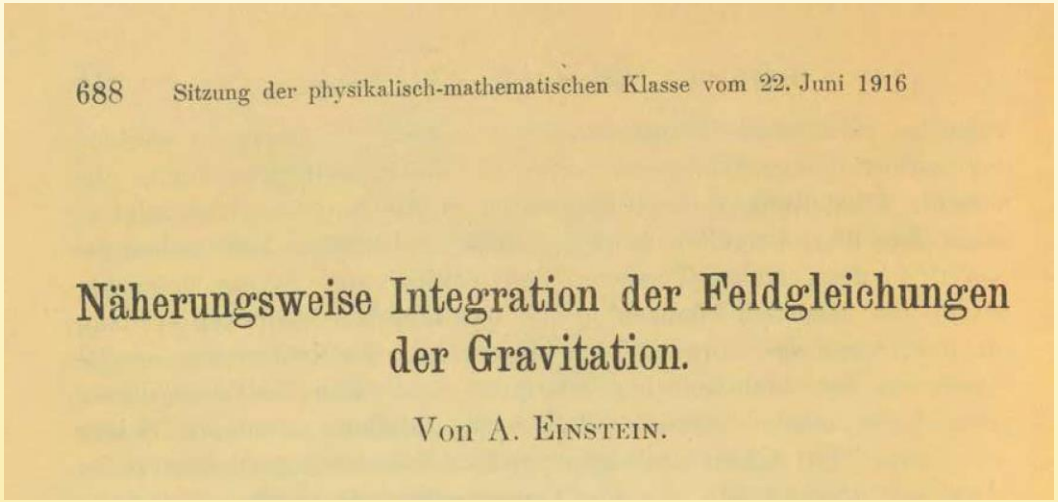
# Amplitudes for Gravitational Waves

David A. Kosower
Institut de Physique Théorique, CEA–Saclay

Amplitudes 2026
Queen Mary University of London
June 29–July 3, 2026

# Gravitational Waves

are a very old field



688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

**Näherungsweise Integration der Feldgleichungen der Gravitation.**

Von A. Einstein.

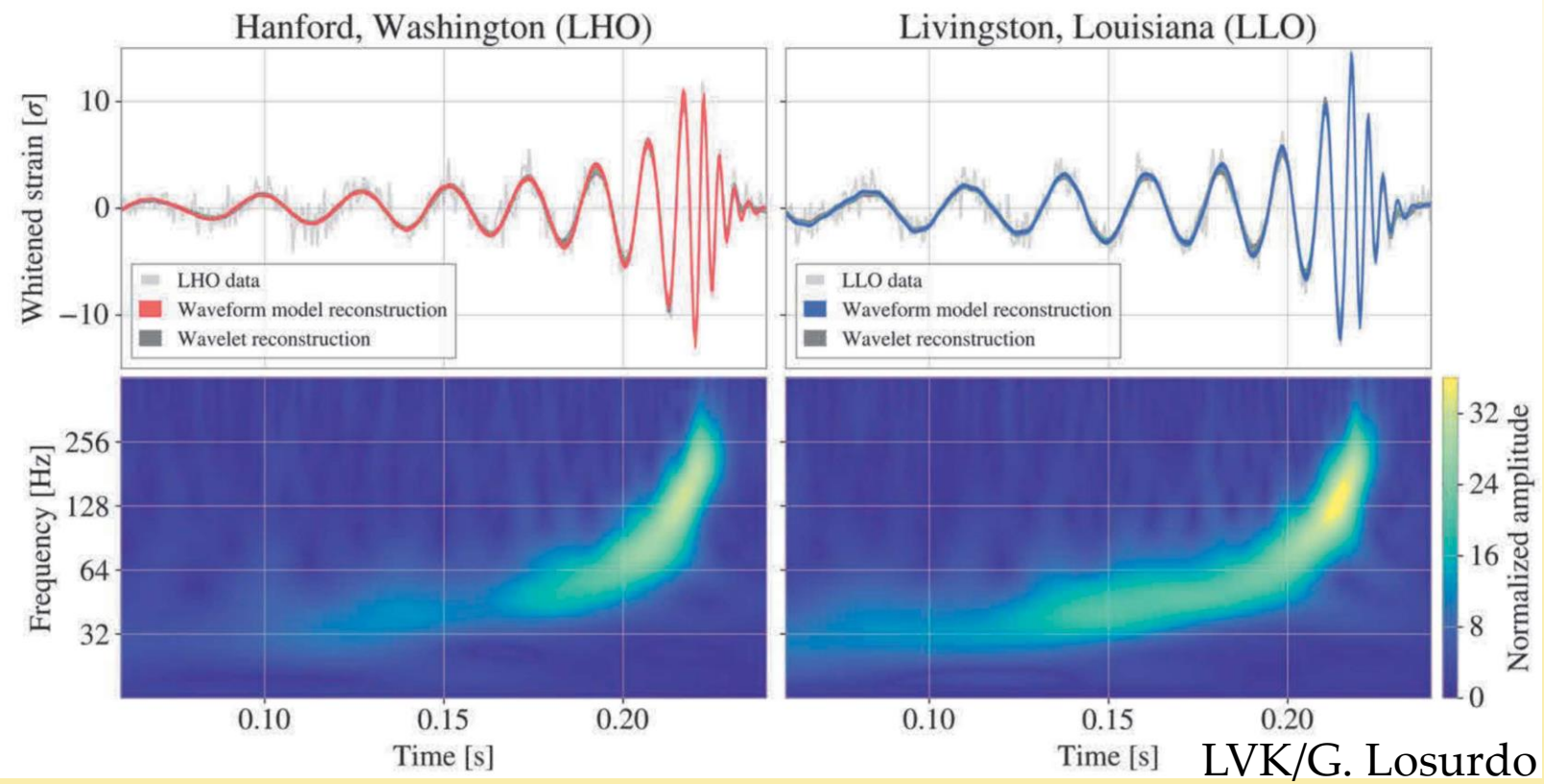
⋮

§ 2. Ebene Gravitationswellen.

