

Soft-Collinear Effective Theory

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Allergic to SCET?

Relief at leading power.

For sudden reactions to:

- Too many modes
- Separate gauge symmetries
- Multipole expansions



Side effects may include factorization and resummation.

PARODY • For theoretical use only.

SCET: what?

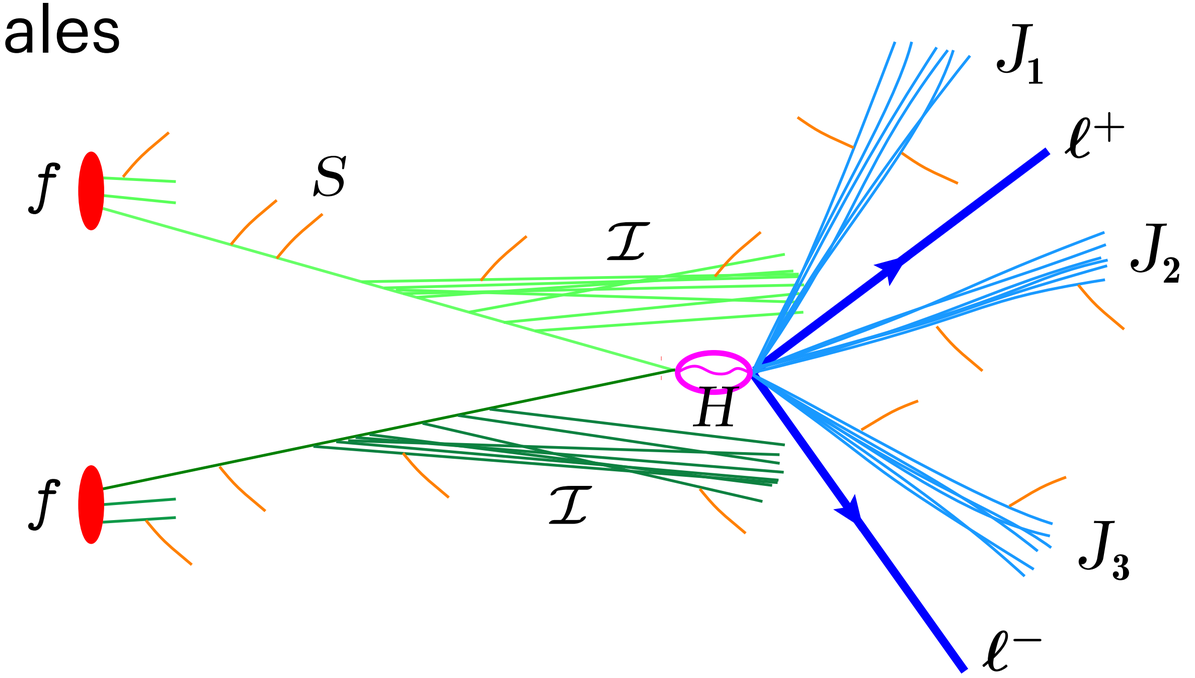
- All LHC predictions rely on factorization

  $d\sigma = f\, f \otimes d\hat{\sigma}$

- Essential to separate nonperturbative.
- SCET factorizes radiation at different scales

  $d\hat{\sigma} = H\, \mathcal{I}\mathcal{I} \otimes S \otimes \prod_i J_i$

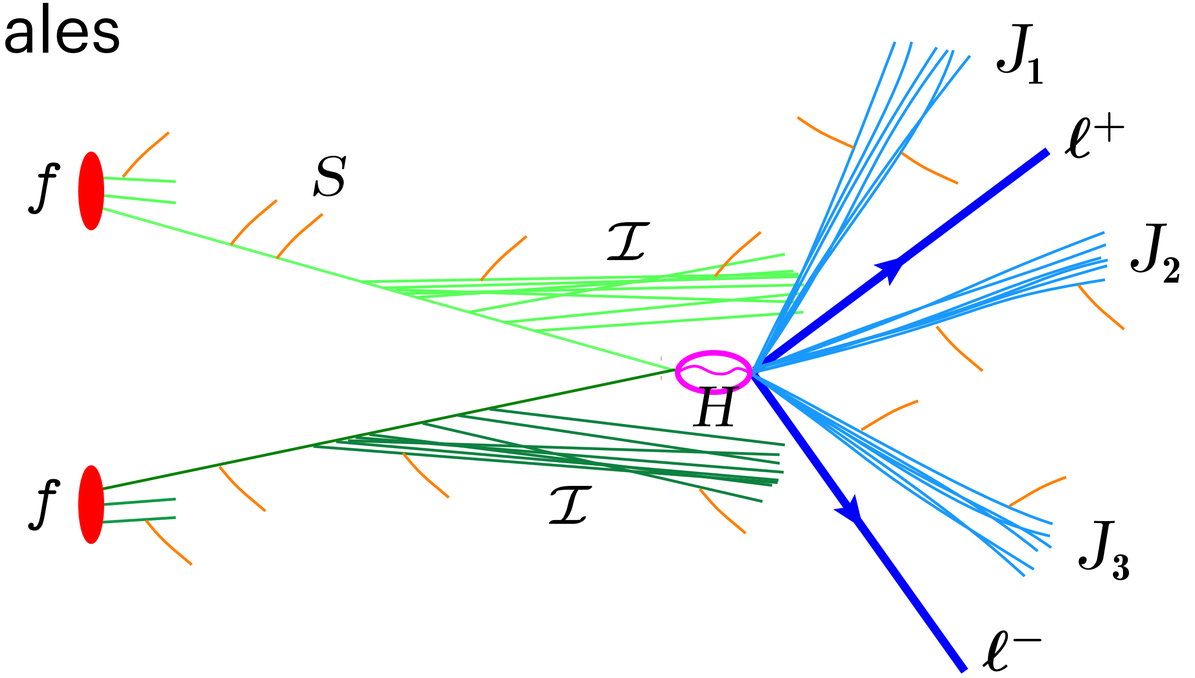
PDFs
Initial-state radiation
Hard scattering
Final-state radiation
Soft radiation



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- Essential to separate nonperturbative.
- SCET factorizes radiation at different scales
  $d\hat{\sigma} = H\, \mathcal{I}\mathcal{I} \otimes S \otimes \prod_i J_i$
- SCET > method of regions.
- Wide range of applications: B physics, event shapes, jet substructure, TMDs, diffraction, slicing...

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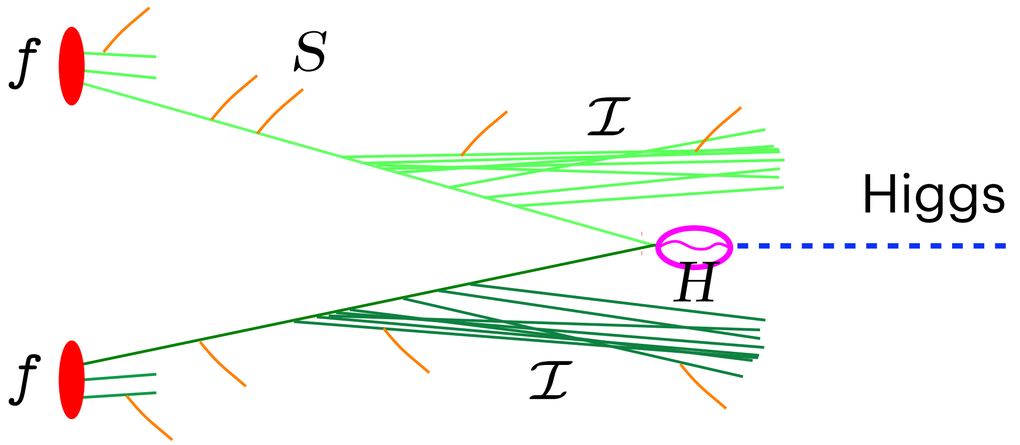


SCET: why?

