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Factorization and virtuality evolution of jet functions in nuclear collisions

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In collaboration with Carlos Salgado and Bin Wu

21/10/26

Based on
[2609.35653]

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Yoctosecond imaging of QCD collectivity using jet observables
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Introduction and motivation

Jets are fundamental probes of the nuclear matter created in heavy ion collisions



The **virtuality evolution**, such as TMD evolution, **of vacuum jets is well understood**

A consistent treatment of the **virtuality evolution** of a particle **inside a QCD medium is still pending**

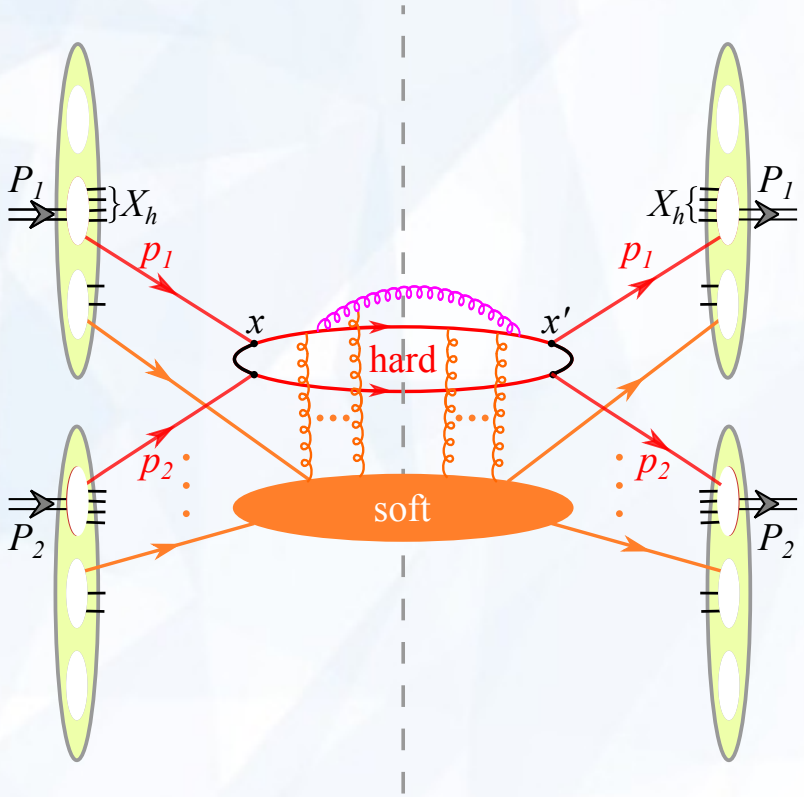
Cao and Majumder [1712.10055]

We develop a factorization formalism to compute virtuality resolved jet functions in HI collisions using standard perturbative techniques

Armesto, Cougolic and Wu [2407.19243]



The collision setup



We focus on $\gamma + Jet$ events

We model the colliding nuclei as **uncorrelated nucleons: Glauber model**

Miller et al. [nucl-ex/0701025] **Kovchegov and Levin [10.1017]**

We **expand the cross section at LO in $Q = p_T$ and medium length**, with initial state effects absorbed in the nPDF

The hard process is point-like so **underlying event only couples to the jet and its radiation**

The factorized cross section

Thickness function times PDF

Projecting up to leading order in Q

$$\frac{d\sigma}{d^2\mathbf{b}dy_J d^2\mathbf{p}_T d\eta_\gamma d^2\mathbf{p}_\gamma} = \sum_{ijk}\int d\xi d\xi' \int d^2\mathbf{X} T_{i/A_1}(\mathbf{X},\xi) T_{j/A_2}(\mathbf{X}-\mathbf{b},\xi')$$

$$\times \int \frac{d^4p_I}{(2\pi)^4} J_k(p_I;p_J,\mathbf{X}) \frac{d\hat{\sigma}_{ij\to\gamma k}}{d\eta_I d^2\mathbf{p}_I d\eta_\gamma d^2\mathbf{p}_\gamma}(\xi P_1, \xi' P_2 \to p_\gamma, p_k)$$

Jet function



Hard cross section with the initial jet momentum p_I^μ replaces by the on-shell momentum p_k^μ

The jet function

Contains all the **information about the jet evolution** and interaction with the remaining nucleons

$$J(p_I; p_J; \mathbf{X}) \equiv \int d^4x\, e^{ip_I\cdot x} \int \prod_{l\in\mathcal{X}} [d\Gamma_{k_l}] J(x/2, -x/2; p_J, \{k_l\}; \mathbf{X})$$

with

$$J(x_0', x_0; p_J, \{k_l\}; \mathbf{X}) = \frac{P_{\alpha\beta}(p_I)}{d_C} \langle\langle 0 \,|\, \bar{T}\Phi_\alpha^c(x_0') \,|\, p_J, \{k_l\}\rangle\langle p_J, \{k_l\} \,|\, T\Phi_\beta^{\dagger c}(x_0) \,|\, 0\rangle\rangle$$

Double brackets indicate **medium configuration average**, applicable to both hot and cold nuclear matter

Calculation of the jet function

Jet functions are computed on the **jet system of coordinates**

$$p_I^+ \equiv \bar{n}\cdot p_I/\sqrt{2}, \qquad p_I^- \equiv n\cdot p_I/\sqrt{2} = \frac{m_I^2}{2p_I^+}, \qquad p_I^\mu = \left(p_I^+, \frac{m_I^2}{2p_I^+}, \mathbf{0}\right)$$

We consider that the hard process happens on average at $X^+ = 0$ and the medium is created after a **thermalization time** τ_0 and lives for L^+

We choose typical values $\tau_0 = 0.85$ fm and $L^+ = 14$ fm. We estimate $\hat{q} = 3T^3$ and use $T = 0.2$ GeV.

