q_T/Q correction for unpolarized Drell-Yan process in TMD factorization

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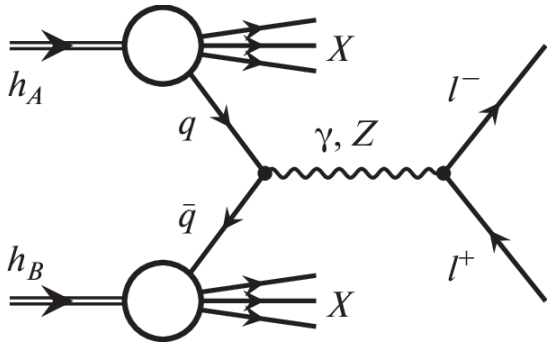
Outline

1. Introduction to TMD factorization theorem
2. Power corrections in TMD factorization
3. Leading TMD approximation
4. Final remarks

Introduction to TMD factorization theorem

Drell-Yan reaction

- $h_A(p_1) + h_B(p_2) \to \gamma^*/Z^*(q) + X \to \ell^-(l) + \ell^+(l') + X.$



- Differential cross section in Collins-Soper frame.and angular coefficients.

$$\frac{d\sigma}{d^4q d\Omega} = \frac{3}{16\pi} \sum_n S_n(\theta, \phi) \Sigma_n$$

Angular coefficients.

$$\Sigma_n = \frac{4\pi\alpha_{\rm em}^2}{3s}(-Q^2) \sum_{G,G'} \mathfrak{L}_n^{\mu\nu} W_{\mu\nu}^{GG'} z_{n\ell}^{GG'} \Delta_G^* \Delta_{G'}$$

- Hadronic tensor.

$$W_{GG'}^{\mu\nu} = \int \frac{d^4y}{(2\pi)^4} e^{-iqy} \sum_X \langle p_1, p_2 | J_G^\mu(y) | X \rangle \langle X | J_{G'}^\nu(0) | p_1, p_2 \rangle, \qquad q_\mu W_{GG'}^{\mu\nu} = 0$$

Power counting rules

- Split original QCD fields into two background and one dynamical component.

$$q = q_{\bar{n}} + q_n + \psi, \qquad A^\mu = A^\mu_{\bar{n}} + A^\mu_n + B^\mu$$

- Λ is a low-energy scale of QCD. TMD factorization is derived in the limit

$$Q^2 = 2q^+q^- - \boldsymbol{q}_T^2 \gg \Lambda^2, \qquad Q^2 \gg \boldsymbol{q}_T^2.$$

- $\bar{n}$-collinear and n-collinear modes ($\{k^+, k^-, k_T\}$).

$$k^\mu_{\bar{n}} \lesssim \{1, \lambda^2, \lambda\}Q, \qquad k^\mu_n \lesssim \{\lambda^2, 1, \lambda\}Q, \qquad \lambda \sim \frac{\Lambda}{Q}$$

Same power counting for gluon fields components.

$$\xi_{\bar{n}} = \frac{\gamma^-\gamma^+}{2} q_{\bar{n}} \sim \lambda, \qquad \eta_{\bar{n}} = \frac{\gamma^+\gamma^-}{2} q_{\bar{n}} \sim \lambda^2, \quad \eta_{\bar{n}} = -\frac{\gamma^+}{2} \frac{1}{D_+[A_{\bar{n}}]} \not{D}_T[A_{\bar{n}}]\xi_{\bar{n}}$$

TMD factorization at LP (I)

- Hadronic tensor as a series in powers of λ.

$$W^{\mu\nu} = W^{\mu\nu}_{\rm LP}[\lambda^2] + W^{\mu\nu}_{\rm kNLP}[\lambda^3] + W^{\mu\nu}_{\rm gNLP}[\lambda^3] + \mathcal{O}(\lambda^4)$$

- Leading term of hadronic tensor. Definition: $x_1 = q^+/p_1^+$, $x_2 = q^-/p_2^-$.

$$W^{\mu\nu}_{\rm LP} = \frac{1}{4N_c}\mathbb{C}_{\rm LP}\left(\frac{2q^+q^-}{\mu^2}\right)\int\frac{d^2b}{(2\pi)^2}e^{-iq_T b}\sum_{n,m}$$
$$\times\left\{\mathrm{Tr}\left(\gamma_G^\mu\overline{\Gamma}^+_m\gamma_{G'}^\nu\overline{\Gamma}^-_n\right)\Phi_{11}^{[\Gamma^+_n]}(x_1,b;\mu,\zeta)\overline{\Phi}_{11}^{[\Gamma^-_m]}(x_2,b;\mu,\bar\zeta)\right.$$
$$\left.+\,\mathrm{Tr}\left(\gamma_G^\mu\overline{\Gamma}^-_n\gamma_{G'}^\nu\overline{\Gamma}^+_m\right)\overline{\Phi}_{11}^{[\Gamma^+_n]}(x_1,b;\mu,\zeta)\Phi_{11}^{[\Gamma^-_m]}(x_2,b;\mu,\bar\zeta)\right\}.$$

- Twist-2 TMD correlator: two "good" spinoral components.

$$\Phi_{11}^{[\Gamma]}(x,b) = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-ix\lambda p^+}\langle p,s|\bar\xi W^\dagger(\lambda n+b)\frac{\Gamma}{2}W\xi(0)|p,s\rangle$$

TMD factorization at LP (II)

