



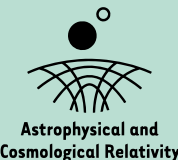
# Black Hole Response Theory and its Shockwave Limit

Based on [2604.22009] & upcoming work with Carl Jordan Eriksen, Jitze Hoogeveen, Gustav Uhre Jakobsen and Jan Plefka

**Amplitudes, Queen Mary University of London**

Lara Bohnenblust

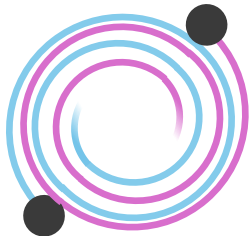
29. June 2026



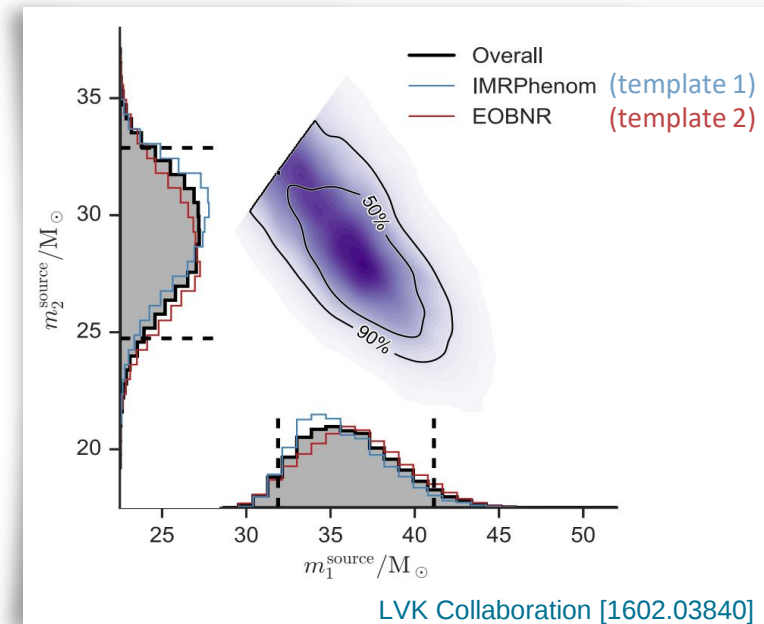
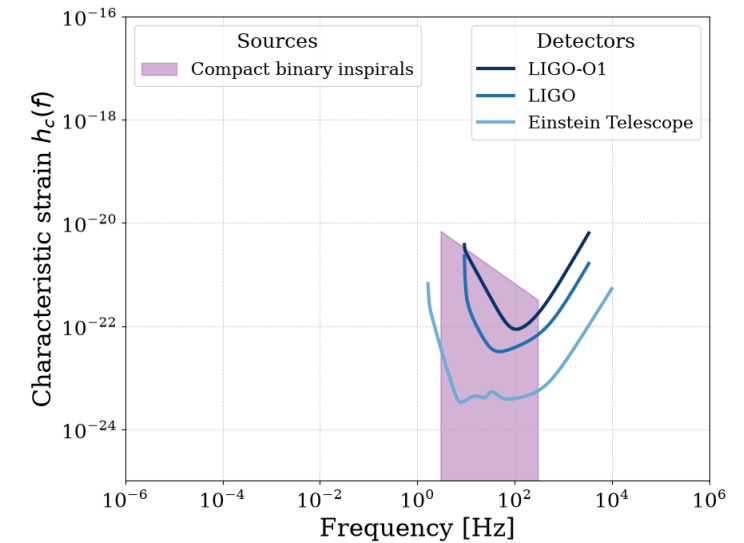
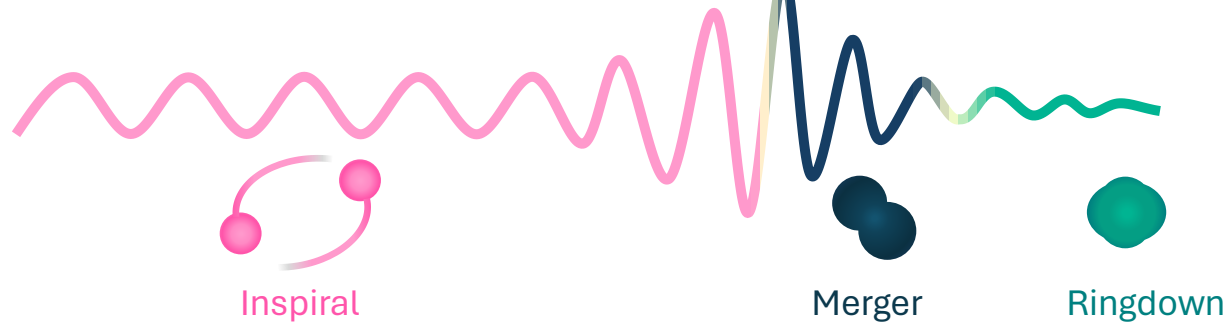
# Gravitational wave modeling



Currently  
detectable

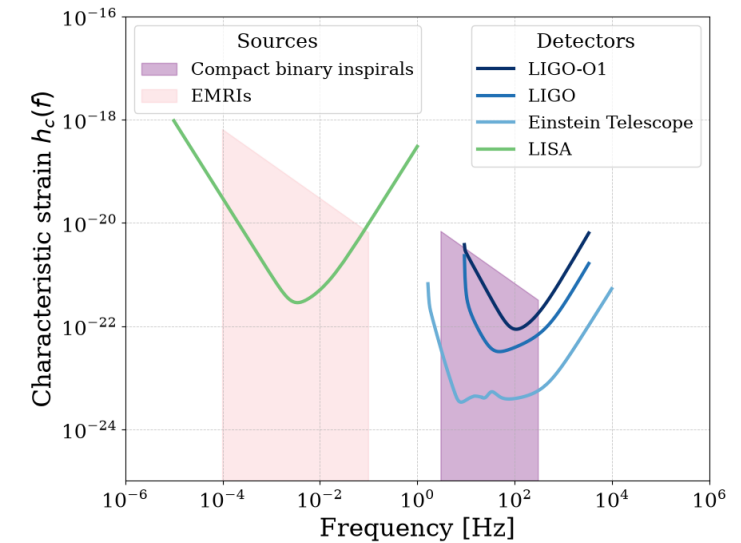
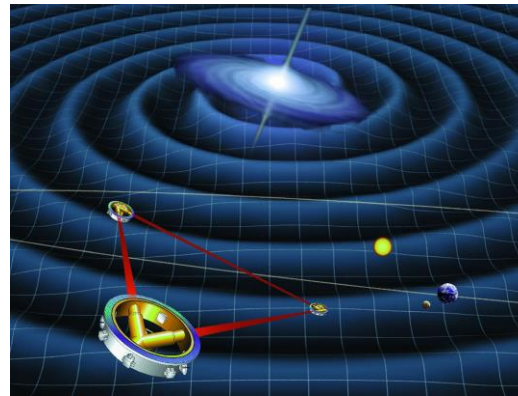


Quasi-circular waveform model

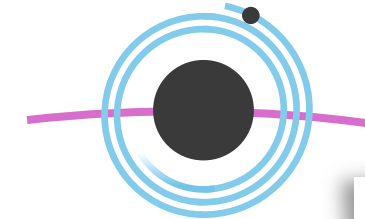
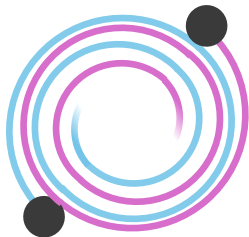


Precise and  
accurate  
models are  
crucial!

# Gravitational wave modeling

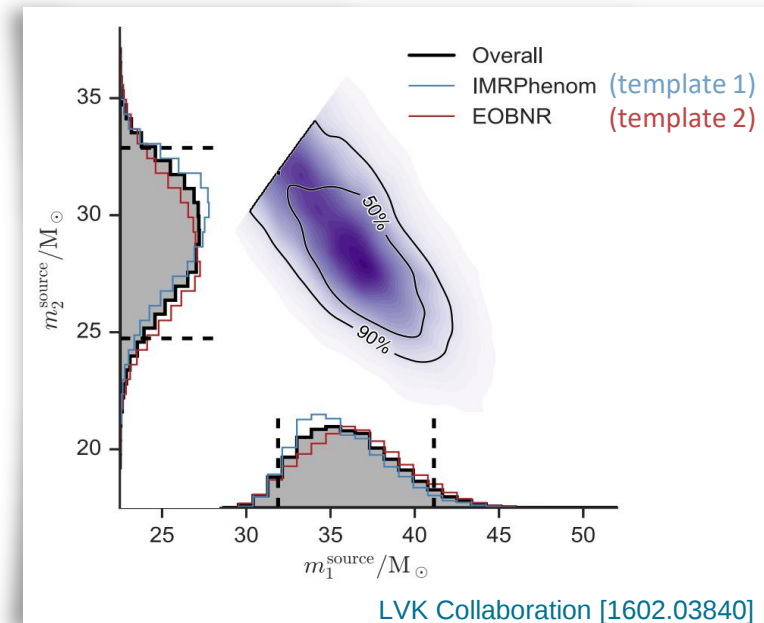
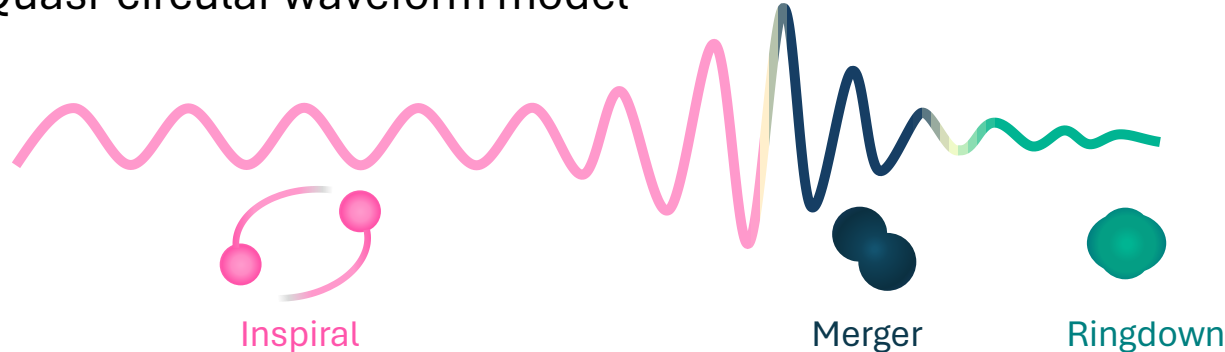


Currently detectable



Detectable in the future!