Apollinario, Lee and Winn [2203.16352]

The jet-medium interaction

Similarly one can use **GBW for cold nuclear matter effects**

We model the interaction with the bulk using **background gluon fields** within the **BDMPS-Z formalism**

$$\langle A^{a-}(x_1^+, \mathbf{x}) A^{b-}(x_2^+, \mathbf{y}) \rangle = \delta(x_1^+ - x_2^+) \delta^{ab} n(x^+) \sigma(\mathbf{x} - \mathbf{y})$$

and the **harmonic oscillator approximation**

$$g^2 C_R n(x^+)[\sigma(\mathbf{0}) - \sigma(\mathbf{x})] \approx \frac{1}{4\sqrt{2}} \hat{q}_R(x^+) \mathbf{x}^2$$

Baier et al. [hep-ph/9608322] Baier et al. [hep-ph/9607355] Zakharov [hep-ph/9607440]

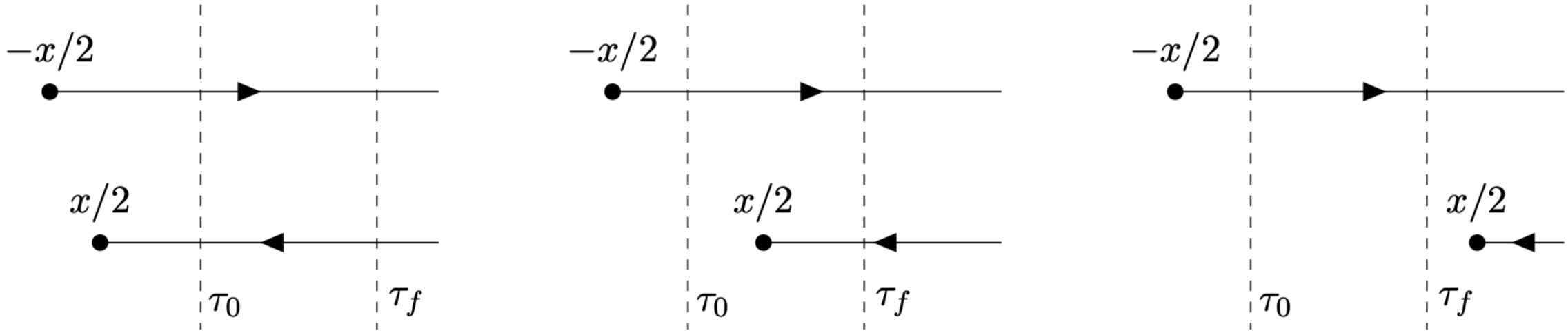
Longitudinal momentum is conserved

$$J_k(p_I; p_J, \mathbf{X}) \equiv \int d^3x \, e^{ip_I \cdot x} \int \prod_{l \notin jet} [d\Gamma_{k_l}] \, 2\pi\delta(p_I^+ - p_J^+ - \sum_{l \notin jet} k_l^+) \tilde{J}_k(x/2, -x/2; p_J, \{k_l\}; \mathbf{X})$$

Jet functions at LO

Transverse **momentum broadening**

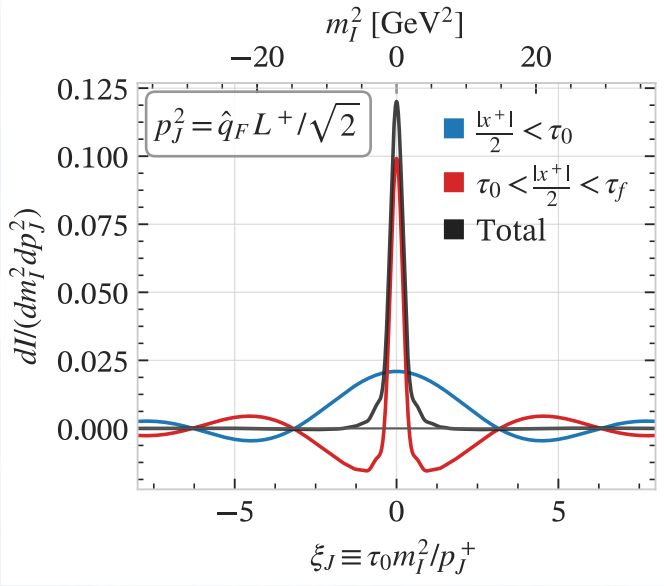
$$\frac{dI_k^{LO}}{dm_I^2 d^2\mathbf{p}_J} = \int \frac{d^3x}{(2\pi)^3 2p_J^+} e^{i\frac{m_I^2}{2p_J^+}x^+} \tilde{J}_k^{LO}(x/2, -x/2; p_J, \mathbf{X})$$

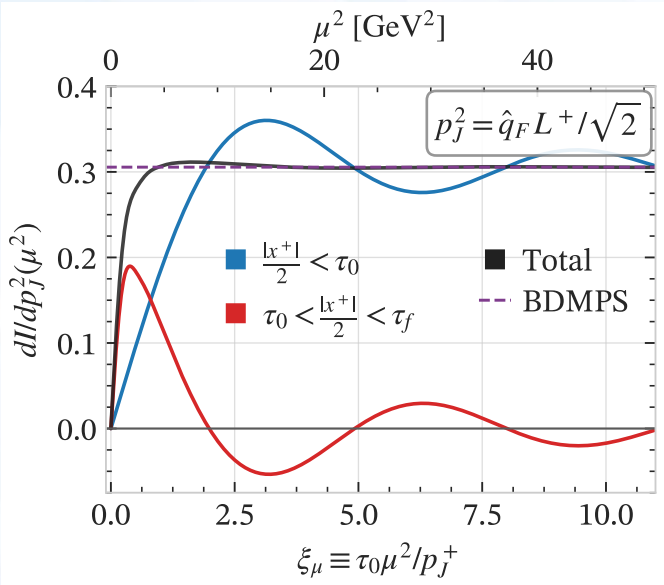


(a) $|x^+| < 2\tau_0$ (b) $2\tau_0 < |x^+| < 2\tau_f$ (c) $2\tau_f < |x^+|$

Transverse momentum distribution

$$I(\mu^2) \equiv \int_{-\mu^2}^{\mu^2} dm_I^2 \frac{dI}{dm_I^2}$$





- Destructive interference between inside and before cases

Baier et al. [hep-ph/9608322]

