High-Precision Predictions for Collider Physics

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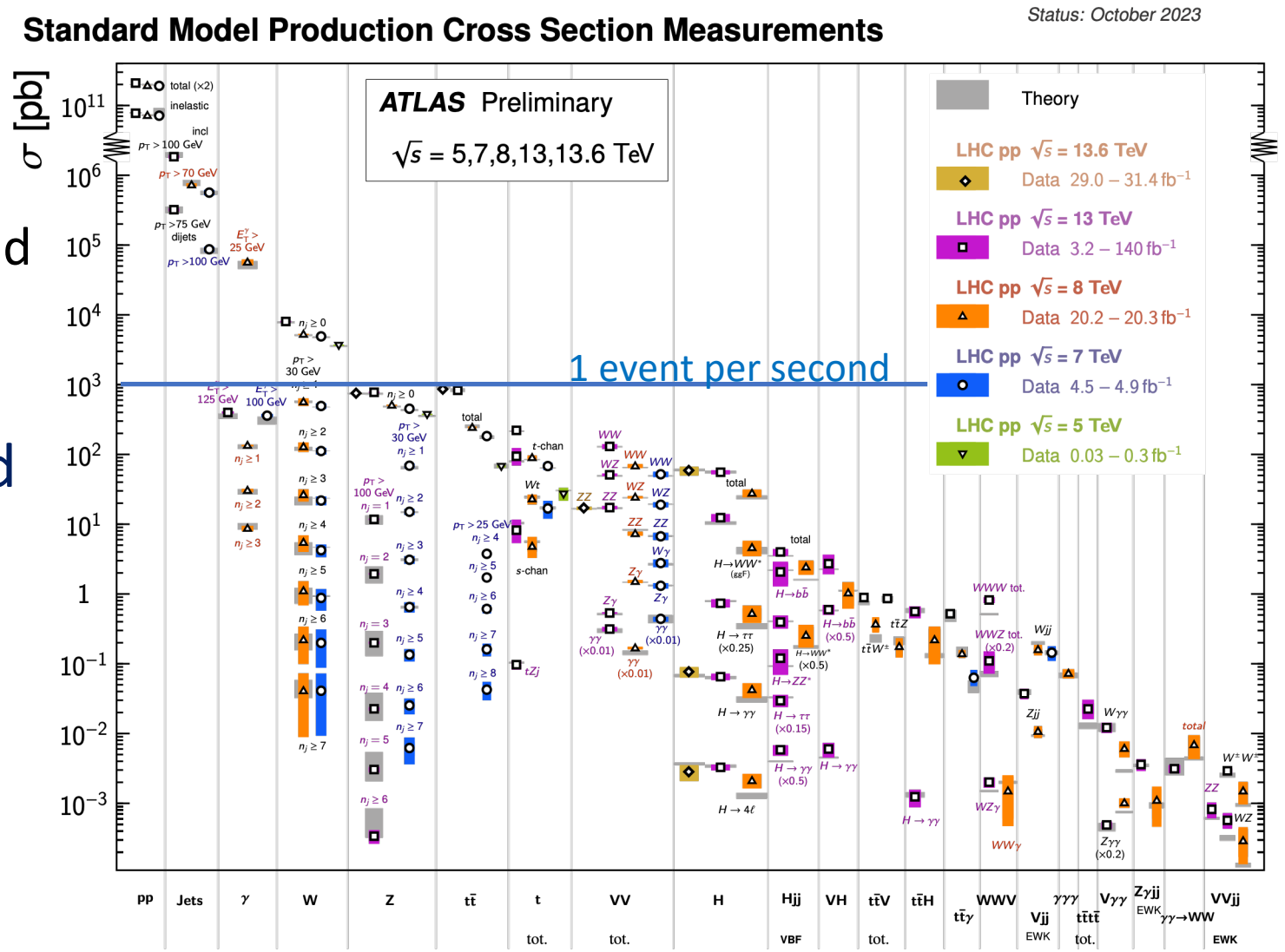
Precision physics at LHC

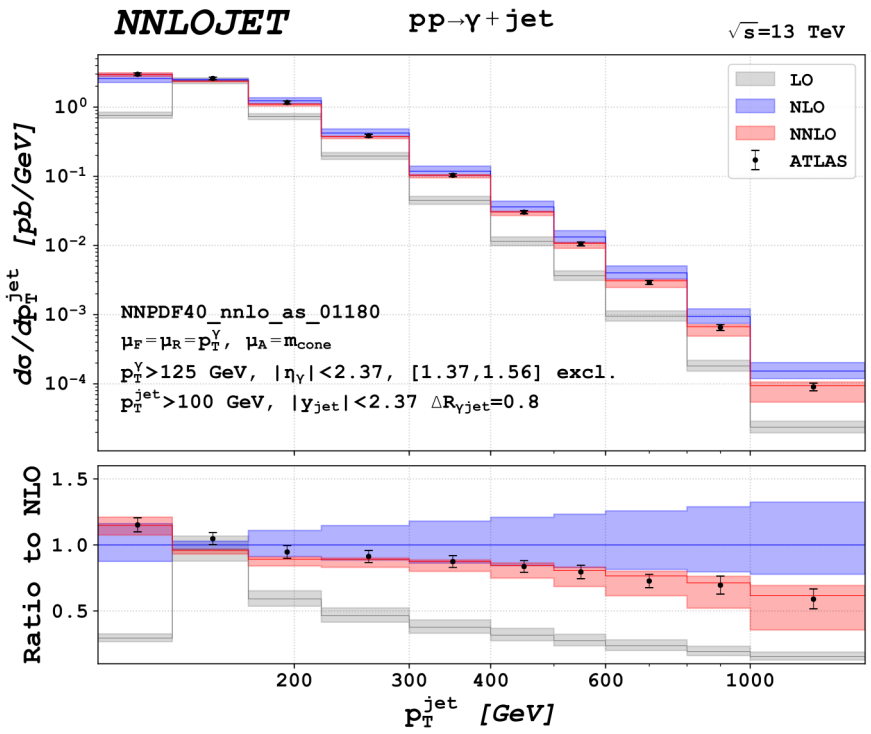
- Precision tests of the Standard Model
  - Measurements of masses and couplings
  - Searches for deviations
- Interplay of calculations and measurements
  - Theory accuracy on most cross sections $\gtrsim$5%
  - Limited by PDFs, QCD corrections



Precision Predictions: State-of-the-art

- NNLO QCD predictions for $2 \longrightarrow 2$ processes (NNLO revolution, 2015 $\longrightarrow$)
  - accomplished during past 10 years on case-by-case basis
  - as parton-level event generators (full final state information)
  - computationally expensive
  - current frontier at NNLO: $2 \longrightarrow 3$

- Typical size of corrections and uncertainty
  - NLO corrections: 10..100%, uncertainty: 10..30%
  - NNLO corrections: 2..15%, uncertainty: 3..8%
  - expect N3LO to yield uncertainty at level of 1%.



