Event-axis TMD measurements in e^+e^- and SIDIS

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Goals

We present the framework for obtaining precise predictions for the transverse momentum of hadrons with respect to the thrust axis in single inclusive $e^+e^- \to hX$ processes. To this end, starting from the established factorization theorems for this observable, we

- Complete the operator-level formulation of the soft ingredients.
- Perform one-loop checks using the δ-regulator.
- Propose a model that captures both the event-shape dependence and correlations between the event-shape and transverse-momentum measurements.
- Compare to simulated e^+e data from Pythia8.3.



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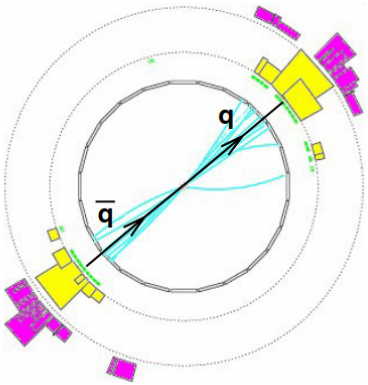


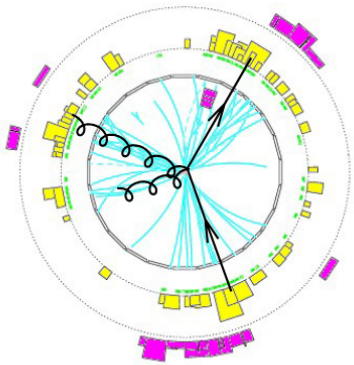
Thrust

For e^+e^- thrust is defined like

$$T = 1 - \tau = \max_{\hat{t}} \frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|}$$

τ ranges from 0 to 1/2:





pencil-like ⟶ spherical

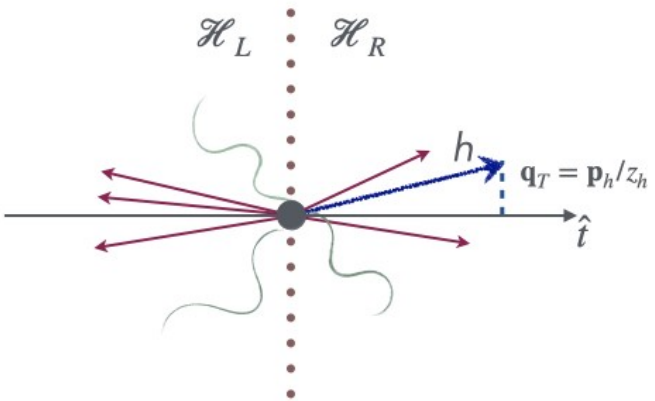


Thrust axis

We choose coordinates such that $\hat{t} = (0, 0, 1)$ and in terms of light-cone coordinates

$$\tau \equiv 1 - T = \frac{\sqrt{2}}{Q} \sum_j \min\{p_j^+, p_j^-\}, \qquad \boldsymbol{p}_h = - \sum_{i \in \text{hemi. with } i \neq h} \boldsymbol{p}_i$$

We define the transverse momentum with respect to the thrust axis $q_T^2 = \boldsymbol{p}_{\boldsymbol{h}}^2 / z_h^2$ where $\boldsymbol{p}_h$ is the hadron momentum and $z_h = 2E_h/Q$.





Kinematic Regions

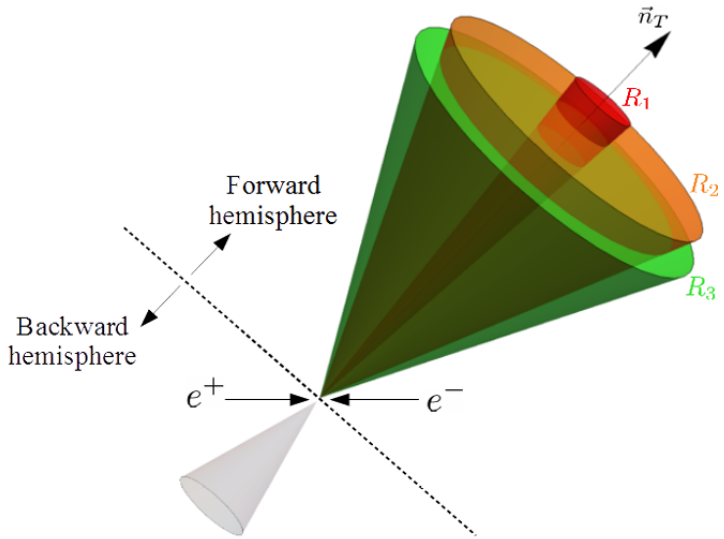


Figure: [M. Boglione and A. Simonelli, JHEP 09 (2023) 006]