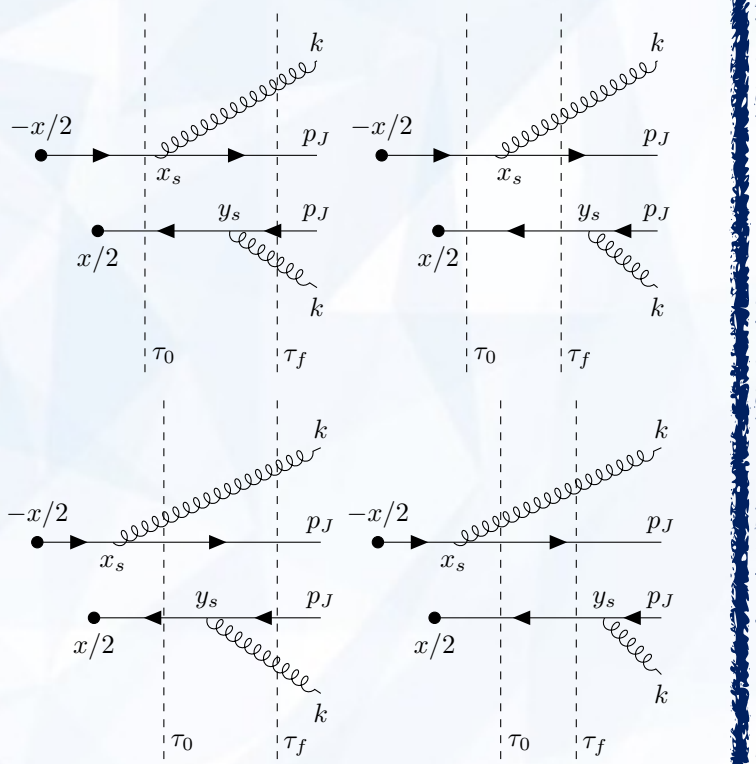
- Before case **converges to BDMPS momentum broadening spectrum** when integrating over virtuality while inside case vanishes
- Total jet function already **reaches the BDMPS-Z result for** $\xi_\mu \equiv 1$

If $t_J \equiv p_J^+/|m_I^2| \ll \tau_0$ **the jet can not enter the medium without radiating** and will not contribute to the LO spectrum

