Bridging Scattering Amplitudes and Black Hole Perturbation Theory —Toward a Fundamental Gravitational Action

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Based on the work 2503.20538, 2506.19705,
2602.06947, 26??.?????w/

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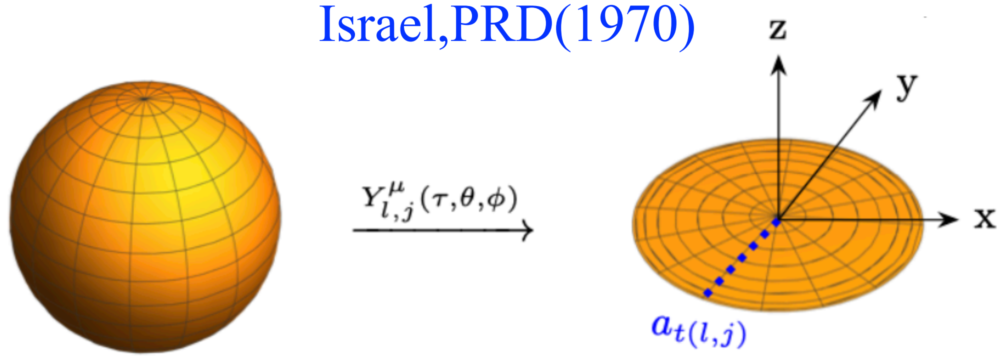
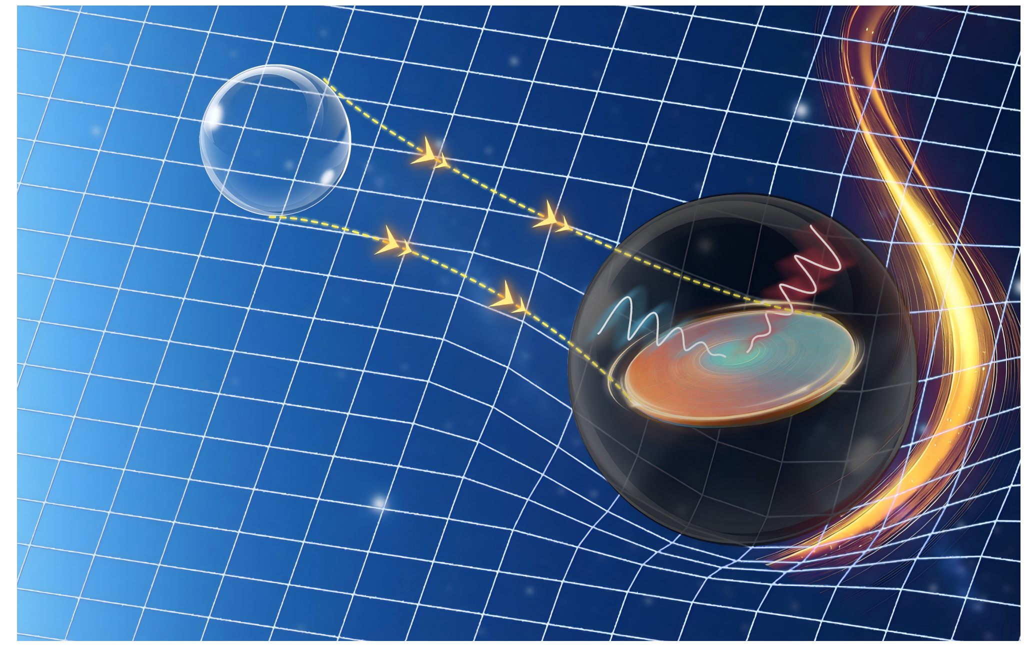
Part I: Hypersurface Model

- Gravitational wave from first-principles
- Evolution of Black hole in Binary System

