Bootstrapping QCD amplitudes

Dmitry Chicherin

LAPTh, CNRS
Annecy-le-Vieux

based on joint works `Phys.Rev.Lett. 136 (2026) 18` and `2602.02783`

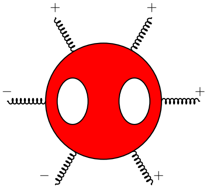
with Sérgio Carrôlo, Johannes Henn, Qinglin Yang, Yang Zhang

and `2512.17881` in collaboration with

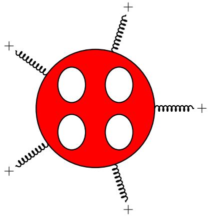
Johannes Henn, Yongqun Xu, Shun-Qing Zhang, Yang Zhang

Amplitudes 2026, London
30 June 2026

Bootstrap Six-gluon Two-loop MHV amplitudes in QCD and Five-gluon Four-loop all-plus amplitude in pure Yang-Mills

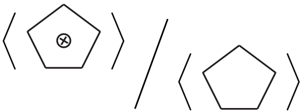


planar six-gluon
MHV amplitudes in QCD



planar five-gluon
all-plus amplitudes in YM

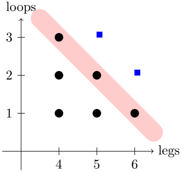
$\Updownarrow$



Correlation function
of null Wilson loops and
Lagrangian (half-BPS operator)
in planar $\mathcal{N} = 4$ sYM

We are guided by the **maximal transcendentality principle**

Loop amplitudes in gauge theories



- Huge progress in calculation of five-point amplitudes in the last decade see talk by Simone Zoia
- Phenomenology of high energy physics see talk by Thomas Gehrmann
- Hidden structures of gauge theories and deep connection to mathematics
- We focus on “the most complicated terms” of planar amplitudes

$$\mathcal{A}^{(L)} = \sum_i \mathcal{R}_i f_i$$

Transcendental weight w	Rational prefactors $\mathcal{R}$	Transcendental functions f
$w = 0$	Hard	Easy
$\vdots$	$\vdots$	$\vdots$
$w = 2L$	Easy	Hard

Proliferation of rational prefactors and their algebraic complexity is a major bottleneck at high miltiplicity

Enormous success of the **bootstrap** for six-, seven-, … point amplitudes and three-, four-point form factors of half-BPS operators in $\mathcal{N} = 4$ **super-Yang-Mills theory**

The Steinmann Cluster Bootstrap for $\mathcal{N} = 4$ Super Yang-Mills Amplitudes

Simon Caron-Huot,[1] Lance J. Dixon,[2] James M. Drummond,[3] Falko Dulat,[2] Jack Foster,[3] Ömer Gürdoğan,[3,4] Matt von Hippel,[5,6] Andrew J. McLeod[2,6] and Georgios Papathanasiou*[7]

weight n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
First entry	1	3	9	26	75	218	643	1929	5897	?	?	?	?	?
Steinmann	1	3	6	13	29	63	134	277	562	1117	2192	4263	8240	?
Ext. Stein.	1	3	6	13	26	51	98	184	340	613	1085	1887	3224	5431

Table 1: The dimensions of the hexagon, Steinmann hexagon, and extended Steinmann hexagon spaces at symbol level.

How much can be done for massless scattering in planar **Yang-Mills/QCD**?

- Success of the bootstrap depends on the balance between the ansatz size and strength of physical constraints

$$\mathcal{A}^{(L)} = \sum_{ij} c_{ij}\, \mathcal{R}_i\, f_j$$

- Reach kinematics of non-DCI amplitudes
- The rational prefactors $\mathcal{R}_i$ are less constrained in nonsupersymmetric theories
- The transcendental functions f_j are constructed from complicated symbol alphabets with many letters. For example, **167** letters for two-loop six-point amplitudes and **56** letters for three-loop five-point amplitudes