- Calculations with multiple scales lead to large logarithms. E.g. Higgs with jet veto

$$\hat{\sigma}(H + 0 \text{ jets}) \propto 1 - \frac{6\alpha_s}{\pi} \ln^2 \frac{m_H}{p_T^{\text{cut}}} + \dots$$

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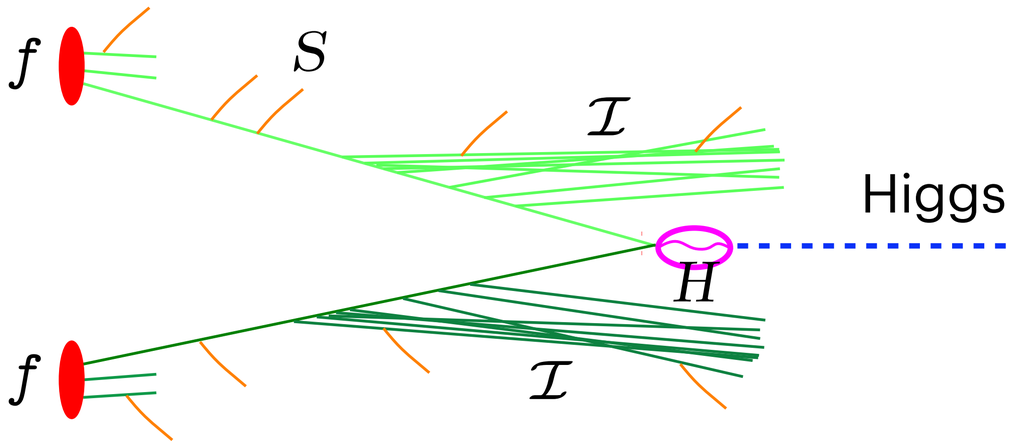
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- Factorization turns this into single scale calculations

$$\hat{\sigma}(H + 0 \text{ jets}) = H(m_H)\, \mathcal{I}(p_T^{\text{cut}})\, \mathcal{I}(p_T^{\text{cut}})\, S(p_T^{\text{cut}})$$

- Resummation by evolving with renormalization group to common scale

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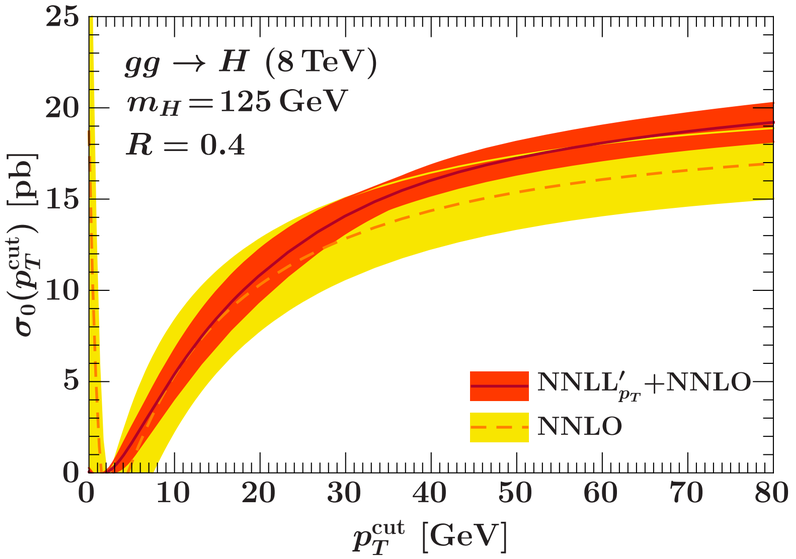
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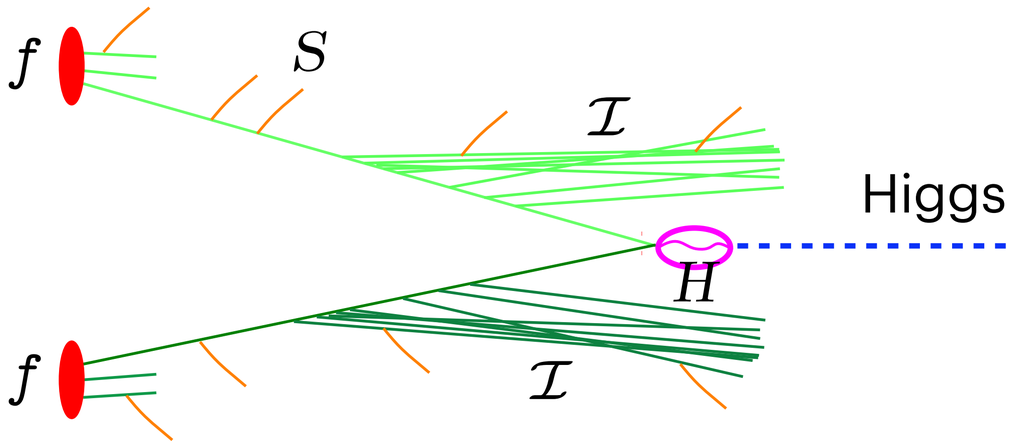
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- Resummation by evolving with renormalization group to common scale

[Stewart, Tackmann, Walsh, Zuberi]





SCET: how?

1. Process & observable
2. Modes & power counting
3. Factorization:
   - all orders in α_s
   - matrix element definition of ingredients.
4. Evolution using renormalization group gives resummation.



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- Will illustrate this for thrust and q_T
- REF → FER 😀

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Classic references by:

Bauer, Fleming, Pirjol, Rothstein and Stewart; Beneke, Chapovsky, Diehl and Feldmann.

Lectures:

Stewart (OpenCourseWare)
Becher (Les Houches)

Book:

Becher, Broggio and Ferroglia

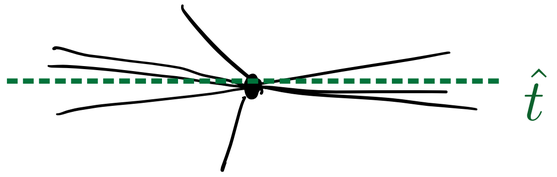
Thrust event shape

- Process & observable: $e^+e^- \to 2$ jets.
- Light-cone coordinates:

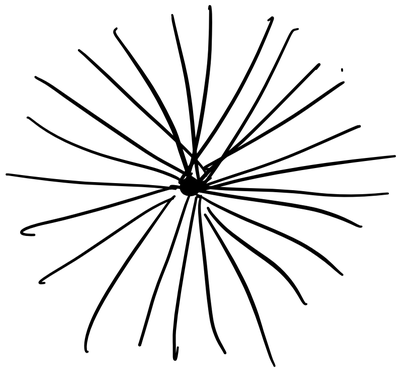
$$p^\mu = \frac{p^- n^\mu}{2} + \frac{p^+ \bar{n}^\mu}{2} + p_\perp^\mu = (p^+, p^-, p_\perp),$$
$$n^\mu = (1,0,0,1), \quad \bar{n}^\mu = (1,0,0,-1).$$

