Recent progress in Generalized Parton Distributions

- ❑ GPDs – Correlations between hadron properties and parton spatial distributions
  – "See" hadron internal structure without breaking it – Hadron Tomography
- ❑ Recent progresses:
  – The x-dependence of GPDs, Shadowed GPDs, SDHEPs & new processes
  – Global QCD analysis for extracting GPDs – Uniqueness of GPD definitions
  – Contribution from Lattice QCD
  – Hadron properties (gravitational form factors) from x-moments of GPDs
- ❑ Summary and Outlook

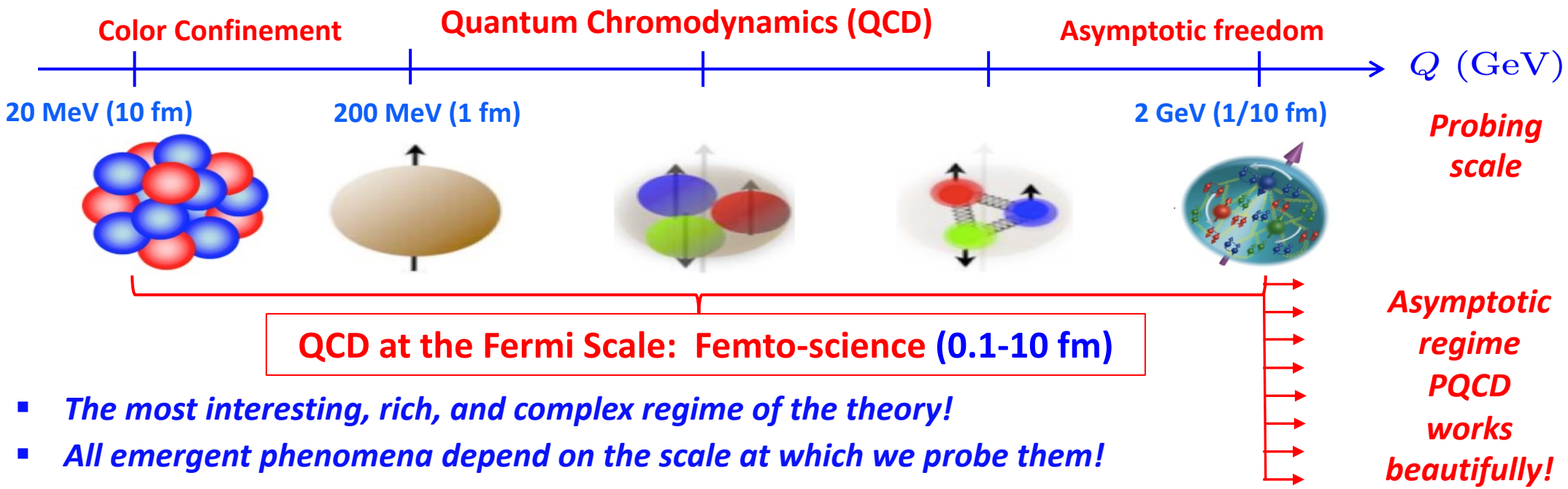
In collaboration with
N. Sato, T. Uji, Z. Yu, Y. Zeng, …

Jianwei Qiu, Theory Center

Jefferson Lab

U.S. DEPARTMENT of ENERGY

QCD and hadron structure at the Fermi-Scale

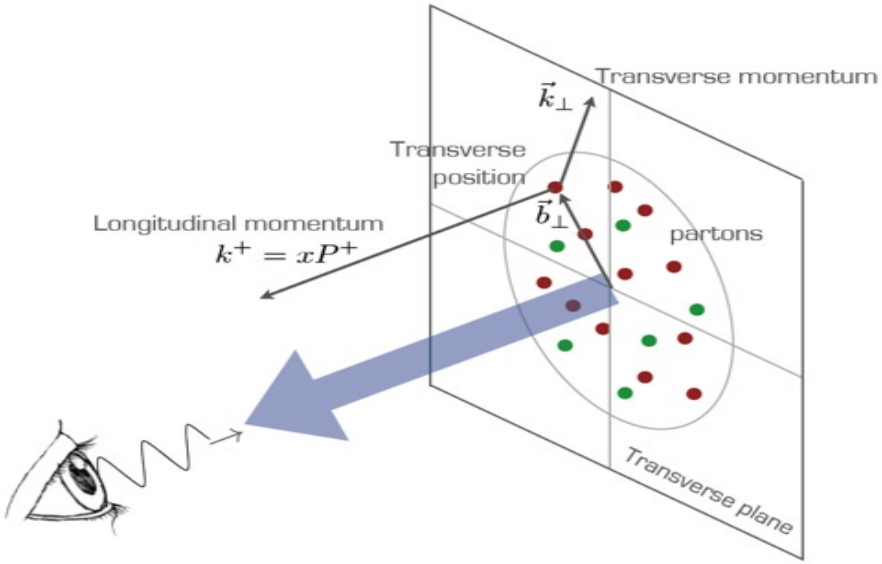


- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

❑ Need new observables with two distinctive scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

- Hard scale: Q_1 to localize the probe to see the particle nature of quarks/gluons
- "Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm

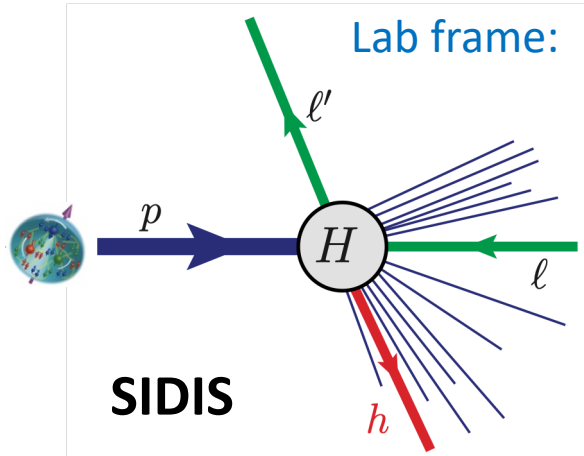


❑ Need new facility to "see" nucleon/nuclei without breaking it!

Explore Hadron's 3D Structure with/without Breaking the Hadron

Inclusive scattering

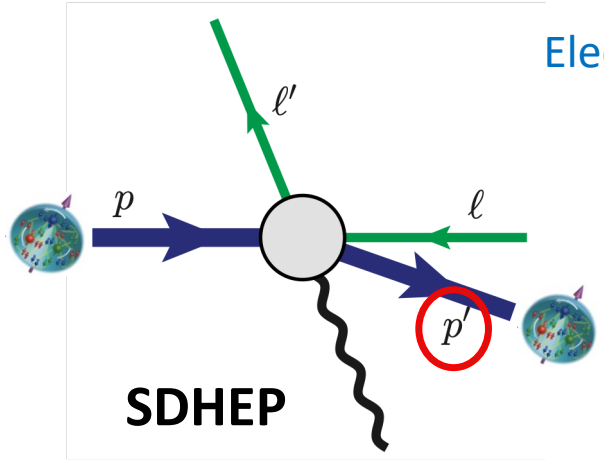
Scattering Amplitude



Lab frame: $|\vec{\ell}'_T| \sim |\vec{P}_{h_T}| \gg |\vec{\ell}'_T + \vec{P}_{h_T}|$

Breit frame: $Q^2 \gg P_{h_T}^2$

Exclusive diffraction



Electroproduction of a Real Photon

$Q^2 = -(\ell - \ell')^2$

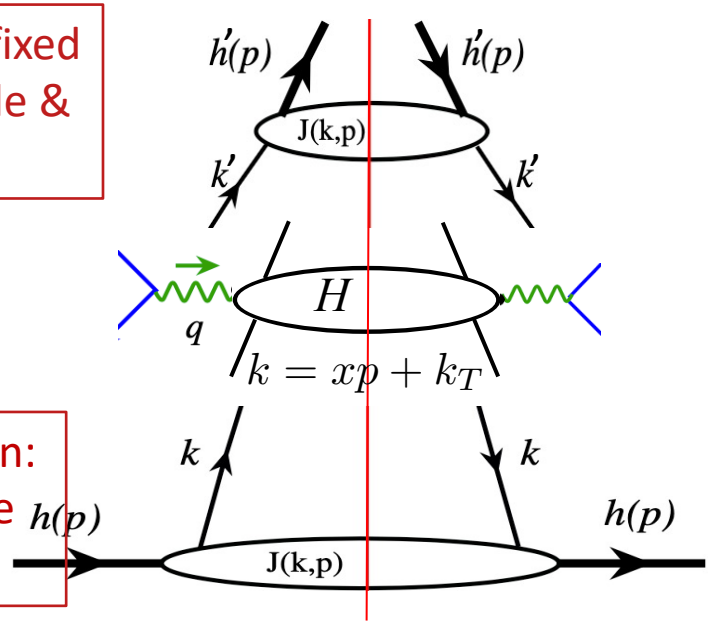
$t = (p - p')^2 < 0$

$Q^2 \gg -t$

Min: $2 \to 3$

Cross Section

Range of x fixed by hard scale & kinematics!



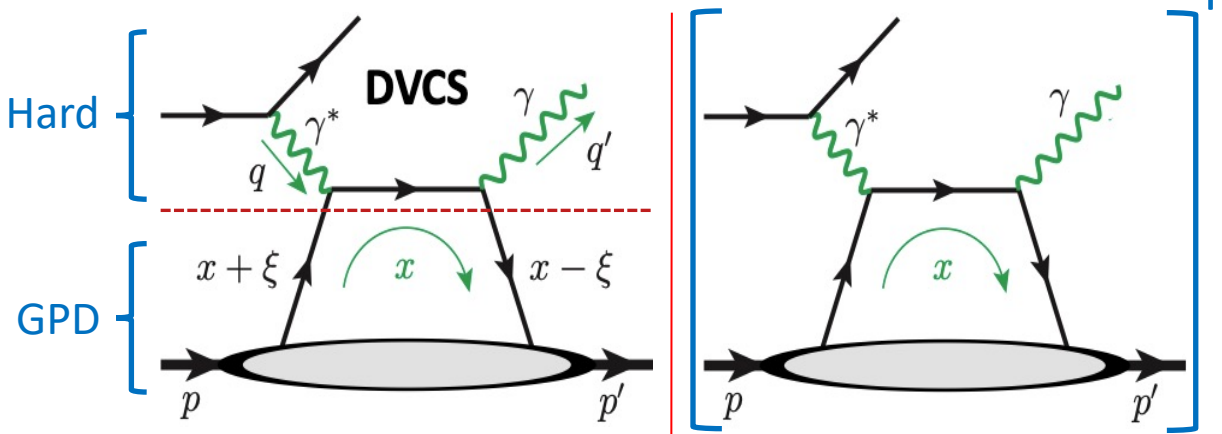
TMD FF

Hard

TMD PDF

Factorization: Perturbative pinch

Range of x is always the same!



