BH scattering in N=8 supergravity to all loops

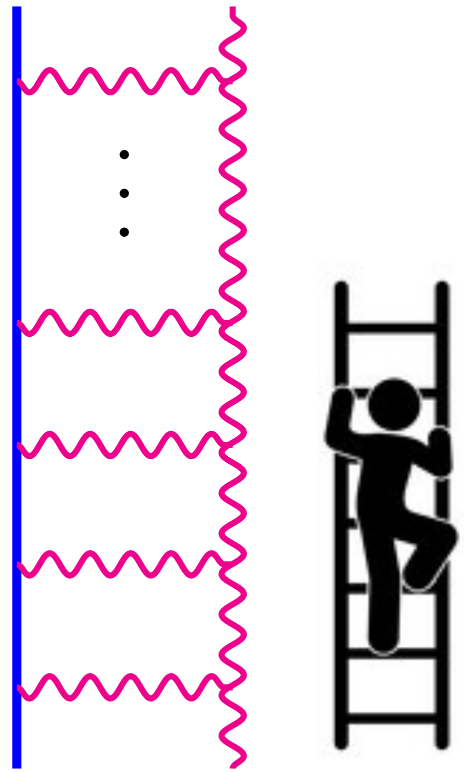
UPPSALA
UNIVERSITET

Henrik Johansson

Uppsala University

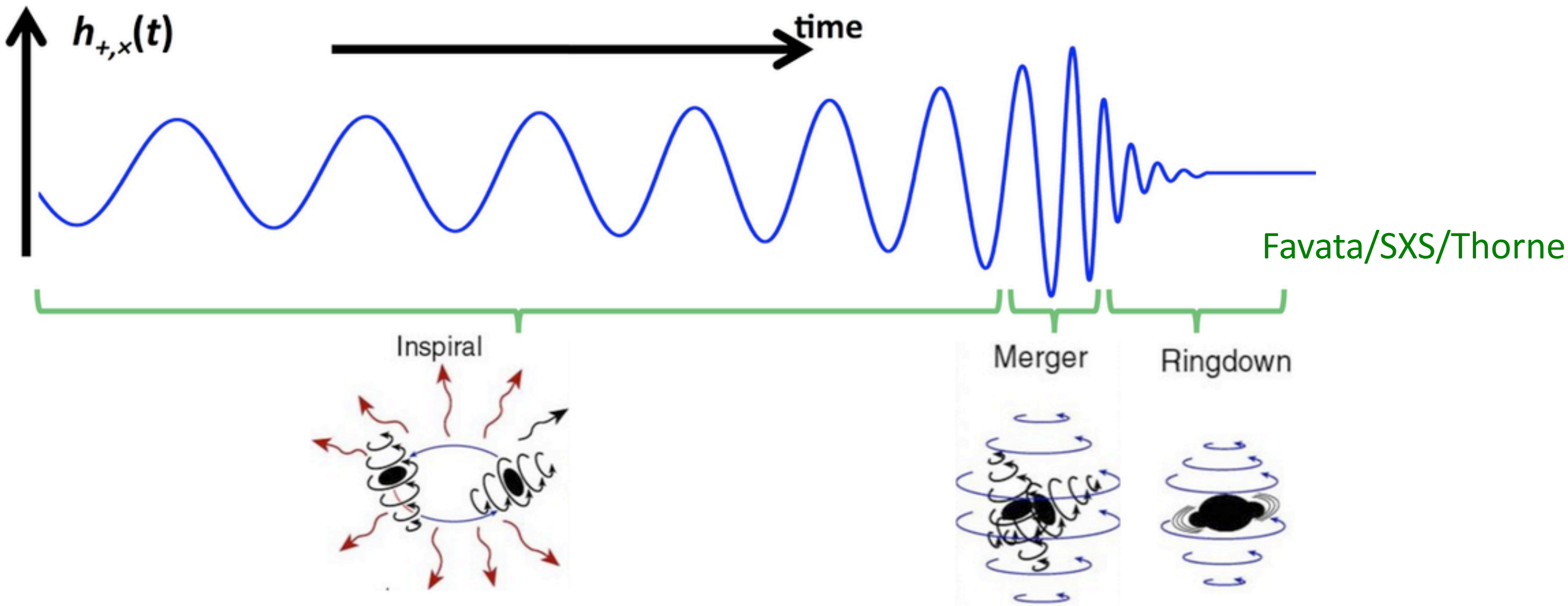
June 30, 2026

AMPLITUDES 2026



WIP: **Maor Ben-Shahar, Lucile Cangemi, HJ, Chia-Hsien Shen**

Motivation: gravitational waves



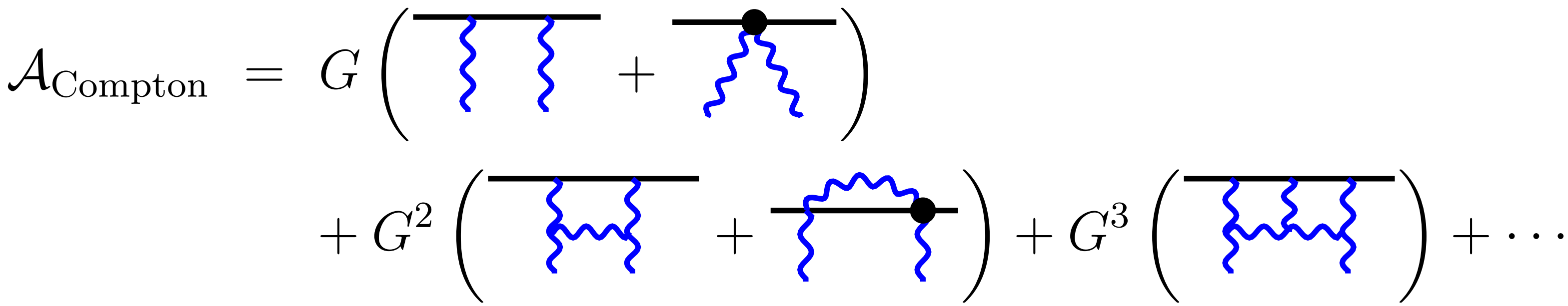
PM expansion: Westfahl et al.; Portilla; Ledvinka et al.; Damour; Bini, Damour; Vines; Bern, Cheung, Roiban, Shen, Solon, Zeng; Kälin, Liu, Porto; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Ruf, Parra-Martinez; Brandhuber, Chen, Travaglini, Wen; Driesse, Jakobsen, Mogull, Plefka, Sauer, [....]

