Reassessing the Connection Between the Color Glass Condensate and TMD PDFs

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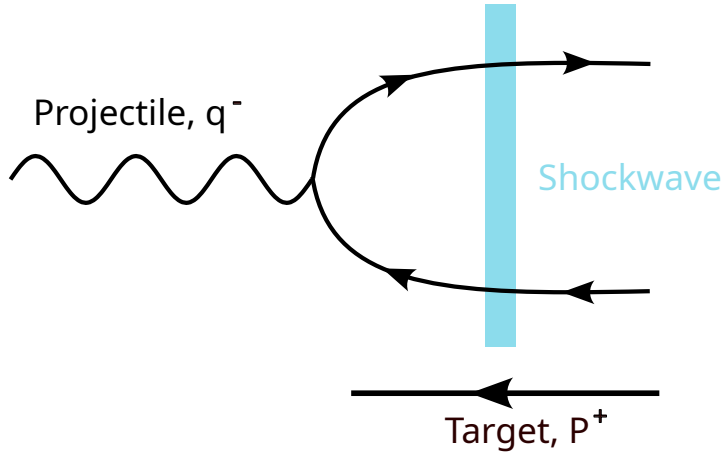
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Based on arXiv:2610.10189

Color Glass Condensate effective theory



The CGC is an effective theory built to describe the small-x regime where the gluon occupation number is large.

Shockwave: any number of soft classical gluons exchanged between the projectile and the target:

$$U_{\text{cgc}}(\boldsymbol{x}_\perp) = P\exp\left(ig\int_{-\infty}^{+\infty} dx^- A^{+,a}_{\text{cl}}(x^-,\boldsymbol{x}_\perp)[\rho]t^a\right)$$

The source ρ of $A^{+,a}_{\text{cl}}$ are the target partons having $k^+ > \Lambda^+ = \Lambda P^+$.

Color Glass Condensate effective theory

The cross section is expressed in terms of multipoles averaged over ρ

$$\sigma_{\mathrm{DIS}} = \sum_{n>1} \int |\psi_n|^2_\Lambda \langle \mathscr{O}_n \rangle_\Lambda,$$

with $\mathscr{O}_n$ a sum of products of $0 < m < n^2$ (conjugate) Wilson lines and

$$\langle \mathscr{O} \rangle_\Lambda = \int [\mathscr{D}\rho] W_\Lambda[\rho] \mathscr{O}[\rho].$$

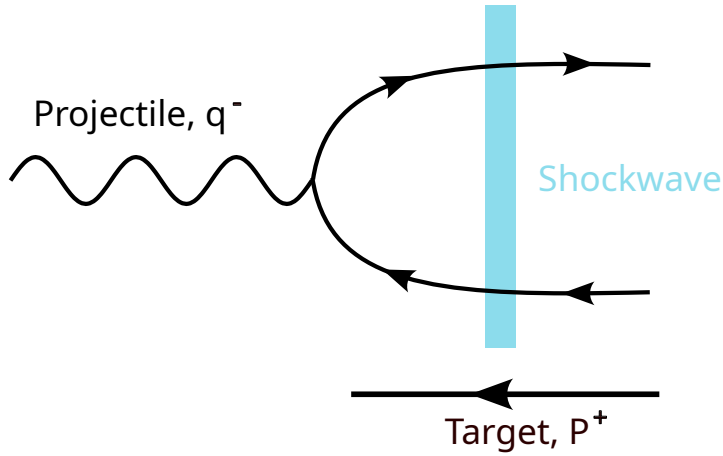
$W_\Lambda[\rho]$ is the weight functional.

The rapidity cutoff Λ is a renormalization scale:

$$\frac{d\sigma}{d\Lambda} = 0$$

$\Rightarrow$ Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, and Kovner (JIMWLK) evolution for $W_\Lambda[\rho]$.