Aus den Gleichungen (6) und (9) folgt, daß sich Gravitationsfelder stets mit der Geschwindigkeit 1, d. h. mit Lichtgeschwindigkeit. fortpflanzen. Ebene, nach der positiven $x$-Achse fortschreitende Gravitationswellen sind daher durch den Ansatz zu finden

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + i x_4) = \alpha_{\mu\nu} f(x - t). \qquad (15)$$

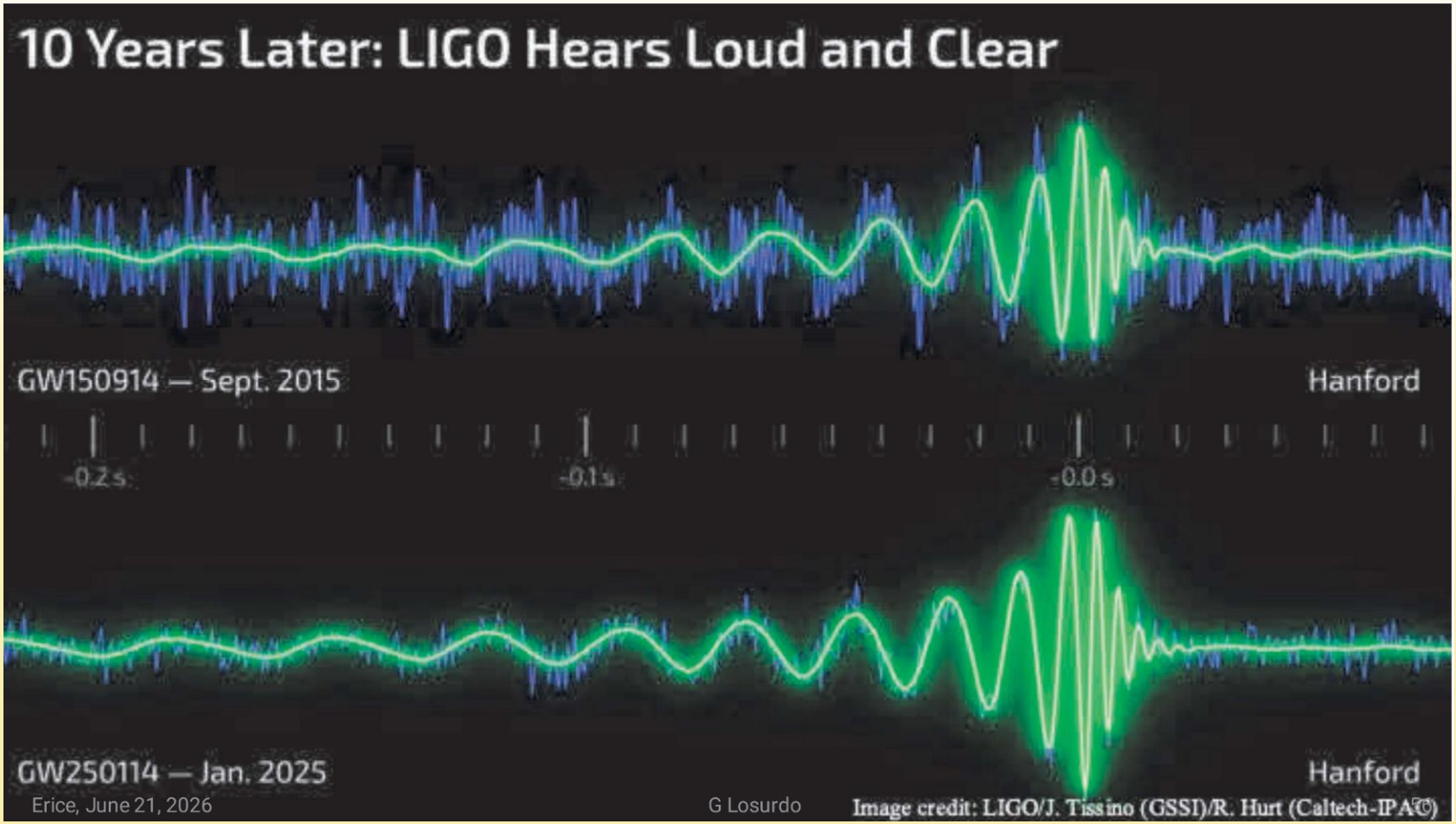
But also a very young field observationally




LVK/G. Losurdo

Have you seen these traces before?