- Example: unpolarized case, $\Phi_{11}^{[\gamma^+]}(x,b) = f_1(x,b)$. [A. Vladimirov, 2307.13054]

$$W^{\mu\nu}_{\text{LP},f_1 f_1} = \frac{-g_T^{\mu\nu}}{N_c}\mathbb{C}_{\text{LP}}\left(\frac{2q^+q^-}{\mu^2}\right)\int d^2k_{1T}d^2k_{2T}\,\delta^{(2)}(q_T - k_{1T} - k_{2T})$$
$$\times\left[f_1(x_1,b;\mu,\zeta)\bar{f}_1(x_2,b;\mu,\bar{\zeta}) + \bar{f}_1(x_1,b;\mu,\zeta)f_1(x_2,b;\mu,\bar{\zeta})\right.$$

- Gauge invariance is preserved at LP, but broken at NLP.

$$q_\mu W^{\mu\nu}_{\text{LP}} \sim q_T^\nu f_1 f_1 = \mathcal{O}(\lambda^3)$$

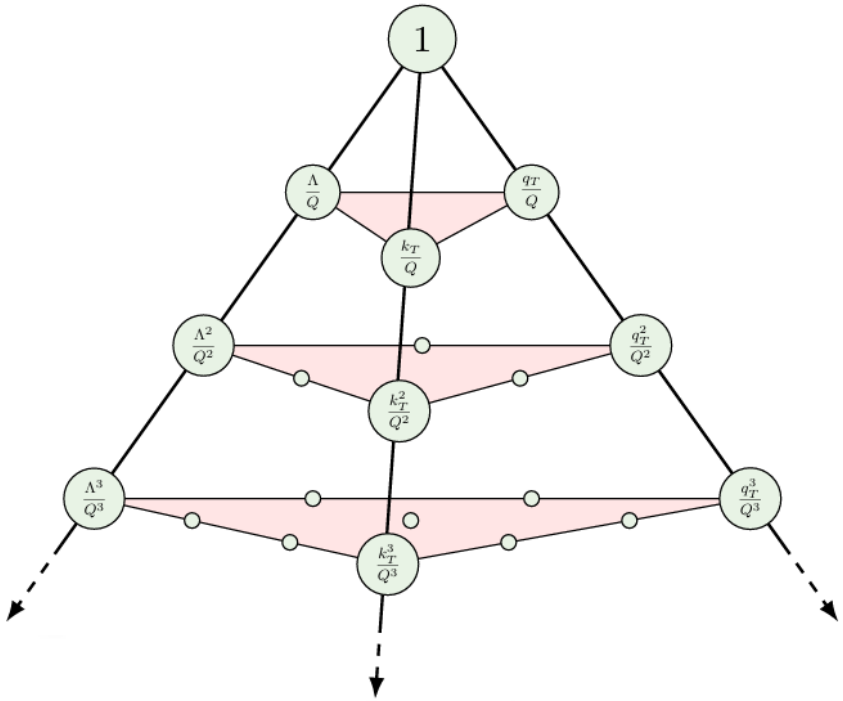
  This is a feature of the TMD factorization theorem.

- By construction, LP term is valid only for $q_T \ll Q \to$ power corrections!

Power corrections in TMD factorization

Power corrections

- Three types of power corrections. Each red triangle of the pyramid has the same power counting in λ. [A. Vladimirov, 2307.13054]
  - $\star$ $\Lambda/Q \to$ higher-twist contributions.
  - $\star$ $k_T/Q \to$ parton transverse momentum.
  - $\star$ $q_T/Q \to$ transverse component of gauge boson momentum.



Kinematic Power Corrections (I)

- k_T/Q, k_T^2/Q^2, etc. correspond to transverse derivatives of twist-two TMD correlators. For example, at NLP

$$W^{\mu\nu}_{\text{kNLP}}$$

$$= \frac{i}{4N_c}\mathbb{C}_{\text{LP}}\left(\frac{2q^+q^-}{\mu^2}\right)\int\frac{d^2b}{(2\pi)^2}e^{-iq_T b}\sum_{n,m}\Bigg\{$$

$$\frac{\bar{n}^\mu \text{Tr}(\gamma_G^\rho \overline{\Gamma}_m^+ \gamma_{G'}^\nu \overline{\Gamma}_n^-) + \bar{n}^\nu \text{Tr}(\gamma_G^\mu \overline{\Gamma}_m^+ \gamma_{G'}^\rho \overline{\Gamma}_n^-)}{q^-}\Phi_{11}^{[\Gamma_n^+]}\left(\partial_\rho + \frac{\partial_\rho \mathcal{D}}{2}\ln\left(\frac{\bar{\zeta}}{\zeta}\right)\right)\overline{\Phi}_{11}^{[\Gamma_m^-]}$$

$$+\ldots\Bigg\}$$

This series can be summed up.

Kinematic Power Corrections (II)

- Unpolarized case. [A. Vladimirov, 2307.13054]

$$W^{\mu\nu}_{\mathrm{KPC},f_1 f_1} = -\frac{4p_1^+ p_2^-}{N_c}\mathbb{C}_{\mathrm{LP}}\left(\frac{Q^2}{\mu^2}\right)\int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2\, \delta^{(4)}(q-k_1-k_2)$$
$$\times \delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)\,\boxed{((k_1k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu)}$$
$$\times [f_1(\xi_1,k_T;\mu,Q^2)\bar{f}_1(\xi_2,k_T;\mu,Q^2) + \bar{f}_1(\xi_1,k_T;\mu,Q^2)f_1(\xi_2,k_T;\mu,Q^2)]$$

  where $\xi_{1,2} = \xi_{1,2}(\boldsymbol{k}_{1T}^2, \boldsymbol{k}_{2T}^2)$.