Hypersurface dynamics in binary system

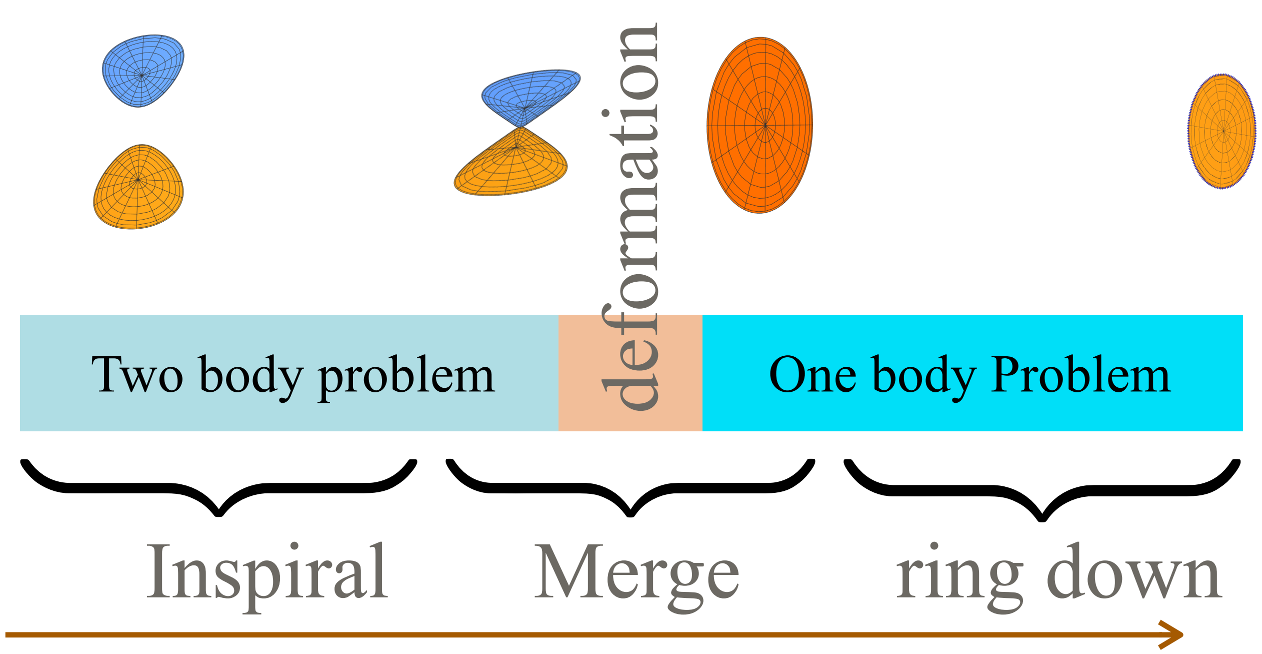
disk topology of stress-energy tensor support — Israel,PRD(1970)

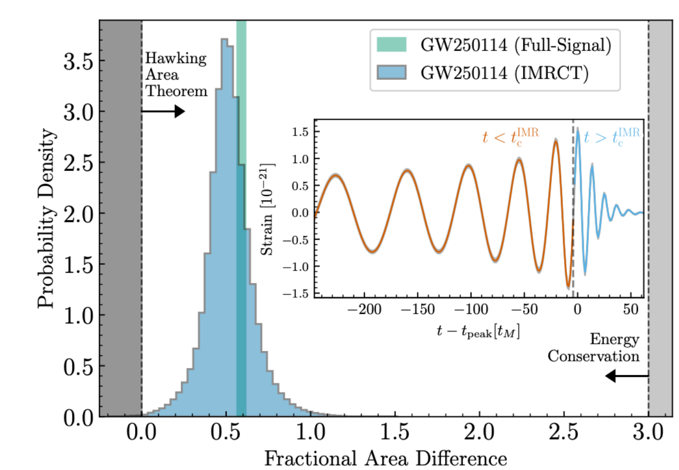




General Action $S_{\mathrm{HE}} + S_{\mathrm{Kerr}}$

Evolution of stress-energy support





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1.black-hole spectroscopy
2.Multi quasi-normal