Scattering waveforms: Georgoudis, Heissenberg, Ingrid Vazquez-Holm; Rodolfo Russo, Herderschee, Roiban, Teng; Brandhuber, Brown, De Angelis, Gowdy, Chen, Travaglini; Bohnenblust, Ita, Kraus, Schlenk, [....]

See talks by: Kosower, Jakobsen, Bohnenblust, Ruf, Isabella, Chen, Shi, Brown, O'Connell

This talk: BH Compton scattering

- **Compton scattering probes detailed structure of BH:**

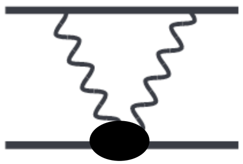


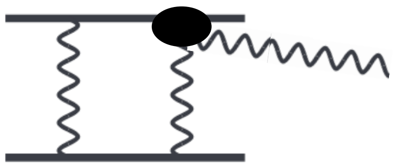
- **Spin multipoles, Love/tidal, finite-size, dissip. effects (BHPT)**

Caron-Huot, Miguel Correia, Isabella, Solon; Bjerrum-Bohr, Chen, Eriksen, Shah; Akpinar, del Duca, Gonzo; Akpinar; Bautista, Driesse, Haddad, Jakobsen; Ivanov, Li, Parra-Martinez, Zhou; Bohnenblust, Eriksen, Hoogeveen, Jakobsen, Plefka; Brunello, Meo, Smith [....]
Aoude, Haddad, Helset; Bautista, Guevara, Kavanagh, Vines; Bjerrum-Bohr, Chen, Skowronek; Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov; Ben-Shahar, Cangemi, HJ [...]

→ talks by: Jakobsen, Bohnenblust, Isabella, Chen

- Needed for N^{k}LO calculations:





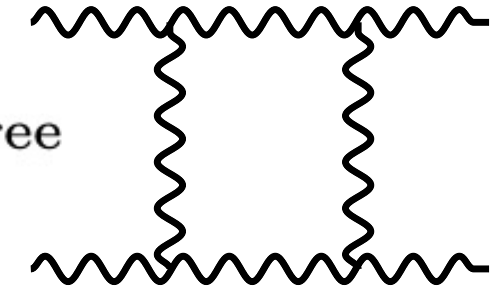
A physical toy model: $\mathcal{N}=8$

- $\mathcal{N}=8$ supergravity is the ”best” point-particle gravity model (not an unphysical toy model)
- Quantum 4D theory: UV finite, at least up to 6 loops
- Many BPS BH solutions, natural stringy extensions
- Will focus on ½-BPS BH's: no-triangle property → finite size effects absent in 4D   Caron-Huot, Zahraee ('18)
- BH solutions valid in $D < 11$ dimensions, goes well with dim. reg.
- Lots of experience with $\mathcal{N}=8$ amplitudes (up to 5 loops)

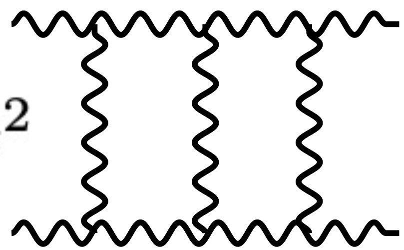
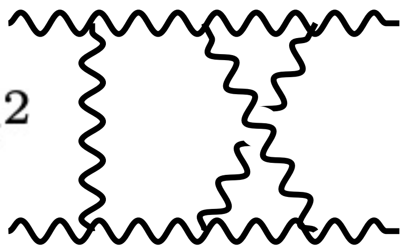
  Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, Zeng;
  Di Vecchia, Naculich, Russo, Veneziano, White; Heissenberg;
  Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng; [...]

$\mathcal{N}$=8 supergravity amplitudes

Massless 4-graviton amplitudes:

$stu\mathcal{M}^{\text{tree}}$  + perms

Green, Schwarz, Brink ('82)

$stu\mathcal{M}^{\text{tree}} \Big(s^2$  $+ s^2$  + perms $\Big)$

Bern, Dixon, Dunbar,
Perelstein, Rozowsky ('98)

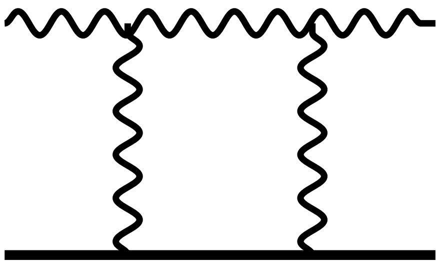
Valid in general dimension:

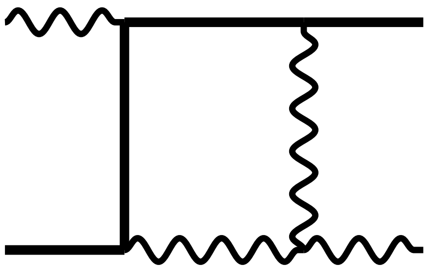
$$2 \leq D \leq 11$$

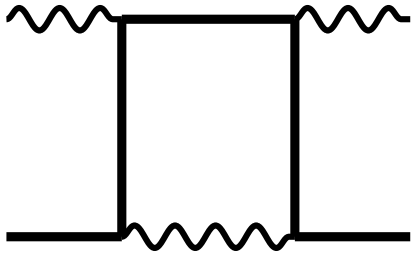
Kaluza-Klein compactification

$$SO(1,10) \to SO(1,d) \times SO(10-d)$$

Massive multiplet $\mathcal{N}=8$ KK momentum: $p^\mu \to (p^\mu, M, 0, \ldots, 0)$

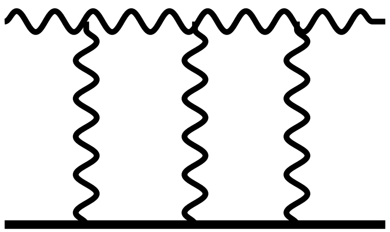






Classical Limit: $\mathcal{O}(\hbar^0)$ $\mathcal{O}(\hbar^1)$ $\mathcal{O}(\hbar^2)$

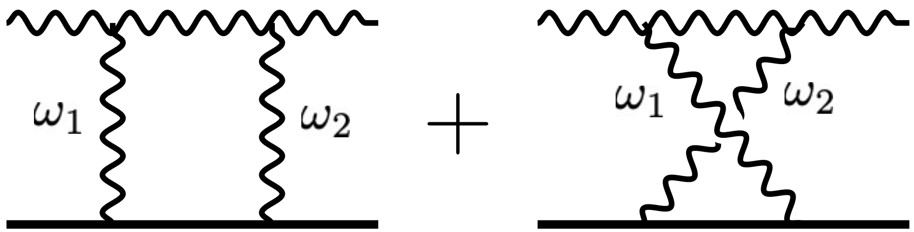
At 2 loops:



$\mathcal{O}(\hbar^0)$

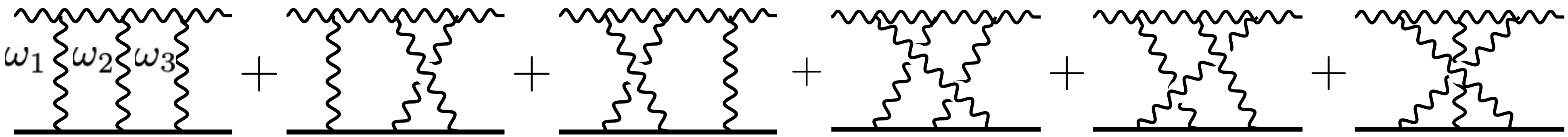
Details of classical limit

Sum over classical ladder and crossed ladder:



BH background conserves graviton energy:

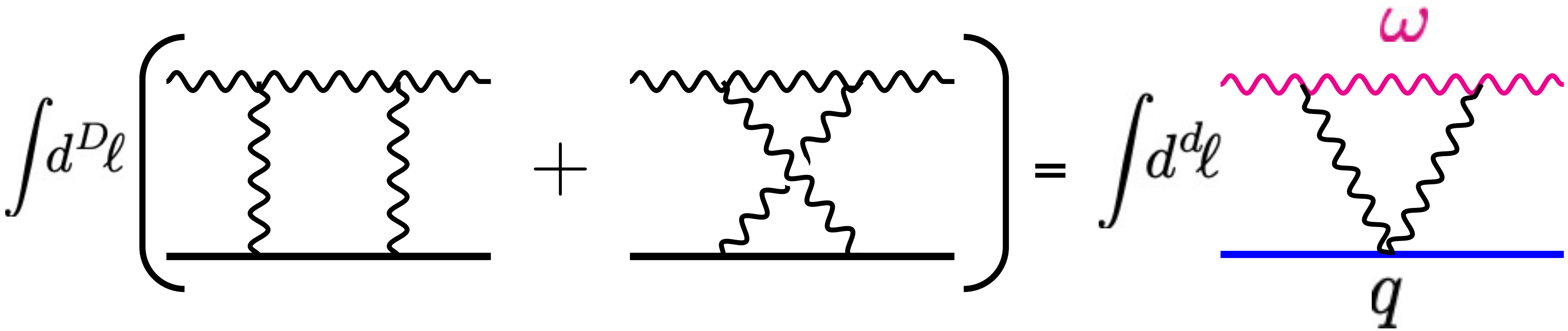
$$\hat{\delta}(\omega_1+\omega_2)\left[\frac{1}{\omega_1-i0}+\frac{1}{\omega_2-i0}\right]=i\hat{\delta}(\omega_1)\hat{\delta}(\omega_2)$$



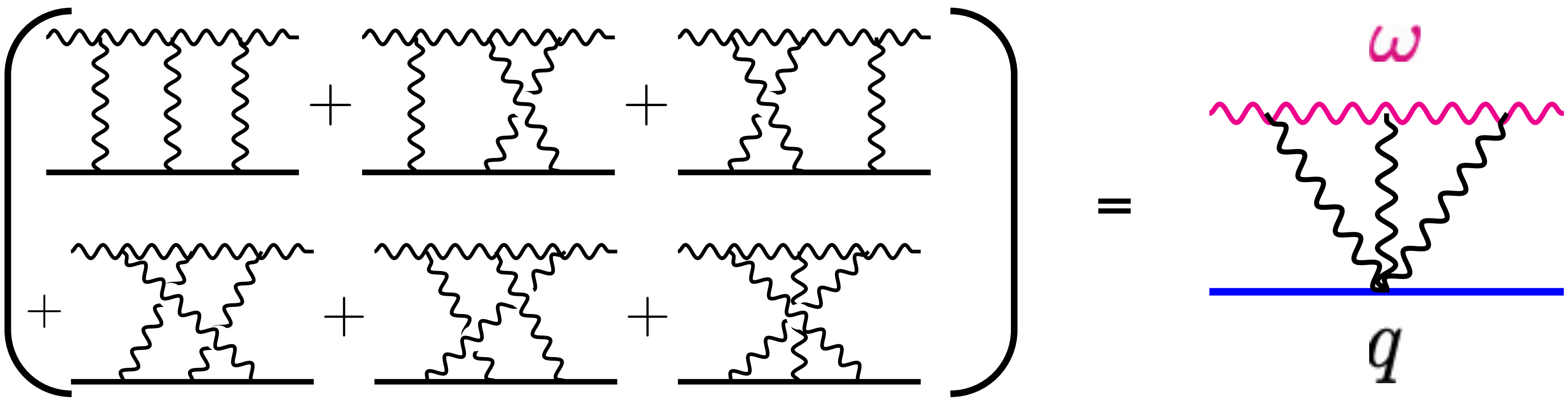
$$\hat{\delta}(\omega_1+\omega_2+\omega_3)\left[\frac{1}{\omega_1-i0}\frac{1}{\omega_1+\omega_2-i0}+\text{perm}\right]=-\hat{\delta}(\omega_1)\hat{\delta}(\omega_2)\hat{\delta}(\omega_3)$$

Saotome, Akhoury ('12)

Ladders become fan diagrams



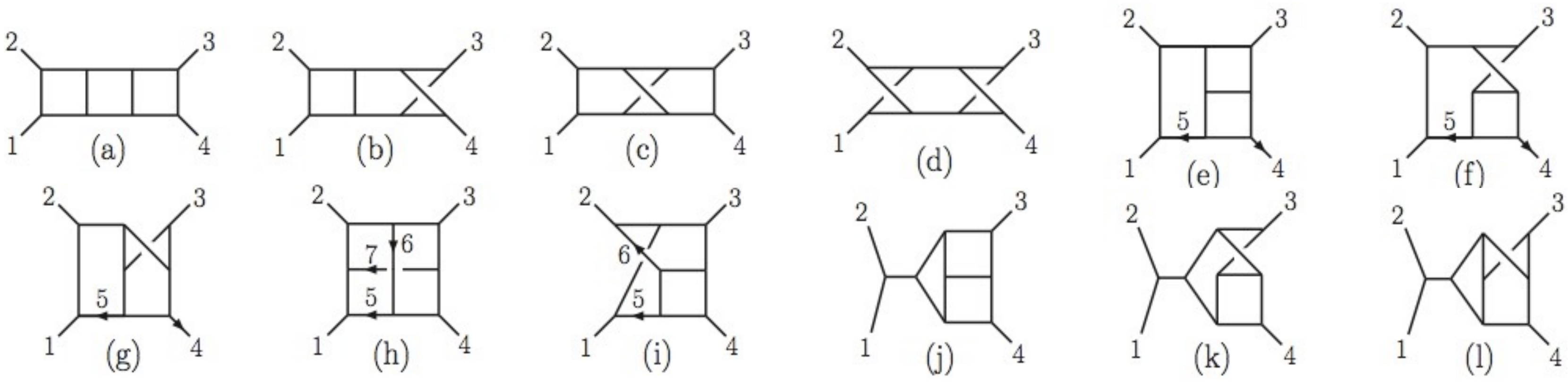
Only spatial integral: $d = D - 1$



Graviton energy ω $\rightarrow$ effective mass (tachyonic)

What about Numerators?