- Restoration of gauge invariance and frame invariance.

$$q_\mu W^{\mu\nu}_{\mathrm{KPC},f_1 f_1} = (k_{1\mu} + k_{2\mu})W^{\mu\nu}_{\mathrm{KPC},f_1 f_1} = 0$$

- Numerical improvement as $q_T \to 0$, but also for small values of Q.

Leading TMD approximation

Genuine NLP hadronic tensor

- gNLP hadronic tensor.

$$W^{\mu\nu}_{\text{gNLP}} = -\frac{i}{4N_c}\int\frac{d^2b}{(2\pi)^2}\int_{-1}^{1}du_1du_2du_3\sum_{n,m}\Big\{$$
$$[\mathbb{C}_R T^{\mu\nu\rho}_{-}(\bar{n},n) - i\pi\mathbb{C}_I T^{\mu\nu\rho}_{+}(\bar{n},n)]\delta(x_2-u_3)\Phi_{11}^{[\Gamma^+_n]}\overline{\mathbf{\Phi}}^{[\Gamma^-_m]}_{\rho,\oplus}(u_{1,2,3},b;\mu,\bar{\zeta})$$
$$+\ldots\Big\}$$

- Twist-3 TMD correlators: two "good" spinorial components and a transverse component gluon field.

$$\Phi^{[\Gamma]}_{21,\mu}(\{z\},b) = g\langle p,s|\bar{\xi}[z_1n+b,z_2n+b]F_{\mu+}W^{\dagger}(z_2n+b)\frac{\Gamma}{2}W\xi(z_3n)|p,s\rangle,$$
$$\Phi^{[\Gamma]}_{12,\mu}(\{z\},b) = g\langle p,s|\bar{\xi}W^{\dagger}(z_1n+b)\frac{\Gamma}{2}W(z_2n)F_{\mu+}[z_2n,z_3n]\xi|p,s\rangle$$

- These twist-3 TMD correlators behave as $1/b$ in the $b \to 0$ limit. The idea is to make a redefinition of twist-three TMD correlators.

$$\Phi_{3,\mu}^{[\Gamma]} = \widehat{\Phi}_{3,\mu}^{[\Gamma]} + \frac{b^\nu}{b^2}[S \otimes \Phi_2]_{\mu\nu}^{[\Gamma]}, \qquad \lim_{b\to 0} \widehat{\Phi}_{3,\mu}^{[\Gamma]} = \text{finite}$$

  The kernel S can be computed perturbatively.

- Integrating over the transverse distance yields

$$d\sigma^{\text{gNLP}} = \widehat{d\sigma}^{\text{gNLP}} + \frac{q_T}{Q} d\sigma^{\text{L-TMD}}, \qquad \widehat{d\sigma}^{\text{gNLP}} \sim \frac{\Lambda}{Q}$$

Small-b expansion

1. Write the twist-3 TMD correlators in the path integral formalism and apply background field method.

$$q \to q + \psi, \qquad A_\mu \to A_\mu + B_\mu$$

The QCD action splits as

$$S_{\rm QCD}[q+\psi, A+B] = S_{\rm QCD}[q, A] + S_{\rm cov.}[B, \psi] + S_{\rm int}$$

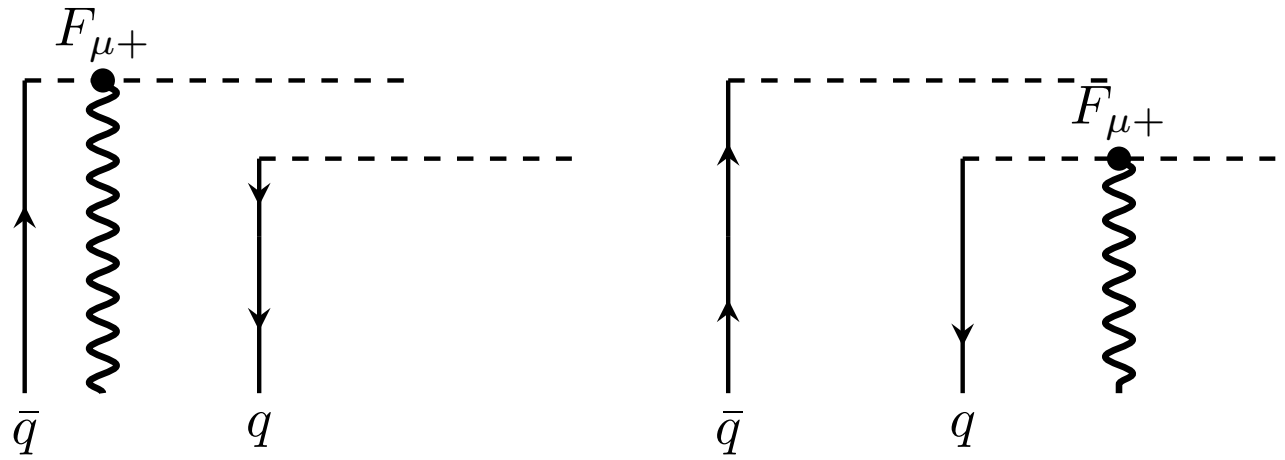
Last term contains interaction terms between background and dynamical fields.

2. Take leading order (LO) term and fix light-cone gauge ($A_+ = 0$). Take terms from $S_{\rm int}$ and contract dynamical fields.