Key Properties from the Action

Worldvolumn metric: γ rigid equivalent class

$$ds^2 = d\tau^2 - a_w^2\left(d\theta^2 + f(\theta)d\phi^2\right) \qquad f(\theta) = \sin^2(\theta)$$

Vacuum solution

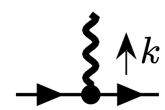
$$Z(\sigma) = X(\tau) + Y(\sigma)$$

$$\frac{\delta S|_{g_{\mu\nu}\to\eta_{\mu\nu}}}{\delta Z} = 0 \longrightarrow Z_0 = x + v\tau + \sum_{l>j\geq 0} Y^{\mu}_{l,j}$$

(2, 2), (2, 1)

(1, 1)

$$\Big\downarrow (1,1)$$



$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{0,\mu\nu} + \cdots \longleftarrow T^{\mu\nu}(x) = \frac{m}{4\pi a_w^2\sqrt{-g}}\int d^3\sigma\sqrt{\gamma}\,\delta^4(x-Z)(\partial_a Z^{\mu}\gamma^{ab}\partial_b Z^{\nu}) + \ldots,$$

Stable condition

$$h_{0,\mu\nu}|_{r\to 0} = 0$$

Fundamental action for Kerr

$$S_{\mathrm{Kerr}} = -\frac{m}{8\pi a_w^2}\int_{\mathbb{S}_2\times\mathbb{R}} d^3\sigma\sqrt{\gamma}\Bigg[(\mathcal{D}Z)^2 + 1 + a_w^2\mathcal{R}_{\mu\nu\rho\lambda}(Z)\partial_a Z^{\mu}\partial_b Z^{\rho}\partial_c Z^{\nu}\partial_d Z^{\lambda} \times\left(\frac{1}{3}\gamma^{ab}\gamma^{cd} - \frac{1}{3^4}a_w^2\varrho^{acbd} + \frac{4}{3^4}a_w^2\varrho^{acbd}(\mathcal{D}Z)^2\right)\Bigg]$$

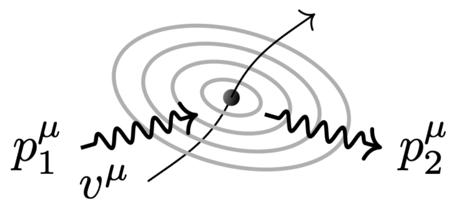
Three extra terms

consistency checks

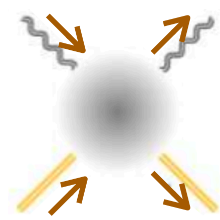
1.Leading three-point amplitude for Kerr

2.Generating Kerr metric: Event & Cauchy horizon

From Action to Classical Compton Amplitude



$\Longrightarrow \quad \mathcal{M} =$



$$\mathrm{h}_{\mu\nu} = h_{\mathbf{0}\,\mu\nu} + \kappa\, h_{\mathbf{1},\mu\nu} + \kappa^2\, h_{\mathbf{2}\,\mu\nu} + \cdots$$

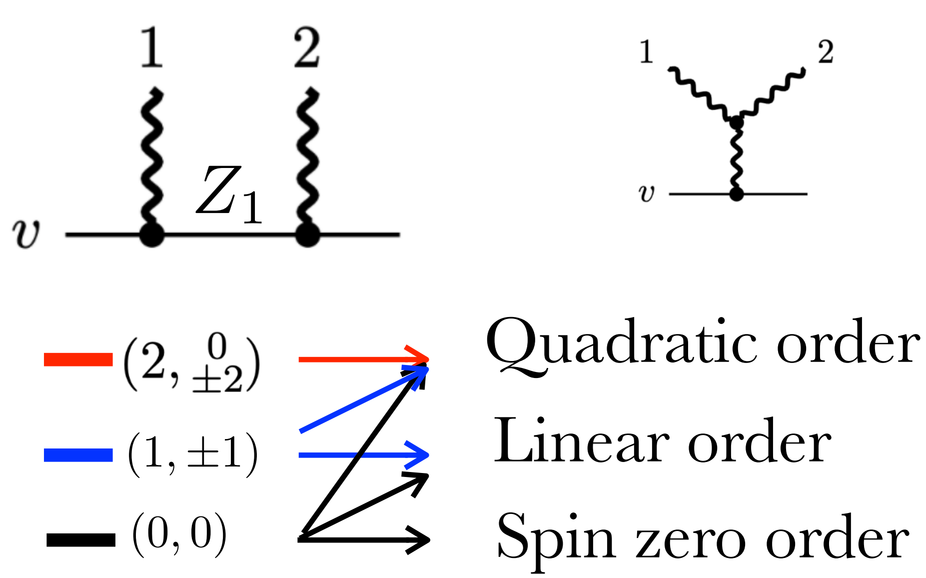
$$Z^\mu(\sigma) = Z_{\mathbf{0}}^\mu(\sigma) + \kappa\lambda Z_{\mathbf{1}}^\mu(\sigma) + \cdots$$