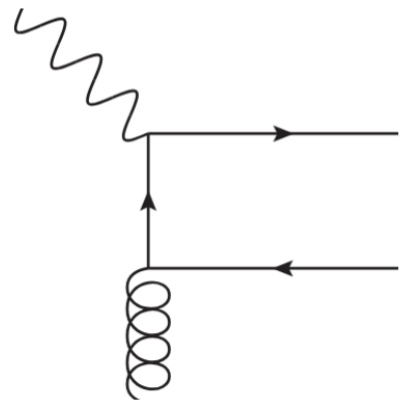
Color Glass Condensate effective theory



- ⋆ In the eikonal CGC, classical gluons have $k^- = 0$ (conservation of the minus component across the shockwave).
- ⋆ It is an extreme case of Glauber gluons, defined here by $k^+ k^- \ll k_\perp^2$.
- ⋆ The CGC resums these contributions.

TMD factorization

- The initial gluon is target-collinear.
- Resummation of soft and collinear gluon exchanges.



This resummation leads to

$$\text{PDF} = \langle P|\text{field } U_c \text{ field}|P\rangle, \quad \text{soft factor} = \langle 0|U_s^\dagger U_s|0\rangle.$$

It cannot resum Glauber gluons:
Their scaling breaks the approximation required to turn the propagator into an eikonal factor.

In DIS, a contour deformation allows to avoid the Glauber region (does not apply to classical gluons).

As they stand, these formalisms cannot be directly compared.

Standard construction of small-x PDFs

1. Dijet cross section:

$$d\sigma^{\gamma^*_{T,L}A\to q\bar{q}X} \propto \int d\Pi \psi\psi^* \left(1 + S^{(4)}_\Lambda - S^{(2)}_\Lambda - S^{(2)}_\Lambda\right) e^{-i\ldots}$$

2. Correlation limit (expansion of $S^{(4)}_\Lambda$) + relation between $\partial_i U_{\text{cgc}}$ and $\partial_i A^+$

$$xG(x,k_\perp) = \int d^3\xi_1 d^3\xi_2 e^{-ik_\perp.(\xi_{1\perp}-\xi_{2\perp})} \left\langle \text{Tr}\, F^{+i}(\xi_1) U^{[+]\dagger}_{\text{cgc}} F^{+i}(\xi_2) U^{[+]}_{\text{cgc}} \right\rangle_{\Lambda=x}$$

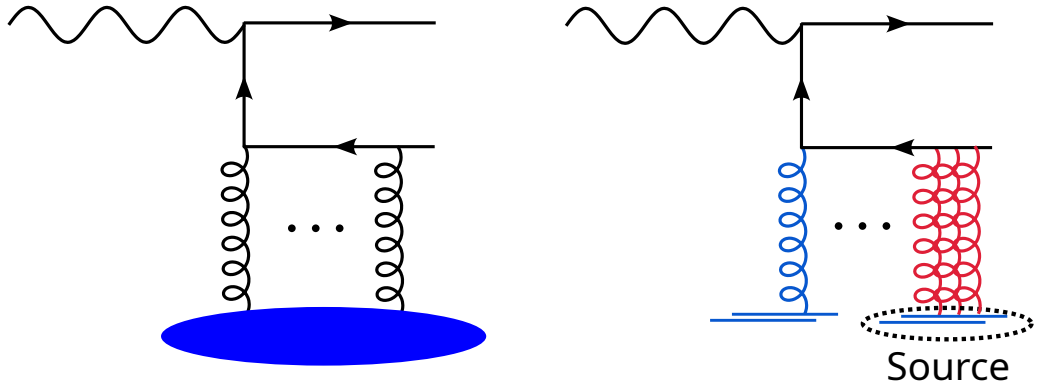
No dependence on x in the rhs without $\Lambda = x$. Very similar to the message of 2604.14049 for $\sigma^{\text{DIS}}(x_b)$.

3. Compare with the TMD gluon density after taking the limit $x \to 0$ in the r.h.s (and no soft factor)

$$xG(x,k_\perp) = \frac{1}{\langle P|P\rangle} \int d^3\xi_1 d^3\xi_2 \, e^{\cancel{ixP^+(\xi_1^- - \xi_2^-)}}$$
$$\times e^{-ik_\perp.(\xi_{1\perp}-\xi_{2\perp})} \langle P| \text{Tr}\, F^{+i}(\xi_1) U^{[-]\dagger} F^{+i}(\xi_2) U^{[-]} |P\rangle$$

4. $\frac{\langle P|\ldots|P\rangle}{\langle P|P\rangle} \longleftrightarrow \langle \ldots \rangle_x$