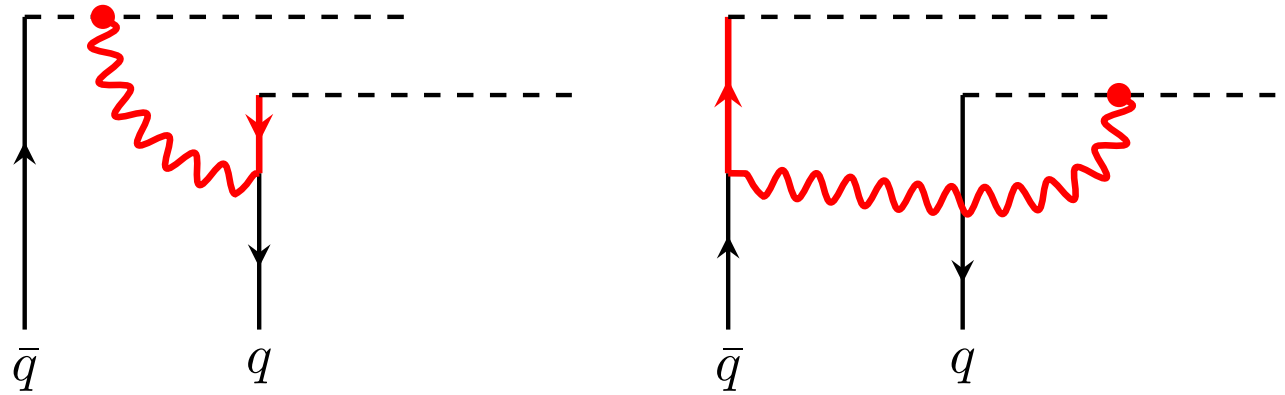
3. After computing the integral, restore gauge invariance.

Quark contribution

- Twist-3 TMD correlator.

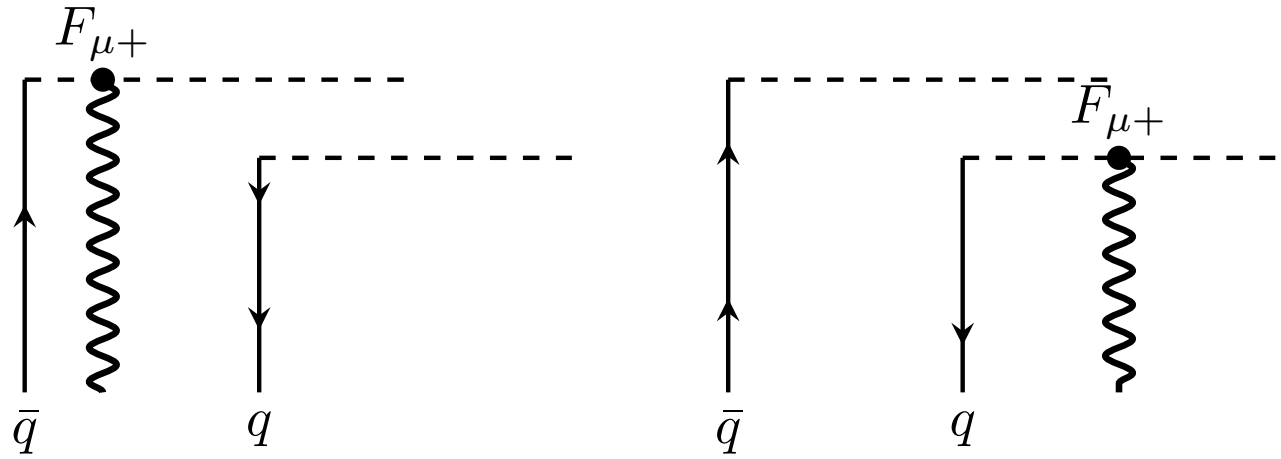


- Projection onto twist-2 quark TMD correlator.

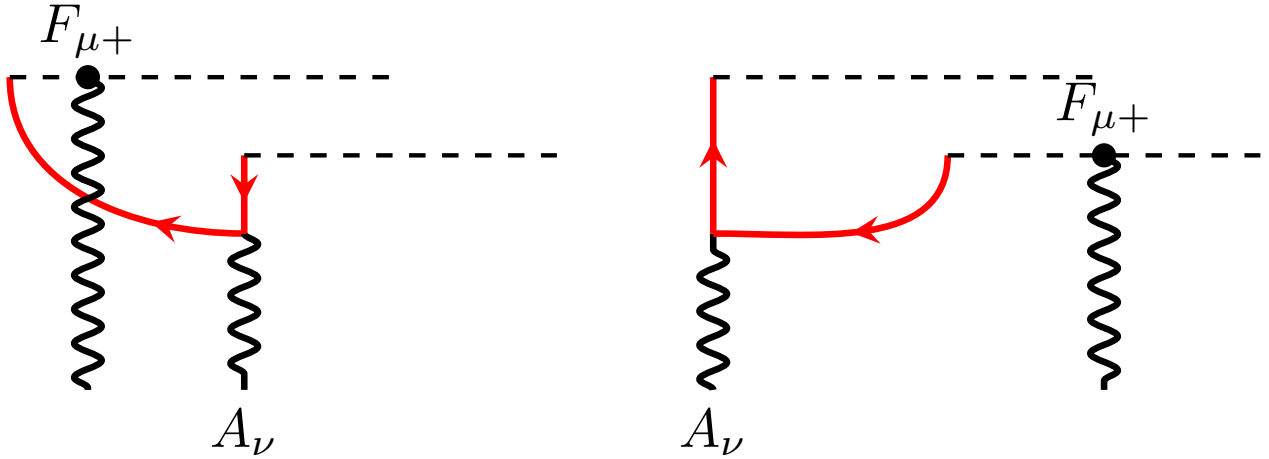


Gluon contribution

- Twist-3 TMD correlator.



- Projection onto twist-2 gluon TMD correlator.



