Charge Charge Correlation in QCD

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DESY

REF 2026, Santiago de Compostela

Correlation functions for QCD dynamics

- Energy-energy correlation: [Basham, Brown, Ellis, Love, PRL. 41 (1978) 1585 & PRD. 19 (1979) 2018]

$$\mathrm{EEC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma \, \frac{E_a E_b}{Q^2} \, \delta(\cos\chi - \cos\theta_{ab})$$

The energy weight emphasizes the hard radiation and makes the EEC IRC safe.

- Charge-charge correlation: [T. D. Lee, 1981]

$$\mathrm{QQC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma \, Q_a Q_b \, \delta(\cos\chi - \cos\theta_{ab})$$

Probes charge flow in the final state, but is generally IRC unsafe beyond LO in QCD.

In terms of correlation functions of energy and charge flow operators [Hofman, Maldacena, 0803.1467]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt \, n^i T^0_i(t, r\vec{n})$$

$$Q(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt \, n^i J_i(t, r\vec{n})$$

EEC: correlation of energy flow operators $\langle J^\mu(x)\mathcal{E}(\vec{n})\mathcal{E}(\vec{n}')J_\mu(0)\rangle$

QQC: correlation of charge flow operators $\langle J^\mu(x)Q(\vec{n})Q(\vec{n}')J_\mu(0)\rangle$

[P. F. Monni, G. Vita, Z. Xu and H. X. Zhu, arXiv:2508.00977]

[H. Chen, P. F. Monni, G. Vita, Z.Y. Pang and H. X. Zhu, arXiv:2510.07377]

[H. Chen, P. F. Monni, G. Vita, Z.Y. Pang and H. X. Zhu, arXiv:2603.22388]

Motivations for understanding charge dynamics

Jet charge:

$$Q_\kappa = \sum_{i \in J} Q_i \, z_i^\kappa \,, \qquad z_i = \frac{E_i}{E_J}$$

$\kappa = 0$: retains the initiating parton's electric charge. [Field, Feynman, Nucl. Phys. B 136 (1978)]

$\kappa > 0$: the energy weight suppresses soft hadrons.

[Krohn, Lin, Schwartz, Waalewijn, arXiv:1209.2421; Waalewijn, arXiv:1209.3019]

- Physics applications:

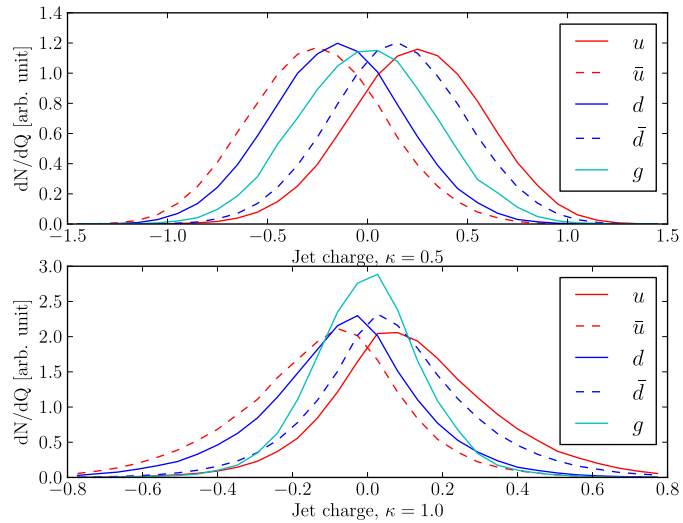
  statistically distinguish quark / antiquark / gluon jets

  identify the charge of decaying resonances.

- Experimental advantages:

  tracking provides finer angular resolution

  suppresses pile-up contamination.



Simulated jet charge for $u, \bar{u}, d, \bar{d}, g$ jets.

Different distributions provide statistical flavor discrimination.

[Krohn, Lin, Schwartz, Waalewijn, arXiv:1209.2421]

Jet charge is IRC unsafe and requires nonperturbative hadronization input.

Factorization separates this input from the calculable dependence on jet energy and radius.

[Chang, Procura, Thaler, Waalewijn, arXiv:1303.6637; Waalewijn, arXiv:1209.3019]

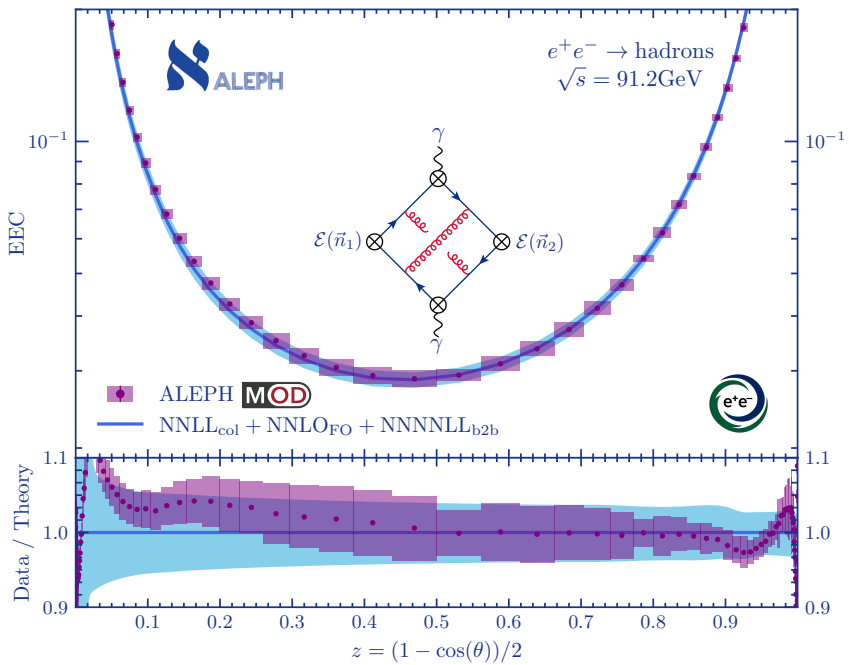
Measurements on charged particles with track function