Our bootstrap is informed by the Feynman integral calculation

Scenario A

Symbol alphabet

$\Downarrow$

Functions
for the alphabet
& Adjacencies

$\Downarrow$

Bootstrap

$\Downarrow$

Observable

<u>Scenario B</u>

Function space
of Master Integrals

$\Downarrow$

Bootstrap

$\Downarrow$

Observable

Scenario C

Master Integrals
& The integrand

$\Downarrow$

IBP

$\Downarrow$

Observable

I. Six-gluon Two-loop MHV amplitudes in QCD at leading color

Setup of the calculation: infrared finite two-loop hard function

$$\mathcal{H}^{(2)}_{\mathrm{QCD}} = \lim_{\epsilon\to 0}\left(\mathcal{A}^{(2)} - I^{(1)}\mathcal{A}^{(1)} + \frac{1}{2}\left(I^{(1)}\right)^2\mathcal{A}^{(0)}\right)$$

- Subtract the universal IR divergences with dipole $I^{(1)} = -\frac{1}{\epsilon^2}\sum_i \left(-\frac{s_{ii+1}}{\mu^2}\right)^{-\epsilon}$

[Catani '98]

- Hard functions are less sensitive to dimensional-regularization than amplitudes
- Leading color approximation: $N_c, N_f \gg 1$ and N_f/N_c is fixed

$$\mathcal{H}^{(2)}_{\mathrm{QCD}} = \mathcal{H}^{(2)}_{\mathrm{YM}} + \left(\frac{N_f}{N_c}\right)\mathcal{H}_{\substack{\text{quark}\\ \text{loop}}} + \left(\frac{N_f}{N_c}\right)^2\mathcal{H}_{\substack{\text{two quark}\\ \text{loops}}}$$

- Several independent MHV helicities

$$--++++ \qquad -+-+++ \qquad -++-++$$

Maximal transcendentality projection and prescriptive unitarity

$$\mathcal{A}^{(L)}\Big|_{\substack{\text{max.}\\ \text{transc.}}} = \sum_k R_k^{(L)} \times \underbrace{I_{L,k}}_{\text{IR free}} + \sum_{\ell=0}^{L-1} \sum_k \underbrace{R_k^{(\ell)} \times I_{\ell,k}}_{\substack{\text{affected by}\\ \text{IR subtraction}}}$$

- rational prefactor $R_k^{(\ell)}$ appears for the first time at ℓ-loop order
- pure integrals $I_{\ell,k}$ evaluate to functions of maximal transcendentality
- d-log integrand evaluates to a function of the maximal transcendentality

[Arkani-Hamed, Bourjaily, Cachazo, Trnka '10]

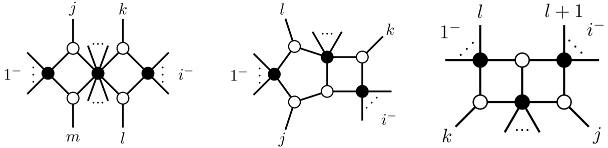
- rational prefactors $R_k^{(\ell)}$ (for the hard function!) are computed by 4D maximal residues [Bobadilla, Henn '21]

Prescriptive unitarity refines the generalized unitarity: optimal integral basis

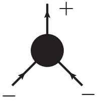
[Bourjaily, Herrmann, Trnka '17]

IR-finite maximal cut contours for MHV gauge theory amplitudes have been classified

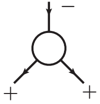
[Bourjaily, Herrmann, Langer, McLeod, Trnka '19]



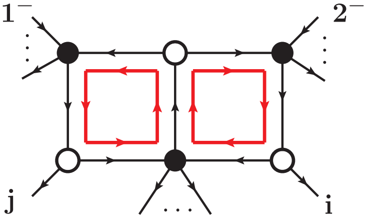
Rational prefactors of the hard function are computed by 4D on-shell diagrams