Master Equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$T^{\mu\nu}(x) = \frac{m}{4\pi a_w^2\sqrt{-g}}\int d^3\sigma\sqrt{\gamma}\delta^4(x-Z)(\partial_a Z^\mu\gamma^{ab}\partial_b Z^\nu) + \dots,$$

$$\frac{\delta S_{\mathrm{Kerr}}}{\delta Z^\rho} = \frac{-m}{8\pi a_w^2}\frac{\delta\left[\sqrt{\gamma}\partial_a Z^\mu\partial^a Z^\nu \mathcal{G}_{\mu\nu}(Z) + \cdots\right]}{\delta Z^\rho} = 0\,,$$



Advantages

- amplitude related inertial d.o.f. (a, m)
- no Mathisson-Papapetrou-Dixon (MPD) equation
- hypersurface coordinate equation is reduced to algebra equation

More finite size effects(spin, tidal, absorption)

- double Riemann tensor coupling $\mathcal{RR}(\mathcal{D}Z\cdots\mathcal{D}Z)$
- dynamic of worldvolumn metric

Fundamental action?

Finite terms,

Combinational Coefficient

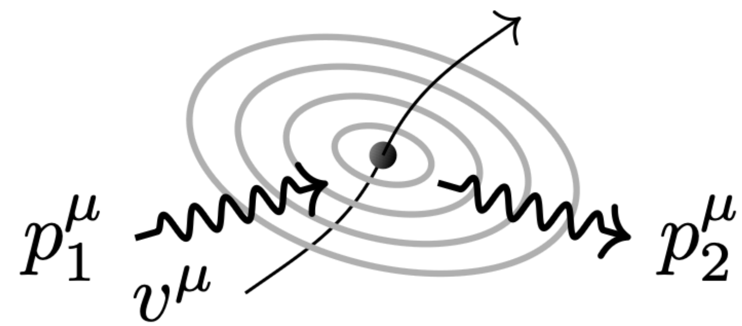
Part II: Bridging amplitude and BHPT

- Bridging gravitational solutions and PM-expanded amplitudes
- Generate the Compton amplitude with finite size effect

Scattering amplitude in BHPT

Amplitude extracted from asymptotic behaviour GR wave

$$r^3 B_{lm}^{(\mathrm{ref})} e^{i\omega r} + \frac{1}{r} B_{lm}^{(\mathrm{inc})} e^{-i\omega r}$$



Properties

- In partial wave
- Work in lab frame
- Parity dependent

$$e^{2i\delta_{lm}^{\pm}} = (-1)^{l+1} \frac{C_{\mathrm{TS}}^{(0)}}{16\omega^4} \frac{B_{lm}^{(\mathrm{ref})}}{B_{lm}^{(\mathrm{inc})}},$$

$$C_{\mathrm{TS}}^{(0)} = l(l^2-1)(l+2) \pm 6i\bar{\epsilon}$$

BHPT amplitude

$$f_{\mathrm{B}}(\theta) = \frac{\pi e^{-i\Phi}}{i\omega} \sum_{l=2}^{\infty} {}_{-2}Y_{l2}(0)\, {}_{-2}Y_{l2}(\theta)\, \Delta_l^{+},$$

$$g_{\mathrm{B}}(\theta) = \frac{\pi e^{-i\Phi}}{i\omega} \sum_{l=2}^{\infty} (-1)^l\, {}_{-2}Y_{l2}(0)\, {}_{-2}Y_{l2}(\pi-\theta) \Delta_l^{-},$$