- Track function $T_i(x, \mu)$: probability density for the fraction x of parton i's energy carried by charged hadrons,

$$\int_0^1 \mathrm{d}x\, T_i(x, \mu) = 1$$

Track functions absorb the collinear singularities introduced by restricting the measurement to charged particles. [Chang, Procura, Thaler, Waalewijn, arXiv:1303.6637]

- EEC measured on tracks



Inclusive EEC is IRC safe.

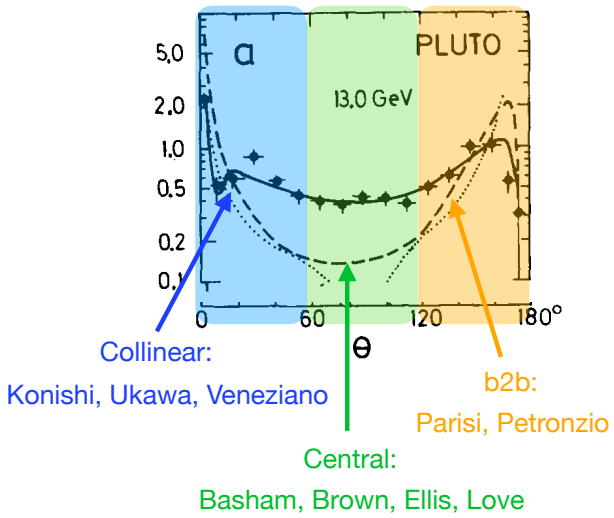
Track-based EEC retains nonperturbative input through a finite set of track-function moments.

[Bossi, Y. Chen, Y.-C. Chen, Jaarsma, Li, Zhang, Moult, Waalewijn, Zhu, Badea, Baty, McGinn, Innocenti, Maggi, Lee, arXiv:2511.00149]

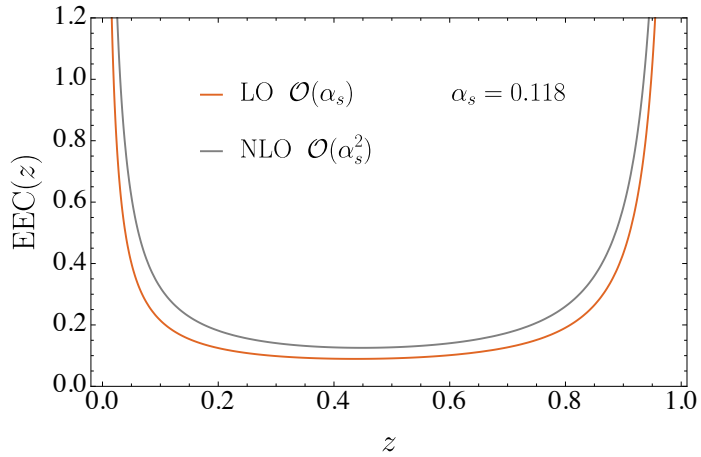
[Jaarsma, Li, Moult, Waalewijn, Zhu, arXiv:2512.11950]

EEC with resummation from factorization $z = (1 - \cos\chi)/2$

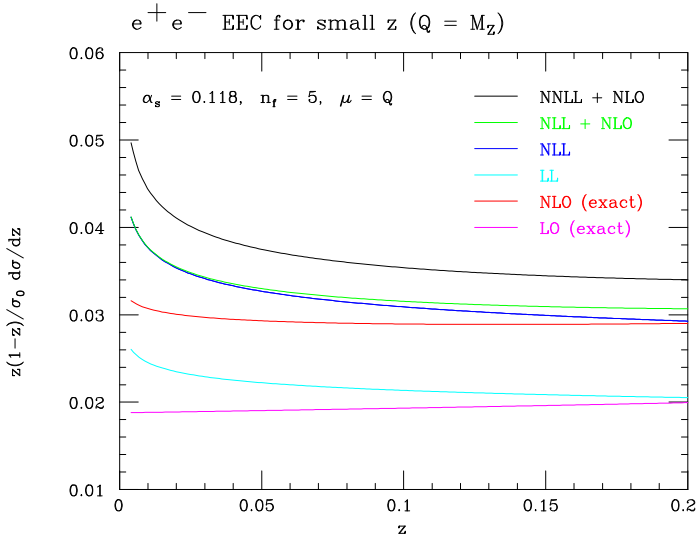
Early experimental measurement:
[PLUTO Collaboration, Z. Phys. C 28 (1985) 365]