Quasi-circular waveform model

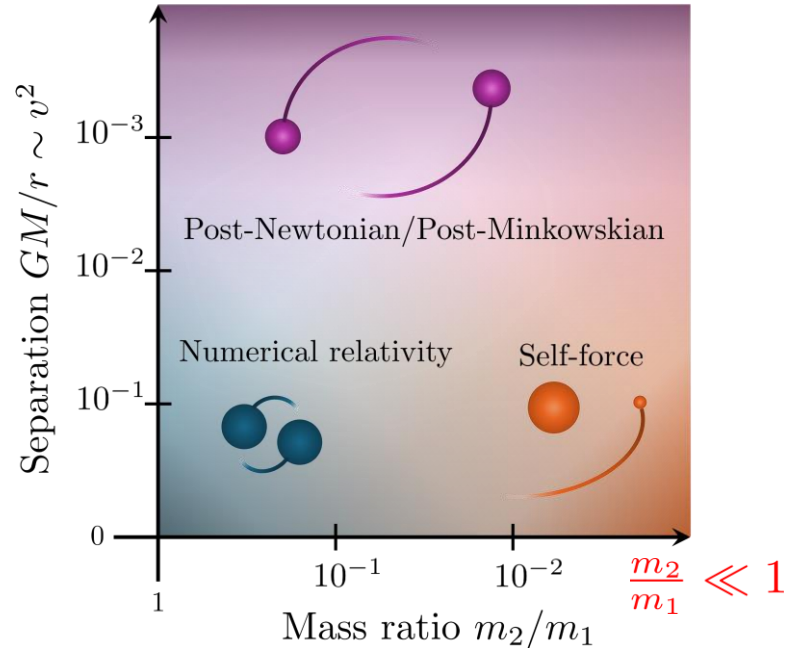


Precise and accurate models are crucial but **do not exist for generic EMRIs!**

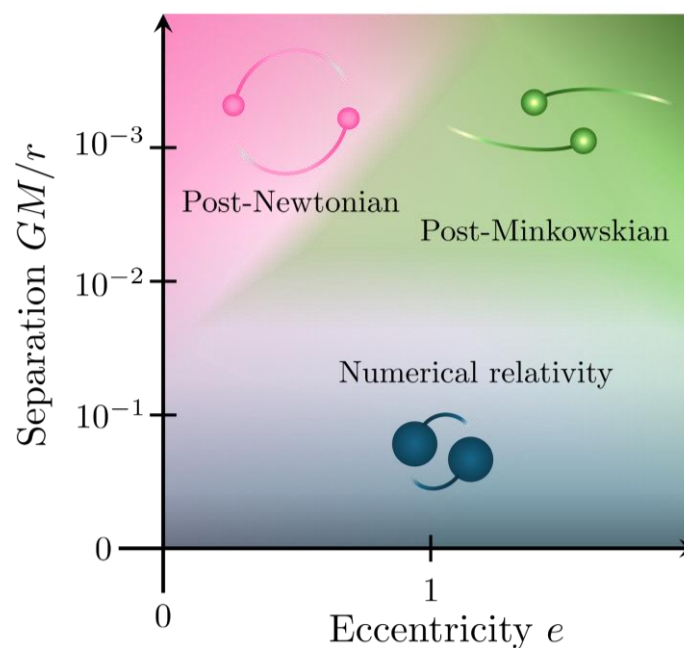
# Gravitational wave modeling

→ Solving **non-linear** Einstein field equations  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$  → **approximate in small parameter**

$$v^2 \sim \frac{GM}{r} \ll 1$$

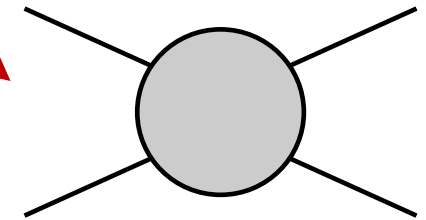


$$v^2 \sim \frac{GM}{r} \ll 1$$



$$\frac{GM}{r} \ll 1$$

From amplitudes:  
**Post-Minkowskian (PM)**  
 $\chi = G\chi_{(1)} + G^2\chi_{(2)} + \dots$   
**Weak coupling expansion**



Classical **worldline QFT** amplitudes

see Gustav Jakobsen's talk

For EMRIs: **self-force (SF)**

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Kerr}} + \frac{m_2}{m_1} h_{\mu\nu}^{(1)} + \left(\frac{m_2}{m_1}\right)^2 h_{\mu\nu}^{(2)} + \dots$$

**Expansion around background metric and geodesic**

**Black hole response theory**

**Organize** amplitudes and **resum** to obtain results in the strong field!

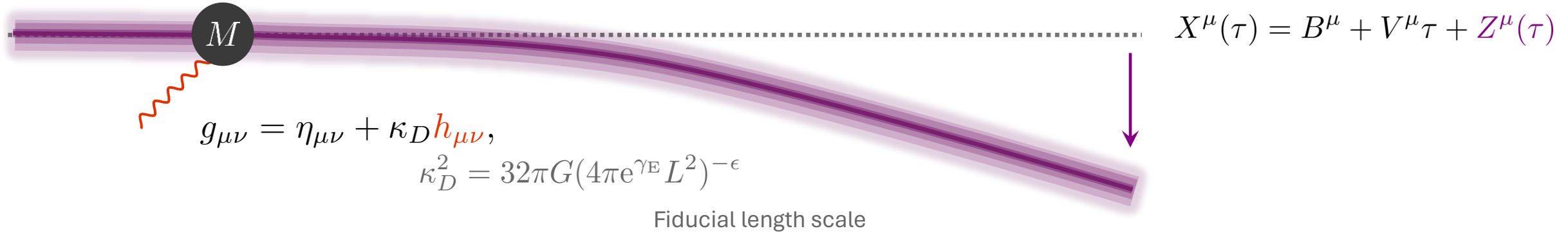
# WQFT of a single black hole

Mogull, Plefka, Steinhoff  
[2010.02865]

$$S_{\text{BH}}[X, g] = -\frac{M}{2} \int d\tau g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu - \int d^D x \frac{2}{\kappa_D^2} \sqrt{|g|} R$$

$$g_{\mu\nu}(X) = \eta_{\mu\nu}$$

Cheung, Parra-Martinez, Rothstein,  
Shah, Wilson-Gerow [2406.14770]



**Quantize deviation** from background solution  $\rightarrow$  Classical solution to EOM are **tree-level diagrams**

**Feynman rules for “in-in” (Schwinger-Keldysh) prescription:**

$$\begin{array}{c} Z^\mu(-\omega) \quad Z^\nu(\omega) \\ \cdots \bullet \longrightarrow \bullet \cdots \end{array} = \frac{-i\eta^{\mu\nu}}{M(\omega + i0^+)^2}$$

$$\begin{array}{c} h_{\mu\nu}(-k) \quad h_{\rho\sigma}(k) \\ \bullet \cdots \longrightarrow \bullet \cdots \end{array} = \frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{k^2 + i0^+(W \cdot k)}$$

$$\begin{array}{c} \cdots \bullet \cdots \\ | \\ \text{wavy line} \\ | \\ \bullet \cdots \\ h_{\mu\nu}(k) \end{array} \sim M\kappa_D$$

$$\begin{array}{c} Z^\rho(\omega) \cdots \bullet \cdots \\ | \\ \text{wavy line} \\ | \\ \bullet \cdots \\ h_{\mu\nu}(k) \end{array} \sim M\kappa_D$$

$$\begin{array}{c} \text{wavy line} \\ | \\ \bullet \\ | \\ \text{wavy line} \end{array} \sim \kappa_D$$

$$\begin{array}{c} \text{wavy line} \\ | \\ \bullet \\ | \\ \text{wavy line} \end{array} \sim \kappa_D^2$$

# Black hole response theory

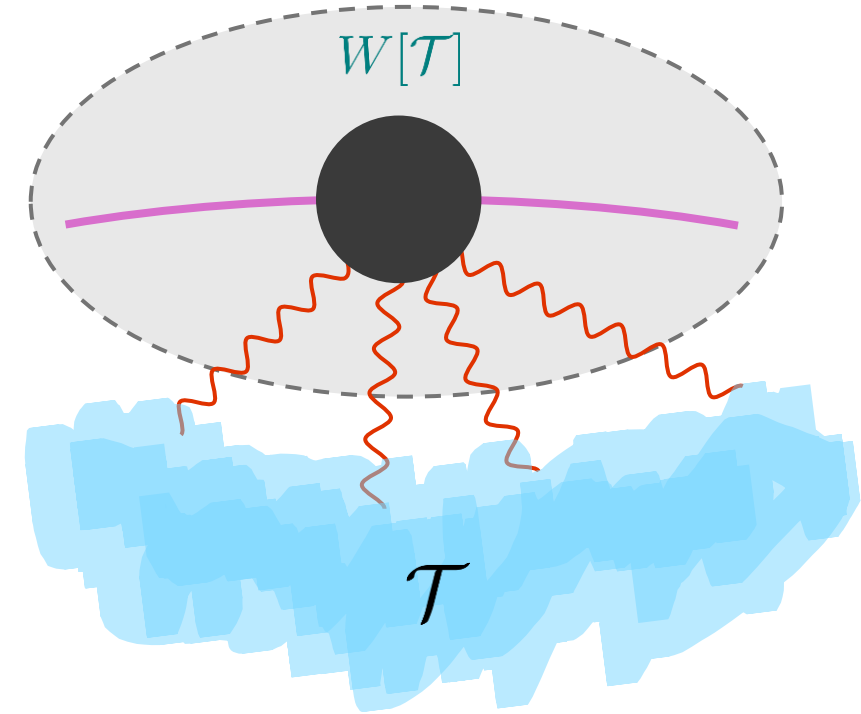
Integrate out worldline and graviton fluctuation

→ **Black hole response (BHR) effective action** encoding response to arbitrary source

$$e^{\frac{i}{\hbar} W[\mathcal{T}]} = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}}$$

Expansion in source yields **response functions**:

$$\begin{aligned} iW[\mathcal{T}] = & -i\kappa_D \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{R}_{\alpha\beta}(k_1) - \frac{\kappa_D^2}{2} \int_{k_1, k_2} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \mathcal{R}_{\alpha\beta\mu\nu}(k_1, k_2) \\ & + \frac{i\kappa_D^3}{6} \int_{k_1, k_2, k_3} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \mathcal{T}^{\rho\sigma}(k_3) \mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1, k_2, k_3) + \dots \end{aligned}$$