Factorization: Color neutral hadron(s)

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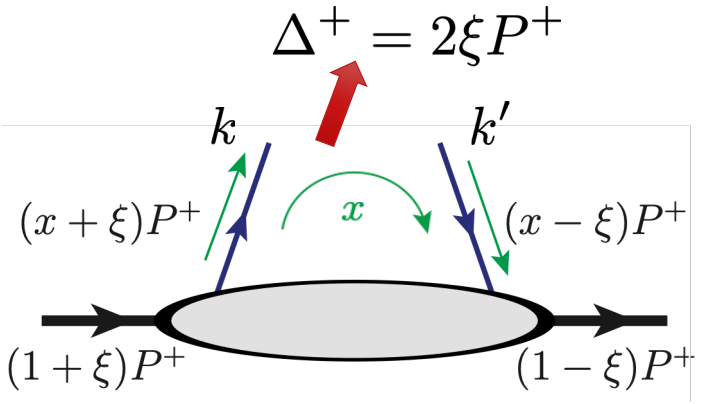
Generalized Parton Distributions (GPDs)

❑ Definition – Collinear Factorization:

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$
$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$



$$P^+ = \frac{p^+ + p'^+}{2}$$
$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

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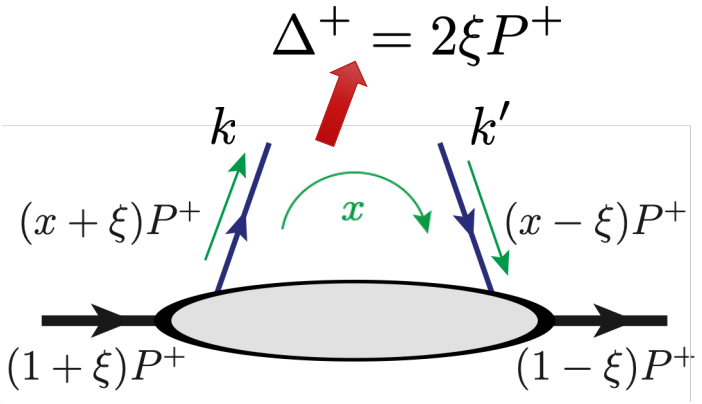
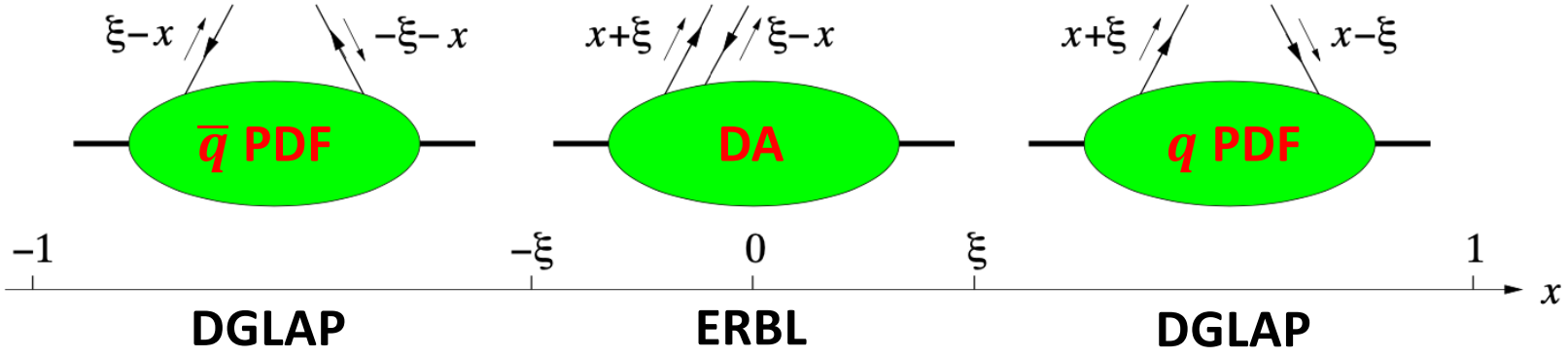


$$P^+ = \frac{p^+ + p'^+}{2}$$
$$\Delta = p - p' \qquad t = \Delta^2$$

❑ Combine *PDFs (p'=p)* and *Form Factors (z=0)* :

Forward limit $\xi = t = 0$: $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



Similar definition for gluon GPDs

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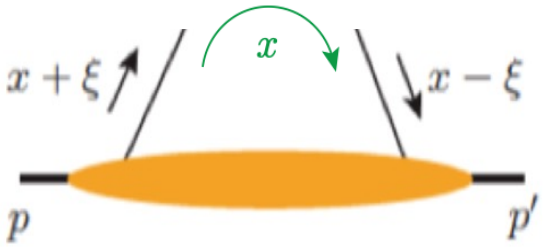
What Do We Learn from GPDs – Partonic ?

- **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

➡ **Quark density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$



Measurement of p' fixes (t, ξ)

x = momentum flow between the pair

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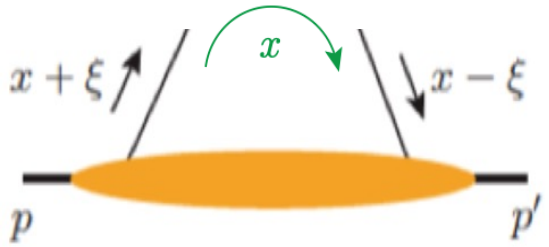

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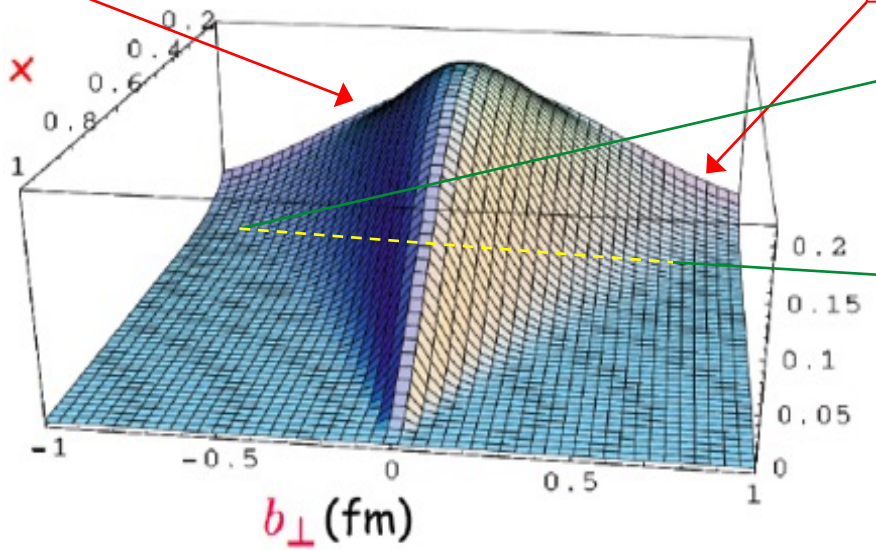
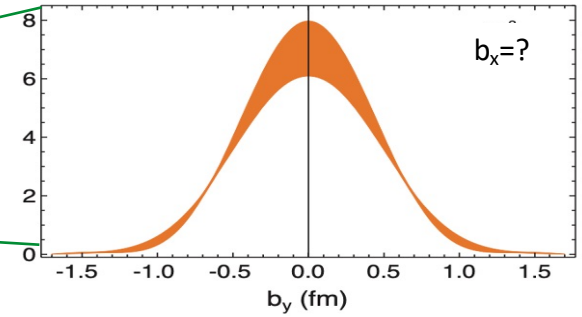
How fast does glue density fall?

Tomographic image of hadron in slice of x

How far does glue density spread?





Modeled by M. Burkdart, PRD 2000

Slice in (x,Q)

$$\langle q_\perp^N \rangle \equiv \int db_\perp b_\perp^N\, q(x, b_\perp, Q)$$

➡ **Proton radii,** $r_q(x)$ **&** $r_g(x)$**, from *q* & *g* spatial density**

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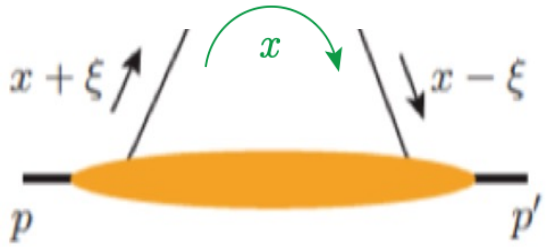

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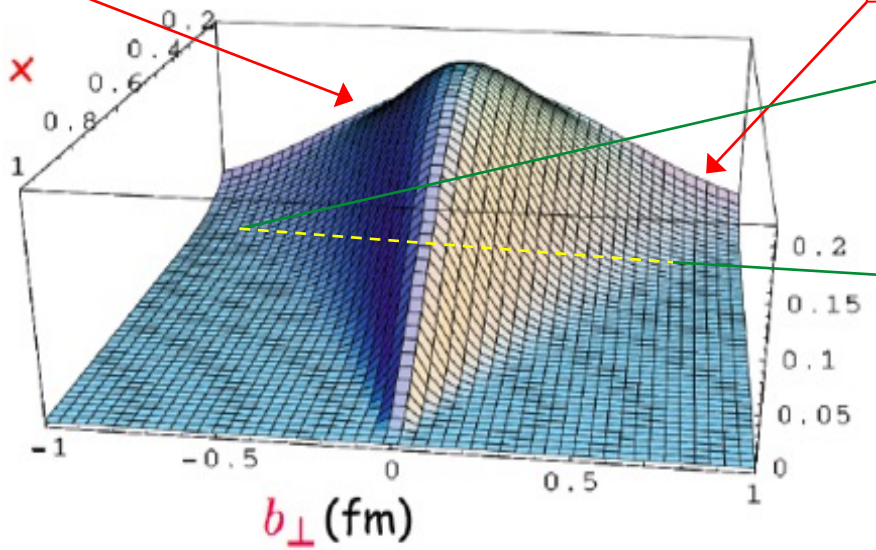

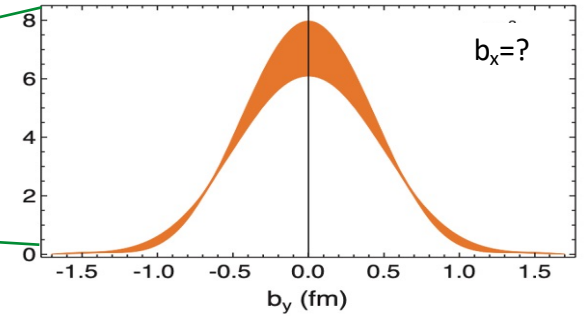

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➡ **Proton radii,** $r_q(x)$ **&** $r_g(x)$**, from *q* & *g* spatial density**

Need to know the x-dependence of GPDs to construct the proper radii !

- **Should** $r_q(x) > r_g(x)$**, or vice versa?**
- **How do they compare with known radius (EM charge radius, mass radius, …), & why?**

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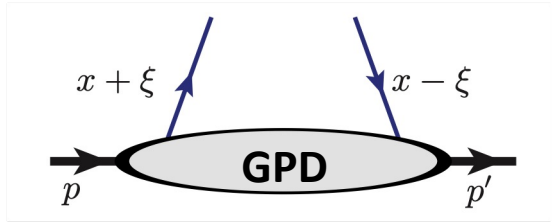
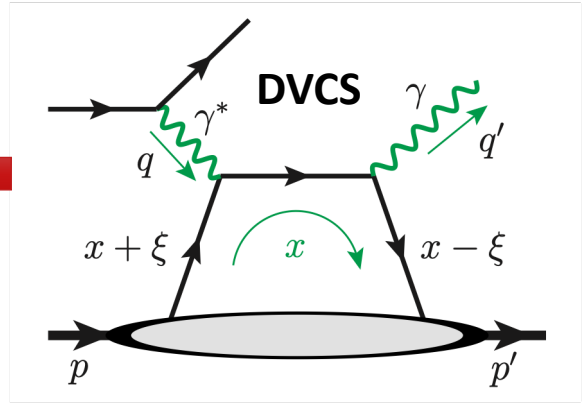
The GPD's x-Dependence is very *Difficult* to Measure – Why?