Missing collinear sector

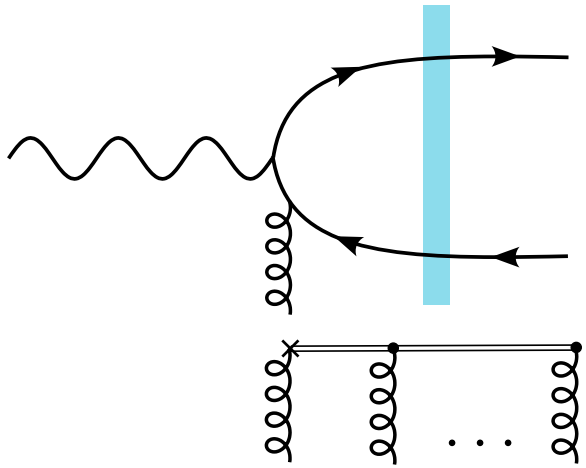


- Gluons integrated out: source of "soft" (classical, Glauber) gluons
- Exchanges not considered: collinear gluons

Collinear exchanges should not be ignored in comparisons with TMD PDFs (or in any observables where they are relevant)

⚠ A collinear parton $[k \sim Q(1, \lambda^2, \lambda)]$ stays collinear in the Breit frame even when $\xi \sim x_b \to 0$.

Proposed modifications



1. Keep the CGC Wilson lines unexpanded $\rightarrow$ "soft" factor
2. Keep one collinear gluon explicit, the others build U_c. We don't want U_{cgc} in the proton matrix element

- Standard factorization tricks apply to collinear gluons.
- In general, the CGC is applied at Q^2 large enough for a hard region to exist (we consider only those cases).

Construction of small-x PDFs

Step one: wave function with collinear insertion:

- Use the CGC Feynman rules
- Calculation performed for a longitudinally polarized photon
- We consider $k_\perp/Q$ small and neglect $k_\perp$ wherever possible

$$|\psi|^2 = 64 z^3 (1-z)^3 (q^-)^2 Q^2 \left[K_0\left(|u|\sqrt{\Delta_1}\right) - K_0\left(|u|\sqrt{\Delta_0}\right)\right] \times \left[K_0^*\left(|u'|\sqrt{\Delta_1}\right) - K_0^*\left(|u'|\sqrt{\Delta_0}\right)\right],$$

with

$$\Delta_1 = z(1-z)Q^2\left(1 - \frac{\xi}{x_b}\right), \quad \Delta_0 = z(1-z)Q^2.$$

The Q^2 dependence cannot be completely factorized, so we define

$$\int dz |\psi|^2 = h(Q^2) c(u, \xi/x_b, Q^2)$$

Construction of small-x PDFs

Step two: upper diagram and the "CGC" cross section:

$p_\perp$: momentum imbalance of the $q\bar{q}$ pair

$P_\perp$: individual quark momentum

After integration over $P_\perp$:

$$\sigma^{\mathrm{CGC}} \propto \int d^2u \left[\int dz \sum_{\lambda\alpha\beta} \left| \psi^{\lambda}_{\alpha\beta}\left(z,u,Q^2,\xi/x_b\right) \right|^2 \right] \int \frac{d^2v}{(2\pi)^2} \frac{d^2v'}{(2\pi)^2} e^{ip_\perp\cdot(v'-v)}$$

$$\times \left[\frac{1}{2N_c} + S^{(4)}_\Lambda - S^{(2)}_\Lambda - S^{(2)}_\Lambda \right]_{u=u'}$$

After the change of variables $v'-v=b$ we obtain

$$\sigma^{\mathrm{CGC}} \propto h(Q^2) \int \frac{d^2b}{(2\pi)^2} e^{ip_\perp\cdot b} \int \frac{d^2u}{(2\pi)^2} c(u,\xi/x_b,Q^2) S_{\mathrm{CGC}}(b,u;\Lambda),$$

with

$$S_{\mathrm{CGC}}(b,u;\Lambda) = \int \frac{d^2v}{(2\pi)^2} \left[\frac{1}{2N_c} + S^{(4)}_\Lambda - S^{(2)}_\Lambda - S^{(2)}_\Lambda \right]_{u=u'}$$