Ingredients to fixed order calculations

- Matrix elements with extra real (R) or virtual (V) partons

	Matrix elements	Parton evolution
LO	Born	1-loop
NLO	R, V	2-loop
NNLO	RR, RV, VV	3-loop
N3LO	RRR, RRV, RVV, VVV	4-loop

- Infrared singularities in all R-type and V-type subprocesses
  - sum of all subprocesses finite
  - require procedure to arrange IR cancellations between subprocesses
- Incoming hadrons: parton distributions
  - mass factorization of initial-state radiation and parton evolution

From amplitudes to cross sections

- Different final state multiplicity for real and virtual corrections
  - R: n+1 particles; V: n particles
  - application of event selection, fiducial cuts: evaluate separately
- Upcycling of lower-order calculations
  - only purely virtual correction (V, VV, VVV, ...) genuinely new
  - real radiation corrections from higher-multiplicity calculations at lower order
  - e.g. Higgs boson production: NNLO RV contribution = NLO V contribution to H+jet
  - stability: use analytic one-loop amplitudes if available
- Cancellation of infrared singularities between subprocesses
  - must evaluate integrals of type

$$\mathcal{I} = \lim_{\epsilon \to 0} \left[\int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right]$$

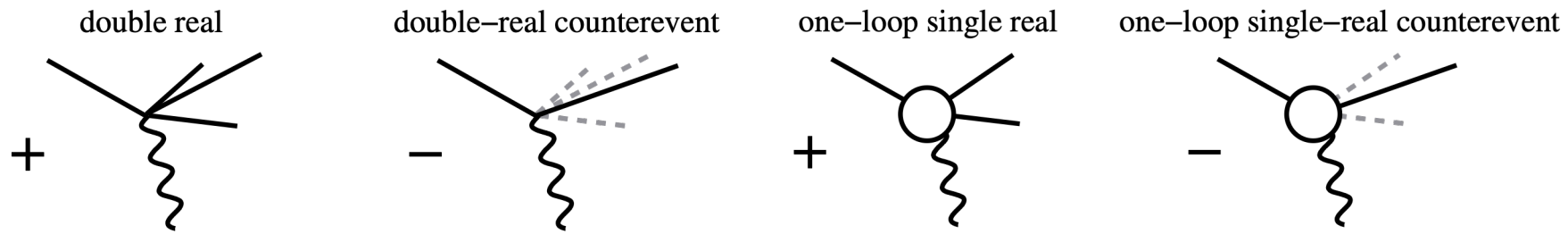
Real radiation: methods

Infrared cancellations: challenges

- subtraction $$\mathcal{I} = \lim_{\epsilon\to 0}\left[\int_0^1 \frac{dx}{x}x^\epsilon\left(F(x)-F(0)\right)+F(0)\int_0^1\frac{dx}{x}x^\epsilon-\frac{1}{\epsilon}F(0)\right]$$
  - construction of subtraction term (completeness, overcompensation)
  - integration of building blocks (analytical or numerical)
- slicing $$\mathcal{I}\approx\lim_{\epsilon\to 0}\left[\int_\delta^1 \frac{dx}{x}x^\epsilon F(x)+F(0)\int_0^\delta\frac{dx}{x}x^\epsilon-\frac{1}{\epsilon}F(0)\right]$$
  - analytic computation of below-cut contribution
  - numerical importance of power-suppressed terms, value of slicing parameter

Real radiation: methods

- Slicing
  - introduce auxiliary observable δ to identify singular real radiation regions
  - typically using some resummation variable: below-cut contribution from resummation
  - e.g.: q_T-slicing (S. Catani, M. Grazzini)
    N-jettiness slicing (R. Boughezal, X. Liu, F. Petriello; J. Gaunt, M. Stahlhofen, F. Tackmann, J. Walsh)
- Subtraction



  - construct subtraction terms describing real radiation in all singular regions
  - e.g.: Residue subtraction (M. Czakon; F. Caola, K. Melnikov, R. Röntsch)
    Local analytic subtraction (V. Del Duca, G. Somogyi, et al.; L. Magnea, C. Signorile-Signorile, P. Torrielli et al.)
    Antenna Subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
    Projection-to-Born (M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi)

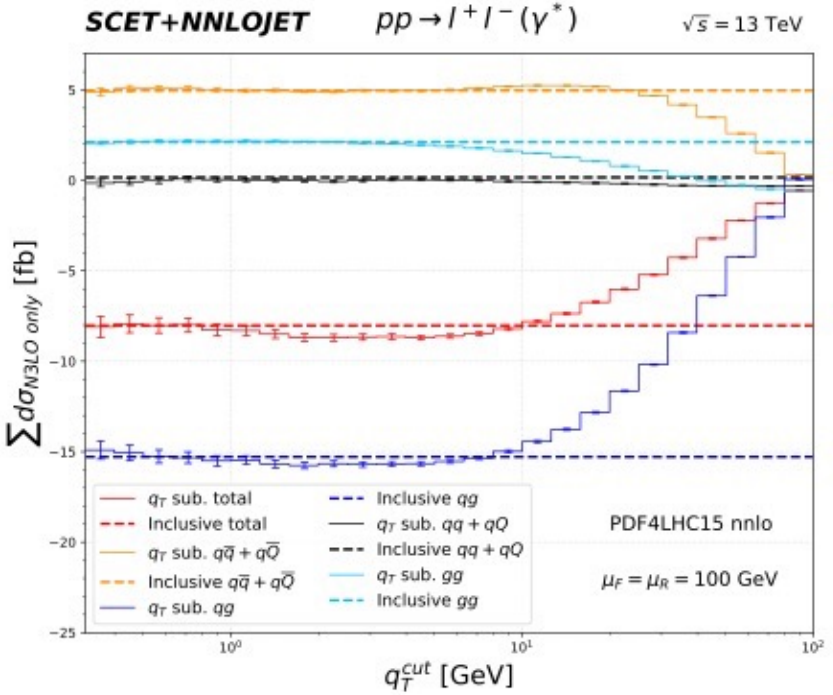
Real radiation: slicing

Slicing parameter: transverse momentum (q_T slicing) (S.Catani, M.Grazzini)

$$\frac{d\sigma_X^{N3LO}}{dO} = \mathcal{H}_{N3LO} \otimes \frac{d\sigma_X^{LO}}{dO} + \left[\int_{q_{T,X}} \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X,CT}^{NNLO}}{dO}(q_T) \right]$$

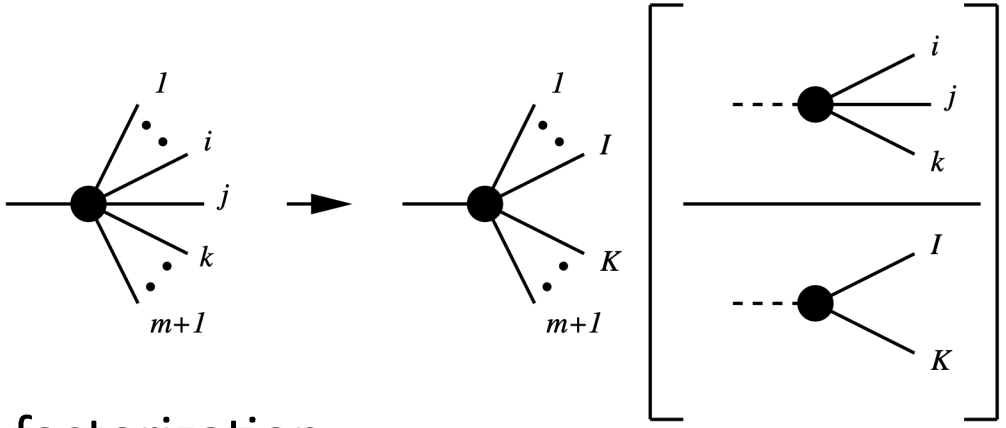
N3LO Drell-Yan production

- below-cut contribution from expansion of N3LL q_T resummation to $O(\alpha_s^3)$ (W. Bizon, P. Monni, E. Re, P. Torrielli; S. Camrada, L. Cieri, G. Ferrera; T. Becher, T. Neumann; W.L. Ju, M. Schönherr)
- ingredients: three-loop soft and beam functions (Y. Li, H.X. Zhu; M. Ebert, B. Mistlberger, G. Vita; M.X. Luo, T.Z. Yang, Y.J. Zhu)
- check: independence on $q_{T,cut}$ slicing parameter (X. Chen, E.W.N. Glover, A. Huss, T.Z. Yang, H.X. Zhu, TG)
- check: reproduce inclusive coefficient functions (no ingredients or methodology in common!) (C. Duhr, F. Dulat, B. Mistlberger)



Real radiation: subtraction

- Subtraction terms extract singular real radiation
  - Example: antenna subtraction for emission between two partons