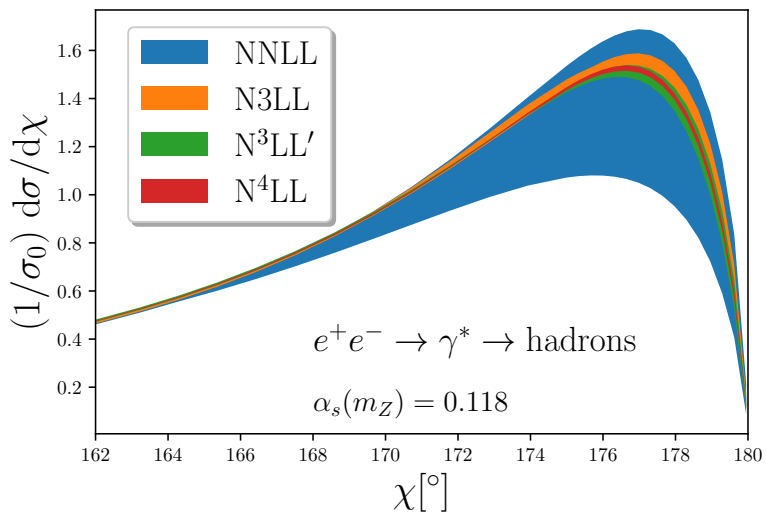
First event-shape calculated analytically at NLO,
[Dixon, Luo, Shtabovenko, Yang, Zhu, PRL. 120 (2018) 102001]



NNLL resummation in collinear limit $z \to 0$:
[Dixon, Moult, Zhu, arXiv:1905.01310]



N^{4}LL resummation in back-to-back limit $z \to 1$:
[Moult, Zhu, arXiv:1801.02627]
[Duhr, Mistlberger, Vita, arXiv:2205.02242]



Factorization of EEC in the back-to-back limit

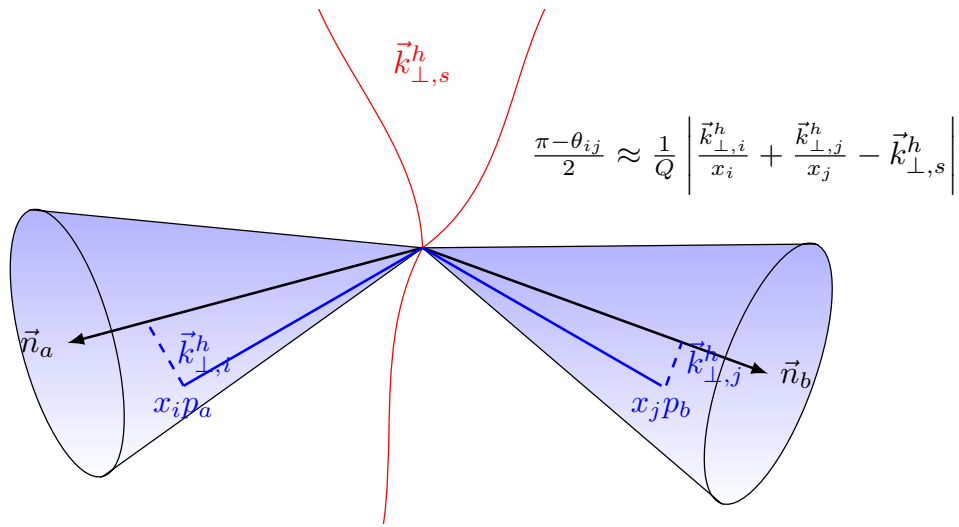
[Moult, Zhu, arXiv:1801.02627]

Energy-weighted sum of the dihadron cross section, In the back-to-back limit $\bar{z} = 1 - z \simeq q_T^2/Q^2 \ll 1$.

[Collins, Soper, Nucl. Phys. B 197 (1982)]

$$\frac{\mathrm{d}\sigma^{\mathrm{EEC}}}{\mathrm{d}z} \overset{z\to 1}{=} \sum_{h_a,h_b} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, \frac{x_a x_b}{4} \int \mathrm{d}^2 q_\perp \, \delta\left(\bar{z} - \frac{q_\perp^2}{Q^2}\right) \frac{\mathrm{d}\sigma_{e^+e^-\to h_a h_b + X}}{\mathrm{d}x_a \, \mathrm{d}x_b \, \mathrm{d}^2 q_\perp}$$

$$\sum_q \sigma_0^q \, H_{q\bar{q}}(Q) \int \frac{\mathrm{d}^2 \vec{b}_T}{(2\pi)^2} \, e^{\mathrm{i}q_\perp \cdot \vec{b}_T} \times D_{h_a/q}(x_1, b_T) D_{h_b/\bar{q}}(x_2, b_T) S_q(b_T)$$



measurement on soft radiation suppressed by energy weights, soft radiation contributes via recoils.

S_q is the standard TMD soft function describing transverse recoil.

