TMD factorisation and quarkonium

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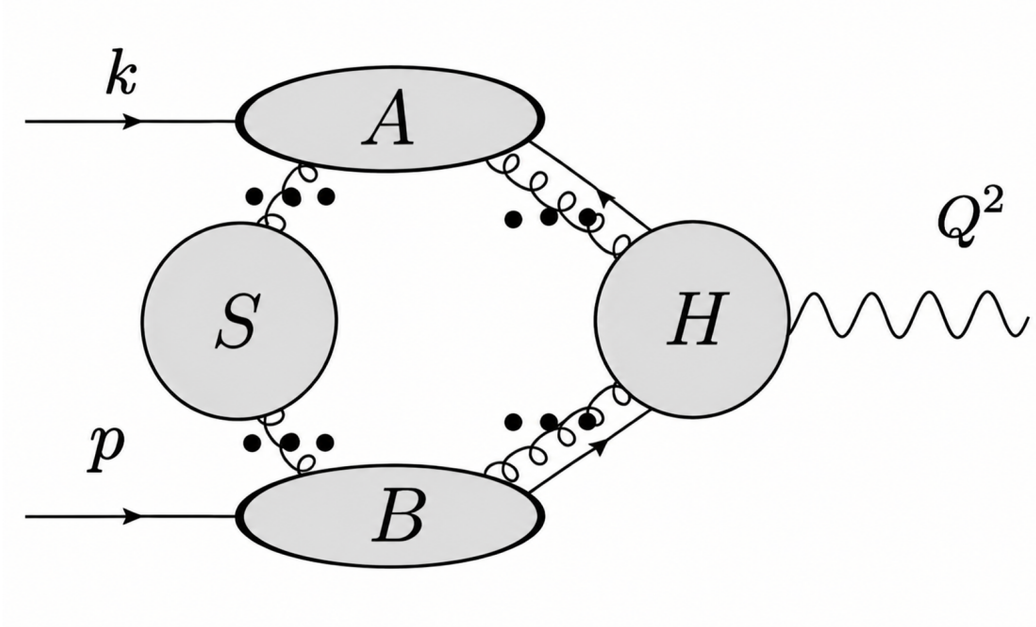
REF 2026, IGFAE Santiago de Compostela

Proof of factorisation of the process

$$p + p \to \mathcal{Q} + \mathcal{Q} + X$$

with Samuel Romera and Miguel Echevarria (Bilbao)

Sudakov form factor



Building block for DIS, SIDIS, DY, …

$$Q^2 \gg k^2 \sim p^2 \sim \Lambda^2_{\rm QCD}$$

Identify *leading twist* contribution:
up to $\Lambda^2_{\rm QCD}/Q^2 \sim p^2/Q^2 \sim k^2/Q^2$

Scale separation and resummation of large logs:

$$F(Q^2, k^2, p^2) = H(Q^2, \mu^2) \otimes A(k^2, \mu^2) \otimes B(p^2, \mu^2) \otimes S\Big(\frac{p^2 k^2}{Q^2}, \mu^2\Big) + \mathcal{O}(1/Q^2)$$