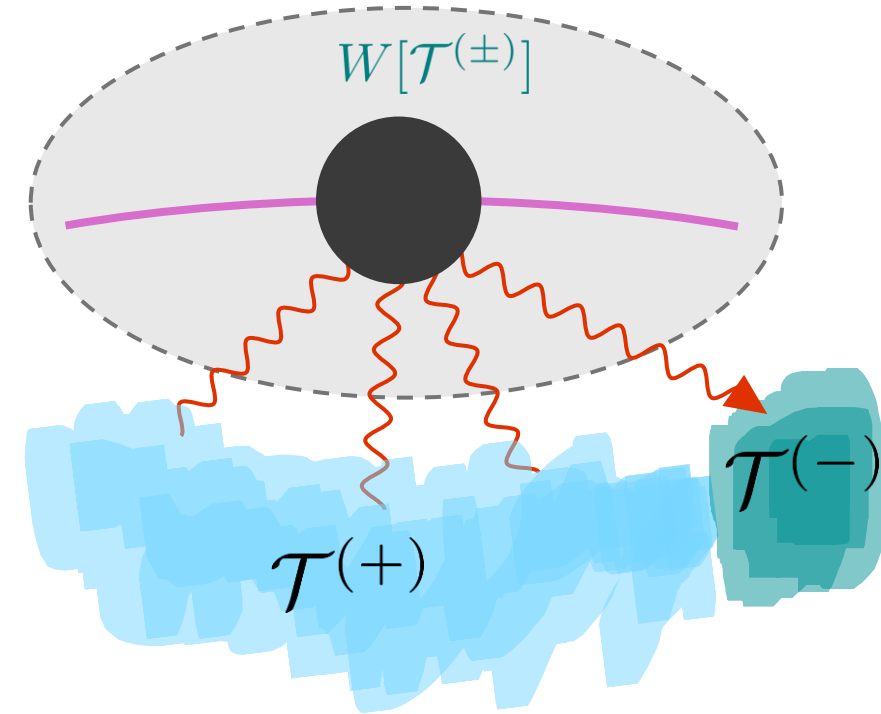


# Black hole response theory with “in-in”

Integrate out worldline and graviton fluctuation (**Keldysh-Schwinger**)

→ **Black hole response (BHR) effective action** encoding response to arbitrary source in Keldysh basis  $\mathcal{F}^{(\pm)} \sim \mathcal{F}^{(1)} \pm \mathcal{F}^{(2)}$

$$e^{\frac{i}{\hbar} W[\mathcal{T}^{(1,2)}]} = \int \mathcal{D}[h^{(1,2)}, Z^{(1,2)}] e^{\frac{i}{\hbar} S[Z^{(1)}, h^{(1)}, \mathcal{T}^{(1)}] - \frac{i}{\hbar} S[Z^{(2)}, h^{(2)}, \mathcal{T}^{(2)}]} \Big|_{\text{tree}}$$



Expansion in source yields **response functions**:

$$\begin{aligned} iW[\mathcal{T}^{(\pm)}] = & -i\kappa_D \int_{k_1} \mathcal{T}^{(-)\alpha\beta}(k_1) \mathcal{R}_{\alpha\beta}(k_1) - \kappa_D^2 \int_{k_1, k_2} \mathcal{T}^{(+)\alpha\beta}(k_1) \mathcal{T}^{(-)\mu\nu}(k_2) \mathcal{R}_{\alpha\beta\mu\nu}(k_1, k_2) \\ & + \frac{i}{2} \kappa_D^3 \int_{k_1, k_2, k_3} \mathcal{T}^{(+)\alpha\beta}(k_1) \mathcal{T}^{(+)\mu\nu}(k_2) \mathcal{T}^{(-)\rho\sigma}(k_3) \mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1, k_2, k_3) + \dots \end{aligned}$$

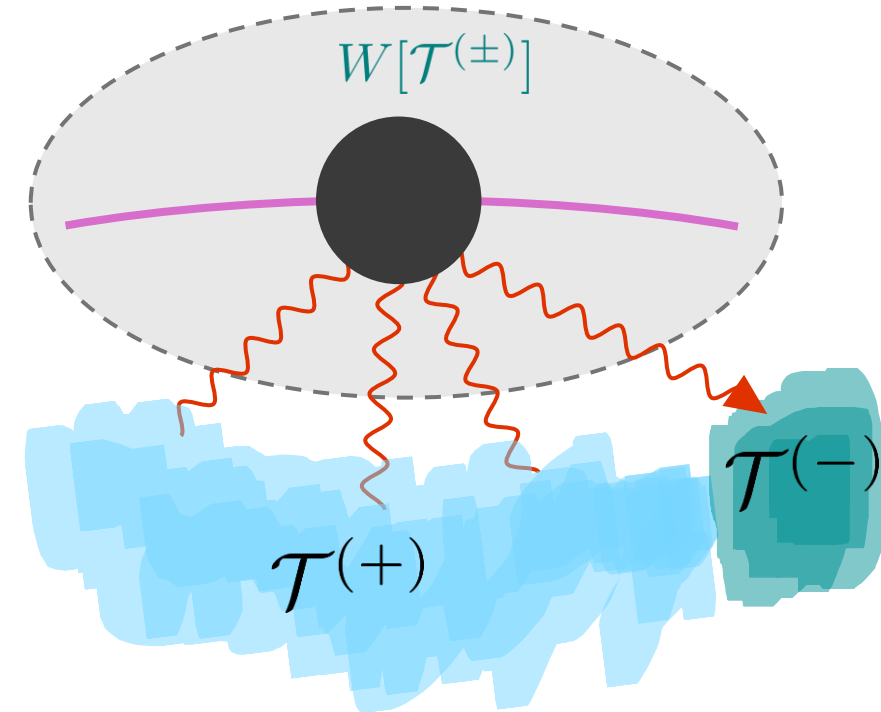
Many incoming (+) legs and one outgoing (−) leg

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$$e^{\frac{i}{\hbar} W[\mathcal{T}^{(1,2)}]} = \int \mathcal{D}[h^{(1,2)}, Z^{(1,2)}] e^{\frac{i}{\hbar} S[Z^{(1)}, h^{(1)}, \mathcal{T}^{(1)}] - \frac{i}{\hbar} S[Z^{(2)}, h^{(2)}, \mathcal{T}^{(2)}]} \Big|_{\text{tree}}$$



Expansion in source yields **response functions**:

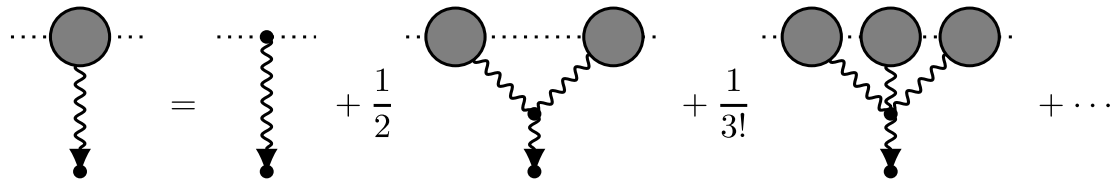
$$W[\mathcal{T}^{(\pm)}] = -i\kappa_D \int_{k_1} \left[ \text{diagram with 1 incoming (+) leg and 1 outgoing (-) leg} \right] - \kappa_D^2 \int_{k_1, k_2} \left[ \text{diagram with 2 incoming (+) legs and 1 outgoing (-) leg} \right] + \frac{i\kappa_D^3}{2} \int_{k_1, k_2, k_3} \left[ \text{diagram with 3 incoming (+) legs and 1 outgoing (-) leg} \right] + \dots$$

Many incoming (+) legs and one outgoing (−) leg

# Response functions

$$e^{\frac{i}{\hbar} W[\mathcal{T}]} = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}}$$

One-point response is equivalent to **background metric**

$$\mathcal{R}_{\mu\nu} = -\kappa_D^{-1} \frac{\delta W[J]}{\delta \mathcal{T}^{\mu\nu}(x)} \Big|_{J=0} =$$


$$= \langle h_{\mu\nu}(x) \rangle_{J=0}$$

# Response functions

$$e^{\frac{i}{\hbar} W[\mathcal{T}]} = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}}$$

One-point response is equivalent to **background metric**

$$\mathcal{R}_{\mu\nu} = -\kappa_D^{-1} \frac{\delta W[J]}{\delta \mathcal{T}^{\mu\nu}(x)} \Big|_{J=0} = \text{[Feynman diagrams]} = \langle h_{\mu\nu}(x) \rangle_{J=0}$$

The Feynman diagrams represent the perturbative expansion of the one-point response function. They consist of a series of terms: a single vertical wavy line with a downward arrow, followed by a term with a coefficient of 1/2 and a diagram of two wavy lines meeting at a vertex with a downward arrow, then a term with a coefficient of 1/3! and a diagram of three wavy lines meeting at a vertex with a downward arrow, and finally a term with a coefficient of 1/2 and a diagram of four wavy lines meeting at a vertex with a downward arrow, followed by an ellipsis. An arrow points from the boxed term in the top equation to the final term in this series.