Factorization of EEC in the back-to-back limit

[Moult, Zhu, arXiv:1801.02627]

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$$\sum_q \sigma_0^q \, H_{q\bar{q}}(Q) \int \frac{\mathrm{d}^2 \vec{b}_T}{(2\pi)^2} \, e^{\mathrm{i}q_\perp \cdot \vec{b}_T} \times D_{h_a/q}(x_1, b_T) D_{h_b/\bar{q}}(x_2, b_T) S_q(b_T)$$

- The energy moment removes the nonperturbative collinear FF by momentum conservation:

$$\begin{aligned} J_q^{\mathrm{EEC}}(b_T) &\equiv \sum_h \int_0^1 \mathrm{d}x \, x \, D_{h/q}(x, b_T) \\ &= \sum_i \int_0^1 \mathrm{d}t \, t \, \mathcal{K}_{qi}(t, b_T) \underbrace{\left[\sum_h \int_0^1 \mathrm{d}\xi \, \xi \, d_{h/i}(\xi)\right]}_{=1 \text{ (momentum sum rule)}} + O\left(b_T^2 \Lambda_{\mathrm{QCD}}^2\right) \end{aligned}$$

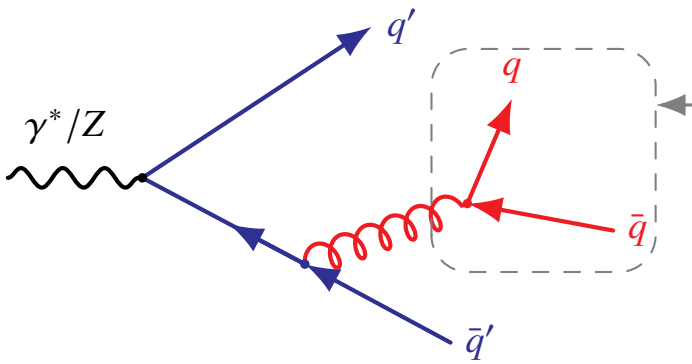
- The resulting factorization, with an explicit recoil soft function, is

$$\frac{\mathrm{d}\sigma^{\mathrm{EEC}}}{\mathrm{d}z} \overset{z \to 1}{=} \sum_q H_{q\bar{q}}(Q) \int_0^\infty \mathrm{d}(b_T Q)^2 \, J_0\left(b_T Q \sqrt{\bar{z}}\right) J_q^{\mathrm{EEC}}(b_T) \, J_{\bar{q}}^{\mathrm{EEC}}(b_T) \, S_q(b_T)$$

[More in Damiano's talk for Sudakov factorization for energy correlators in top decays]

Analysis of IRC unsafety in QQC

The problematic contribution starts at NLO / two loop: a soft gluon splits into $q\bar{q}$.



Tree-level double-soft emission

$q, \bar{q}$: soft pair
$q', \bar{q}'$: hard pair

Correlated pairs:

Measured soft pair $(q, \bar{q})$

↓

IRC unsafe at generic angles

Power suppressed in the back-to-back limit

Measured hard pair $(q', \bar{q}')$

↓

Sensitive to soft transverse recoil

Requires the TMD soft function $S_q(b_T)$

▹ Suppression of double-soft current in the back-to-back limit
[Catani, Grazzini, 1999; Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler, 2306.07314]

$$|\mathcal{M}_{q\bar{q}}|^2 \propto \frac{2k_{T,q}k_{T,\bar{q}} - (k_{T,q}^2 + k_{T,\bar{q}}^2)\cos\varphi + |\vec{k}_{T,q} - \vec{k}_{T,\bar{q}}|^2 \cosh Y}{k_{T,q}k_{T,\bar{q}}\,(k_{T,q}^2 + k_{T,\bar{q}}^2 + 2k_{T,q}k_{T,\bar{q}}\cosh Y)^2(\cosh Y - \cos\varphi)^2} \xrightarrow{Y\gg 1} \boxed{\frac{|\vec{k}_{T,q} - \vec{k}_{T,\bar{q}}|^2}{k_{T,q}^3 k_{T,\bar{q}}^3}\, e^{-3Y}}$$

Back-to-back QQC from dihadron production

For $\bar{z} = 1 - z \simeq q_T^2/Q^2 \ll 1$, sum dihadron production with charge weights:

$$\frac{\mathrm{d}\sigma^{\mathrm{QQC}}}{\mathrm{d}z} \overset{z\to 1}{=} \sum_{h_a,h_b} Q_{h_a} Q_{h_b} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int \mathrm{d}^2 q_\perp \, \delta\left(\bar{z} - \frac{q_\perp^2}{Q^2}\right) \frac{\mathrm{d}\sigma_{e^+e^-\to h_a h_b + X}}{\mathrm{d}x_a \, \mathrm{d}x_b \, \mathrm{d}^2 q_\perp}$$

- Same TMD factorization as for the EEC: [Collins, Soper, Nucl. Phys. B 193 (1981) 381]

$$\frac{\mathrm{d}\sigma_{e^+e^-\to h_a h_b + X}}{\mathrm{d}x_a \, \mathrm{d}x_b \, \mathrm{d}^2 q_\perp} \overset{z\to 1}{=} \sum_q \sigma_0^q H_{q\bar{q}}(Q) \int \frac{\mathrm{d}^2 \vec{b}_T}{(2\pi)^2} \, e^{\mathrm{i} q_\perp \cdot \vec{b}_T} \, D_{h_a/q}(x_a, b_T) D_{h_b/\bar{q}}(x_b, b_T) S_q(b_T)$$

Soft charges cancel or are power suppressed; soft recoil gives the same S_q.

- At small b_T, TMD matching and the charge sum rule give [Collins, Soper, Nucl. Phys. B 194 (1982) 445]

$$\begin{aligned} J_q^{\mathrm{QQC}}(b_T) &\equiv \sum_h Q_h \int_0^1 \mathrm{d}x \, D_{h/q}(x, b_T) \\ &= \sum_i \int_0^1 \mathrm{d}t \, \mathcal{K}_{qi}(t, b_T) \underbrace{\left[\sum_h Q_h \int_0^1 \mathrm{d}\xi \, d_{h/i}(\xi)\right]}_{= Q_i \text{ (charge sum rule)}} + \mathcal{O}\left(b_T^2 \Lambda_{\mathrm{QCD}}^2\right) \end{aligned}$$

The longitudinal FFs drop out due to charge sum rule, exposing the parton charge.