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

$$\tau = 1 - T = \frac{1}{Q} \sum_i \min(p_i^+, p_i^-)$$



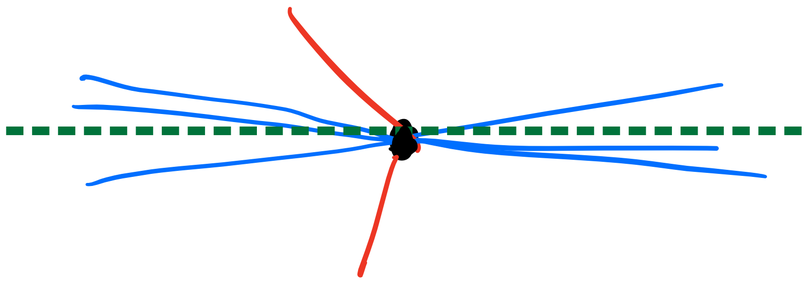
$\tau \ll 1$



$\tau \approx \frac{1}{2}$

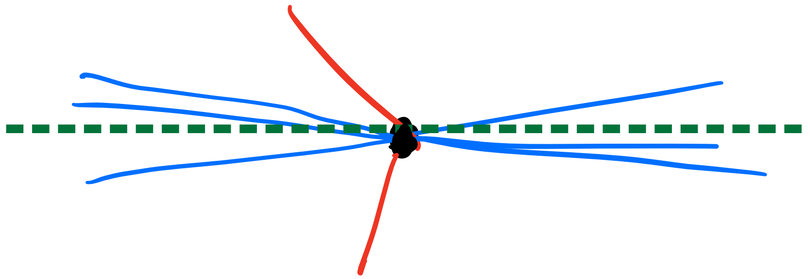
Thrust: modes and power counting

- In light-cone coordinates: $\tau = 1 - T = \frac{1}{Q}\sum_i \min(p_i^+, p_i^-) \ll 1$
- Collinear radiation: $p_n^\mu \sim Q(\lambda^2, 1, \lambda) \to \tau \sim \min(\lambda^2, 1) = \lambda^2$



Thrust: modes and power counting

- In light-cone coordinates: $\tau = 1 - T = \frac{1}{Q}\sum_i \min(p_i^+, p_i^-) \ll 1$
- Collinear radiation: $p_n^\mu \sim Q(\lambda^2, 1, \lambda) \to \tau \sim \min(\lambda^2, 1) = \lambda^2$
- Soft radiation: $p_s^\mu \sim Q(\lambda_s, \lambda_s, \lambda_s) \to \tau \sim \min(\lambda_s, \lambda_s) = \lambda_s \sim \lambda^2$
- EFT philosophy: can add modes as long as overlap is subtracted (zero bin).



SCET Lagrangian

- $\mathcal{L} = \mathcal{L}_{\rm SCET} + \mathcal{L}_{\rm hard}$
  - $\mathcal{L}_{\rm hard}$ encodes hard process with virtuality Q^2
  - $\mathcal{L}_{\rm SCET}$ describes dynamics of SCET fields.

Focus on collinear quark:

$$\psi_n = \xi_n + \eta_n, \qquad \xi_n = \frac{\not{n}\bar{\not{n}}}{4}\psi_n, \qquad \eta_n = \frac{\bar{\not{n}}\not{n}}{4}\psi_n$$

$$\mathcal{L}_\xi = \bar{\xi}_n \frac{\bar{\not{n}}}{2}\, in\cdot D\, \xi_n + \bar{\eta}_n \frac{\not{n}}{2}\, i\bar{n}\cdot D\, \eta_n + \bar{\xi}_n i\not{D}_\perp \eta_n + \bar{\eta}_n i\not{D}_\perp \xi_n.$$

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- Power counting fields:

  $\mathrm{d}^4x \sim \lambda^{-4} \to \xi \sim \lambda, \quad \eta \sim \lambda^2.$

- Similarly, $A_s^\mu \sim \lambda^2, q_s \sim \lambda^3$

  $A_n^\mu \sim (\lambda^2, 1, \lambda)$

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Collinear quark Lagrangian

$$\mathcal{L}_\xi = \bar{\xi}_n \frac{\bar{n}\!\!\!/}{2}\, in\cdot D\, \xi_n + \bar{\eta}_n \frac{n\!\!\!/}{2}\, i\bar{n}\cdot D\, \eta_n + \bar{\xi}_n i D\!\!\!/_\perp \eta_n + \bar{\eta}_n i D\!\!\!/_\perp \xi_n .$$

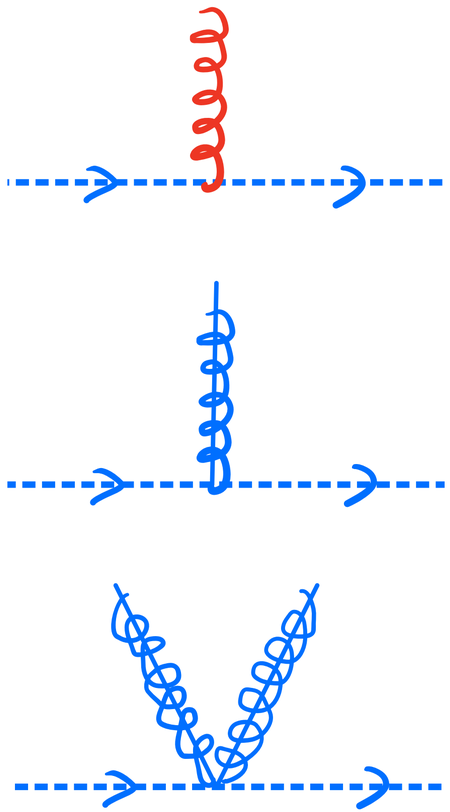
- Integrate out suppressed η_n with equation of motion:

$$\eta_n = \frac{1}{i\bar{n}\cdot D}\, iD\!\!\!/_\perp \frac{\bar{n}\!\!\!/}{2}\, \xi_n \sim \lambda\, \xi_n$$

$$\mathcal{L}_\xi = \bar{\xi}_n \left[in\cdot D + iD\!\!\!/_\perp \frac{1}{i\bar{n}\cdot D} iD\!\!\!/_\perp \right] \frac{\bar{n}\!\!\!/}{2}\, \xi_n$$

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- The soft gluon only enters through

  $in\cdot D = in\cdot\partial + g\, n\cdot A_n + g\, n\cdot A_s$

  and is suppressed in the other components of D.

- Alternative derivation uses gauge & reparametrization invariance (later).

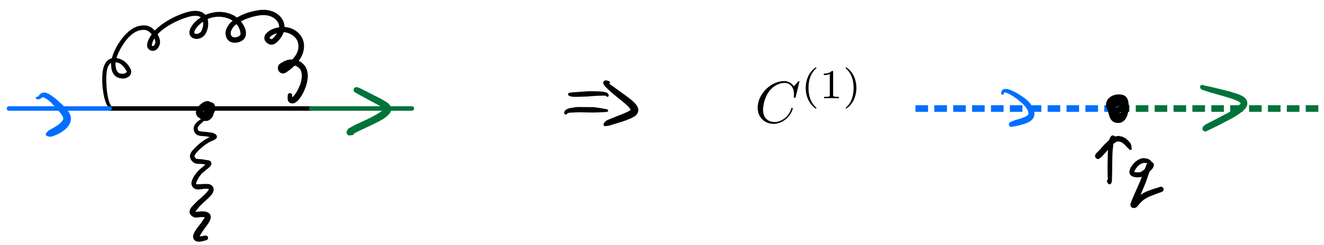
Hard matching

- Integrate out interactions between n and $\bar{n}$-collinear, since $(p_n + p_{\bar{n}})^2 \sim Q^2$

  $$J^\mu_{\rm QCD} = \bar{\psi}\gamma^\mu\psi = (\bar{\xi}_{\bar{n}} + \bar{\eta}_{\bar{n}})\left(n^\mu\frac{\bar{\not{n}}}{2} + \bar{n}^\mu\frac{\not{n}}{2} + \gamma^\mu_\perp\right)(\xi_n + \eta_n) \quad \to \quad J^\mu_{\rm SCET} = \bar{\xi}_{\bar{n}}\gamma^\mu_\perp\xi_n$$

  since $\not{n}\xi_n = 0 = \bar{\xi}_{\bar{n}}\bar{\not{n}}$

- Beyond tree-level, $J^\mu_{\rm SCET} = C(Q^2,\mu)\,\bar{\xi}_{\bar{n}}W_{\bar{n}}\gamma^\mu_\perp W^\dagger_n\xi_n$