$\mathcal{N}$=8 supergravity has complicated numerators:



$\mathcal{N}=8$ Supergravity **non-BCJ numerators:**
$[s^2]^2$
$[s\,(l_1+k_4)^2]^2$
$(sl_{1,2}^2+tl_{3,4}^2-st)^2-s^2(2(l_{1,2}^2-t)+l_5^2)l_5^2-t^2(2(l_{3,4}^2-s)+l_6^2)l_6^2$ $-s^2(2l_1^2l_8^2+2l_2^2l_7^2+l_1^2l_7^2+l_2^2l_8^2)-t^2(2l_3^2l_{10}^2+2l_4^2l_9^2+l_3^2l_9^2+l_4^2l_{10}^2)+2stl_5^2l_6^2$
$(sl_{1,2}^2-tl_{3,4}^2)^2-(s^2l_{1,2}^2+t^2l_{3,4}^2+\frac{1}{3}stu)l_5^2$

Bern, Carrasco, Dixon, HJ, Kosower, Roiban ('08)

Numerators simplify!

In the classical limit:

→ observe that all *N* =8 supergravity ladder numerators become

”’rung-rule” numerators:

$$\sim stu\mathcal{M}^{\text{tree}}(s^2)^{L-1} \qquad \rightarrow \qquad q^2\mathcal{M}^{\text{tree}}(2M\omega)^{2L}$$

→ and all non-ladder diagrams vanish (quantum)

→ However, standard 3-4 loop BCJ numerators not well behaved (super-classical and quantum terms mix). [Open problem!]

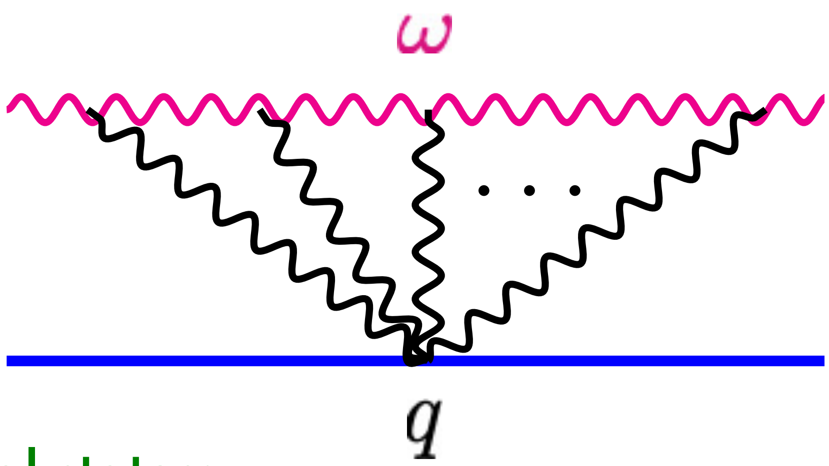
→ No proof that ladder ”’rung-rule” numerators always emerge, but one can make heuristic arguments e.g. Carrasco, Edison, HJ

Recap: $\mathcal{N}$=8 simplifications

Ben-Shahar, Cangemi, HJ, Shen

The one diagram that rules them all:

$$q^2\mathcal{M}^{\text{tree}} \times (4GM\omega^2)^L$$



Many options for selecting external states:

- e.g. gravitons & spinning BH:

$$\mathcal{M}^{\text{tree}}(h^+ \to h^\pm) = \frac{2GM(u\cdot\chi_\pm)^4}{q^2\omega^2}e^{a\cdot q_\pm} + \mathcal{O}(\hbar)$$

- e.g. scalars & spherically-symmetric BH:

$$\mathcal{M}^{\text{tree}}(\phi \to \phi) = \frac{2GM\omega^2}{q^2} + \mathcal{O}(\hbar)$$

all unified by the same diagram!

$$\boxed{\begin{aligned} q_\pm &= k \pm k' \\ \chi^\mu_\pm &= \{[k'|u\sigma^\mu|k], \langle k'|\sigma^\mu|k]\} \end{aligned}}$$

Integrate by solving the wave equation!

$$\Box\phi = 0$$

What is the metric?

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = 0$$

Integrate by solving the wave equation!

$$\Box\phi = 0$$

What is the metric? $$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = 0$$

Consider a perturbative solution to: $$\Delta\phi = 0$$

$$\Delta = \Delta_0 + r_S\Delta_1 + r_S^2\Delta_2 + \ldots$$

$$\phi = \phi_0 + \phi_1 + \phi_2 + \ldots \qquad \phi_0(x) = e^{-ip\cdot x}$$

$$\phi_1 = -r_S\Delta_0^{-1}\Delta_1\phi_0$$

$$\phi_2 = -r_S^2\Delta_0^{-1}\Delta_2\phi_0 + \underbrace{r_S^2\Delta_0^{-1}\Delta_1\Delta_0^{-1}\Delta_1\phi_0}_{\text{ladder/fan diagram}}$$

Integrate by solving the wave equation!

$$\Box\phi = 0$$

What is the metric? $$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = 0$$

Consider a perturbative solution to: $$\Delta\phi = 0$$

$$\Delta = \Delta_0 + r_S\Delta_1 + r_S^2\Delta_2 + \dots$$

$$\phi = \phi_0 + \phi_1 + \phi_2 + \dots \qquad \phi_0(x) = e^{-ip\cdot x}$$

$$\phi_1 = -r_S\Delta_0^{-1}\Delta_1\phi_0$$

$$\phi_2 = -r_S^2\Delta_0^{-1}\Delta_2\phi_0 + \underbrace{r_S^2\Delta_0^{-1}\Delta_1\Delta_0^{-1}\Delta_1\phi_0}_{\text{ladder/fan diagram}}$$

Effective metric linear in coupling!

$$\Delta_1 = \frac{1}{4\pi r}(-\partial_t^2) \xrightarrow{F.T.} \frac{1}{\vec{\ell}^2}\omega^2 \qquad \Delta_2 = \Delta_3 = 0$$

Finding the metric!