# Response functions

$$e^{\frac{i}{\hbar} W[\mathcal{T}]} = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}}$$

**One-point response** is equivalent to **background metric**

$$\mathcal{R}_{\mu\nu} = -\kappa_D^{-1} \frac{\delta W[J]}{\delta \mathcal{T}^{\mu\nu}(x)} \Big|_{J=0} = \text{[diagram: circle with wavy line]} = \text{[diagram: wavy line]} + \frac{1}{2} \text{[diagram: two circles with wavy lines]} + \frac{1}{3!} \text{[diagram: three circles with wavy lines]} + \dots = \langle h_{\mu\nu}(x) \rangle_{J=0}$$

**Two-point response** acts as **propagator** on curved spacetime

$$\text{[diagram: circle with wavy lines]} = \text{[diagram: wavy line]} + \sum_{n=1}^{\infty} \left[ \text{[diagram: two circles with wavy lines]} + \sum_{m=1}^{\infty} \frac{1}{m!} \text{[diagram: m circles with wavy lines]} \right]^n$$

**Three-point response** (and higher) can be recovered from lower-point responses

$$\text{[diagram: circle with wavy lines]} = \text{[diagram: three circles with wavy lines, first and third circled]} + \left[ \text{[diagram: two circles with wavy lines, first circled]} + \text{[diagram: two circles with wavy lines, second circled]} + (\text{two permutations}) \right]$$

With vertex rule

$$\text{[diagram: circle with wavy lines]} = \sum_{m=0}^{\infty} \frac{1}{m!} \text{[diagram: m circles with wavy lines]}$$

# Self-force expansion in black hole response theory

Couple to **light secondary black hole**  $\mathcal{T}_{\text{sec}}^{\mu\nu}(x) = \frac{m}{2} \int d\tau \delta^{(D)}(x - x(\tau)) \dot{x}^\mu(\tau) \dot{x}^\nu(\tau)$

$$S_{\text{BHR}}[x] = S_{\text{kin}}[x] + W[\mathcal{T}_{\text{sec}}(x)]$$

natural **self-force** expansion

$$= S_{\text{kin}}[x] - \frac{1}{2} m \kappa_D \int d\tau_1 \text{ (diagram)} + \frac{i}{8} m^2 \kappa_D^2 \int d\tau_1 d\tau_2 \text{ (diagram)} + \frac{1}{48} m^3 \kappa_D^3 \int d\tau_1 d\tau_2 d\tau_3 \dots$$

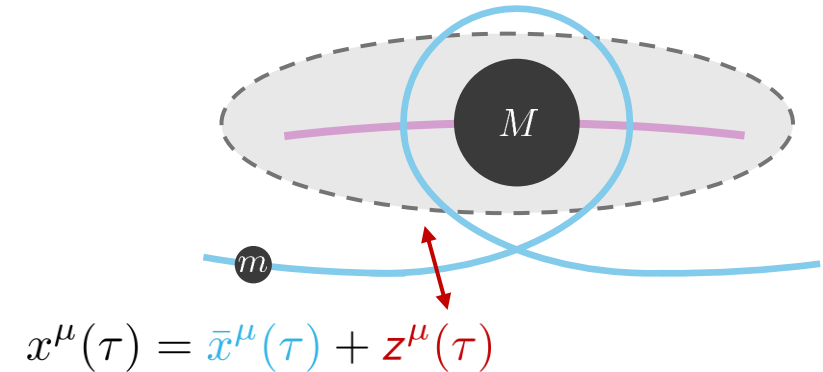
# Self-force expansion in black hole response theory

Couple to **light secondary black hole**  $\mathcal{T}_{\text{sec}}^{\mu\nu}(x) = \frac{m}{2} \int d\tau \delta^{(D)}(x - x(\tau)) \dot{x}^\mu(\tau) \dot{x}^\nu(\tau)$

$$S_{\text{BHR}}[x] = S_{\text{kin}}[x] + W[\mathcal{T}_{\text{sec}}(x)] \quad \text{natural **self-force** expansion}$$

$$= S_{\text{kin}}[x] + \underbrace{\dots \text{ (1 loop) } \dots}_{\text{OSF}} + \frac{1}{2} \dots \text{ (2 loops) } \dots + \frac{1}{3!} \dots \text{ (3 loops) } \dots + \dots$$

**OSF** solved by **geodesic**  $\bar{x}^\mu(\tau)$



Expand around **OSF**

**Feynman vertices** resumming background motion

$$h_{\mu\nu}(k) \begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} z^{\alpha_1}(\omega_1) \\ \text{---} z^{\alpha_2}(\omega_2) \\ \vdots \\ \text{---} z^{\alpha_n}(\omega_n) \end{array} = \sum_{m=0}^{\infty} \frac{1}{m!} \begin{array}{c} \overbrace{\dots \text{ (m loops) } \dots}^m \\ \text{---} \otimes \text{---} \\ \text{---} z^{\rho_1}(\omega_1) \\ \text{---} z^{\rho_2}(\omega_2) \\ \vdots \\ \text{---} z^{\rho_n}(\omega_n) \end{array}$$

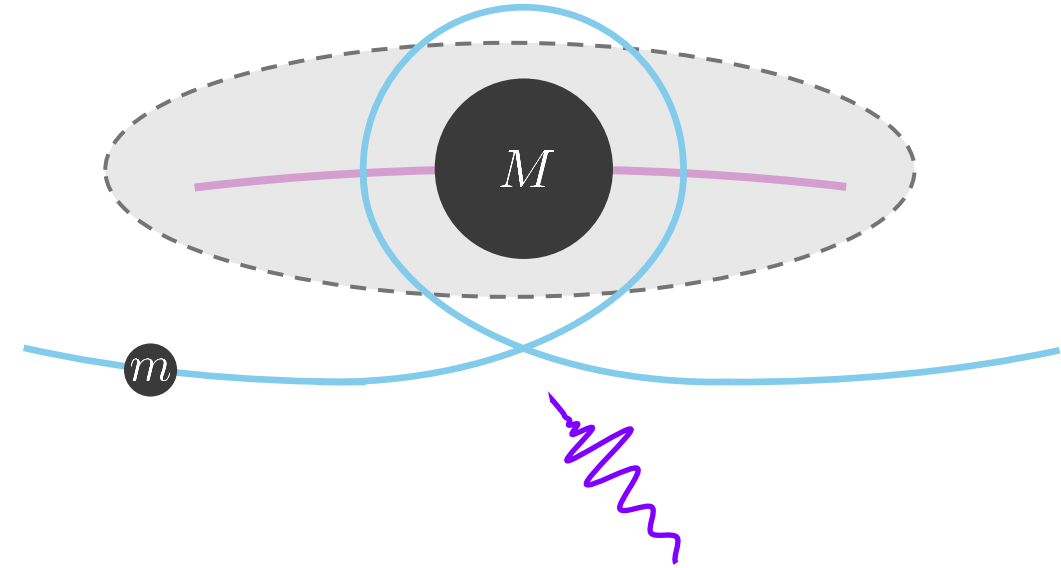
$$= -i\kappa_D \frac{\delta^n \mathcal{T}_{\text{sec}}^{\mu\nu}(k)}{\delta x^{\alpha_1}(\omega_1) \delta x^{\alpha_2}(\omega_2) \dots \delta x^{\alpha_n}(\omega_n)} \Big|_{x^\mu \rightarrow \bar{x}^\mu}$$

# Observables in the self-force expansion

## Trajectory/Impulse

$$\langle z^\rho(\omega) \rangle =$$

The diagrams for  $\langle z^\rho(\omega) \rangle$  are organized into two rows. The first row contains a tree-level diagram with a mass  $m^1$  and a  $\frac{1}{2}$  order diagram with two mass  $m^2$  vertices. The second row contains a  $\frac{1}{2}$  order diagram with two mass  $m^2$  vertices and a higher-order term  $\mathcal{O}\left(\frac{m^3}{M^3}\right)$ .



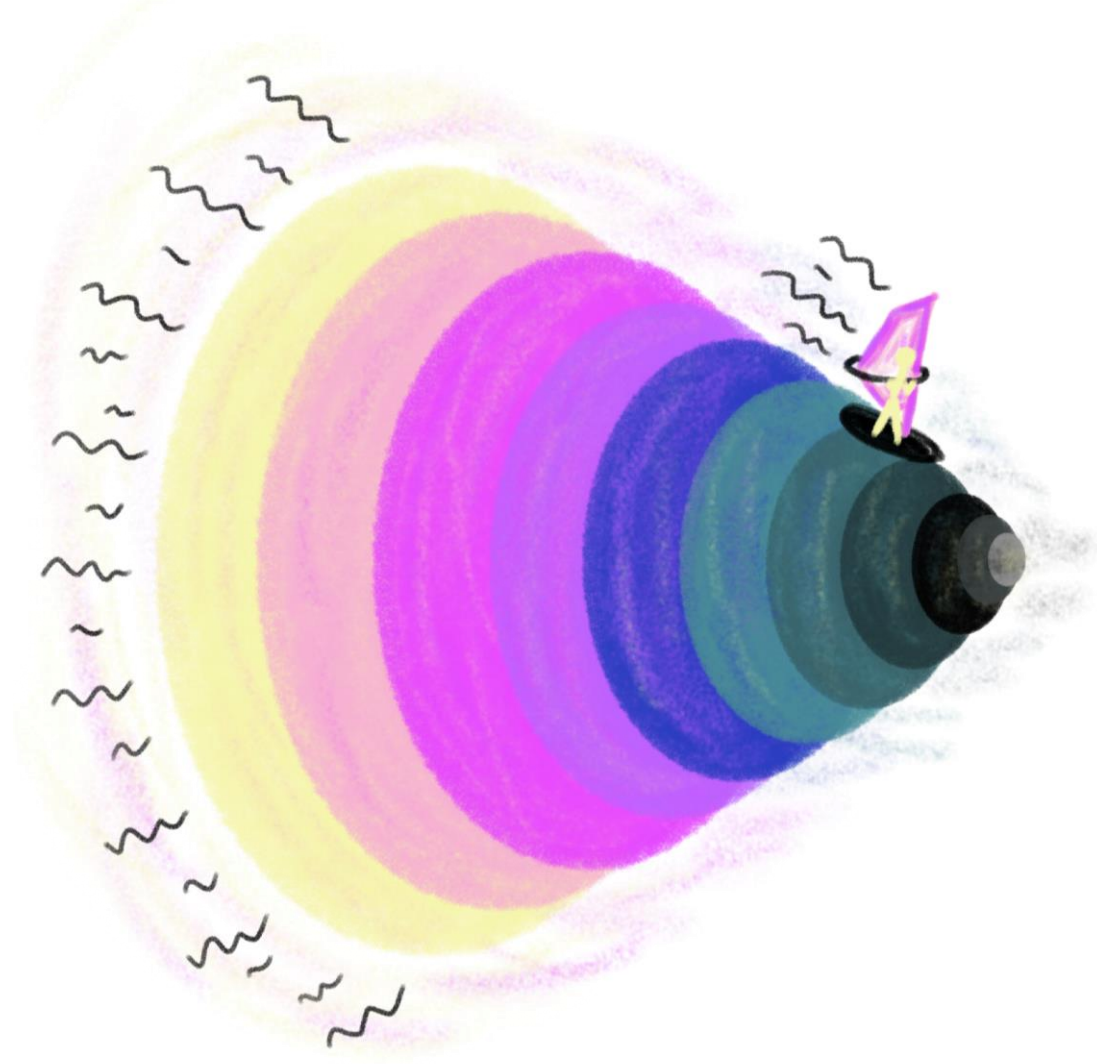
## Waveform

$$\langle h_{\mu\nu}(k) \rangle =$$

The diagrams for  $\langle h_{\mu\nu}(k) \rangle$  are organized into two rows. The first row contains a tree-level diagram with a mass  $m^1$  and a  $\frac{1}{2}$  order diagram with two mass  $m^2$  vertices. The second row contains a  $\frac{1}{2}$  order diagram with two mass  $m^2$  vertices and a higher-order term  $\mathcal{O}\left(\frac{m^3}{M^3}\right)$ .