MHV_3



$\overline{\mathrm{MHV}}_3$

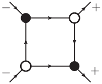


Concise expressions for the two-loop rational prefactors

$$R_{i,j}/\mathrm{PT} = -1 + 12u_{ij} - 30u_{ij}^2 + 20u_{ij}^3$$

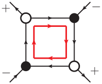
$$u_{ij} \equiv \frac{\langle 1i\rangle\langle 2j\rangle}{\langle 12\rangle\langle ij\rangle}$$

- All maximal residues are computed by on-shell diagrams [Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka '12]
- All MHV maximal residues in $\mathcal{N}=4$ sYM are Parke-Taylor prefactors



$$\sim \quad \mathrm{PT}$$

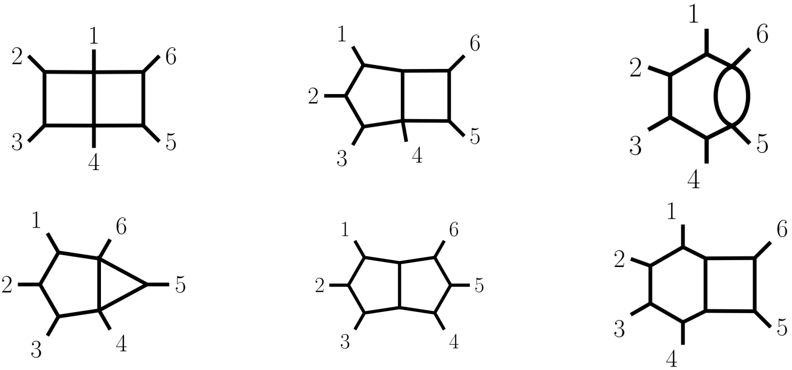
- Non-singlet on-shell diagrams evaluate to non-PT rational prefactors [Bourjaily, Herrmann, Langer, Patatoukos, Trnka, Zheng '21]



$$\sim \quad \mathrm{PT}\left(\frac{\langle 12\rangle\langle 34\rangle}{\langle 13\rangle\langle 24\rangle}\right)^4$$

- Conformal invariance of the 4D rational prefactors $\mathbb{K}_{\alpha\dot\alpha} = \sum_i \frac{\partial^2}{\partial\lambda_i^\alpha \partial\tilde\lambda_i^{\dot\alpha}}$

Canonical differential equations for Two-loop Six-point Planar Feynman integrals



[Samuel Abreu, Pier Monni, Ben Page, Johann Usovitsch '24]

[J. Henn, Antonela Matijašić, Julian Miczajka, Tiziano Peraro, Yingxuan Xu, Yang Zhang '21 '22 '24 '25]

see talk by Yang Zhang

- 7 kinematic variables
- 167 alphabet letters (in four dimensions, for finite parts)
- 639 transcendental functions* for finite part of amplitudes

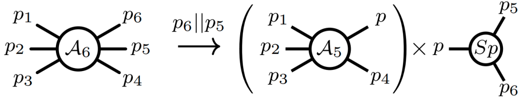
* weight-4 symbols of the Feynman integrals

Example: Bootstrapping the split-helicity $--++++$ MHV hard function in YM from physical limits

ansatz $\mathcal{H}^{(2)}_{\mathrm{YM}}$	2412
scaling	996
no spurious poles	
$\langle 36\rangle = 0$	333
$\langle 35\rangle = 0$	614
$\langle 34\rangle = 0$	629
$\langle 45\rangle = 0$	343
collinear	
$--++++$	1785
$+--+++$	1307
$++--++$	1785
$+++--+$	1646
$-++++-$	1646
triple collinear	
$--++++$	1836
$++--++$	724
Total	**2412**