  - Phase space factorization

$$d\Phi_{m+1}(p_1,\ldots,p_{m+1};q) = d\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_K,\ldots,p_{m+1};q)\cdot d\Phi_{X_{ijk}}(p_i,p_j,p_k;\tilde{p}_I+\tilde{p}_K)$$

  - Integrated subtraction term

$$\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$$

Analytical phase space integrals

- integrated subtraction terms
  - contain infrared real-radiation singularities
- translate phase space to loop integrals
  - reverse unitarity (C. Anastasiou, L.J. Dixon, K. Melnikov, F. Petriello)

$$\delta(p^2) \to \frac{1}{2\pi i}\left[\frac{1}{p^2 - i0} - \frac{1}{p^2 + i0}\right]$$

- can include complicated constraints
  - initial and final state kinematics, cuts, identified objects, ...
- use multi-loop machinery
  - IBP reduction to phase space master integrals
  - canonical differential equations

$$\begin{aligned}
D_1 &= (q - k_p)^2\,, \\
D_2 &= (p_i + q - k_p)^2\,, \\
D_3 &= (p_i - k_l)^2\,, \\
D_4 &= (q - k_l)^2\,, \\
D_5 &= (p_i + q - k_l)^2\,, \\
D_6 &= (q - k_p - k_l)^2\,, \\
D_7 &= (p_i - k_p - k_l)^2\,, \\
D_8 &= (k_p + k_l)^2\,, \\
D_9 &= k_p^2\,, \\
D_{10} &= k_l^2\,, \\
D_{11} &= (q + p_i - k_p - k_l)^2\,, \\
D_{12} &= (p_i - k_p)^2 + Q^2\frac{z}{x}\,,
\end{aligned}$$

RR integrals for identified particles at NNLO

family	master	deepest pole	at $x = 1$	at $z = 1$
	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
A	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3,5,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,3,4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1,3,5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1,3,8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1,4,5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2,4,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4,7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3,4,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3,5,8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4,5,7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4,5,8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

NNLO cross section

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \int_{n+2} \left(\mathrm{d}\sigma^{RR} - \mathrm{d}\sigma^{S}\right) + \int_{n+1} \left(\mathrm{d}\sigma^{RV} - \mathrm{d}\sigma^{T} + \mathrm{d}\sigma^{MF,1}\right) + \int_{n} \left(\mathrm{d}\sigma^{VV} + \mathrm{d}\sigma^{MF,2} + \int_{2} \mathrm{d}\sigma^{S} + \int_{1} \mathrm{d}\sigma^{T}\right)$$

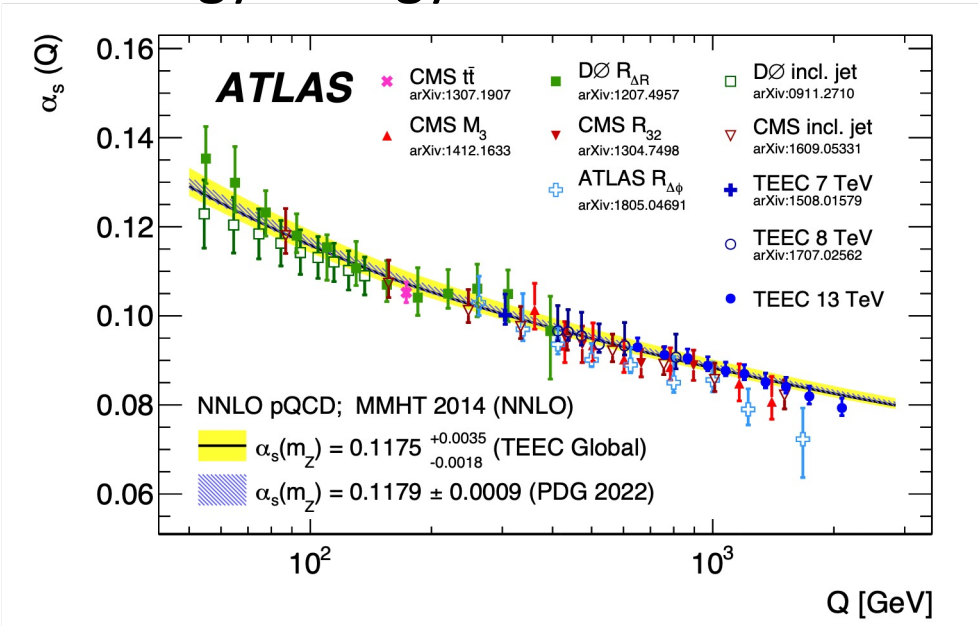
- Real and virtual contributions: $\mathrm{d}\sigma^{RR},\ \mathrm{d}\sigma^{RV},\ \mathrm{d}\sigma^{VV}$
- Singular double real radiation: $\mathrm{d}\sigma^{S}$
- Singular one-loop single real radiation: $\mathrm{d}\sigma^{T}$
- Mass factorization terms: $\mathrm{d}\sigma^{MF,1},\ \mathrm{d}\sigma^{MF,2}$

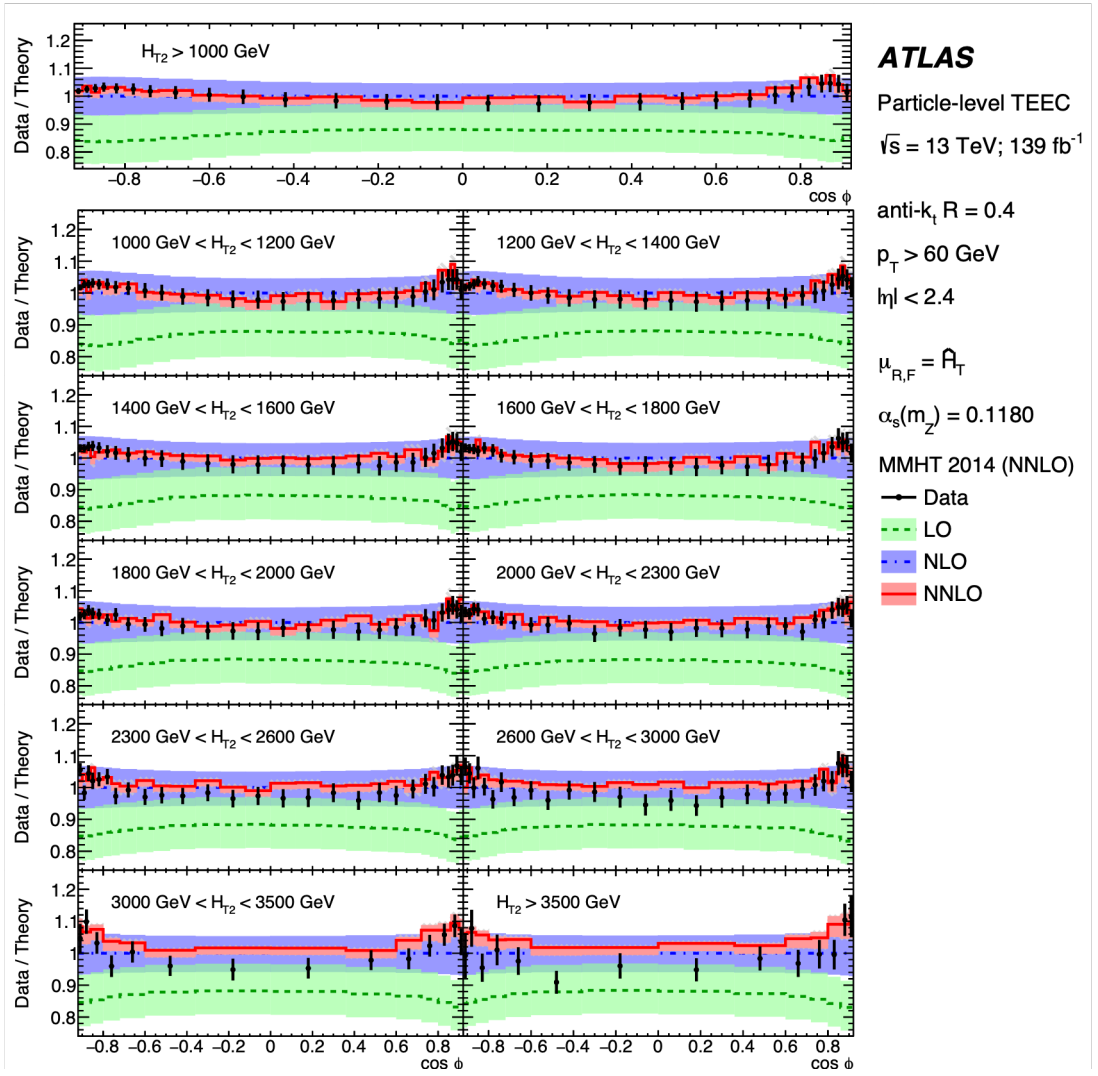
- Each line finite and free of poles → numerical implementation
- Implementation: parton-level event generator at NNLO