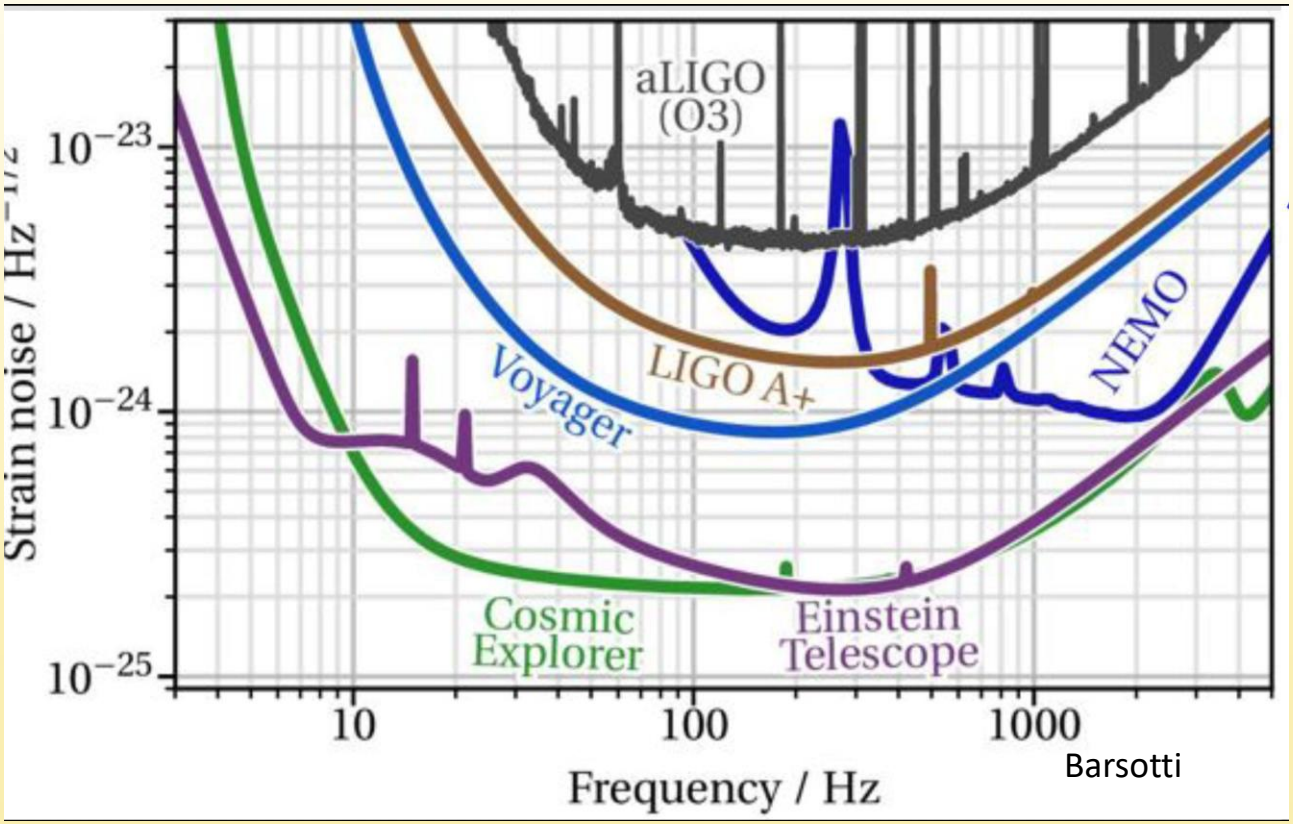
# Continuous Improvement





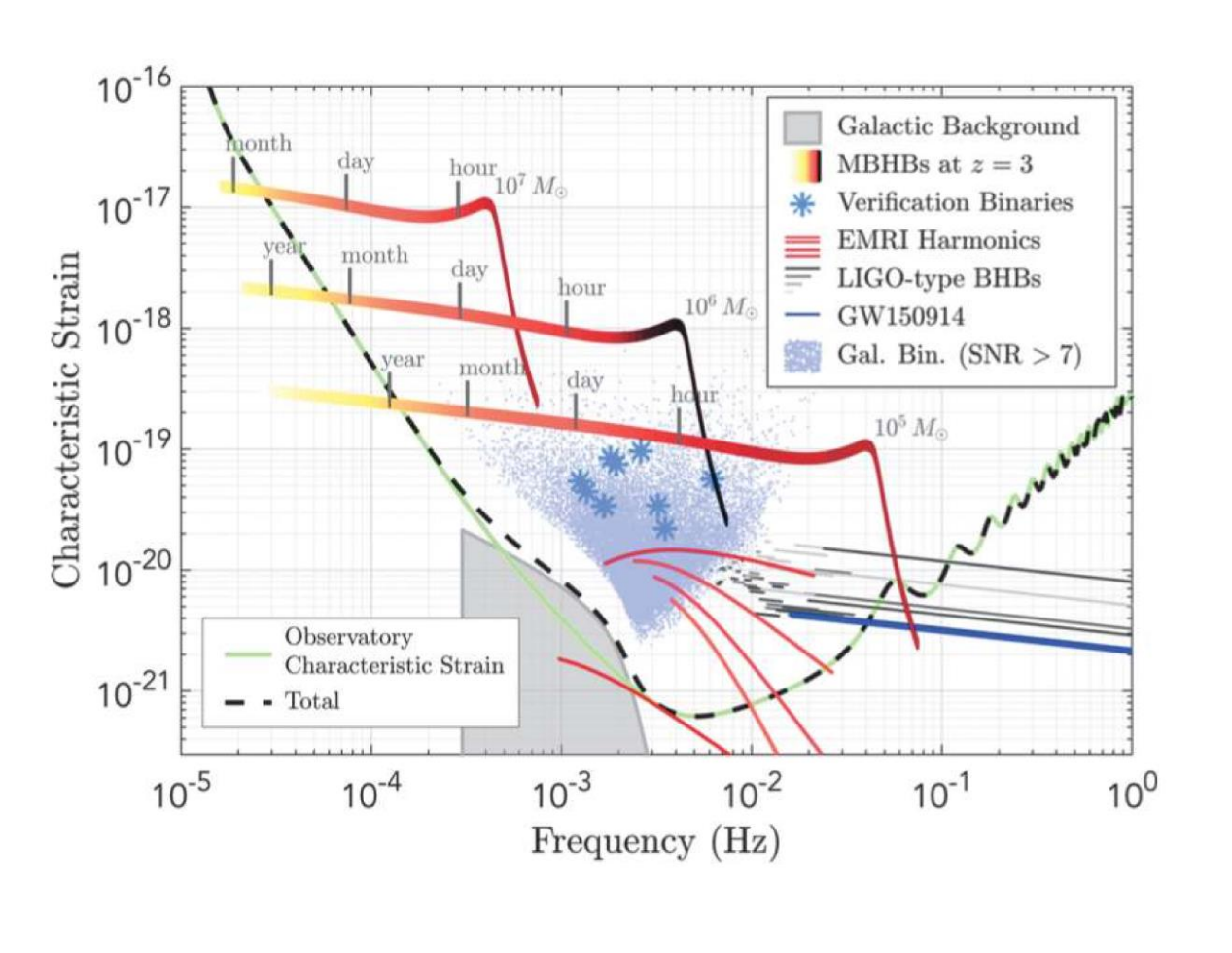

courtesy LVK/G. Losurdo

# Future Terrestrial GWOs




Path to higher precision

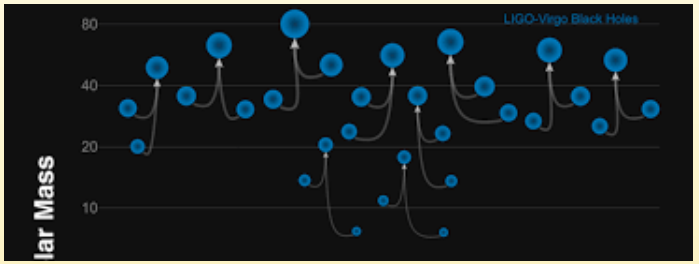
# LISA




From LISA L3 Mission proposal (via Robson–Cornish–Liu [1803.01944])

# Science Goals

- Determine presence and distribution of heavy compact objects
  - Already discovered unexpected class of black hole binaries
  - More surprises?
- Neutron stars
  - Determine equation of state
  - Contribution to transferric element production
  - Insights into Gamma-Ray Bursters
- Measure Hubble constant
  - Bright candles as individual events
  - Dark candles statistically
- Limits on exotic compact objects
- Precision strong-field tests of Einstein gravity