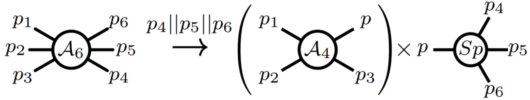
Inhomogeneous input

— splitting functions $h \to h_5 h_6$



Consistency in the triple collinear limit

— kinematics factorizes



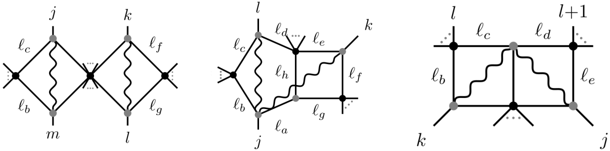
167 letters in the ansatz

but **137** letters in the hard function

All MHV helicities and prescriptive unitarity

$$\mathcal{H}^{(2)} = \sum_k R_k^{(2)} \times \underbrace{I_k}_{\text{IR finite}} + \sum_k R_k^{(1)} \times \underbrace{f_k}_{\text{IR subtracted}} + \text{PT} \times f_0$$

where $R_k^{(\ell)}$ are genuinely ℓ-loop rational prefactors, and I_k are IR-finite prescriptive unitarity integrals

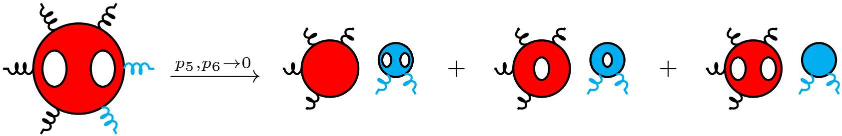


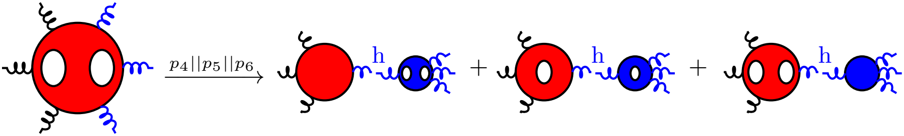
[Bourjaily, Herrmann, Langer, McLeod, Trnka '19]

Symbol bootstrap for IR-subtracted part of the hard function and IBP reductions for IR-finite 4D prescriptive unitarity integrals

helicity	$--++++$	$-+-+++$	$-++-++$
IR-subtracted ansatz	238	872	582

Two-loop triple collinear and double-soft splitting functions from the bootstrapped amplitudes





Agreement of the two-loop triple-collinear splittings with recent results for $gg \to Hgg$

[De Laurentis, Ita, Kuschke, Ruf, Sotnikov 2605.04009]

Maximal transcendentality principle for collinear/soft splittings:

Outlook and Future directions

- Maximal transcendentality principle for the hard functions

$$\mathcal{H}^{(2)}_{\mathrm{YM}}\Big|_{\substack{\text{max.}\\ \text{transc.}}} \xrightarrow[\text{all } R\to 1]{\text{double-boxes}\to 0} \mathcal{H}^{(2)}_{\mathcal{N}=4\,\mathrm{sYM}}$$

- Extend the two-loop symbol bootstrap to functions
- Six-gluon two-loop NMHV amplitudes and amplitudes with quark legs
- Contributions of lower transcendentalities: proliferation of rational prefactors
- Nonplanar contributions and their rational prefactors?
- Higher multiplicity amplitudes? For example, the form of n-gluon two-loop split-helicity MHV amplitude is very restricted by the *prescriptive unitarity*
- Better understanding of the six-parton MHV function space? Cluster algebras and adjacencies? [Bossinger, Drummond, Glew, Gürdoğan, Wright '25; Pokraka, Spradlin, Volovich, Weng '25] Additional letters for NMHV amplitudes

II. Three-loop Pentagonal Wilson with Lagrangian insertion in $\mathcal{N} = 4$ super-Yang-Mills and its duality with Four-loop Five-gluon all-plus planar Yang-Mills amplitude

All-Plus is "the simplest" amplitude in pure Yang-Mills

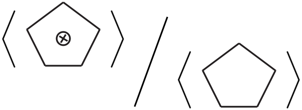
$$\mathcal{A}_{\mathrm{YM},n}^{(L),\text{all-plus}} = \sum_{\sigma \in S_n/Z_n} \mathrm{tr}\left(T^{a_{\sigma_1}} \ldots T^{a_{\sigma_n}}\right) A_{\mathrm{YM},n}^{(L)}(\sigma_1^+, \ldots, \sigma_n^+) + \mathcal{O}\left(N_c^{-1}\right)$$