## Ingredients:

- Background metric
- Geodesics and cross vertices
- $n$ -point response function for  $(n-1)$  self-force observable



Application of black hole response theory  
to the **shockwave background**

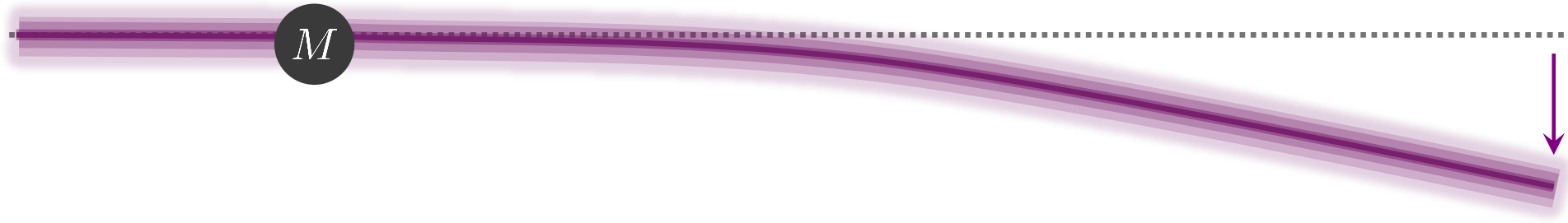
# Massless WQFT

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Brink-Di Vecchia-Howe or Polyakov form for massless limit

$$S = -\frac{1}{2} \int d\tau \left( e^{-1} g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu + e M^2 \right) \xrightarrow{M \rightarrow 0, e^{-1} \rightarrow E} -\frac{E}{2} \int d\tau g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu$$

**Infinitely boosted black hole** with energy  $E$

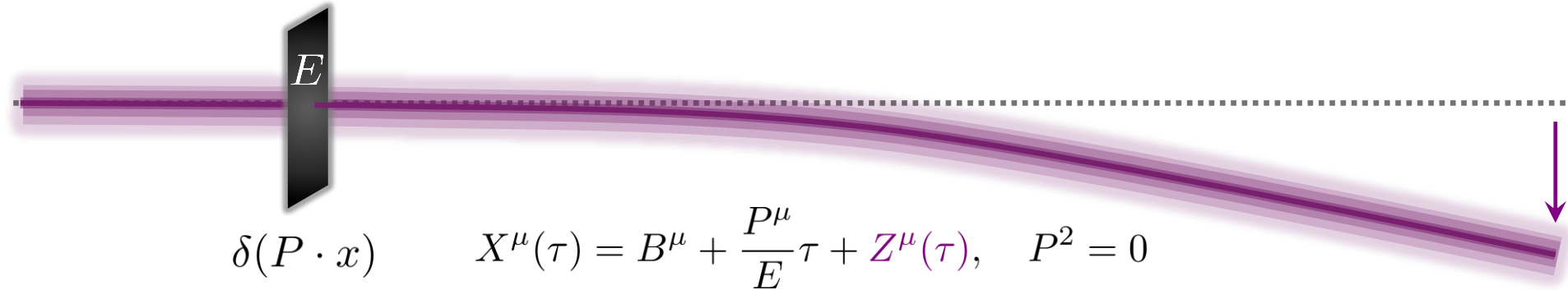


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**Infinitely boosted black hole with energy  $E$**



$$\begin{array}{c} \text{Diagram 1: A wavy line labeled } h_{\mu\nu}(k) \text{ enters from the left and splits into three curved lines labeled } Z^{\rho_1}(\omega_1), Z^{\rho_2}(\omega_2), \text{ and } Z^{\rho_n}(\omega_n). \\ \text{Diagram 2: A wavy line labeled } h_{\mu\nu}(k) \text{ enters from the left and splits into three curved lines labeled } Z^{\rho_1}(\omega_1), Z^{\rho_2}(\omega_2), \text{ and } Z^{\rho_n}(\omega_n). \end{array} = \begin{array}{c} \text{Diagram 3: A wavy line labeled } h_{\mu\nu}(k) \text{ enters from the left and splits into three curved lines labeled } Z^{\rho_1}(\omega_1), Z^{\rho_2}(\omega_2), \text{ and } Z^{\rho_n}(\omega_n). \end{array}$$

$\left. \vphantom{\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array}} \right| \begin{array}{l} M \rightarrow E \\ V^\mu \rightarrow P^\mu / E \end{array}$

$$\begin{array}{c} \text{Diagram 1: A horizontal line with a dot labeled } \mu \text{ on the left and a dot labeled } \nu \text{ on the right. The line is labeled } \omega \text{ below it. To the right of the diagram is the equation } = \frac{-i\eta^{\mu\nu}}{E(\omega + i0^+)^2}. \\ \text{Diagram 2: A wavy line with a dot labeled } h_{\mu\nu}(-k) \text{ on the left and a dot labeled } h_{\rho\sigma}(k) \text{ on the right. To the right of the diagram is the equation } = \frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{k^2 + i0^+(P \cdot k)}. \end{array}$$

# The shockwave metric

$$\text{Vertex} = \text{Vertex} + \frac{1}{2} \text{Vertex} + \frac{1}{3!} \text{Vertex} + \dots$$

**1PM contribution**

$$\kappa_D \langle h_{\mu\nu}(x) \rangle_{1\text{PM}} = \kappa_D \int_k e^{-ik \cdot x} \text{Diagram} = \frac{\kappa_D^2}{2} P_\mu P_\nu \int_k e^{-ik \cdot x} \frac{\delta(k \cdot P)}{k^2} = 4G \delta(P \cdot x) \Gamma(-\epsilon) \overbrace{\left( \frac{x_\perp^2}{4e^{\gamma_E} L^2} \right)^\epsilon}^{f(x_\perp^2)}$$

Diagram: A wavy line with a dot at the top and an arrow pointing down, labeled  $h_{\mu\nu}(k)$ .

**2PM contribution**

$$\text{Diagram} = \int_{\ell_1} \frac{\delta(P \cdot \ell_1) \delta(P \cdot \ell_2) P^{\mu_1} P^{\nu_1} P^{\mu_2} P^{\nu_2} \Omega_{\mu_1 \nu_1 \mu_2 \nu_2; \mu\nu}(\ell_1, \ell_2)}{\ell_1^2 \ell_2^2} \sim P^2, P \cdot \ell_1, P \cdot \ell_2 = 0$$

Diagram: A wavy line with two dots at the top, labeled  $\ell_1$  and  $\ell_2$ . The dot  $\ell_2$  is circled in blue. A blue label  $\sim P^\mu P^\nu$  is next to it. An arrow points down from the wavy line, labeled  $h_{\mu\nu}(k)$ .

Light-cone decomposition

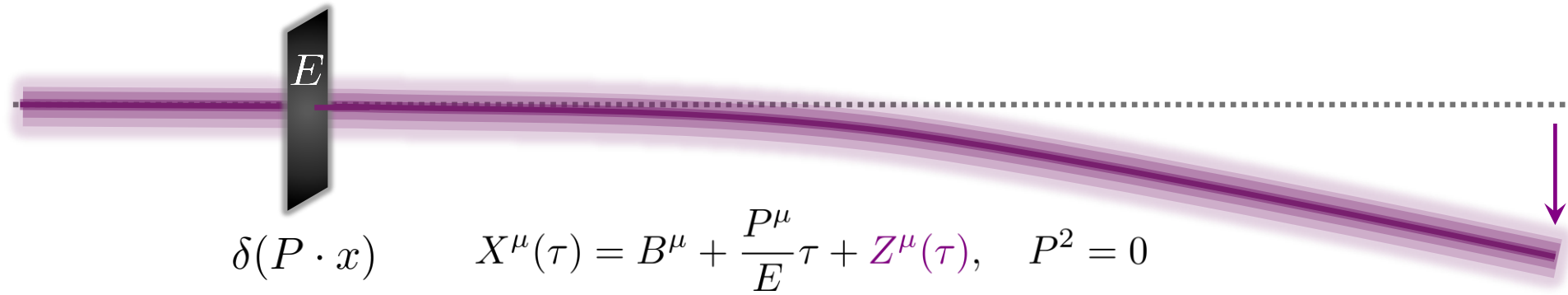
$$\ell^\mu = \frac{\ell^+ e^\mu + \ell^- \bar{e}^\mu}{2} + \ell_\perp^\mu$$

$$\sim \int d^D \ell_1 \frac{\delta(\ell_1^-)}{\ell_1^2} = \underbrace{\left( \int \frac{d\ell_1^+}{2} \right)}_{\text{Non-regularised divergence}} \int d^{D-2} \ell_{1\perp} \frac{1}{\ell_{1\perp}^2}$$

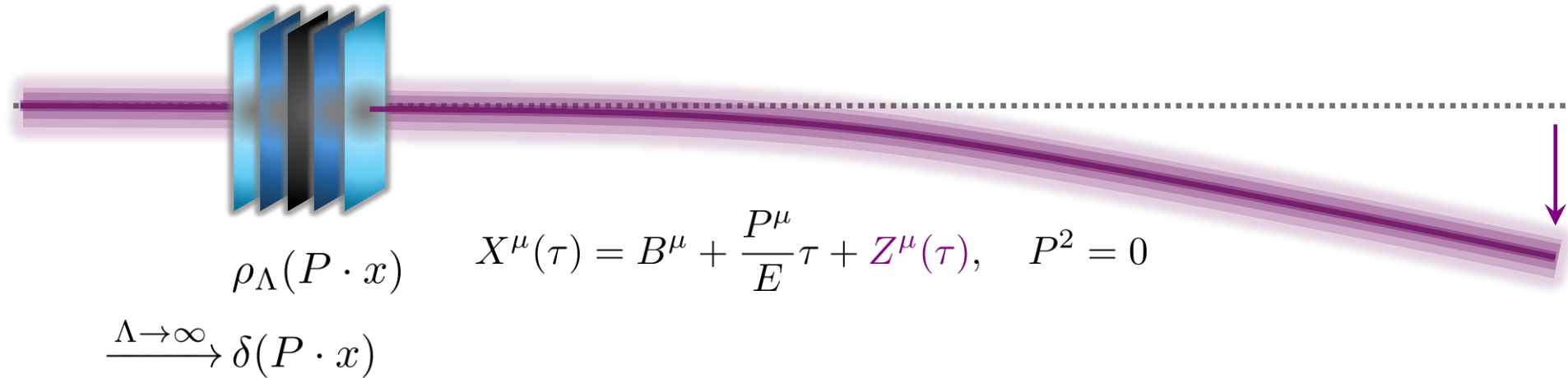
Non-regularised divergence

# Regularizing massless WQFT

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# Regularizing massless WQFT