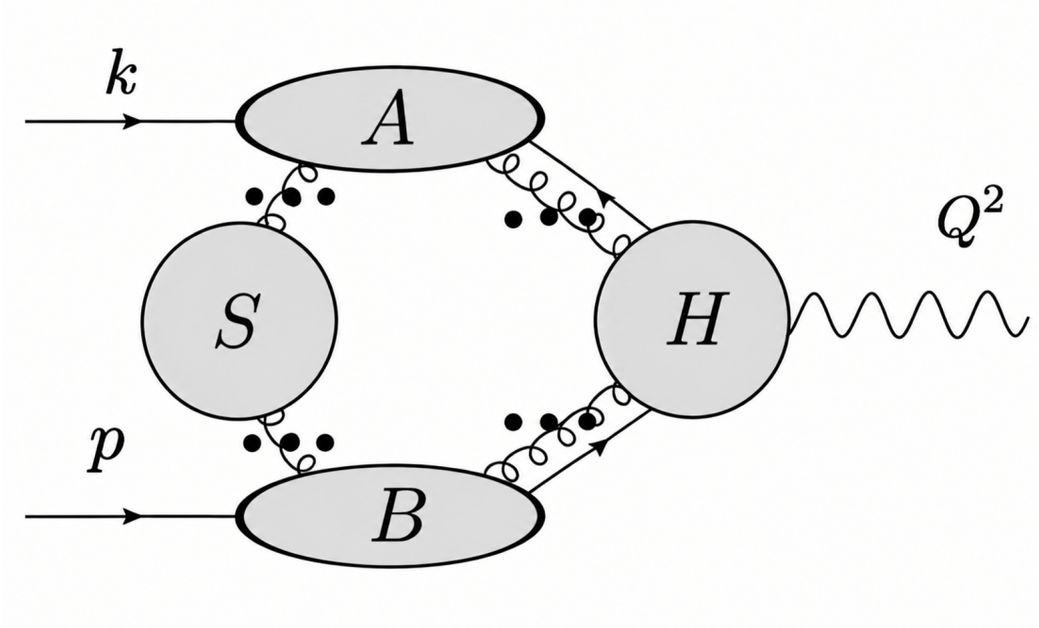
Highly virtual corrections w/o collinear or soft enhancement

quarks/gluons collinear to k or p

soft gluon radiation

See, e.g.: Bertone, *PhD Course* (2019);
Collins, *Sudakov form factors* (2003);
Sterman, *TASI lectures* (1995)

Soft-Collinear Effective Theory (SCET)



LC coordinates: $\ell^\mu = (\ell^+, \ell^-, \boldsymbol{\ell}_\perp)$

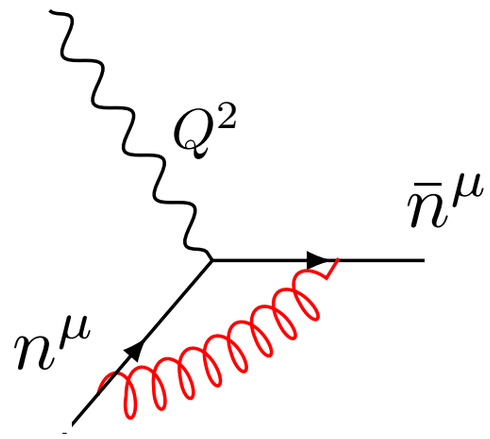
$n^\mu = (1, 0, \mathbf{0}_\perp); \quad \bar{n}^\mu = (0, 1, \mathbf{0}_\perp)$

Consider scaling of momentum components

$$p^\mu = Q(1, \lambda^2, \lambda) \ \to \text{collinear}$$
$$k^\mu = Q(\lambda^2, 1, \lambda) \ \to \text{anticollinear}$$
$$q^\mu = Q(1, 1, \lambda) \ \to \text{hard}$$
$$s^\mu = Q(\lambda, \lambda, \lambda) \to \text{soft}$$

$$\lambda \ll 1$$

Instead of extracting collinear, soft or hard contributions from full QCD calculations:



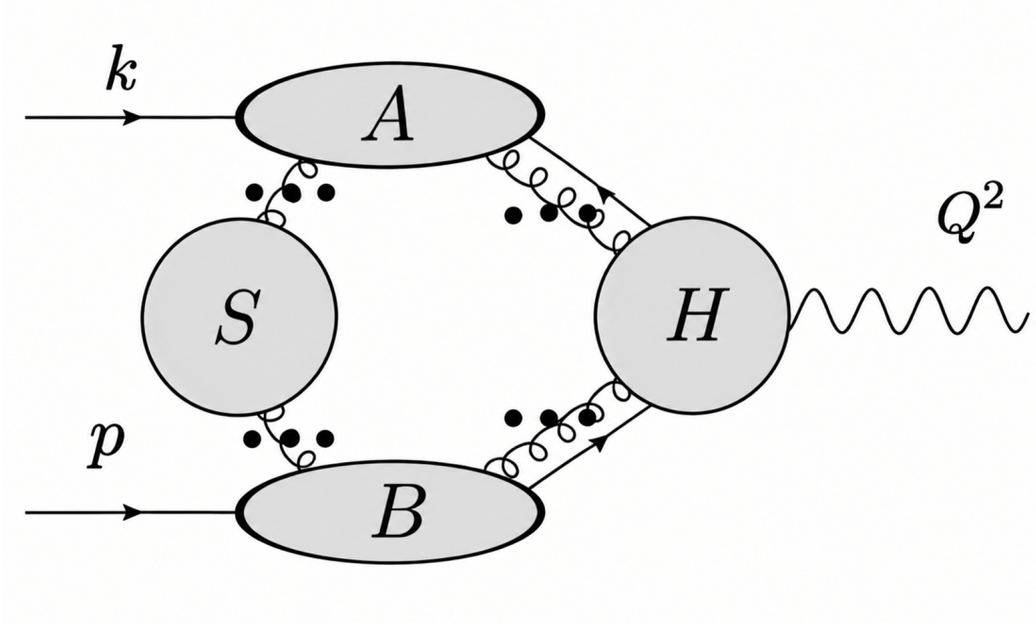
$$\bar{\psi}\gamma^\mu\psi \to \bar{\psi}_{\bar{n}}\Gamma^\mu\psi_n$$

… split QCD degrees of freedom themselves!

$$\psi \to \psi_n + \psi_{\bar{n}} + \psi_s \qquad A^\mu \to A^\mu_n + A^\mu_{\bar{n}} + A^\mu_s$$

Bauer, Fleming, Luke, Pirjol, Stewart (2000-2002)
See, e.g.: Becher, Broggio, Ferroglia (2014)

Soft-Collinear Effective Theory (SCET)



Work with operators that directly reflect factorization at leading power

Determine effective couplings (=Wilson/matching coefficients) by comparing with full QCD à la operator product expansion

$$\lim_{x\to 0} \hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0) = \sum_i C_i(x)\hat{\mathcal{A}}_i(0) + \text{power corrections}$$