- **Amplitude nature: exclusive processes**



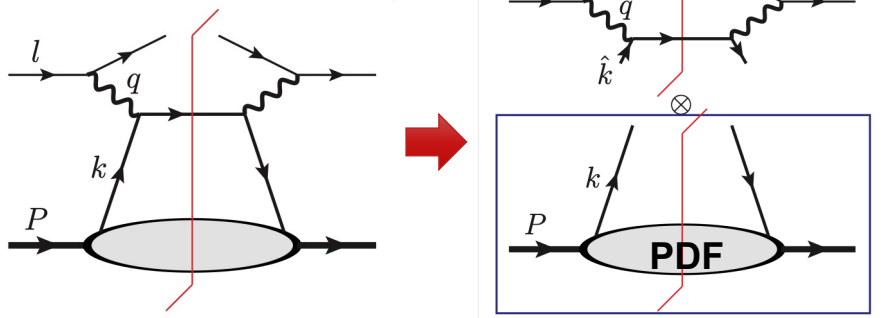


$x \sim$ virtual loop momentum

$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t)\cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x=0;\quad x=\pm\xi$

Compare with DIS



***cross section*: cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

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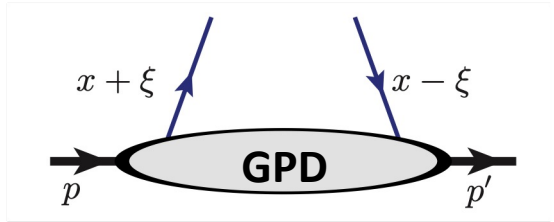

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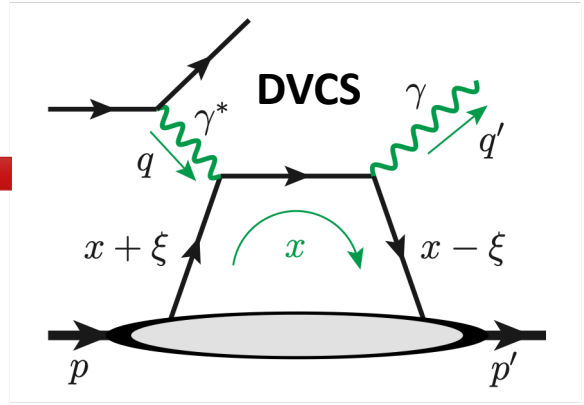

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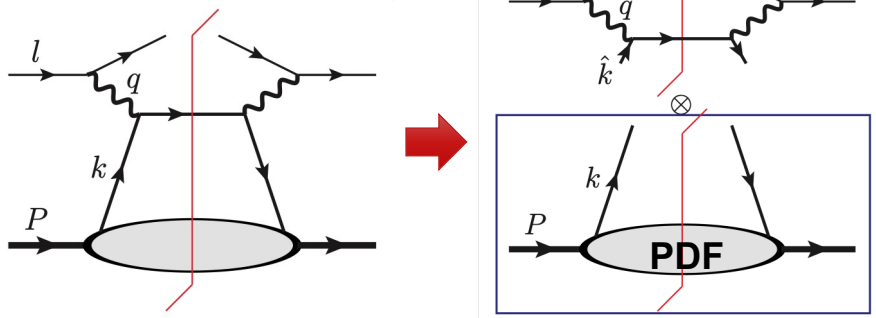
Full range of x, including $x=0;\quad x=\pm\xi$

❑ **Sensitivity to x: comes from $C(x,\xi;Q/\mu)$**

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

$$\Rightarrow\ i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$ **“x-moment”**

Compare with DIS



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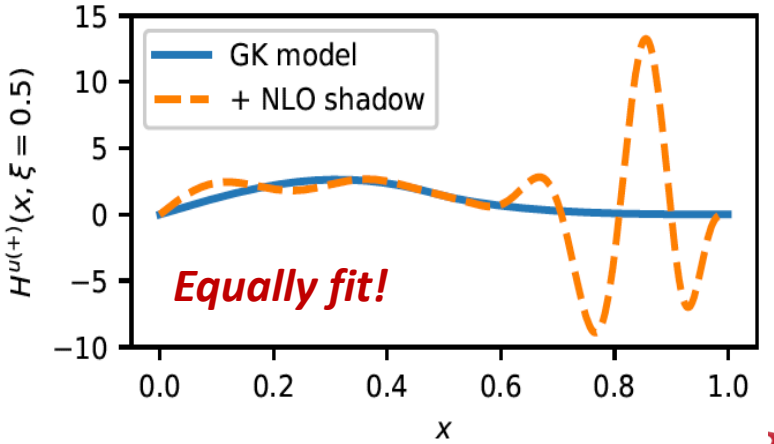

Shadowed GPDs

[Bertone et al. PRD `21]

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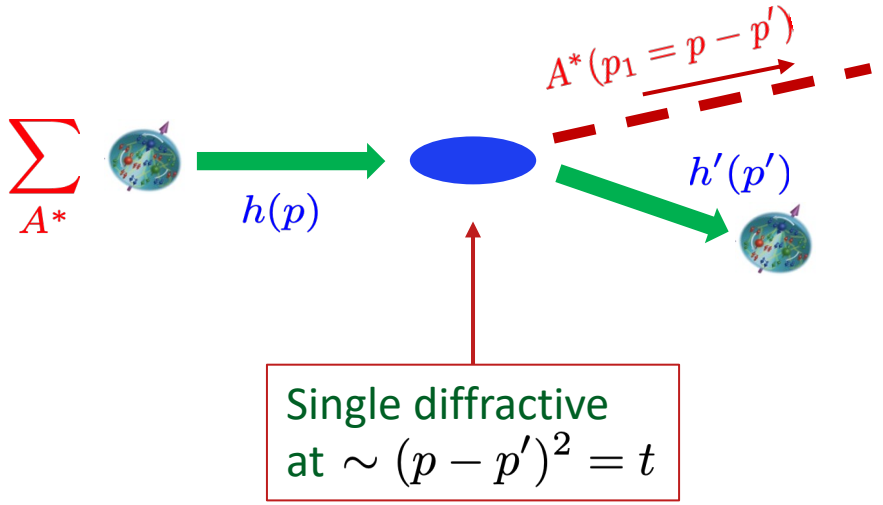
How to Find Physical Processes to be Sensitive to GPDs?

- **Need an exclusive process – Not to break the hadron:**

Single diffractive: $h(p) \to h'(p') + A^*(p_1 = p - p')$ ← Momentum, spin, composition of particles (or partons)

A "soft" scale: $t = (p - p')^2 \equiv \Delta^2$



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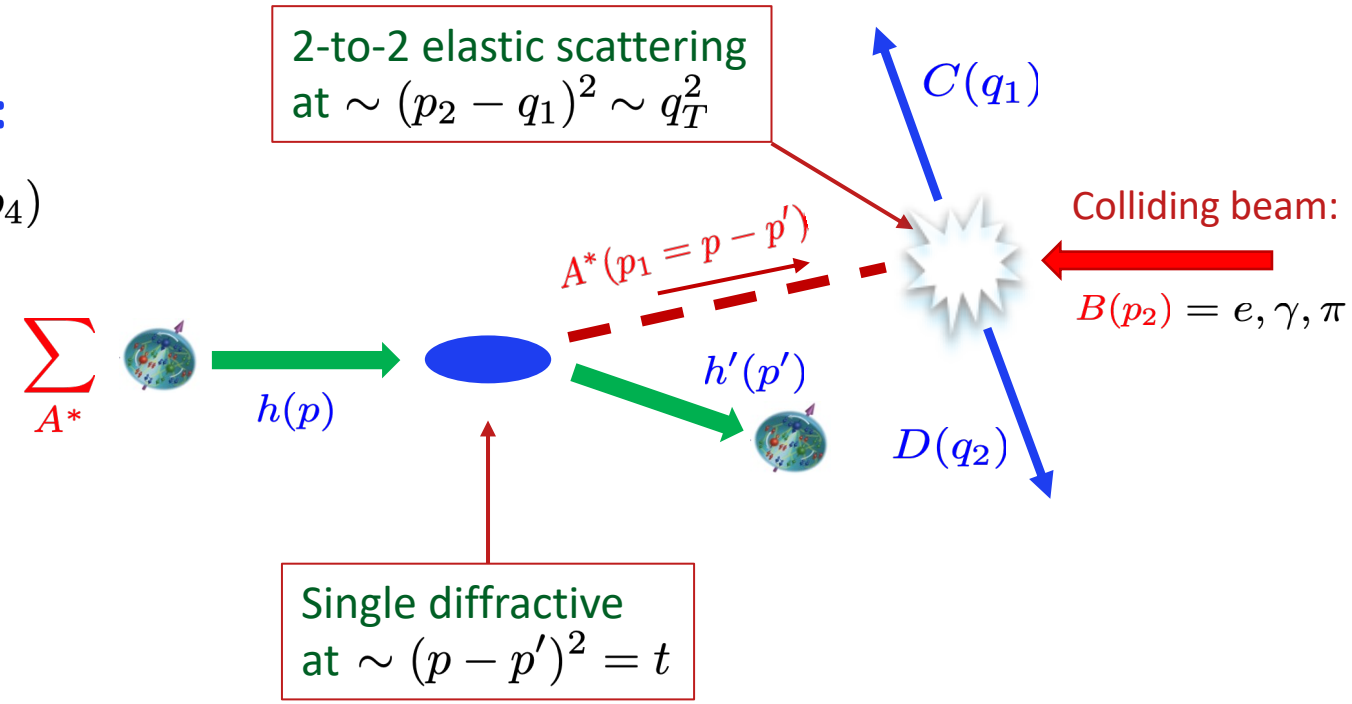
❑ Need a hard collision – to see the parton(s):

Hard exclusive: $A^*(p_1) + B(p_2) \to C(p_3) + D(p_4)$

($2 \to 2$ minimum)

A "hard" scale: $\hat{s} = (p_1 + p_2)^2 \gg \Lambda^2_{\rm QCD}$

$\hat{t} = (p_1 - q_1)^2 \sim (p_1 - q_2)^2 = \hat{u}$



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$\hat{t} = (p_1 - q_1)^2 \sim (p_1 - q_2)^2 = \hat{u}$