Projection of twist-3 onto twist-2 (I)

- Notation: $\theta_{ij} = \theta(x_i, x_j) - \theta(-x_i, -x_j)$. Twist-(1+2) TMD correlator:

$$\Phi^{[\Gamma]}_{12,\mu}\Big|_{\text{quark}} = a_s \frac{C_F}{2} \frac{b^\nu}{b^2} \theta_{12} \text{Tr}\left[\overline{\Gamma}^-_k \left(\gamma_{T\mu}\gamma_{T\nu} - \frac{x_1}{x_3}\gamma_{T\nu}\gamma_{T\mu}\right)\Gamma\right] \Phi^{[\Gamma^+_k]}_{11}(x_3, b),$$

$$\Phi^{[\Gamma]}_{12,\mu}\Big|_{\text{gluon}} = -\frac{a_s}{4} \frac{b_\nu}{b^2} \theta_{13} \text{Tr}\left[\left(\frac{x_1}{x_2}\gamma^\nu_T\gamma^\alpha_T - \frac{x_3}{x_2}\gamma^\alpha_T\gamma^\nu_T\right)\gamma^-\Gamma\right] \Phi_{\mu\alpha}(-x_2, b)$$

- Technical note: special rapidity divergence must be subtracted. Substitute the parametrization for the twist-2 TMD correlators.

$$\Phi^{[\gamma^+]}_{11}(x, b) = f_1(x, b) + \ldots,$$

$$\Phi^{\mu\nu}(x, b) = -g^{\mu\nu}_T f_g(x, b) + \ldots.$$

- Find the corresponding expressions for $\Gamma \in \{\gamma^+, \gamma^+\gamma^5, \mathrm{i}\sigma^{\alpha+}\gamma^5\}$.

Projection of twist-3 onto twist-2 (II)

- Compare with the parametrization for the twist-(1+2) TMD correlator.

$$\begin{aligned}
\mathbf{\Phi}^{[\gamma^+]}_{12,\mu}(x_{1,2,3},b) &= ib^\mu M^2 \mathbf{f}^\perp_{12}(x_{1,2,3},b),\\
\mathbf{\Phi}^{[\gamma^+\gamma^5]}_{12,\mu}(x_{1,2,3},b) &= -i\epsilon_T^{\mu\nu} b_\nu M^2 \mathbf{g}^\perp_{12}(x_{1,2,3},b) + \dots,\\
\mathbf{\Phi}^{[\mathrm{i}\sigma^{\alpha+}\gamma^5]}_{12,\mu}(x_{1,2,3},b) &= \epsilon_T^{\mu\alpha} M \mathbf{h}_{12}(x_{1,2,3},b) + \dots\\
&\quad + (b^\mu \epsilon_T^{\alpha\beta} + b^\alpha \epsilon_T^{\mu\beta}) b_\beta M^3 \mathbf{h}^\perp_{12}(x_{1,2,3},b) + \dots
\end{aligned}$$

- After some combinations of distributions, we get twist-3 TMDPDFs in terms of twist-2 ones.

$$\mathbf{f}_2^\perp\Big|_{\text{L-TMD}} = \frac{a_s}{b^2M^2}\left[4C_F\theta_{23}(f_1(-x_1,b) - f_1(x_3,b)) + \frac{x_1}{x_2}\theta_{13} f_g(-x_2,b)\right],$$

$$\bar{\mathbf{f}}_2^\perp\Big|_{\text{L-TMD}} = \frac{-a_s}{b^2M^2}\left[4C_F\theta_{23}(\bar{f}_1(-x_1,b) - \bar{f}_1(x_3,b)) + \frac{x_1}{x_2}\theta_{13} f_g(-x_2,b)\right]$$

Final result

- gNLP coefficients in the L-TMD approximation.

$$\Sigma_1^{\text{L-TMD}} = \frac{8\pi\alpha_{\text{em}}^2}{3N_c s Q^2} \sum_{f,G,G'} Q^4 \Delta_G^* \Delta_{G'} \boxed{\frac{-2|\boldsymbol{q}_T|}{Q}} z_{+\ell}^{GG'} z_{+f}^{GG'} \mathcal{J}_+^{\text{L-TMD}},$$

$$\Sigma_3^{\text{L-TMD}} = \frac{8\pi\alpha_{\text{em}}^2}{3N_c s Q^2} \sum_{f,G,G'} Q^4 \Delta_G^* \Delta_{G'} \boxed{\frac{-4|\boldsymbol{q}_T|}{\tau}} z_{-\ell}^{GG'} z_{-f}^{GG'} \mathcal{J}_-^{\text{L-TMD}}$$

- Convolutions finite at $q_T \to 0$. First TMDPDF evaluated at (x_1, b). Second one evaluated at (x_2, b).