The jet function becomes a finite perturbative charge moment of the matching kernel at leading power in small b_T.

Factorization and QQC jet function to three-loop

- Leading-power factorization for $\bar{z} = 1 - z \ll 1$:

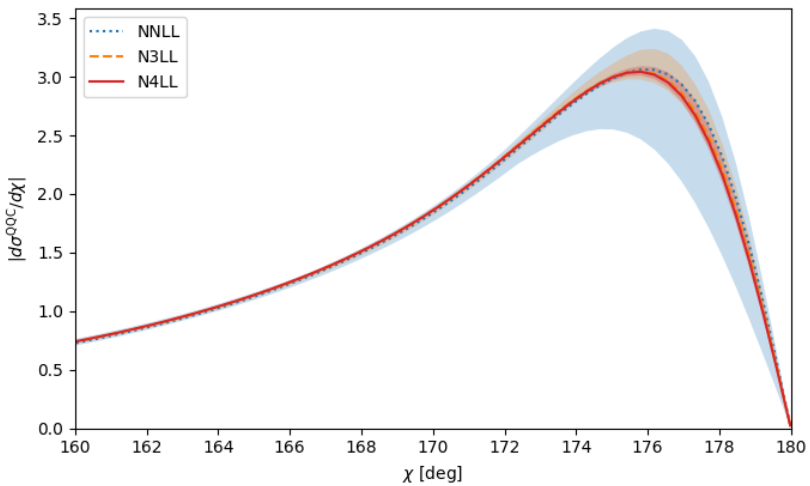
$$\frac{\mathrm{d}\sigma^{\mathrm{QQC}}}{\mathrm{d}z} \overset{z \to 1}{=} \sum_q \frac{\sigma_0^q}{2} H_{q\bar{q}}(Q,\mu) \int_0^\infty \mathrm{d}(b_T Q)^2 \, J_0\left(b_T Q \sqrt{\bar{z}}\right) J_q^{\mathrm{QQC}}(b_T) \, J_{\bar{q}}^{\mathrm{QQC}}(b_T) \, S_q(b_T)$$

The genuinely new object: the QQC jet function:

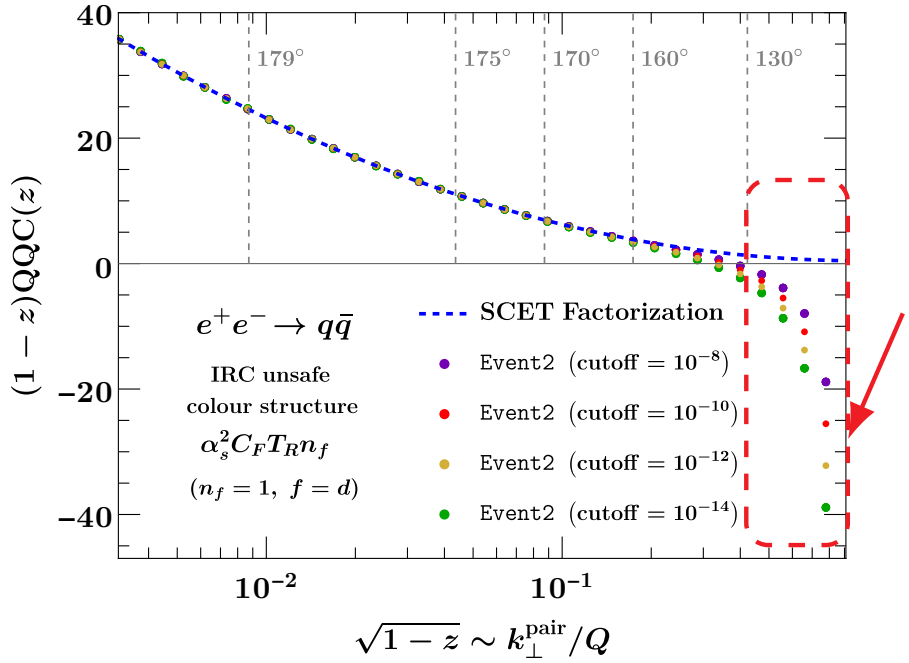
$$J_q^{\mathrm{QQC}}(b_T) = \sum_i Q_i \int_0^1 \mathrm{d}t \, \mathcal{K}_{qi}(t, b_T) = \sum_{q'} Q_{q'} \int_0^1 \mathrm{d}t \left[\mathcal{K}_{qq'}(t, b_T) - \mathcal{K}_{q\bar{q}'}(t, b_T)\right]$$

$$\lim_{x\to 0} \mathcal{K}_{qi} = O\left(1/x\right) \qquad \lim_{x\to 0} \left[\mathcal{K}_{qq'} - \mathcal{K}_{q\bar{q}'}\right] = O\left(x^0\right)$$

- Opposite charges cancel the common small-x singularity. The difference is integrable through three loops. [Luo, Yang, Zhu, Zhu, arXiv:2012.03256; Ebert, Mistlberger, Vita, arXiv:2012.07853]
- New three-loop structure $N_{F,V}$ contributing to a finite charge asymmetry
  - Hard function known to four loops
  - Soft function known to three loops
  - QQC jet function calculated to three loops
- RG consistency checked to three loops
- Determines the resummation to N^{4}LL