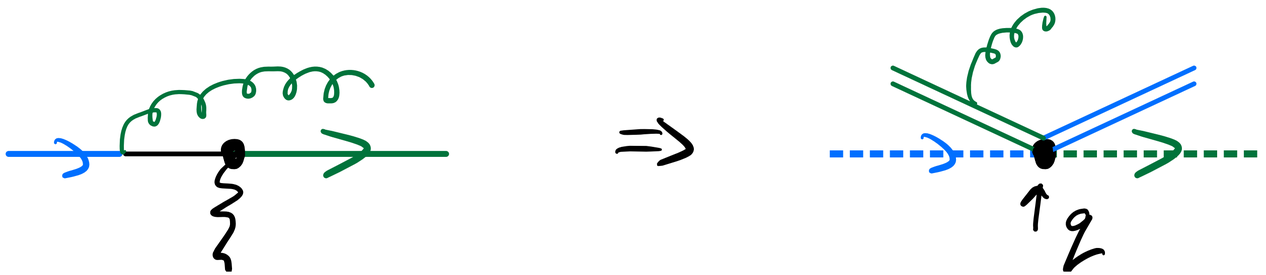
Collinear gauge invariance

- Beyond tree-level, $J^\mu_{\rm SCET} = C(Q^2,\mu)\,\bar\xi_{\bar n} W_{\bar n}\gamma^\mu_\perp W^\dagger_n \xi_n \equiv C(Q^2,\mu)\,\bar\chi_{\bar n}\gamma^\mu_\perp \chi_n$
- For longitudinal gluons, $A_n^- \sim \lambda^0$, emissions build Wilson lines:

$$W_n(x) = P\exp\left[ig\int_{-\infty}^0 ds\,\bar n\cdot A_n(x+s\bar n)\right]$$

- Structure dictated by n-collinear gauge invariance:

$$\xi_n \to U_n\xi_n, \quad W_n \to U_n W_n, \quad \bar\xi_{\bar n} \to \bar\xi_{\bar n}$$



collinear Wilson lines along opposite direction,
can't draw them on top of (anti)quark

Soft decoupling

- Expanding soft field in covariant derivative:

$$in\cdot D = in\cdot\partial + g\,n\cdot A_n(x) + g\,n\cdot A_s(x^-)\,,\quad x_\mu^- = (\bar n\cdot x)\frac{n_\mu}{2}$$

- Use a soft Wilson line:

$$Y_n(x) = \overline{P}\exp\left[-ig\int_0^\infty ds\,n\cdot A_s(x+sn)\right],\quad [in\cdot\partial + g\,n\cdot A_s(x)]Y_n(x) = 0,$$

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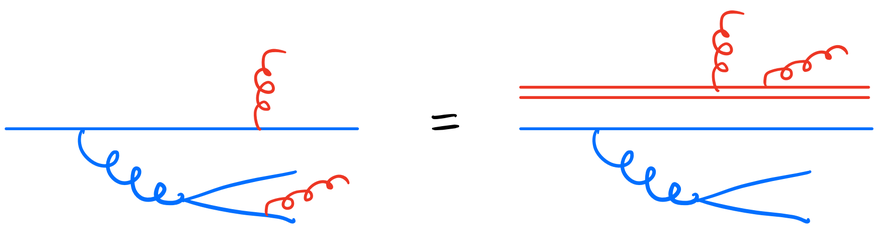
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- We can decouple the soft and collinear through a field redefinition:

$$\xi_n(x) = Y_n(x_-)\xi_n^{(0)}(x),\qquad A_n^\mu(x) = Y_n(x_-)A_n^{(0)\mu}(x)Y_n^\dagger(x_-)$$

[Bauer, Pirjol, Stewart]

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- Soft decouples in $\mathcal{L}_\xi \ni \bar\xi_n \not{\bar n}\xi_n \to \bar\xi_n^{(0)} Y_n^\dagger \not{\bar n} Y_n \xi_n^{(0)} = \bar\xi_n^{(0)}\not{\bar n}\xi_n^{(0)}$ since $Y_n^\dagger Y_n = 1$.

Thrust: factorization

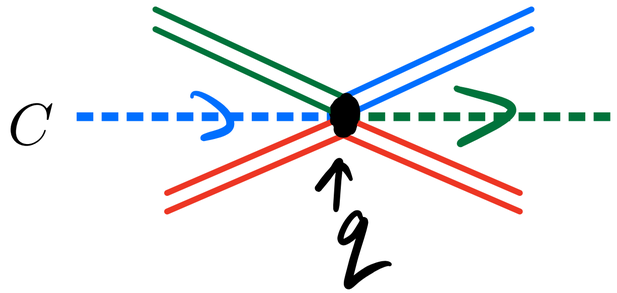
- Soft decouples in $\mathcal{L}_{\rm SCET}$, which enters often, justifying

  $|X\rangle = |X_n\rangle \otimes |X_{\bar n}\rangle \otimes |X_s\rangle$

- It does not in the hard scattering, which fortunately is only needed once:

  $\mathcal{L}_{\rm hard} \ni \bar\chi_{\bar n} \gamma_\perp^\mu \chi_n \to \bar\chi_{\bar n} Y_{\bar n}^\dagger \gamma_\perp^\mu Y_n \chi_n, \quad Y_{\bar n}^\dagger Y_n \neq 1$

- This leads to $\dfrac{d\sigma}{d\tau} \sim \displaystyle\int d\Phi_X \, |C \, \langle X_n| \otimes \langle X_{\bar n}| \otimes \langle X_s| \bar\chi_{\bar n} \gamma_\perp^\mu Y_{\bar n}^\dagger Y_n \chi_n |0\rangle|^2 \delta(\tau - \tau[X])$



soft Wilson lines along same direction

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- Thrust is additive: $\tau[X] = \dfrac{s_n + s_{\bar n}}{Q^2} + \dfrac{k}{Q},$

$$s_n = p_{X_n}^2, \quad s_{\bar n} = p_{X_{\bar n}}^2, \quad k = \sum_{i \in X_s} \min(k_i^+, k_i^-).$$

- The rest is Fierzing and rearranging...

Thrust: factorization

- Factorization at leading power: [Bauer, Fleming, Lee, Sterman; Becher Schwartz]

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu)\int ds_n\, ds_{\bar n}\, dk\, J(s_n,\mu)\, J(s_{\bar n},\mu)\, S_\tau(k,\mu)\,\delta\Big(\tau - \frac{s_n+s_{\bar n}}{Q^2} - \frac{k}{Q}\Big)$$

- Convolutions can be eliminated by taking the Laplace transform.
- Factorization separates physical scales. At order α_s

$$\ln^2\tau = 2\ln^2\Big(\frac{Q}{\mu}\Big) - 4\ln^2\Big(\frac{\sqrt{\tau}Q}{\mu}\Big) + 2\ln^2\Big(\frac{\tau Q}{\mu}\Big)$$

- Evaluate each ingredient at its natural scale

$$\mu_H = Q, \quad \mu_J = \sqrt{\tau}Q, \quad \mu_S = \tau Q$$

Thrust: resummation

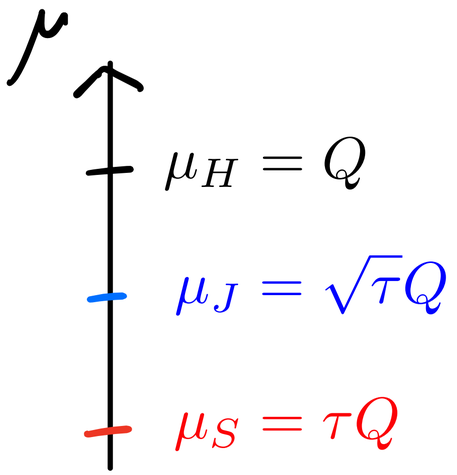
- Hard function contains no large logs when evaluated at natural scale:

$$H(Q^2,\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-L^2 + 3L - 8 + \frac{7\pi^2}{6}\right] + \mathcal{O}(\alpha_s^2), \quad L = \ln\frac{Q^2}{\mu^2}$$

- Use renormalization group to evolve H, J, S to a common scale. E.g.