- Dihedral symmetry
- Vanishes at tree level, $A_{\mathrm{YM},n}^{(0)} = 0$
- $(L+1)$-loop all-plus amplitude has the functional complexity of L-loop MHV amplitude. For example, "the most complicated terms" of four-loop all-plus amplitude are of transcendental weight 6
- Conformal invariance of one-loop amplitude $A_{\mathrm{YM},n}^{(1)}$ and of the rational prefactors from "the most complicated terms" of L-loop amplitudes

Polygonal Null Wilson loop with Lagrangian insertion in planar $\mathcal{N} = 4$ super-Yang-Mills

- Light-like polygonal contour $(x_i - x_{i+1})^2 = 0$
- Wilson loop $W[x_1, \ldots, x_n] \equiv \mathrm{tr}_{\mathrm{F}} \mathrm{Pexp}(\oint dx^\mu A_\mu(x))$
- chiral form of the Lagrangian $\mathcal{L}$ is a half-BPS operator of dimension $\Delta = 4$

$$F_n \equiv \frac{\langle W[x_1, \ldots, x_n] \mathcal{L}(x_0) \rangle}{\langle W[x_1, \ldots, x_n] \rangle} \equiv$$



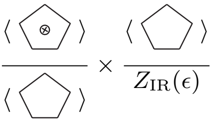
introduced in [Alday, Buchbinder, Tseytlin '11]

- Conformal (co)variant function well-defined in four dimensions,
  no regularization/subtraction required!
- Thanks to the Wilson loop/Amplitude duality the L-loop integrand of F is the $(L+1)$-loop integrand of the logarithm of the MHV amplitude

$$g^2 \partial_{g^2} \log(\mathcal{A}_{\mathrm{MHV}}) \sim \int d^D x_0 \, F(x_0)$$

Conjectured duality with All-Plus amplitude in pure Yang-Mills

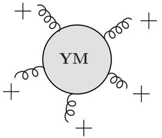
Non-DCI kinematics in the frame $x_0 \to \infty$ [DC, J. Henn '22]



$$\boxed{F \times \mathcal{R}^{\text{MHV}} \quad \sim \quad \mathcal{H}^{\text{all-plus}}_{\text{YM}}}$$

$\leq L$-loop in super-YM

$(L+1)$-loop in pure YM



- duality ($\sim$) of the maximal transcendental parts, in the leading color limit
- IR divergences of gauge theory amplitudes

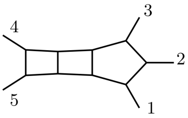
$$\mathcal{R}^{\text{MHV}} = \lim_{\epsilon \to 0} Z^{-1}_{\text{IR}}(\epsilon)\, \mathcal{A}^{\text{MHV}}_{\mathcal{N}=4 \text{ sYM}}(\epsilon)$$

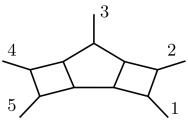
$$\mathcal{H}^{\text{all-plus}}_{\text{YM}} = \lim_{\epsilon \to 0} Z^{-1}_{\text{IR}}(\epsilon)\, \mathcal{A}^{\text{all-plus}}_{\text{YM}}(\epsilon)$$

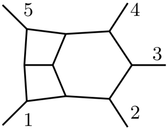
  are subtracted $Z_{\text{IR}} = \exp(g^2 I(\epsilon))$ by the dipole $I(\epsilon) = -\frac{1}{2\epsilon^2} \sum_i (s_{i\,i+1}/\mu^2)^{-\epsilon}$

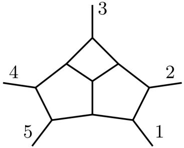
- Agreement with available data for four-gluon three-loop [Q. Jin, H. Luo '19] and n-gluon two-loop [Dunbar, Jehu, Perkins '16] all-plus amplitudes
- Another manifestation of the maximal transcendentality principle
- We employ this duality as a powerful constraint in our three-loop bootstrap of $F^{(3)}$ and obtain predictions for four-loop $\mathcal{H}^{\text{all-plus}}_{\text{YM}}$!