QCD operators

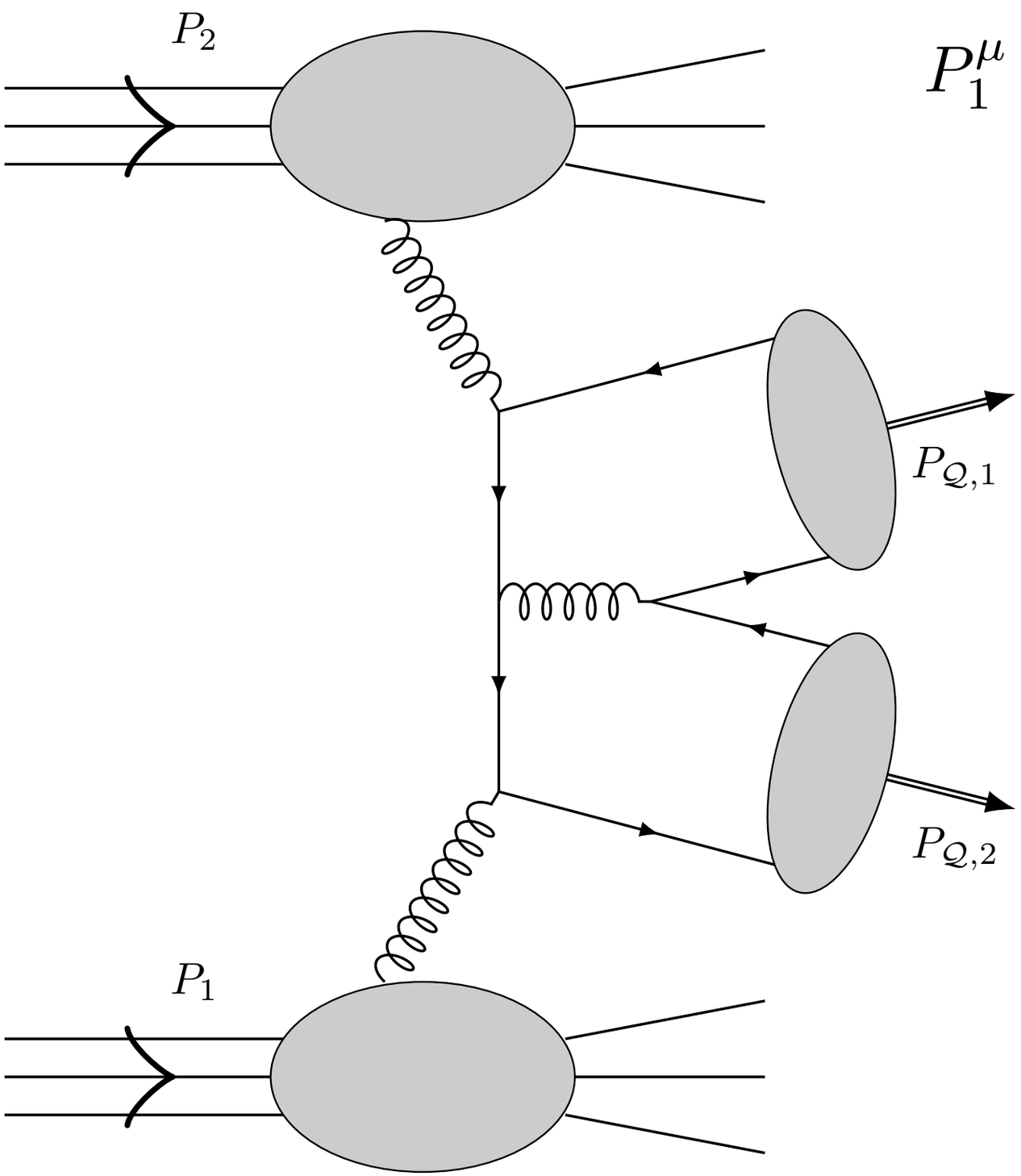
matching coefficients

effective theory operators

Effective theory needs to reproduce long-distance structure of QCD order-by-order

Bauer, Fleming, Luke, Pirjol, Stewart (2000-2002)
See, e.g.: Becher, Broggio, Ferroglia (2014)

Kinematics



$$P_1^\mu = \sqrt{\frac{s}{2}} n^\mu \quad \text{and} \quad P_2^\mu = \sqrt{\frac{s}{2}} \bar{n}^\mu$$

$$Q^2 \equiv (P_{\mathcal{Q},1} + P_{\mathcal{Q},2})^2$$

$$\mathbf{q}_T \equiv \mathbf{p}_T^{\mathrm{di}-J/\psi} \equiv \mathbf{P}_{\mathcal{Q},1} + \mathbf{P}_{\mathcal{Q},2}$$

$$\mathbf{q}_T^2 \ll Q^2$$

Cross section & effective current

$$\mathrm{d}\sigma = \frac{1}{2s}\sum\!\!\!\!\!\!\int_X \mathrm{d}\mathcal{PS}(P_{\mathcal{Q}1}, P_{\mathcal{Q}2})(2\pi)^4\delta^{(4)}\big(P_1 + P_2 - P_{\mathcal{Q}1} - P_{\mathcal{Q}2} - P_X\big)$$
$$\times \langle N_1 N_2|J^\dagger(0)|\mathcal{Q}_1\mathcal{Q}_2 X\rangle\langle \mathcal{Q}_1\mathcal{Q}_2 X|J(0)|N_1 N_2\rangle$$

Ansatz for the current:

$$J(0) = \Gamma_{\alpha\beta\mu\nu}(Q,\mu)\big[S_n S_{\bar n}\mathcal{B}^\alpha_{n\perp}\mathcal{B}^\beta_{\bar n\perp}\mathcal{O}^{\mu\nu}\big](0)$$

Effective current

$$J(0) = \Gamma_{\alpha\beta\mu\nu}(Q,\mu)\left[S_n S_{\bar{n}} \mathcal{B}^{\alpha}_{n\perp} \mathcal{B}^{\beta}_{\bar{n}\perp} \mathcal{O}^{\mu\nu}\right](0)$$

