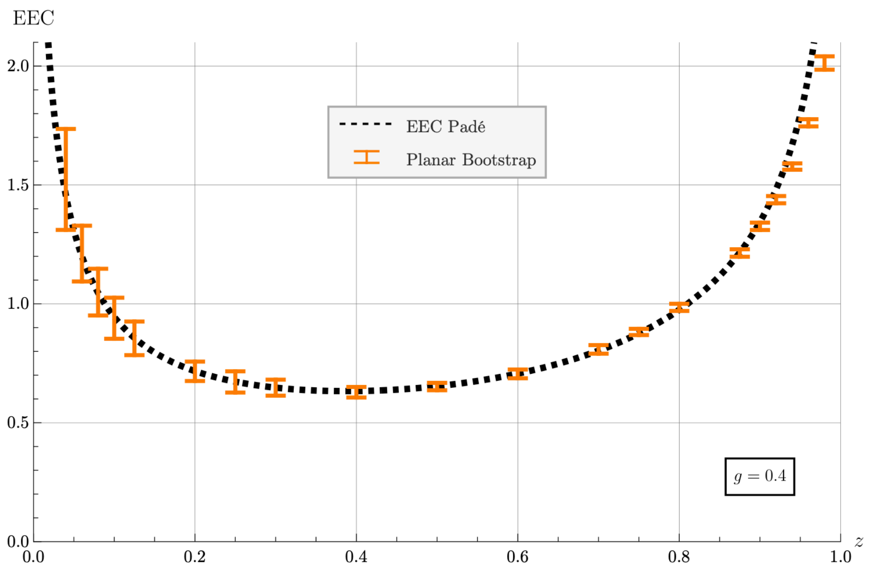
[uniqueness of the stress tensor is not imposed]

[Caron-Huot,Coronado,Trinh,Zahraee '22-'24]

Planar $\mathcal{N} = 4$ SYM: bootstrapping the full EEC

[Dempsey,Karlsson,Pufu,Zahraee,AZ '25 and wip]





$$\psi_{n,\delta}(z) = \Theta(|z| \leq \delta)(z-\delta)^n(z+\delta)^n$$

input:

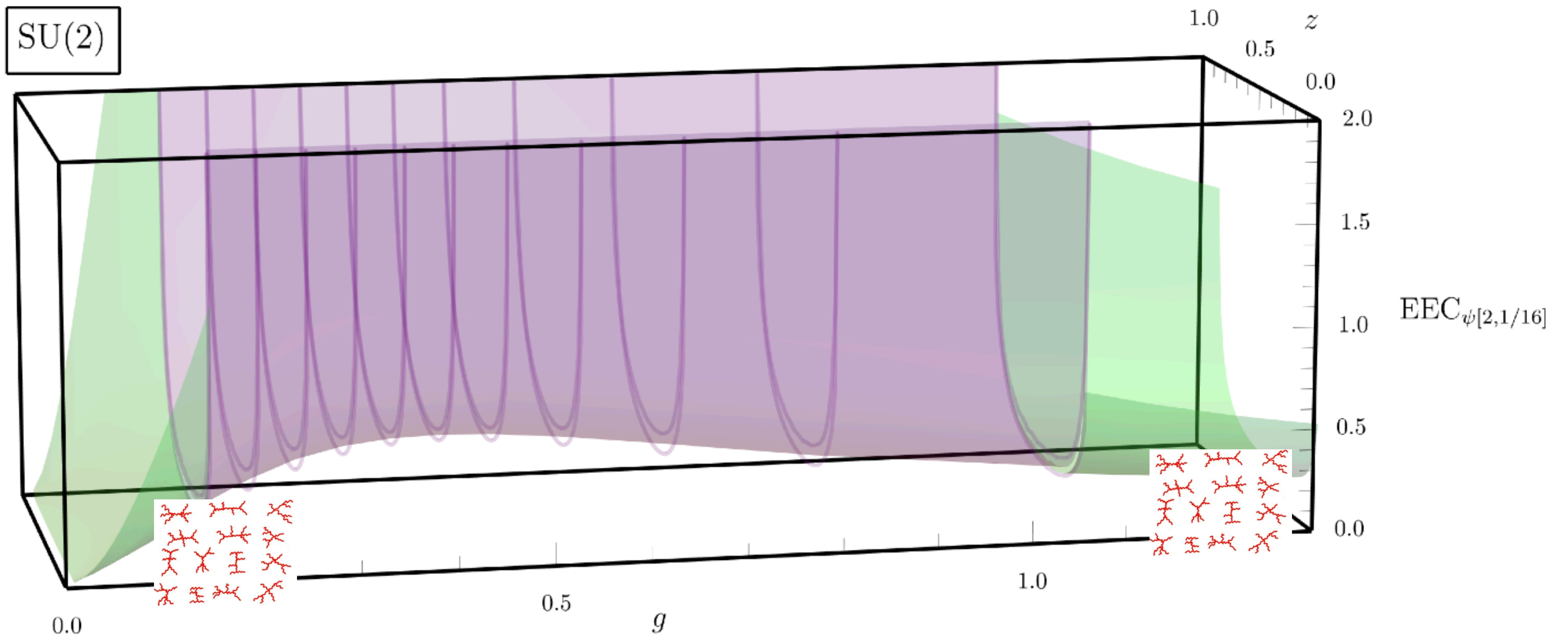
- SUSY localization
- crossing symmetry
- dispersion relations
- integrability