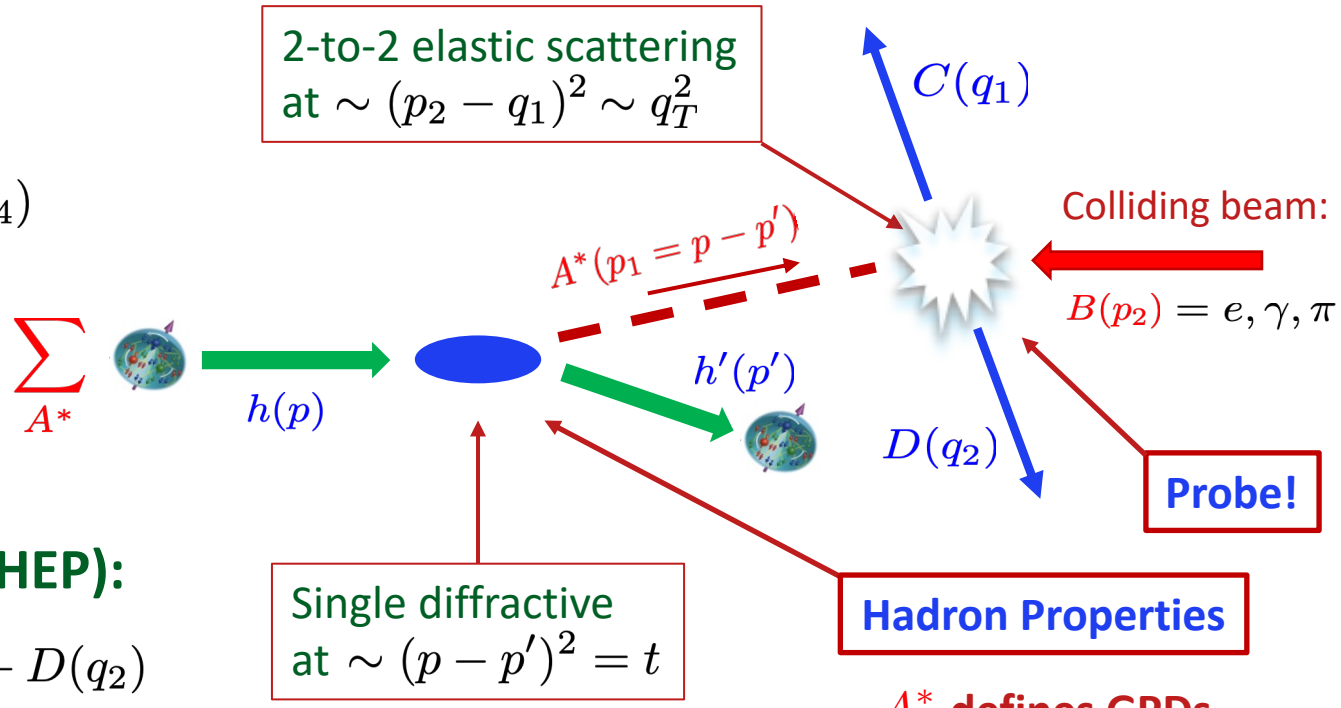
❑ Single diffractive hard exclusive process (SDHEP):

SDHEP Process: $h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$

(Need a minimum $2 \to 3$ SDHEPs)

Factorization: $Q^2 \sim \hat{s} \sim -\hat{t} \sim -\hat{u} \sim q_T^2 \gg -t$

A necessary condition!



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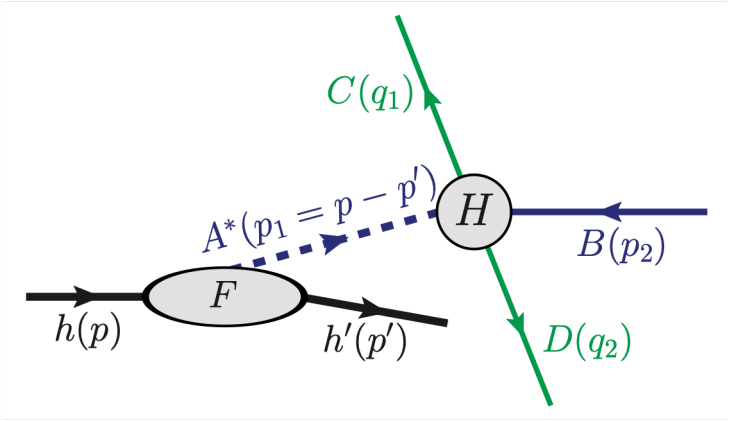
Sum of the Exchange State A* = An Expansion in Powers of $\sqrt{-t}/q_T$

$\sum_{A^*}$



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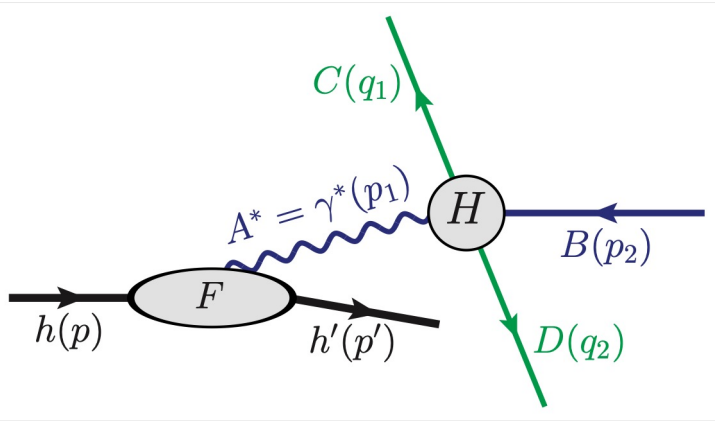
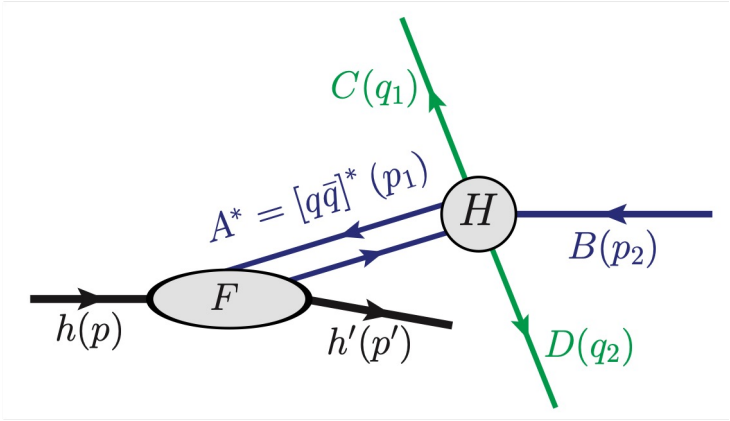


Leading Power in $\frac{\sqrt{-t}}{Q}$

\+



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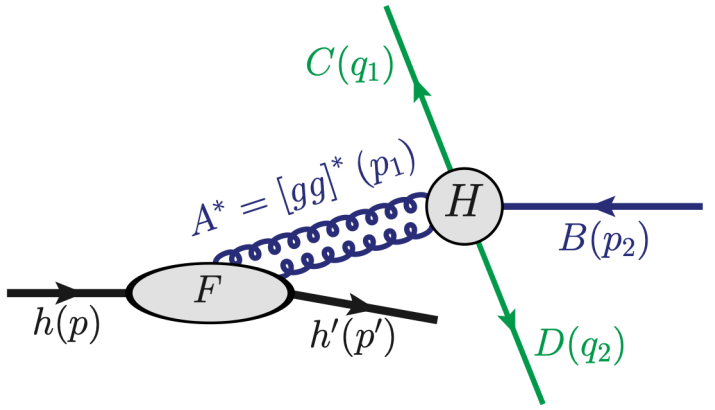


Next Leading Power in $\frac{\sqrt{-t}}{Q}$

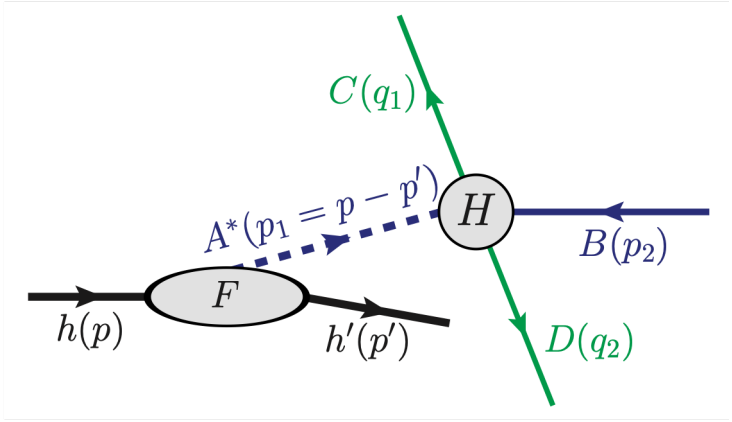
\+ ⋯

≥ 3 parton connection: further Power suppressed

If A* cannot be factorized in terms of partons, one might interpretate them as quasi-states, like pomeron, odderon, …, from the diffractive scattering

Sum of the Exchange State A* = An Expansion in Powers of $\sqrt{-t}/q_T$

$\sum_{A^*}$



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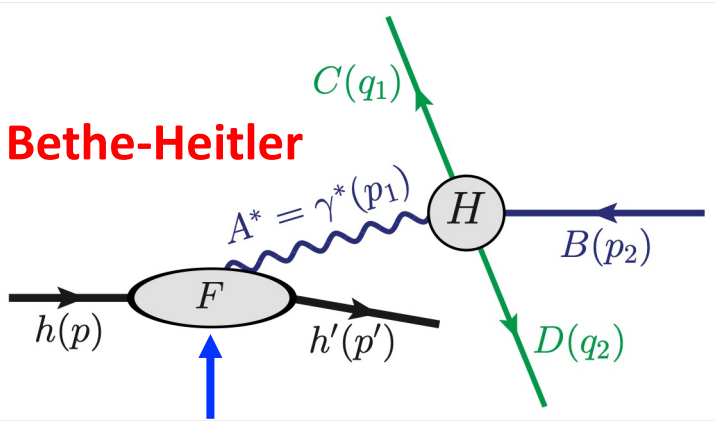
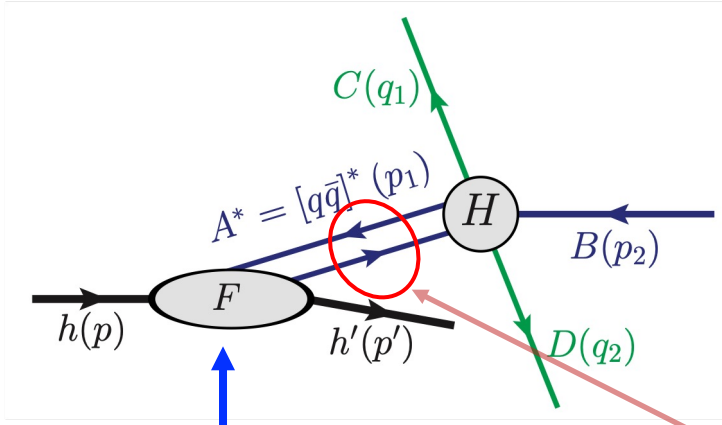