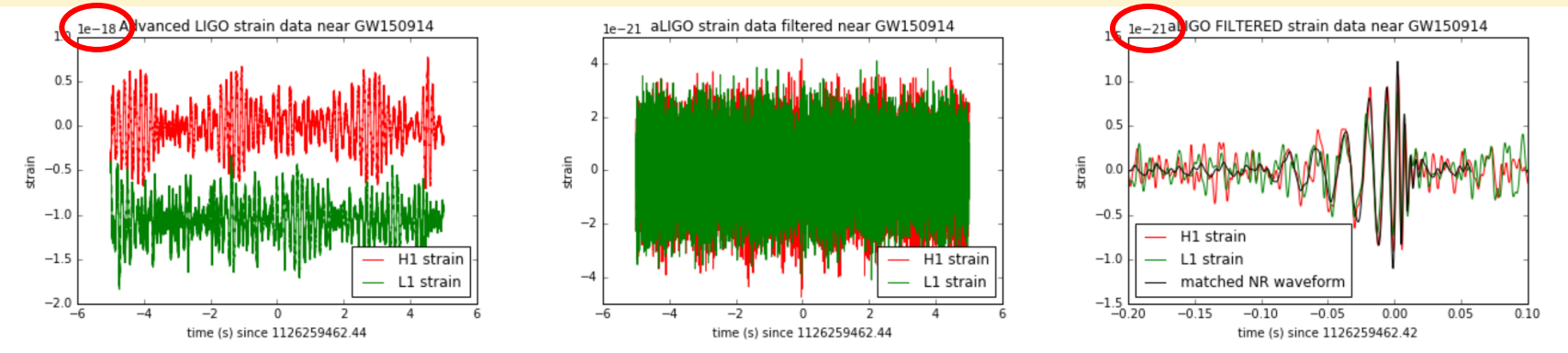
# Science Goals

- Insight into galactic evolution through mergers of supermassive black holes

- Insight into quantum gravity?
  - with exotic black hole models
  - with cosmological gravitational waves

# Theorists' Role

- One aspect of detection & analysis
- High noise level: 400 times larger than signal! Rehman [1711.07421]




- To extract the signal, we need theoretical templates
- That's where theorists come into the picture

# Theorists

- Waveform Templates
- Detection
  - Extended mass/spin range for standard events
  - Fainter events
  - New classes of events (IMRIs, EMRIs)
  - Exotic events (capture, early phase, supernovae, …)
- Measurement
  - Parameter extraction
  - Avoid biases in population-level estimates

# What Do We Need?

$$
\begin{aligned}
&G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)
\end{aligned}
$$

Snowmass White Paper: Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng (2022)

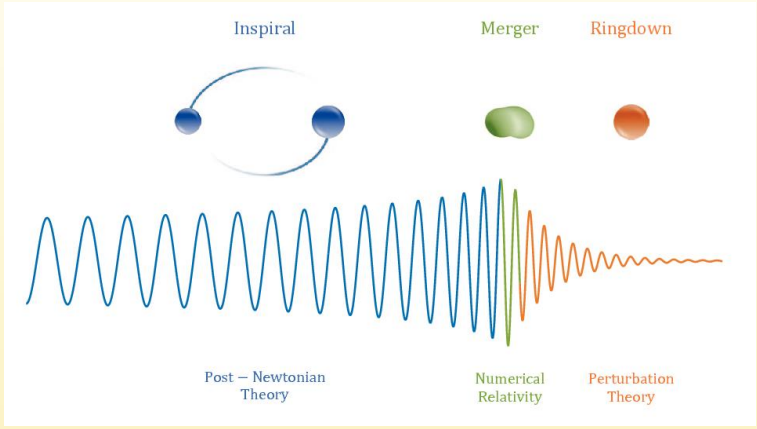
*I've been rich and I've been poor. Believe me, honey, rich is better!*

—Sophie Tucker

More is better

# Basic Setup in Mergers

- Two slowly inspiralling compact objects
  - Ringdown
  - Merger
- Inspiral
  - Weak-field perturbation theory




Antelis et al

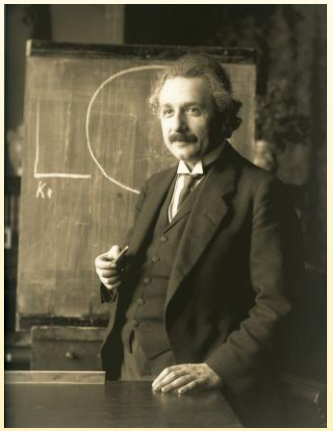
- Relativistic perturbative expansion in the scattering domain
- ~~"Post-Minkowskian Expansion"~~
- Relativistic Perturbation Theory (RPT)

# Numerical Relativity

- Key tool for merger phase
- Tool for cross-checking other calculations
- Computationally expensive
  - Not realistic for 100Ks of templates needed
- Accumulates errors over long inspirals
- Hard to do large mass ratios
- Can't be the only tool

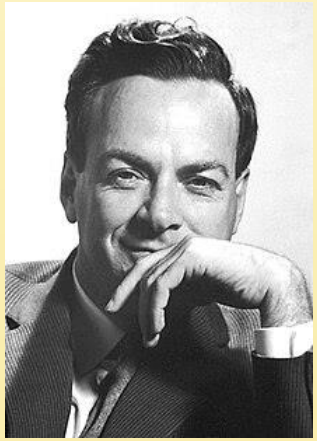
# Scattering Amplitudes Approach

- ~~Gravitation = General Relativity ← geometry~~



- Inspiration from Feynman

"What I cannot create, I do not understand."



- Gravitation = spin-2 field theory

Before proceeding, let's review some of the experimental facts which a Venutian theorist would have to play with in constructing a theory for the amazing new effect.

# Analysis Pipelines

Vallisneri

- Three dominant pipelines
  - Phenomenological: post-Newtonian (PN) based, Numerical Relativity (NR) calibrated; fast, mass ratios up to 1:10
  - Effective One-Body (EOB): resummed PN, NR calibrated, self-force (SF) matched; slow
  - Surrogate: interpolated NR; slowest, limited mass ratios

- EOB only one (so far) to use relativistic perturbative (RPT) information

# EFT Matching

Cheung, Rothstein, Solon

- Start with generic Hamiltonian

$$\sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{b}),\ V = \sum_{j=1}^{\infty} c_j \left(\frac{G}{\boldsymbol{b}}\right)^j$$

- Build amplitudes in this EFT

- Match to classical limit of four-point amplitudes computed in perturbative gravity

- Or match using world-line field theory computation

- Gives conservative dynamics (scattering angle)
- Feed back to bound-state predictions

# Progress

- $\mathcal{O}(G^2)$: classical GR result

  Westphal (1985)

- $\mathcal{O}(G^3)$: First amplitudes contribution

  Bern, Cheung, Roiban, Shen, Solon, Zeng (2019)
  Kälin, Liu, Porto (2021)
  Herrmann, Parra-Martinez, Zeng (2021)

- $\mathcal{O}(G^4)$: Industrialization

  Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2021)
  Dlapa, Kälin, Liu, Porto (2021)
  Jakobsen, Mogull, Plefka, Sauer, Xu (2023)

- $\mathcal{O}(G^5)$: Cutting edge

  Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, Sauer, Usovitsch (2024–6) ["conservative"]
  Bern, Herrmann, Roiban, Ruf, Smirnov, Smith, Zeng (2025) [partial]
  Dlapa, Kälin, Liu, Porto (2025–6) [nonlocal-in-time conservative]