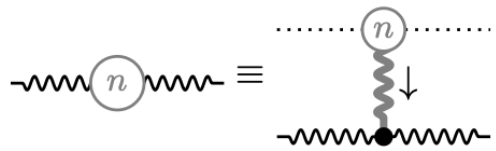
$$\Delta_l^{\pm} = e^{2i\delta_{l2}^{+}} \pm e^{2i\delta_{l2}^{-}}$$

$$\Phi \equiv 2\bar{\epsilon} \ln(2\bar{\epsilon}) - \bar{\epsilon}$$

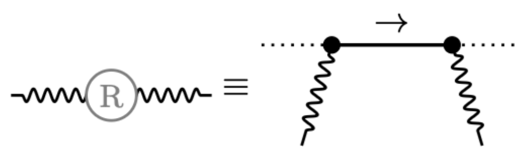
Classical Gravitational Compton Amplitude

Curved Space Rules

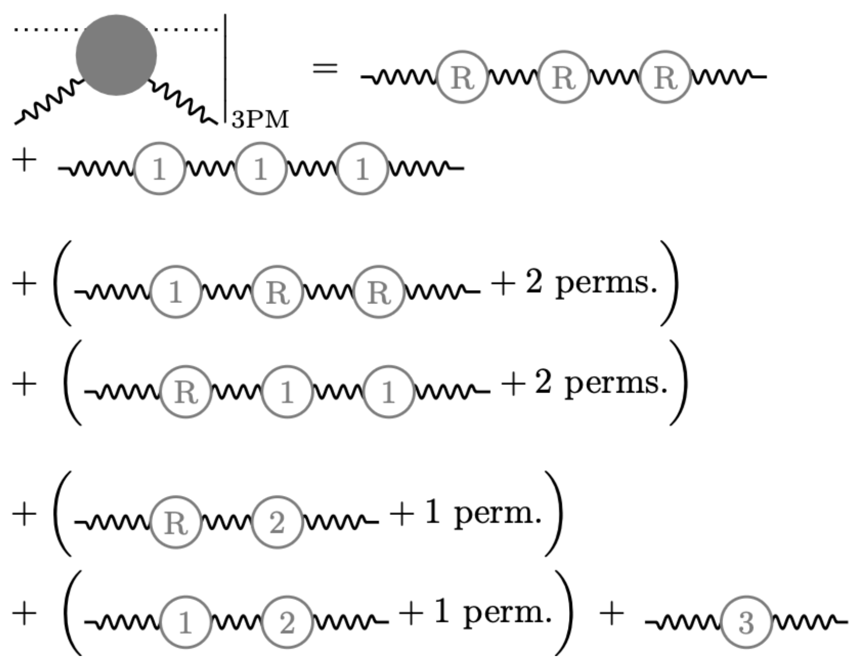
metric insertions



deflection



3PM-13 diagrams



WQFT and HEFT both work

Amplitude agrees in the same day works:
Bjerrum-Bohr, Chen, Eriksen, Shah 2026
Bautista, Driesse, Haddad, Jakobsen 2026
Ivanov, Li, Parra-Martinez, Zhou 2026

General structures

- Compton amplitude expansion

$$\mathcal{M} = \mathcal{M}_0^{1\mathrm{PM}} + \sum_{n=2}^{\infty} \sum_{j=1-n}^{\infty} \epsilon^j \mathcal{M}_j^{n\mathrm{PM}},$$