Leading Power in $\frac{\sqrt{-t}}{Q}$

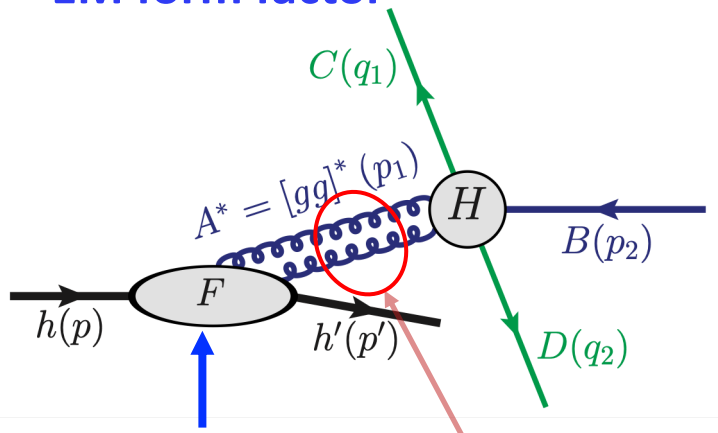
EM form factor

+



Quark GPD

+



Gluon GPD

Next Leading Power in $\frac{\sqrt{-t}}{Q}$

$+\cdots$

$\geq$ 3 parton connection: further Power suppressed

Ensured by $\sqrt{-t}/q_T \ll 1$

Energy $\gg$ virtuality

Factorization into GPDs

GPDs are defined by the quantum states of *A**

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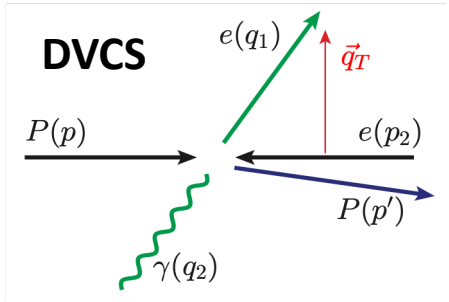
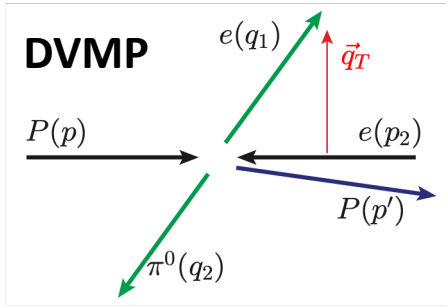
All Known Processes for Extracting GPDs – Fit the Kinematics of SDHEPs

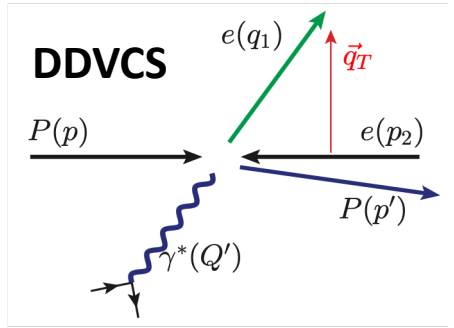
Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

❑ Electro-production (JLab, EIC, …)



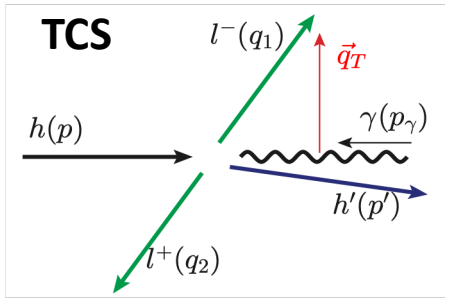


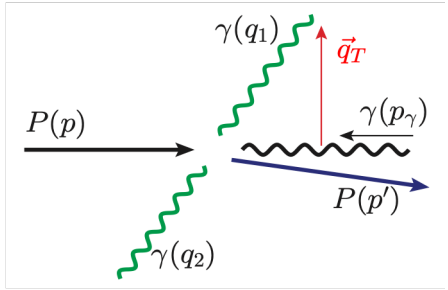
add virtuality

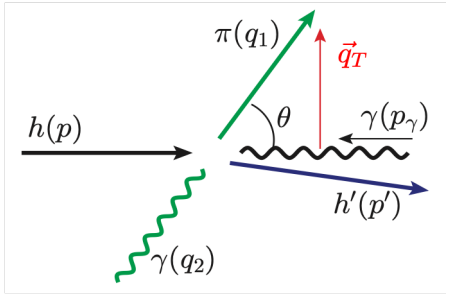


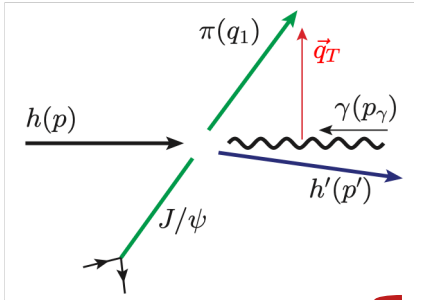
DHQP…

❑ Photo-production (JLab, EIC, …)



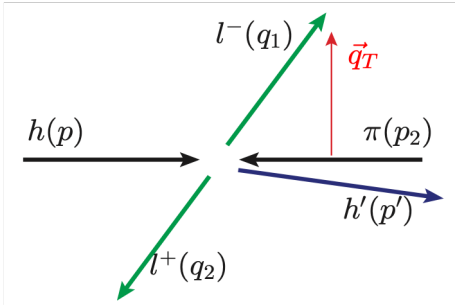


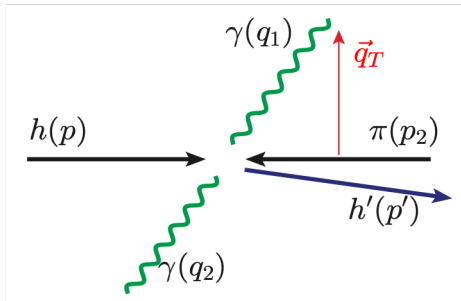




All of them are $2 \to 3$ exclusive Processes !

❑ Meso-production (AMBER, J-PARC, …)





…

Unified frame for all these observables

Same "n" along p_2

- Unified definition for all GPDs
- Ready for Global analysis of GPDs
- Easy to implement experimentally

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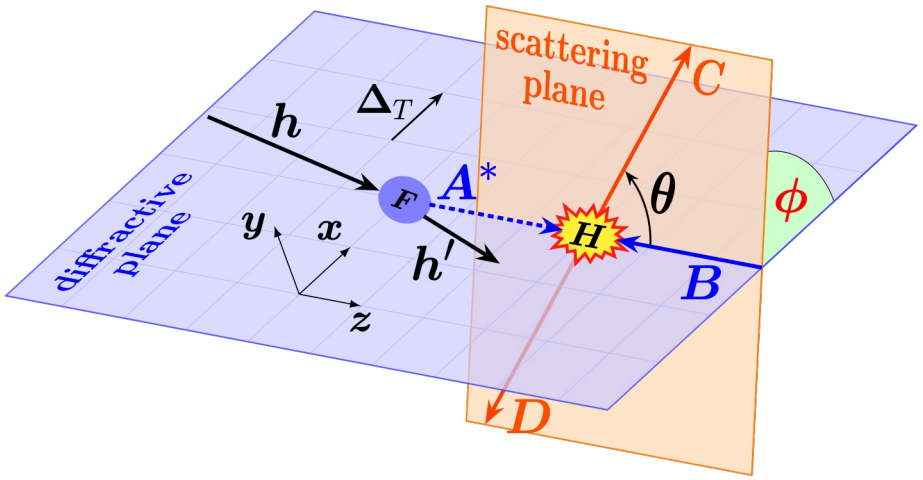
Where could the x-sensitivity Come From?

❑ **Single Diffractive Hard Exclusive Process (SDHEP):** $h(p) \to h'(p') + A^*(p_1 = p - p')$ – at scale t



Two stages at t and Q

$A^*(p_1) + B(p_2) \to C(p_3) + D(p_4)$ – at scale Q

x-sensitivity ⇔ $2 \to 2$ hard scattering:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⟸ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\,\sin\theta$ ⟺ x
3. ϕ ⟸ (A^*B) spin states

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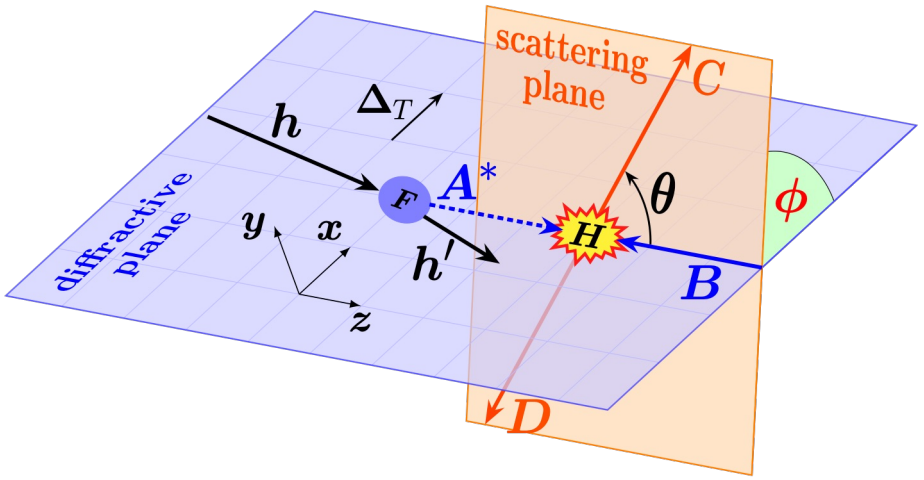

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$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$ [suppressing t and ξ dependence]

- **Moment-type sensitivity:** $C(x;Q) = G(x)\cdot T(Q)$ ⟹ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q Scaling for F_G**

  ⟹ **Inversion problem:** ***Shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

- **Enhanced sensitivity:** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟹ $d\sigma/dQ \sim |C(x;Q) \otimes_x F(x,\xi,t)|^2$

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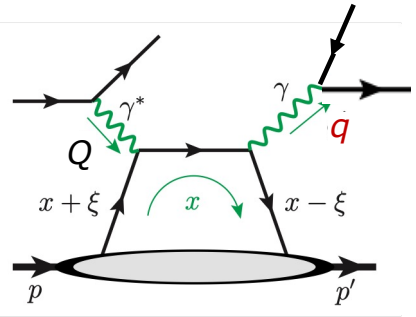
What Kind of Process Could be Sensitive to the *x*-Dependence?

- **Create an entanglement between the internal *x* and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x - x_p(\xi,q) + i\varepsilon}$$

Change external *q* to sample different part of x.



- **Double DVCS (two scales):**

$$x_p(\xi,q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

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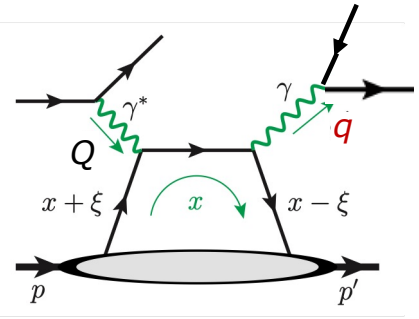

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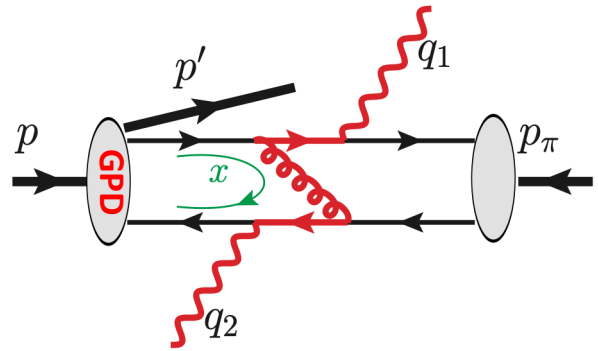
- **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

Hard scale: $q_T \gg \Lambda_{\mathrm{QCD}}$ Soft scale: $t \sim \Lambda^2_{\mathrm{QCD}}$

Qiu & Yu
JHEP 08 (2022) 103



- **Factorization:**

[*suppressing pion DA factor*]

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t;\mu)\cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T)$$

$x \leftrightarrow q_T$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\ \mathrm{d}\xi\ \mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

q_T distribution is "conjugate" to the x distribution !