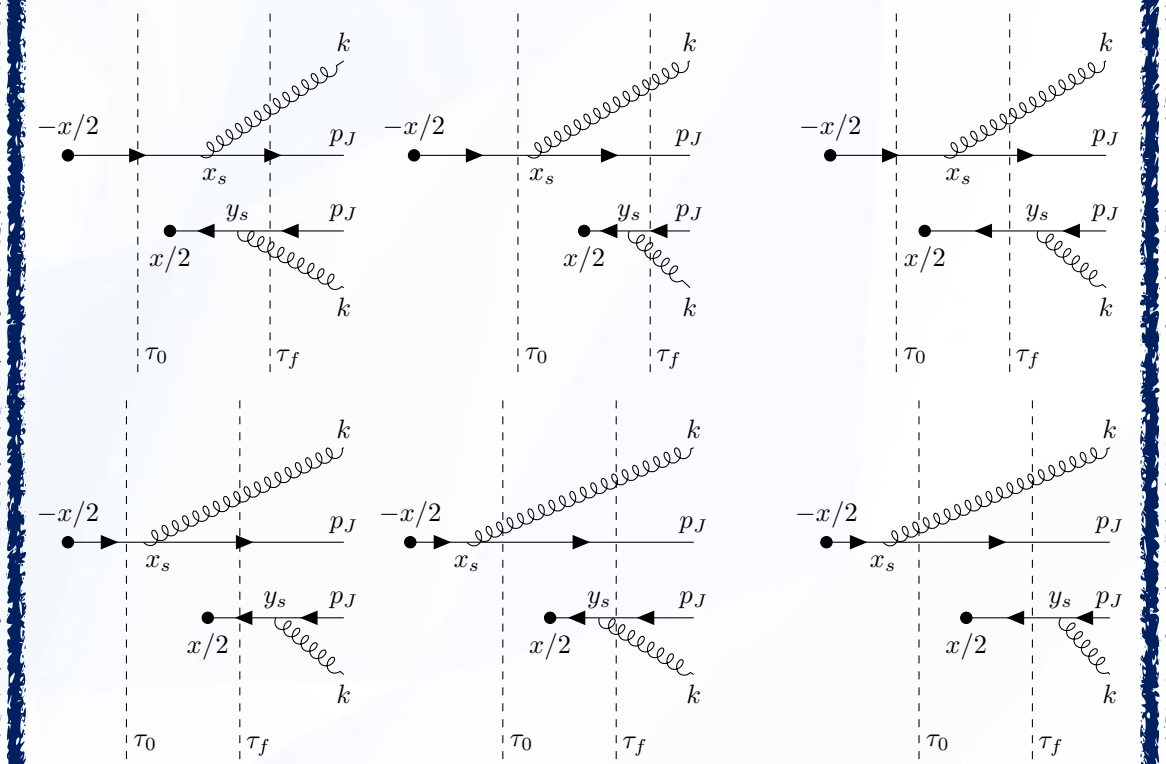
Jet functions at NLO

Medium-induced soft gluon radiation spectrum

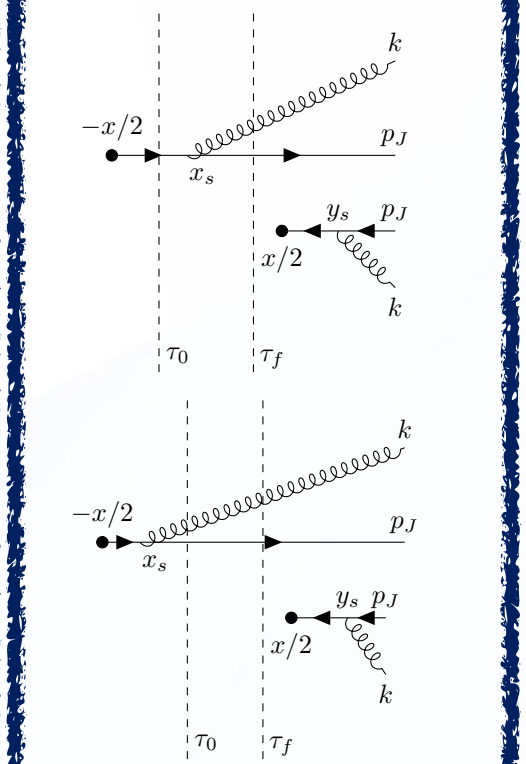
$$k^+\frac{dI^{NLO}}{dk^+dm_I^2}=\frac{1}{4\pi}\int\frac{d^3x}{(2\pi)2p_J^+}e^{i\frac{m_I^2}{2p_J^+}x^+}\int\frac{d^2\mathbf{p}_J}{(2\pi)^2}\frac{d^2\mathbf{k}}{(2\pi)^2}[\tilde{J}_q^{NLO}(x/2,-x/2;p_J,k)-(vac)]$$



$|x^+|<2\tau_0$

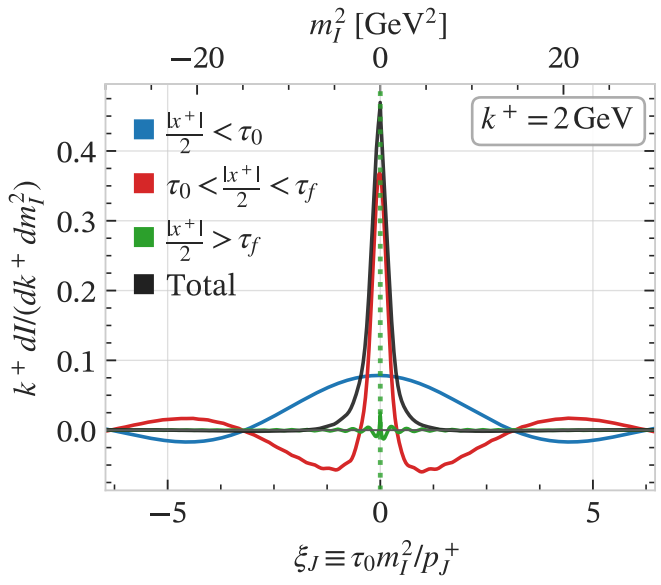


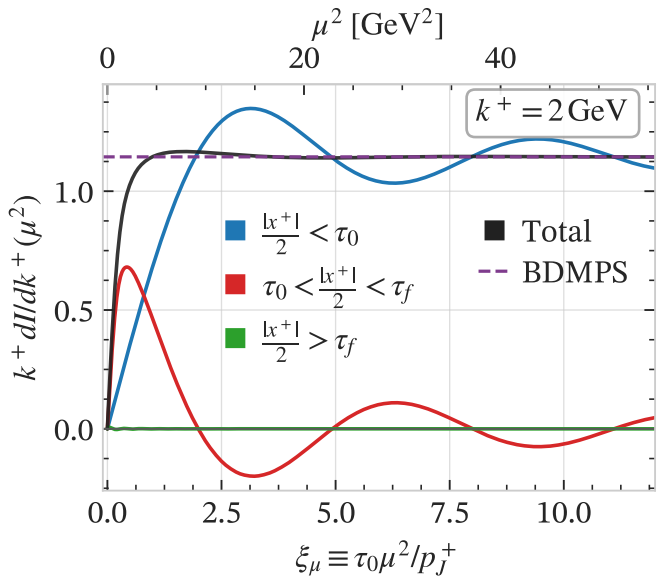
$2\tau_0<|x^+|<2\tau_f$



$|x^+|<2\tau_f$

Soft gluon radiation spectrum





- Vacuum emissions have been subtracted, only medium induced radiation

Andres et al. [2211.10161]

- Before case **converges to BDMPS-Z result** when integrating over virtuality while inside case vanishes
- Total jet function already **reaches the BDMPS-Z result for** $\xi_\mu \equiv 1$

If $t_J \equiv p_J^+/m_I^2 \ll \tau_0$ **the jet radiates before entering the medium** and medium induced radiation has to come at higher orders in α_s

The general picture of the heavy ion-collision

- Jets are created with **high initial virtualities** and they will generate **vacuum-like parton showers** in the very early times Caucal et al. [1801.09703]
- If jets **interact with pre-equilibrium matter** there will be a **competition between medium-induced and vacuum emissions**. The relation between λ_R and t_J parametrically indicates which effect is dominating
- Jet partons can also **exchange virtuality with the medium** besides from loosing virtuality via splittings



A detailed calculation of the virtuality evolution of jets inside the medium requires going further from the BDMPS-Z formalism