  MATRIX: M. Grazzini, S. Kallweit, M. Wiesemann; MCFM: J. Campbell, T. Neumann, R.K. Ellis, S. Seth; NNLOCAL: V. Del Duca et al.; NNLOJET: A. Huss et al., MiNNLOPS: P. Monni, P. Nason, E. Re, M. Wiesemann, G. Zanderighi; GENEVA: S. Alioli et al.

Three-jet production

- 3-jet production and related event shapes at hadron colliders at NNLO
  (M. Czakon, A. Mitov, R. Poncelet)
  - using residue subtraction (M. Czakon)
  - enabled ATLAS α_s determination based on energy-energy correlation functions





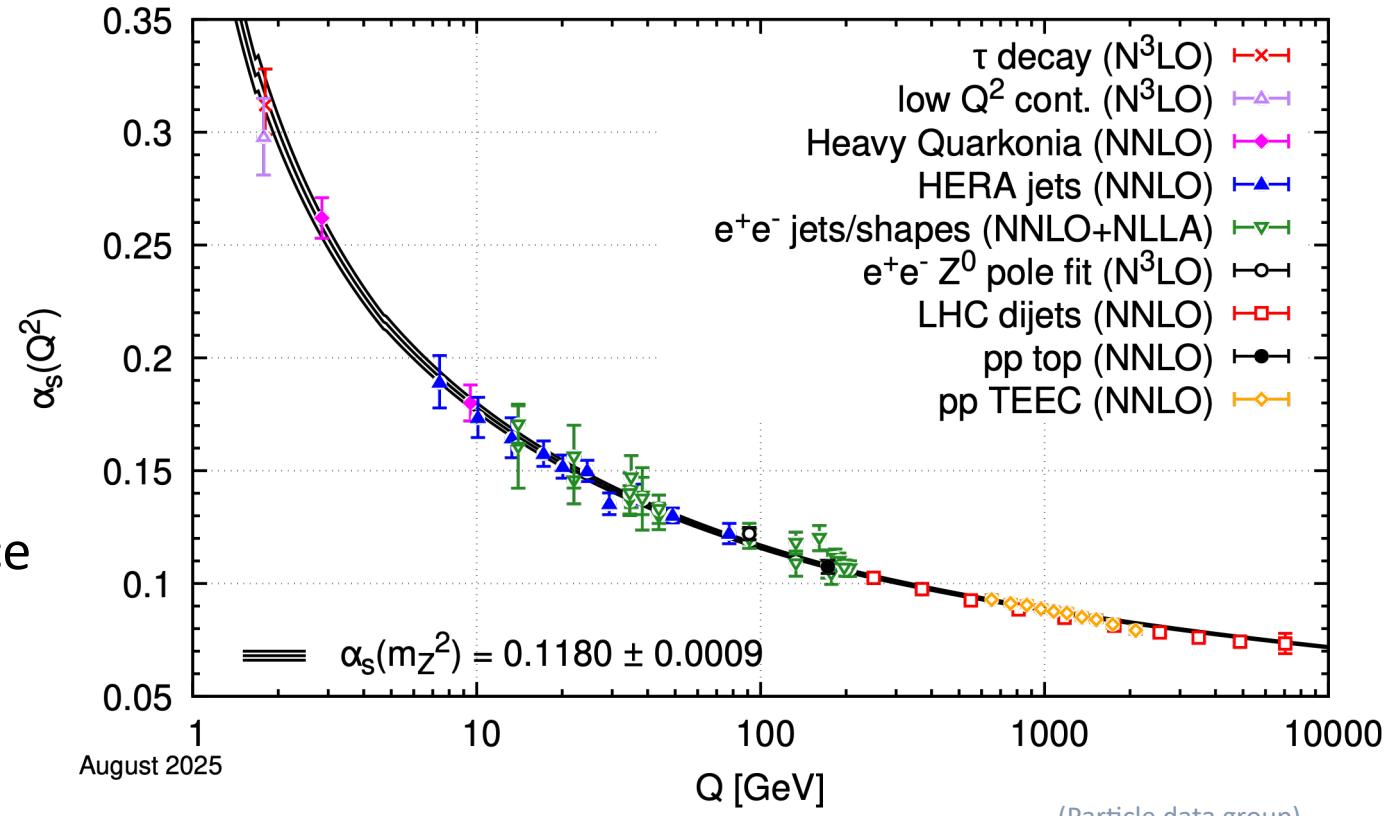
NNLOJET code

- NNLO parton level event generator
  - Based on antenna subtraction
- Provides infrastructure
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing (MPI) support for warm-up and production
  - ApplGrid/fastNLO interfaces
- Processes implemented at NNLO
  - Z+(0,1)jet, γ+1 jet, H+(0,1)jet, W+(0,1)jet
  - DIS-2j, LHC-2j
  - Typical runtimes: 60’000-250’000 core-hours

NNLOJET project:
A.Huss, L.Bonino, O.Braun-White, S.Caletti, X.Chen, J.Cruz-Martinez, J.Currie, W.Feng, G.Fontana, E.Fox, R.Gauld, A.Gehrmann-De Ridder, E.W.N.Glover, M.Höfer, P.Jakubcik, M.Jaquier, M.Löchner, F.Lorkowski, I.Majer, M.Marcoli, P.Meinzinger, J.Mo, T. Morgan, J.Niehues, J.Pires, C.Preuss, A.Rodriguez-Gracia, K.Schönwald, R.Schürmann, V.Sotnikov, G.Stagnitto, D.Walker, J.Whitehead, T.Z.Yang, H.Zhang, TG

Di-jet production

- 2-jet production at LHC
  - most basic hard QCD process
  - large cross section: high-statistics measurement
  - combine ATLAS/CMS data sets
  - group data according to $\mu=M_{2j}$
- correlation of α_s with PDFs
  - vary PDF through APPLgrid interface
  - combine with DIS 2-jet (HERA)
- reach: 7.4 GeV ... 7.0 TeV



(Particle data group)

$\alpha_s(M_Z) = 0.1178 \pm 0.0014_{exp,pdf} \pm 0.0017_{th}$

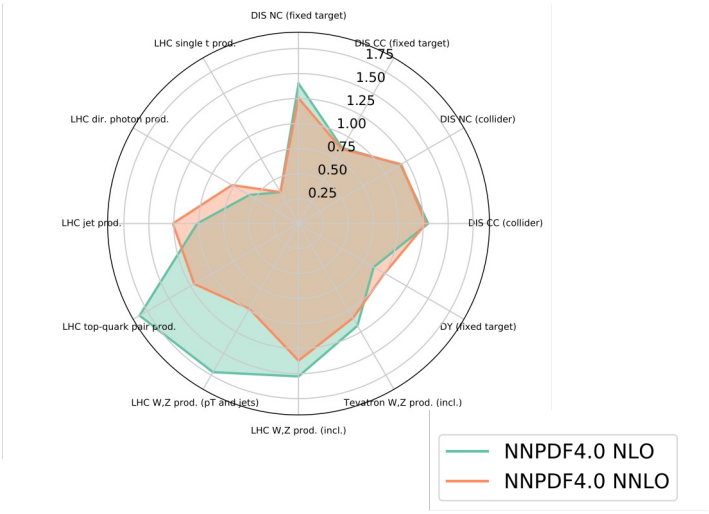
(APPLfast+NNLOJET: F. Ahamdova et al.)