The soft factor cancels due to the fact that all hadrons are color neutral !

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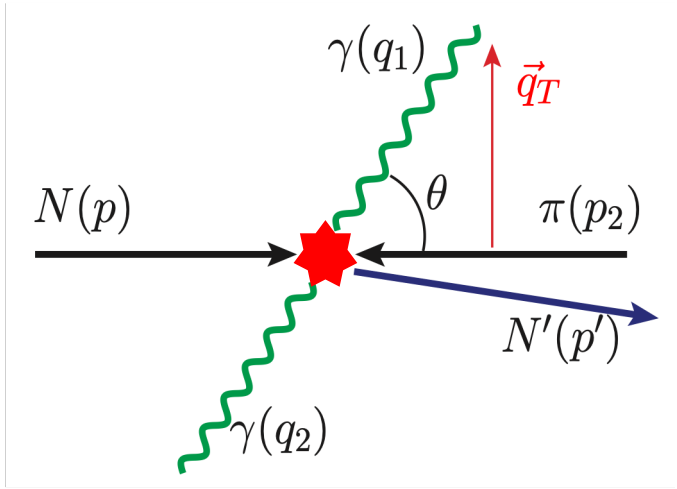
New Physical Processes to have Enhanced x-Sensitivity

❑ Diphoton process $N\pi \to N'\gamma\gamma:\ (1)\ p\pi^- \to n\gamma\gamma;\ (2)\ n\pi^+ \to p\gamma\gamma$



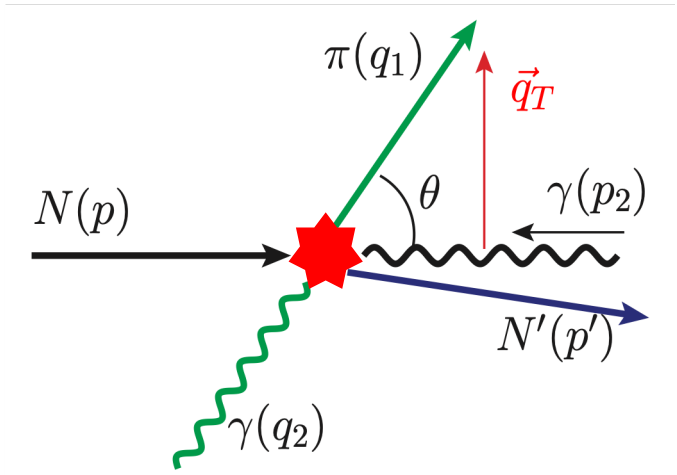
$i\mathcal{M}$ **also contains** $I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho(z;\theta) + i\epsilon\, \mathrm{sgn}\left[\cos^2(\theta/2) - z\right]}$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1 - z + \tan^2(\theta/2)\, z}{1 - z - \tan^2(\theta/2)\, z}\right] \in (-\infty, -\xi] \cup [\xi, \infty)$$



Covers the DGLAP region

❑ γ-π Pair Photoproduction:



$i\mathcal{M}$ **also contains:** $I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$



Covers the ERBL region

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How to Choose the Light-cone Vector "n" to Define GPDs?

❑ Choose the z-axis of the hard collisions – a choice of frame:

Recall – Inclusive DIS: $h(p) + l(\ell) \to l'(\ell') + X$

Let p and ℓ define the **z**-axis of the collision & p along the **+z** direction and ℓ long the **–z** direction,

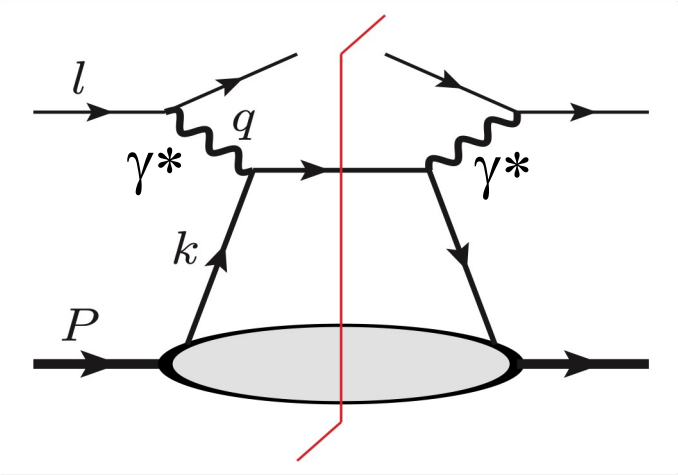
$p^\mu = (E, \mathbf{0}_\perp, p_z) \to (p^+, p^\perp, p^-)$ with

$p^+ = (E + p_z)/\sqrt{2}$ and $p^- = (E - p_z)/\sqrt{2} = m^2/2p^+$

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$

along the direction of $l(\ell)$ to pick up large component of p

Breit Frome = γ^* - Beam Frame:



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along the direction of $l(\ell)$ to pick up large component of p

❑ Choose the z-axis of the hard exclusive collision – SDHEP frame:

Let momentum of *A**, $p_1 = p - p'$ and the momentum of colliding beam, p_2

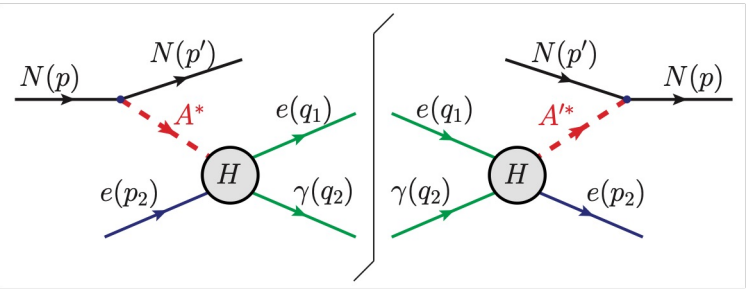
define the **z**-axis of the hard exclusive collision with p_1 along with **+z** direction

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$

along the direction of $B(p_2)$ to pick up large component of $p_1 = p - p'$

$p_1^+ = (p - p')^+ = 2\,\xi P^+ = \xi\,(p + p')^+$ and $p_1^- = t\,/\,2p_1^+$

SDHEP Frame = *A** **- Beam Frame:**



"Breit" frame with *A** replacing γ^*

A^* defines GPDs that we are probing!

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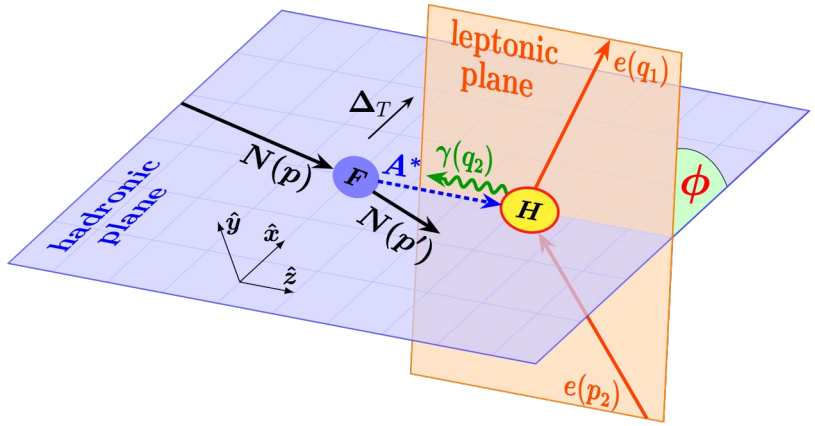
SDHEP Frame vs. Breit Frame – Use DVCS as an Example

❑ **Breit Frome = γ^* - Beam Frame:**



$$h(p) + e(p_2) \to h(p') + e(q_1) + \gamma(q_2)$$

1) $e(p_2) \to e(q_1) + \gamma^*(q)$

DVCS

$$h(p) + \gamma^*(q) \to h(p') + \gamma(q_2)$$

2) $h(p) \to h(p') + \gamma^*(\Delta)$

BH

$$e(p_2) + \gamma^*(\Delta) \to e(q_1) + \gamma(q_2)$$

$$q = p_2 - q_1$$

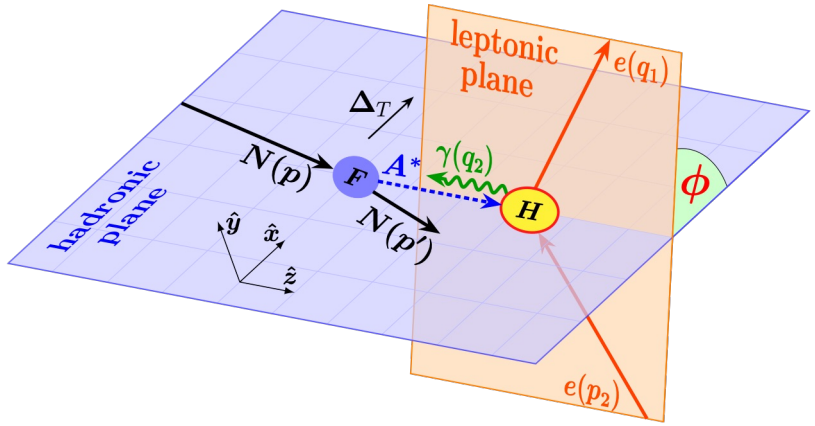
$$Q^2 \equiv -q^2 = -(p_2 - q_1)^2$$

Physics at both Q^2 and $t = (p - p')^2$

$$\Delta = p - p'$$

SDHEP Frame vs. Breit Frame – Use DVCS as an Example

❑ Breit Frome = γ^* - Beam Frame:



$$h(p) + e(p_2) \to h(p') + e(q_1) + \gamma(q_2)$$

1) $e(p_2) \to e(q_1) + \gamma^*(q)$

DVCS

$$h(p) + \gamma^*(q) \to h(p') + \gamma(q_2)$$

2) $h(p) \to h(p') + \gamma^*(\Delta)$

BH

$$e(p_2) + \gamma^*(\Delta) \to e(q_1) + \gamma(q_2)$$

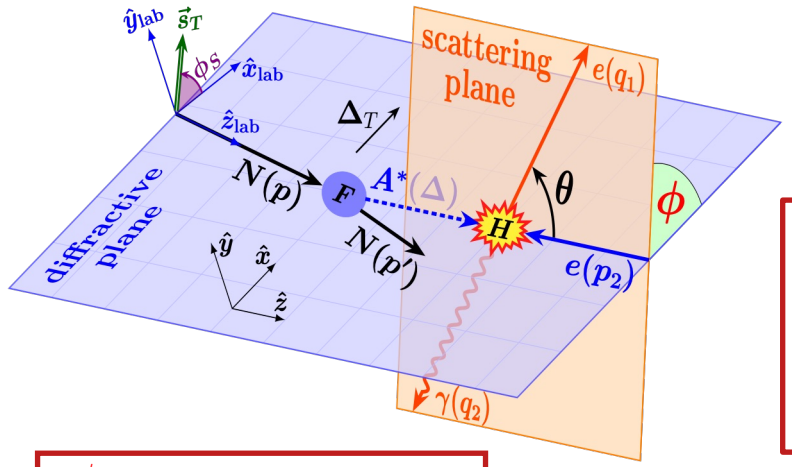
$$q = p_2 - q_1$$

$$Q^2 \equiv -q^2 = -(p_2 - q_1)^2$$

Physics at both Q^2 and $t = (p - p')^2$

$$\Delta = p - p'$$

❑ SDHEP Frame = *A** - Beam Frame:



$$h(p) \to h(p') + A^*(\Delta)$$

Physics at t

SDHEP — Include both DVCS & BH subprocesses

$$e(p_2) + A^*(\Delta) \to e(q_1) + \gamma(q_2)$$

Physics at Q^2

Hadron plays the role of the lepton in Breit frame

$$\gamma^*(q) \to A^*(\Delta)$$

- Easy to implement
- A* defines the GPDs
- Angular modulation to separate GPDs
- Same for all GPD finding processes!