Ben-Shahar, Cangemi, HJ, Shen

Wave equation must take the form:

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = \frac{1}{\sqrt{-g}}\Big(\Box_0 + \frac{4GM}{r}(u\cdot\partial)^2\Big)\phi = 0\,,$$

Gives inverse metric: $$g^{\mu\nu} = \frac{1}{\sqrt{-g}}\Big(\eta^{\mu\nu} + 4GM\frac{u^\mu u^\nu}{r}\Big)$$

and metric: $$g_{\mu\nu} = \sqrt{|g|}\Big(\eta_{\mu\nu} - \frac{4GM}{4GM+r}u_\mu u_\nu\Big)$$

Finding the metric!

Ben-Shahar, Cangemi, HJ, Shen

Wave equation must take the form:

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = \frac{1}{\sqrt{-g}}\left(\Box_0 + \frac{4GM}{r}(u\cdot\partial)^2\right)\phi = 0\,,$$

Gives inverse metric: $$g^{\mu\nu} = \frac{1}{\sqrt{-g}}\left(\eta^{\mu\nu} + 4GM\frac{u^\mu u^\nu}{r}\right)$$

and metric: $$g_{\mu\nu} = \sqrt{|g|}\left(\eta_{\mu\nu} - \frac{4GM}{4GM + r}u_\mu u_\nu\right)$$

line element form: $$ds^2 = \frac{1}{\sqrt{1+\frac{4GM}{r}}}dt^2 - \sqrt{1+\frac{4GM}{r}}\,(d\vec{x}^2)$$

volume element/warp factor: $$\sqrt{-g} = \sqrt{1+4GM/r}$$

The D-dimensional metric

Since the BH amplitude exists up to $D = 10$, let's generalize:

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = \frac{1}{\sqrt{-g}}\left(\Box_0 + \frac{\lambda}{r^{D-3}}(u\cdot\partial)^2\right)\phi = 0$$

Gives inverse metric:
$$g^{\mu\nu} = \frac{1}{\sqrt{|g|}}\left(\eta^{\mu\nu} + \lambda\frac{u^\mu u^\nu}{r^{D-3}}\right)$$

and metric:
$$g_{\mu\nu} = \sqrt{|g|}\left(\eta_{\mu\nu} - \frac{\lambda}{\lambda + r^{D-3}}u_\mu u_\nu\right)$$

The D-dimensional metric

Since the BH amplitude exists up to $D = 10$, let's generalize:

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\phi = \frac{1}{\sqrt{-g}}\Big(\Box_0 + \frac{\lambda}{r^{D-3}}(u\cdot\partial)^2\Big)\phi = 0$$

Gives inverse metric:
$$g^{\mu\nu} = \frac{1}{\sqrt{|g|}}\Big(\eta^{\mu\nu} + \lambda\frac{u^\mu u^\nu}{r^{D-3}}\Big)$$

and metric:
$$g_{\mu\nu} = \sqrt{|g|}\Big(\eta_{\mu\nu} - \frac{\lambda}{\lambda + r^{D-3}}u_\mu u_\nu\Big)$$

Mass parameter:
$$\lambda = 4GM\frac{\Gamma(\frac{1}{2}(D-3))}{\pi^{\frac{1}{2}(D-3)}}$$

volume element/warp factor:
$$\sqrt{|g|} = \left(1 + \lambda/r^{D-3}\right)^{\frac{1}{D-2}}$$

This is the known (KK) metric of a D0 brane in maximal supergravity!

Solving the wave 4D equation to all loops

The BH wave equation $$\left(\Box_0 + \frac{4GM}{r}(u\cdot\partial)^2\right)\phi = 0$$

is now equivalent to the Coloumb wave eqn in flat space

$$\left(\nabla^2 + \omega^2 + \frac{4GM\omega^2}{r}\right)\phi = 0$$

The radial equation $v(r) = rR(r)$

$$\frac{d^2v}{dr^2} + \left(\omega^2 + \frac{4GM\omega^2}{r} - \frac{\ell(\ell+1)}{r^2}\right)v = 0$$

have well-known solutions (hypergeometric/Whittaker)

Solving the wave 4D equation to all loops

The BH wave equation $\left(\Box_0 + \frac{4GM}{r}(u\cdot\partial)^2\right)\phi = 0$

is now equivalent to the Coloumb wave eqn in flat space

$$\left(\nabla^2 + \omega^2 + \frac{4GM\omega^2}{r}\right)\phi = 0$$

The radial equation $v(r) = rR(r)$

$$\frac{d^2v}{dr^2} + \left(\omega^2 + \frac{4GM\omega^2}{r} - \frac{\ell(\ell+1)}{r^2}\right)v = 0$$

have well-known solutions (hypergeometric/Whittaker)

The all loop amplitude is then

$$\mathcal{M}(\theta) = \frac{1}{2i|\omega|}\sum_{\ell=0}^{\infty}(2\ell+1)(e^{2i(\delta_\ell+\delta_{\rm IR})}-1)P_\ell(\cos\theta)$$

with the partial-wave phase shift $e^{2i\delta_\ell} = \frac{\Gamma(\ell+1+i2GM\omega)}{\Gamma(\ell+1-i2GM\omega)}$

Resumming the 4D $\mathcal{N}$=8 Compton amplitude

Ben-Shahar, Cangemi, HJ, Shen

Shift-enter directly gives:

$$\mathcal{M}(\theta) = -\frac{GM}{2\sin^2(\theta/2)} \exp\left[-2iGM\omega \ln\sin\frac{\theta}{2} + 2i\delta_0 + 2i\delta_{\mathrm{IR}}\right]$$

Using $\sin^2\frac{\theta}{2} = -q^2/(4\omega^2)$ one can massage it into

$$\begin{aligned}\mathcal{M}(\theta) &= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \ln\frac{4\omega^2}{-q^2} + 2i\delta_0 + 2i\delta_{\mathrm{IR}}\right] \\ &= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega\left(\frac{1}{\epsilon} - \ln\frac{-q^2}{\mu^2}\right) + 2i\delta_0\right]\end{aligned}$$

Resumming the 4D $\mathcal{N}$=8 Compton amplitude

Ben-Shahar, Cangemi, HJ, Shen

Shift-enter directly gives:

$$\mathcal{M}(\theta) = -\frac{GM}{2\sin^2(\theta/2)} \exp\left[-2iGM\omega \ln\sin\frac{\theta}{2} + 2i\delta_0 + 2i\delta_{\rm IR}\right]$$

