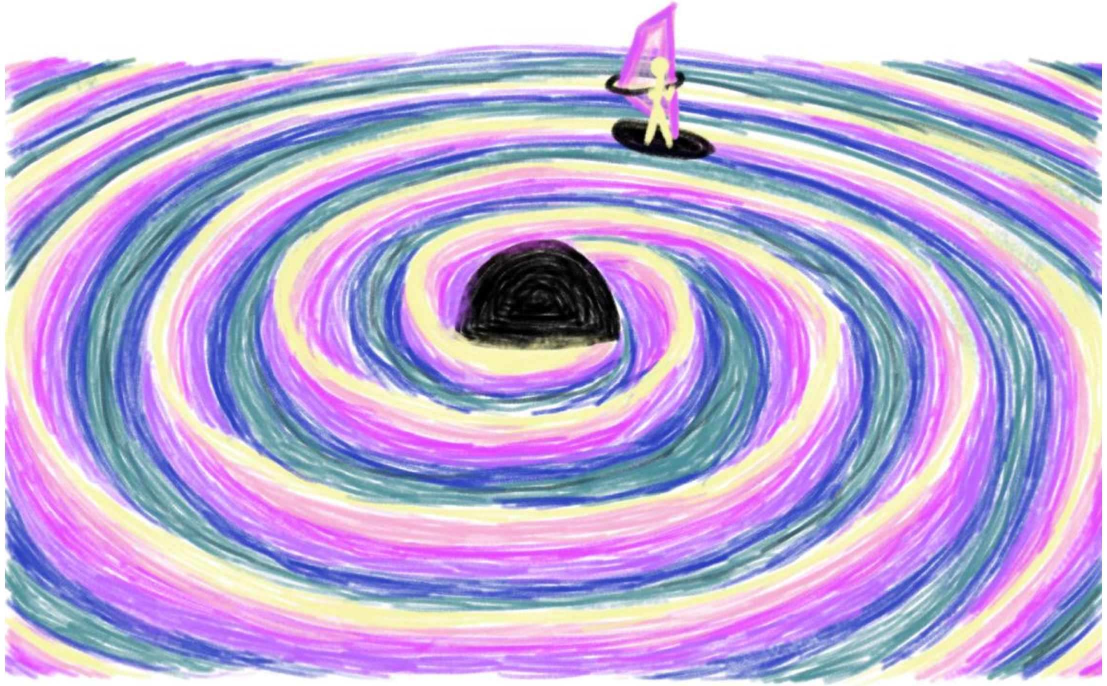


Black Hole Response Theory and its Shockwave Limit

Based on **[2604.22009]** & **upcoming work** with Carl Jordan Eriksen, Jitze Hoogeveen, Gustav Uhre Jakobsen and Jan Plefka

HUMBOLDT-UNIVERSITÄT ZU BERLIN

RTG 2575: Rethinking Quantum Field Theory

Amplitudes, Queen Mary University of London

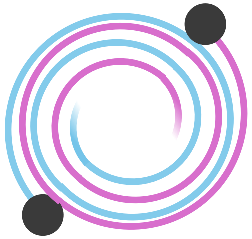
Lara Bohnenblust

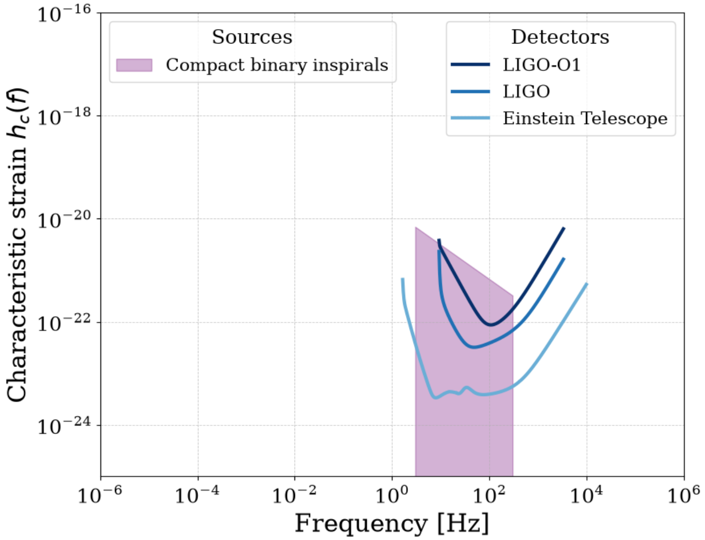
29. June 2026

Astrophysical and Cosmological Relativity

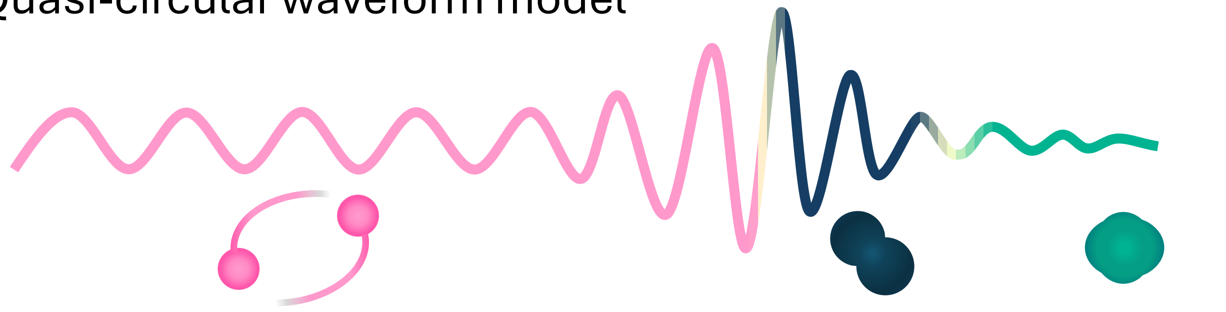
Gravitational wave modeling

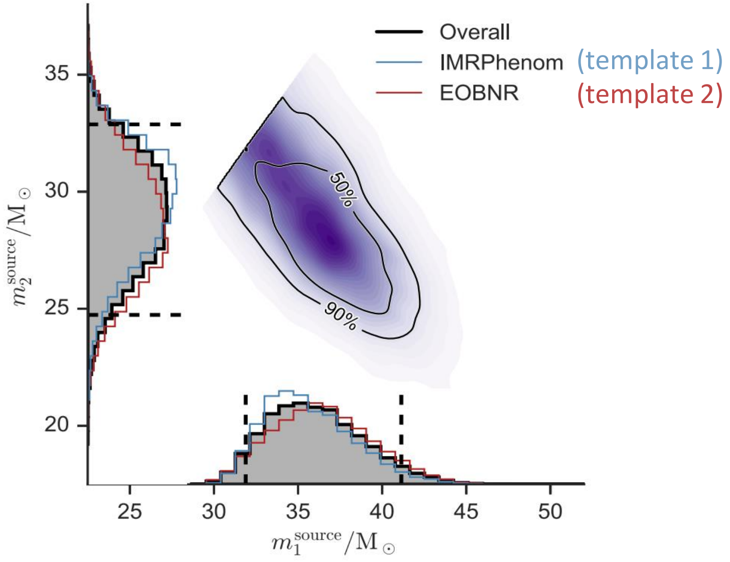
Currently detectable





Quasi-circular waveform model

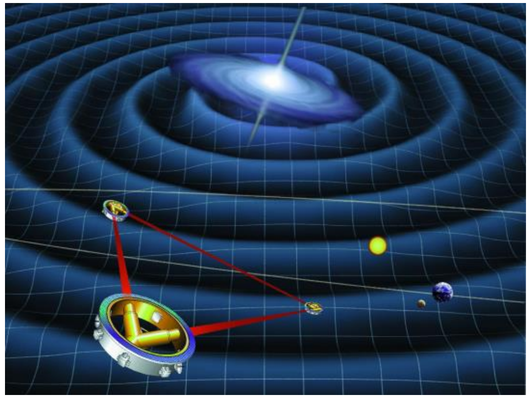


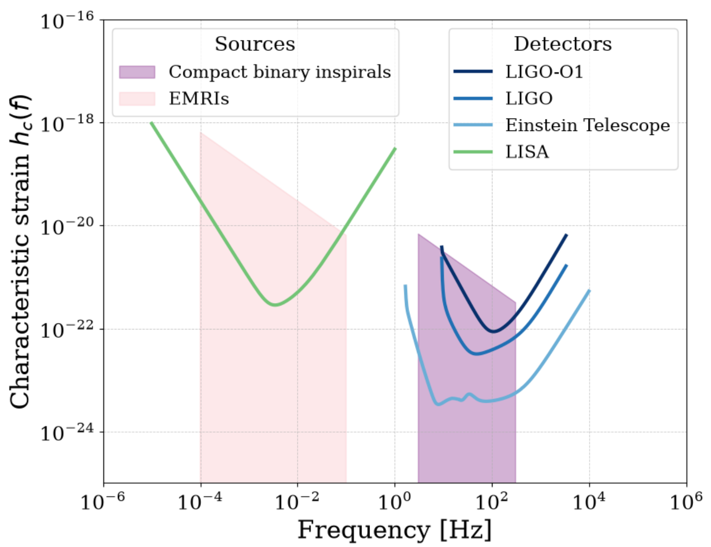


LVK Collaboration [1602.03840]

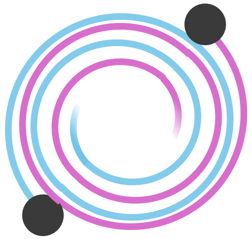
Precise and accurate models are crucial!

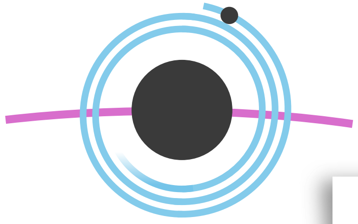
Gravitational wave modeling





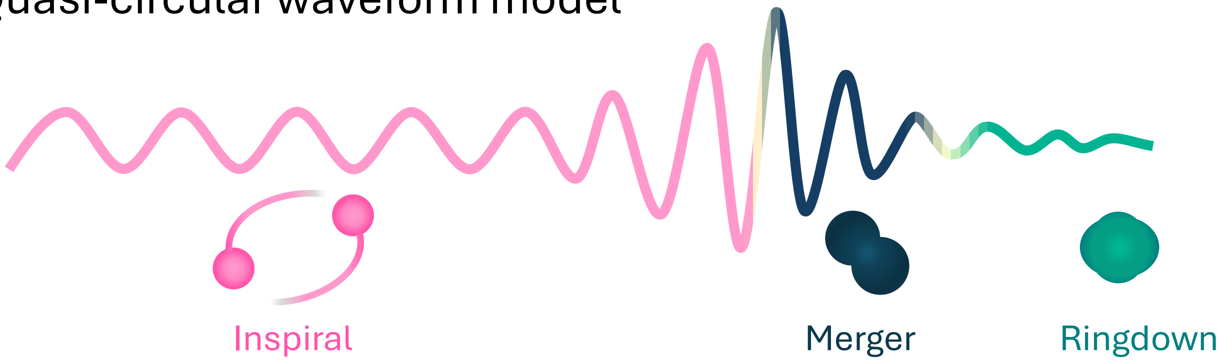
Currently detectable

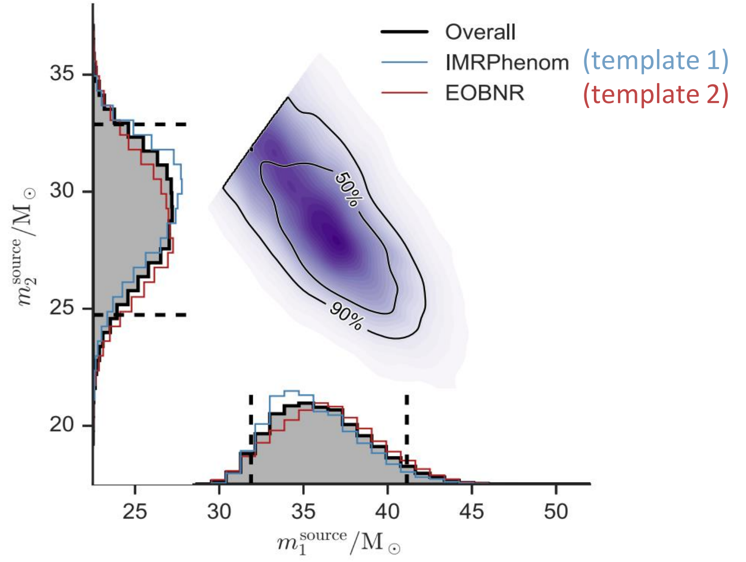




Detectable in the future!

Quasi-circular waveform model





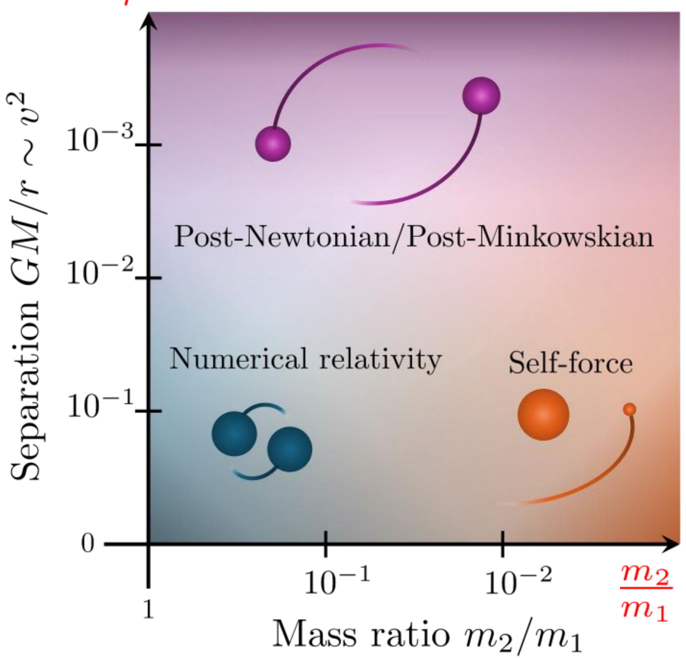
LVK Collaboration [1602.03840]

Precise and accurate models are crucial but **do not exist for generic EMRIs**!

Gravitational wave modeling

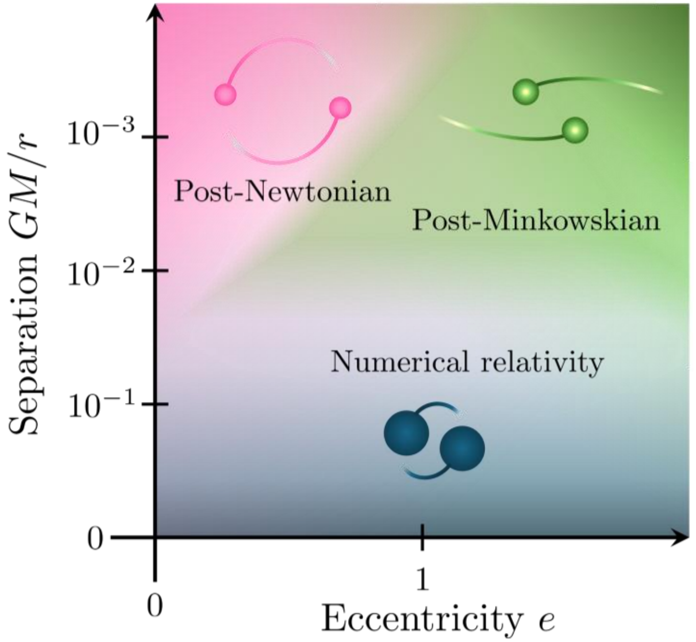
→ Solving **non-linear** Einstein field equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$ → **approximate in small parameter**

$v^2 \sim \frac{GM}{r} \ll 1$



$\frac{m_2}{m_1} \ll 1$

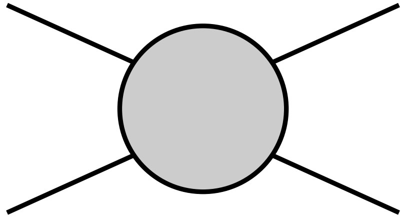
$v^2 \sim \frac{GM}{r} \ll 1$ $\frac{GM}{r} \ll 1$



From amplitudes:
Post-Minkowskian (PM)

$$\chi = G\chi_{(1)} + G^2\chi_{(2)} + \cdots$$

Weak coupling expansion



Classical **worldline QFT** amplitudes

see Gustav Jakobsen's talk

For EMRIs: self-force (SF)

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Kerr}} + \frac{m_2}{m_1} h_{\mu\nu}^{(1)} + \left(\frac{m_2}{m_1}\right)^2 h_{\mu\nu}^{(2)} + \cdots$$

Expansion around background metric and geodesic

Black hole response theory

Organize amplitudes and **resum** to obtain results in the strong field!