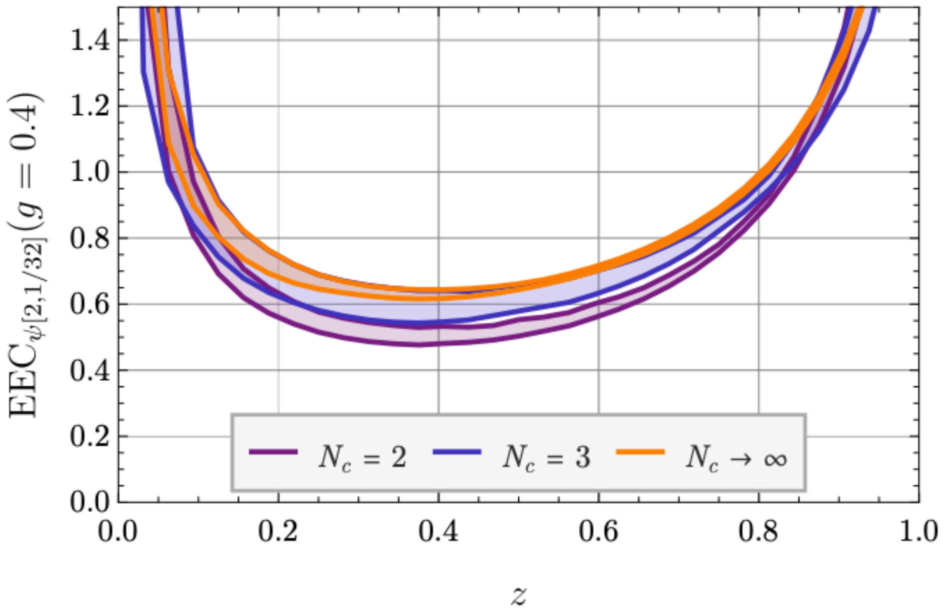
output:

- two-sided bounds
- weak/strong transition
- predictions for $z \to 0,1$
- tests of analyticity

Finite N_c $\mathcal{N} = 4$ SYM: bootstrapping the full EEC

[Dempsey,Karlsson,Pufu,Zahraee,AZ ’25 and wip]





$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2}$$

S-duality: $g_{YM} \to \dfrac{4\pi}{g_{YM}}$

input:

- SUSY localization
- crossing symmetry
- dispersion relations

output:

- two-sided bounds
- predictions for $z \to 0{,}1$
- θ-independence
- max coupling

Conclusions

What is a scattering amplitude?

S-matrix element

2d celestial conformal correlator

worldsheet correlator

what Feynman diagrams sum to

canonical form/volume

null Wilson loop

solution to bootstraps

CY period

...

What is an energy correlator?

collider observable

matrix element of light-ray operators

gravitational scattering

weighted cross section

measure on the space of celestial measures

RG monotone

...

solution to bootstrap

null Wilson loop

Modern bootstrap methods allow to probe collider phyics in a novel nonperturbative regime.

Open directions

- **Detector algebra**:

  space of detectors, light-ray OPEs, renormalization/mixing, positivity, soft/BMS charges, horizontal trajectories, light-ray algebras

- **States&theories**:

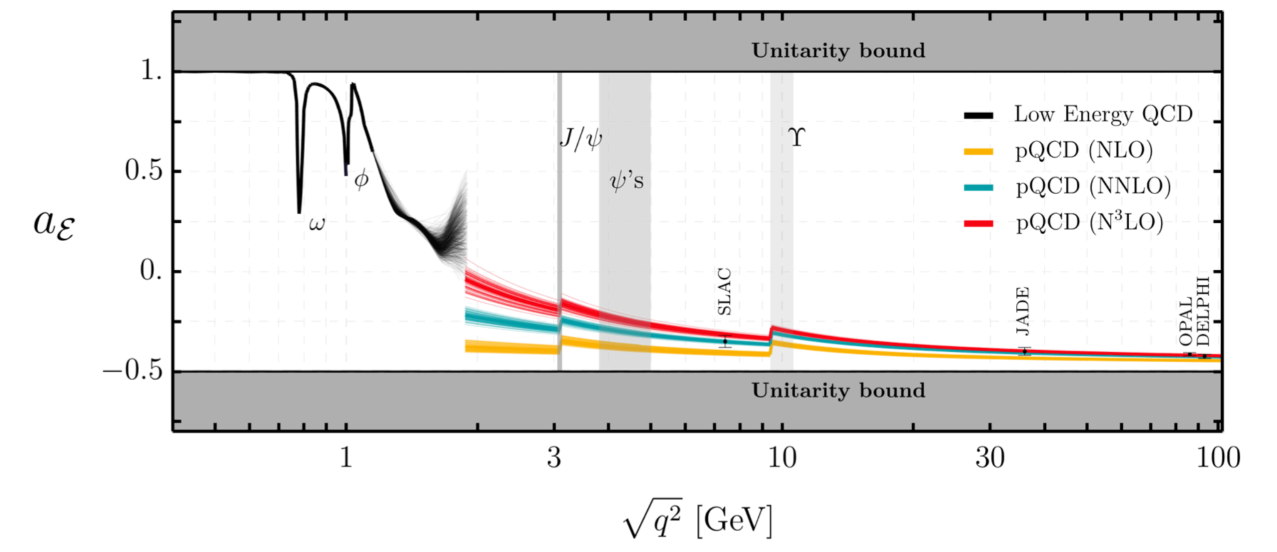
  spinning sources, heavy sources, Love numbers, finite mass effects, media, other CFTs (3d Ising/M-theory collider)

- **Positivity, angular RG**:

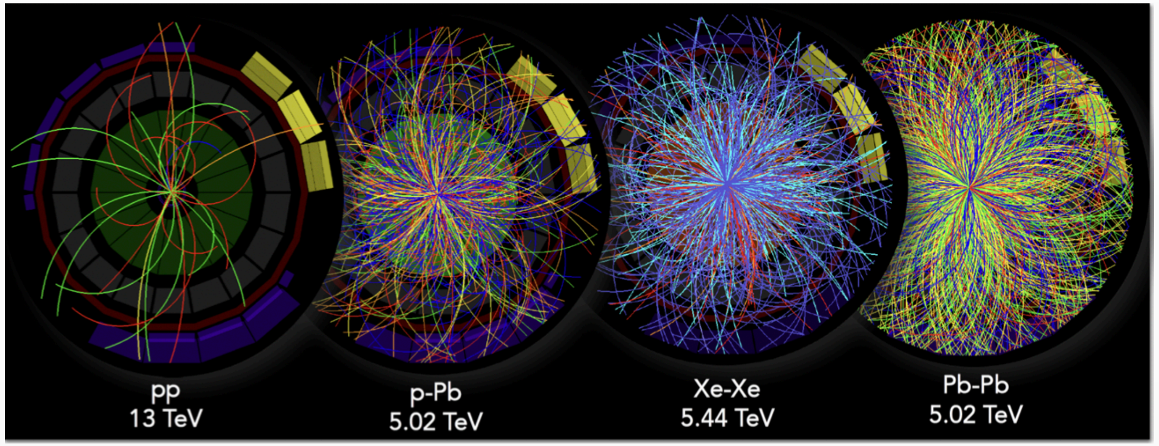
  higher-point correlators, RG monotones, moment problem, constraints on the AdS EFTs

Thank you for your attention!

Extra slides



[Riembau, Son '25]



Many new measurements:

- e^+e^- Colliders: Re-analysis using ALEPH archival data (Bossi *et al.*, 2025a,b).
- *ep* Colliders: Measurement by HERA (H1).
- Proton-proton Colliders: Measurements by the CMS (Hayrapetyan *et al.*, 2024a), ALICE (Acharya *et al.*, 2024a, 2025; Hwang) and STAR (Tamis, 2024; STA, 2025; Shen) collaborations.
- Proton-Pb Colliders: Measurement by the ALICE collaboration (Nambrath, a,b).
- Pb-Pb Colliders: Measurements by the ALICE (Rai) and CMS (CMS, 2023, 2025; Chekhovsky *et al.*, 2025) collaborations.

Measurement of energy correlators inside jets and determination of the strong coupling $\alpha_S(m_Z)$

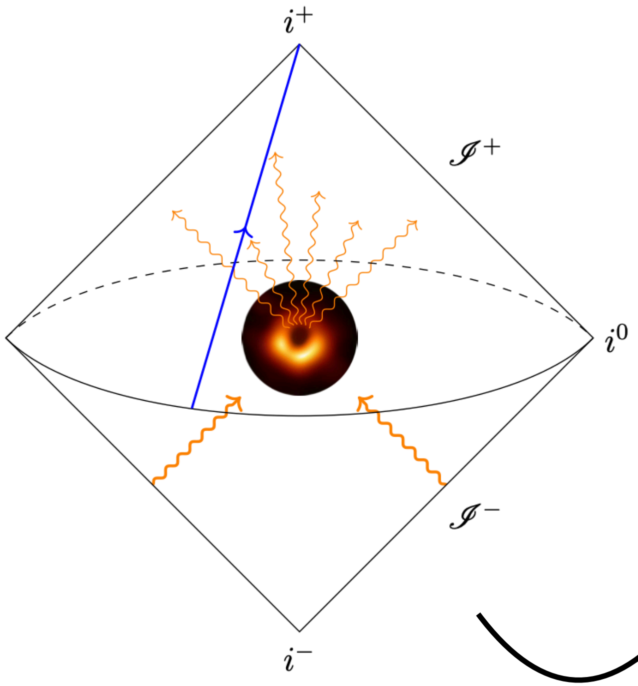
The CMS Collaboration*

finement and asymptotic freedom. By comparing the ratio of the measured three- and two-particle energy correlator distributions with theoretical calculations that resum collinear emissions at approximate next-to-next-to-leading logarithmic accuracy matched to a next-to-leading order calculation, the strong coupling is determined at the Z boson mass: $\alpha_S(m_Z) = 0.1229^{+0.0040}_{-0.0050}$, the most precise $\alpha_S(m_Z)$ value obtained using jet substructure observables.

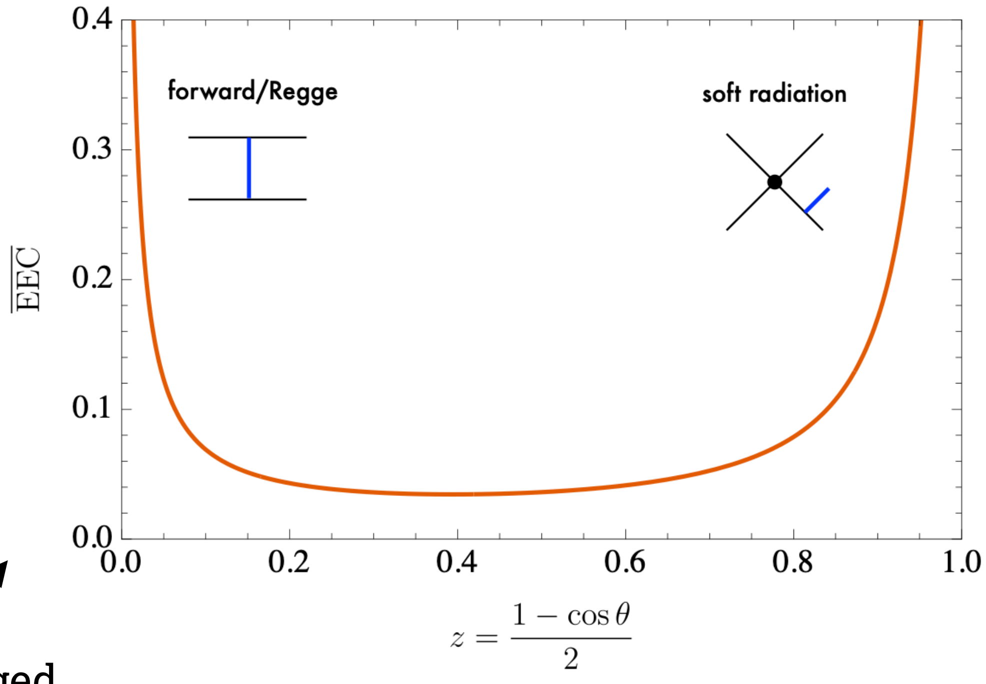
Four-dimensional gravity EEC

[Gonzo,Pokraka '20]
[Herrmann,Kologlu,Moult '24]
[Chicherin,Korchemsky,Sokatchev,AZ '25]