- Weinberg phase factor $\bar{\epsilon} = 2GM\omega$

$$\mathcal{M}_{\mathrm{fin}}(\mu, \varepsilon_1, \varepsilon_2^*) = \left(e^{i\bar{\epsilon}/\epsilon}\mathcal{M}\right)\Big|_{\epsilon\to 0}$$
$$= \mathcal{M}_0^{1\mathrm{PM}} + \mathcal{M}_0^{2\mathrm{PM}} + \left(\mathcal{M}_0^{3\mathrm{PM}} + i\bar{\epsilon}\mathcal{M}_1^{2\mathrm{PM}}\right)$$
$$+ \cdots$$

- Differential cross section only get contribution from Compton

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}_{\mathrm{fin}}(\mu, \varepsilon_1, \varepsilon_2^*)|^2}{(8\pi M)^2}$$

ϵ positive power in amplitude can't be discard a priori

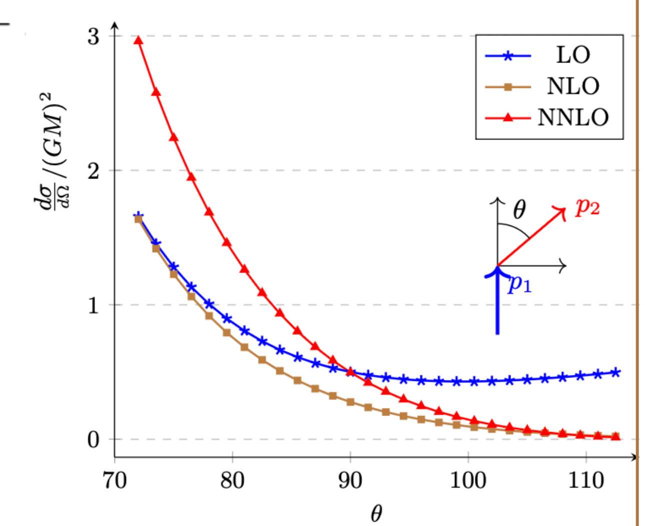
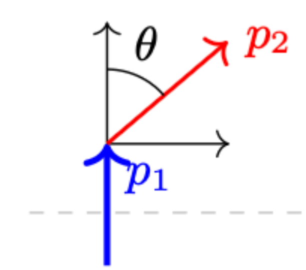


FIG. 2: differential cross section at $\bar{\epsilon} = 0.8$.

Comparing with BHPT

Work in lab frame



Helicity Conserving

$$f_{\rm amp}(\theta) = \frac{\mathcal{M}_{\rm fin}(2e^{-\gamma}\omega, \varepsilon_1^+, \varepsilon_2^{+*})}{8\pi M}$$

Helicity Flip

$$g_{\rm amp}(\theta) = \frac{\mathcal{M}_{\rm fin}(2e^{-\gamma}\omega, \varepsilon_1^+, \varepsilon_2^{-*})}{8\pi M},$$

Project to sphere harmonic function

$$f_{\rm amp}^{(l)} = f_{\rm B}^{(l)} \qquad g_{\rm amp}^{(l)} = g_{\rm B}^{(l)}$$

$$(f)^{(l)} \equiv 2\pi \int_0^\pi f(\theta)\,{}_{-2}Y_{l2}(\theta)\sin(\theta)d\theta,$$

$$(g)^{(l)} \equiv 2\pi \int_0^\pi g(\theta)\,{}_{-2}Y_{l2}(\pi-\theta)\sin(\theta)d\theta$$

A relation from BHPT

$$g_{\rm B}^{(l)} = \frac{6i\bar{\epsilon}}{(-1)^l l(l^2-1)(l+2)} f_{\rm B}^{(l)}$$

Compton amplitude

Related

$$\mathcal{M}(+,+) \longleftrightarrow \mathcal{M}(+,-)$$

Isospectrality

Novel non-perturbative relation

Open question: Universal? (QED,QCD)