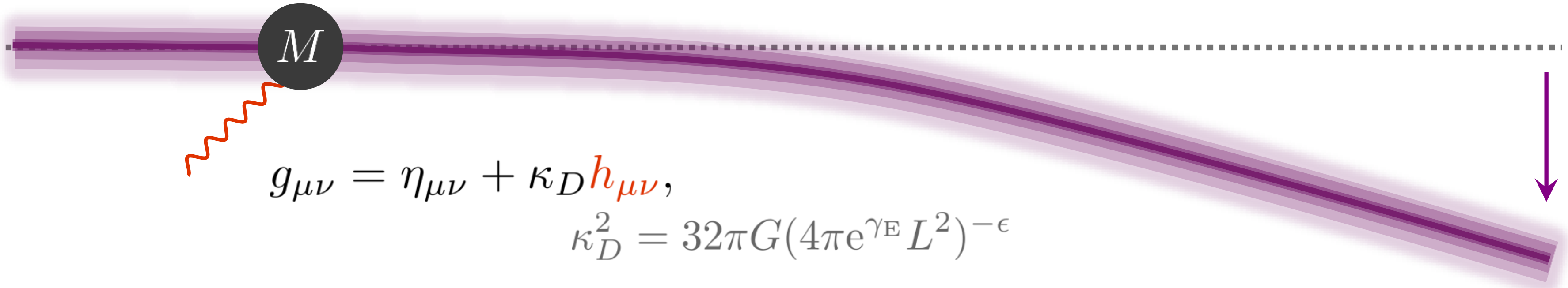
WQFT of a single black hole

Mogull, Plefka, Steinhoff [2010.02865]

$$S_{\mathrm{BH}}[X,g] = -\frac{M}{2}\int \mathrm{d}\tau\, g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu - \int \mathrm{d}^D x\, \frac{2}{\kappa_D^2}\sqrt{|g|}R$$

$$g_{\mu\nu}(X) = \eta_{\mu\nu}$$

Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow [2406.14770]



$$X^\mu(\tau) = B^\mu + V^\mu \tau + Z^\mu(\tau)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_D h_{\mu\nu},$$

$$\kappa_D^2 = 32\pi G(4\pi e^{\gamma_E} L^2)^{-\epsilon}$$

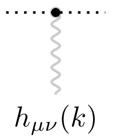
Fiducial length scale

Quantize deviation from background solution → Classical solution to EOM are **tree-level diagrams**

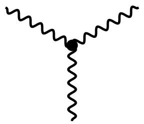
Feynman rules for "in-in" (Schwinger-Keldysh) prescription:

$$Z^\mu(-\omega) \quad Z^\nu(\omega) = \frac{-\mathrm{i}\eta^{\mu\nu}}{M(\omega + \mathrm{i}0^+)^2}$$

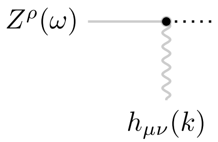
$$h_{\mu\nu}(-k) \quad h_{\rho\sigma}(k) = \frac{\mathrm{i}\mathcal{P}_{\mu\nu\rho\sigma}}{k^2 + \mathrm{i}0^+(W\cdot k)}$$



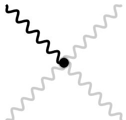
$\sim M\kappa_D$



$\sim \kappa_D$



$\sim M\kappa_D$



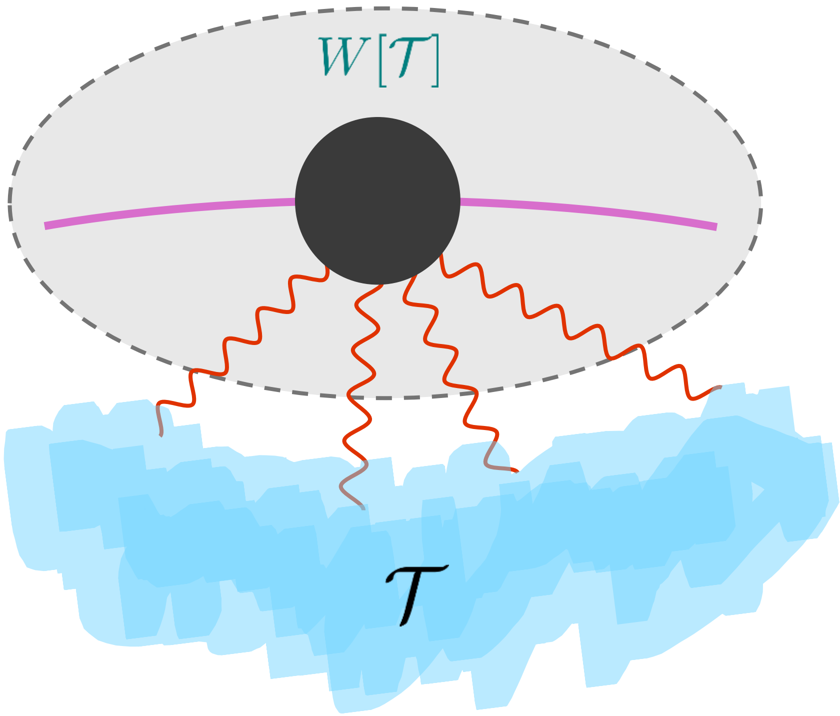
$\sim \kappa_D^2$

Black hole response theory

Integrate out worldline and graviton fluctuation

→ **Black hole response (BHR) effective action** encoding response to arbitrary source

$$e^{\frac{i}{\hbar}W[\mathcal{T}]} = \int \mathcal{D}[h, Z]\, e^{\frac{i}{\hbar}\left(S_{\rm BH}[X,g] - \int d^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}\right)}\Big|_{\rm tree}$$



Expansion in source yields **response functions**:

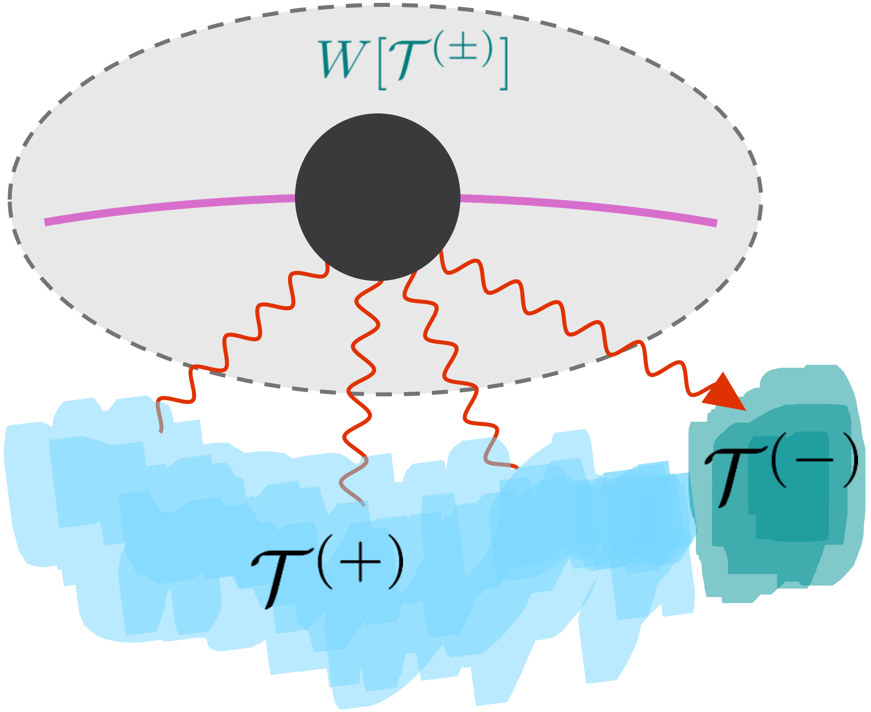
$$iW[\mathcal{T}] = -i\kappa_D \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1)\boxed{\mathcal{R}_{\alpha\beta}(k_1)} - \frac{\kappa_D^2}{2}\int_{k_1,k_2} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\boxed{\mathcal{R}_{\alpha\beta\mu\nu}(k_1,k_2)}$$

$$+ \frac{i\kappa_D^3}{6}\int_{k_1,k_2,k_3} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{T}^{\rho\sigma}(k_3)\boxed{\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1,k_2,k_3)} + \cdots$$

Black hole response theory with "in-in"

Integrate out worldline and graviton fluctuation (**Keldysh-Schwinger**)

→ **Black hole response (BHR) effective action** encoding response to arbitrary source in Keldysh basis $\mathcal{F}^{(\pm)} \sim \mathcal{F}^{(1)} \pm \mathcal{F}^{(2)}$



$$e^{\frac{i}{\hbar}W[\mathcal{T}^{(1,2)}]} = \int \mathcal{D}[h^{(1,2)}, Z^{(1,2)}]\, e^{\frac{i}{\hbar}S[Z^{(1)},h^{(1)},\mathcal{T}^{(1)}] - \frac{i}{\hbar}S[Z^{(2)},h^{(2)},\mathcal{T}^{(2)}]}\Big|_{\text{tree}}$$

Expansion in source yields **response functions**:

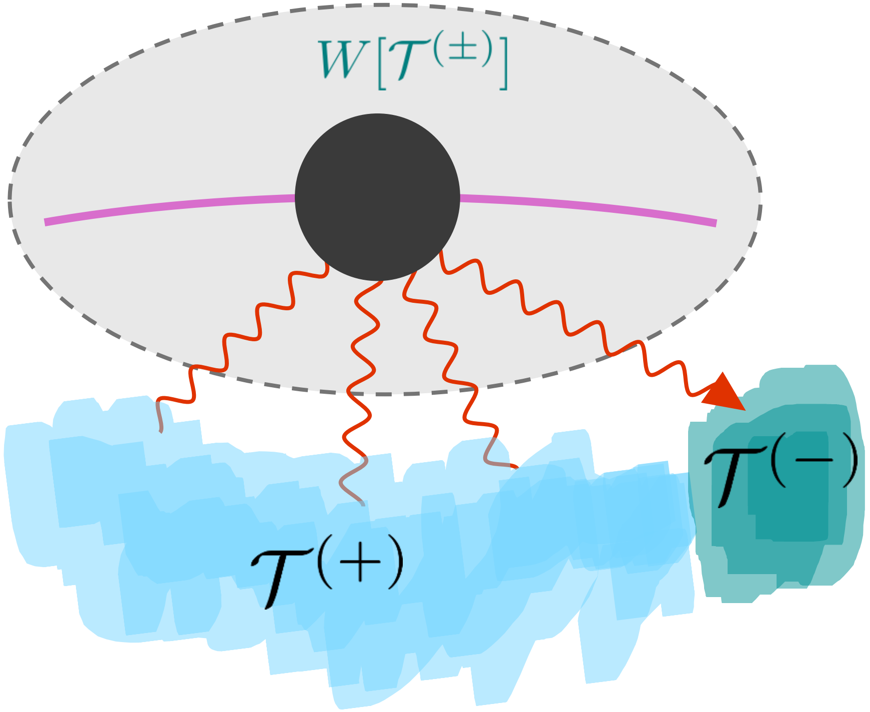
$$\begin{aligned} iW[\mathcal{T}^{(\pm)}] = &-i\kappa_D \int_{k_1} \mathcal{T}^{(-)\alpha\beta}(k_1) \boxed{\mathcal{R}_{\alpha\beta}(k_1)} - \kappa_D^2 \int_{k_1,k_2} \mathcal{T}^{(+)\alpha\beta}(k_1)\mathcal{T}^{(-)\mu\nu}(k_2) \boxed{\mathcal{R}_{\alpha\beta\mu\nu}(k_1,k_2)} \\ &+ \frac{i}{2}\kappa_D^3 \int_{k_1,k_2,k_3} \mathcal{T}^{(+)\alpha\beta}(k_1)\mathcal{T}^{(+)\mu\nu}(k_2)\mathcal{T}^{(-)\rho\sigma}(k_3) \boxed{\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1,k_2,k_3)} + \dots \end{aligned}$$

Many incoming $(+)$ legs and one outgoing $(-)$ leg

Black hole response theory with “in-in”

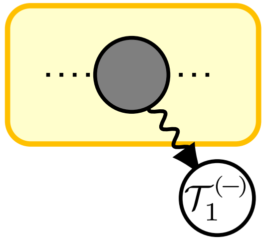
Integrate out worldline and graviton fluctuation (**Keldysh-Schwinger**)

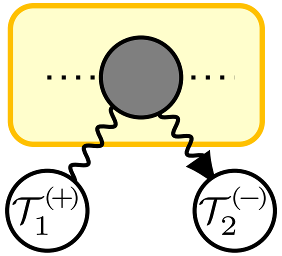
→ **Black hole response (BHR) effective action** encoding response to arbitrary source in Keldysh basis $\mathcal{F}^{(\pm)} \sim \mathcal{F}^{(1)} \pm \mathcal{F}^{(2)}$

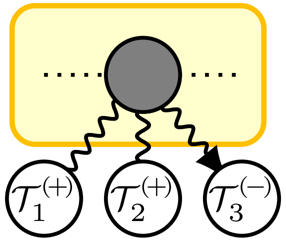


$$\mathrm{e}^{\frac{\mathrm{i}}{\hbar} W[\mathcal{T}^{(1,2)}]} = \int \mathcal{D}[h^{(1,2)}, Z^{(1,2)}]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar} S[Z^{(1)}, h^{(1)}, \mathcal{T}^{(1)}] - \frac{\mathrm{i}}{\hbar} S[Z^{(2)}, h^{(2)}, \mathcal{T}^{(2)}]}\Big|_{\mathrm{tree}}$$

Expansion in source yields **response functions**:

$W[\mathcal{T}^{(\pm)}] = -\mathrm{i}\kappa_D \int_{k_1}$ 

$- \kappa_D^2 \int_{k_1, k_2}$ 

$+ \frac{\mathrm{i}\kappa_D^3}{2} \int_{k_1, k_2, k_3}$ 

$+ \cdots$

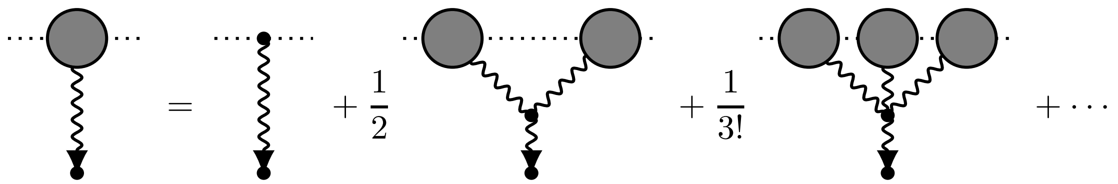
Many incoming $(+)$ legs and one outgoing $(-)$ leg

Response functions

$$e^{\frac{i}{\hbar}W[\mathcal{T}]} = \int \mathcal{D}[h, Z]\, e^{\frac{i}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \boxed{\int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}}\right)}\Big|_{\mathrm{tree}}$$

One-point response is equivalent to **background metric**

$$\mathcal{R}_{\mu\nu} = -\kappa_D^{-1} \frac{\delta W[J]}{\delta \mathcal{T}^{\mu\nu}(x)}\bigg|_{J=0} =$$



$$= \langle h_{\mu\nu}(x) \rangle_{J=0}$$

Response functions

$$\mathrm{e}^{\frac{\mathrm{i}}{\hbar}W[\mathcal{T}]} = \int \mathcal{D}[h, Z]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \boxed{\int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}}\right)}\Big|_{\text{tree}}$$