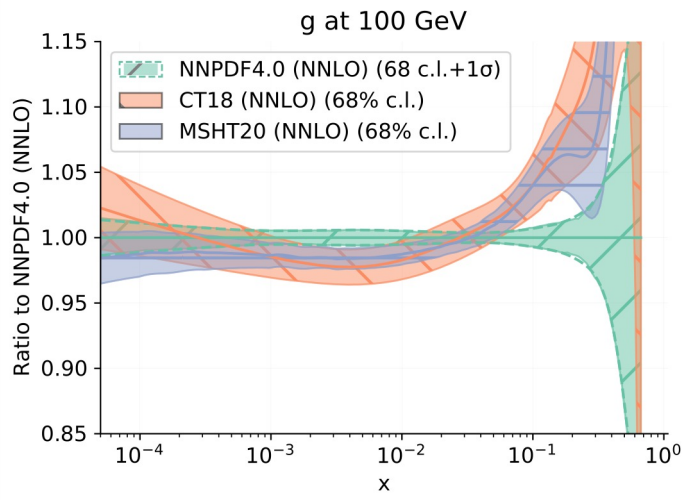
Parton distributions

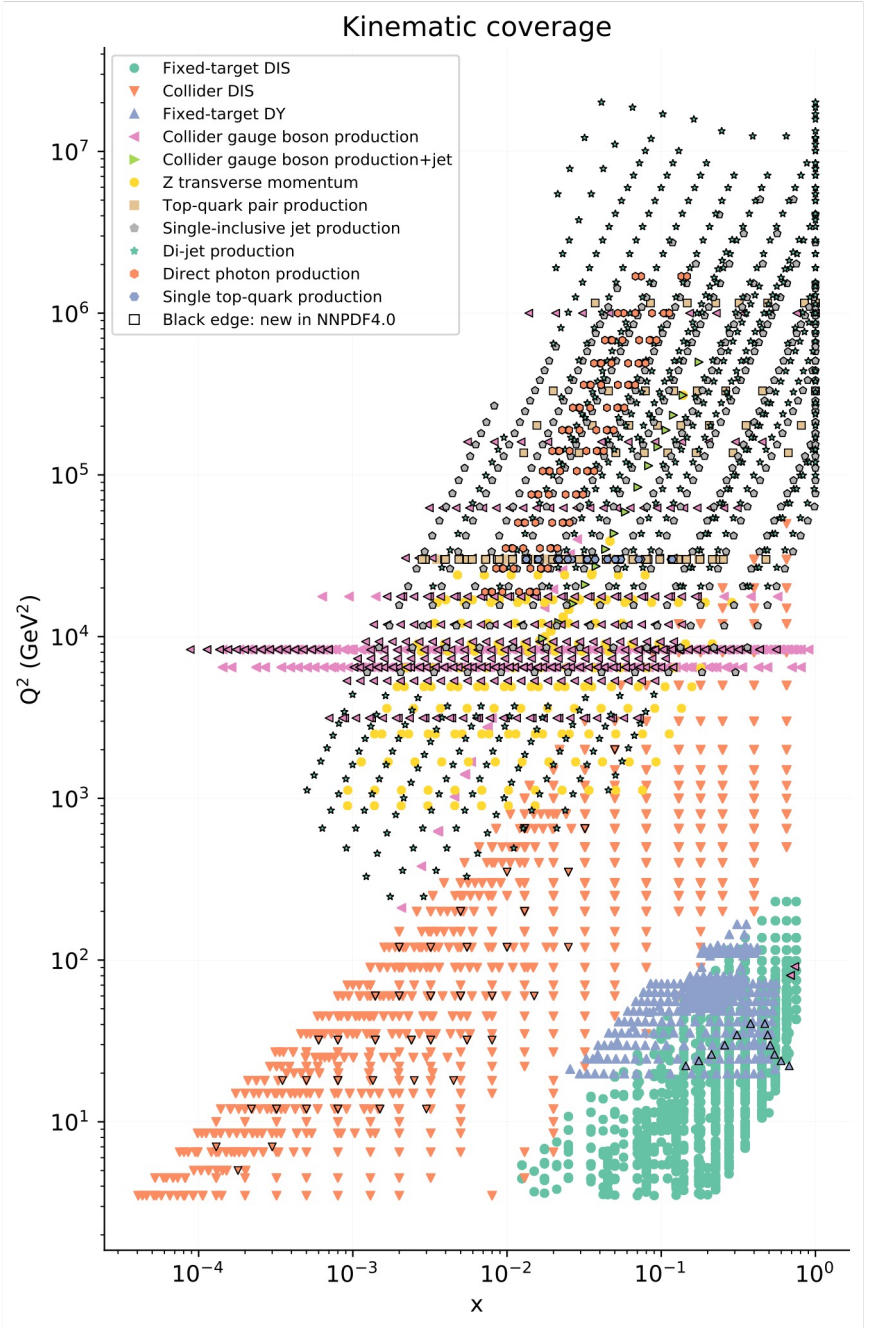
Global fit to collider data at NNLO

(NNPDF: R.Ball et al., MSHT: T.Cridge et al.; CT: A.Ablat et al.; ABMP: S.Alekhin et al.)

- NNLO cross sections, 3-loop parton splitting functions
- include ~4000 data points from ~70 data sets
- constrain quark and gluon distributions to (1..3)% level
- substantial improvement of fit quality from NLO to NNLO



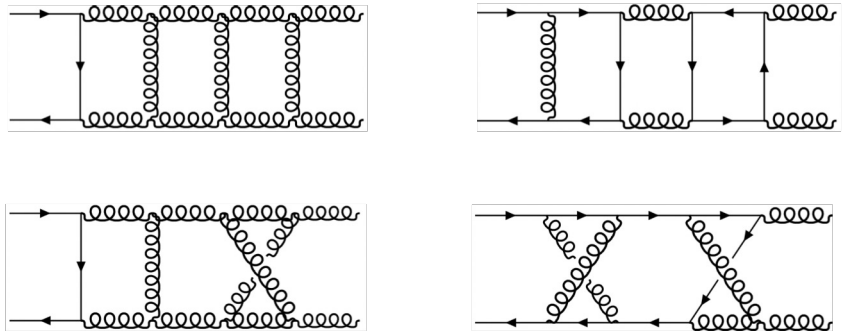




Towards N3LO for fiducial cross sections

Three-loop amplitudes for 2 ⟶ 2 processes (VVV)

- algebraic complexity of integral reduction, computation of master integrals
- first results
  - four-parton amplitudes (F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi)
  - parton-photon amplitudes (P.Bargiela, F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi)
  - V+3-parton amplitudes (planar) (P.Jakubcik, C.Mella, N.Syrrakos, L.Tancredi, TG)
  - H+3-parton amplitudes (leading colour) (X.Chen, X.Guan, B.Mistlberger)



Infrared cancellations at N3LO

- Slicing: q_T (S.Catani, M.Grazzini)
- N3LO subtraction methods under development
  - Projection-to-Born (M.Cacciari, F.Dreyer, A.Karlberg, G.Salam, G.Zanderighi)
  - Antenna subtraction (X.Chen, P.Jakubcik, M.Marcoli, G.Stagnitto; G. Fontana, K. Schönwald, TG)