Energy correlators provide infrared-finite inclusive observables in perturbative 4d gravity.



beam-averaged



- Perturbatively calculable

[pure gravity+matter, $\mathcal{N} = 8$ SUGRA, ST]

- Lorentz-invariant

[depends on $G_N s, \alpha' s$ and z]

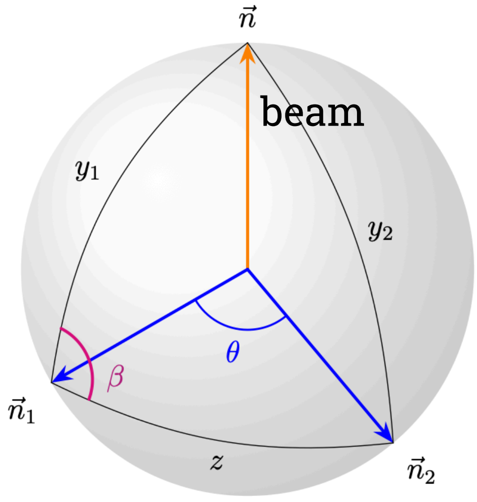
- Positive and analytic

The **beam-averaged EEC** is nonintegrable at $z = 0$, for incoming plane waves ($\sigma_{\text{tot}} = \infty$ forward limit).
Introducing wave packets **requires** dressing (BMS symmetry precludes interference).

[Carney,Chaurette,Neuenfeld,Semenoff '18]

Four-dimensional gravity: EEC limits

Remarkably, the $z \to 1$ limit is nonperturbatively calculable



$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \frac{C(y_1,\beta)}{(1-z)^{1-B_{\rm gr}/2}}$$

Weinberg's Bremsstrahlung function:

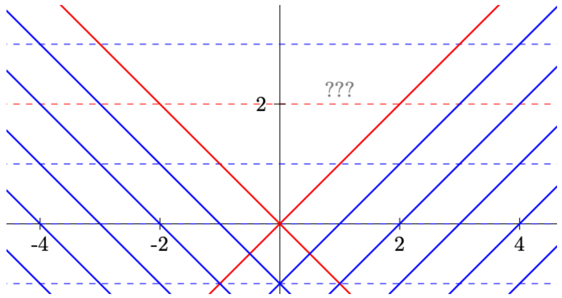
$$B_{\rm gr} = -4\frac{G_N s}{\pi}\left(y_1 \log y_1 + (1-y_1)\log(1-y_1)\right)$$

[Weinberg '65]

$$y_1 = (1-\vec{n}\cdot\vec{n}_1)/2$$

In the collinear limit $z \to 0$, the energy-energy correlator appears regular (absence of collinear divergences).

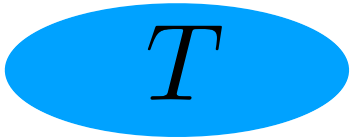
- The algebra of detector operators in gravity?



- Detector soft theorems

[Chang,Chen,Simmons-Duffin,Zhu '25]
[Gonzalez,Salzer '25] [Moult,Narayanan,Pasterski '25]

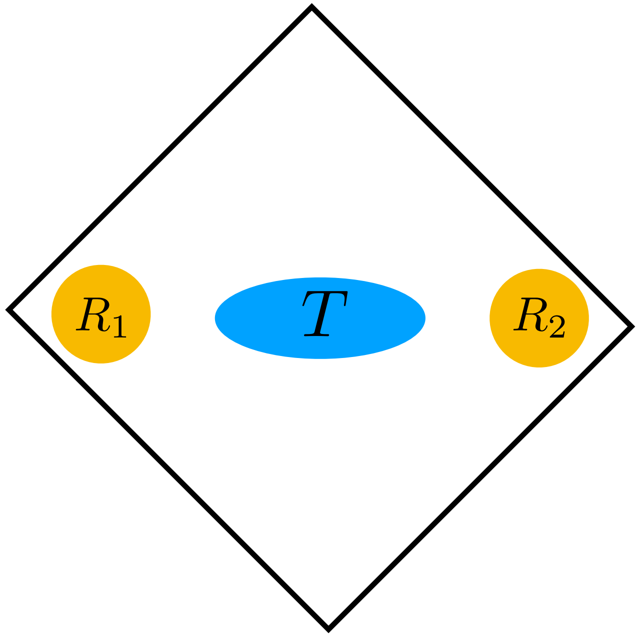
Why is a point different from a null line in QFT?



$$\langle 0|T|0\rangle = 0$$

$$\langle \psi|T|\psi\rangle \geq 0$$

$$|\langle \psi|T|0\rangle|^2 \leq \langle \psi|T|\psi\rangle\langle 0|T|0\rangle = 0$$