Incoming collinear gluon + Wilson lines

$$\mathcal{B}^{\mu}_{n\perp}(x) \equiv \frac{1}{g}\left[W_n^\dagger i D^{\mu}_{n\perp} W_n\right](x) \qquad W_n(x) = \mathcal{P}\exp\left[ig\int_{-\infty}^{0} \mathrm{d}\tau\, \bar{n}\cdot A_n(x+\bar{n}\tau)e^{\tau x\delta^+}\right]$$

Soft Wilson lines: $S_n(x) = \mathcal{P}\exp\left[ig\int_{-\infty}^{0} \mathrm{d}\tau\, n\cdot A_s(x+n\tau)e^{\tau\delta^-}\right]$

Hard coefficient (obtained from matching on full QCD)

Operators describing bound heavy-quarks

Soft-collinear factorisation

$$\langle \mathcal{Q}_1 \mathcal{Q}_2 X | J(0) | N_1 N_2 \rangle$$

$$J(0) = \Gamma_{\alpha\beta\mu\nu}(Q,\mu)\left[S_n S_{\bar{n}} \mathcal{B}^{\alpha}_{n\perp} \mathcal{B}^{\beta}_{\bar{n}\perp} \mathcal{O}^{\mu\nu}\right](0)$$

At leading power, collinear and anti-collinear states decouple

$$|N_1 N_2\rangle = |N_1\rangle \times |N_2\rangle$$

$$|\mathcal{Q}_1 \mathcal{Q}_2 X\rangle = |\mathcal{Q}_1 \mathcal{Q}_2 X_s\rangle \times |X_n\rangle \times |X_{\bar{n}}\rangle$$

The matrix element factorises as follows:

$$\begin{aligned}\langle \mathcal{Q}_1 \mathcal{Q}_2 X | J(0) | N_1 N_2 \rangle = \Gamma_{\alpha\beta\mu\nu}(Q,\mu) \\ \times \langle X_n | \mathcal{B}^{\alpha}_{n\perp}(0) | N_1 \rangle \times \langle X_{\bar{n}} | \mathcal{B}^{\beta}_{\bar{n}\perp}(0) | N_2 \rangle \\ \times \langle \mathcal{Q}_1 \mathcal{Q}_2 X_s | \left[S_n S_{\bar{n}} \mathcal{O}^{\mu\nu}\right](0) | 0 \rangle\end{aligned}$$

Gluon TMD PDFs

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{PS}(P_{\mathcal{Q}1}, P_{\mathcal{Q}2})} = \frac{1}{sQ^2}\left[\Gamma_{\alpha\beta\mu\nu}\Gamma^{\dagger}_{\bar{\alpha}\bar{\beta}\bar{\mu}\bar{\nu}}\right](Q^2;\mu)$$

$$\times \int \mathrm{d}^2\mathbf{b} e^{i\mathbf{b}\cdot\mathbf{q}_\perp} G_n^{(0)\bar{\alpha}\alpha}(x_1,\mathbf{b};\mu) G_{\bar{n}}^{(0)\bar{\beta}\beta}(x_2,\mathbf{b};\mu)$$

$$\times \sum\!\!\!\!\!\!\int_{X_s} \langle 0|\left[S_n^\dagger S_{\bar{n}}^\dagger \mathcal{O}^{\bar{\mu}\bar{\nu}\dagger}\right](\mathbf{b})|\mathcal{Q}_1\mathcal{Q}_2 X_s\rangle\langle \mathcal{Q}_1\mathcal{Q}_2 X_s|\left[S_n S_{\bar{n}}\mathcal{O}^{\mu\nu}\right](0)|0\rangle$$

Hard coefficient (obtained from matching on full QCD)

Bare gluon TMD correlator:

$$G_n^{(0)\bar{\mu}\mu}(x_1,\mathbf{y},\mu) = x_1 P_1^+ \int \frac{\mathrm{d}y^-}{2\pi} e^{-ix_1 P_1^+ y^-} \langle N_1|\mathcal{B}_{n\perp}^{\bar{\mu}\dagger}(y^-,\mathbf{y})\mathcal{B}_{n\perp}^{\mu}(0)|N_1\rangle$$

Quarkonium operators and soft Wilson lines

Factorisation of the quarkonium states

Away from the production threshold $Q^2 = 4M_{\mathcal{Q}}^2$

... the quarkonia are almost back-to-back in transverse space $\mathbf{P}_{\mathcal{Q}1\perp} \simeq -\mathbf{P}_{\mathcal{Q}2\perp}$

otherwise NRQCD itself breaks down...