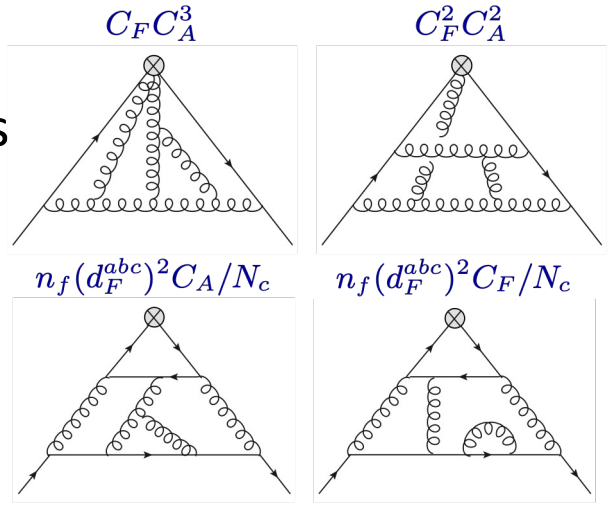
Parton distributions at N3LO

Four-loop Altarelli-Parisi splitting functions

- from four-loop anomalous dimensions of quark and gluon operators

$$O_q(n) = \frac{i^{n-1}}{2}\left[\bar{\psi}\Delta_\mu\gamma^\mu(\Delta\cdot D)^{n-1}\psi\right] \qquad O_g(n) = -\frac{i^{n-2}}{2}\left[\Delta^\mu G^a_{\mu\nu}(\Delta\cdot D)^{n-2}_{ab}\Delta_\rho G^{b\,\rho\nu}\right]$$

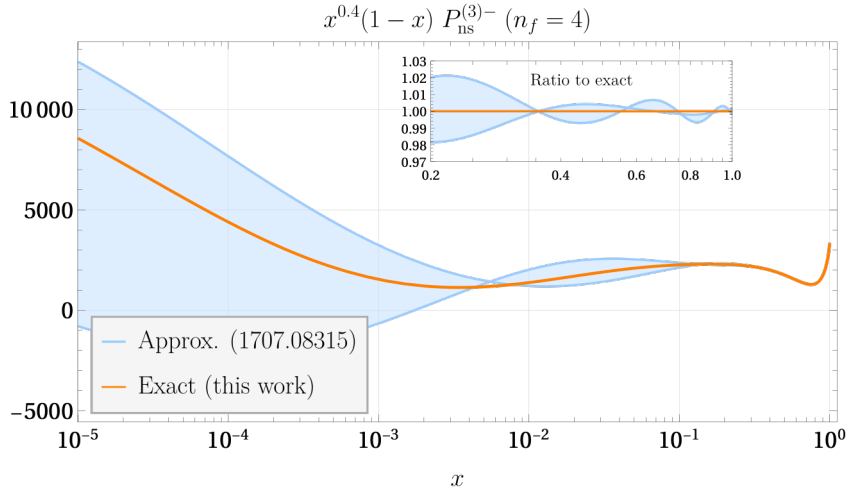
- fixed Mellin moments n≤20, approximate full x-dependence of splitting functions (G.Falcioni, F.Herzog, S.Moch, A.Pelloni, A.Vogt)



- tracing parameter for all-n calculation
  (J.Ablinger, J.Blümlein, A.Hasselhuhn, C.Schneider, A. von Manteuffel)

$$(\Delta\cdot p)^{n-1} \to \sum_{n=1}^{\infty} t^n (\Delta\cdot p)^{n-1} = \frac{t}{1 - t\,\Delta\cdot p}$$

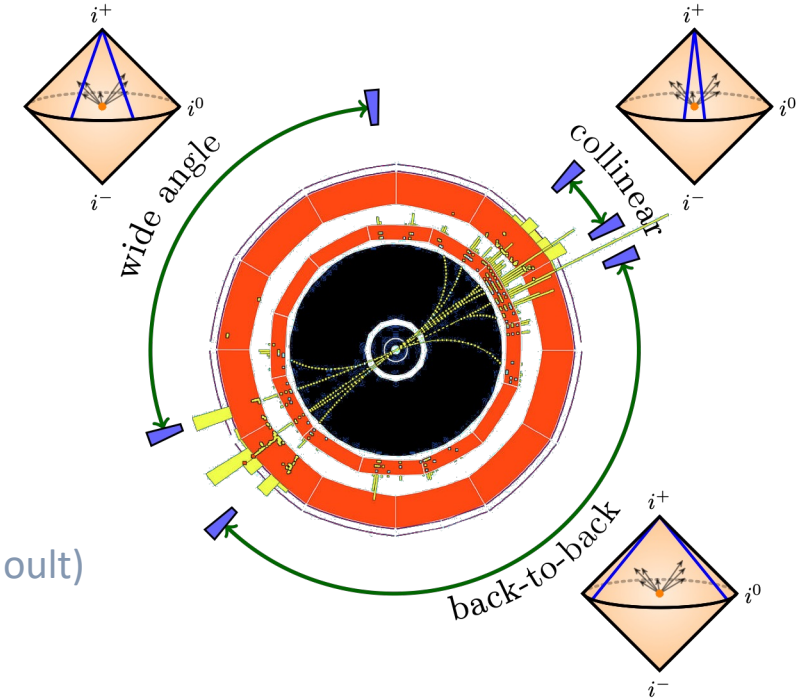
- four-loop quark non-singlet splitting functions for all-n = exact x-dependence
  (A. von Manteuffel, V. Sotnikov, T.Z. Yang, TG)



Precision physics with legacy data

Energy-energy correlators

$$EEC(z) = \sum_{i,j} \int \mathrm{d}\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$