$$\frac{dH}{d\ln\mu} = \left[2\Gamma^q_{\rm cusp}\ln\frac{Q^2}{\mu^2} + \gamma_H\right] H,$$

$$H(Q^2,\mu) = U_H(Q\to\mu)\, H(Q^2,Q),$$



Thrust: resummation

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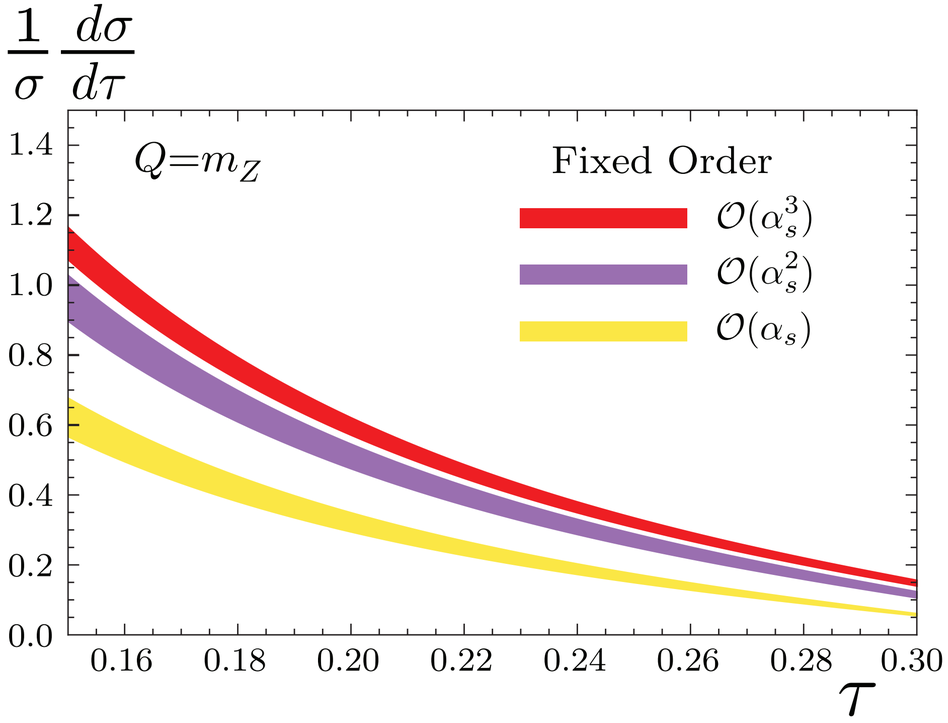
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$$H(Q^2,\mu) = U_H(Q\to\mu)\, H(Q^2,Q), \qquad U_H \overset{\rm LL}{=} \exp\left[-\frac{\alpha_s C_F}{2\pi} L^2\right]$$

- Leading logarithmic expression highlights the exponentiation.
- Cross section is μ independent → consistency of anomalous dimensions.

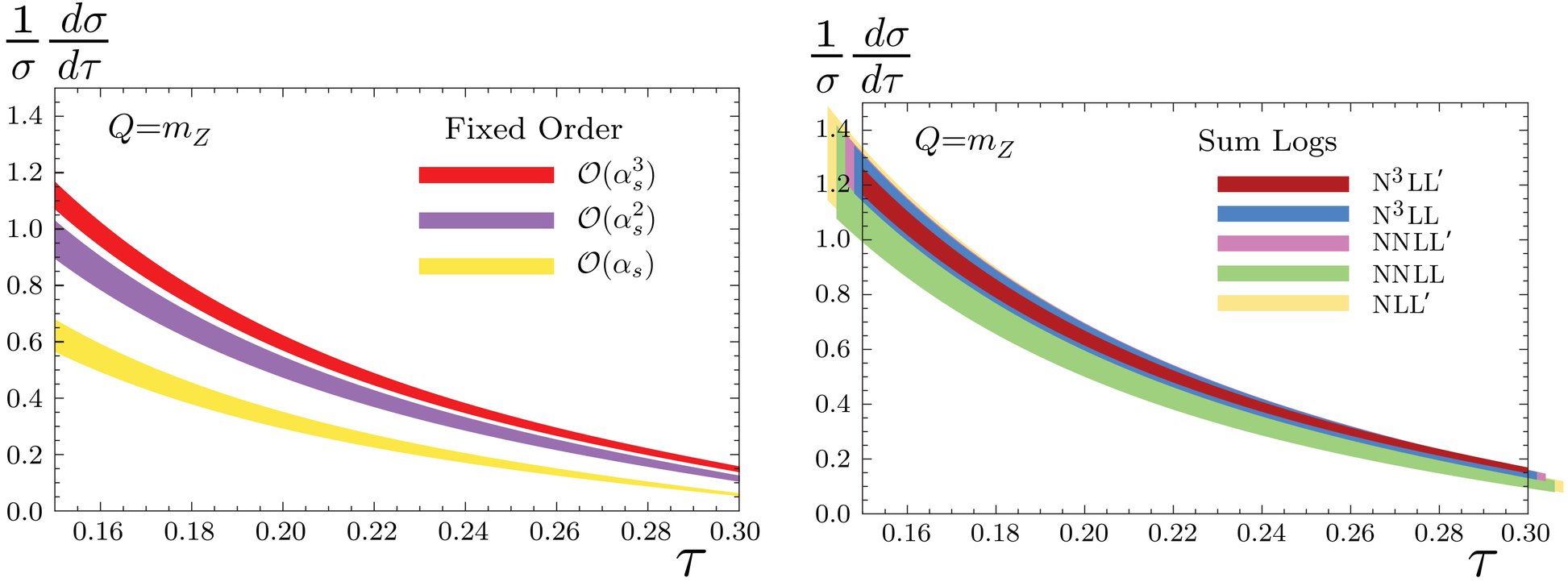
Thrust: resummation results



[Abbate et al]

- Scale variations in fixed-order underestimate the uncertainty.

Thrust: resummation results



[Abbate et al]

- Scale variations in fixed-order underestimate the uncertainty.
- Resummation greatly improves the perturbative convergence.

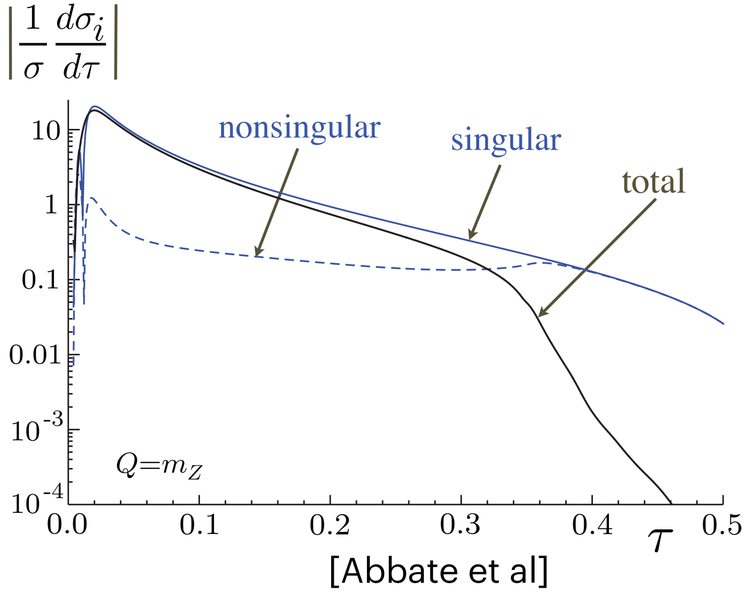
Thrust: fixed-order matching

- Cross section only contains terms singular in τ, add the rest by matching

$$\frac{d\sigma^{\rm matched}}{d\tau} = \frac{d\sigma^{\rm resum}}{d\tau} + \underbrace{\frac{d\sigma^{\rm FO}}{d\tau} - \left[\frac{d\sigma^{\rm resum}}{d\tau}\right]_{\rm expanded\ to\ FO}}_{\text{nonsingular}}$$

singular

- Cancellation between singular and nonsingular requires turning resummation off for large τ.