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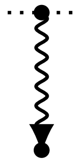
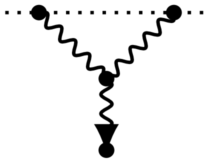
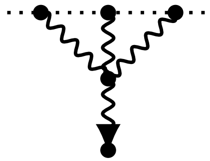


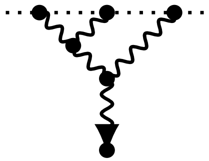
$+ \frac{1}{2}$



$+ \frac{1}{3!}$



$+ \frac{1}{2}$



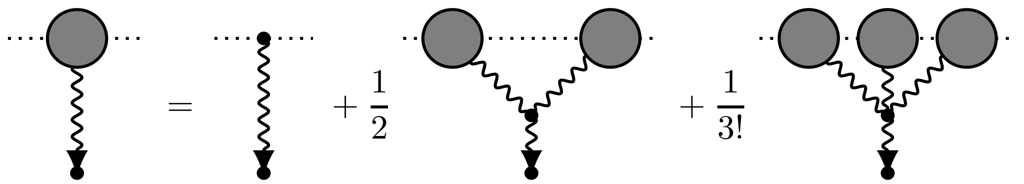
$$+ \cdots = \langle h_{\mu\nu}(x) \rangle_{J=0}$$

Response functions

$$e^{\frac{i}{\hbar}W[\mathcal{T}]} = \int \mathcal{D}[h, Z]\, e^{\frac{i}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \boxed{\int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}}\right)}\Big|_{\text{tree}}$$

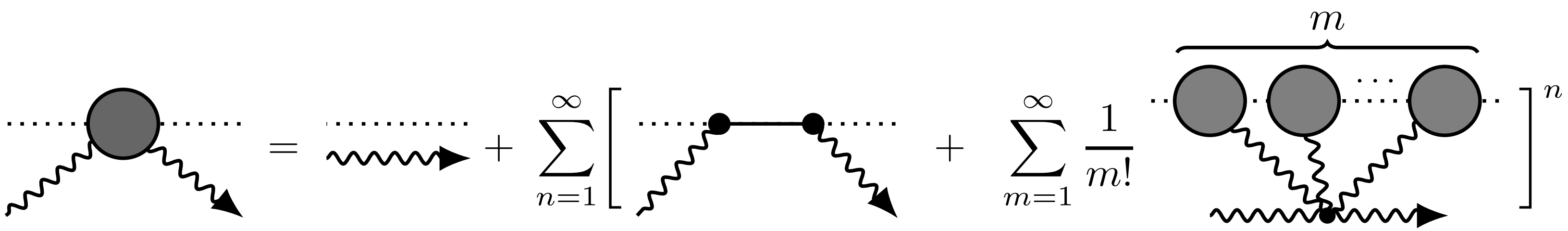
One-point response is equivalent to **background metric**

$$\mathcal{R}_{\mu\nu} = -\kappa_D^{-1} \frac{\delta W[J]}{\delta \mathcal{T}^{\mu\nu}(x)}\bigg|_{J=0} =$$

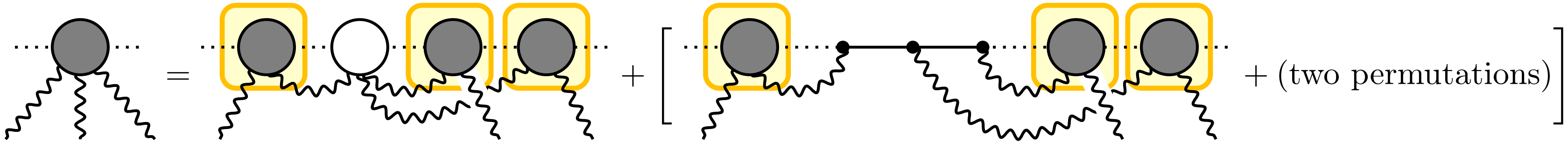


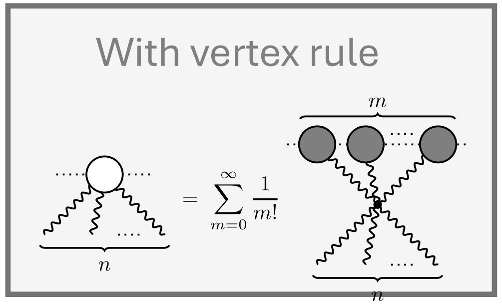
$$= \langle h_{\mu\nu}(x)\rangle_{J=0}$$

Two-point response acts as **propagator** on curved spacetime



Three-point response (and higher) can be recovered from lower-point responses





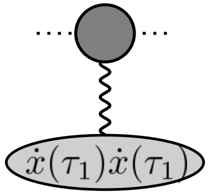
Self-force expansion in black hole response theory

Couple to **light secondary black hole** $\mathcal{T}^{\mu\nu}_{\mathrm{sec}}(x) = \frac{m}{2}\int \mathrm{d}\tau\, \delta^{(D)}(x - x(\tau))\dot{x}^{\mu}(\tau)\dot{x}^{\nu}(\tau)$

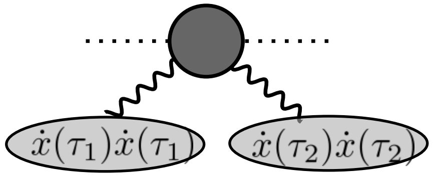
$$S_{\mathrm{BHR}}[x] = S_{\mathrm{kin}}[x] + W[\mathcal{T}_{\mathrm{sec}}(x)]$$

natural **self-force** expansion

$$= S_{\mathrm{kin}}[x] - \frac{1}{2}\boldsymbol{m}\kappa_D \int \mathrm{d}\tau_1$$



$$+ \frac{\mathrm{i}}{8}\boldsymbol{m}^2\kappa_D^2 \int \mathrm{d}\tau_1 \mathrm{d}\tau_2$$



$$+ \frac{1}{48}\boldsymbol{m}^3\kappa_D^3 \int \mathrm{d}\tau_1 \mathrm{d}\tau_2 \mathrm{d}\tau_3 \quad \cdots$$

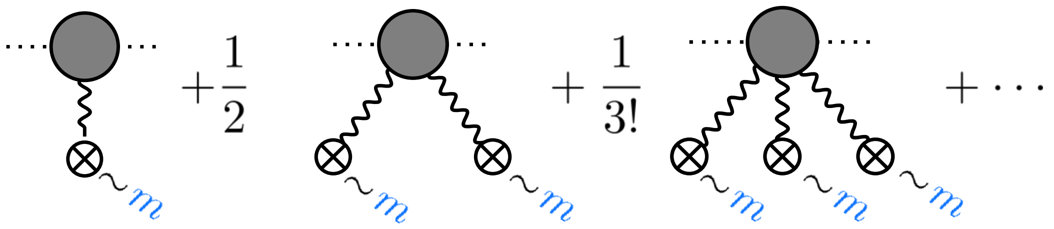
Self-force expansion in black hole response theory

Couple to **light secondary black hole** $\mathcal{T}^{\mu\nu}_{\rm sec}(x) = \frac{m}{2}\int \mathrm{d}\tau\,\delta^{(D)}(x - x(\tau))\dot{x}^\mu(\tau)\dot{x}^\nu(\tau)$

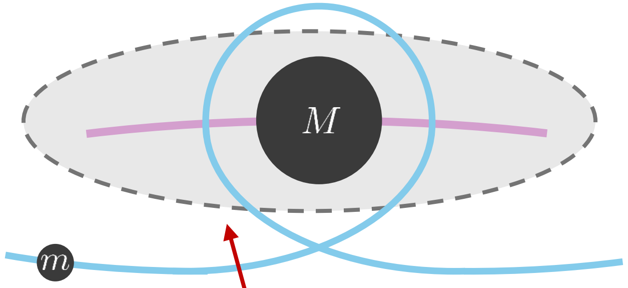
$$S_{\rm BHR}[x] = S_{\rm kin}[x] + W[\mathcal{T}_{\rm sec}(x)]$$

natural **self-force** expansion

$$= S_{\rm kin}[x] + $$



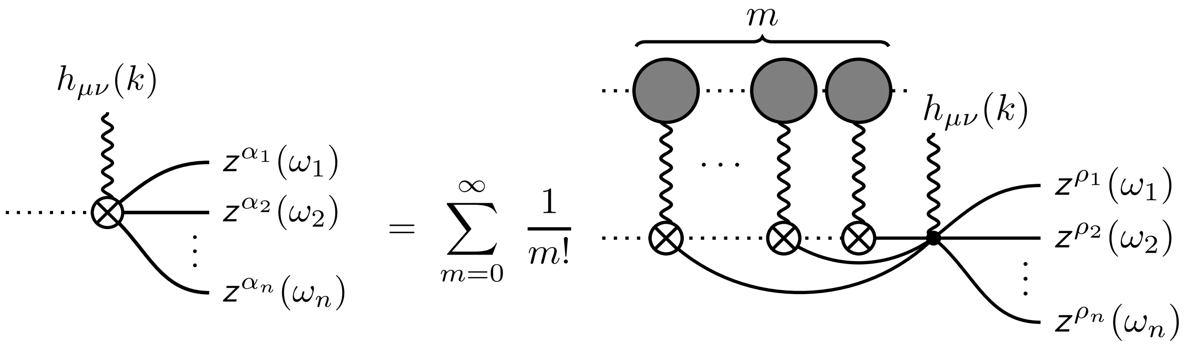
0SF solved by **geodesic** $\bar{x}^\mu(\tau)$



$$x^\mu(\tau) = \bar{x}^\mu(\tau) + z^\mu(\tau)$$

Expand around **0SF**

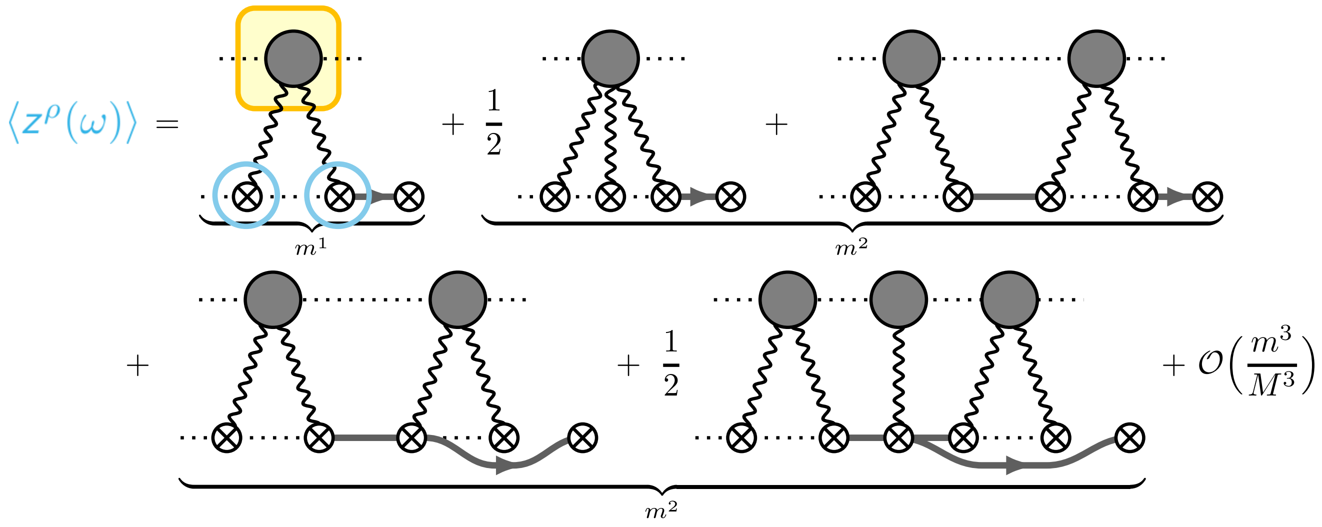
Feynman vertices resumming background motion

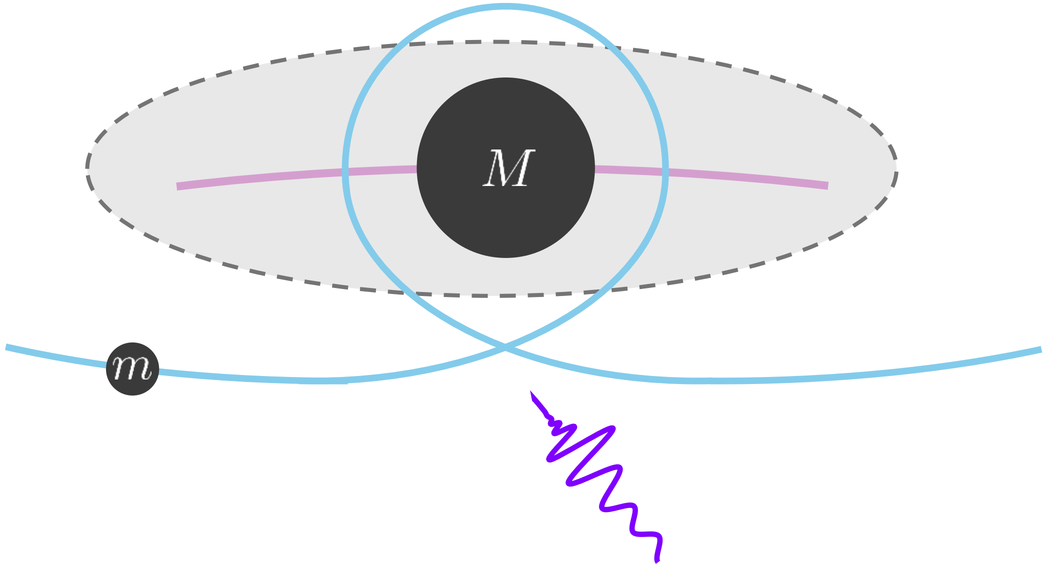


$$= -\mathrm{i}\kappa_D \frac{\delta^n \mathcal{T}^{\mu\nu}_{\rm sec}(k)}{\delta x^{\alpha_1}(\omega_1)\delta x^{\alpha_2}(\omega_2)\ldots\delta x^{\alpha_n}(\omega_n)}\bigg|_{x^\mu \to \bar{x}^\mu}$$