Kinematic bound: $\sqrt{\tau} \gtrsim q_T/Q \gtrsim \tau$. There is a different factorization in each kinematic region [Y. Makris, F. Ringer, and W. J. Waalewijn, JHEP 02, 070]:

- Region 1: $\sqrt{\tau} \gg q_T/Q \sim \tau$
- Region 2: $\sqrt{\tau} \gg q_T/Q \gg \tau$
- Region 3: $\sqrt{\tau} \sim q_T/Q \gg \tau$



Cross section for the three regions

Factorized formulas

Region 1: $$\frac{\mathrm{d}\sigma_1}{\mathrm{d}z_h\,\mathrm{d}\boldsymbol{q}\,\mathrm{d}\tau} = \sum_q \sigma_{0,q}(Q) \int \frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}+u\tau}\, H(Q^2,\mu^2)$$
$$\times J\left(\frac{u}{Q^2},\mu\right) S_{\mathrm{hemi}}\left(\boldsymbol{b},\frac{u}{Q},\mu,\frac{\zeta}{Q^2}\right) D_{q\to h}(z_h,\boldsymbol{b},\mu,\zeta)$$

Region 2: $$\frac{\mathrm{d}\sigma_2}{\mathrm{d}z_h\,\mathrm{d}\boldsymbol{q}\,\mathrm{d}\tau} = \sum_q \sigma_{0,q}(Q) \int \frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}+u\tau}\, H(Q^2,\mu^2)$$
$$\times J\left(\frac{u}{Q^2},\mu\right) S_{\mathrm{thr}}\left(\frac{u}{Q},\mu\right) C\left(\boldsymbol{b},\frac{u}{Q},\mu,\frac{\zeta}{Q^2}\right) D_{q\to h}(z_h,\boldsymbol{b},\mu,\zeta)$$

Region 3: $$\frac{\mathrm{d}\sigma_3}{\mathrm{d}z_h\,\mathrm{d}\boldsymbol{q}\,\mathrm{d}\tau} = \sum_q \sigma_{0,q}(Q) \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i}\, e^{u\tau}\, H(Q^2,\mu^2) J\left(\frac{u}{Q^2},\mu\right)$$
$$\times S_{\mathrm{thr}}\left(\frac{u}{Q},\mu\right) \mathcal{G}_{q\to h}\left(\frac{u}{Q^2},z_h,\boldsymbol{q},\mu\right)$$



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Soft functions

The S_{thr}, and C are defined by the following matrix elements [M. Procura, W.J. Waalewijn and L. Zeune, JHEP 02 (2015) 117]

$$S_{\text{thr}}(k^+) = \frac{1}{N_c} \left\langle 0 \left| \text{Tr}\left[\overline{\mathcal{T}}\left(S_n^\dagger(0) S_{\bar{n}}(0) \right) \delta\left(k^+ - \mathcal{P}_1^+ - \mathcal{P}_2^-\right) \mathcal{T}\left(S_{\bar{n}}^\dagger(0) S_n(0) \right) \right] \right| 0 \right\rangle$$

$$C(\boldsymbol{k}, k^+) = \frac{1}{N_c} \left\langle 0 \left| \text{Tr}\left[\overline{\mathcal{T}}\left(X_n^\dagger(0) V_n(0) \right) \delta\left(k^+ - \mathcal{P}^+\right) \delta^2(\boldsymbol{k} - \boldsymbol{\mathcal{P}}) \mathcal{T}\left(V_n^\dagger(0) X_n(0) \right) \right] \right| 0 \right\rangle$$

where $(\overline{\mathcal{T}})\ \mathcal{T}$ denotes (anti-)time ordering and the operator $\mathcal{P}_1$ ($\mathcal{P}_2$) gives the momentum of the soft radiation going into the hemisphere defined by $p_i^+ < p_i^-$ ($p_i^+ > p_i^-$).