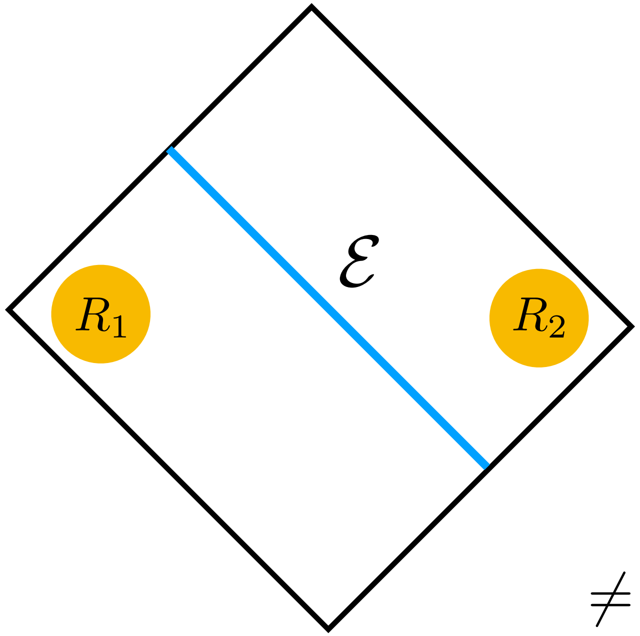
$$\langle 0|O_{R_1} T O_{R_2}|0\rangle = \\ \langle 0|O_{R_1} O_{R_2} T|0\rangle = 0$$

$$\langle \psi_1|T|\psi_2\rangle = 0$$

$$T = 0$$

[Epstein,Glaser,Jaffe '65]

For the line, the argument fails

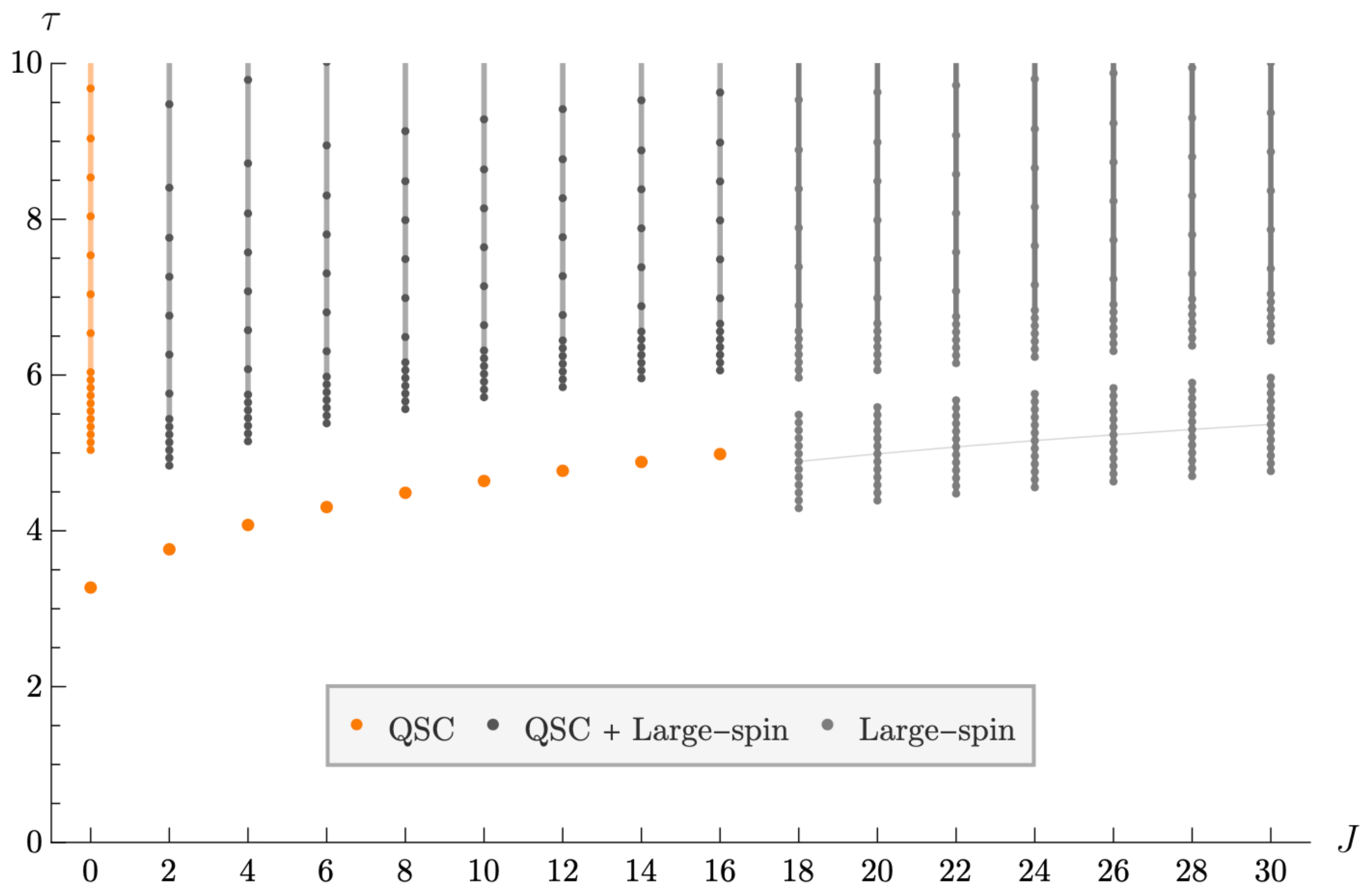


$$\langle 0|O_{R_1}\mathcal{E}O_{R_2}|0\rangle \\ \neq \langle 0|O_{R_1}O_{R_2}\mathcal{E}|0 = 0$$