Function space and Canonical differential equations for Three-loop Five-point Planar Feynman integrals









[Yuanche Liu, Antonela Matijašić, Julian Miczajka, Yingxuan Xu, Yongqun Xu, Yang Zhang '24]

[DC, Yu Wu, Zihao Wu, Yongqun Xu, Shun-Qing Zhang, Yang Zhang '25]

see talk by Yang Zhang

- 5 kinematic variables
- 56 alphabet letters [DC, J. Henn, J. Trnka, S.-Q. Zhang '24]
- 2220 transcendental functions* for finite parts of three-loop amplitudes
- Our three-loop observable $F^{(3)}$ also involves products of lower-loop topologies coming from the integrand of $\log(\mathcal{A}_{\mathrm{MHV}})$

- **4729** transcendental functions* for $F^{(3)}$

* weight-6 symbols

Bootstrap the observable from physical limits and duality with all-plus amplitude

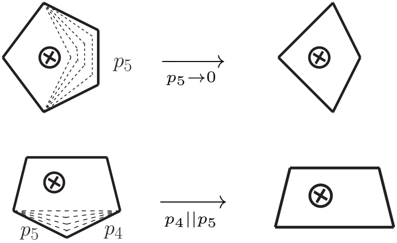
$$F^{(3)} = \sum_{i=0}^{5} r_i\, f_i^{(3)}$$

Rational prefactors r_i are known at any multiplicity and any loop order [DC, Henn '22]

[Brown, Henn, Mazzucchelli, Trnka '25]

ansatz	6×4729
dihedral invariant ansatz	**2914**
soft	237
collinear	682
no spurious poles	1693
scaling	778
duality to all-plus	1182
Total	**2868**

The remaining 46 unknowns are uniquely fixed by the near-collinear expansion



$$\underbrace{F^{(3)}}_{\text{4729 three-loop functions}} + \underbrace{F^{(2)} \times \mathcal{R}^{(1)}_{\text{MHV}} + \ldots}_{\text{lower loop}} \sim \underbrace{\mathcal{H}^{\text{all-plus, 4-loop}}_{\text{YM}}}_{\text{2220 three-loop functions}}$$

56 letters in the ansatz
but **30** letters in the observable

Outlook and Future directions

- Very first application of the three-loop five-point planar Feynman integrals in gauge-theory calculations

- We simultaneously bootstrapped $F^{(3)}$ and four-loop all-plus amplitude relying on the (conjectured) duality between them

- Verified our symbol bootstrap result for $F^{(3)}$ by an honest three-loop IBP reduction calculation!

- A new application of the near-collinear OPE [L.F. Alday, D. Gaiotto, J. Maldacena, A. Sever, P. Vieira '10] and flux tube excitations [B. Basso, A. Sever, P. Vieira '13] for the Wilson Loop with Lagrangian insertion F based on work with Alex Tumanov

- Conjectured positivity $(-1)^{L+1} F^{(L)} > 0$ in the MHV amplituhedron region [Arkani-Hamed, Henn, Trnka '21][DC, Henn '22]

- Negative geometry expansion [Arkani-Hamed, Henn, Trnka '21] reorganizes $F^{(L)}$ according to "connectivity" among loops. Three-loop five-point Ladder Negative geometries [DC, J. Henn, E. Mazzucchelli, J. Trnka, Q. Yang, S-Q. Zhang '25]

- Only **30** out of **56** alphabet letters are needed for $F_5^{(3)}$. What about three-loop five-gluon MHV amplitudes? Previously noticed at two-loops that both six-point $F_6^{(2)}$ [DC, S. Carrôlo, J. Henn, Q. Yang, Y. Zhang '25] and six-gluon MHV amplitudes involve **137** out of **167** alphabet letters

Thank you!