Hemisphere-Thrust-TMD Soft function

The new hemisphere soft function for this process has not been defined as a matrix element in the literature before, and only been extracted from consistency. At the bare level it is defined as

$$S_{\text{hemi}}(\boldsymbol{k}, k^+) = \frac{1}{N_c} \left\langle 0 \left| \text{Tr}\left[\overline{\mathcal{T}}\left(S_n^\dagger(0) S_{\bar{n}}(0) \right) \delta\left(k^+ - \mathcal{P}_1^+ - \mathcal{P}_2^-\right) \delta^2(\boldsymbol{k} - \boldsymbol{\mathcal{P}}_2)\, \mathcal{T}\left(S_{\bar{n}}^\dagger(0) S_n(0) \right) \right] \right| 0 \right\rangle$$

The soft radiation goes into the hemisphere of the identified hadron only (we have assumed that goes into the hemisphere 2).



δ-Regulator

The factorization of the cross-section introduces rapidity divergences, which we regulate with the δ-regulator [M. G. Echevarria, I. Scimemi, and A. Vladimirov, Phys. Rev. D 93, 054004 (2016)]. It consists on the following modification of the Wilson lines

$$S_n(0) = \boldsymbol{P}\exp\left[ig\int_{-\infty}^{0} ds\; n\cdot A_s(sn)\right] \to \boldsymbol{P}\exp\left[ig\int_{-\infty}^{0} ds\; n\cdot A_s(sn)e^{\delta^+ s}\right]$$

$$S_{\bar{n}}(0) = \boldsymbol{P}\exp\left[-ig\int_{0}^{\infty} ds\; \bar{n}\cdot A_s(s\bar{n})\right] \to \boldsymbol{P}\exp\left[-ig\int_{0}^{\infty} ds\; \bar{n}\cdot A_s(s\bar{n})e^{-\delta^- s}\right]$$

The gauge properties are recovered in the limit $\delta \to 0^+$.



Hemisphere soft function

The final result for the bare hemisphere soft function is

$$S^{(1)}_{\text{hemi}}(u,\boldsymbol{b},\mu,\delta^2) = -\frac{4a_S C_F}{\epsilon^2} - \frac{2a_S C_F}{\epsilon}\ln\left(\frac{e^{2\gamma_E}\mu^2 u^2}{Q^2}\right) - \frac{2a_S C_F}{\epsilon}\ln\left(\frac{\mu^2}{Q^2\delta^2}\right)$$
$$+4a_S C_F\left[-\frac{1}{2}\ln\left(B^2 e^{2\gamma_E}\mu^2\right)\ln\left(\frac{\mu^2}{Q^2\delta^2}\right) + \frac{1}{4}\ln^2\left(B^2 e^{2\gamma_E}\mu^2\right)\right.$$
$$\left.+\frac{B^2Q^2F}{u^2} - \frac{1}{4}\ln^2\left(\frac{B^2Q^2}{u^2}\right) - \frac{1}{4}\ln^2\left(\frac{\mu^2u^2e^{2\gamma_E}}{Q^2}\right) - \frac{\pi^2}{4}\right]$$

where $F = {}_4F_3[\frac{3}{2},1,1,1;2,2,2;-\frac{4B^2Q^2}{u^2}]$. Our definition gives the expected result at one loop [Y. Makris, F. Ringer, and W. J. Waalewijn, JHEP 02, 070].

One-loop relation between soft functions

$$S^{(1)}_{\text{hemi}} = \frac{1}{2}S^{(1)} + \frac{1}{2}S^{(1)}_{\text{thr}}$$



Soft subtraction

The factorization theorems are expressed in terms of the unsubtracted TMDFF. To address the overlap between the collinear and soft radiation, we still have to perform a zero-bin subtraction. This zero-bin is equal to the TMD soft function

$$D_{q\to h}\left(z_h, \boldsymbol{b}, \mu, \frac{\zeta}{Q^2}\right) = \frac{D_{q\to h}(z_h, \boldsymbol{b}, \mu, \delta^2 Q^2)}{\sqrt{S(\boldsymbol{b}, \mu, \delta^2\zeta)}}$$

$$S_{\text{hemi}}\left(\boldsymbol{b}, \frac{u}{Q}, \mu, \frac{\zeta}{Q^2}\right) = \frac{S_{\text{hemi}}(\boldsymbol{b}, u/Q, \mu, \delta^2 Q^2)}{\sqrt{S(\boldsymbol{b}, \mu, \delta^2\zeta)}} = 1 + 4a_S C_F \left[\frac{B^2Q^2F}{u^2}\right.$$
$$\left. - \frac{1}{2}\ln(\mu^2 B^2 e^{2\gamma_E})\ln\left(\frac{\zeta}{Q^2}\right) - \ln^2\left(\frac{u}{BQ}\right) - \ln^2\left(\frac{ue^{\gamma_E}\mu}{Q}\right) - \frac{7\pi^2}{24}\right]$$