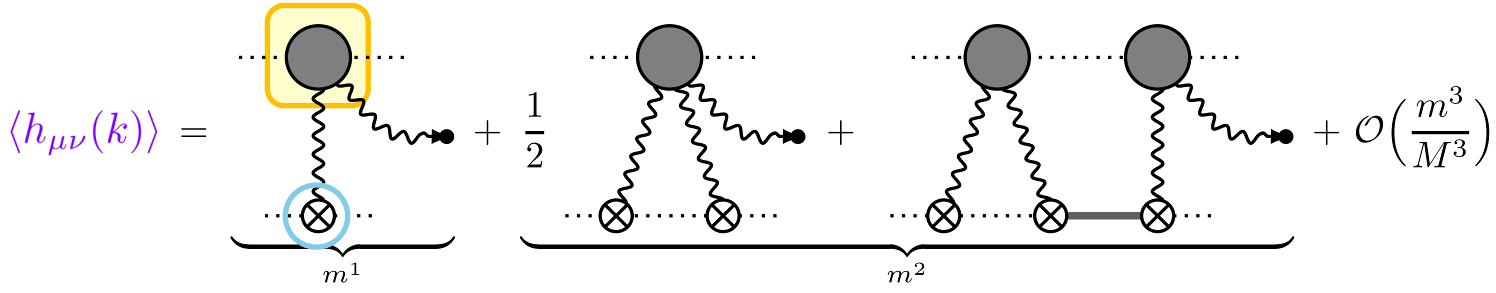
Observables in the self-force expansion

Trajectory/Impulse



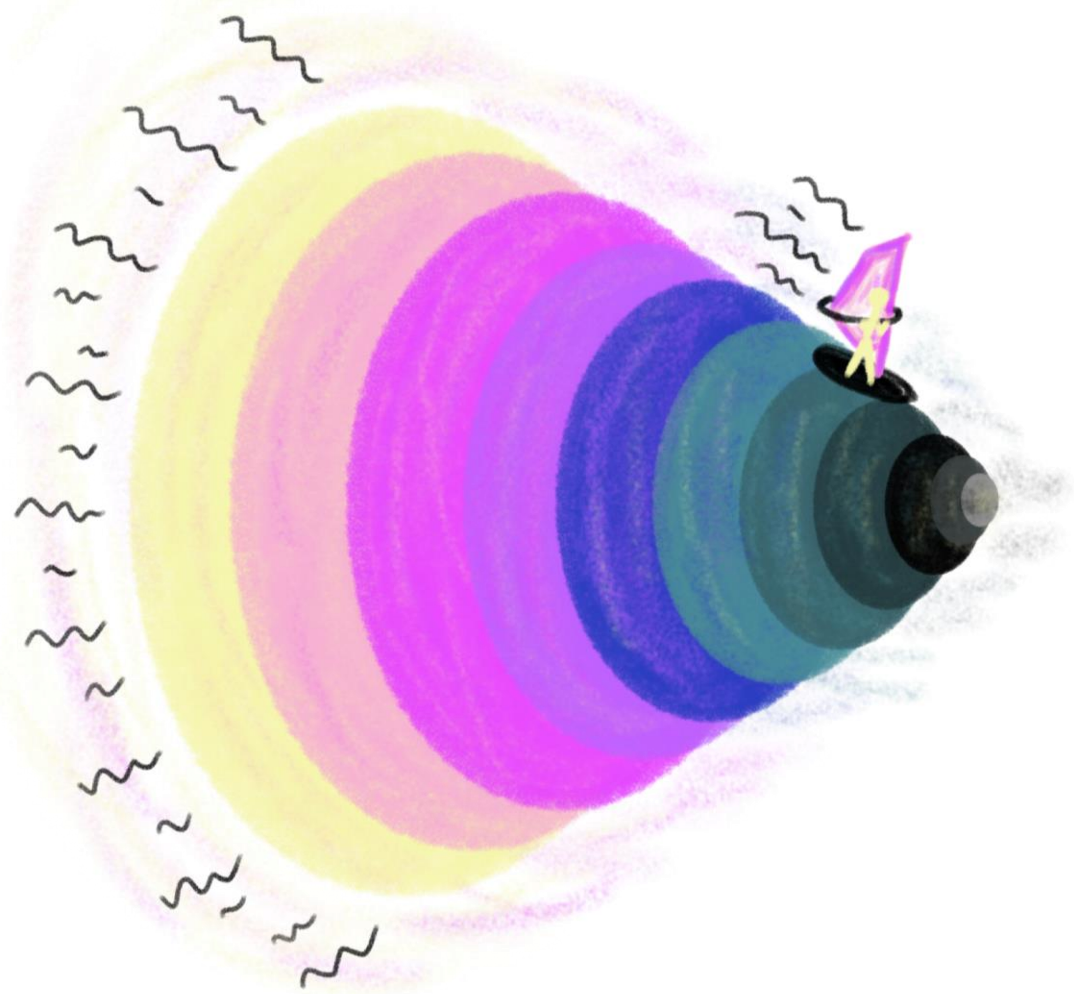


Waveform



Ingredients:

- Background metric
- Geodesics and cross vertices
- *n*-point response function for *(n-1)* self-force observable



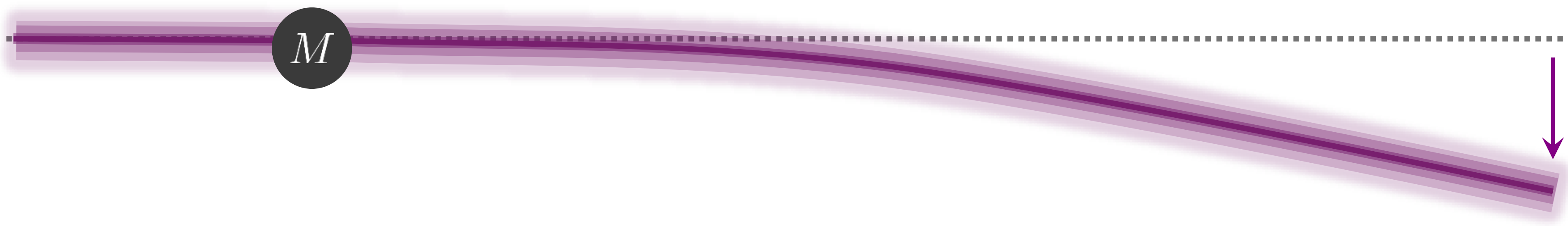
Application of black hole response theory to the **shockwave background**

Massless WQFT

Brink-Di Vecchia-Howe or Polyakov form for massless limit

$$S = -\frac{1}{2}\int \mathrm{d}\tau\left(e^{-1}\,g_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu + eM^2\right) \xrightarrow{M\to 0,\, e^{-1}\to E} -\frac{E}{2}\int \mathrm{d}\tau\; g_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu$$

Infinitely boosted black hole with energy E

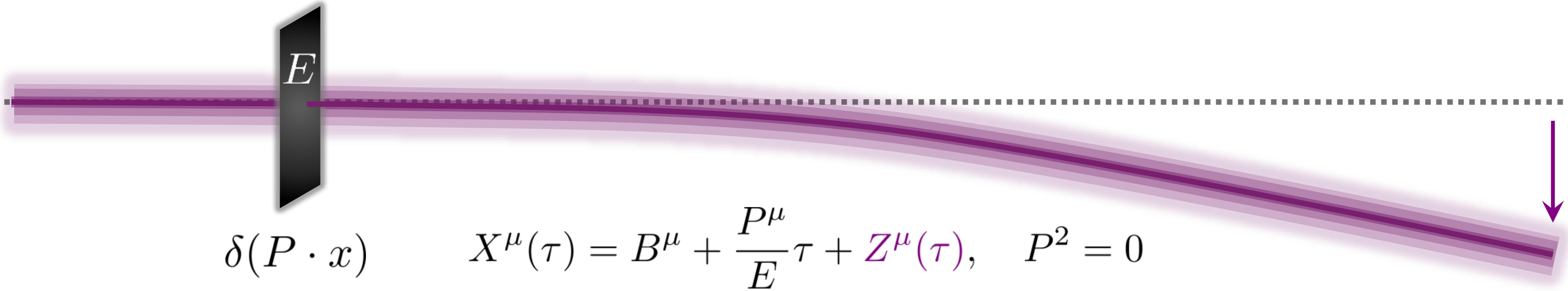


Massless WQFT

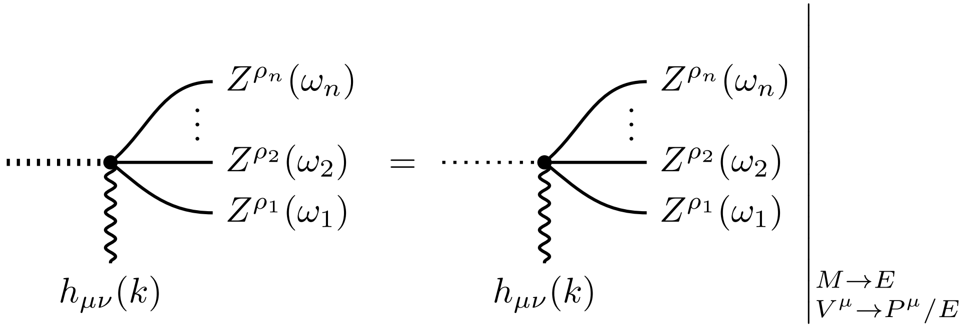
Brink-Di Vecchia-Howe or Polyakov form for massless limit

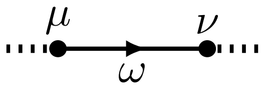
$$S=-\frac{1}{2}\int \mathrm{d}\tau\left(e^{-1}g_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu+eM^2\right)\xrightarrow{M\to0,\,e^{-1}\to E}-\frac{E}{2}\int\mathrm{d}\tau\; g_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu$$

Infinitely boosted black hole with energy E

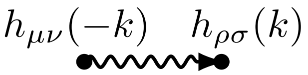


$\delta(P\cdot x)$ $\qquad X^\mu(\tau)=B^\mu+\frac{P^\mu}{E}\tau+Z^\mu(\tau),\quad P^2=0$



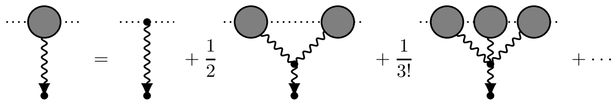


$$=\frac{-\mathrm{i}\eta^{\mu\nu}}{E(\omega+\mathrm{i}0^+)^2}$$



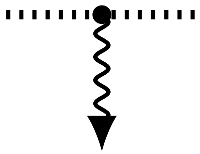
$$=\frac{\mathrm{i}\mathcal{P}_{\mu\nu\rho\sigma}}{k^2+\mathrm{i}0^+(P\cdot k)}$$

The shockwave metric



1PM contribution

$$\kappa_D\langle h_{\mu\nu}(x)\rangle_{1\mathrm{PM}} = \kappa_D\int_k e^{-\mathrm{i}k\cdot x}$$

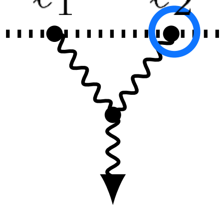


$h_{\mu\nu}(k)$

$$= \frac{\kappa_D^2}{2}P_\mu P_\nu\int_k e^{-\mathrm{i}k\cdot x}\frac{\delta(k\cdot P)}{k^2} = 4G\delta(P\cdot x)\Gamma(-\epsilon)\overbrace{\left(\frac{\boldsymbol{x}_\perp^2}{4e^{\gamma_E}L^2}\right)^{\epsilon}}^{f(\boldsymbol{x}_\perp^2)}$$

2PM contribution

ℓ_1 ℓ_2 $\sim P^\mu P^\nu$



$h_{\mu\nu}(k)$

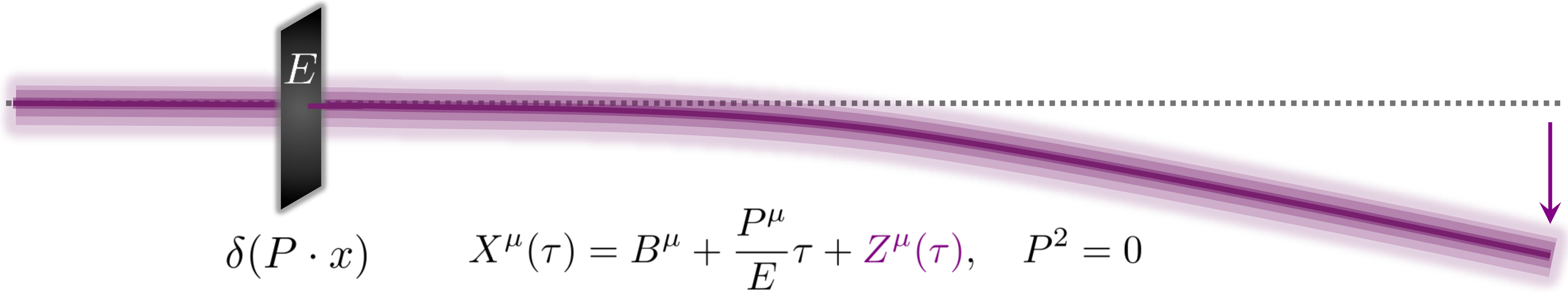
$$= \int_{\ell_1}\frac{\delta(P\cdot\ell_1)\delta(P\cdot\ell_2)P^{\mu_1}P^{\nu_1}P^{\mu_2}P^{\nu_2}\Omega_{\mu_1\nu_1\mu_2\nu_2;\mu\nu}(\ell_1,\ell_2)}{\ell_1^2\ell_2^2} \sim P^2, P\cdot\ell_1, P\cdot\ell_2 = 0$$

Light-cone decomposition

$$\ell^\mu = \frac{\ell^+ e^\mu + \ell^-\bar{e}^\mu}{2} + \boldsymbol{\ell}_\perp^\mu$$

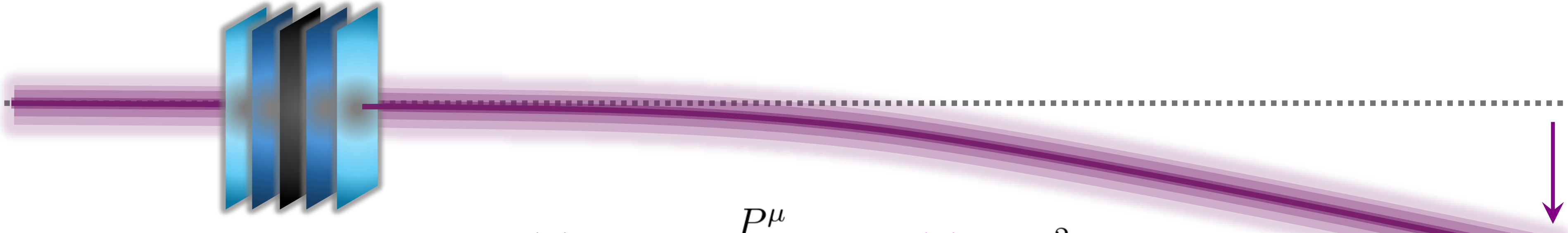
$$\sim \int \mathrm{d}^D\ell_1\frac{\delta(\ell_1^-)}{\ell_1^2} = \underbrace{\left(\int\frac{\mathrm{d}\ell_1^+}{2}\right)}_{\text{Non-regularised divergence}}\int \mathrm{d}^{D-2}\boldsymbol{\ell}_{1\perp}\frac{1}{\boldsymbol{\ell}_{1\perp}{}^2}$$

Regularizing massless WQFT



$$\delta(P \cdot x) \qquad X^\mu(\tau) = B^\mu + \frac{P^\mu}{E}\tau + Z^\mu(\tau), \quad P^2 = 0$$

Regularizing massless WQFT

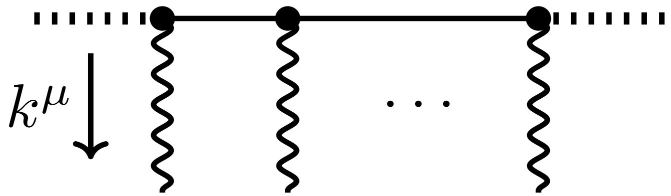


$$\rho_\Lambda(P\cdot x) \xrightarrow{\Lambda\to\infty} \delta(P\cdot x)$$

$$X^\mu(\tau) = B^\mu + \frac{P^\mu}{E}\tau + Z^\mu(\tau), \quad P^2 = 0$$

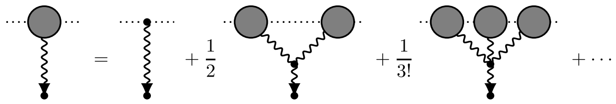
Choose $\rho_\Lambda(\sigma) = \frac{\Lambda}{2}\exp(-\Lambda|\sigma|)$ with Fourier transformation $\tilde{\rho}_\Lambda(\omega) = \frac{\Lambda^2}{\omega^2+\Lambda^2}$

$\Longrightarrow$



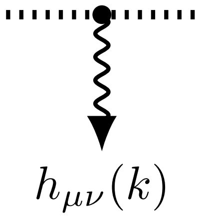
$$\sim \frac{\Lambda^2}{{k^+}^2+\Lambda^2}$$

The shockwave metric



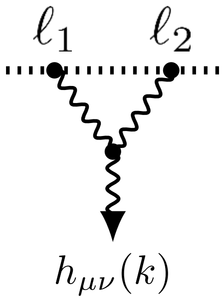
1PM contribution

$$\kappa_D \langle h_{\mu\nu}(x)\rangle_{1\mathrm{PM}} = \kappa_D \int_k e^{-\mathrm{i}k\cdot x}$$



$$= \frac{\kappa_D^2}{2} P_\mu P_\nu \int_k e^{-\mathrm{i}k\cdot x} \frac{\delta(k\cdot P)}{k^2} = 4G\delta(P\cdot x)\Gamma(-\epsilon)\overbrace{\left(\frac{\boldsymbol{x}_\perp^2}{4\mathrm{e}^{\gamma_{\mathrm{E}}} L^2}\right)^\epsilon}^{f(\boldsymbol{x}_\perp^2)}$$