ϕ – dependence ⇔ quantum # of *A**

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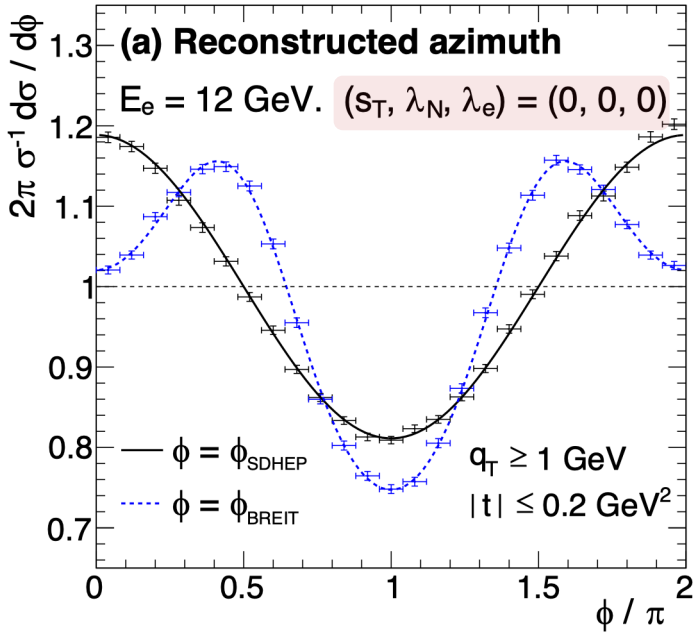
Simple Numerical Examples for Angular Distribution/Modulation

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\text{LP}}+\lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S$$
$$+\left(A_{UU}^{\text{NLP}}+\lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi+\left(\lambda_e A_{UL}^{\text{NLP}}+\lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$
$$+s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi+A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right)$$
$$+\lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi+A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

Generate 10^6 events and reconstruct

unpolarized



(normalized)

→ **SDHEP fit to** $1.00+0.190\cos\phi$ ⇒ $\langle A_{UU}^{\text{NLP}}\rangle = 0.190$

→ **Breit fit to** $1.00+0.15\cos\phi-0.12\cos2\phi-0.01\cos3\phi+0.01\cos4\phi$

- **No straightforward interpretation of the coefficients.**
- **Need to introduce more gears in GPD extraction.**

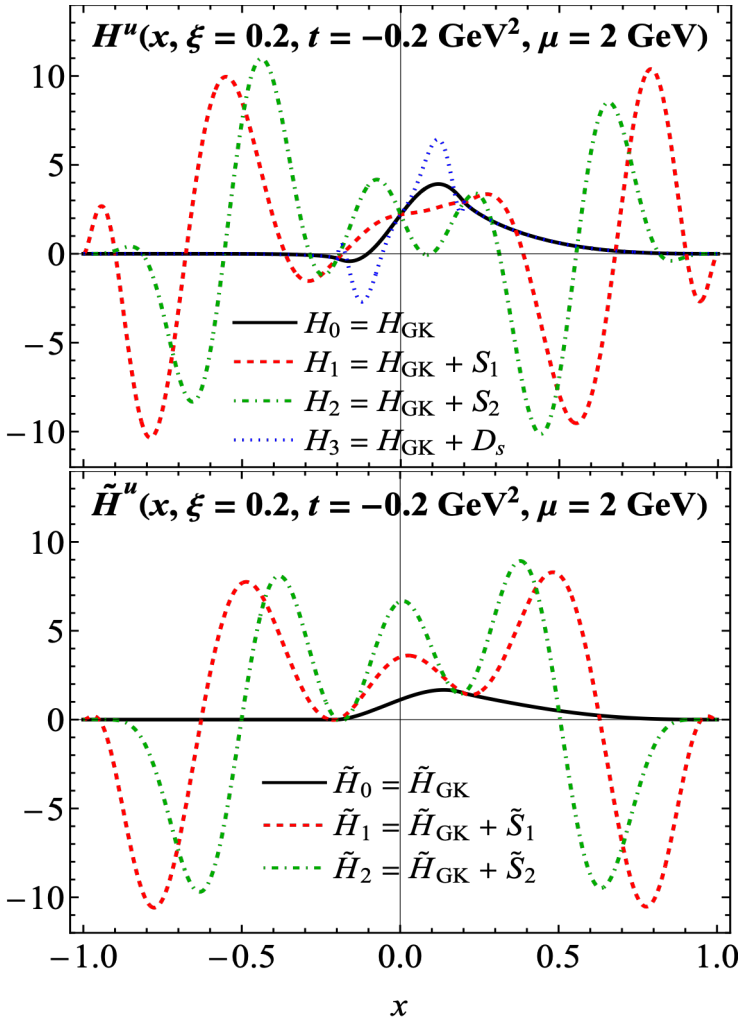
[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
…

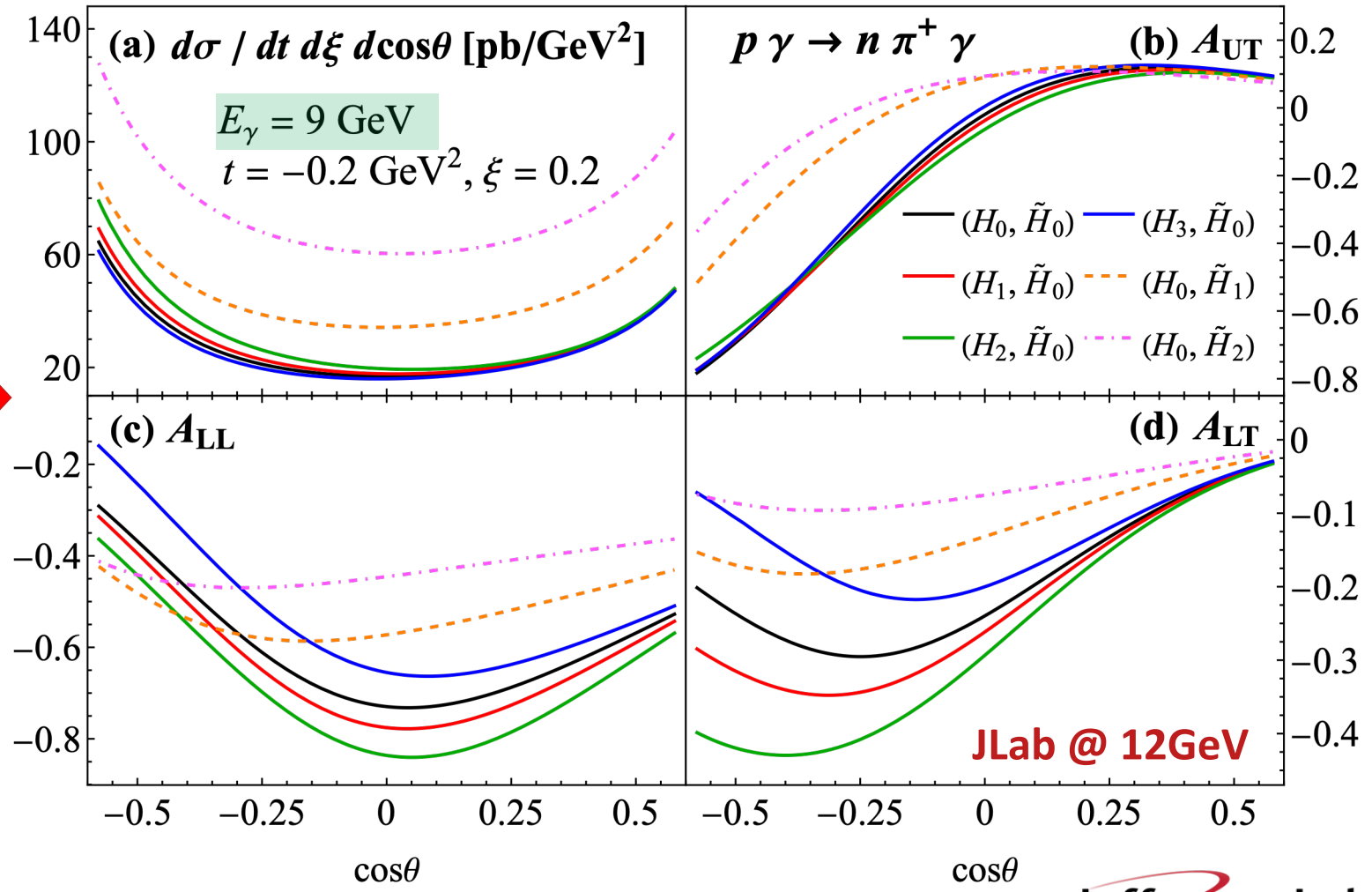
Test the Sensitivity on the GPDs' x-Dependence

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi\pm i\epsilon} = 0$$

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23





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GPDs – Partonic Origin of Hadronic Properties

❑ **QCD energy-momentum tensor:**

$$T^{\mu\nu} = \sum_f \overline{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$\begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

Pressures

Stress tensor

❑ **"Gravitational" form factors (GFFs) of the nucleon:**

$$\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p') \left[\gamma^{(\mu} P^{\nu)} A(t) + \frac{i P^{(\mu} \sigma^{\nu)\rho} \Delta_\rho}{2M} B(t) + \frac{D(t)}{4M} (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) \right] u(p)$$

❑ **Connection to GPD moments:**

$$\int_{-1}^{1} dx\, x\, F_i(x,\xi,t) \propto \langle p'|T_i^{++}|p\rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{(A_i + \xi^2 D_i)}_{\int_{-1}^{1} dx\, x\, H_i(x,\xi,t)} \gamma^+ + \underbrace{(B_i - \xi^2 D_i)}_{\int_{-1}^{1} dx\, x\, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)

Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ **Angular momentum sum rule:**

$$J_i = \lim_{t\to 0} \int_{-1}^{1} dx\, x\, [H_i(x,\xi,t) + E_i(x,\xi,t)]$$

$$i = q, g$$

➡ **3D tomography**
Relation to GFFs
Angular Momentum

➡ ***x*-moments of GPDs! Input for GFFs!**

Need to extract GPD and its x-dependence from DATA to construct the proper moments!