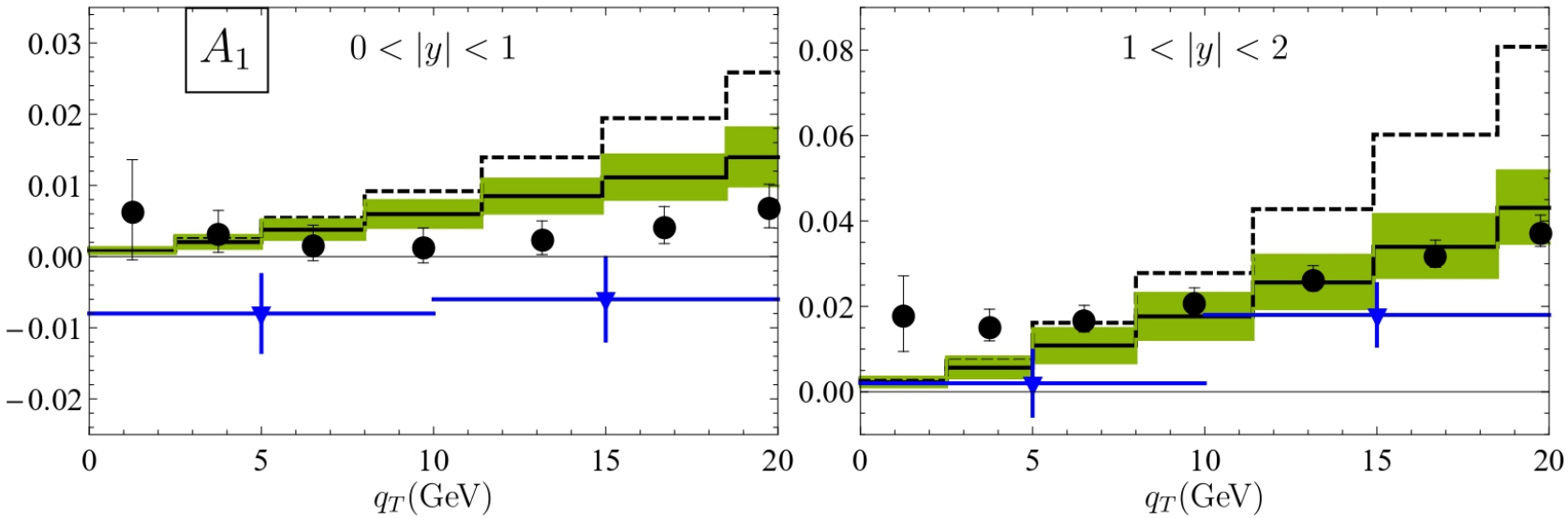
$$\mathcal{J}_\pm^{\text{L-TMD}} = \frac{1}{2}\int_0^\infty \frac{b\,db}{2\pi}(J_0(b|\boldsymbol{q}_T|) + J_2(b|\boldsymbol{q}_T|)) \left(\frac{\tau^2}{\zeta_\mu}\right)^{-2\mathcal{D}(b,\mu)} \Big\{ s_{qq} \otimes f_1\bar{f}_1 \pm s_{qq} \otimes \bar{f}_1 f_1 - f_1 s_{qq} \otimes \bar{f}_1 \mp \bar{f}_1 s_{qq} \otimes f_1 + s_{qg} \otimes f_g(f_1 \pm \bar{f}_1) - (f_1 \pm \bar{f}_1) s_{qg} \otimes f_g \Big\}$$

- Quark and gluon coefficient functions (at a_s order):

$$s_{qq}(x) = \frac{4a_s C_F}{(1-x)_+}, \quad s_{qg}(x) = a_s(1-x), \quad s \otimes f(x) = \int_x^1 \frac{dy}{y} s(y) f\left(\frac{x}{y}\right)$$

Theoretical prediction vs. experimental data (I)

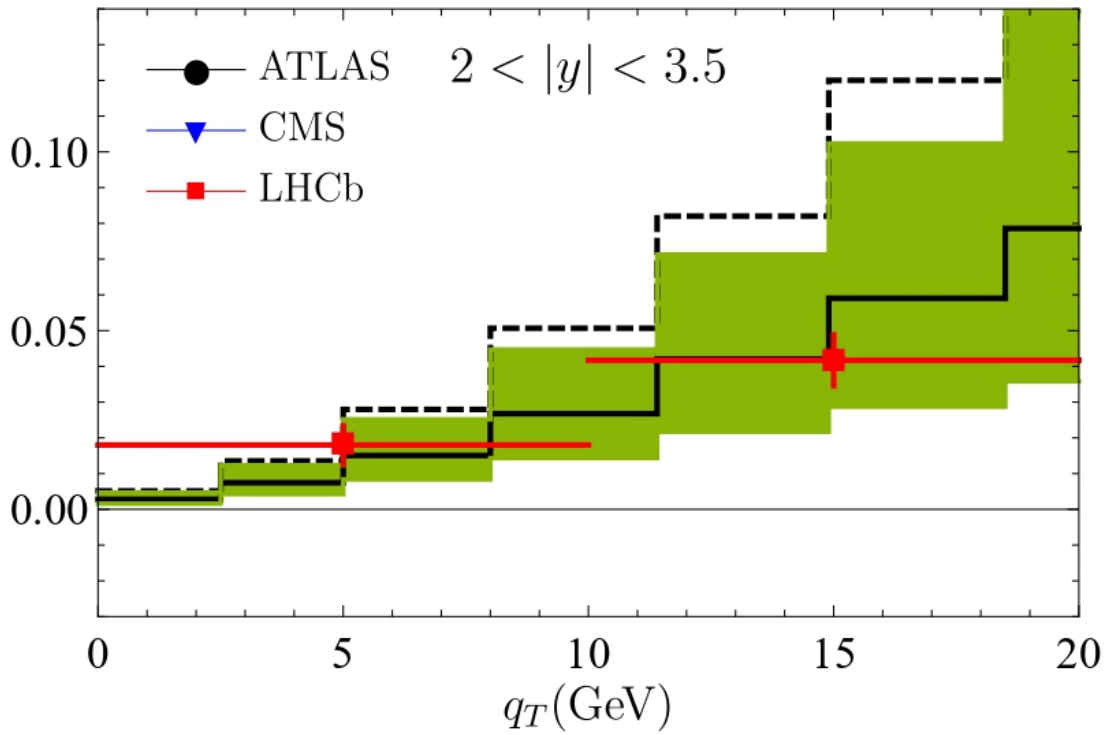
- Coefficient $A_1 = \Sigma_1/\Sigma_U$. $Q \in [80, 100]$ GeV; ATLAS, CMS and LHCb; $\sqrt{s} = 8$ TeV run. [A. Arroyo-Castro, I. Scimemi, A. Vladimirov, 2503.24336].