Construction of small-x PDFs

Step three: complete contribution

$$\sigma_{T,L}(x_b, Q^2, p'_\perp) = h_{T,L}(Q^2;\mu) \int d^2b\, e^{-ip'_\perp .b} f(x_b, Q^2, b;\mu,\Lambda)$$

$\vec{p'}_\perp = \vec{k}_\perp + \vec{p}_\perp$ with $\vec{p}_\perp$ from the shockwave, and

$$f(x_b, Q^2, b;\mu,\Lambda) = \int_0^1 \frac{d\xi}{\xi P^+} \int \frac{d^2u}{(2\pi)^2} c(u, \xi/x_b, Q^2) S_{\mathrm{CGC}}(b,u;\Lambda)$$
$$\times \mathrm{Tr} \int \frac{d\omega^-}{2\pi} e^{-i\xi P^+\omega^-} \langle P| F^{+i}(\omega/2) U_{\mathrm{c}}^{[+]\dagger} F^{+i}(-\omega/2) U_{\mathrm{c}}^{[+]} |P\rangle$$

and $\omega = (0, \omega^-, b)$.

It mimics TMD factorization but is fundamentally different, in particular because of the CGC "soft" factor, which is process dependent.

Comments

1. Soft gluons form the CGC soft factor, absent in earlier derivations.
2. Collinear-gluon exchanges are accounted for by collinear Wilson lines in the proton matrix element, similar to TMD factorization.
3. Using $\frac{\langle P|...|P\rangle}{\langle P|P\rangle} \longleftrightarrow \langle ... \rangle_x$ is no longer needed.
4. It possesses both DGLAP and small-x resummations.

Evolution of small-x PDFs with Λ and μ

$$f(x_b, Q^2, b; \mu, \Lambda) = \int_0^1 \frac{d\xi}{\xi P^+} \int \frac{d^2u}{(2\pi)^2} c(u, \xi/x_b, Q^2) S_{\text{CGC}}(b, u; \Lambda)$$

- ⋆ Nothing changes regarding W_Λ and the CGC correlators.
- ⋆ The answer depends on where we choose to put higher-orders dependence on Λ.
- ⋆ TMD-like: $h_{T,L}(Q^2; \mu)$ and $c(u, \xi/x_b, Q^2; \Lambda)$. In that case:

$$\frac{d}{d\Lambda} f(x_b, Q^2, b; \mu, \Lambda) = 0.$$

- ⋆ Evolution with μ: DGLAP-like, from the UV divergences of the proton matrix element.

Evolution of small-x PDFs with x_b?

$$f(x_b, Q^2, b; \mu, \Lambda) = \int_0^1 \frac{d\xi}{\xi P^+} \int \frac{d^2u}{(2\pi)^2} c(u, \xi/x_b, Q^2) S_{\mathrm{CGC}}(b, u; \Lambda)$$

The x_b dependence is not related to small-x evolutions, in particular to BFKL (until you set $\Lambda = x_b$).
In agreement with Collins, Ellis, Nucl. Phys. B 360; Catani, Ciafaloni, Hautmann, Nucl. Phys. B 366; Catani, Hautmann, Nucl. Phys. B 427, where

$$F_N(k_\perp; \mu) = f_N(\mu) C_N(k_\perp, \mu)$$

is the UGD in Mellin space, f_N the collinear PDF, and

$$C_N(k_\perp, \mu) = \frac{\gamma_N(\alpha_s)}{k_\perp^2} \left(\frac{k_\perp^2}{\mu^2} \right)^{\gamma_N(\alpha_s)}$$

the resummation factor, solution of the BFKL equation.

The evolution of $F(x, k_\perp; \mu)$ with x is not only driven by the BFKL equation.

Conclusion

- ⋆ Classical gluons are an extreme case of Glauber gluons.
- ⋆ A meaningful comparison with TMD factorization requires collinear exchanges.
- ⋆ Adding the missing collinear sector leads to a new definition of small-x PDFs with the following features:
  - Can be cast in a TMD-like form but are fundamentally different.
  - Obey DGLAP (in μ) and small-x (in Λ) equations.
  - The x_b part of $f(x_b, Q^2, b; \mu, \Lambda = x_b)$ is not solely driven by small-x evolutions.

Thanks for your attention!