2PM contribution



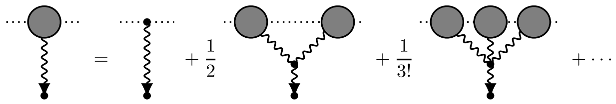
$$= \int_{\ell_1} \frac{\Lambda^4 \delta(P\cdot\ell_1)\delta(P\cdot\ell_2) P^{\mu_1}P^{\nu_1}P^{\mu_2}P^{\nu_2}\Omega_{\mu_1\nu_1\mu_2\nu_2;\mu\nu}(\ell_1,\ell_2)}{({\ell_1^+}^2+\Lambda^2)({\ell_2^+}^2+\Lambda^2)\ell_1^2\ell_2^2} \sim P^2, P\cdot\ell_1, P\cdot\ell_2 = 0$$

Light-cone decomposition

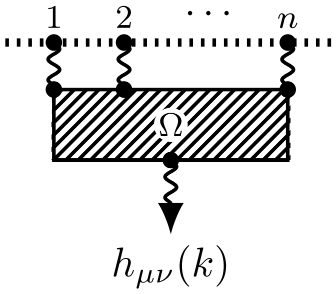
$$\ell^\mu = \frac{\ell^+ e^\mu + \ell^- \bar{e}^\mu}{2} + \boldsymbol{\ell}_\perp^\mu$$

$$\sim \int \mathrm{d}^D\ell_1 \frac{\delta(\ell_1^-)}{\ell_1^2} \frac{\Lambda^2}{{\ell_1^+}^2+\Lambda^2} = \underbrace{\left(\int \frac{\mathrm{d}\ell_1^+}{2} \frac{\Lambda^2}{{\ell_1^+}^2+\Lambda^2}\right)}_{\text{Regularised!}} \int \mathrm{d}^{D-2}\boldsymbol{\ell}_{1\perp} \frac{1}{{\boldsymbol{\ell}_{1\perp}}^2}$$

The shockwave metric



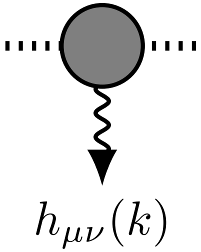
Higher-point contributions **vanish**



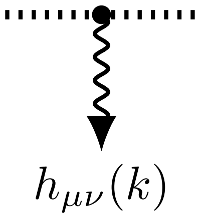
$$= \int_{\ell_1,\dots,\ell_{n-1}} \left(\prod_{i=1}^{n} \frac{\delta(P\cdot\ell_i) P^{\mu_i} P^{\nu_i}}{\ell_i^2} \right) \Omega_{\mu_1\nu_1\cdots\mu_n\nu_n;\mu\nu}(\ell_1,\dots,\ell_n) \sim P^2,\ P\cdot\ell_i = 0$$

Truncation at 1PM

$$\langle h_{\mu\nu}(x)\rangle = \int_k e^{-ik\cdot x}$$



$$= \int_k e^{-ik\cdot x}$$



$$= 4G\delta(P\cdot x)\Gamma(-\epsilon)\overbrace{\left(\frac{\boldsymbol{x}_\perp^2}{4e^{\gamma_E}L^2}\right)^{\epsilon}}^{f(\boldsymbol{x}_\perp^2)}$$

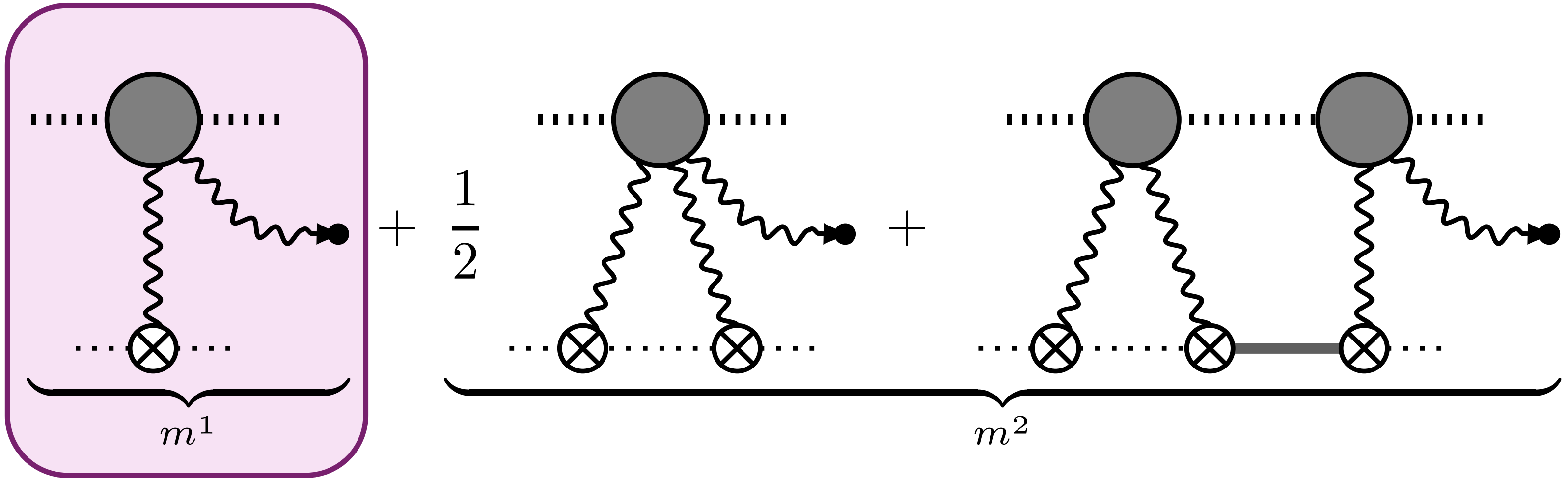
4D limit recovers **Aichelburg-Sexl shockwave background**

$$\boxed{\mathrm{d}s^2 = \mathrm{d}x^-\,\mathrm{d}x^+ - \mathrm{d}\boldsymbol{x}_\perp^2 + 4G\delta(P\cdot x)\log\left(\frac{\boldsymbol{x}_\perp^2}{4L^2}\right)(P\cdot\mathrm{d}x)^2}$$

Aichelburg, Sexl, 1971

Goal: Exact in G waveform at 1SF

$\langle h_{\mu\nu}(k)\rangle =$

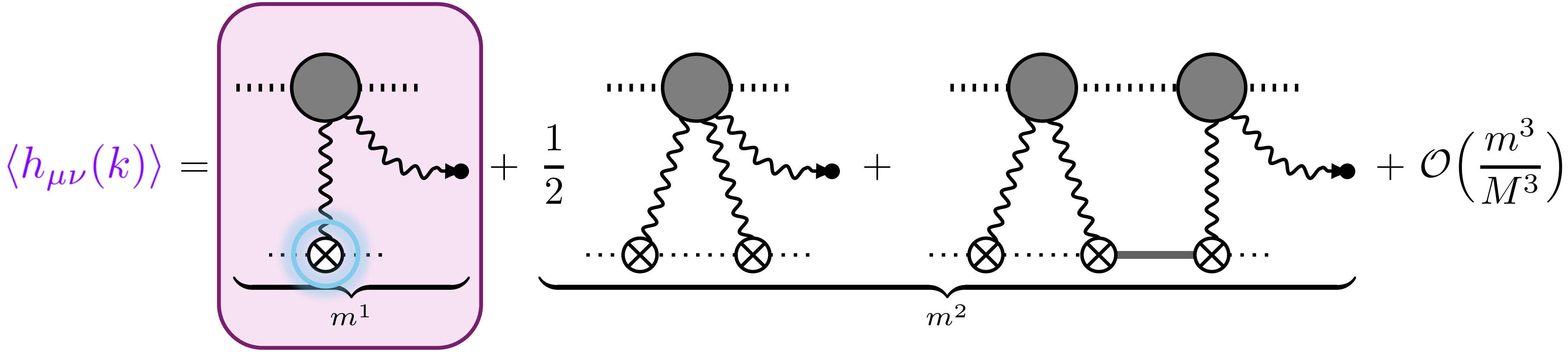


$+ \mathcal{O}\left(\frac{m^3}{M^3}\right)$

Ingredients:

- ☑ Background metric
- ➢ Geodesics and cross vertices
- ➢ ***n*-point response function for *(n-1)* self-force observable**

Goal: Exact in G waveform at 1SF



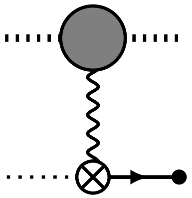
Ingredients:

- ☑ Background metric
- ➢ **Geodesics and cross vertices**
- ➢ ***n*-point response function for *(n-1)* self-force observable**

Shockwave geodesic and geodesic vertices

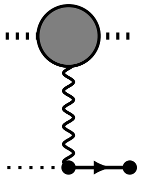
Trajectory is **recursively defined**

$$\langle \bar{x}^\mu(\tau) \rangle = \int_\omega e^{-\mathrm{i}\omega\tau}$$

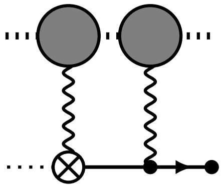


$z^\mu(\omega)$

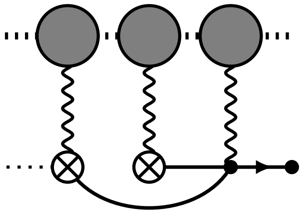
$$= \int_\omega e^{-\mathrm{i}\omega\tau}$$



$+$



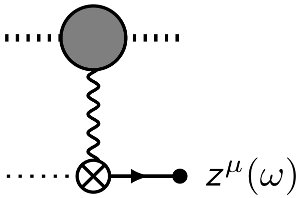
$+\frac{1}{2}$

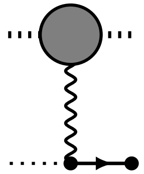
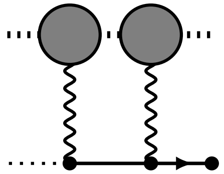


$+\cdots$

Shockwave geodesic and geodesic vertices

Trajectory is **recursively defined** ... but **truncates** at 2PM

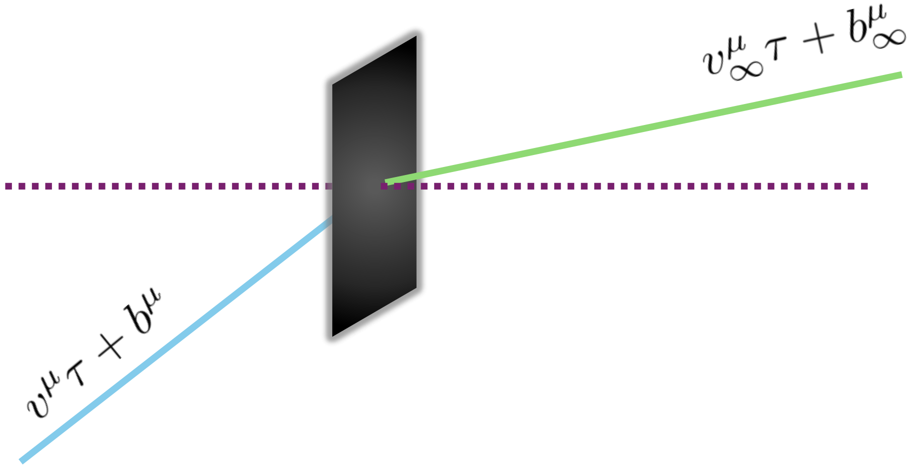
$$\langle \bar{x}^\mu(\tau)\rangle = \int_\omega e^{-\mathrm{i}\omega\tau}$$ 

$$= \int_\omega e^{-\mathrm{i}\omega\tau}$$  $+$  $+$ **supressed** for $\Lambda \to \infty$

$$\bar{x}^\mu(\tau) = \underline{(v^\mu\tau + b^\mu)\theta(\tau_0 - \tau)} + \underline{\big(v^\mu_\infty(\tau - \tau_0) + b^\mu_\infty\big)\theta(\tau - \tau_0)}$$

$$v^\mu_\infty = v^\mu \overbrace{- \frac{4GE\epsilon f(\boldsymbol{b}_\perp^2)}{|b_\perp|^2}\left(b^\mu_\perp - 2G\epsilon f(\boldsymbol{b}_\perp^2)P^\mu\right)}^{\Delta\bar{v}^\mu},$$

$$b^\mu_\infty = b^\mu + v^\mu\tau_0 \underbrace{- 2GP^\mu f(\boldsymbol{b}_\perp^2)}_{\Delta\bar{b}^\mu}$$

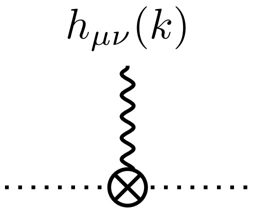


before crossing

after crossing

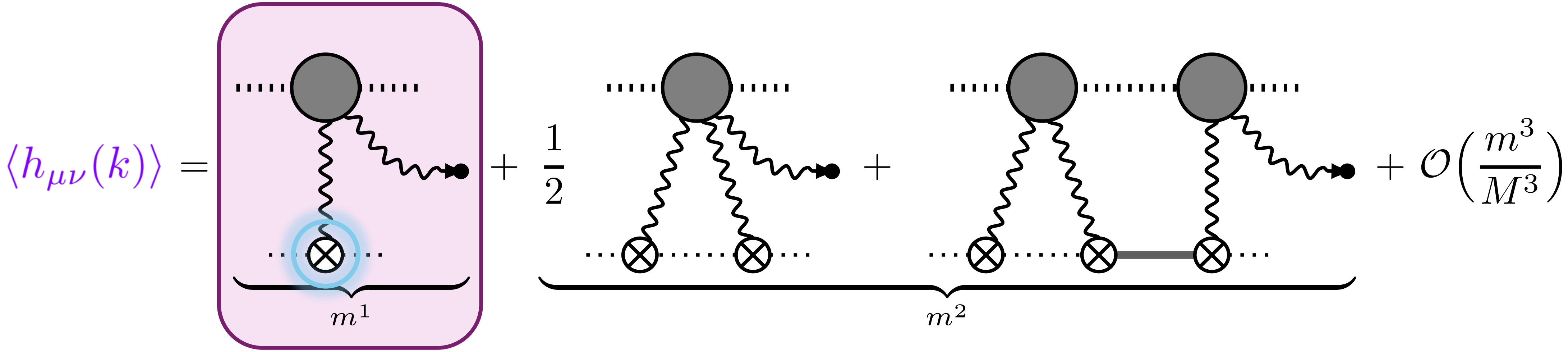
during crossing

Expand action **around geodesic motion** and derive vertices

$$\mathcal{T}^{\mu\nu}_{\text{sec}}(k)\Big|_{z=0} =$$ 

$$= -\frac{m\kappa_D}{2}\mathrm{e}^{\mathrm{i}k\cdot(v\tau_0+b)}\left(\underline{\frac{v^\mu v^\nu}{v\cdot k - \mathrm{i}0^+}} - \underline{\frac{\mathrm{e}^{\mathrm{i}k\cdot\Delta\bar{b}}v^\mu_\infty v^\nu_\infty}{v_\infty\cdot k + \mathrm{i}0^+}}\right) - \underline{\mathrm{i}m\kappa_D\mathrm{e}^{\mathrm{i}k\cdot\bar{x}(\tau_0)}\left(v^{(\mu}\Delta\bar{b}^{\nu)} + \theta(0)\Delta\bar{v}^{(\mu}\Delta\bar{b}^{\nu)}\right)}$$