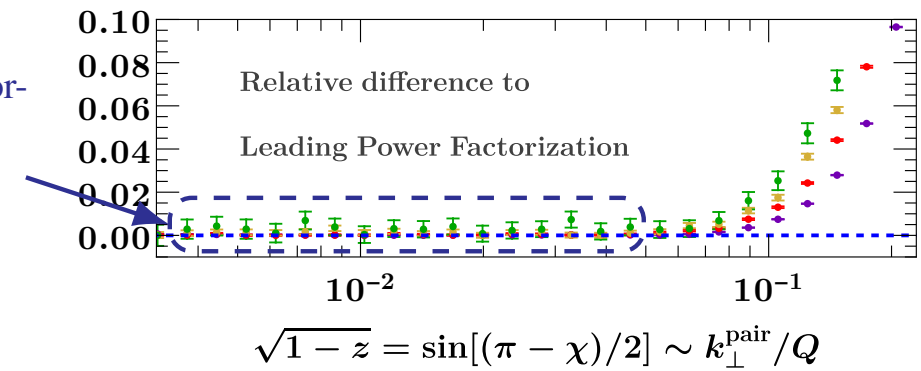


Fixed-order check with Event2



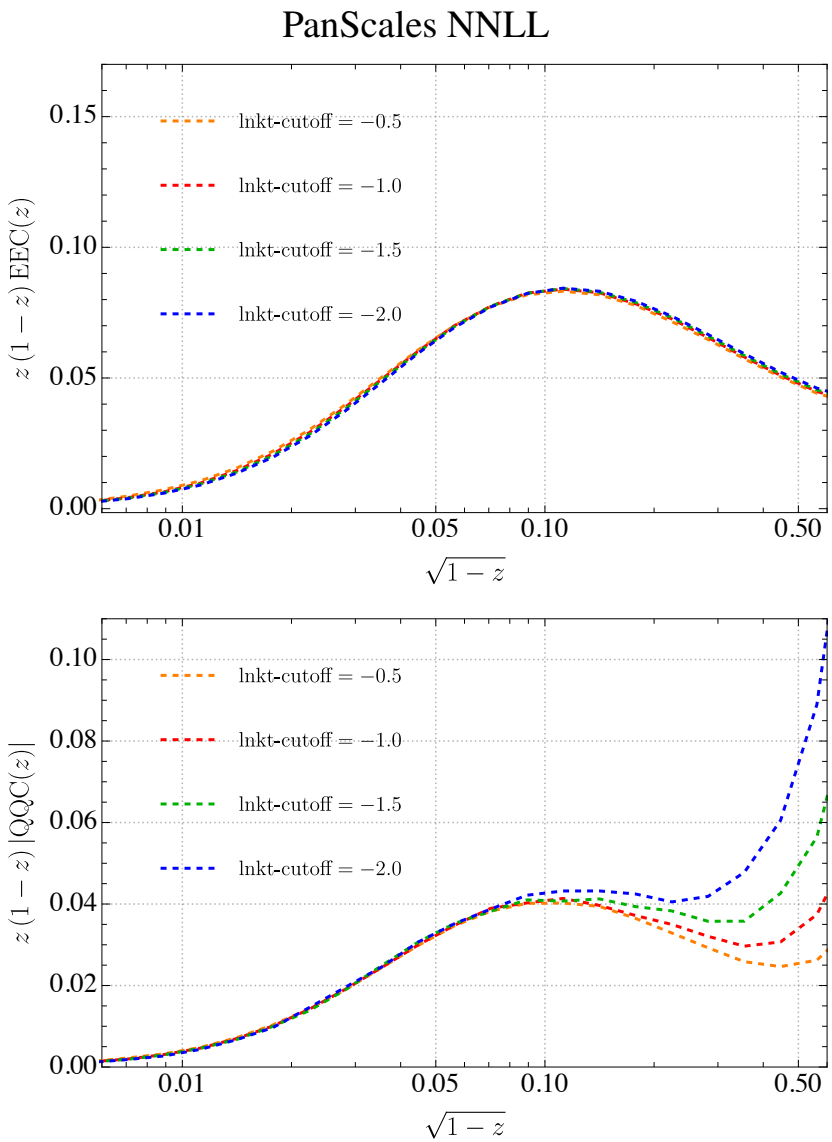
large cut-off dependence
away from back-to-back

perfect agreement with factor-
ization at the back-to-back



Leading-power safety to all orders

EEC and QQC in the PanScales parton shower at NNLL, including the $g \to q\bar{q}$ splittings.



▹ Residual IR unsafety would produce logarithmic dependence on the shower cutoff as it is lowered.

▹ EEC distribution shows no dependence on the cutoffs, as expected for an IRC-safe observable.

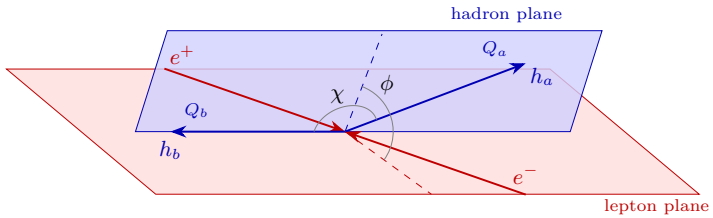
▹ The QQC curves converge as $z \to 1$, while cutoff dependence becomes large away from this limit.