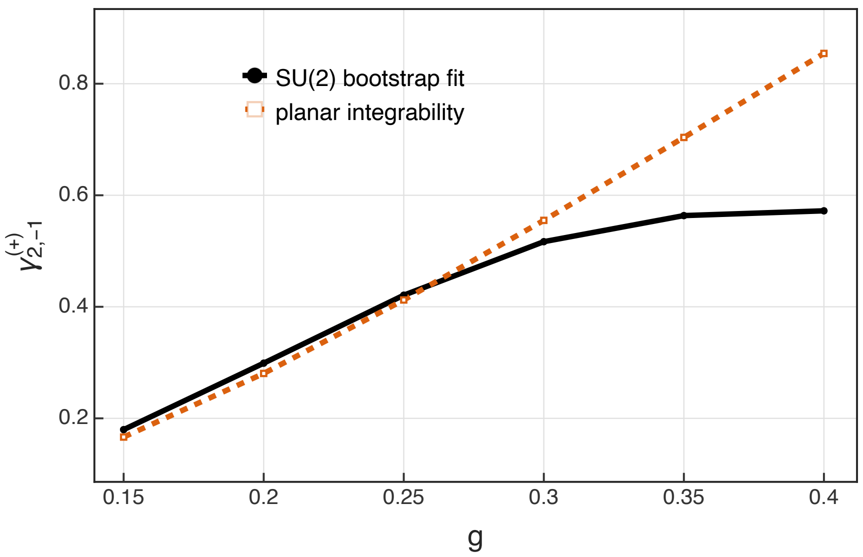
Energy flux is in fact positive!

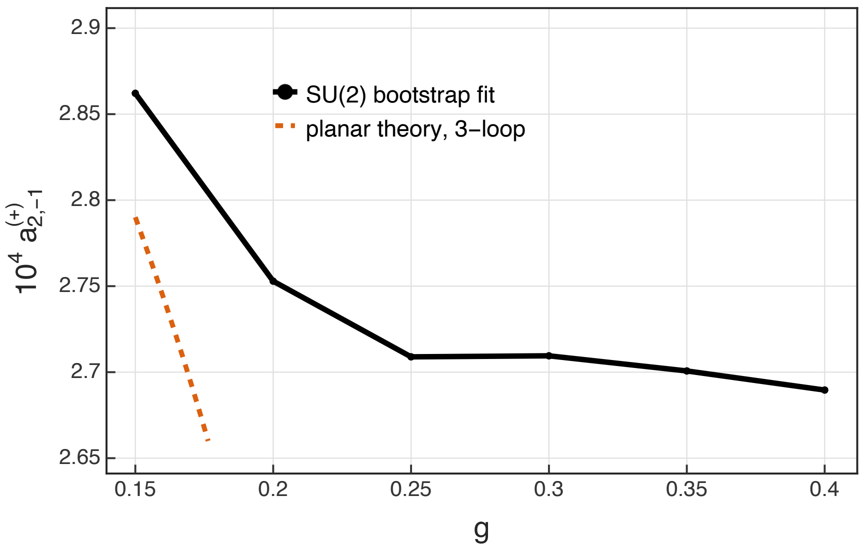
$$\mathcal{E} \geq 0$$

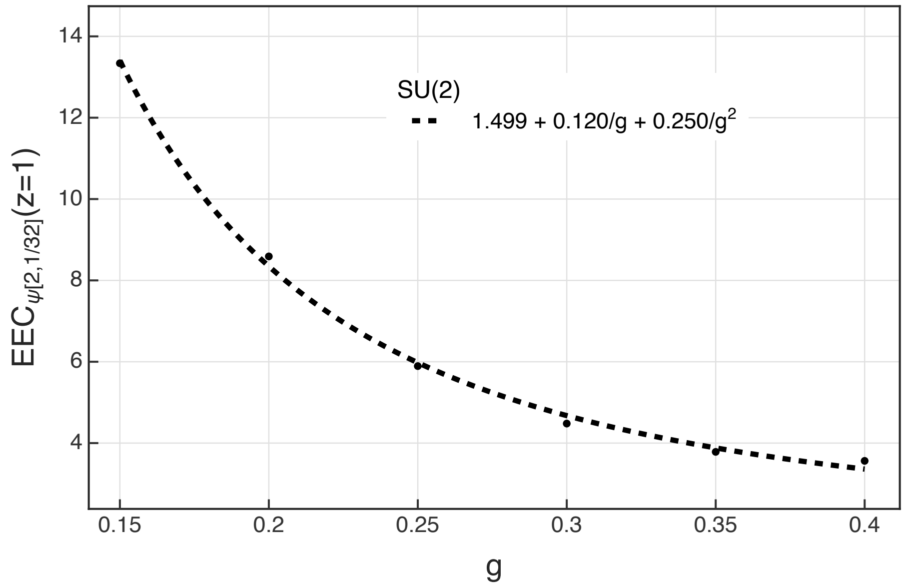
Spectral gap from integrability (planar theory)



Light-ray OPE and back-to-back (at finite Nc)