Summary and Outlook

- GPDs are fundamental, carrying rich information on:
  - Tomographic images of confined quarks and gluons
  - Underline dynamics of hadronic properties

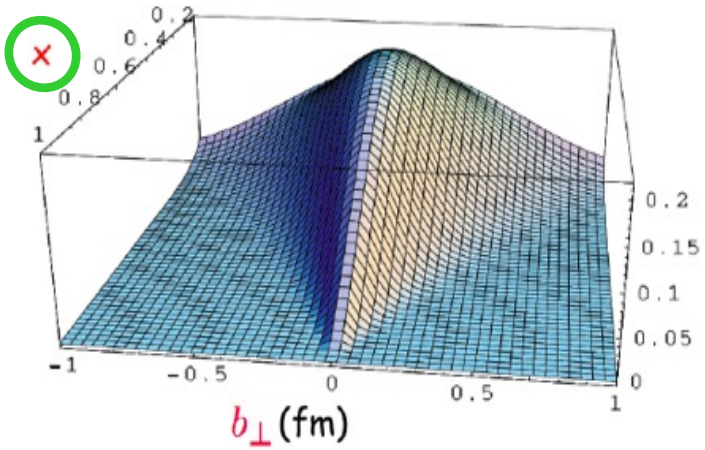
- The $2 \to 3$ SDHEPs are excellent physical processes for extracting of GPDs
  - SDHEP frame is the right one for evaluating angular modulations
  - Need SDHEPs with ***x*** of GPDs entangled with measured hard scales!

- QCD Global analyses to extract GPDs – more data from JLab and EIC:
  - With $p \neq p'$, the choice of "+" component is not unique
  - SDHEP frame for all known SDHEPs provides a unique way to define the GPDs, necessary for Global analyses
  - Need to identify more factorizable SDHEPs for extracting GPDs through Global analyses
  - LQCD can help – same factorization + inverse problem!
  - …

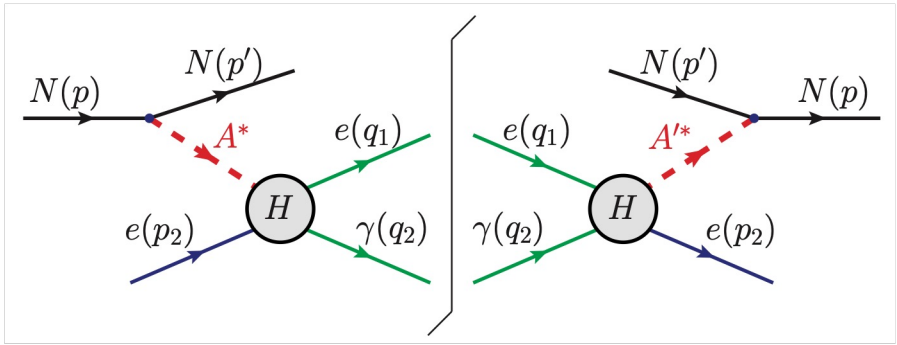
A long, exciting but challenging way to get full images of the proton !

Thanks!

3D image



SDHEP Frame = *A - Beam Frame:**



Two stages for physics at two different scales, plus natural power expansion

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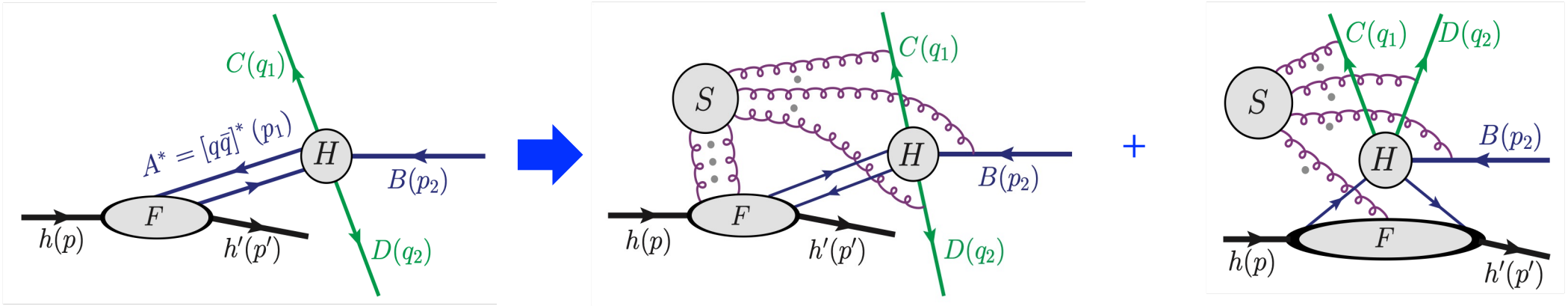
Backup Slides

Factorization for SDHEP in the Two-stage Paradigm

❑ Factorization for 2-parton channel factorization:

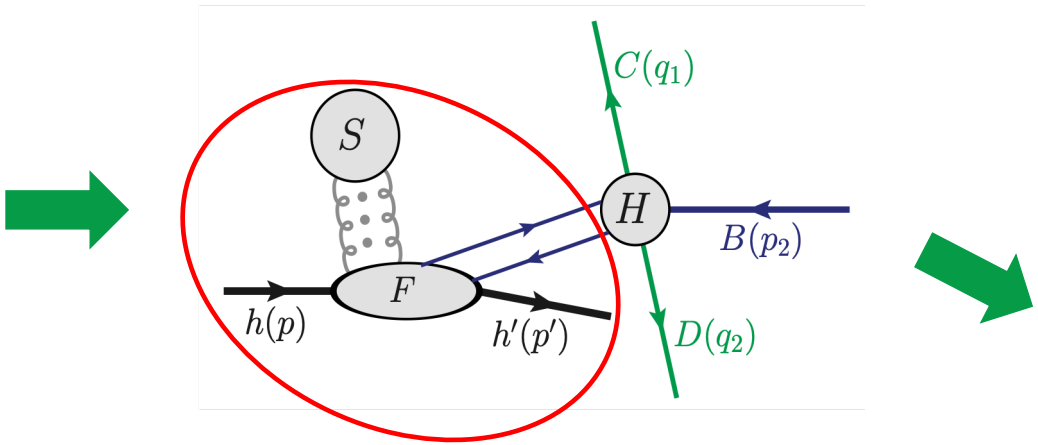
Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1



ERBL region: $[q\bar{q}']$ ~ meson

DGLAP region: Glauber pinch

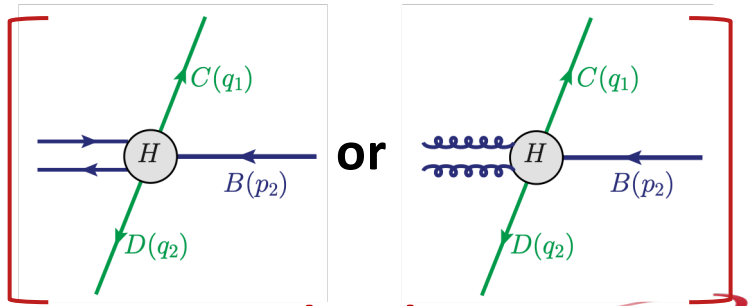
❑ Soft gluons cancel when coupling to color neutral hadrons:



Glauber gluons of SDHEP:

$$k_s = (\lambda^2, \lambda^2, \lambda) \rightarrow (1, \lambda^2, \lambda)$$ **Collinear gluons**

$$\text{GPD} \otimes \left[\text{or} \right]$$



Hard probes

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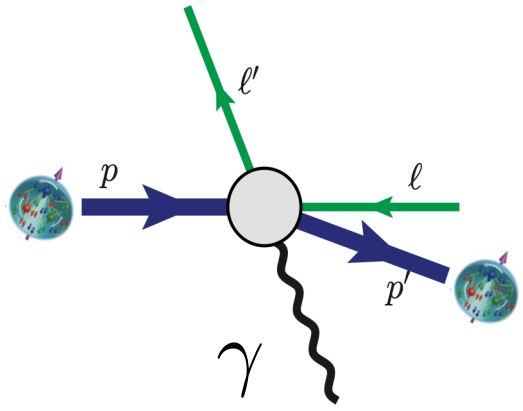
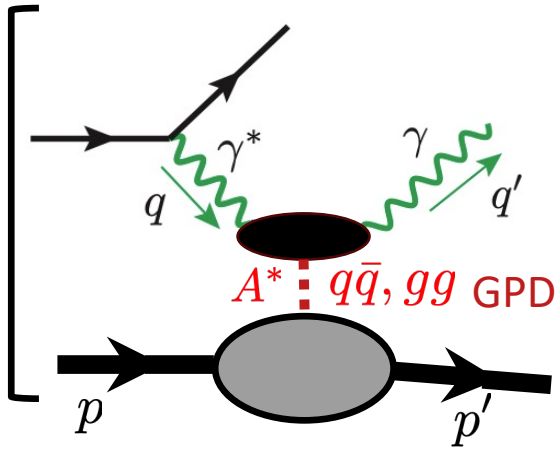
SDHEP Frame vs. Breit Frame – Use DVCS as an Example

❑ Breit Frome = γ^* - Beam Frame:



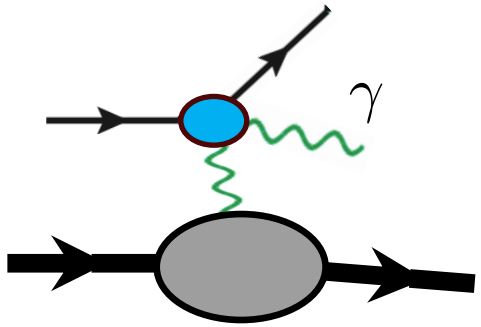
Breit frame is a natural extension from the treatment of DIS, SIDIS

Angular distribution between lepton plane ($\ell \to \ell'$) and hadron plane ($p \to p'$)
DOES not have a simple relation to GPDs to help their extraction, due to mixing of BH!



DVCS: in terms of factorized GPDs

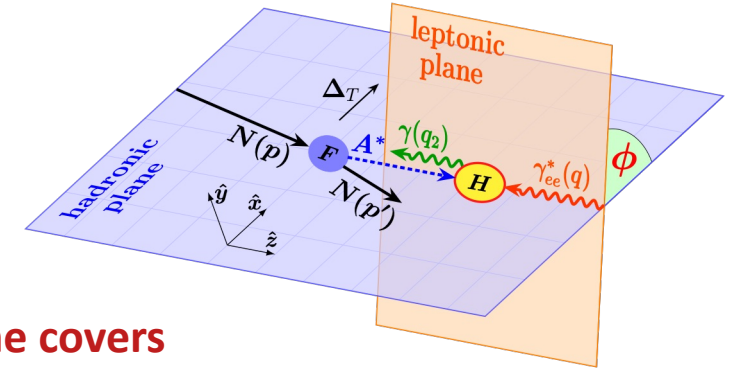
\+



2 EM form factors

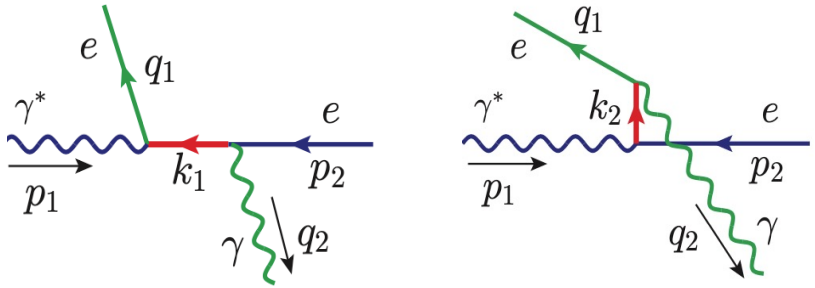
Bethe-Heitler subprocess in terms of EM form factors

2



Hadronic plane covers Both hard and soft physics

BH is not a "t"-channel process:



Propagators of k_1 & k_2 have ϕ-dependence!

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