Choose  $\rho_\Lambda(\sigma) = \frac{\Lambda}{2} \exp(-\Lambda|\sigma|)$  with Fourier transformation  $\tilde{\rho}_\Lambda(\omega) = \frac{\Lambda^2}{\omega^2 + \Lambda^2}$

$\Rightarrow$ 
 $\sim \frac{\Lambda^2}{k^{+2} + \Lambda^2}$

# The shockwave metric

**1PM contribution**

$$\kappa_D \langle h_{\mu\nu}(x) \rangle_{1\text{PM}} = \kappa_D \int_k e^{-ik \cdot x} \text{ (diagram: wavy line with dot at top and arrow at bottom) } = \frac{\kappa_D^2}{2} P_\mu P_\nu \int_k e^{-ik \cdot x} \frac{\delta(k \cdot P)}{k^2} = 4G \delta(P \cdot x) \Gamma(-\epsilon) \overbrace{\left( \frac{x_\perp^2}{4e^{\gamma_E} L^2} \right)^\epsilon}^{f(x_\perp^2)}$$

**2PM contribution**

$$\text{ (diagram: two wavy lines meeting at a vertex, with dots at top and arrow at bottom) } = \int_{\ell_1} \frac{\Lambda^4 \delta(P \cdot \ell_1) \delta(P \cdot \ell_2) P^{\mu_1} P^{\nu_1} P^{\mu_2} P^{\nu_2} \Omega_{\mu_1 \nu_1 \mu_2 \nu_2; \mu\nu}(\ell_1, \ell_2)}{(\ell_1^{+2} + \Lambda^2)(\ell_2^{+2} + \Lambda^2) \ell_1^2 \ell_2^2} \sim P^2, P \cdot \ell_1, P \cdot \ell_2 = 0$$

Light-cone decomposition

$$\ell^\mu = \frac{\ell^+ e^\mu + \ell^- \bar{e}^\mu}{2} + \ell_\perp^\mu$$

$$\sim \int d^D \ell_1 \frac{\delta(\ell_1^-)}{\ell_1^2} \frac{\Lambda^2}{\ell_1^{+2} + \Lambda^2} = \underbrace{\left( \int \frac{d\ell_1^+}{2} \frac{\Lambda^2}{\ell_1^{+2} + \Lambda^2} \right)}_{\text{Regularised!}} \int d^{D-2} \ell_{1\perp} \frac{1}{\ell_{1\perp}^2}$$

# The shockwave metric

$$= \dots + \frac{1}{2} \dots + \frac{1}{3!} \dots + \dots$$

Higher-point contributions **vanish**

$$= \int_{\ell_1, \dots, \ell_{n-1}} \left( \prod_{i=1}^n \frac{\delta(P \cdot \ell_i) P^{\mu_i} P^{\nu_i}}{\ell_i^2} \right) \Omega_{\mu_1 \nu_1 \dots \mu_n \nu_n; \mu \nu}(\ell_1, \dots, \ell_n) \sim P^2, P \cdot \ell_i = 0$$

$h_{\mu\nu}(k)$

**Truncation at 1PM**

$$\langle h_{\mu\nu}(x) \rangle = \int_k e^{-ik \cdot x} \text{ [Diagram: circle with wavy line out]} = \int_k e^{-ik \cdot x} \text{ [Diagram: wavy line out]} = 4G\delta(P \cdot x) \Gamma(-\epsilon) \overbrace{\left( \frac{x_{\perp}^2}{4e^{\gamma_E} L^2} \right)^{\epsilon}}^{f(x_{\perp}^2)}$$

$h_{\mu\nu}(k)$        $h_{\mu\nu}(k)$

4D limit recovers **Aichelburg-Sexl shockwave background**

$$ds^2 = dx^- dx^+ - d\mathbf{x}_{\perp}^2 + 4G\delta(P \cdot x) \log\left(\frac{\mathbf{x}_{\perp}^2}{4L^2}\right) (P \cdot dx)^2$$

Aichelburg, Sexl, 1971

Goal: Exact in  $G$  waveform at 1SF

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{Diagram 1}}_{m^1} + \frac{1}{2} \underbrace{\text{Diagram 2} + \text{Diagram 3}}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

### Ingredients:

- ✓ Background metric
- Geodesics and cross vertices
- $n$ -point response function for  $(n-1)$  self-force observable

Goal: Exact in  $G$  waveform at  
1SF

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{Diagram 1}}_{m^1} + \frac{1}{2} \underbrace{\text{Diagram 2} + \text{Diagram 3}}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

### Ingredients:

- ✓ Background metric
- Geodesics and cross vertices
- $n$ -point response function for  $(n-1)$  self-force observable

# Shockwave geodesic and geodesic vertices

Trajectory is **recursively defined**

$$\langle \bar{x}^\mu(\tau) \rangle = \int_{\omega} e^{-i\omega\tau} \left[ \text{Diagram 1} \right] = \int_{\omega} e^{-i\omega\tau} \left[ \text{Diagram 2} + \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} + \dots \right]$$

The diagrams represent Feynman-like diagrams for a trajectory. Each diagram consists of a horizontal line with a right-pointing arrow, ending in a black dot. The label  $z^\mu(\omega)$  is placed below the first diagram. Vertical wavy lines connect the horizontal line to gray circular vertices. Dotted lines extend from the top and bottom of these vertices. Diagram 1 has one vertex. Diagram 2 has two vertices. Diagram 3 has three vertices, with a curved line connecting the first and second vertices. Ellipses indicate the series continues.

# Shockwave geodesic and geodesic vertices

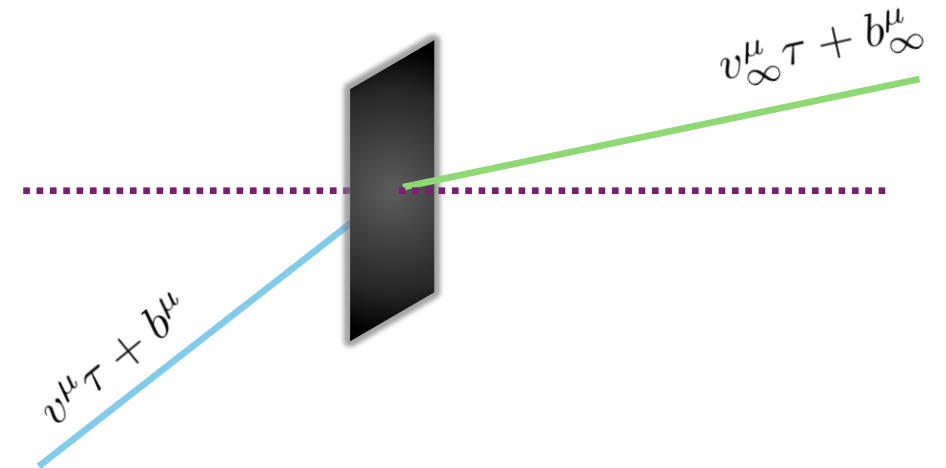
Trajectory is **recursively defined** ... but **truncates** at 2PM

$$\langle \bar{x}^\mu(\tau) \rangle = \int_\omega e^{-i\omega\tau} \text{ [diagram: vertex with incoming wavy line and outgoing straight line labeled } z^\mu(\omega) \text{]} = \int_\omega e^{-i\omega\tau} \text{ [diagram: vertex with incoming wavy line and outgoing straight line]} + \text{ [diagram: two vertices with incoming wavy lines and outgoing straight lines]} + \text{supressed for } \Lambda \rightarrow \infty$$

$$\bar{x}^\mu(\tau) = \underbrace{(v^\mu \tau + b^\mu) \theta(\tau_0 - \tau)}_{\Delta \bar{v}^\mu} + \underbrace{(v_\infty^\mu (\tau - \tau_0) + b_\infty^\mu) \theta(\tau - \tau_0)}_{\Delta \bar{b}^\mu}$$

$$v_\infty^\mu = v^\mu - \frac{4GE\epsilon f(\mathbf{b}_\perp^2)}{|\mathbf{b}_\perp|^2} \left( b_\perp^\mu - 2G\epsilon f(\mathbf{b}_\perp^2) P^\mu \right),$$

$$b_\infty^\mu = b^\mu + v^\mu \tau_0 - \underbrace{2GP^\mu f(\mathbf{b}_\perp^2)}_{\Delta \bar{b}^\mu}$$



before crossing  
after crossing  
during crossing

Expand action **around geodesic motion** and derive vertices

$$\mathcal{T}_{\text{sec}}^{\mu\nu}(k) \Big|_{z=0} = \text{[diagram: vertex with incoming wavy line labeled } h_{\mu\nu}(k) \text{]} = -\frac{m\kappa_D}{2} e^{ik \cdot (v\tau_0 + b)} \left( \underbrace{\frac{v^\mu v^\nu}{v \cdot k - i0^+}}_{\text{before crossing}} - \underbrace{\frac{e^{ik \cdot \Delta \bar{b}} v_\infty^\mu v_\infty^\nu}{v_\infty \cdot k + i0^+}}_{\text{after crossing}} \right) - \underbrace{im\kappa_D e^{ik \cdot \bar{x}(\tau_0)} \left( v^{(\mu} \Delta \bar{b}^{\nu)} + \theta(0) \Delta \bar{v}^{(\mu} \Delta \bar{b}^{\nu)} \right)}_{\text{during crossing}}$$

Goal: Exact in  $G$  waveform at  
1SF

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{Diagram 1}}_{m^1} + \frac{1}{2} \underbrace{\text{Diagram 2} + \text{Diagram 3}}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

### Ingredients:

- ✓ Background metric
- ✓ **Geodesics and cross vertices**

➤  $n$ -point response function for  
( $n-1$ ) self-force observable

Goal: Exact in  $G$  waveform at  
1SF

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{Diagram 1}}_{m^1} + \frac{1}{2} \underbrace{\text{Diagram 2} + \text{Diagram 3}}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

### Ingredients:

- ✓ Background metric
- ✓ Geodesics and cross vertices
- $n$ -point response function for  $(n-1)$  self-force observable