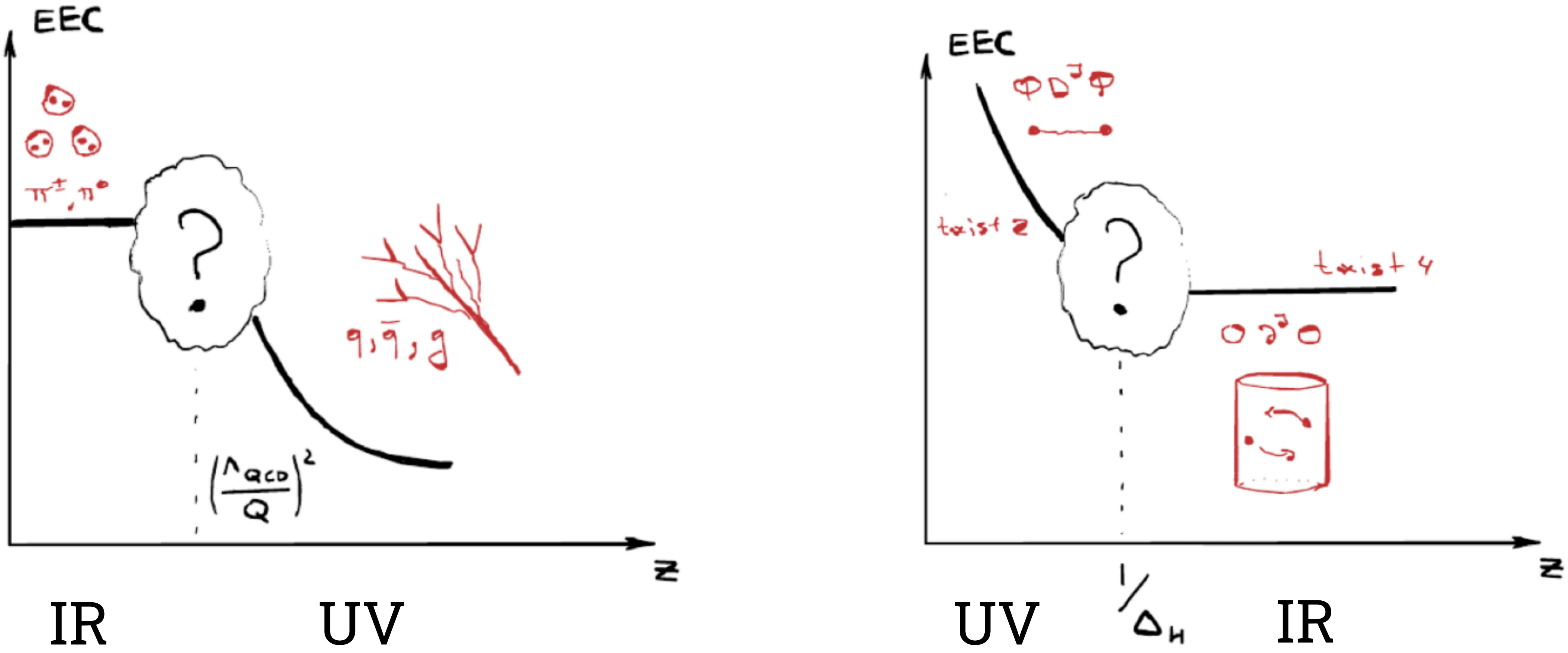


starts at 30

$$g = 0$$

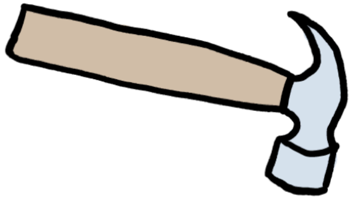
[Dempsey et al. , to appear]

Angular RG


EEC
π±, π0
q, q̄, g
(ΛQCD/Q)²
z
IR
UV
EEC
φD^Jφ
twist 2
twist 4
O∂^JO
1/ΔH
z
UV
IR

Local operators vs light-ray operators

Local operators/measure at a point

 $= \sum_i h_i \mathcal{O}_i$

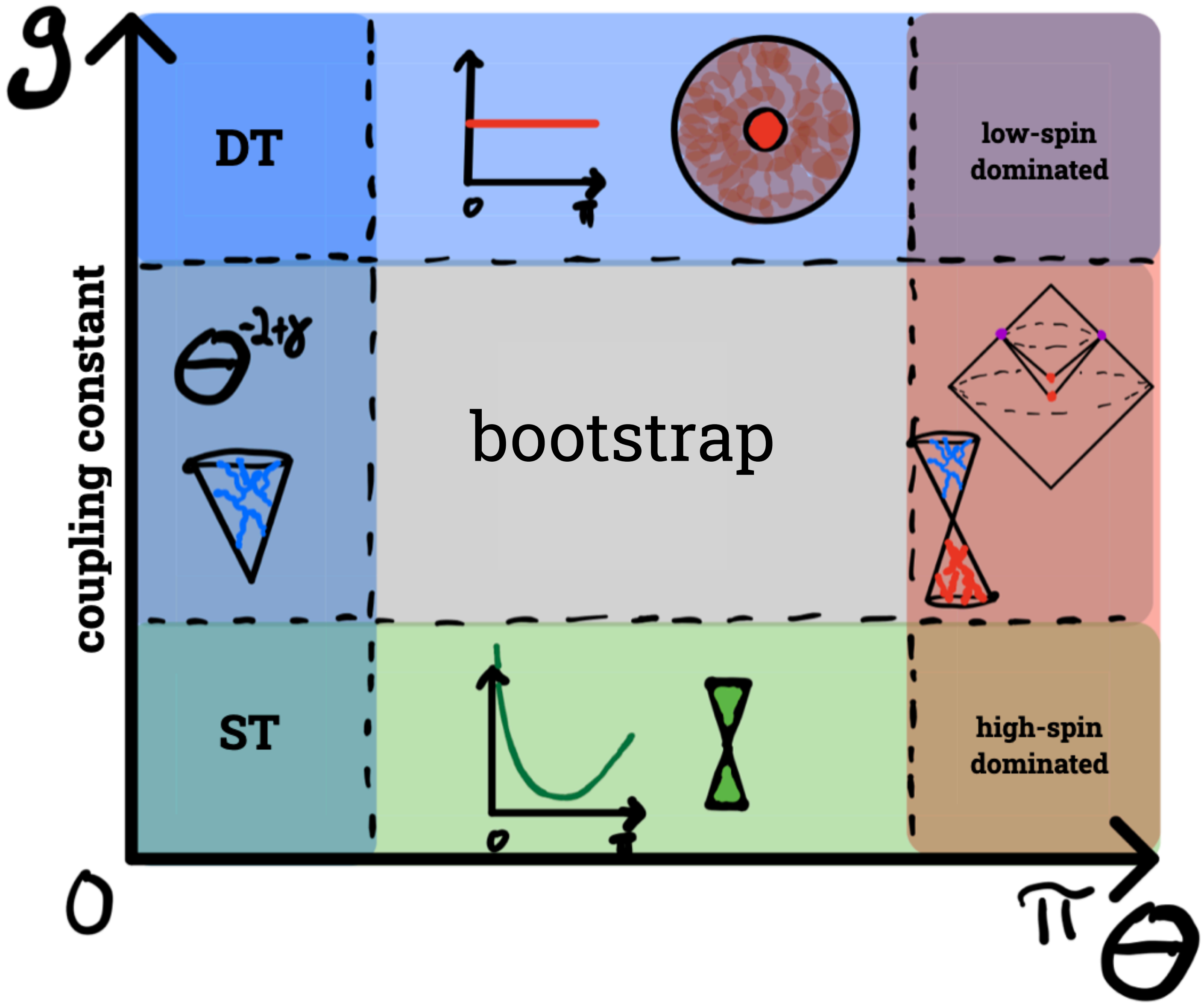
- integer spin [theory-dependent]
- changes sign
- radial quantization
  [operator/state correspondence]
- UV divergence
- Renormalization/mixing
- OPE
- line defect