- Boundary to Bound Mapping

  Cho, Kälin, & Porto (2019—2021)

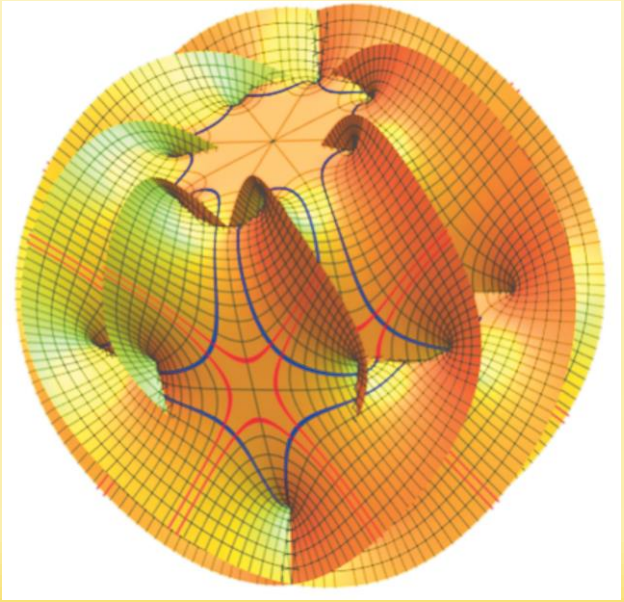
- Also work on spin, which I won't discuss

- Organize in self-force orders

$$\frac{\tilde{I}_{r,5}}{m_1^4 m_2^4} = \frac{(m_1^4+m_2^4)}{m_1^2 m_2^2}\tilde{I}^{0\mathrm{SF}}_{r,5} + \frac{(m_1^2+m_2^2)}{m_1 m_2}\tilde{I}^{1\mathrm{SF}}_{r,5} + \tilde{I}^{2\mathrm{SF}}_{r,5}$$

Bern, Herrmann, Roiban, Ruf, Smirnov, Smith, & Zeng (2025)

- Calabi-Yau integrals make an appearance



Driesse, Jakobsen,
Klemm, & Mogull (2024)

# Workflow

- Integrand Generation (+classical limit)
- Integration-by-Parts (+tensor) Reduction
  - Key difficulty
- Integration (differential equations, boundary conditions)
  - Novel geometries at $\mathcal{O}(G^5)$
- Observables (scattering angles)

$\Rightarrow$ talks by Jakobsen, Bern

# Progress in IBPs

- Many different ideas explored: improvements to existing codes and new directions

Wu, Boehm, Ma, Xu, & Zhang; Hu, Feng, Shen, & Zhang; Liao; Wang & Yang; von Hippel & Wilhelm; Liu & Mitov; Wu, Böhm, Ma, Usovitsch, Xu, & Zhang; Bree, Gasparotto, Matijašić, Mazloumi, Melnichenkod, Pögel, Teschke, Wang, Weinzierl, Wu, & Xu; Wu & Xu; Song; Smirnov & Zeng; Belitsky, Kokosinskaya, Smirnov, Voevodin, & Zeng; Song, Yang, Cao, Luo, & Zhu; Zeng; Ma, Qin, Tan, & Yan; Smith & Zeng; Chen; Feng, Li, Liu, Ma, & Zhang; Coro, Novichkov, Page, & Song; de la Cruz & DAK; Huang, Ma, Wang, & Yang; von Gersdorff & Lessa; Britto, Grimm, & Hoefnagels; Berman, Charton, Luna, Wilhelm, & Zeng

- **Same** problem and **same** codes for QCD/EW and gravitational waves
- Most intense and sensible use of machine learning and AI

# Topics I Won't Be Discussing

- All have seen important progress in the last few years
- Post-Newtonian calculations

Narikawa; Mandal, Mastrolia, Silva, Patil, & Steinhoff; Georgoudis, Heissenberg, & Russo; Siddhant, Grant, & Nichols; Skoupý & Witzany; Bini, Damour, & Geralico; Placidi, Rettegno, & Nagar; Blanchet, Faye, & Trestini; Bautista, Khalil, Sergola, Kavanagh, & Vines; Cunningham, Kavanagh, Pound, Trestini, Warburton, & Neef; Witzany, Skoupý, Stein, Tanay; Sridhar, Bhattacharyya, Paul, & Mishra; Gamboa, Khalil, & Buonanno; Dones, Henry, Bernard; Islam & Venumadhav; Morras, Pratten, & Schmidt; Fumagalli, Loutrel, Gerosa, & Boschini; Trestini; Brunello, Mandal, Mastrolia, Patil, Pegorin

- Self-Force calculations & formalism

Honet, Pound, Compère; Honet, Mathews, Compère, Pound, Wardell; Brunello, Mandal, Mastrolia, Patil, Pegorin, Ronca, Smith, Steinhoff, & Torres Bobadilla; Castillo, Evans, Kavanagh, Neef, Wardell; Liu & Cao; Bini, Geralico, & Rufrano Aliberti; De Falco & Gallo; Yoo, Boyle, & Deppe; Placidi, Sebastiani, & Grignani; Henry & Heffernan; Aykroyd & Bourgoin; Henry; Blanchet, Faye, Seraille, & Trestini; Laxman, Fujita, & Mishra; Colin, Tanay, & Bernard; Bini & Di Russo; Porto & Riva; Morras, Pratten, Schmidt, & Buonanno; Brunello, Mandal, Mastrolia, Patil, & Pegorin; Sun, Bonga, Stein, & Da Re; Cheung, Parra-Martinez, Rothstein, Shah, & Wilson-Gerow

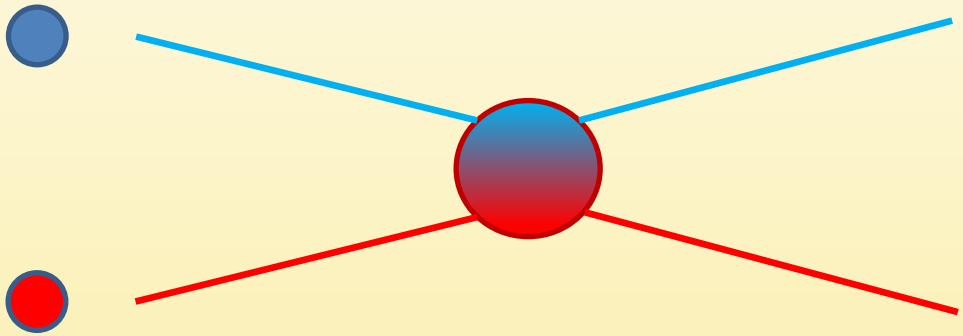
# Topics I Won't Be Discussing

- Eikonal, Resummation, Tidal, & Other

Heissenberg & Russo; Fernandes & Lin; Georgoudis, Heissenberg, & Russo; Cipriani, Di Russo, Fucito, Francisco Morales, Poghosyan, & Poghossian; Ivanov, Li, Parra-Martinez, & Zhou; Parra-Martinez & Podo; Schulze, Bernuzzi, Rettegno, Fontbuté, Placidi, & Damour; Caron-Huot, Correia, Isabella, & Solon