Limitations of the BDMPS-Z formalism

In the BDMPS-Z formalism **interactions with the medium are instantaneous**

$$\langle A^{a-}(x_1^+, \mathbf{x}) A^{b-}(x_2^+, \mathbf{y}) \rangle \propto \delta(x_1^+ - x_2^+),$$

so by means or the uncertainty principle $\Delta x^+ \Delta q^- \sim 1$ **the virtuality transfer between the medium and the jet is not regulated by the medium characteristic scales**



A **consistent calculation of virtuality evolution** inside the medium requires relaxing the instantaneous scattering approximation and **develop a q^- dependent potential**



A similar problem appears in small-x calculations in the shockwave picture

Conclusions…

- Developed a **factorization formalism** that allows to compute **virtuality resolved jet functions** inside a QCD medium
- Computed the **virtuality differential momentum broadening** (at LO) and **medium-induced radiation** (at NLO) spectra
- Identified the ratio between the jet formation time $t_J \sim p_J^+ / |m_J^2|$ and the medium formation time τ_0 as the **fundamental scale controlling which virtualities contribute to the jet function** at fixed order

… and outlook

- Relax the BDMPS-Z correlator to study the **virtuality evolution of jets inside the medium**
- Go beyond fixed order calculations and develop **in-medium parton showers**
- Use the factorized cross section to **estimate phenomenological observables**

BACKUP SLIDES!

The BDMPS-Z formalism

Baier et al. [hep-ph/9608322]
Baier et al. [hep-ph/9607355]
Zakharov [hep-ph/9607440]
Casalderrey and Salgado [0712.3443]

Particle propagation is expressed in terms of the **dressed propagator**

$$G(x;y;p^+) = \int_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(x^+)=\mathbf{x}} \mathscr{D}\mathbf{r} \exp\left\{ i\frac{p^+}{2}\int_{y^+}^{x^+} dt\dot{\mathbf{r}}^2 \right\} W(\mathbf{r};x^+,y^+)$$

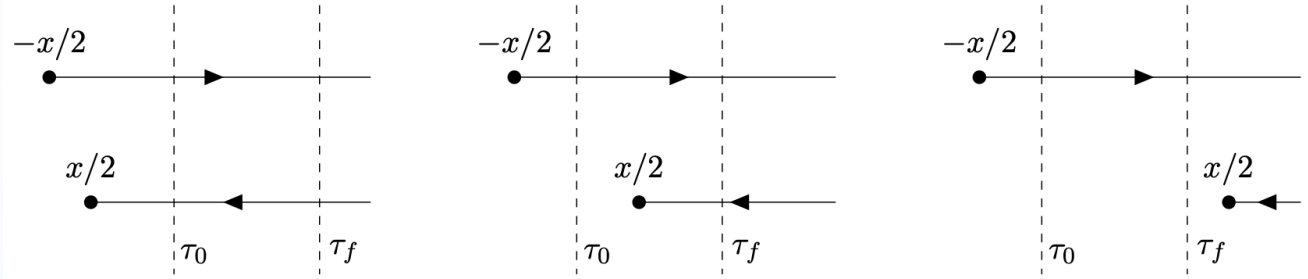
with the Wilson line

$$W(\mathbf{r};t_2,t_1) \equiv \mathscr{P}\exp -ig\int_{t_1}^{t_2} dx^+ A^-(x^+,\mathbf{r})$$

We will use the harmonic oscillator approximation

$$\frac{1}{d_R}Tr\langle W^\dagger(\mathbf{y})W(\mathbf{x})\rangle = \exp\left\{ -\frac{1}{4\sqrt{2}}\int dx^+ \hat{q}_R(x^+)(\mathbf{y}-\mathbf{x})^2 \right\} \qquad \frac{1}{d_R}Tr\langle W(\mathbf{x})\rangle = \exp\left\{ -\frac{1}{2}\int_{t_1}^{t_2}\frac{dt}{\lambda_R(t)} \right\}$$

Jet functions at LO



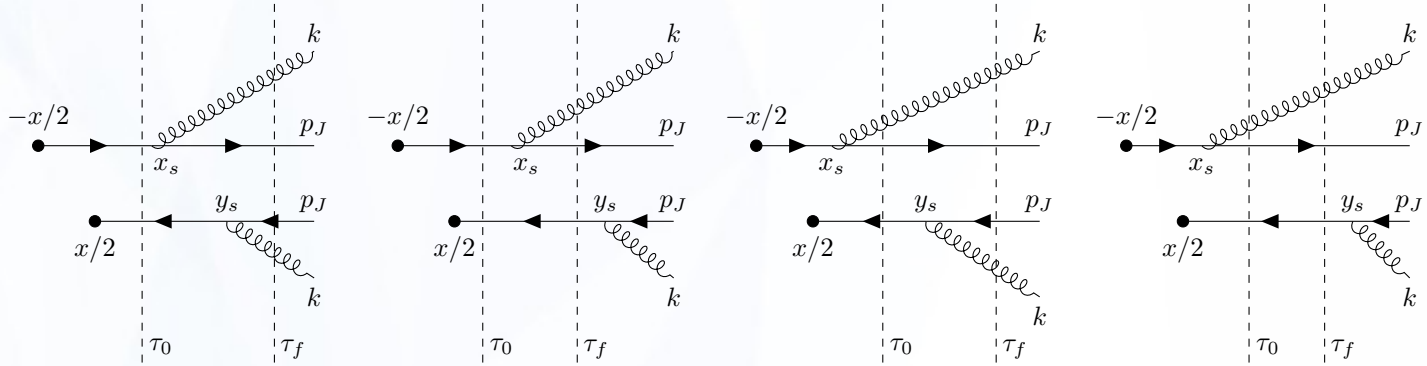
(a) $|x^+| < 2\tau_0$ (b) $2\tau_0 < |x^+| < 2\tau_f$ (c) $2\tau_f < |x^+|$