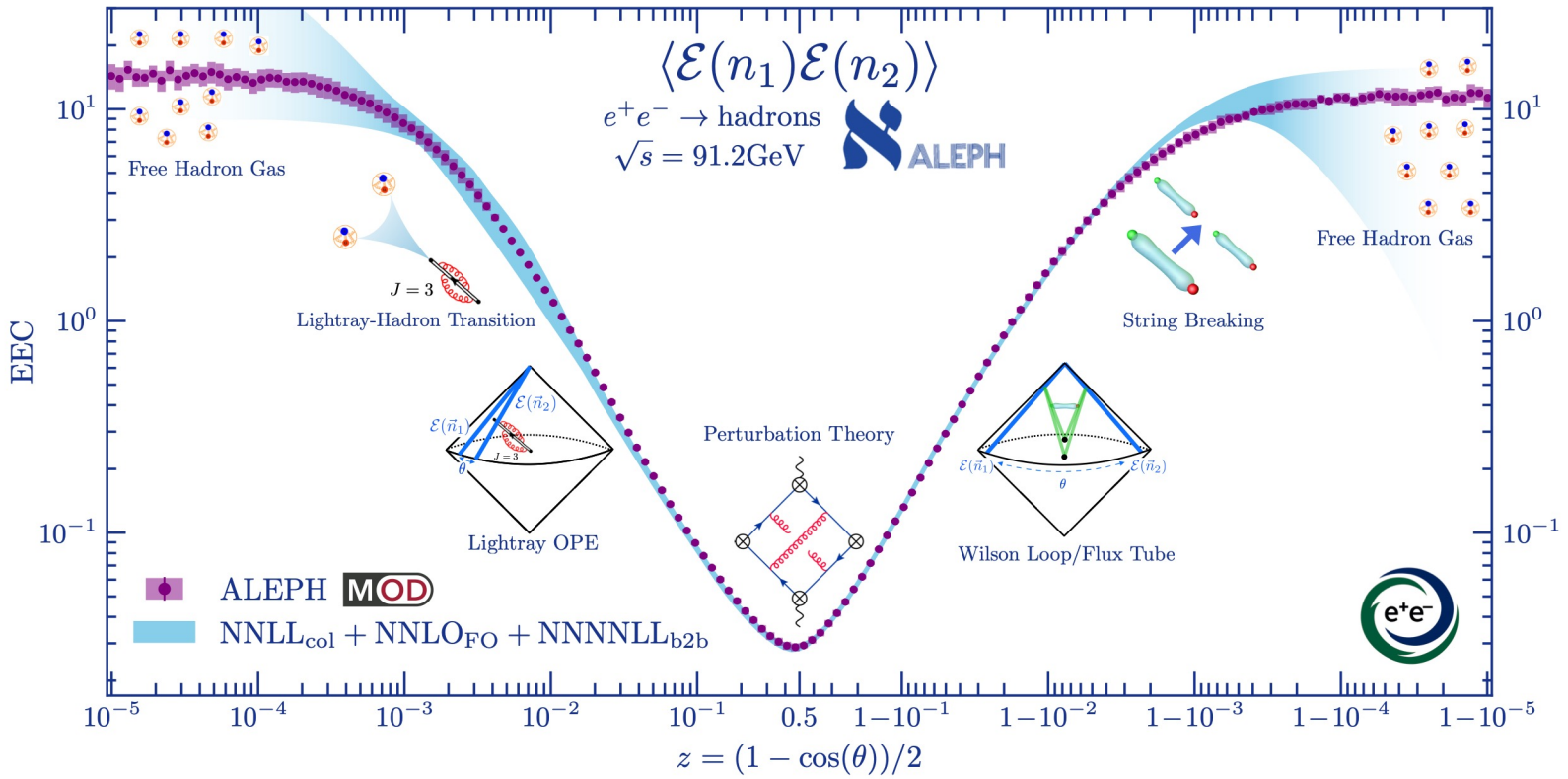


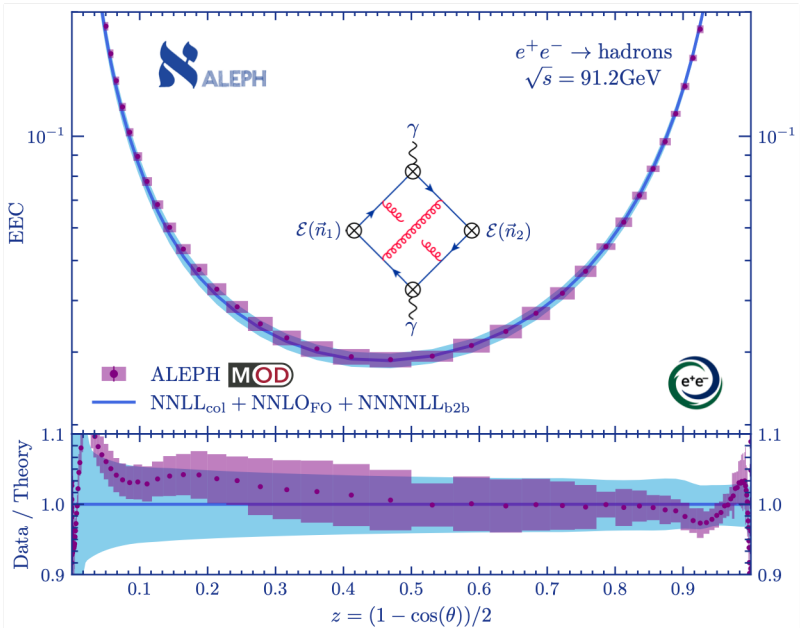
- theory progress leads to renewed interest
  - Resummation at N4LL (C.Duhr, B.Mistlberger, G.Vita)
  - Renormalon subtraction (S.Schindler, I.Stewart, Z.Sun)
  - Light-ray OPE (H.Chen, I.Moult, X.Zhang, H.X.Zhu; K.Lee, B.Macaj, I.Moult)
  - EEC for charged tracks (P.F.Monni, G.Vita, Z.Xu, H.X.Zhu)
- rich phenomenology in e^+e^- annihilation
- reanalysis of LEP data (Electron-positron alliance: H.Bossi et al.)

Precision physics with legacy data

Energy-energy correlators: ALEPH data reanalyzed

(Electron-positron alliance: H.Bossi et al.)





Precision physics with legacy data

NNLO corrections to $e^+e^- \longrightarrow 4$ jets

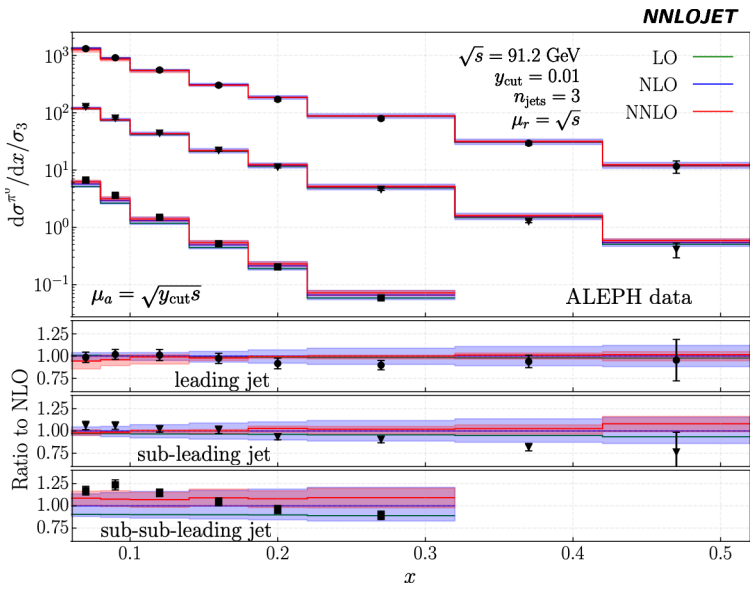
(X.Chen, D.Chicherin, E.Fox, N.Glover, M.Marcoli, V.Sotnikov, H.Sun, H.Zhang, S.Zoia)

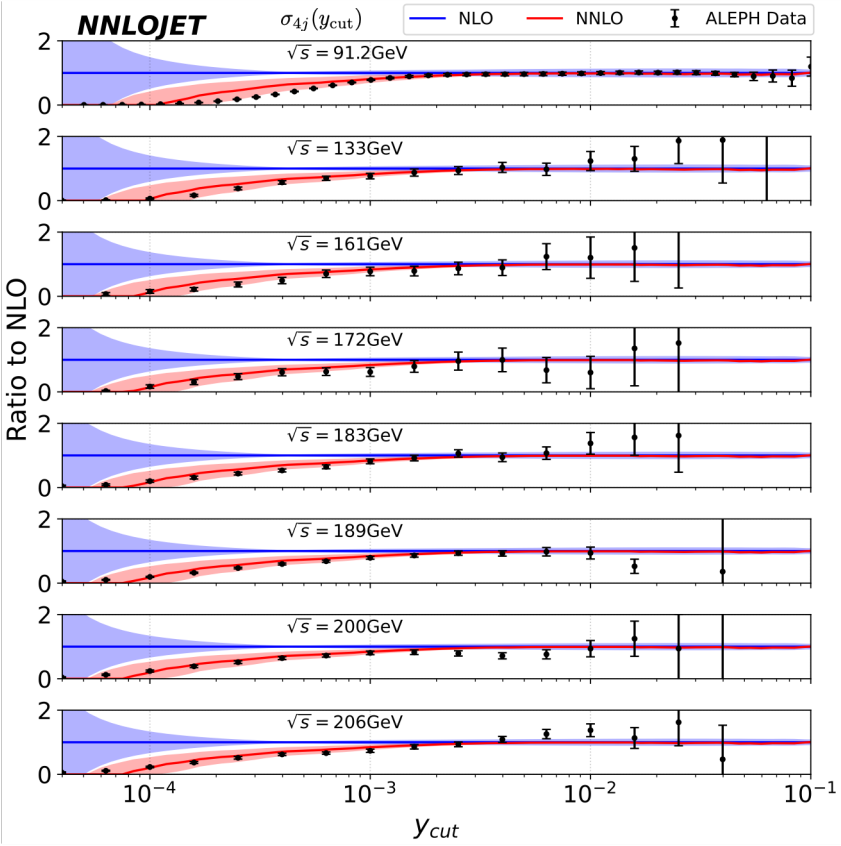
- two-loop $1 \longrightarrow 4$ amplitudes and efficient subtraction
- confront LEP precision data