$$C\left(\boldsymbol{b}, \frac{u}{Q}, \mu, \frac{\zeta}{Q^2}\right) = \frac{C(\boldsymbol{b}, u/Q, \mu, \delta^2 Q^2)}{\sqrt{S(\boldsymbol{b}, \mu, \delta^2\zeta)}}$$
$$= 1 + 2a_S C_F \left[2\ln\left(\frac{ue^{\gamma_E}\mu}{\sqrt{\zeta}}\right)\ln(\mu^2 B^2 e^{2\gamma_E}) - \frac{1}{2}\ln^2(\mu^2 B^2 e^{2\gamma_E}) - \frac{\pi^2}{12}\right]$$



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Resummation

The large logarithms in q_T/Q and τ are resummed by evaluating the ingredients at their natural scale and evolving them to a common scale using the RGEs. We consider three different approaches:

- Choosing scales in terms of the conjugate variables $\boldsymbol{b}$ and u.
- Using $\boldsymbol{b}$ but employ τ instead of u in our choice of scales.
- Choosing a common initial condition for all functions that have u-dependence, called the $\mathcal{M}$-scheme [M.G. Echevarria, P.A. Gutierrez García, and I. Scimemi, arXiv:2603.00375].



$\mathcal{M}$-scheme

All formulas simplify if we write

$$\mathcal{M}(\boldsymbol{b}, u, Q, \mu, \zeta) = J\left(\frac{u}{Q^2}, \mu\right) S_{\text{thr}}\left(\frac{u}{Q}, \mu\right) C\left(\boldsymbol{b}, \frac{u}{Q}, \mu, \zeta\right)$$

Then the cross-section for region 2 ($\tau Q \ll q_T \ll \sqrt{\tau} Q$) can be written as

$$\begin{aligned}\frac{\mathrm{d}\sigma_2}{\mathrm{d}z_h\,\mathrm{d}\boldsymbol{q}\,\mathrm{d}\tau} &= \sum_q \sigma_{0,q}(Q) \int \frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i} e^{i\boldsymbol{b}\cdot\boldsymbol{q}+u\tau}\, H(Q^2,\mu^2) \\ &\quad \times \Big(R[\boldsymbol{b}, (\mu_0,\zeta_0) \to (Q, Q^2)]\Big)^2 \mathcal{M}(\boldsymbol{b}, u)\, D_{q\to h}(z_h, \boldsymbol{b})\end{aligned}$$



Double scale evolution

For the TMD as well as for $\mathcal{M}$, there are two different scales and therefore, two RGEs.

$$\mu \frac{d}{d\mu}\mathcal{M} = \gamma^{\mathcal{M}}(\mu,\zeta)\mathcal{M}$$
$$\zeta \frac{d}{d\zeta}\mathcal{M} = -\mathcal{D}(\mu,b)\mathcal{M}$$

Its anomalous dimension is

$$\gamma^{\mathcal{M}} = \gamma^J + \gamma^{S_{\text{thr}}} + \gamma^S = \Gamma_{\text{cusp}}(a_s)\ln\left(\frac{\mu^2}{\zeta}\right) - \gamma^V(a_s) = \gamma^F$$

So the TMD and $\mathcal{M}$ have the same anomalous dimension. Since it does not depend on u and ζ-prescription can be applied straightforwardly.



Double scale evolution

The solution of the above eqs

$$\mathcal{M}(u, b, \mu_f, \zeta_f) = R_{\mathcal{M}}[b, (\mu_{\mathcal{M}}, \zeta_{\mathcal{M}}) \to (\mu_f, \zeta_f)]\mathcal{M}(u, b)$$

where the general form of the evolution factor is

$$R_{\mathcal{M}}[b, (\mu_{\mathcal{M}}, \zeta_{\mathcal{M}}) \to (\mu_f, \zeta_f)] = \exp\left[\int_P (\gamma^{\mathcal{M}}(\mu, \zeta)\frac{\mathrm{d}\mu}{\mu} - \mathcal{D}(\mu, b)\frac{\mathrm{d}\zeta}{\zeta})\right]$$

where $(\mu_{\mathcal{M}}, \zeta_{\mathcal{M}})$ and (μ_f, ζ_f) refer respectively to an initial and final set of scales. While perturbatively R and $R_{\mathcal{M}}$ are equal, there could be a difference in their nonperturbative contribution. As a starting point, we will take $R = R_{\mathcal{M}}$.