Detector operators/measure in cross sections

 $= \sum_j c_j \mathcal{D}_j$

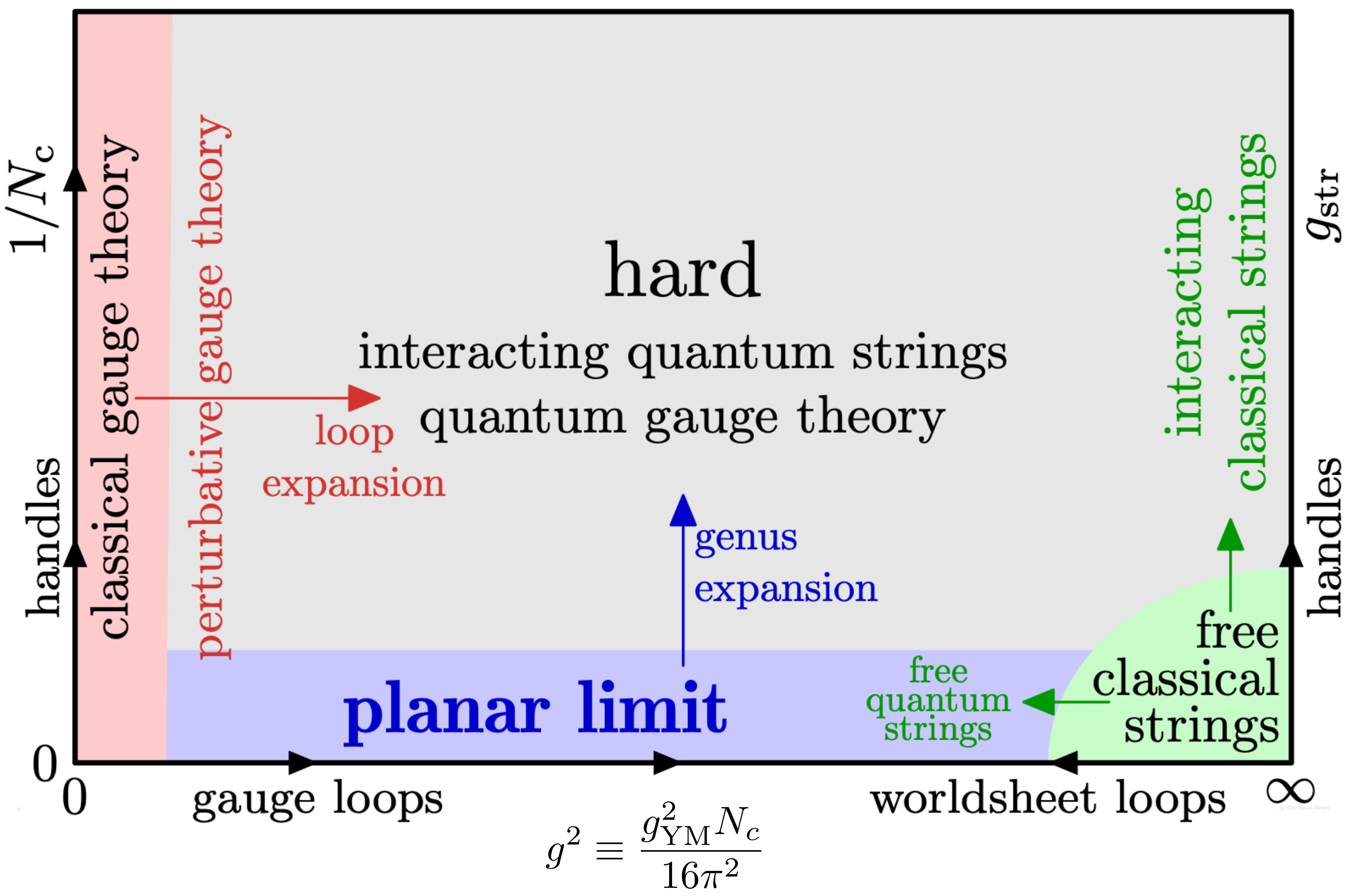
- complex spin [theory-dependent]
- positivity [ANEC, higher-spin ANEC]
- detector algebra?
- IR divergence
- Renormalization/mixing
  [Caron-Huot,Kologlu,Kravchuk,Meltzer,Simmons-Duffin '22]
- light-ray OPE
  [Chang,Kologlu,Kravchuk,Simmons-Duffin,AZ '19-20]
- null defect?
  [Erramilli,Kulp,Popov '25]

CFT: changing degrees of freedom across coupling



- **analogy**: parton/hadron in QCD and weak/strong in large-N CFTs
- the crossover tracks changing effective degrees of freedom in the OPE

Different regimes of the gauge theory: (N_c, g)



[Integrability in AdS/CFT Review, Beisert et al]