# Set-up

- Scatter two massive objects, treated as point particles



impact parameter $b$
distinguishable particles

# Numerical Relativity Meets Scattering

- Numerical relativists have taken up serious interest in scattering & in comparing to perturbative calculations
  - Rettegno, Pratten, Thomas, Schmidt, & Damour (2023), using Einstein Toolkit
  - Kankani & McWilliams (2024), B2B doesn't work nonperturbatively
  - Swain, Pratten, & Schmidt (2024), using Einstein Toolkit
  - Long, Pfeiffer, Buonanno, Jakobsen, Mogull, Ramos-Buades, Rüter, Kidder, & Scheel (2025), using SPEC
  - Long, Pfeiffer, Kidder, & Scheel (2025), using SPEC

- In some ways observables are cleaner in scattering than in bound state

# What About the Bound Domain?

- Towards waveform mapping

Gonzo and Mogull

- More to come in the next 6–12 months…
- Some steps in D. O'Connell's talk later today

# KMOC Approach

DAK, Maybee, O'Connell; **Cristofoli**, **Gonzo**, DAK, O'Connell

- Start with physical observables in quantum theory

- Initial state of two localized massive particles

- Observables will have the form

$$\text{Final} - \text{Initial} = \langle\psi|S^\dagger \mathbb{O} S|\psi\rangle - \langle\psi|\mathbb{O}|\psi\rangle$$

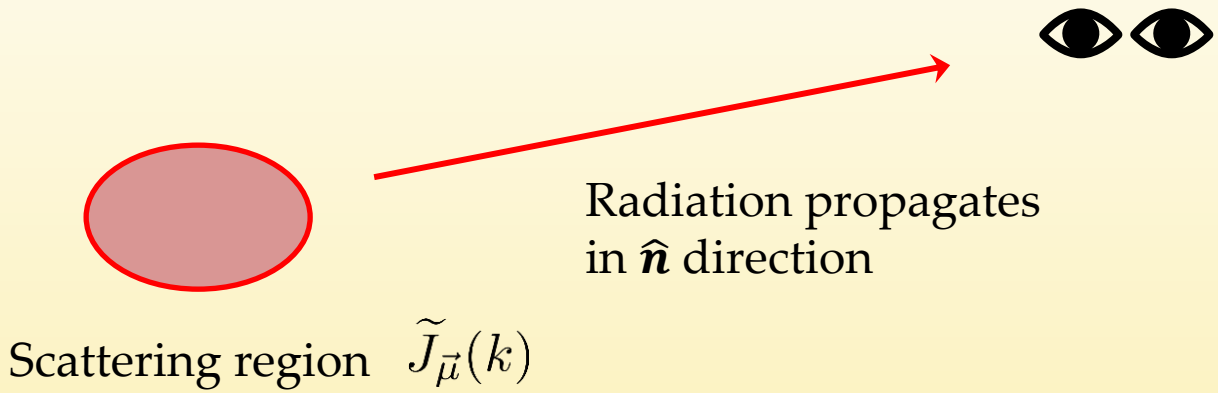
- All-orders master formula

$$\langle \Delta O\rangle = i\langle\psi|[\mathbb{O},T]|\psi\rangle + \langle\psi|T^\dagger[\mathbb{O},T]|\psi\rangle$$

$$= \quad I_{(1)} \quad + \quad I_{(2)}$$

loop cut

# Local Observables




- Local radiation observable

$$R(x) = i \int d\Phi(k) \left[ \widetilde{J}_{\vec{\mu}}(k) e^{-ik\cdot x} - \widetilde{J}_{\vec{\mu}}(-k) e^{+ik\cdot x} \right]$$

- Waveform is leading large-distance behavior

$$R(x) = \frac{1}{|\boldsymbol{x}|} W(t, \hat{\boldsymbol{n}}; x_0)$$

# Waveform

- Formally, the Riemann tensor

$$\langle R^{\text{out}}_{\mu\nu\lambda\sigma}(x)\rangle = \langle\psi|\mathbb{R}_{\mu\nu\lambda\sigma}(x)|\psi\rangle + 2\ \text{Re}\ i\langle\psi|\mathbb{R}_{\mu\nu\lambda\sigma}(x)T|\psi\rangle + \langle\psi|T^{\dagger}\mathbb{R}_{\mu\nu\lambda\sigma}(x)T|\psi\rangle$$

- At lowest order only the first term contributes $[\mathcal{O}(g^3)]$

- The second term enters at next-to-leading order $[\mathcal{O}(g^5)]$

# Waveform

- LO: classical GR + reproduction

  Kovacs & Thorne (1978)
  Jakobsen, Mogull, Plefka, & Steinhoff (2021)
  Mougiakakos, Riva, & Vernizzi (2021)

- NLO & related

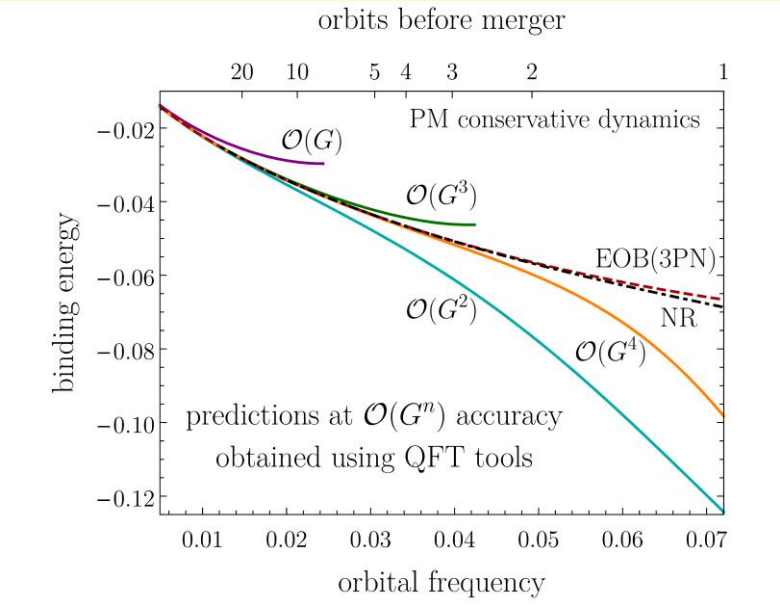
  Brandhuber, Brown, Chen, De Angelis, Gowdy, & Travaglini (2023)
  Herderschee, Roiban, & Teng (2023)
  Georgoudis, Heissenberg, & Vazquez-Holm (2023)
  Caron-Huot, Giroux, Hannesdottir, & Mizera (2023)
  Bohnenblust, Ita, Kraus, & Schlenk (2023)
  Elkhidir, O'Connell, Sergola, & Vazquez-Holm (2023)
  Brunello, De Angelis, & DAK (2025)