Resummation of Leading Forward Divergence

Resum of leading forward divergence

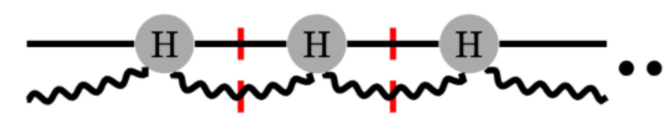
$x = \frac{|q|}{2\omega} = \sin\left(\frac{\theta}{2}\right) \qquad 0 \le x \le 1$

scale parameter fixed

$$\mathcal{M}_{\text{fin}}(2e^{-\gamma}\omega, \varepsilon_1, \varepsilon_2^*) = \boxed{32\pi G M^2 \frac{\Gamma(1-i\bar{\epsilon})}{\Gamma(1+i\bar{\epsilon})} \frac{\mathcal{N}_{\text{tree}}}{4x^{2(1-i\bar{\epsilon})}}}$$

$$+ \sum_{n=2}^{\infty} \mathcal{M}^{n\text{PM}}_{\text{fin,rm}}(2e^{-\gamma}\omega, \varepsilon_1, \varepsilon_2^*)$$

resum $\frac{\mathcal{N}_{\text{tree}}}{x^2}$



sub-divergent terms

Newtonian contribution

Similar structure in Shockwave (Bohnenblust, Eriksen, Hoogeveen, Jakobsen, Plefka, 2026)

Projecting Mix PM orders

$$f^{\text{nt}}_{\text{amp}}(\theta) = GM\left(1-x^2\right)^2 x^{-2+2i\bar{\epsilon}} \frac{\Gamma(1-i\bar{\epsilon})}{\Gamma(1+i\bar{\epsilon})} \xrightarrow[\int dx]{\text{projecting}} \frac{1}{\bar{\epsilon}}(\cdots) + (\cdots)$$

4PM

1PM

$$f^{\text{rm}(2)}_{\text{amp}} = \frac{\left(-28805 + 12\gamma(1715 + 1284 i\pi) - 16050 i\pi + 2568\pi^2\right)\bar{\epsilon}^3}{15120\omega} + \frac{(1284\pi - 1715 i)\bar{\epsilon}^2}{2520\omega},$$

$$f^{\text{nt}(2)}_{\text{amp}} = \frac{\bar{\epsilon}^3\left(\boxed{-1152\zeta(3)} - 2304\gamma^3 + 7200\gamma^2 - 9960\gamma + 5845\right)}{1728\omega} + \frac{\left(415 - 600\gamma + 288\gamma^2\right) i\bar{\epsilon}^2}{144\omega} + \frac{(24\gamma - 25)\bar{\epsilon}}{12\omega} \boxed{- \frac{i}{\omega}}$$

$$f^{(2)}_{B} = \frac{\bar{\epsilon}^3\left(-13440\zeta(3) + 29785 - 88760\gamma + 84000\gamma^2 - 26880\gamma^3 - 21400 i\pi + 20544 i\gamma\pi + 3424\pi^2\right)}{20160\omega}$$

$$+ \frac{i\left(11095 - 21000\gamma + 10080\gamma^2 - 2568 i\pi\right)\bar{\epsilon}^2}{5040\omega} + \frac{(24\gamma - 25)\bar{\epsilon}}{12\omega} - \frac{i}{\omega}$$

Extend to Kerr black hole

Kerr Compton

$$\mathcal{M}_{\text{fin.}}(2e^{-\gamma}\omega,\varepsilon_1,\varepsilon_2^*) = 32\pi G M^2\Big(-\frac{\Gamma(1-i\bar{\epsilon})}{\Gamma(1+i\bar{\epsilon})}\frac{\mathcal{N}_0\mathcal{N}_a}{4x^{2(1-i\bar{\epsilon})}} - \frac{1}{4\omega^2}\mathcal{N}_r + \mathcal{N}_c\Big) + \sum_{n=2}^{\infty}\mathcal{M}^{n\text{PM}}_{\text{fin.,rm}}(2e^{-\gamma}\omega,\varepsilon_1,\varepsilon_2^*)\,.$$

$$|a\omega| \to 0$$

Extend from bootstrap in the effective point-particle(EPP) limit, Bjerrum-Bohr, Chen, Skowronek 2023