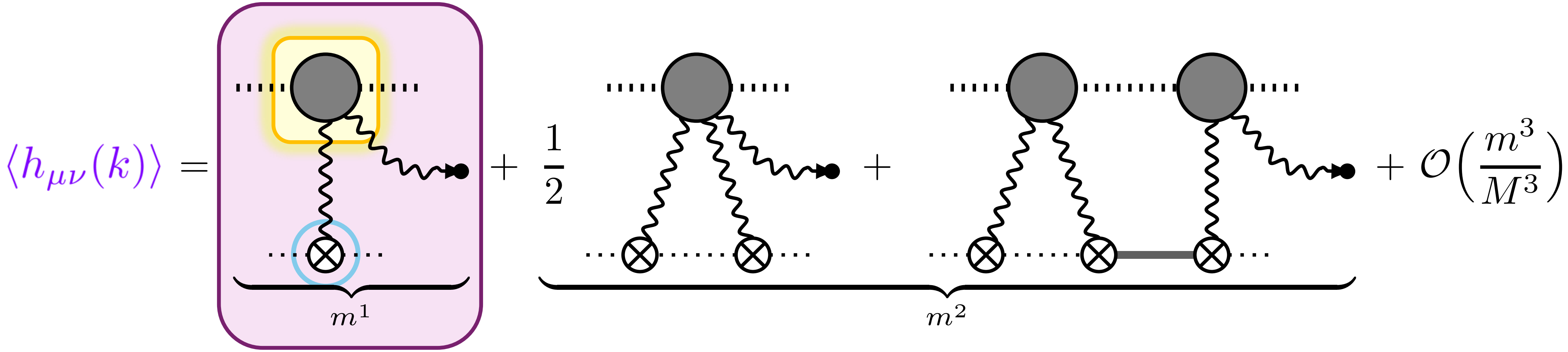
Goal: Exact in G waveform at 1SF



Ingredients:

- ☑ Background metric
- ☑ **Geodesics and cross vertices**
- ➢ *n*-point response function for *(n-1)* self-force observable

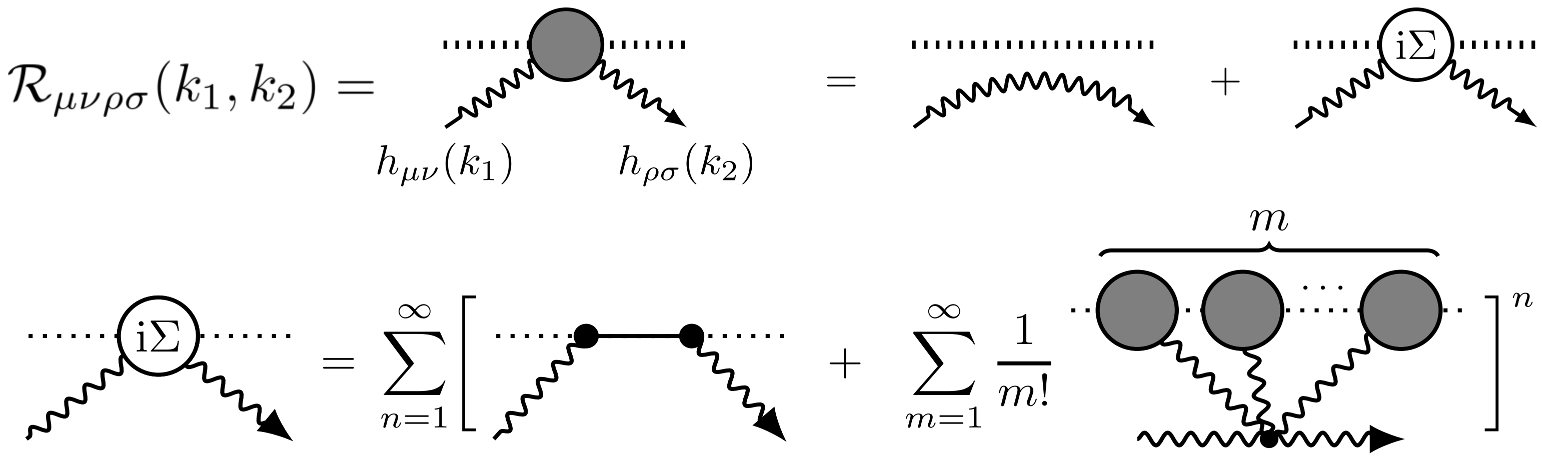
Goal: Exact in G waveform at 1SF



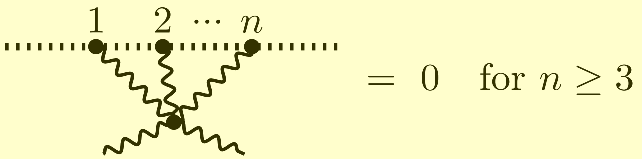
Ingredients:

- ☑ Background metric
- ☑ Geodesics and cross vertices
- ➢ ***n*-point response function for *(n-1)* self-force observable**

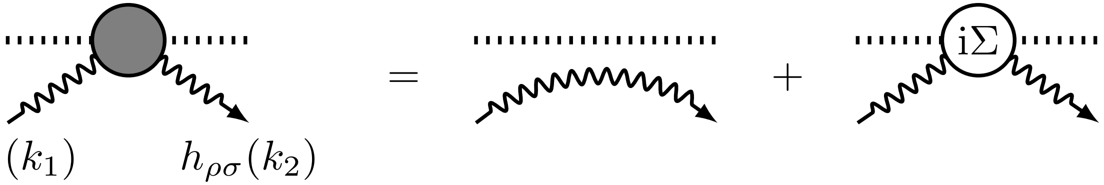
Two-point shockwave response

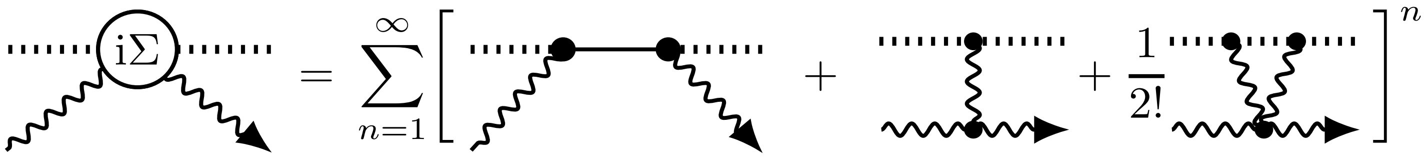


Shockwave self-energy vertices truncate:

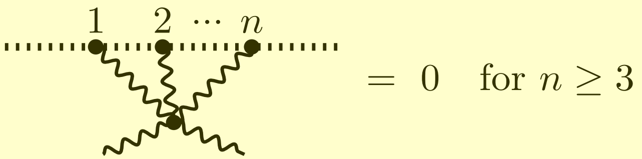


Two-point shockwave response

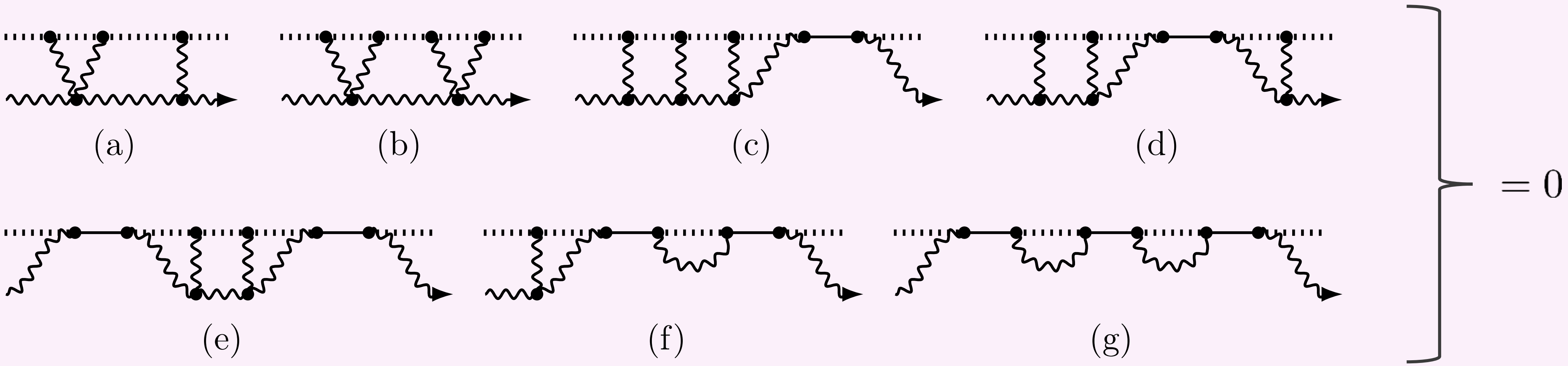
$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) =$ 



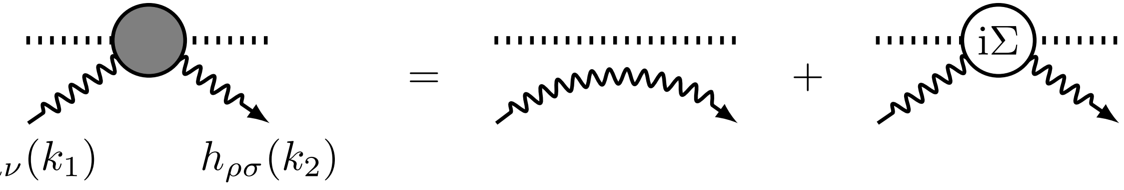
Shockwave self-energy vertices truncate:

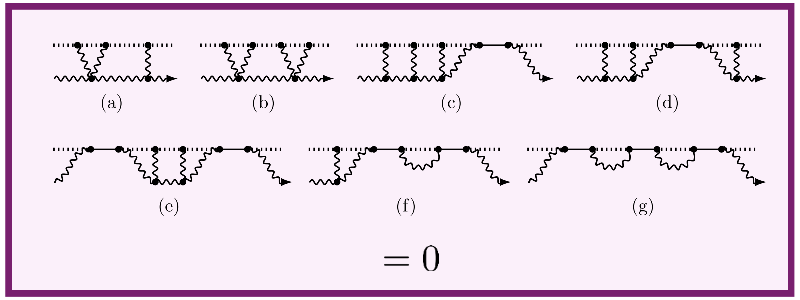


Large class of **vanishing diagrams**

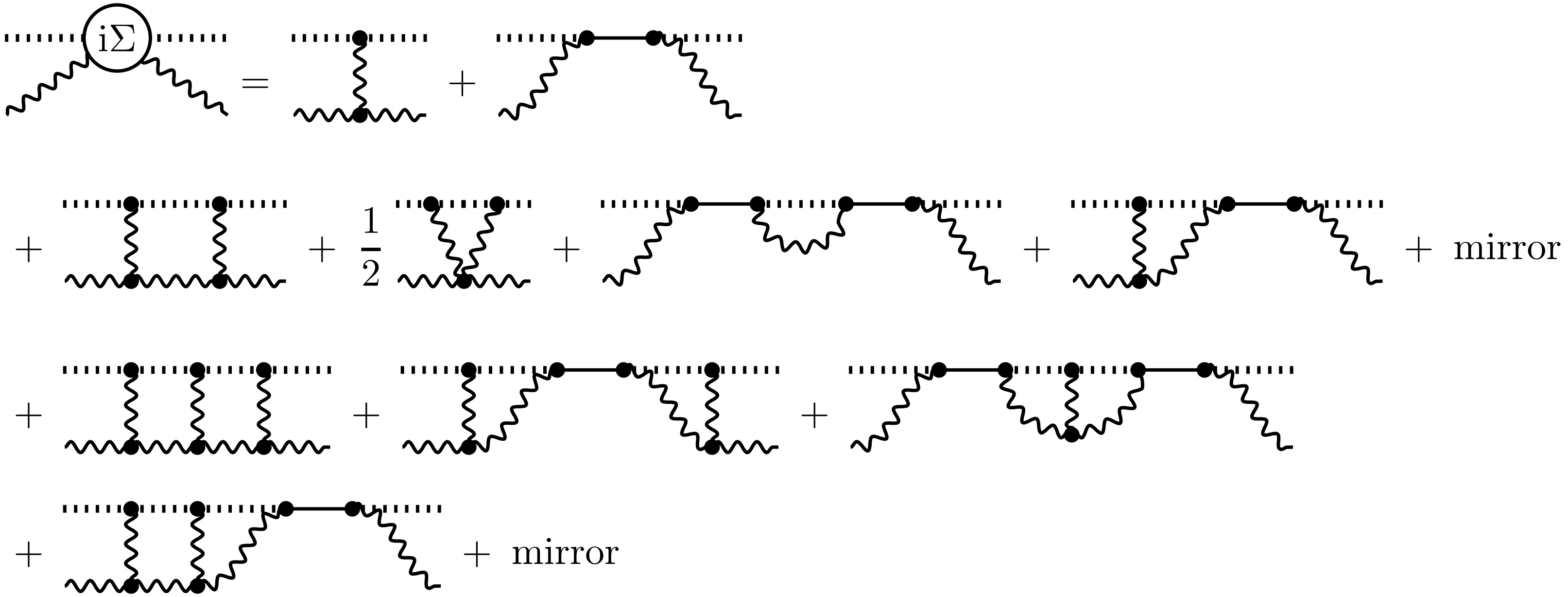


Two-point shockwave response

$\mathcal{R}_{\mu\nu\rho\sigma}(k_1,k_2) =$ 



- **All-order expansion in PM**

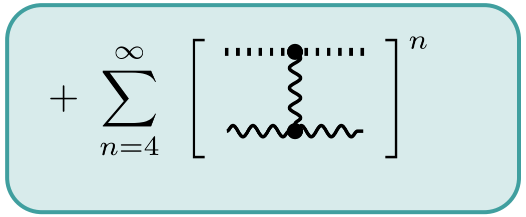


(1PM)

(2PM)

(3PM)

3PM exact remainder $\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}(k_1,k_2)$

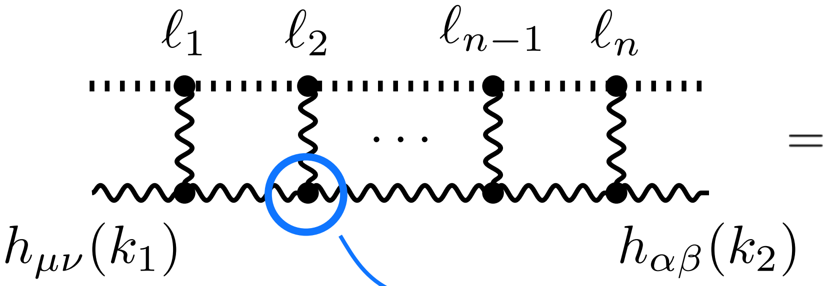


Resum to $\Sigma^{\mu\nu\alpha\beta}_{\text{sum}}(k_1,k_2)$

($\geq$ 4PM)

Two-point shockwave response

$$\mathcal{L}_n^{\mu\nu\alpha\beta} =$$



$$= \frac{\kappa_D^{2n}}{2^n}(P\cdot k_1)^{2n}\int_{\ell_1,\ldots,\ell_{n-1}} \Omega_n^{\mu\nu\alpha\beta}(k_1,k_2,P,\ell_1,\cdots,\ell_{n-1})\,\mathcal{I}_{n-1},$$

231 terms

Simplification at 4PM and beyond!
(using de Donder gauge)

$$\Omega_{n\geq 4}^{\mu\nu\alpha\beta}(k_1,k_2,P) = -\mathrm{i}\,\Pi^{\mu\nu}{}_{\kappa\lambda}(k_1)\Pi^{\kappa\lambda\alpha\beta}(k_2)$$

→ Independent of loop momentum

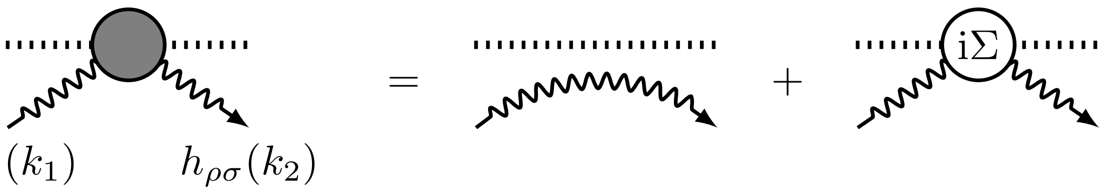
Only scalar integrals
→ Computed with **method of regions**