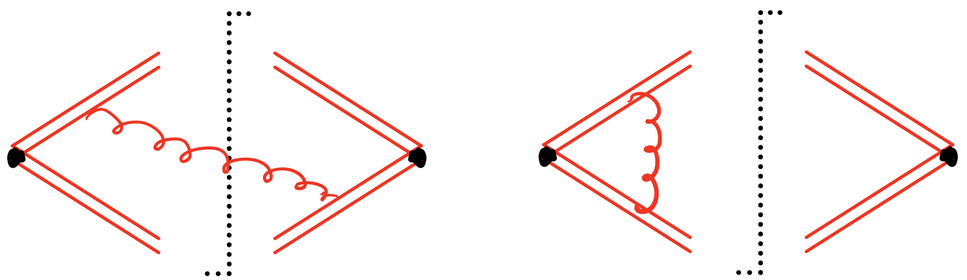
[Abbate et al]

Operator definitions of ingredients

- Soft function:

$$S_\tau(k) = \frac{1}{N_c} \sum_{X_s} \mathrm{tr}\left[\langle 0|\bar{T}\{Y_{\bar{n}}^\dagger Y_n\}|X_s\rangle\langle X_s|T\{Y_n^\dagger Y_{\bar{n}}\}|0\rangle\right]\delta\big(k - k(X_s)\big)$$

- One-loop diagrams:

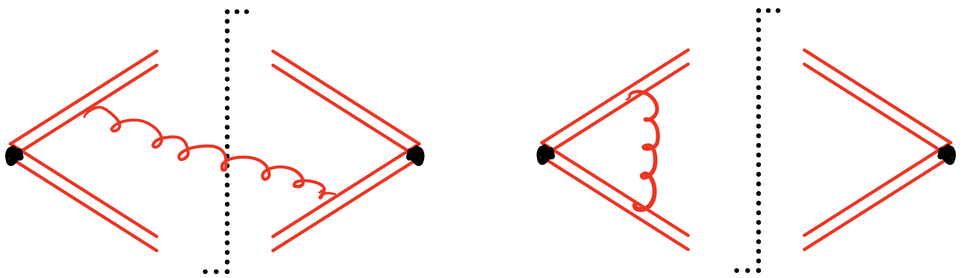


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- One-loop diagrams:



- Note: after factorization we can use QCD instead of SCET Feynman rules.
- For thrust, all ingredients are IR finite. Using dim. reg., virtual graphs convert IR in UV poles.

Thrust: nonperturbative corrections

- Soft function has lowest scale, so its nonperturbative effects dominate

$$S_\tau(k,\mu) = \int d\ell\, S_\tau^{\rm pert}(k-\ell,\mu) F_\tau(\ell), \quad \int d\ell\, F_\tau(\ell) = 1, \quad \Omega_\tau \equiv \int d\ell\, \ell F_\tau(\ell) \sim \Lambda_{\rm QCD}$$

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- Expanding in $\dfrac{\Lambda_{\rm QCD}}{\tau Q}$, we obtain $F_\tau(\ell) = \delta(\ell) - \Omega_\tau\, \delta'(\ell)$ [Lee, Sterman]
- This leads to a shift in τ of the spectrum:

$$S_\tau(k,\mu) = S_\tau^{\rm pert}(k,\mu) - \Omega_\tau \frac{d}{dk} S_\tau^{\rm pert}(k,\mu) = S_\tau^{\rm pert}(k-\Omega_\tau,\mu)$$

- Caveat: hadron mass scheme, only holds strictly in 2-jet region of τ.
  [Nason, Zanderighi]

Reparametrization Invariance

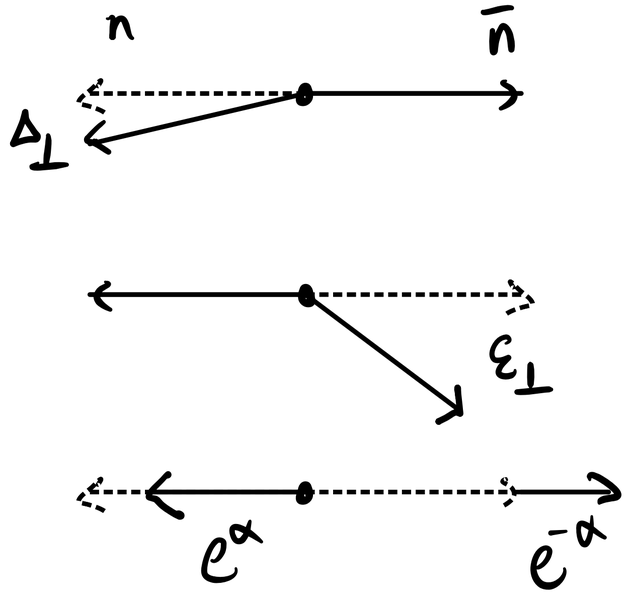
- Other choices for $n, \bar{n}$ keep $n^2 = \bar{n}^2 = 0,\ n \cdot \bar{n} = 2$ and power counting.

[Manohar, Mehen, Pirjol, Stewart]

I. $n \to n + \Delta_\perp,$

II. $\bar{n} \to \bar{n} + \epsilon_\perp,$ Infinitesimal version

III. $n \to e^{\alpha} n, \quad \bar{n} \to e^{-\alpha} \bar{n},$



Reparametrization Invariance

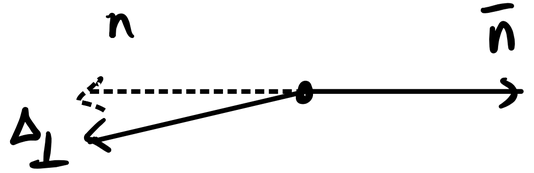
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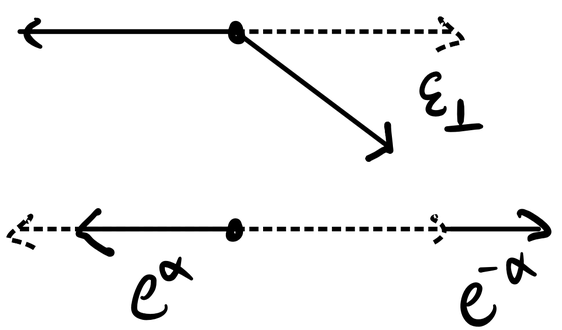
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- Relates for example the two terms in $\mathcal{L}_\chi$:

$$\delta_{\rm I}\left(\bar{\xi}_n\, in\cdot D\,\frac{\bar{n}\!\!\!/}{2}\,\xi_n\right) = +\bar{\xi}_n\, i\Delta_\perp\cdot D_\perp\frac{\bar{n}\!\!\!/}{2}\,\xi_n$$

$$\delta_{\rm I}\left(\bar{\xi}_n\, iD\!\!\!/_\perp\frac{1}{i\bar{n}\cdot D}iD\!\!\!/_\perp\,\frac{\bar{n}\!\!\!/}{2}\,\xi_n\right) = -\bar{\xi}_n\, i\Delta_\perp\cdot D_\perp\frac{\bar{n}\!\!\!/}{2}\,\xi_n$$

Drell-Yan qT: modes and power counting

- $\vec{q}_T = -\sum_{i \neq Z} \vec{p}_{T,i}$
- Collinear radiation: $p_n^\mu \sim Q(\lambda^2, 1, \lambda) \to q_T \sim \lambda Q$
- Soft radiation: $p_s^\mu \sim Q(\lambda_s, \lambda_s, \lambda_s) \to q_T \sim \lambda_s Q \to \lambda_s \sim \lambda$

Mode	$\mathrm{SCET_I}$	$\mathrm{SCET_{II}}$
n-collinear	$(\lambda^2, 1, \lambda)$	$(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$(1, \lambda^2, \lambda)$	$(1, \lambda^2, \lambda)$
soft	$(\lambda^2, \lambda^2, \lambda^2)$	$(\lambda, \lambda, \lambda)$

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n-collinear	$(\lambda^2, 1, \lambda)$	$(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$(1, \lambda^2, \lambda)$	$(1, \lambda^2, \lambda)$
soft	$(\lambda^2, \lambda^2, \lambda^2)$	$(\lambda, \lambda, \lambda)$

- Collinear and soft no longer interact, because $(p_n + p_s)^2 \sim \lambda Q^2 \gg \lambda^2 Q^2$
- Same Wilson line structure from $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$ matching.