Comparing with BHPT $\rightarrow$

Fix tree Compton Uniquely

Agree in EPP limit

Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov 2023
Bautista, Guevara, Kavanagh, Vines 2022
Bern, Kosmopoulos, Luna, Roiban, Teng 2022
Aoude, Haddad, Helset 2022
Chen, Chung, Huang, Kim 2021

Beyond EPP limit : S^5 S^6

$$\mathcal{N}_c^{(5)} = -\frac{2}{45}i(64\zeta(3)-64\zeta(5)-7)a{\cdot}a\,(p_{12}{\cdot}v)\,(a{\cdot}av{\cdot}F_1{\cdot}F_2{\cdot}v - a{\cdot}F_1{\cdot}F_2{\cdot}a)\,(a{\cdot}F_2{\cdot}\widetilde{F}_1{\cdot}v)$$
$$-\frac{1}{90}i(4\zeta(3)-16\zeta(5)+5)a{\cdot}aa{\cdot}F_1{\cdot}va{\cdot}F_2{\cdot}v(p_{12}{\cdot}v)\left(a{\cdot}\widetilde{F}_1{\cdot}F_2{\cdot}v + a{\cdot}F_2{\cdot}\widetilde{F}_1{\cdot}v\right)$$

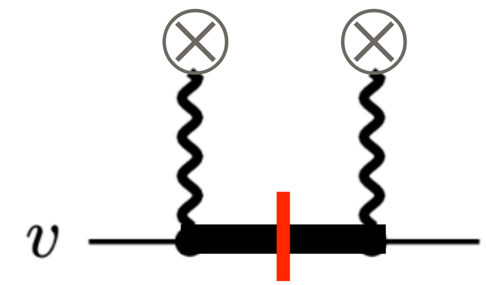
$$\mathcal{N}_c^{(6)} = \frac{a{\cdot}a(p_1{\cdot}v)^2}{\bar{\epsilon}}\Big[\frac{1}{360}i(120\zeta(2)-1152\zeta(4)+1024\zeta(6)-11)(a{\cdot}a)^2\text{tr}\,(F_1{\cdot}F_2)\,v{\cdot}F_1{\cdot}F_2{\cdot}v$$
$$-\frac{1}{90}i(60\zeta(2)-600\zeta(4)+544\zeta(6)-5)(a{\cdot}a)^2\,(v{\cdot}F_1{\cdot}F_2{\cdot}v)^{\,2}$$
$$+\frac{1}{120}i(120\zeta(2)-1152\zeta(4)+1024\zeta(6)-1)(a{\cdot}a)\,(a{\cdot}F_1{\cdot}F_2{\cdot}v + a{\cdot}F_2{\cdot}F_1{\cdot}v)^{\,2}$$
$$-\frac{1}{15}i(10\zeta(2)-92\zeta(4)+80\zeta(6)-1)\,(a{\cdot}F_1{\cdot}F_2{\cdot}a)^{\,2}.\Big]$$
$$\ldots$$

Properties

- Vanishing in EPP limit
- Non-Combinational coefficients
- Spin-related PM order mixing
- Indicate extended body dynamical Model(non-local field theory)

	S^0	S^1	S^2	S^3	S^4	S^5	S^6
Tree	1PM	2PM	3PM	4PM	5PM	6PM	7PM+6PM

Outlook



- Connection to Green function in Curved background and quasi-normal modes (QNM) (separating PM expansion and frequency expansion, aim to obtain resumed finite size effect)

- Higher-point amplitudes and non-linear QNM dynamics(Beyond BHPT)

- Developing a classical non-local field theory based on hypersurface model(Fix double Riemann tensor couplings and worldvolumn dynamics by bridging with BHPT)

- Toward a quantum description of black holes and quantum gravity