Double scale evolution

The initial scale is chosen with ζ-prescription. In particular we work with the special null-evolution line, a line through which the distribution does not evolve and that passes through the saddle point. That is, a solution of

$$\gamma_{\mathcal{M}}(u, \mu, \zeta_\mu(b)) = 2\mathcal{D}(b, \mu)\frac{\mathrm{d}\ln\zeta_\mu(b)}{\mathrm{d}\ln\mu^2}$$

$$\gamma_{\mathcal{M}}(u, \mu_{\mathcal{M}}, \zeta_{\mathcal{M}}) = 0$$
$$\mathcal{D}(\boldsymbol{b}, \mu_{\mathcal{M}}) = 0$$

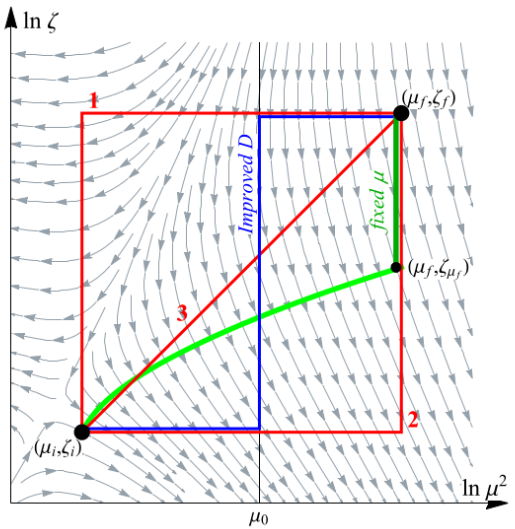


Figure: Null Evolution curve

Double scale evolution

Under these conditions we have

$$\mathcal{M}(u, b, \mu_f, \zeta_f) = R_{\mathcal{M}}[u, b, (\mu_{\mathcal{M}}, \zeta_{\mathcal{M}}) \to (\mu_f, \zeta_f)]\mathcal{M}(u, b)$$

where

$$R_{\mathcal{M}}[u, b, (\mu_{\mathcal{M}}, \zeta_{\mathcal{M}}) \to (\mu_f, \zeta_f)] = \left(\frac{\zeta_f}{\zeta_{\mu_f}(\mu_{\mathcal{M}}, \zeta_{\mathcal{M}})}\right)^{-\mathcal{D}(b,\mu_f)}$$

$$\mathcal{M}(u, b, \mu_{\mathcal{M}}, \zeta_{\mathcal{M}}) = \mathcal{M}(u, b), \qquad \mu_{\mathcal{M}} = \mu_0 \sim 1/b_T, \qquad \zeta_{\mathcal{M}} = \zeta_0 \sim 1/b_T^2$$

The explicit form of $\zeta_{\mu_f}(\mu, \zeta)$ and $\mathcal{D}(b, \mu)$ can be found at [I. Scimemi and A. Vladimirov, 1803.11089].



Non perturbative model

While our goal is also to extract the TMDs and evolution kernels, the description of these measurements also requires taking into account the nonperturbative effects for the thrust measurement (which appears in addition to the nonperturbative contributions to TMDs)

$$\mathcal{M}^{\rm NP}(b,u) = J^{\rm NP}\left(\frac{u}{Q^2},\mu_0\right) S^{\rm NP}_{\rm thr}\left(\frac{u}{Q},\mu_0\right) C^{\rm NP}\left(b,\frac{u}{Q}\right)$$

In this paper we take a first step towards a fit with real data, by proposing a parametrization for $\mathcal{M}^{\rm NP}$ and testing it using simulated data. The specific model we use is

$$\mathcal{M}^{\rm NP}(b,u) = \frac{1}{(1+\mu_1 u)^{\mu_2}} \frac{1}{(1+\mu_3\, b\, u)} \frac{1+\mu_5\, b^{\mu_6}}{\cosh(\mu_4 b)}$$



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$e^+e^- \to \pi^+X$ fit results

In order to test our formalism, both the perturbative predictions as well as the nonperturbative modeling, we compare to events generated by PYTHIA 8.3 for $e^+e^- \to \pi^+X$.