# Two-point shockwave response

$$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) = \begin{array}{c} \text{diagram: incoming wavy line } h_{\mu\nu}(k_1) \text{ and outgoing wavy line } h_{\rho\sigma}(k_2) \text{ meeting at a grey circle vertex} \\ h_{\mu\nu}(k_1) \quad h_{\rho\sigma}(k_2) \end{array} = \begin{array}{c} \text{diagram: incoming wavy line } h_{\mu\nu}(k_1) \\ \text{diagram: outgoing wavy line } h_{\rho\sigma}(k_2) \end{array} + \begin{array}{c} \text{diagram: incoming wavy line } h_{\mu\nu}(k_1) \text{ and outgoing wavy line } h_{\rho\sigma}(k_2) \text{ meeting at a circle vertex labeled } i\Sigma \end{array}$$
  

$$\begin{array}{c} \text{diagram: incoming wavy line } h_{\mu\nu}(k_1) \text{ and outgoing wavy line } h_{\rho\sigma}(k_2) \text{ meeting at a circle vertex labeled } i\Sigma \end{array} = \sum_{n=1}^{\infty} \left[ \begin{array}{c} \text{diagram: incoming wavy line } h_{\mu\nu}(k_1) \text{ and outgoing wavy line } h_{\rho\sigma}(k_2) \text{ meeting at a vertex with } n \text{ internal lines} \end{array} + \sum_{m=1}^{\infty} \frac{1}{m!} \left[ \begin{array}{c} \text{diagram: } m \text{ grey circle vertices connected by a horizontal line, with } n \text{ wavy lines attached} \end{array} \right]^n \right]$$

Shockwave self-energy vertices truncate:

$$\begin{array}{c} \text{diagram: } n \text{ vertices labeled } 1, 2, \dots, n \text{ connected by a horizontal line, with } n \text{ wavy lines attached} \end{array} = 0 \quad \text{for } n \geq 3$$

.....

$$\begin{array}{c} \text{---} \\ | \\ \circlearrowleft i\Sigma \\ | \\ \text{---} \\ / \end{array} = \sum_{n=1}^{\infty} \left[ \begin{array}{c} \text{---} \\ | \\ \bullet \text{---} \bullet \\ | \\ / \end{array} + \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \backslash \\ \bullet \end{array} + \frac{1}{2!} \begin{array}{c} \text{---} \\ | \\ \bullet \quad \bullet \\ | \quad | \\ \backslash \quad / \\ \bullet \quad \bullet \end{array} \right]^n$$

$$= 0 \quad \text{for } n \geq 3$$

Figure 1 displays seven Feynman diagrams, labeled (a) through (g), representing the decay of a scalar particle into two photons. The diagrams are arranged in two rows. The top row contains diagrams (a), (b), (c), and (d). The bottom row contains diagrams (e), (f), and (g). A large curly bracket on the right side of the figure groups diagrams (a) through (d), with an equals zero symbol next to it, indicating that the sum of these four diagrams is zero. Each diagram shows a scalar particle (wavy line) decaying into two photons (wavy lines) via various internal fermion (dashed line) and scalar (solid line) loops. The diagrams are arranged in two rows: (a)-(d) on top and (e)-(g) on the bottom.

# Two-point shockwave response

$$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) = \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ h_{\mu\nu}(k_1) \quad h_{\rho\sigma}(k_2) \end{array} = \begin{array}{c} \text{---} \text{---} \\ \diagdown \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} \text{---} \text{---}$$

## ➤ All-order expansion in PM

$$\begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} = \begin{array}{c} \text{---} \bullet \\ \diagdown \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array}$$

$$+ \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \bullet \text{---} \bullet \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \text{mirror}$$

$$+ \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array}$$

$$+ \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} + \text{mirror}$$

$$+ \sum_{n=4}^{\infty} \left[ \begin{array}{c} \text{---} \bullet \text{---} \\ \diagdown \\ \text{---} \end{array} \right]^n$$

Resum to  $\Sigma_{\text{sum}}^{\mu\nu\alpha\beta}(k_1, k_2)$

$$\begin{array}{cccc} \text{(a)} & \text{(b)} & \text{(c)} & \text{(d)} \\ \text{(e)} & \text{(f)} & \text{(g)} & \end{array} = 0$$

(1PM)

(2PM)

(3PM)

3PM exact  
remainder  
 $\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2)$

( $\geq 4\text{PM}$ )

# Two-point shockwave response

$$\mathcal{L}_n^{\mu\nu\alpha\beta} = \text{Diagram} = \frac{\kappa_D^{2n}}{2^n} (P \cdot k_1)^{2n} \int_{\ell_1, \dots, \ell_{n-1}} \Omega_n^{\mu\nu\alpha\beta}(k_1, k_2, P, \ell_1, \dots, \ell_{n-1}) \mathcal{I}_{n-1},$$

Diagram: A Feynman diagram with two external wavy lines labeled  $h_{\mu\nu}(k_1)$  and  $h_{\alpha\beta}(k_2)$ . Between them are  $n$  internal wavy lines. The first internal line is circled in blue. Vertices are labeled  $\ell_1, \ell_2, \dots, \ell_{n-1}, \ell_n$ . A blue arrow points from the circled vertex to the text "231 terms".

**Simplification at 4PM and beyond!**  
(using de Donder gauge)

$$\Omega_{n \geq 4}^{\mu\nu\alpha\beta}(k_1, k_2, P) = -i \Pi^{\mu\nu}{}_{\kappa\lambda}(k_1) \Pi^{\kappa\lambda\alpha\beta}(k_2)$$

→ Independent of loop momentum

Only scalar integrals  
→ Computed with  
**method of regions**

With the *TT*-projector  $\Pi^{\mu\nu\alpha\beta}(k_i) = \pi_i^{\mu(\rho} \pi_i^{\alpha)\beta} - \frac{1}{D-2} \pi_i^{\mu\nu} \pi_i^{\alpha\beta}$

and the *transverse metric*  $\pi_i^{\mu\nu} = \eta^{\mu\nu} + \frac{k_i^2 P^\mu P^\nu - 2(P \cdot k_i) k_i^{(\mu} P^{\nu)}}{(P \cdot k_i)^2}$

.....

➤ **All-order expansion in PM**

33

.....

➤ **All-order expansion in PM**

34

# Two-point shockwave response

3PM and below, ladders do **not** follow the pattern observed at higher orders ...

$$\text{Ladder diagram} \neq -i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \Big|_{n=3}$$

... but the **sum of all 3PM diagrams** does!

$$\text{Ladder} + \text{Crossed} + \text{Box} + \text{Triangle} + \text{Mirror} = -i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \Big|_{n=3}$$

Same pattern as high-orders!

Summing low PM order:

$$\Sigma^{\mu\nu\alpha\beta} \Big|_{\leq 3\text{PM}} = \sum_{n=1}^3 \left[ \underbrace{-i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1}}_{\text{Follows high-PM pattern}} \right] + \underbrace{\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}}_{\text{2PM exact Remainder}}$$

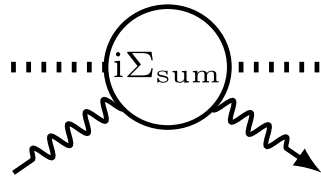
Vanishes on-shell

Include in resummed response  $\Sigma_{\text{sum}}^{\mu\nu\alpha\beta}(k_1, k_2)$

$$\varepsilon_{1\mu}^{(h_1)} \varepsilon_{1\nu}^{(h_1)} \Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2) \bar{\varepsilon}_{2\alpha}^{(h_2)} \bar{\varepsilon}_{2\beta}^{(h_2)} \Big|_{\text{on-shell}} = 0$$

# Two-point shockwave response

Sum up ...



$$\equiv \sum_{n=1}^{\infty} \underbrace{\left[ -i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \right]}_{\text{High-order ladder pattern}} = 2\delta[P \cdot (k_1 - k_2)] (P \cdot k_1) [\Pi_1 \cdot \Pi_2]^{\mu\nu\alpha\beta} \mathcal{F}(q_{\perp}; \epsilon),$$

... into an **expanded form**

$$\mathcal{F}(q_{\perp}; \epsilon) = \left( \frac{4\pi}{q_{\perp}^2} \right)^{1-\epsilon} \sum_{n=1}^{\infty} \frac{(iW)^n}{n!} \frac{\Gamma(-\epsilon)^n}{(q_{\perp}^2 e^{\gamma_E} L^2)^{n\epsilon}} \frac{\Gamma(1 + (n-1)\epsilon)}{\Gamma(-n\epsilon)} = \int d^{D-2} \mathbf{x}_{\perp} e^{-i\mathbf{q}_{\perp} \cdot \mathbf{x}_{\perp}} \left[ e^{iW\Gamma(-\epsilon) \left( \frac{x_{\perp}^2}{4e^{\gamma_E} L^2} \right)^{\epsilon}} - 1 \right],$$

... into an **resummed form**

Same structure as calculation in eikonal approximation  
Raj, Venugopalan [2406.10483]

Weinberg factor

$$W = 2G(P \cdot k_1)$$

**4-dimensional limit**

$$\Sigma_{\text{sum}}^{\mu\nu\alpha\beta} = \frac{\Gamma(1 - iW)}{\Gamma(1 + iW)} \frac{e^{-iW/\epsilon}}{(q_{\perp}^2 L^2)^{-iW}} \Sigma_{\text{sum}}^{\mu\nu\alpha\beta}|_{\text{1PM}}$$

Pure phase factor

Equivalent structure observed by 't Hooft, 1987 for scalar ultra-high energy amplitude

Goal: Exact in  $G$  waveform at  
1SF

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{Diagram } m^1}_{\text{1SF}} + \frac{1}{2} \underbrace{\text{Diagram } m^2}_{\text{2SF}} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

### Ingredients:

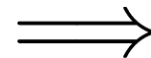
- ✓ Background metric
- ✓ Geodesics and cross vertices
- ✓  $n$ -point response function for  $(n-1)$  self-force observable

Goal: Exact in  $G$  waveform at 1SF

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{Diagram 1}}_{m^1} + \frac{1}{2} \underbrace{\text{Diagram 2} + \text{Diagram 3}}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

### Ingredients:

- ✓ Background metric
- ✓ Geodesics and cross vertices
- ✓  $n$ -point response function for  $(n-1)$  self-force observable