With the *TT-projector* $\Pi^{\mu\nu\alpha\beta}(k_i) = \pi_i^{\mu(\rho}\pi_i^{\alpha)\beta} - \frac{1}{D-2}\pi_i^{\mu\nu}\pi_i^{\alpha\beta}$

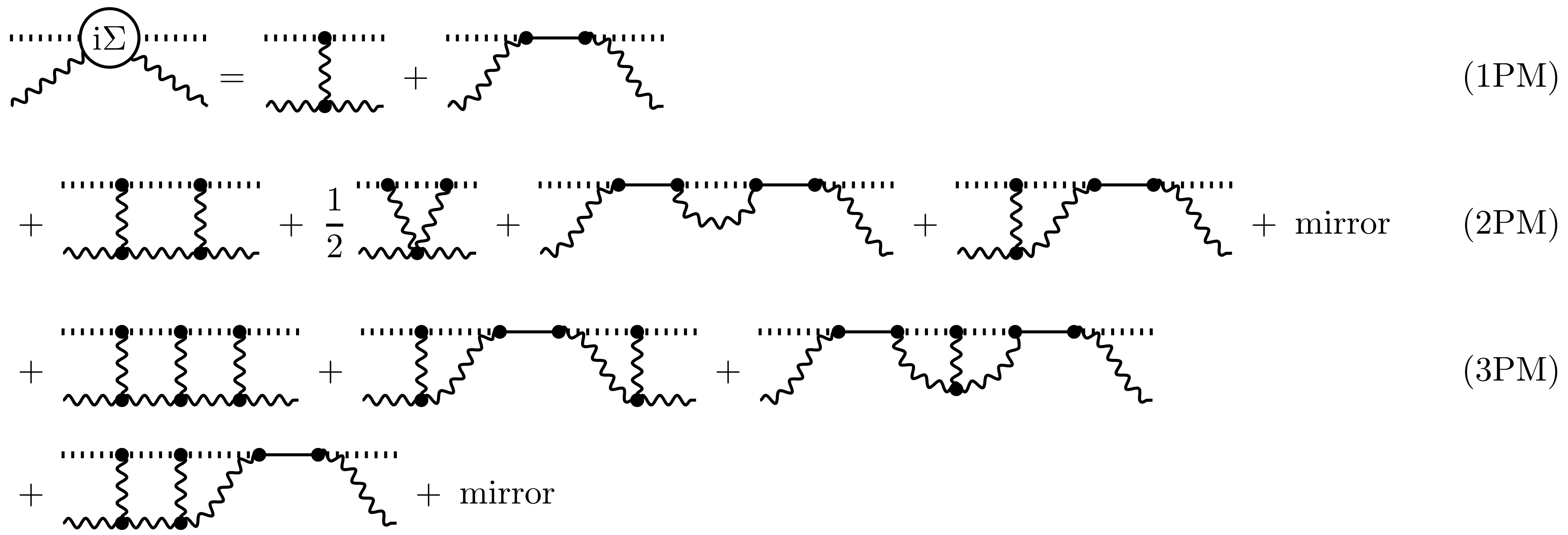
and the *transverse metric* $\pi_i^{\mu\nu} = \eta^{\mu\nu} + \frac{k_i^2 P^\mu P^\nu - 2(P\cdot k_i)k_i^{(\mu}P^{\nu)}}{(P\cdot k_i)^2}$

Two-point shockwave response

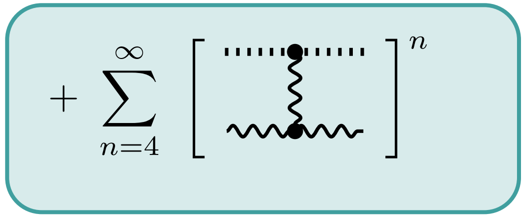
$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) =$



➢ **All-order expansion in PM**



3PM exact remainder $\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2)$

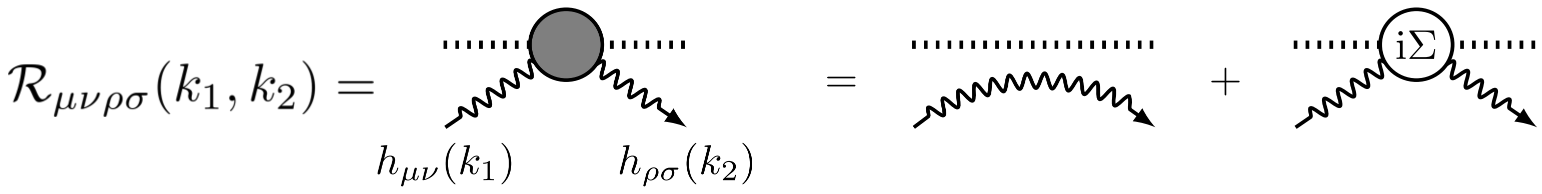


$\longrightarrow \Sigma_{\text{sum}}^{\mu\nu\alpha\beta}(k_1, k_2) = \sum_{n=4}^{\infty} \mathcal{L}_n^{\mu\nu\alpha\beta}$

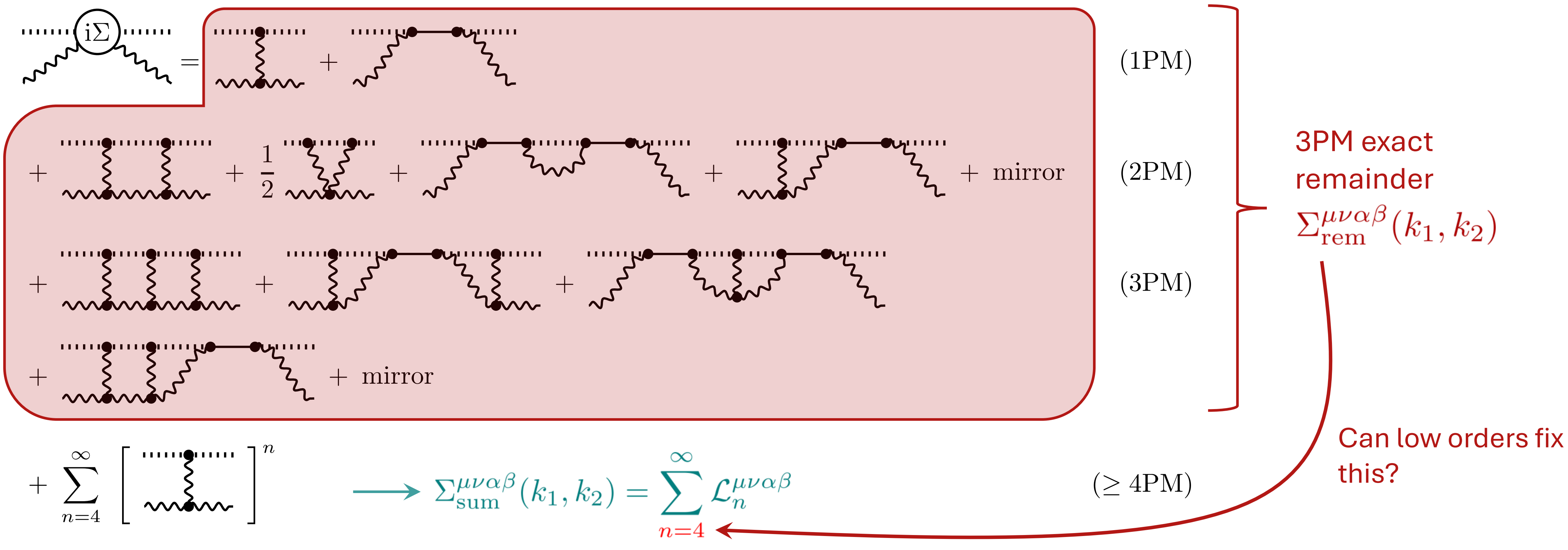
$(\geq 4\text{PM})$

Can low orders fix this?

Two-point shockwave response

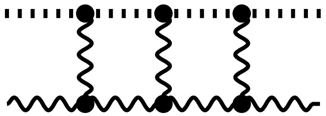


- **All-order expansion in PM**



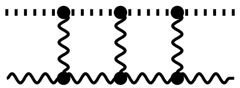
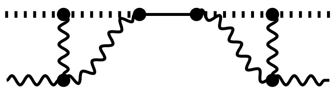
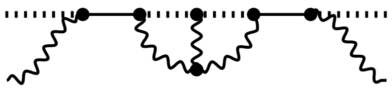
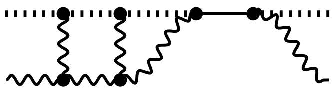
Two-point shockwave response

3PM and below, ladders do **not** follow the pattern observed at higher orders ...



$$\neq -\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\Big|_{n=3}$$

... but the **sum of all 3PM diagrams** does!

 $+$  $+$  $+$ 

$$+\ \text{mirror} = -\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\Big|_{n=3}$$

Same pattern as high-orders!

Summing low PM order:

$$\Sigma^{\mu\nu\alpha\beta}\Big|_{\leq 3\mathrm{PM}} = \underbrace{\sum_{n=1}^{3}\left[-\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\right]}_{\text{Follows high-PM pattern}} + \underbrace{\Sigma_{\mathrm{rem}}^{\mu\nu\alpha\beta}}_{\text{2PM exact Remainder}}$$

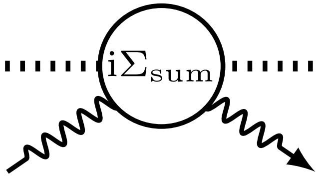
Vanishes on-shell

Include in resummed response $\Sigma_{\mathrm{sum}}^{\mu\nu\alpha\beta}(k_1,k_2)$

$$\boxed{\varepsilon_{1\mu}^{(h_1)}\varepsilon_{1\nu}^{(h_1)}\Sigma_{\mathrm{rem}}^{\mu\nu\alpha\beta}(k_1,k_2)\bar{\varepsilon}_{2\alpha}^{(h_2)}\bar{\varepsilon}_{2\beta}^{(h_2)}\Big|_{\text{on-shell}} = 0}$$

Two-point shockwave response

Sum up ...



$$\equiv \underbrace{\sum_{n=1}^{\infty} \left[-\,\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n} (\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\right]}_{\text{High-order ladder pattern}} = 2\hat{\delta}[P\cdot(k_1-k_2)]\,(P\cdot k_1)\big[\Pi_1\cdot\Pi_2\big]^{\mu\nu\alpha\beta}\mathcal{F}(q_\perp;\epsilon)\,,$$

... into an **expanded form** ... into an **resummed form**

$$\mathcal{F}(q_\perp;\epsilon) = \left(\frac{4\pi}{\boldsymbol{q}_\perp^2}\right)^{1-\epsilon}\sum_{n=1}^{\infty}\frac{(\mathrm{i}W)^n}{n!}\frac{\Gamma(-\epsilon)^n}{(\boldsymbol{q}_\perp^2\,\mathrm{e}^{\gamma_\mathrm{E}}L^2)^{n\epsilon}}\frac{\Gamma(1+(n-1)\epsilon)}{\Gamma(-n\epsilon)} = \int \mathrm{d}^{D-2}\boldsymbol{x}_\perp\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp}\Big[\mathrm{e}^{\mathrm{i}W\Gamma(-\epsilon)\left(\frac{\boldsymbol{x}_\perp^2}{4\mathrm{e}^{\gamma_\mathrm{E}}L^2}\right)^{\epsilon}}-1\Big],$$

Weinberg factor

$$W = 2G(P\cdot k_1)$$

Same structure as calculation in eikonal approximation
Raj, Venugopalan [2406.10483]

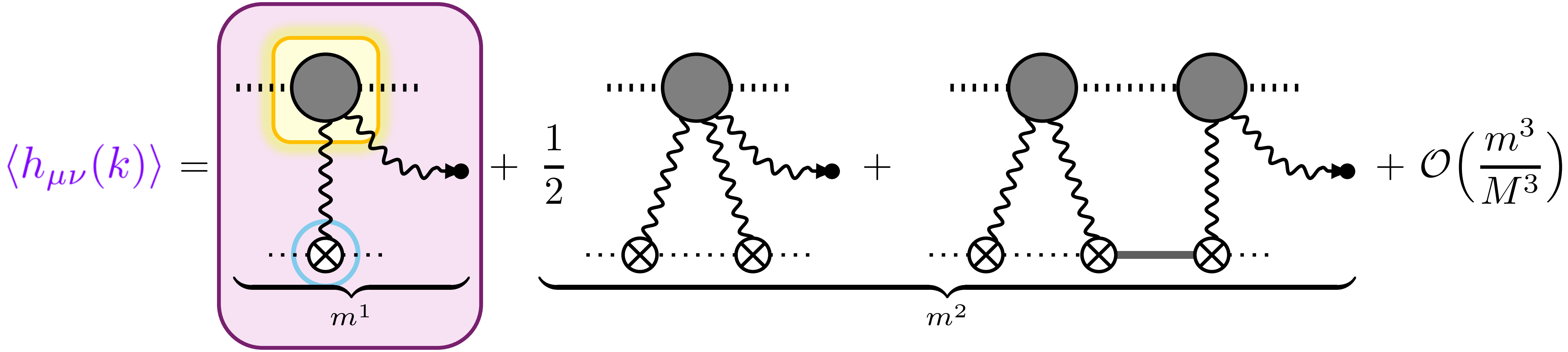
4-dimensional limit

$$\Sigma^{\mu\nu\alpha\beta}_{\mathrm{sum}} = \frac{\Gamma(1-\mathrm{i}W)}{\Gamma(1+\mathrm{i}W)}\frac{\mathrm{e}^{-\mathrm{i}W/\epsilon}}{(\boldsymbol{q}_\perp^2L^2)^{-\mathrm{i}W}}\,\Sigma^{\mu\nu\alpha\beta}_{\mathrm{sum}}\big|_{\mathrm{1PM}}$$

Pure phase factor

Equivalent structure observed by 't Hooft, 1987 for scalar ultra-high energy amplitude

Goal: Exact in GW waveform at 1SF

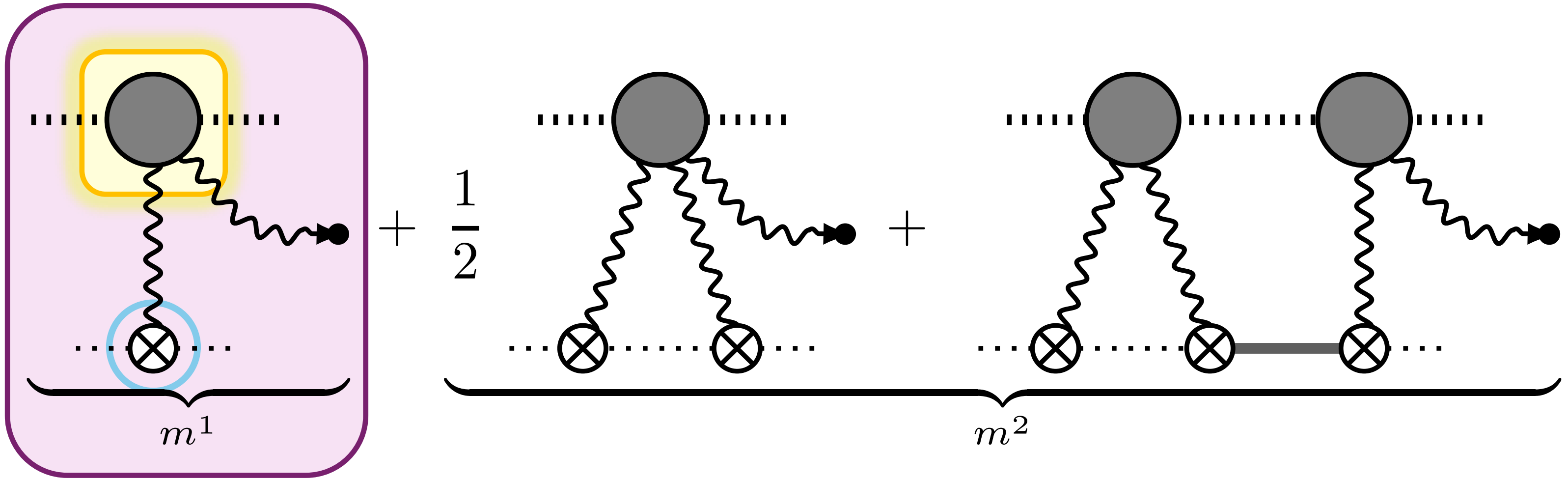


Ingredients:

- ☑ Background metric
- ☑ Geodesics and cross vertices
- ☑ ***n*-point response function for *(n-1)* self-force observable**

Goal: Exact in G waveform at 1SF

$\langle h_{\mu\nu}(k) \rangle =$



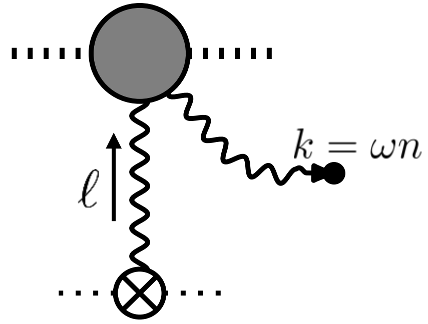
$+ \mathcal{O}\left(\frac{m^3}{M^3}\right)$

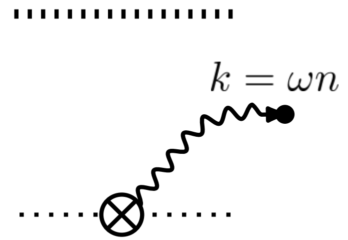
Ingredients:

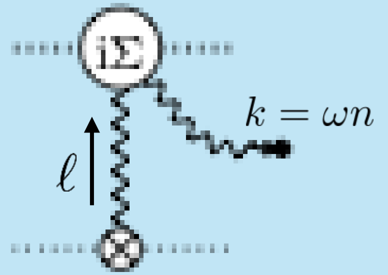
- ✓ Background metric
- ✓ Geodesics and cross vertices
- ✓ *n*-point response function for *(n-1)* self-force observable

$\Longrightarrow$ **Assemble and integrate!**

Two-point shockwave response

$$\langle h_{\mu\nu}(u,\boldsymbol{n})\rangle_{1\mathrm{SF}} = \int_\omega \mathrm{e}^{-\mathrm{i}\omega u}\int_\ell$$ 

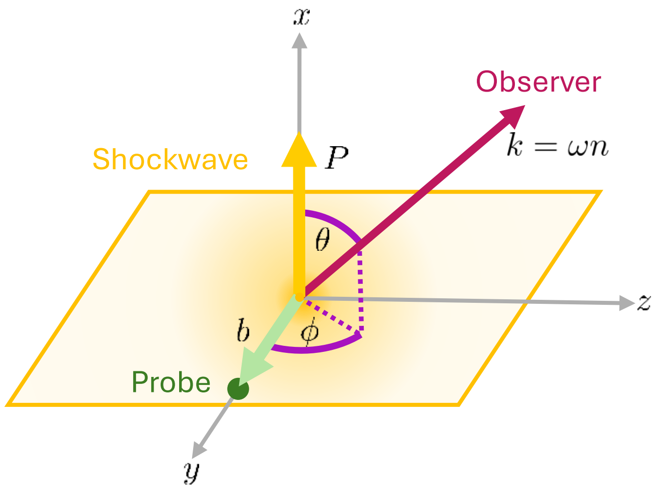
$$= \int_\omega \mathrm{e}^{-\mathrm{i}\omega u}\Bigg[$$ 

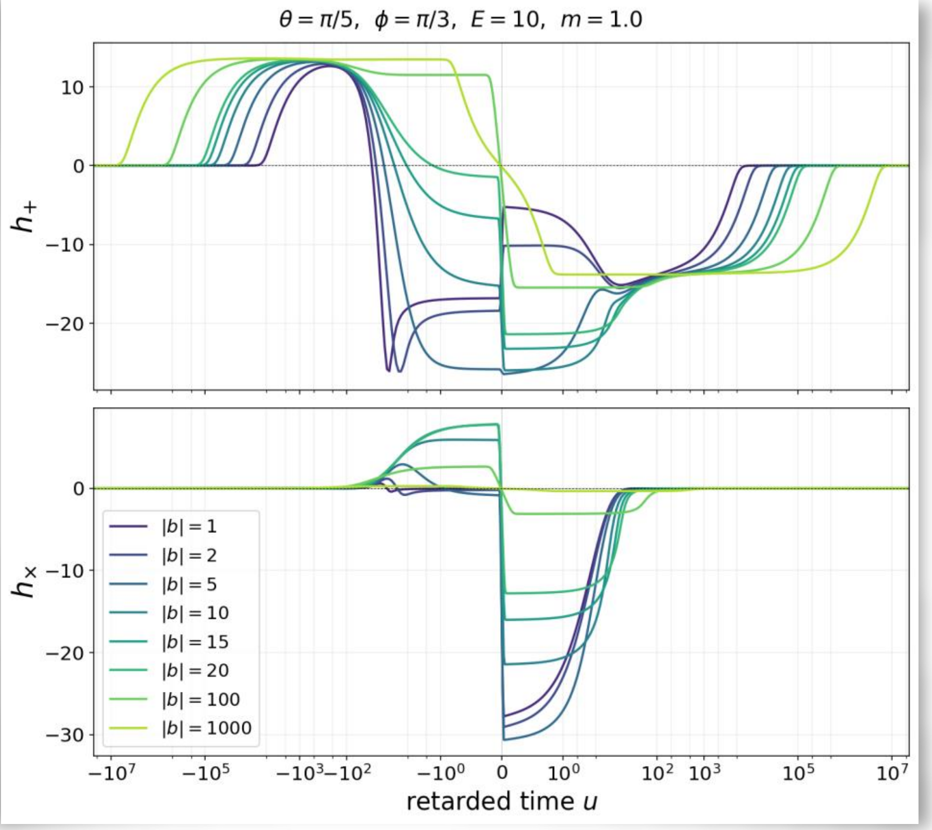
$$+\int_\ell$$ 

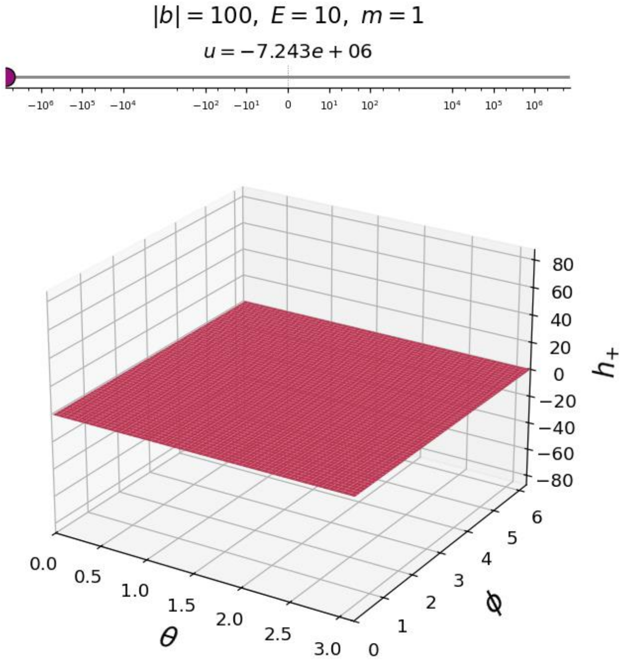
$$\Bigg]$$

Purely memory

No memory

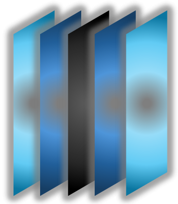




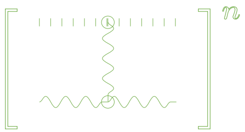


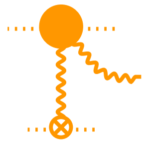
Summary and Outlook

In this talk



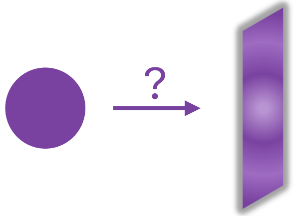
- Novel way of obtaining **SF expansion with WQFT**!
- Proof of concept by applying the formalism to the **shockwave**
- Obtained the **first exact in G** two-point response function and **waveform**

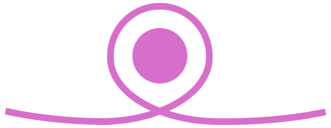




Outlook

- Compute more observables, such as **impulse** for the shockwave metric
- Connection to **high-energy limit** of massive particle?
- Response functions of **massive WQFT**?





Thanks!

Backup slides

Generating functional, background fields and vertices

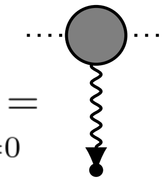
Generalize to generic sources

$$\mathrm{e}^{\frac{\mathrm{i}}{\hbar}W\overbrace{[\mathcal{T},f]}^{J_A}} = \int \mathcal{D}\overbrace{[h,Z]}^{\phi_A}\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g]-\int \mathrm{d}^D x\,\mathcal{T}^{\mu\nu}(x)h_{\mu\nu}(x)-\int \mathrm{d}\tau\, f_\mu(\tau)Z^\mu(\tau)\right)}\Big|_{\text{tree}} = \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\overbrace{\left(S_{\mathrm{BH}}[\langle\phi\rangle_J]-\langle\phi_A\rangle_J J_A\right)}^{\int \mathrm{d}^D x\,\mathcal{T}^{\mu\nu}(x)\langle h_{\mu\nu}(x)\rangle_{\{\mathcal{T},f\}}+\int \mathrm{d}\tau\, f_\mu(\tau)\langle Z^\mu(\tau)\rangle_{\{\mathcal{T},f\}}}}$$

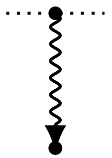
Classical action as functional

$W[J]$ generates **response fuction**

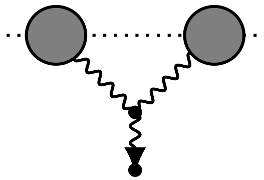
$$\langle h_{\mu\nu}(x)\rangle_{J=0} = -\kappa_D^{-1}\frac{\delta W[J]}{\delta\mathcal{T}^{\mu\nu}(x)}\Big|_{J=0} =$$



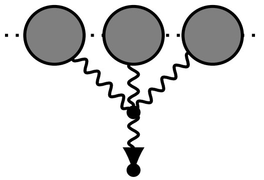
$=$



$+\frac{1}{2}$



$+\frac{1}{3!}$



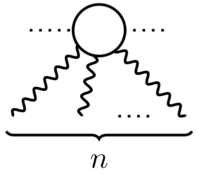
$+\cdots,$

$$\langle Z^\mu(\tau)\rangle_{J=0} = -\frac{\delta W[J]}{\delta f_\mu(\tau)}\Big|_{J=0} = 0$$

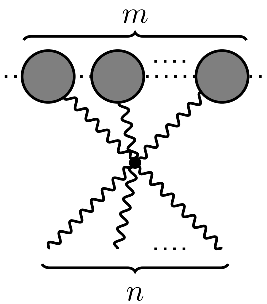
Background metric

$S_{\mathrm{BH}}[[\langle\phi\rangle_J]]$ generates **Feyman vertices** on curved spacetime

$$\frac{\delta^n \mathrm{i}S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle h_{\mu_1\nu_1}\rangle_J\delta\langle h_{\mu_2}\nu_2\rangle_J\cdots\delta\langle h_{\mu_n\nu_n}\rangle_J}\Big|_{J=0} =$$



$=\sum_{m=0}^{\infty}\frac{1}{m!}$

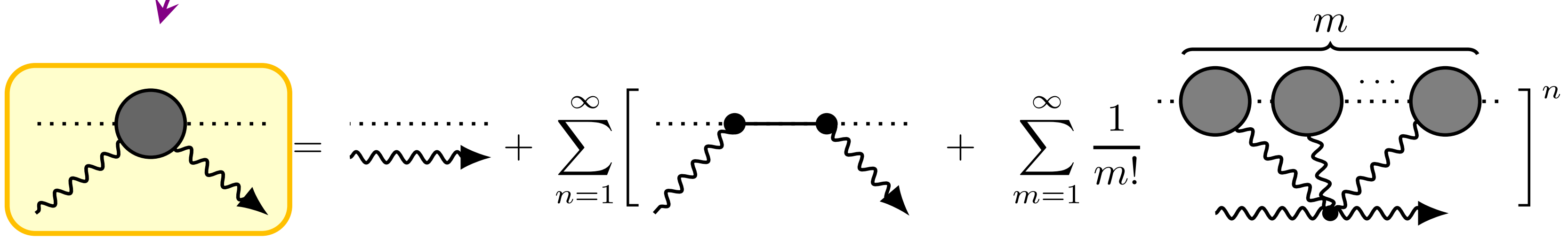


Propagator and higher-point response functions

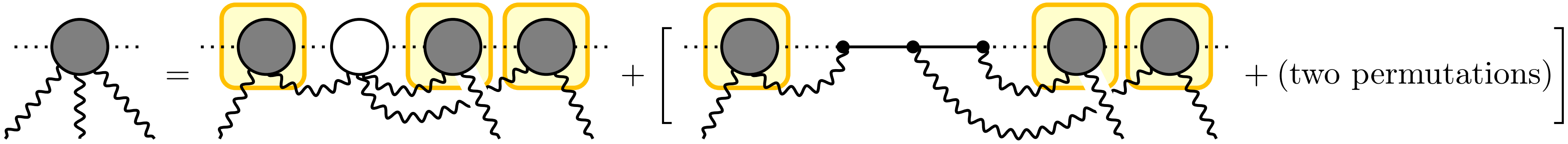
Two-point response acts as **propagator** on curved spacetime due to **Green's function-like relation**

$$\boxed{\overbrace{\frac{\delta^2 S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle\phi_A\rangle_J\,\delta\langle\phi_B\rangle_J}}^{\text{Two-point vertex on curved background}}\;\underbrace{\frac{\delta^2 W[J]}{\delta J_B\,\delta J_C}}_{\text{Response Function}}\Bigg|_{J=0} = -\delta_{AC}}$$

Solved by a geometric series:



Three-point response (and higher) can be recovered from lower-point response functions



The on-shell limit and *N*-operator

On-shell response function (Compton amplitude)

$$\langle k_2, h_2|\mathrm{i}\hat{T}|k_1, h_1\rangle = \mathrm{i}\varepsilon_{1\mu}^{(h_1)}\varepsilon_{1\nu}^{(h_1)}\Sigma^{\mu\nu\alpha\beta}\bar{\varepsilon}_{2\alpha}^{(h_2)}\bar{\varepsilon}_{2\beta}^{(h_2)}\Big|_{\text{on-shell}} = 2\delta[P\cdot(k_1-k_2)]\,\frac{(P\cdot F_1^{(h_1)}\cdot F_2^{(h_2)}\cdot P)^2}{(P\cdot k_1)^3}\mathcal{F}(q_\perp;\epsilon)\,,$$

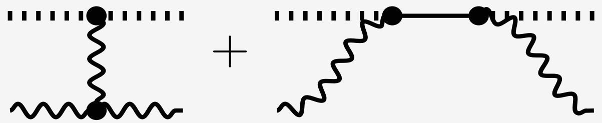
with the linearised field-strength tensor $F_i^{(h_i)\mu\nu} = 2k_i^{[\mu}\varepsilon_i^{(h_i)\nu]}\,.$

Double copy

Matrix element of **Magnus operator** $\hat{S} = \mathbb{1} + \mathrm{i}\hat{T} = e^{\mathrm{i}\hat{N}}$

Magnus, 1954
Damgaard, Planté, Vanhove
[2107.12891]

$$\langle k_2, h_2|\mathrm{i}\hat{N}|k_1, h_1\rangle = \langle k_2, h_2|\mathrm{i}\hat{T}|k_1, h_1\rangle\Big|_{\text{1PM}} =$$



$$= \frac{16\pi\mathrm{i}G}{(4\pi e^{\gamma_E}L^2)^\epsilon}\delta[P\cdot(k_1-k_2)]\,\frac{(P\cdot F_1^{(h_1)}\cdot F_2^{(h_2)}\cdot P)^2}{(P\cdot k_1)}$$

1PM exact and ϵ - finite!