Using $\sin^2\frac{\theta}{2} = -q^2/(4\omega^2)$ one can massage it into

$$\mathcal{M}(\theta) = \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \ln\tfrac{4\omega^2}{-q^2} + 2i\delta_0 + 2i\delta_{\rm IR}\right]$$

$$= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega\left(\frac{1}{\epsilon} - \ln\tfrac{-q^2}{\mu^2}\right) + 2i\delta_0\right]$$

$$= \mathcal{M}^{\rm tree} \exp\left[i\frac{\mathcal{M}^{(1)}}{\mathcal{M}^{\rm tree}} + 2i\delta_0\right]$$

BDS-like form!

$$\delta_0 = -GM\omega\gamma_E + (GM\omega)^3\frac{\zeta_3}{3} - (GM\omega)^5\frac{\zeta_5}{5} + (GM\omega)^7\frac{\zeta_7}{7} + \dots$$

IR divergence -- sleight of hand

$$\mathcal{M}(\theta) = \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \ln \tfrac{4\omega^2}{-q^2} + 2i\delta_0 + 2i\delta_{\mathrm{IR}}\right]$$
$$= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega\Big(\frac{1}{\epsilon} - \ln \tfrac{-q^2}{\mu^2}\Big) + 2i\delta_0\right]$$

The 4D IR divergent phase is matched to dim. reg. by sleight of hand

$$\delta_{\mathrm{IR}} = -GM\omega \ln(4\omega^2 R^2) \overset{!}{=} -\frac{GM\omega}{\epsilon}\Big(\frac{\mu^2}{4\omega^2}\Big)^{-\epsilon} = -GM\omega\Big(\frac{1}{\epsilon} + \ln\frac{4\omega^2}{\mu^2} + \mathcal{O}(\epsilon)\Big)$$

IR divergence -- sleight of hand

$$\mathcal{M}(\theta) = \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \ln \tfrac{4\omega^2}{-q^2} + 2i\delta_0 + 2i\delta_{\mathrm{IR}} \right]$$
$$= \frac{2GM\omega^2}{q^2} \exp\left[iGM\omega \Big(\frac{1}{\epsilon} - \ln \tfrac{-q^2}{\mu^2} \Big) + 2i\delta_0 \right]$$

The 4D IR divergent phase is matched to dim. reg. by sleight of hand

$$\delta_{\mathrm{IR}} = -GM\omega \ln(4\omega^2 R^2) \stackrel{!}{=} -\frac{GM\omega}{\epsilon}\Big(\frac{\mu^2}{4\omega^2}\Big)^{-\epsilon} = -GM\omega\Big(\frac{1}{\epsilon} + \ln\frac{4\omega^2}{\mu^2} + \mathcal{O}(\epsilon)\Big)$$

→ Should be done more carefully!

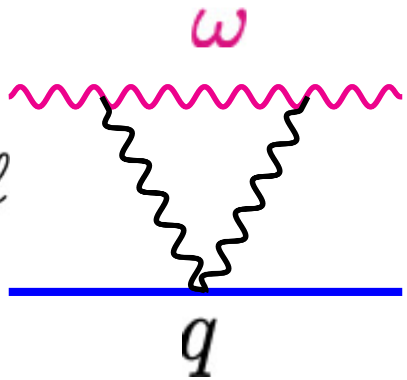
→ Proper treatment in $4-2\epsilon$ dimensions!

→ *N*=8 BH exists up to *D* = 10 dimensions, why not give it a go?

D-dimensional warm-up at 1-loop

Ben-Shahar, Cangemi, HJ, Shen

Consider the 1-loop classical Compton amplitude in *d*-dimensions:

$\int d^d\ell$ 

Observations:

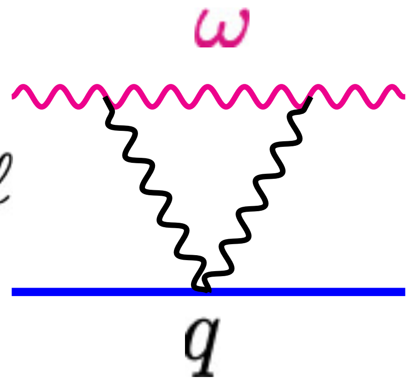
- IR divergent: $d \leq 3$
- UV divergent: $d \geq 6$
- divide by $\Gamma(3 - d/2)$ renders it UV finite!
- evaluates to log() in integer dimensions $d \geq 5$

D-dimensional warm-up at 1-loop

Ben-Shahar, Cangemi, HJ, Shen

Consider the 1-loop classical Compton amplitude in *d*-dimensions:

$$\int d^d\ell$$



Observations:

- IR divergent: $d \leq 3$
- UV divergent: $d \geq 6$
- divide by $\Gamma(3-d/2)$ renders it UV finite!
- evaluates to log() in integer dimensions $d \geq 5$

Work out the integer dimensional results → closed *d*-dimensional formula

$$f^{(d)} = \frac{(1+y^2)^{(3-d)/2}}{d-4}\Bigg[(y^2)^{d/2-2}\Big(\ln(-y^2) + \sum_{k=1}^{(d-5)/2}\frac{(1+y^2)^k}{k}\Big)$$

$$-\, i^{1-d}2^{(d-3)/2}\sqrt{\pi}\,\Gamma(d/2-1)\Big(\frac{(-y^2)^{d/2-2}}{(d-4)!!} - \sum_{k=0}^{(d-5)/2}\frac{(-1)^k(1+y^2)^k}{2^k k!(d-4-2k)!!}\Big)\Bigg]$$

$$\propto \left(\tfrac{4\omega^2}{q^2}\right)^{d/2-2}\Phi\left(1+\tfrac{4\omega^2}{q^2}, 1, \tfrac{d-3}{2}\right) - B\left(\tfrac{1}{2}, \tfrac{d}{2}-1\right)\,{}_2F_1\left(\tfrac{1}{2}; 1; \tfrac{d-1}{2}, 1+\tfrac{4\omega^2}{q^2}\right)$$

Partial-wave amplitude at 1-loop

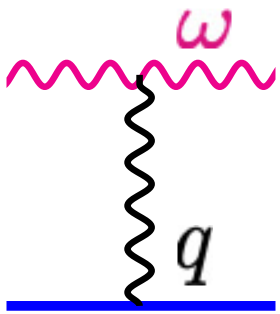
In general *d* Gegenbauer polynomials describe amplitude

$$\mathcal{M}(\theta) = \frac{1}{2i|\omega|}\sum_{\ell=0}^{\infty}\frac{2\ell+d-2}{d-2}(e^{2i(\delta_\ell+\delta_{\rm IR})}-1)\,C_\ell^{(d/2-1)}(\cos\theta)$$