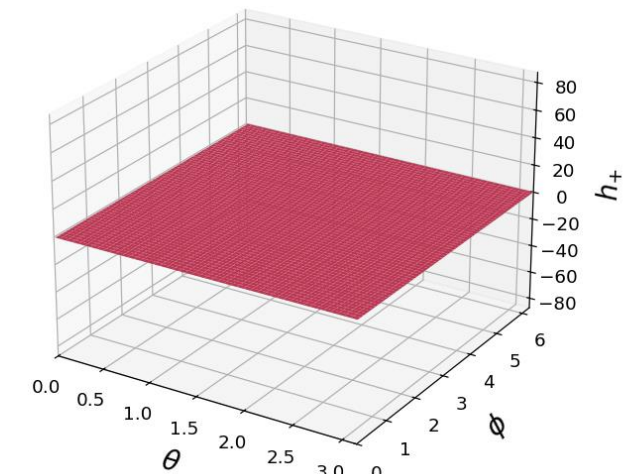
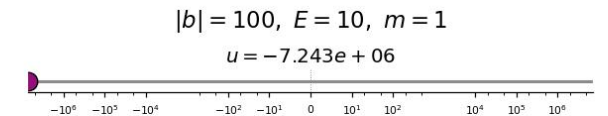
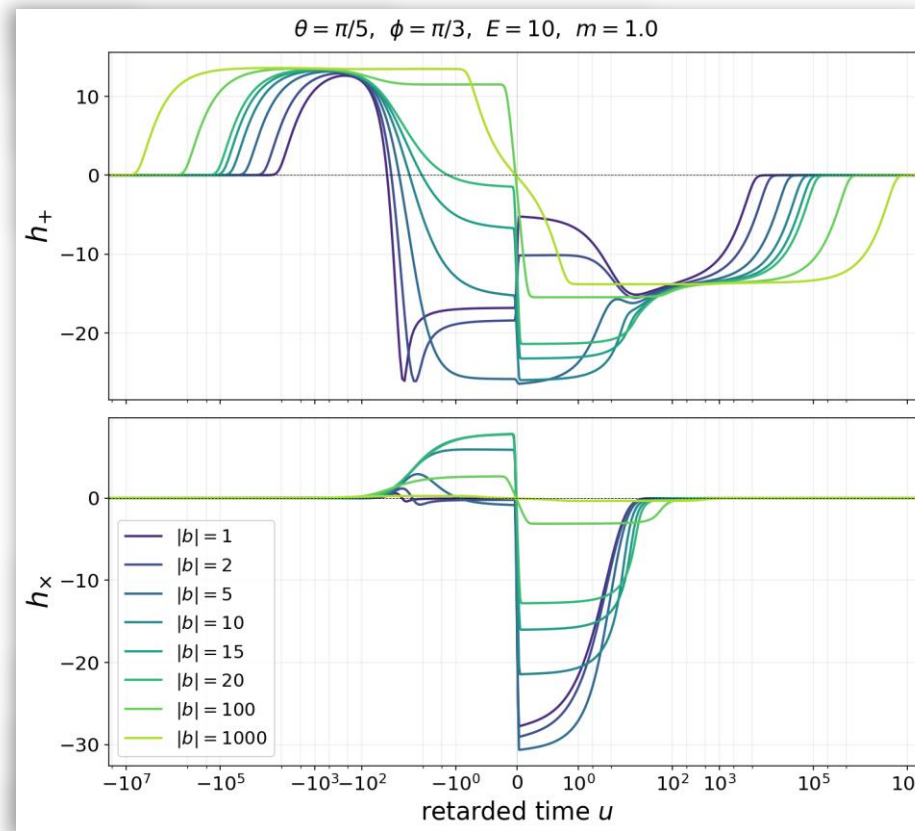
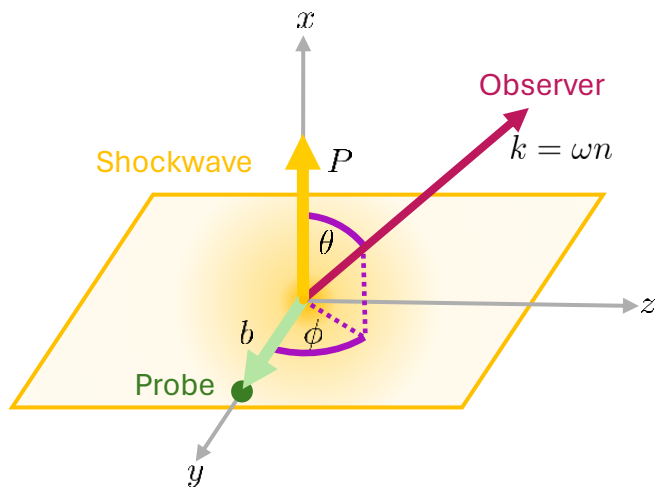
**Assemble and integrate!**

# Two-point shockwave response

$$\langle h_{\mu\nu}(u, \mathbf{n}) \rangle_{\text{1SF}} = \int_{\omega} e^{-i\omega u} \int_{\ell} \text{[Diagram: Shockwave source emitting a wave with momentum } k = \omega n \text{ and distance } \ell \text{]} = \int_{\omega} e^{-i\omega u} \left[ \text{[Diagram: Purely memory]} + \int_{\ell} \text{[Diagram: No memory]} \right]$$

Purely memory

No memory

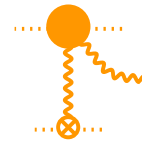
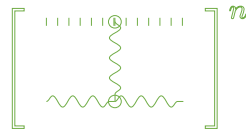
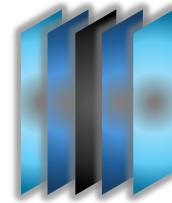


# Summary and Outlook

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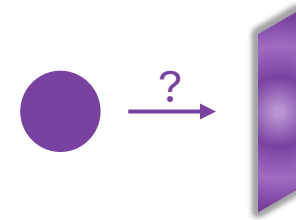
## In this talk

- ▶ Novel way of obtaining **SF expansion with WQFT!**
- ▶ Proof of concept by applying the formalism to the **shockwave**
- ▶ Obtained the **first exact in G** two-point response function and **waveform**



## Outlook

- ▶ Compute more observables, such as **impulse** for the shockwave metric
- ▶ Connection to **high-energy limit** of massive particle?
- ▶ Response functions of **massive WQFT**?



Thanks!

# Backup slides

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# Generating functional, background fields and vertices

Generalize to generic sources

$$e^{\frac{i}{\hbar} W[\mathcal{T}, f]} = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu}(x) h_{\mu\nu}(x) - \int d\tau f_\mu(\tau) Z^\mu(\tau))} \Big|_{\text{tree}} = e^{\frac{i}{\hbar} \left( \underbrace{\int d^D x \mathcal{T}^{\mu\nu}(x) \langle h_{\mu\nu}(x) \rangle_{\{\mathcal{T}, f\}} + \int d\tau f_\mu(\tau) \langle Z^\mu(\tau) \rangle_{\{\mathcal{T}, f\}}}_{\text{Classical action as functional}} \right)}$$

Classical action as functional

$W[J]$  generates **response function**

$$\langle h_{\mu\nu}(x) \rangle_{J=0} = -\kappa_D^{-1} \frac{\delta W[J]}{\delta \mathcal{T}^{\mu\nu}(x)} \Big|_{J=0} = \text{diagrammatic expansion} = \dots, \quad \langle Z^\mu(\tau) \rangle_{J=0} = -\frac{\delta W[J]}{\delta f_\mu(\tau)} \Big|_{J=0} = 0$$

**Background metric**

$S_{\text{BH}}[[\langle \phi \rangle_J]]$  generates **Feynman vertices** on curved spacetime

$$\frac{\delta^n i S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle h_{\mu_1 \nu_1} \rangle_J \delta \langle h_{\mu_2 \nu_2} \rangle_J \cdots \delta \langle h_{\mu_n \nu_n} \rangle_J} \Big|_{J=0} = \text{diagrammatic expansion} = \sum_{m=0}^{\infty} \frac{1}{m!} \text{diagrammatic expansion}$$

# Propagator and higher-point response functions

**Two-point response** acts as **propagator** on curved spacetime due to **Green's function-like relation**

Two-point vertex on curved background

$$\frac{\delta^2 S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle \phi_A \rangle_J \delta \langle \phi_B \rangle_J} \frac{\delta^2 W[J]}{\delta J_B \delta J_C} \Big|_{J=0} = -\delta_{AC}$$

Response Function

Solved by a **geometric series**:

$$\boxed{\text{Vertex}} = \text{Wavy line} + \sum_{n=1}^{\infty} \left[ \text{Diagram 1} + \sum_{m=1}^{\infty} \frac{1}{m!} \text{Diagram 2} \right]^n$$

**Three-point response** (and higher) can be recovered from lower-point response functions

$$\text{Vertex} = \text{Diagram 1} + \left[ \text{Diagram 2} + (\text{two permutations}) \right]$$

# The on-shell limit and $N$ -operator

**On-shell** response function (Compton amplitude)

$$\langle k_2, h_2 | \hat{T} | k_1, h_1 \rangle = i \varepsilon_{1\mu}^{(h_1)} \varepsilon_{1\nu}^{(h_1)} \Sigma^{\mu\nu\alpha\beta} \bar{\varepsilon}_{2\alpha}^{(h_2)} \bar{\varepsilon}_{2\beta}^{(h_2)} \Big|_{\text{on-shell}} = 2\delta[P \cdot (k_1 - k_2)] \frac{(P \cdot F_1^{(h_1)} \cdot F_2^{(h_2)} \cdot P)^2}{(P \cdot k_1)^3} \mathcal{F}(q_\perp; \epsilon),$$

with the linearised field-strength tensor  $F_i^{(h_i)\mu\nu} = 2k_i^{[\mu} \varepsilon_i^{(h_i)\nu]}$ . Double copy

Matrix element of **Magnus operator**  $\hat{S} = \mathbb{1} + i\hat{T} = e^{i\hat{N}}$

Magnus, 1954  
Damgaard, Planté, Vanhove  
[2107.12891]

$$\begin{aligned} \langle k_2, h_2 | i\hat{N} | k_1, h_1 \rangle &= \langle k_2, h_2 | i\hat{T} | k_1, h_1 \rangle \Big|_{\text{1PM}} = \text{diagram 1} + \text{diagram 2} \\ &= \frac{16\pi i G}{(4\pi e^{\gamma_E} L^2)^\epsilon} \delta[P \cdot (k_1 - k_2)] \frac{(P \cdot F_1^{(h_1)} \cdot F_2^{(h_2)} \cdot P)^2}{(P \cdot k_1)} \end{aligned}$$

**1PM exact** and  $\epsilon$  - finite!