Rapidity divergences

- Collinear and soft no longer separated in invariant mass.
- Leads to rapidity divergences. E.g. one-loop soft function

$$S^{(1)}(\boldsymbol{k}_T) \propto \alpha_s \int d^4k\, \theta(k^0)\delta(k^2) \frac{\delta^2(\boldsymbol{k}_T - \boldsymbol{k}_\perp)}{k^+k^-} = \frac{\alpha_s}{\boldsymbol{k}_T^2} \int \frac{dk^+}{k^+} = \frac{\alpha_s}{\boldsymbol{k}_T^2} \int dy$$

where $k^+ = k_T\, e^{-y}$

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- Rapidity regulators add $e^{-\eta|y|}$, making the integral convergent [See Scimemi's talk]
- Other example: electroweak Sudakov [Jenkins, Manohar et al].

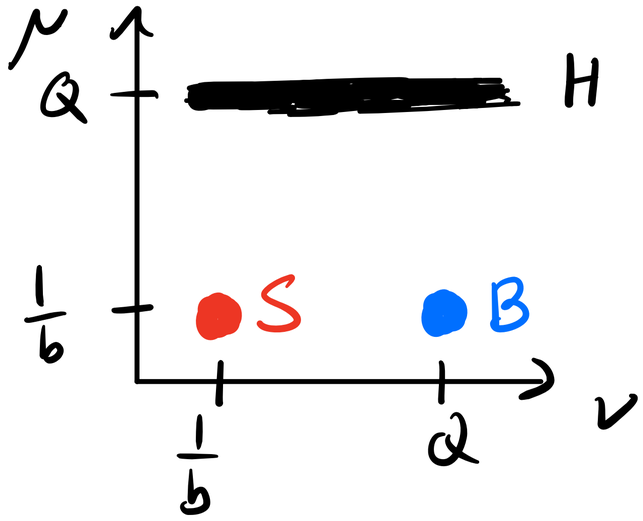
Drell-Yan qT: factorization and resummation

$$\frac{d\sigma}{dQ^2\,dY\,d^2\boldsymbol{q}_T} = H(Q^2,\mu)\int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, B_n(x_1,b_T,\mu,\nu)\, B_{\bar{n}}(x_2,b_T,\mu,\nu)\, S(b_T,\mu,\nu)$$

[Becher, Neubert; Echevarria, Idilbi, Scimemi; Chiu, Jain, Neill, Rothstein]

- Natural scales:

$$\mu_H = Q, \quad \mu_B = \mu_S = \frac{b_0}{b_T}, \qquad \nu_B = Q, \quad \nu_S = \mu_S$$



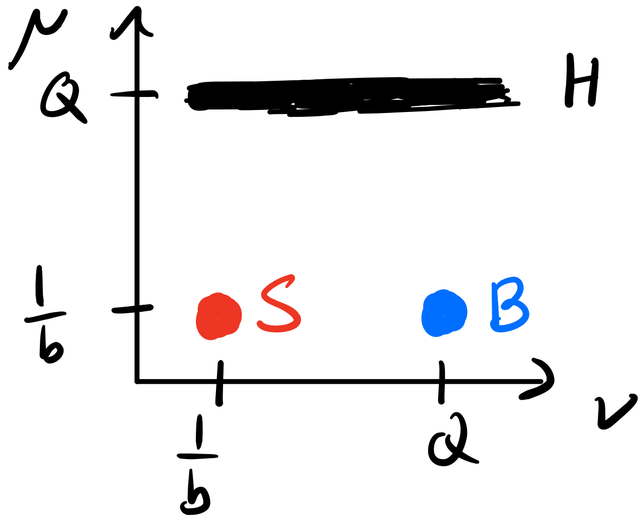
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$$\mu_H = Q, \quad \mu_B = \mu_S = \frac{b_0}{b_T}, \qquad \nu_B = Q, \quad \nu_S = \mu_S$$



- Renormalization group equations

$$\frac{d\ln\widetilde{F}}{d\ln\mu} = \gamma^F_\mu, \qquad \frac{d\ln\widetilde{F}}{d\ln\nu} = \gamma^F_\nu$$

- Resummation is (formally) path independent:

$$\frac{d\gamma^F_\mu}{d\ln\nu} = \frac{d\gamma^F_\nu}{d\ln\mu}$$

Joint measurements: qT and (beam) thrust

Regime	$\mathrm{SCET_I}$ $\mathcal{T} \ll q_T \sim \sqrt{Q\mathcal{T}}$		
n-collinear	$(\mathcal{T}, Q, \sqrt{Q\mathcal{T}})$		
$\bar{n}$-collinear	$(Q, \mathcal{T}, \sqrt{Q\mathcal{T}})$		
n-collinear-soft	–		
$\bar{n}$-collinear-soft	–		
soft	$(\mathcal{T}, \mathcal{T}, \mathcal{T})$		

- (Beam) thrust $\mathcal{T} = \tau Q$

[Procura, WW, Zeune; see talks by Fernandez and Shao]

Joint measurements: qT and (beam) thrust

Regime	$\mathrm{SCET_I}$ $\mathcal{T} \ll q_T \sim \sqrt{Q\mathcal{T}}$		$\mathrm{SCET_{II}}$ $\mathcal{T} \sim q_T \ll Q$
n-collinear	$(\mathcal{T}, Q, \sqrt{Q\mathcal{T}})$		$(q_T^2/Q, Q, q_T)$
$\bar{n}$-collinear	$(Q, \mathcal{T}, \sqrt{Q\mathcal{T}})$		$(Q, q_T^2/Q, q_T)$
n-collinear-soft	–		–
$\bar{n}$-collinear-soft	–		–
soft	$(\mathcal{T}, \mathcal{T}, \mathcal{T})$		(q_T, q_T, q_T)

- (Beam) thrust $\mathcal{T} = \tau Q$

[Procura, WW, Zeune; see talks by Fernandez and Shao]

- $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$ near phase-space boundaries

Joint measurements: qT and (beam) thrust

Regime	$\mathrm{SCET_I}$ $\mathcal{T} \ll q_T \sim \sqrt{Q\mathcal{T}}$	$\mathrm{SCET_+}$ $\mathcal{T} \ll q_T \ll \sqrt{Q\mathcal{T}}$	$\mathrm{SCET_{II}}$ $\mathcal{T} \sim q_T \ll Q$
n-collinear	$(\mathcal{T}, Q, \sqrt{Q\mathcal{T}})$	$(q_T^2/Q, Q, q_T)$	$(q_T^2/Q, Q, q_T)$
$\bar{n}$-collinear	$(Q, \mathcal{T}, \sqrt{Q\mathcal{T}})$	$(Q, q_T^2/Q, q_T)$	$(Q, q_T^2/Q, q_T)$
n-collinear-soft	–	$(\mathcal{T}, q_T^2/\mathcal{T}, q_T)$	–
$\bar{n}$-collinear-soft	–	$(q_T^2/\mathcal{T}, \mathcal{T}, q_T)$	–
soft	$(\mathcal{T}, \mathcal{T}, \mathcal{T})$	$(\mathcal{T}, \mathcal{T}, \mathcal{T})$	(q_T, q_T, q_T)

- (Beam) thrust $\mathcal{T} = \tau Q$

[Procura, WW, Zeune; see talks by Fernandez and Shao]

- $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$ near phase-space boundaries, with $\mathrm{SCET_+}$ interpolating