For fixed ℓ, d it is easy to transform to partial waves:

First, at tree-level we get that the q propagator transform to

$$e^{2i(\delta_\ell+\delta_{\rm IR})} = 1 - \frac{2iGM|\omega|\ell!}{(d-3)(d-1)_{\ell-1}} + \mathcal{O}(G^2)$$



IR divergent in $d \leq 3$

Partial-wave amplitude at 1-loop

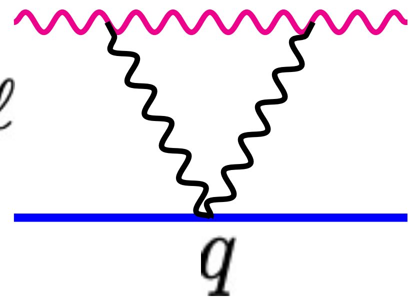
In general *d* Gegenbauer polynomials describe amplitude

$$\mathcal{M}(\theta) = \frac{1}{2i|\omega|}\sum_{\ell=0}^{\infty}\frac{2\ell+d-2}{d-2}\left(e^{2i(\delta_\ell+\delta_{\mathrm{IR}})}-1\right)C_\ell^{(d/2-1)}(\cos\theta)$$

For fixed ℓ, d it is easy to transform to partial waves:

At 1-loop level we get that triangle transforms to

$$\int d^d\ell$$



$$\frac{d-2}{2\ell+d-2}\frac{\mathcal{M}_\ell^{(1)}}{G^2M^2\omega^2} = -\frac{(-i)^d 2^{2d-11}}{(d-4)(d-2)}\left[\frac{\ell!}{(d-3)(d-1)_{\ell-1}}\right]^2 + (-1)^\ell 2^{d-\ell-8}B\left(\tfrac{1}{2},\tfrac{d}{2}-1\right)\frac{\Gamma(d-4)\Gamma(d/2)\Gamma(2d-5)}{\Gamma(d+\ell-2)^2\Gamma(3d/2+\ell-4)}\mathcal{P}^{(\ell)}(d)$$

Recursion formula for polynomial:

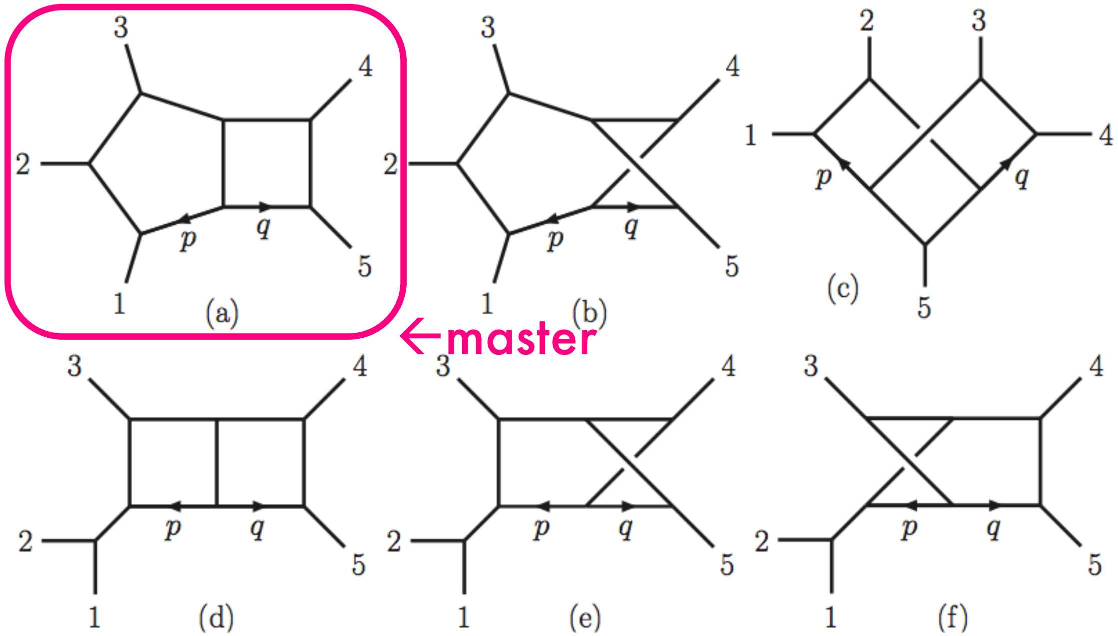
$$\mathcal{P}^{(\ell+2)} = (2\ell+d)\left(d^2-2\ell^2-2d(\ell+5)-16\right)\mathcal{P}^{(\ell+1)} - (\ell+1)^2(\ell+d-2)^2(2\ell-d+4)(2\ell+3d+8)\mathcal{P}^{(\ell)},$$

Conclusion

- N = 8 supergravity is a testbed for advanced classical calc.
- Compton ampl. is dramatically simper than: 2→2, waveform
- Showed that all-loop resummation is possible for ½ BPS BH
- Identified the metric with D0 brane (11D KK mode)
- BH has no horizon, no finite-size corrections. Exact PP.
- Calculation captures infinitely many ladder diagrams of GR
- Further work required for higher-loop D-dimensional solution
- Outlook: Analyzing spin multipoles; UV divergences/finite-size corrections in higher dimensions, connection to ¼-BPS BH.