Validated IRC safety in the back-to-back limit and the factorization. Consider possible applications.

Azimuthal QQC probes Collins fragmentation

Keep the angle ϕ between the detector and lepton planes, with $\bar{z} = 1 - z \ll 1$.

$$\frac{\mathrm{d}\Sigma_{\mathrm{QQC}}}{\mathrm{d}z\,\mathrm{d}\phi} = F_{UU}(z) + \cos(2\phi)\, F_{\mathrm{Coll}}(z), \qquad A^{\mathrm{Coll}}_{\mathrm{QQC}}(z) = \frac{F_{\mathrm{Coll}}(z)}{F_{UU}(z)}$$



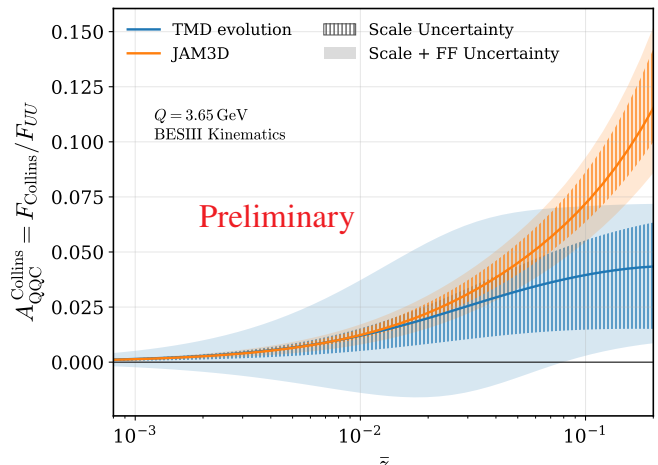
- Leading-power TMD factorization, with the soft factor absorbed into the jets:

$$\frac{\mathrm{d}\Sigma_{\mathrm{QQC}}}{\mathrm{d}z\,\mathrm{d}\phi} \overset{z\to 1}{=} \sum_q \sigma_0^q H_{q\bar{q}} \int_0^\infty \frac{\mathrm{d}(b_T Q)^2}{8\pi} \Big[J_0(b_T Q\sqrt{\bar{z}})\, J_q^{\mathrm{QQC}}\, J_{\bar{q}}^{\mathrm{QQC}} + \cos(2\phi)\, \frac{b_T^2}{8}\, J_2(b_T Q\sqrt{\bar{z}})\, J_q^{\mathrm{QQC},\perp}\, J_{\bar{q}}^{\mathrm{QQC},\perp} \Big]$$

- The charge-weighted Collins jet function

$$J_q^{\mathrm{QQC},\perp}(b_T) = \sum_h Q_h \int_0^1 \mathrm{d}x\, \widetilde{H}^{\perp}_{1,h/q}(x, b_T)$$

At small b_T, it matches onto twist-three FFs.



- Charge weighting combines opposite-sign π^+ and π^- Collins functions.
- JAM3D and TMD evolution matched to the twist-three Collins function agree in the dijet window.

Collins charge correlations in DIS

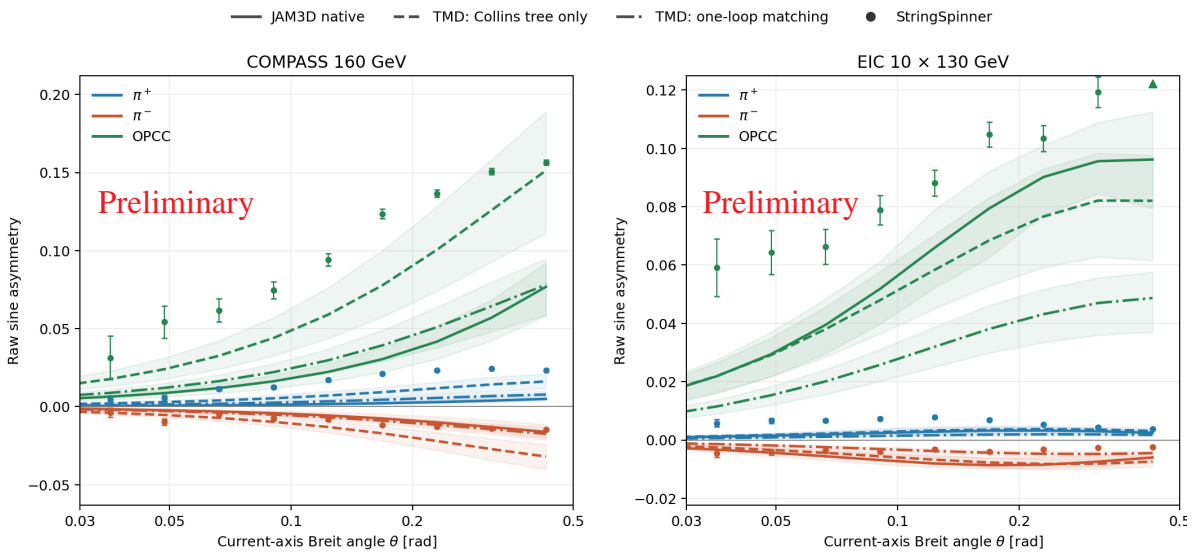
[W. Li, Z. Kang and Z. Xu, in progress]