Matching with SCET+

- Matching $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$ onto $\mathrm{SCET_+}$ gives, up to power corrections,

$$B(t, x, \vec{k}_\perp, \mu) = \int \mathrm{d}^2\vec{k}'_\perp \, B_i(x, \vec{k}'_\perp, \mu, \nu) \, \mathscr{S}\big(t/p^-, \vec{k}_\perp - \vec{k}'_\perp, \mu, \nu\big),$$

$$S(k^+, \vec{k}_\perp, \mu, \nu) = \int \mathrm{d}^2\vec{k}'_\perp \int \mathrm{d}k'^+ \, \mathrm{d}k''^+ \, \mathscr{S}(k'^+, \vec{k}'_\perp, \mu, \nu) \, \mathscr{S}\big(k''^+, \vec{k}_\perp - \vec{k}'_\perp, \mu, \nu\big) \, S(k^+ - k'^+ - k''^+, \mu).$$

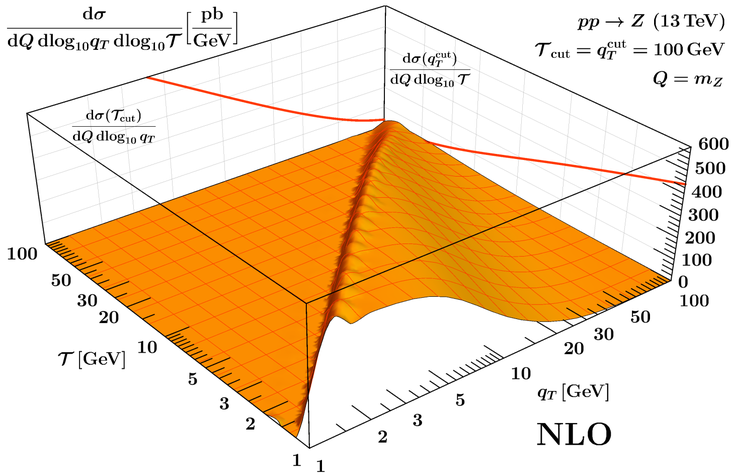
Matching with SCET+

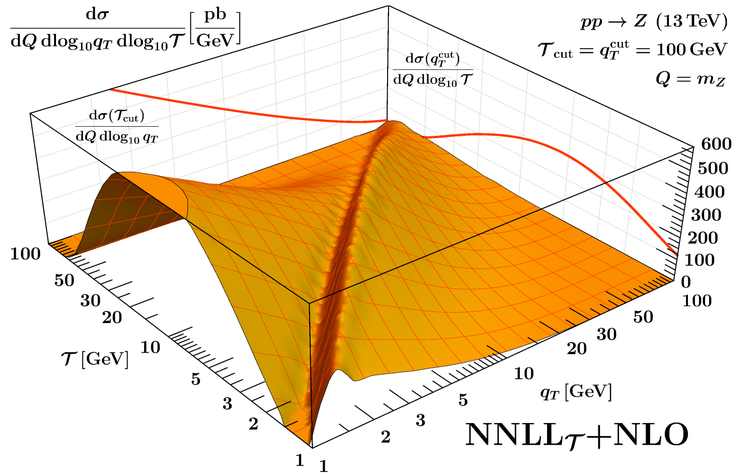
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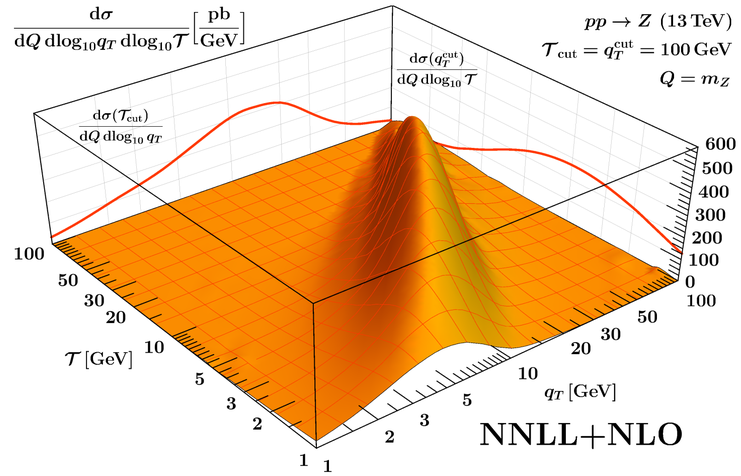
$$B(t, x, \vec{k}_\perp, \mu) = \int \mathrm{d}^2\vec{k}'_\perp \, B_i(x, \vec{k}'_\perp, \mu, \nu) \, \mathscr{S}\big(t/p^-, \vec{k}_\perp - \vec{k}'_\perp, \mu, \nu\big),$$

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- Fixed-order vs. single resummation, vs. double resummation:





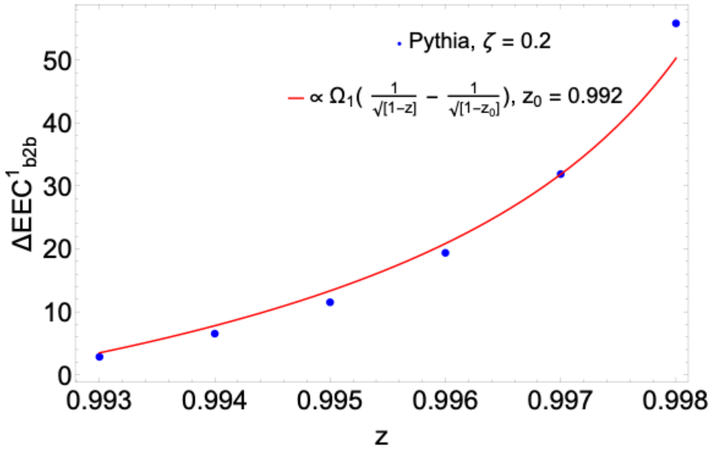


[Lustermans, Michel, Tackmann, WW]

SCET: what's next?

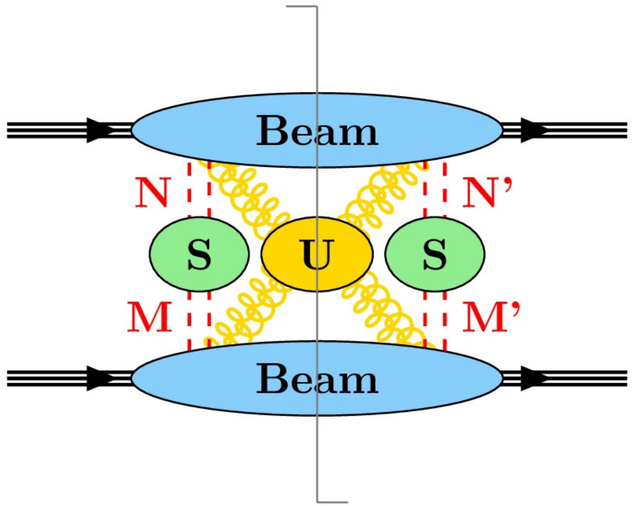
Selection of topics from SCET workshop 2026

- Heavy flavor: B physics, quarkonia and top.
- Non-global logarithms: beyond LL.
- TMDs: beyond leading power.
- Energy correlators: a canvas for new ideas.
- Nonperturbative corrections.
- Glauber exchange: Regge and diffraction.



Nonperturbative from higher-point back-to-back energy correlators

[Talk by Jiang]



Factorization for (double) diffractive pp

[Talk by Aretz]

Conclusions

- SCET is an EFT with collinear and soft degrees of freedom:
  - Process and observable determine the modes and power counting.
  - Hard fluctuations are integrated out.
  - Field redefinitions decouple soft interactions at leading power.
- Factorization enables resummation:
  - Evaluate each ingredient at its natural scale and use RG evolution.
- Wide range of applications, with ongoing developments in resummation, evolution and factorization.

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Thank you!