- Power spectrum to $\mathcal{O}(G^3)$, $\mathcal{O}(G^4)$

- Additional information away from circular orbits

# Proof of the Pudding

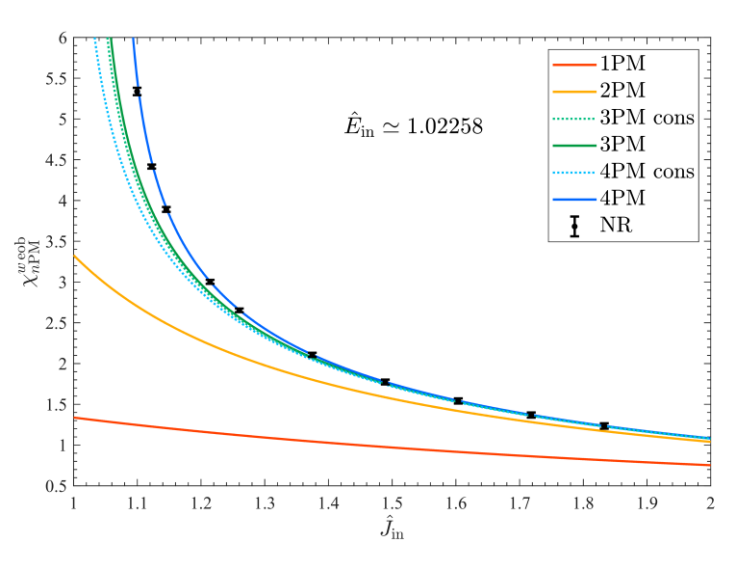
- Incorporating input from RPT gets binding energy predictions closer to the "gold standard" (NR)




Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng (2022)

# Proof of the Pudding

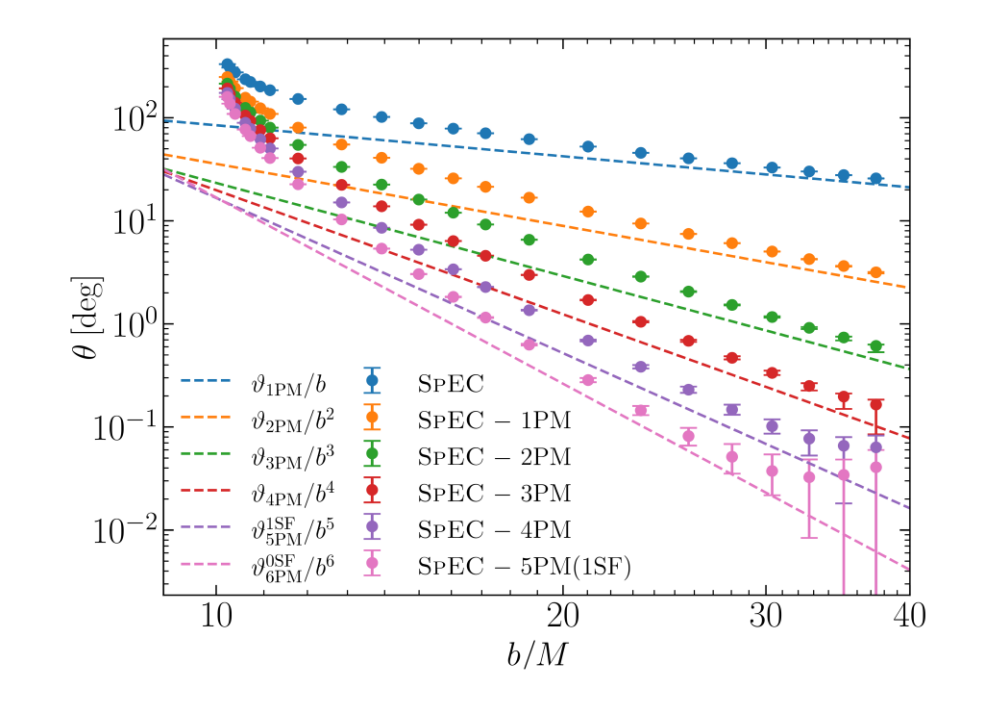
- Gets EOB resummation closer to the "gold standard" in scattering




Damour, Rettegno (2022)