Extra slides

Example: 2-loop 5-pts $\mathcal{N}$=4 SYM & $\mathcal{N}$=8 SG



Carrasco, HJ
1106.4711

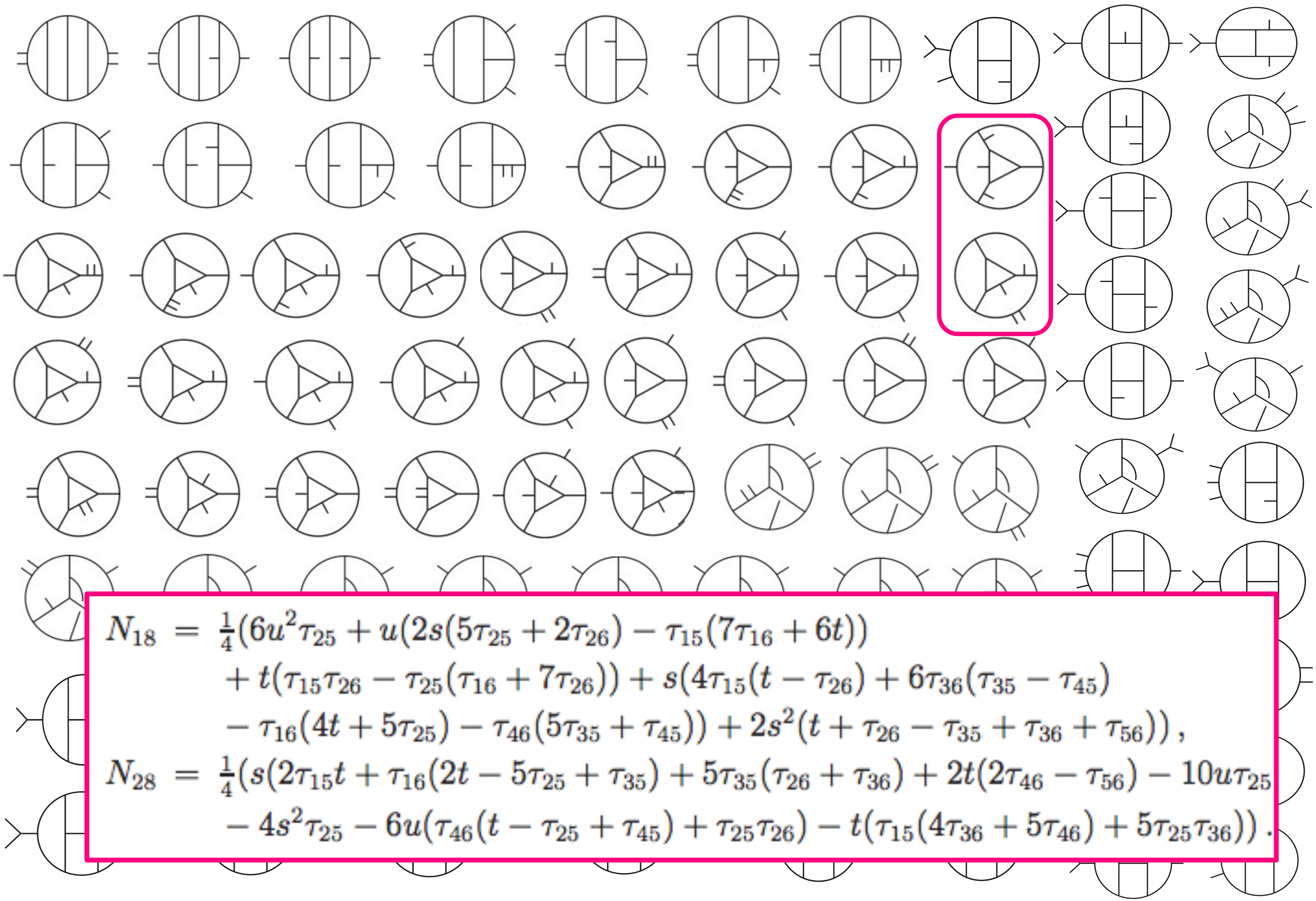
color-kinematics duality
+ unitarity method
→ simple numerators

$\mathcal{I}^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a),(b)	$\frac{1}{4}\Big(\gamma_{12}(2s_{45}-s_{12}+\tau_{2p}-\tau_{1p})+\gamma_{23}(s_{45}+2s_{12}-\tau_{2p}+\tau_{3p}) + 2\gamma_{45}(\tau_{5p}-\tau_{4p})+\gamma_{13}(s_{12}+s_{45}-\tau_{1p}+\tau_{3p})\Big)$
(c)	$\frac{1}{4}\Big(\gamma_{15}(\tau_{5p}-\tau_{1p})+\gamma_{25}(s_{12}-\tau_{2p}+\tau_{5p})+\gamma_{12}(s_{34}+\tau_{2p}-\tau_{1p}+2s_{15}+2\tau_{1q}-2\tau_{2q}) + \gamma_{45}(\tau_{4q}-\tau_{5q})-\gamma_{35}(s_{34}-\tau_{3q}+\tau_{5q})+\gamma_{34}(s_{12}+\tau_{3q}-\tau_{4q}+2s_{45}+2\tau_{4p}-2\tau_{3p})\Big)$
(d)-(f)	$\gamma_{12}s_{45}-\frac{1}{4}\Big(2\gamma_{12}+\gamma_{13}-\gamma_{23}\Big)s_{12}$

$$\tau_{ip}=2k_i\cdot p$$

direct calculation in GR would naively give $\sim 10^{18}$ terms

4-loops: 85 diagrams, 2 masters



$$
\begin{aligned}
N_{18} \;=\;& \tfrac{1}{4}(6u^2\tau_{25} + u(2s(5\tau_{25}+2\tau_{26}) - \tau_{15}(7\tau_{16}+6t)) \\
&+ t(\tau_{15}\tau_{26} - \tau_{25}(\tau_{16}+7\tau_{26})) + s(4\tau_{15}(t-\tau_{26}) + 6\tau_{36}(\tau_{35}-\tau_{45}) \\
&- \tau_{16}(4t+5\tau_{25}) - \tau_{46}(5\tau_{35}+\tau_{45})) + 2s^2(t+\tau_{26}-\tau_{35}+\tau_{36}+\tau_{56}))\,, \\
N_{28} \;=\;& \tfrac{1}{4}(s(2\tau_{15}t + \tau_{16}(2t-5\tau_{25}+\tau_{35}) + 5\tau_{35}(\tau_{26}+\tau_{36}) + 2t(2\tau_{46}-\tau_{56}) - 10u\tau_{25} \\
&- 4s^2\tau_{25} - 6u(\tau_{46}(t-\tau_{25}+\tau_{45}) + \tau_{25}\tau_{26}) - t(\tau_{15}(4\tau_{36}+5\tau_{46}) + 5\tau_{25}\tau_{36}))\,.
\end{aligned}
$$

Kerr/root-Kerr Effective theories

From matching to Kerr/root-Kerr amplitudes one can infer EFTs:

EFTs	$s = {}^{1}\!/_{2}$	$s = 1$	$s = {}^{3}\!/_{2}$	$s = 2$	$s = {}^{5}\!/_{2}$	$s \geq 3$
Kerr	Major.	Proca	Rar.-Sch.	KK grav.	HS	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	HS	HS

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Massless limit exist for $s \leq 2$ Kerr and $s \leq 1$ root-Kerr

Kerr and root-Kerr related by double copy:

$$M(1\phi^{s}, 2\bar{\phi}^{s}, 3h^{\pm}) = iA(1\phi^{s_{\rm L}}, 2\bar{\phi}^{s_{\rm L}}, 3A^{\pm})A(1\phi^{s_{\rm R}}, 2\bar{\phi}^{s_{\rm R}}, 3A^{\pm})$$

For $s > 2$ Kerr dynamics = higher-spin and higher-derivative EFTs