Dashed line: only kinematic power corrections (KPCs).

Solid line: KPCs + q_T/Q correction.

Theoretical prediction vs. experimental data (II)



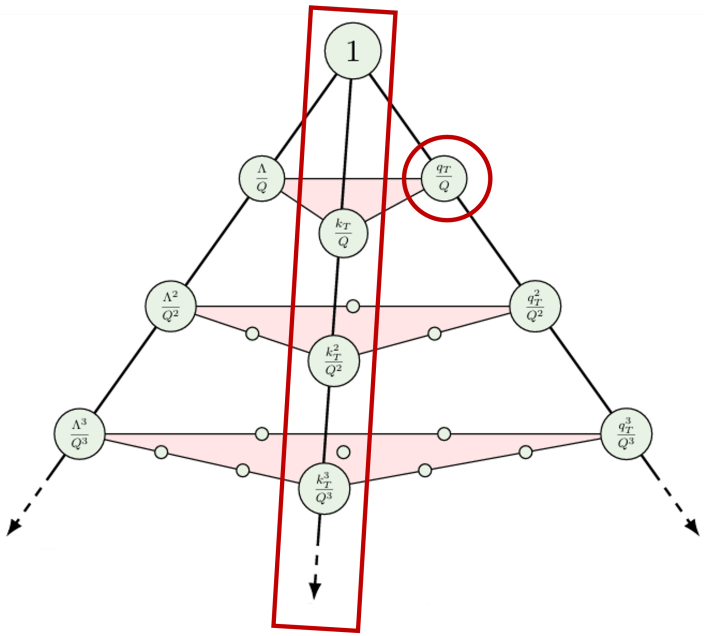
Final remarks

Final remarks (I)

- **Conclusions**.

q_T/Q correction is "hidden" in twist-three terms

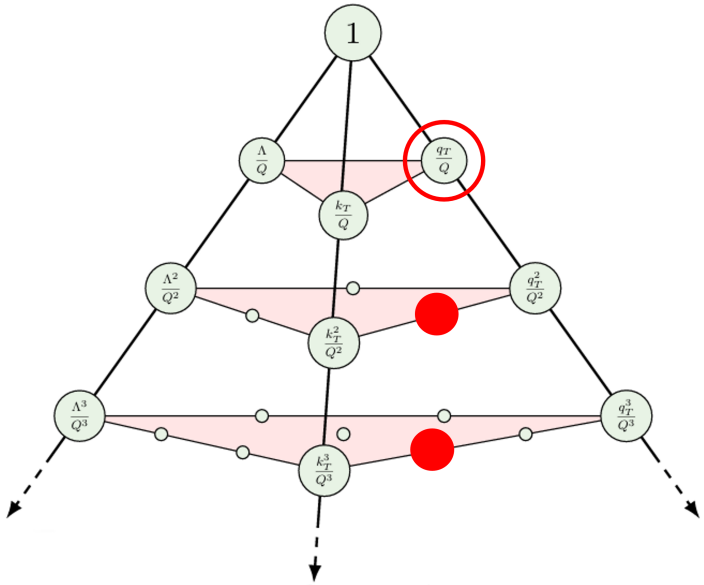
Leading TMD approximation is in complete agreement with data



Final remarks (II)

- **What are we working on?**

Compute $k_T q_T/Q^2$, $k_T^2 q_T/Q^3$, $k_T^3 q_T/Q^4$, etc.



- **Long-term goals?** Compute q_T^2/Q^2, q_T^3/Q^3, etc.

Thank you for your attention!

Back up

Composite background field method

- Path integral. (Anti-)causal sectors.
- We split fields into background (which obey EOMs) and dynamical components:

$$q^{(\pm)} = q_{\bar{n}}^{(\pm)} + q_n^{(\pm)} + \psi^{(\pm)},$$
$$A_\mu^{(\pm)} = A_{\bar{n},\mu}^{(\pm)} + A_{n,\mu}^{(\pm)} + B_\mu^{(\pm)}.$$

- Background field gauge:

$$D_\mu[A_{\bar{n}}^{(\pm)} + A_n^{(\pm)}]B^{(\pm)\mu} = 0.$$

- Hadronic tensor is matrix element of effective operator:

$$\mathcal{J}_{GG'}^{\mu\nu}(y) \sim J_G^{\mu(-)}[q](y) J_{G'}^{\nu(+)}[q](0) e^{iS_{\rm int}^{(+)} - iS_{\rm int}^{(-)}}\Big|_{q=\psi+q_{\bar{n}}+q_n}.$$

- Light-cone gauge:

$$A_{\bar{n}}^{+(\pm)} = 0, \quad A_n^{-(\pm)} = 0.$$

$$\frac{Q}{2M}\Pi_1^{\rm gNLP} = z_{+q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_A]$$
$$+z_{+q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_A],$$

$$\frac{\tau}{4M}\Pi_3^{\rm gNLP} = z_{-q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_S] + 2ir_{-q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_S]$$
$$+z_{-q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_S],$$

$$\frac{\tau}{2M}\Pi_6^{\rm gNLP} = z_{-q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_S]$$
$$-z_{-q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_S],$$

$$\frac{Q}{4M}\Pi_7^{\rm gNLP} = z_{+q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_A]$$
$$-z_{+q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_A] + 2r_{+q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_A]$$

Theoretical approach

- New twist-3 TMD correlators:

$$\widehat{\Phi}_{\bullet,\mu}(x_{1,2,3},b;\mu)=\Phi_{\bullet,\mu}(x_{1,2,3},b;\mu)-\frac{b^\nu}{b^2}[S_\bullet\otimes\Phi_{11}]_{\mu\nu}(x_{1,2,3},b;\mu)$$

  such that $\lim_{b\to 0}\widehat{\Phi}^{[\Gamma]}_{\bullet,\mu}(x_{1,2,3},\mu)=\text{finite}$.