- 10 million events generated at $Q = 91$ GeV.
- restrict to region 2 and impose $\tau > 0.03 \longrightarrow$ 1788 data points for our analysis.
- systematic uncertainty of 15% + statistical uncertainty from the Monte Carlo data.

	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	χ^2/N
τ-scheme	0.00458	1.61	0.0397	0.00239	-9.96	0.161	2.57
u-scheme	0.0285	0.319	0.0160	2.68	-2.77	2.32	1.71
$\mathcal{M}$-scheme	0.0120	2.19	$6.30 \cdot 10^{-8}$	0.00682	-7.03	2.94	1.69

Table: Results for the fit parameters of the nonperturbative model in region 2, with $0.2 < z < 0.8$. The rows correspond to the results for the three resummation schemes discussed. Dimensionful parameters are in the appropriate units of GeV.



$e^+e^- \to \pi^+X$ fit results

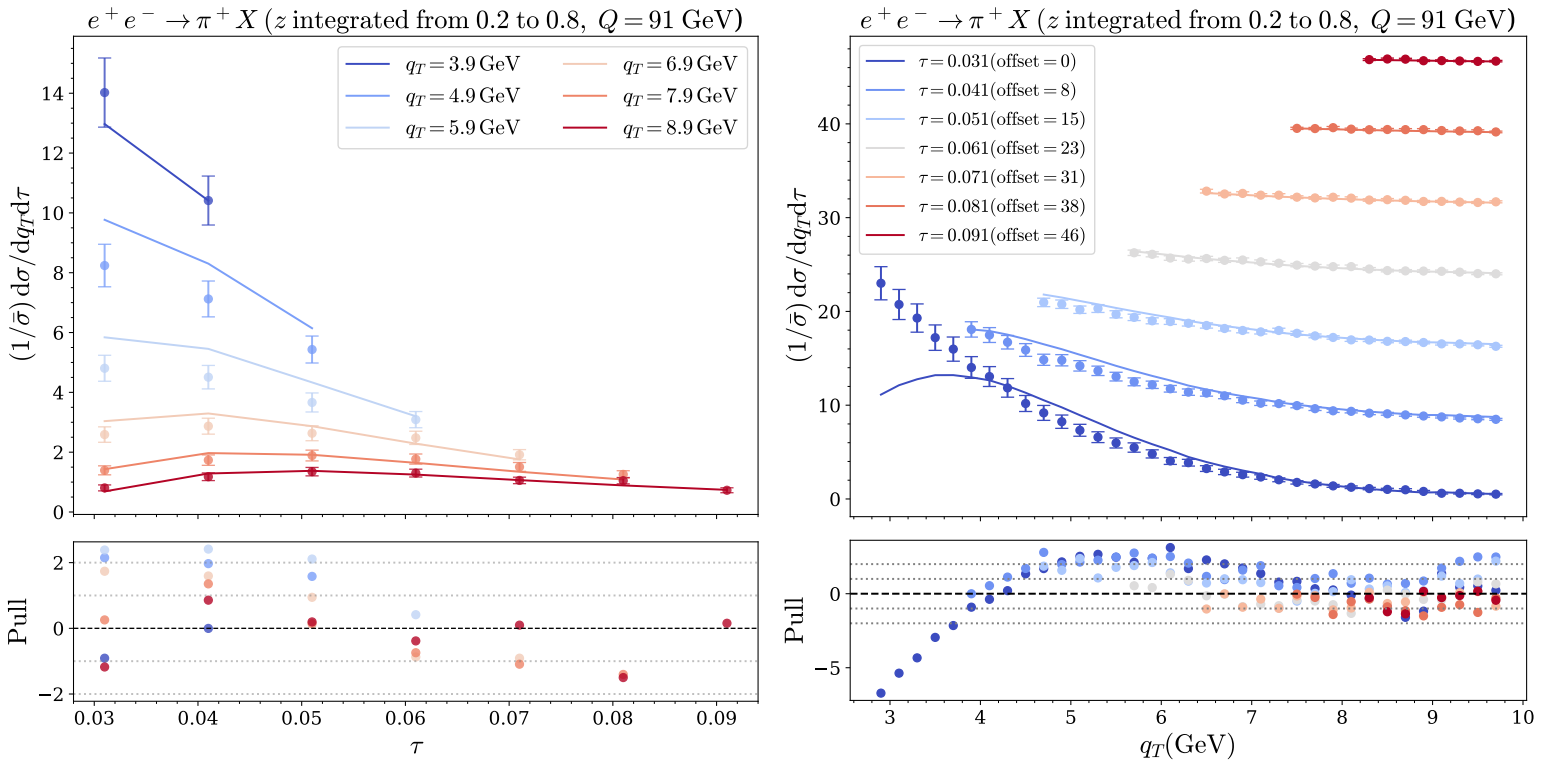


Figure: Shown are the PYTHIA 8.3 data with statistical uncertainties, as well as the fit result. We include an offset (of indicated size) to space out the different curves. The pull is shown in the lower panel.