$$\frac{dI_{|x^+|<2\tau_0}}{dm_I^2 d\mathbf{p}_J^2} = \frac{1}{\pi m_I^2} \sin\left(\frac{m_I^2}{p_J^+}\tau_0\right) \frac{\sqrt{2}}{\hat{q}_R L^+} \exp\left\{-\frac{\sqrt{2}}{\hat{q}_R L^+}\mathbf{p}_J^2\right\}$$

$$\frac{dI_{2\tau_0<|x^+|<2\tau_f}}{dm_I^2 d\mathbf{p}_J^2} = 2\Re \int_{2\tau_0}^{2\tau_f} \frac{dx^+}{2p_J^+(2\pi)} e^{i\frac{m_I^2}{2p_J^+}x^+} e^{-\frac{\frac{x^+}{2}-\tau_0}{2\lambda_R}} \frac{\sqrt{2}}{\hat{q}_R(\tau_f - \frac{x^+}{2})} \exp\left\{-\frac{\sqrt{2}}{\hat{q}_R(\tau_f - \frac{x^+}{2})}\mathbf{p}_J^2\right\}$$

$$\frac{dI_{2\tau_f<|x^+|}}{dm_I^2 d\mathbf{p}_J^2} = \delta(\mathbf{p}_J^2) e^{-\frac{L^+}{2\lambda_R}} \left[\delta(m_I^2) - \frac{1}{\pi m_I^2} \sin\left(\frac{m_I^2}{p_J^+}\tau_f\right)\right]$$

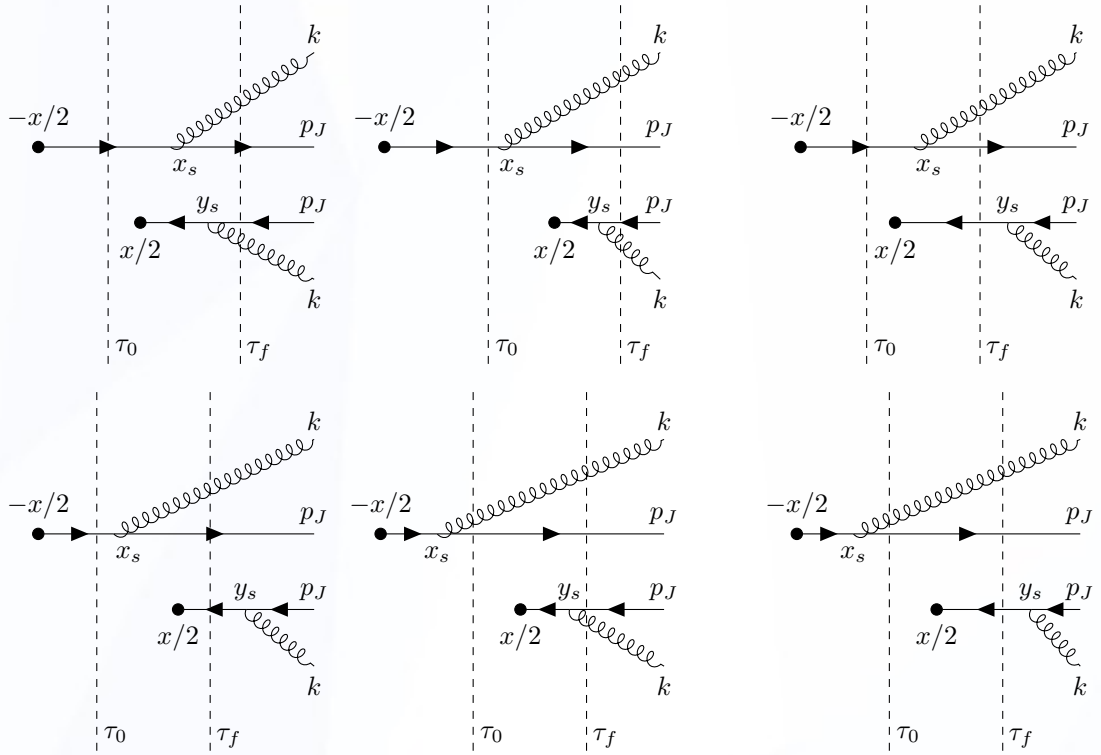
Jet functions at NLO



$$k^+\frac{dI_{|x^+|<2\tau_0}}{dk^+dm_I^2}=\frac{2\alpha_s C_R}{\pi}\Re\int_{-2\tau_0}^{2\tau_0}\frac{dx^+}{2p_J^+2\pi}e^{i\frac{m_I^2}{2p_J^+}x^+}\log\left[\cos(\Omega L^+)-\left(\tau_0+\frac{x^+}{2}\right)\Omega\sin(\Omega L^+)\right]$$

$$k^+\frac{dI_{|x^+|<2\tau_0}}{dk^+dm_I^2}=\frac{2\alpha_s C_R}{\pi}\Re\frac{1}{2\pi i}\frac{1}{m_I^2}\left\{e^{i\frac{m_I^2}{p_J^+}\tau_0}\log\left(\cos(\Omega L^+)-2\Omega\tau_0\sin(\Omega L^+)\right)-e^{-i\frac{m_I^2}{p_J^+}\tau_0}\log(\cos(\Omega L^+))-e^{-i\frac{m_I^2}{p_J^+}\left(\tau_0-\frac{\cot(\Omega L^+)}{\Omega}\right)}\right.$$
$$\left.\times\left[E_1\left(-i\frac{m_I^2}{p_J^+}\left(\frac{\cot(\Omega L^+)}{\Omega}-2\tau_0\right)\right)-E_1\left(-i\frac{m_I^2}{p_J^+}\frac{\cot(\Omega L^+)}{\Omega}\right)\right]\right\}$$