Nayak, Qiu, Sterman (2007)

Soft radiation decouples from quarkonium states *probed at zero radial separation*

$$r^2 \sim 1/Q^2$$

Fleming, Mehen (2006)
Brambilla, Chung, Vairo, Wang (2023)
Copeland, Vitev (2026)

Up to corrections of order $\mathcal{O}(v)$

$$|\mathcal{Q}_1 \mathcal{Q}_2 X_s\rangle = |\mathcal{Q}_1\rangle \times |\mathcal{Q}_2\rangle \times |X_s\rangle$$

Factorisation of the quarkonium states

$$\langle 0|\left[S_n^\dagger S_{\bar{n}}^\dagger \mathcal{O}^{\mu\nu\dagger}\right](\mathbf{b})|\mathcal{Q}_1\mathcal{Q}_2 X_s\rangle = \Lambda_i^{\mu\dagger}\Lambda_j^{\nu\dagger}\langle 0|\left[S_n^\dagger S_{\bar{n}}^\dagger\right](\mathbf{b})|X_s\rangle$$
$$\times \langle 0|\left[\chi^\dagger \sigma^i \psi\right](0)|\mathcal{Q}_1\rangle$$
$$\times \langle 0|\left[\chi^\dagger \sigma^j \psi\right](0)|\mathcal{Q}_2\rangle$$

Lorentz transformations to quarkonium rest frame

Soft Wilson lines in incoming proton directions

NRQCD heavy-quark Pauli fields

Factorised cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{PS}(P_{\mathcal{Q}1}, P_{\mathcal{Q}2})} = \frac{1}{sQ^2} H_{\mu\nu\rho\sigma}(Q^2;\mu)\langle \mathcal{O}_1^{\mathcal{Q}}(^3S_1)\rangle^2$$
$$\times \int \mathrm{d}^2\mathbf{b} e^{i\mathbf{b}\cdot\mathbf{q}_\perp} G_n^{(0)\mu\nu}(x_1,\mathbf{b};\mu) G_{\bar{n}}^{(0)\rho\sigma}(x_2,\mathbf{b};\mu) S(\mathbf{b})$$

(Collinear) Long-Distance Matrix Elements

Bodwin, Braaten, Lepage (1995)

Conclusions

EFT (SCET + NRQCD) ‘proof' of factorisation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{PS}(P_{\mathcal{Q}1}, P_{\mathcal{Q}2})} = \frac{1}{sQ^2} H_{\mu\nu\rho\sigma}(Q^2;\mu)\langle \mathcal{O}_1^{\mathcal{Q}}({}^3S_1)\rangle^2 \times \int \mathrm{d}^2\mathbf{b} e^{i\mathbf{b}\cdot\mathbf{q}_\perp} G_n^{\mu\nu}(x_1,\mathbf{b};\mu) G_{\bar{n}}^{\rho\sigma}(x_2,\mathbf{b};\mu)$$

$$G^{\mu\nu}(x_i,k_{Ti}) = \frac{1}{2x_i}\Big[-g_T^{\mu\nu} f_1^g(x_i,\mathbf{k}_{Ti}^2) + \frac{k_T^\mu k_T^\nu - \frac{1}{2}k_T^2 g_T^{\mu\nu}}{M_p^2} h_1^{\perp g}(x_i,\mathbf{k}_{Ti}^2)\Big]$$

Must be verified by matching unto full QCD order-by-order

works at NLO! Sun (2023-2024)

Caveat: valid up to power corrections $\mathcal{O}(\mathbf{q}_T^2/Q^2)$

$\mathcal{O}(v^2)$ for charmonium, $v^2 \sim 0.3$