$e^+e^- \to \pi^+X$ fit results

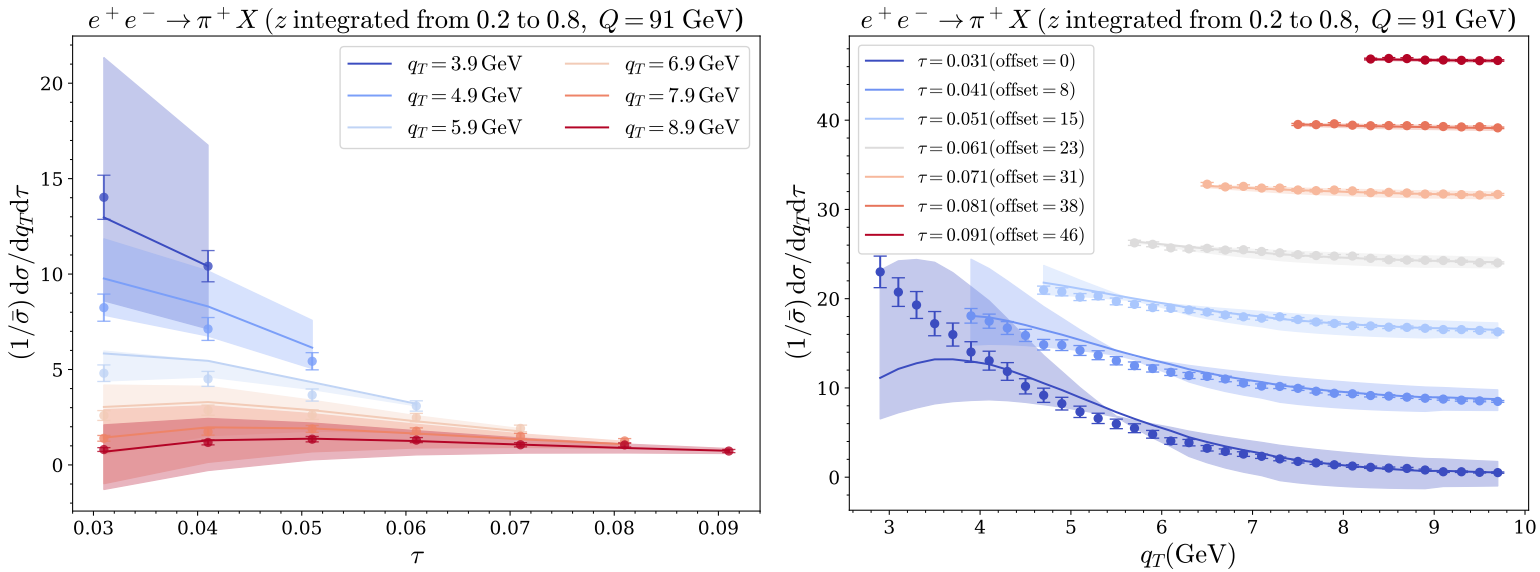


Figure: Same as fig. 3, including the perturbative uncertainty band from the hard scale variation, $\mu \in (2, 1/2)Q$.



Results

Summary:

- The factorization proposed in the literature has been completed by giving the operator definition of the soft function S_{hemi}.
- One-loop check using the δ-regulator has been performed.
- We have tested the e^+e^- case for $\tau Q \ll q_T \ll \sqrt{\tau}Q$, providing a NNLL resummation, discussing nonperturbative effects and testing the framework by fitting to PYTHIA 8.3 data at $Q = 91.0$ GeV for three different resummation schemes.
- The fit with our nonperturbative model is consistent with PYTHIA data, validating the structure of the model we adopted.
- A natural extension of this project is to apply the present formalism to fit the Belle data, and make predictions for the forthcoming Electron Ion Collider.