Jet functions at NLO

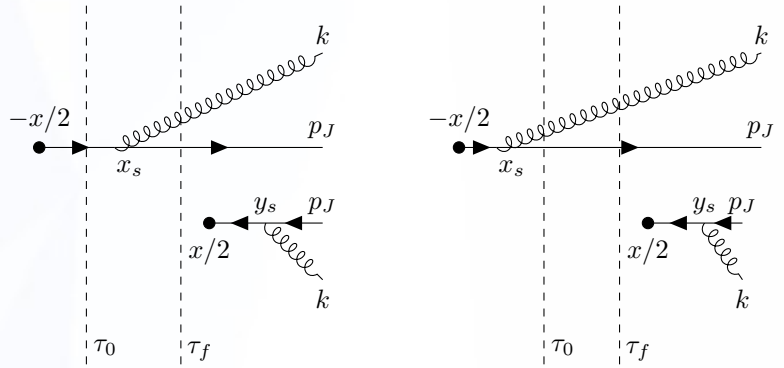


$$k^+\frac{dI^{in-in,|x^+|>2x_s^+}_{2\tau_0<|x^+|<2\tau_f}}{dk^+dm_I^2} = \frac{\alpha_s C_R}{(k^+)^2}2\Re\int_{2\tau_0}^{2\tau_f}\frac{dx^+}{2p_J^+2\pi}e^{i\frac{m_I^2}{2p_J^+}x^+}e^{-\frac{\frac{x^+}{2}-\tau_0}{2\lambda_R}}2\Re\int_{\tau_0}^{x^+/2}dx_s^+$$

$$\times\int_{x^+/2}^{\tau_f}dy_s^+\,\nabla_{\mathbf{u}}\cdot\nabla_{\mathbf{v}}\left[\int d^2\mathbf{x_0}\,\mathscr{K}\left(y_s^+,\mathbf{v};\frac{x^+}{2},\mathbf{x_0};k^+\right)\tilde{\mathscr{K}}\left(\frac{x^+}{2},\mathbf{x_0};x_s^+,\mathbf{u};k^+\right)-\mathscr{K}_0(y_s^+,\mathbf{v};x_s^+,\mathbf{u};k^+)\right]_{\mathbf{u}=\mathbf{v}=\mathbf{0}}$$

No analytic closed form

Jet functions at NLO



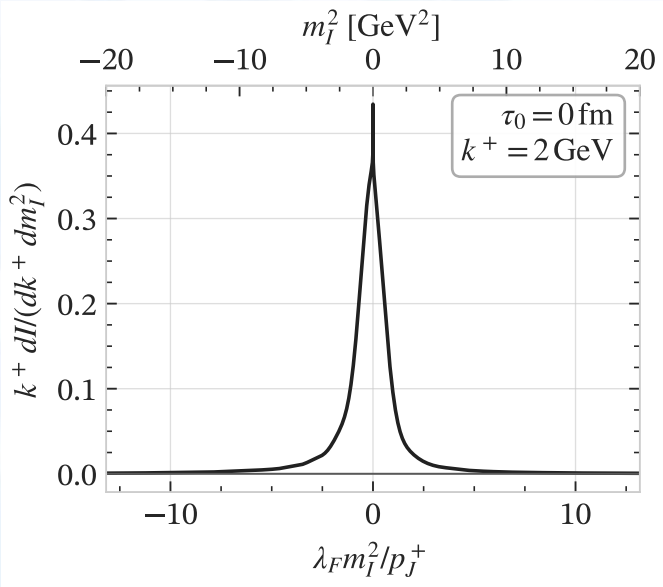
$$d_\pm = \frac{2 - \frac{\Omega}{\sqrt{2}}(\tau_f+\tau_0)\tan(\frac{\Omega L^+}{\sqrt{2}}) \pm \sqrt{4(1+\tan^2(\frac{\Omega L^+}{\sqrt{2}})) + \frac{\Omega^2}{2}(\tau_f+\tau_0)^2\tan^2(\frac{\Omega L^+}{\sqrt{2}})}}{\sqrt{2}\Omega\tan(\frac{\Omega}{\sqrt{2}}L^+)}$$

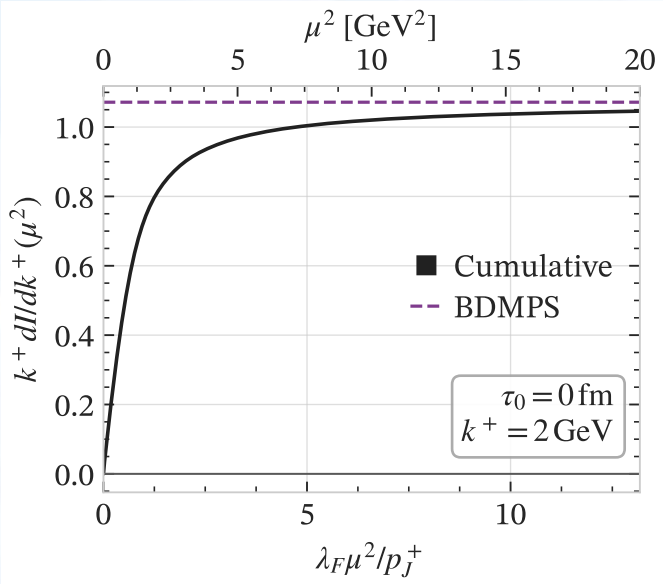
$$k^+\frac{dI_{2\tau_f<|x^+|}}{dk^+dm_I^2} = \frac{2\alpha_s C_R}{\pi}e^{\frac{-L^+}{2\lambda_R}}\Re\int_{2\tau_f}^{+\infty}\frac{dx^+}{2p_J^+2\pi}e^{i\frac{m_I^2+i\varepsilon}{2p_J^+}x^+}\log\left[\frac{\frac{\Omega}{\sqrt{2}}x^+\left(1-\Sigma_0\frac{\Omega}{\sqrt{2}}\tan(\frac{\Omega}{\sqrt{2}}L^+)\right)}{\frac{\Omega}{\sqrt{2}}(x^+-L^+)+\left(1+\frac{\Omega^2}{2}\Delta_f\Sigma_0\right)\tan(\frac{\Omega}{\sqrt{2}}L^+)}\right]$$