- Start with SIDIS $\ell(l) + p(P, S_T) \to \ell(l') + h(P_h) + X$, $q = l - l'$, $|S_T| = 1$ transverse to q
- The unpolarized and Collins structure coupled to transversity, convention from [Z. Kang, A. Prokudin, P. Sun, and F. Yuan, Phys. Rev. D 93 (2016) 014009]

$$\mathrm{d}\sigma^h = \mathrm{d}\sigma_0 \left[F^h_{UU} + D_{NN}(y) \sin(\phi_h + \phi_S)\, F^h_{UT} + \cdots \right]$$

- Sum over hadrons weighted by charge Q_h:

$$\frac{\mathrm{d}\Sigma_Q}{\mathrm{d}\theta\, \mathrm{d}\phi} = \sum_h \int \mathrm{d}\sigma^h\, Q_h\, \delta(\theta - \theta_h)\, \delta(\phi - \phi_h)$$
$$= U_Q(\theta) + \sin(\phi_h + \phi_S)\, V_Q(\theta)$$



- unpolarized structure reduced
- polarized structure builds up
- consider breakdown of two effects in asymmetry

Summary

QQC is IRC unsafe at generic angles, but leading-power safe in the back-to-back limit.

Back-to-back factorization for QQC and determined the new charge jet function through three loops.

Checked RG consistency and fixed-order agreement with Event2. PanScales at NNLL provides numerical evidence for leading-power safety beyond fixed order.

Charges and angles measured on tracks connect perturbative QCD, spin-dependent fragmentation and nucleon structure.

Thank you for your attention.

Acknowledgement

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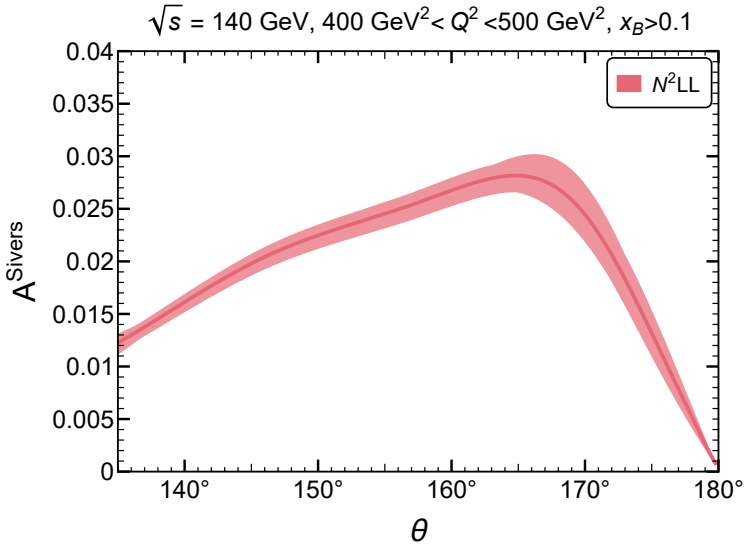
Nucleon spin with charge correlator — backup

- Charge signs and track directions resolve two different spin correlations.

Sivers in transversely polarized DIS

$$\sin(\phi - \phi_S)\, F_{UT}, \qquad F_{UT} \sim f_{1T}^{\perp} \otimes J_Q$$

Modulation from Sivers function coupled to the same perturbative jet function above.



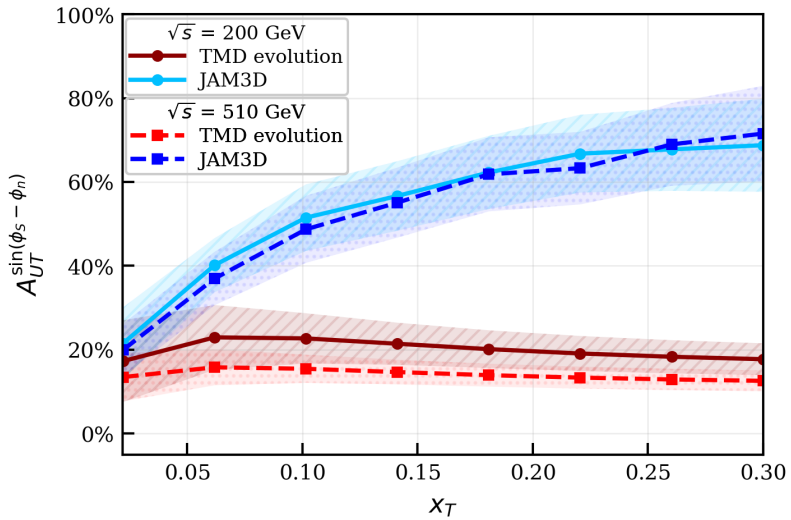
Breit-frame back-to-back limit. Sivers asymmetry at N^{2}LL.

[H. Cao, X. Liu, F. Petriello, arXiv:2605.15280]

Transversity via Collins in $p^{\uparrow}p$

$$\sin(\phi_s - \phi_n)\, Z_{UT}^{Q}, \qquad Z_{UT}^{Q} \sim h_1 \otimes J_Q^{\perp}$$

Transversity couples to the Collins charge dipole, which retains a nonperturbative fragmentation moment.



Charge flow around the jet axis. Charged-pion estimates.

[W. Li, X. Liu, D. Y. Shao, arXiv:2607.16392]