- Operator Product Expansion (OPE) for the twist-2 TMD correlator:

$$\Phi_{11}(x,b)=C_{2/f}(\mathbf{L}_b;\mu_{\rm OPE})\otimes f_{\rm coll}(x;\mu_{\rm OPE})+\mathcal{O}(b^2),\quad \mathbf{L}_b=\ln(\mu^2_{\rm OPE}b^2)$$

  This relation can be inverted perturbatively (set $\mu_{\rm OPE}=\mu$):

$$f_{\rm coll}(x;\mu)=C^{-1}_{2/f}(\mathbf{L}_b;\mu)\otimes\Phi_{11}(x,b)+\mathcal{O}(b^2)$$

- Twist-3 OPE:

$$\Phi^\mu_\bullet(x_{1,2,3},b;\mu)=\frac{b_\nu}{b^2}C^{\mu\nu}_{3/f}(\mathbf{L}_b;\mu)\otimes f_{\rm coll}(x;\mu)+\mathcal{O}(b^0)$$

- Altogether:

$$[S_\bullet\otimes\Phi_{11}]^{\mu\nu}(x_{1,2,3},b;\mu)=C^{\mu\nu}_{3/f}(\mathbf{L}_b;\mu)\otimes C^{-1}_{2/f}(\mathbf{L}_b;\mu)\otimes\Phi_{11}(x,b)$$

Matching of twist-three TMD distributions

$$f_2^\perp = \frac{1}{b^2 M^2}\left[4a_s C_F \theta_{23}\left(f_1(-x_1,b) - f_1(x_3,b)\right) + a_s\theta_{13}\frac{x_1}{x_2} f_g(-x_2,b)\right],$$

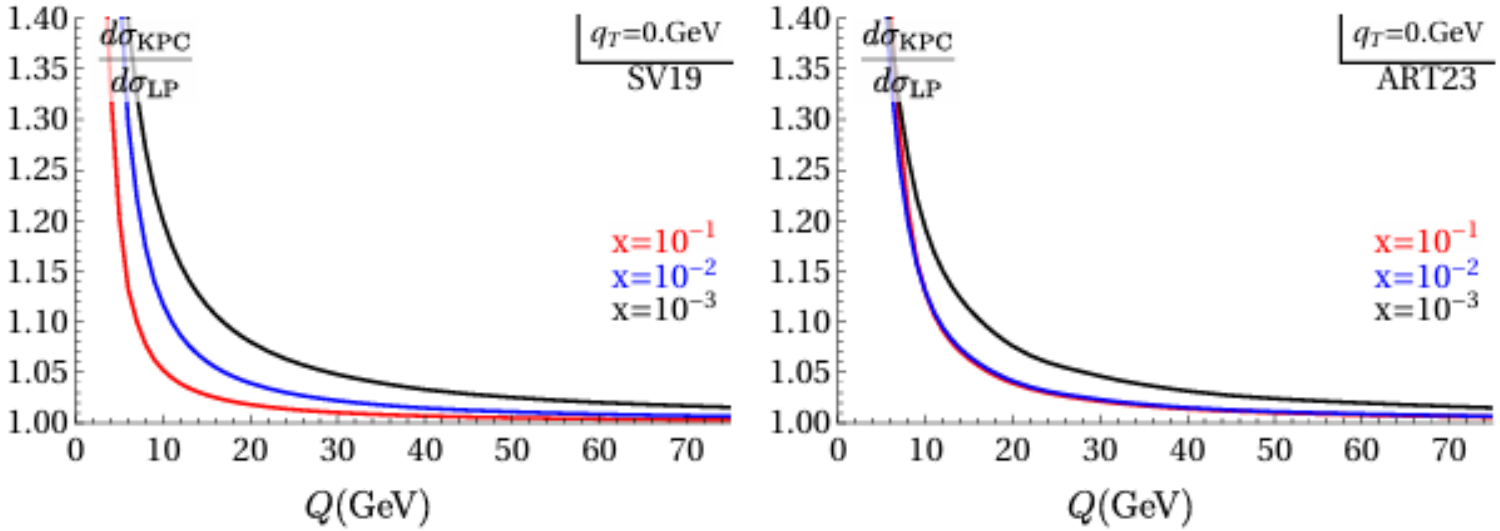
$$\mathbf{g}_2^\perp = 0,$$

$$\bar{\mathbf{f}}_2^\perp = \frac{-1}{b^2 M^2}\left[4a_s C_F \theta_{23}\left(\bar{f}_1(-x_1,b) - \bar{f}_1(x_3,b)\right) + a_s\theta_{13}\frac{x_1}{x_2} f_g(-x_2,b)\right],$$

$$\bar{\mathbf{g}}_2^\perp = 0$$

Kinematic Power Corrections

- Ratio between KPCs and LP Drell-Yan cross section at $q_T = 0$ [A. Vladimirov, 2307.13054].



- Fraction momentum.

$$\xi_1 = \frac{x_1}{2}\left(1 + \frac{\boldsymbol{k}_{1T}^2}{\tau^2} - \frac{\boldsymbol{k}_{2T}^2}{\tau^2} + \frac{\sqrt{\lambda(\boldsymbol{k}_{1T}^2, \boldsymbol{k}_{2T}^2, \tau^2)}}{\tau^2}\right)$$