$$k^+\frac{dI_{2\tau_f<|x^+|}}{dk^+dm_I^2} = \frac{2\alpha_s C_R}{\pi}e^{\frac{-L^+}{2\lambda_R}}\left\{\frac{\log 2}{2}\left[\delta(m_I^2)-\frac{1}{\pi m_I^2}\sin(\frac{m_I^2}{p_J^+}\tau_f)\right]+\frac{1}{2\pi p_J^+}\Re i\frac{p_J^+}{m_I^2}e^{i\frac{m_I^2}{p_J^+}\tau_f}\left[\log\left(\frac{\tau_f\left(\tau_f+\tau_0-\frac{1}{\Omega/\sqrt{2}\tan(L^+\Omega/\sqrt{2})}\right)}{d_+d_-}\right)+e^{-i\frac{m_I^2}{p_J^+}}E_1\left(-i\frac{m_I^2}{p_J^+}\tau_f\right)\right.\right.$$

$$\left.\left.+e^{-i\frac{m_I^2}{p_J^+}\left(\tau_f+\tau_0-\frac{1}{\Omega/\sqrt{2}\tan(L^+\Omega/\sqrt{2})}\right)}E_1\left(-i\frac{m_I^2}{p_J^+}\left(\tau_f+\tau_0-\frac{1}{\Omega/\sqrt{2}\tan(L^+\Omega/\sqrt{2})}\right)\right)-e^{i\frac{m_I^2}{p_J^+}d_+}E_1\left(i\frac{m_I^2}{p_J^+}d_+\right)-e^{i\frac{m_I^2}{p_J^+}d_-}E_1\left(i\frac{m_I^2}{p_J^+}d_-\right)\right]\right\}$$

No thermalization time

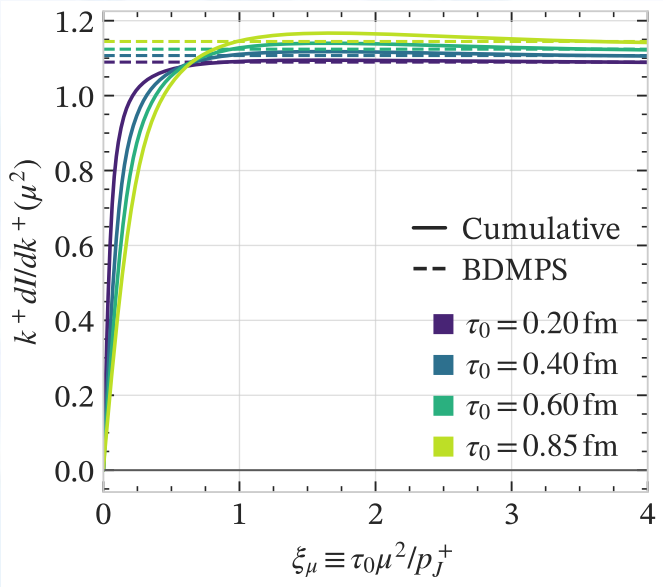


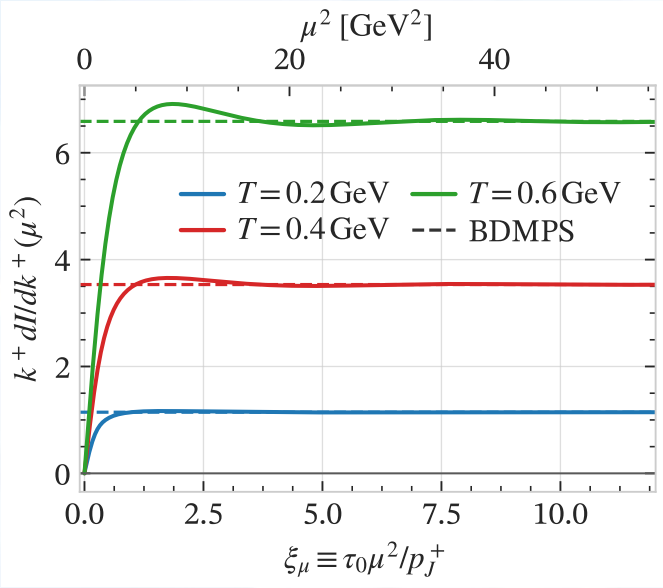


Baier et al. [hep-ph/9607355]

- Jets with much **higher virtualities can contribute to the spectrum** as there is no pre-equilibrium propagation
- The relation between t_J and λ_R parametrically determines wether **vacuum or medium induced radiation is more dominating** although precise calculation is complicated
- Helps understand interaction of jets with **pre-equilibrium stages**

Sensitivity to temperature and medium creation time





Andres et al. [2211.10161]

- Qualitative behavior does **not depend** on the medium temperature of thermalization time
- For shorter thermalization times larger vituality jets can enter the medium, keeping **the ratio between** $t_J = p_J^+ / |m_J^2|$ **and** τ_0 **the relevant scale** that determines the virtualities that can contribute
- For larger temperatures **stronger medium effects** as expected but the **dependence of the jet function on virtuality does not change**