Hadron-in-jet production in e^+e^- annihilation

(L.Bonino, A. Gehrmann-De Ridder, A.Huss, F.Merlotti, G.Stagnitto, TG)

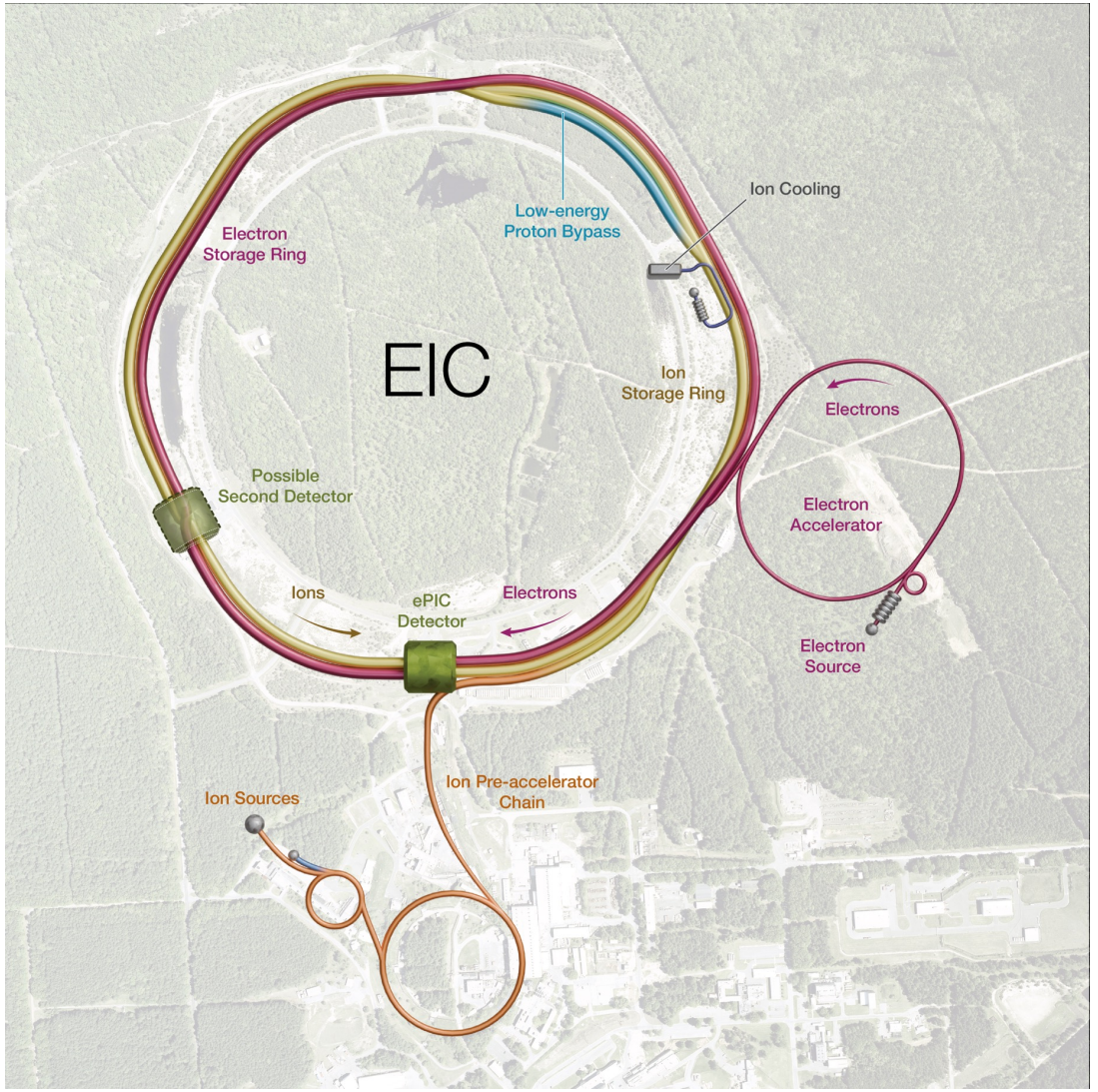
- probe parton-to-hadron transition and jet structure





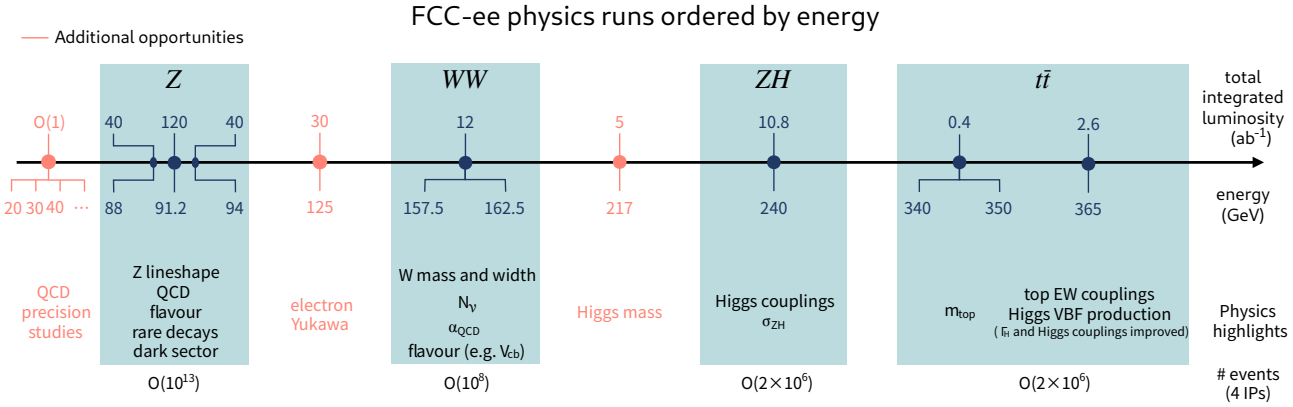
Beyond LHC: EIC

- Electron-Ion Collider at BNL
  - High-luminosity: $10^{33}...10^{34}$ $cm^{-2}s^{-1}$
  - Centre-of-mass energy range: 40..140 GeV
  - Full identification of hadronic final state
  - precision QCD: hadron structure, fragmentation,....
  - Polarized collisions
- Theory challenges
  - NNLO precision for unpolarized and polarized benchmark processes
  - identified hadrons, fully exclusive final states
  - novel types of observables: TMD, GPD, ...

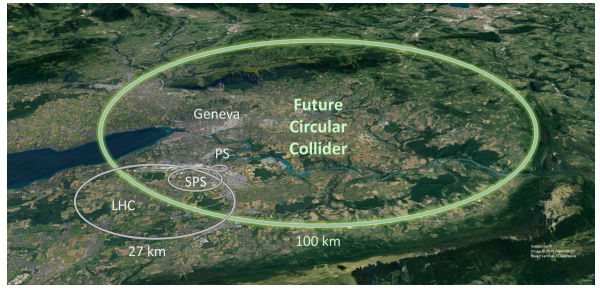


Beyond LHC: FCC-ee

- FCC-ee project at CERN
  - ultrahigh-luminosity e^+e^- collider
  - under consideration
  - start of operation ~2045



- Will require new level of precision predictions
  - electroweak NNLO corrections (multi-scale loop amplitudes)
  - precision QCD: jets, hadrons, energy correlators
  - novel types of observables
  - transition perturbative ↔ non-perturbative



Summary

- LHC is on a decade-long program of precision physics
- Ultimate precision challenge for QCD
  - predictions for complex final states at per-cent level accuracy
- Theory ready to face this challenge
  - NNLO predictions becoming the new standard
  - N3LO techniques, tools and first results emerging
- Exciting opportunities for precision physics at FCC-ee and EIC
  - learn technical experience from LHC calculations
  - develop novel methods and observables
  - recover legacy from previous-generation experiments