# Proof of the Pudding

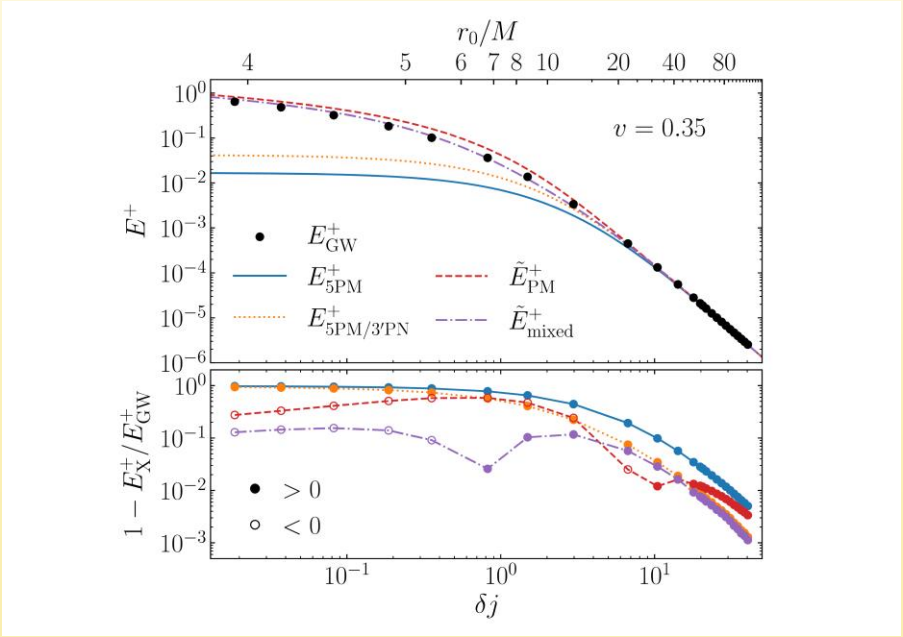
- Scattering angle residuals

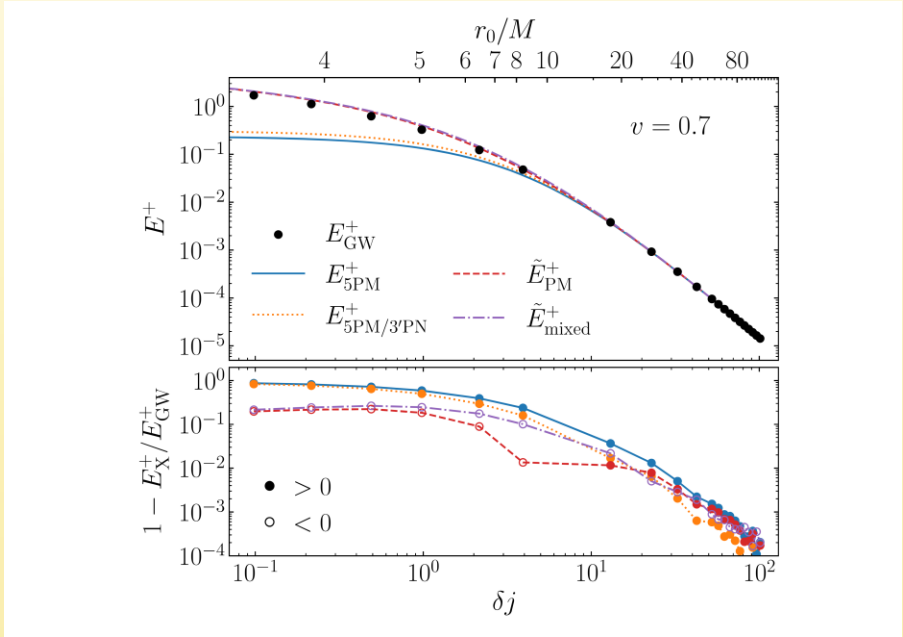



Long, Pfeiffer, Kidder, & Scheel (2025)

# Proof of the Pudding

- Emitted energy in scattering with (non-EOB) resummation




Barack, Gonzo, Leather, Long, & Warburton

# Power Spectra

- LO vs NLO power spectra $\frac{dE}{d\omega} = \frac{2}{\kappa^2}\int d\Omega \sum_h |\omega\, \mathcal{W}_h(\omega,\Omega)|^2$

w/**G. Brunello** and S. De Angelis

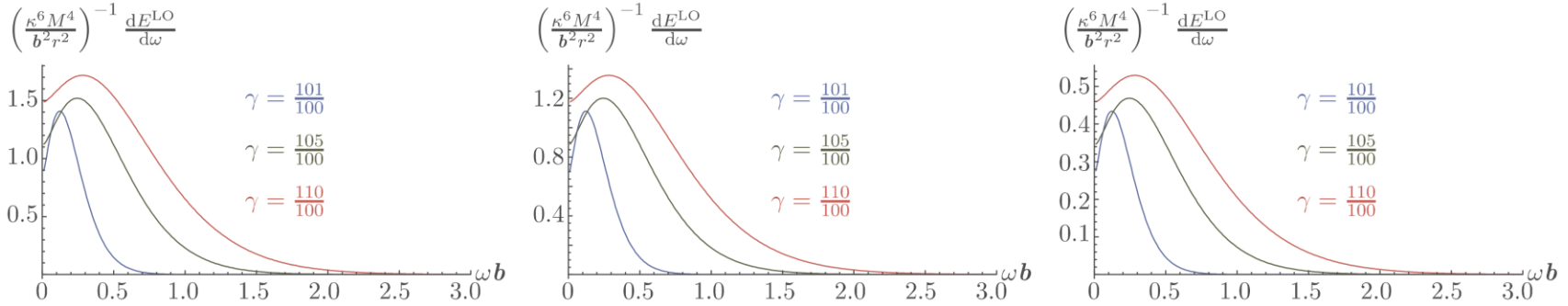



FIG. 6. Plots of the gravitational power spectrum at LO $dE^{\rm LO}/d\omega$ as a function of the energy $\omega$. Each plot shows different values of the boost $\gamma$ with fixed symmetric mass ratios $\nu$, which are, respectively $\{\frac{1}{4}, \frac{2}{9}, \frac{5}{36}\}$.

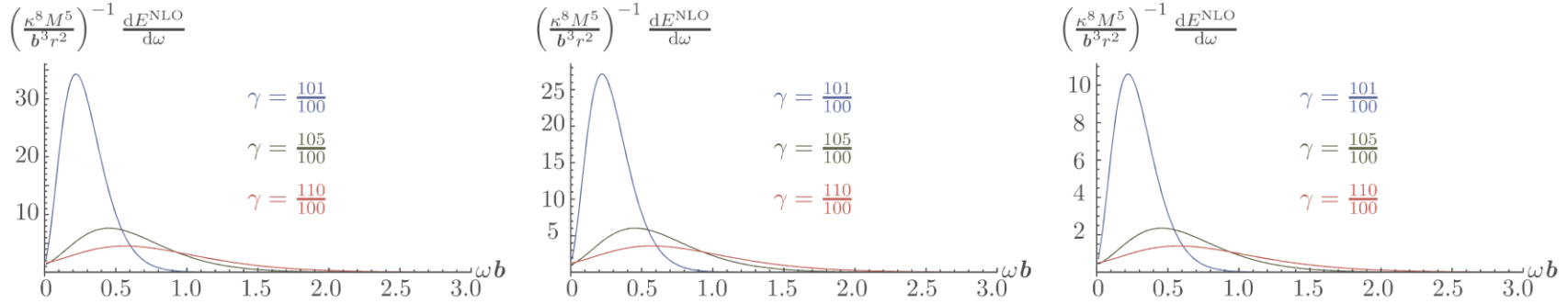



FIG. 7. Plots of the NLO corrections to the gravitational power spectrum $dE^{\rm NLO}/d\omega$ as a function of the energy $\omega$. Each plot shows different values of the boost $\gamma$ with fixed symmetric mass ratios $\nu$, which are, respectively $\{\frac{1}{4}, \frac{2}{9}, \frac{5}{36}\}$.

# Summary & Outlook

- Relativistic perturbation theory for gravitational waves

Challenges

- Higher orders for scattering angle/matching
  - in $G$
  - in spin
- Higher orders for waveforms
  - in $G$
  - in spin
- Interpolating waveforms to bound states
- New ideas for resummation