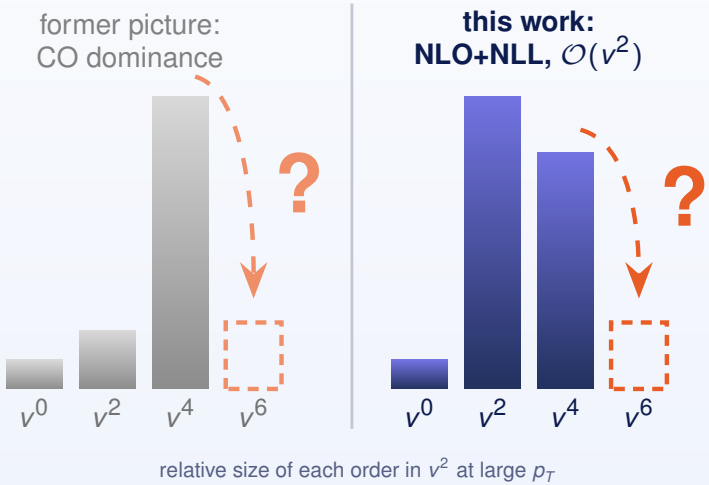
High-p_T charmonium hadroproduction in NRQCD up to $\mathcal{O}(v^2)$ at NLO+NLL

is NRQCD converging better than expected ?



relative size of each order in v^2 at large p_T

V. Bertone, **J.P. Lansberg**,
K. Lynch and T. Rabemananjara

IJCLab Orsay – Paris-Saclay U. – CNRS

Funded by the European Union

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REF2027 – Resummation, Evolution, Factorisation
October 19–23, 2026 · Santiago de Compostela

based on arXiv:2510.06456 and arXiv:2610.02096

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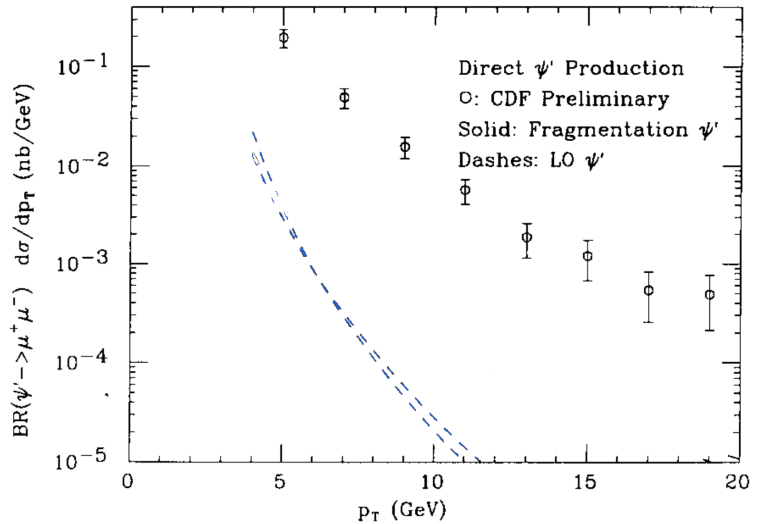

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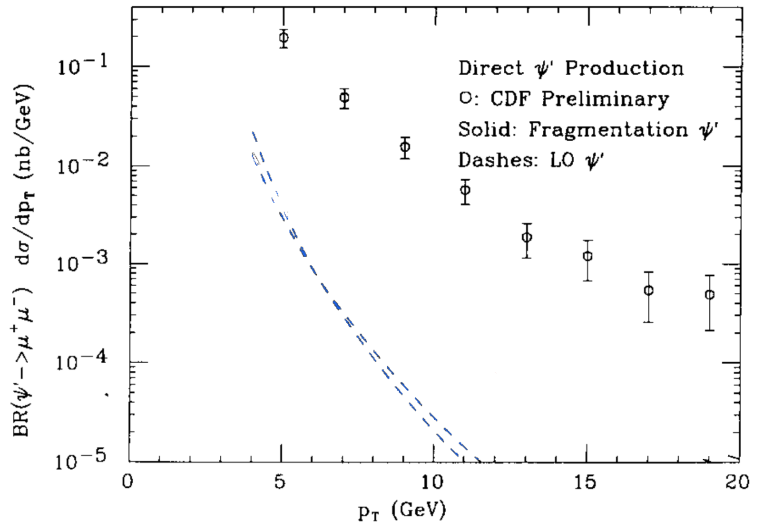

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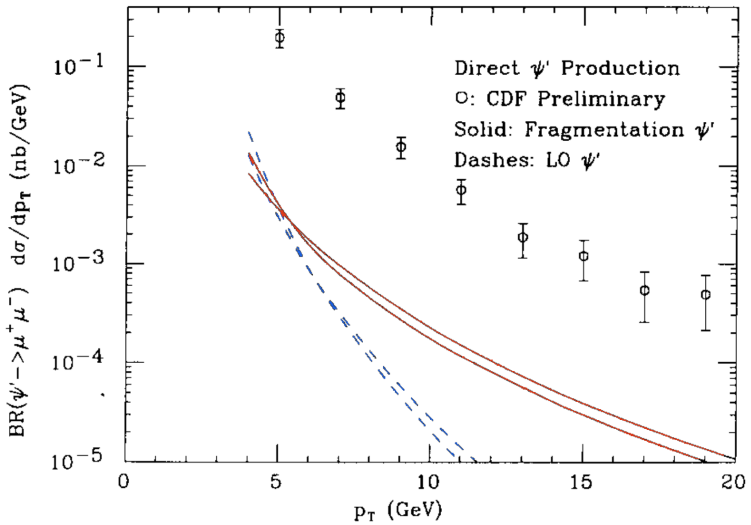

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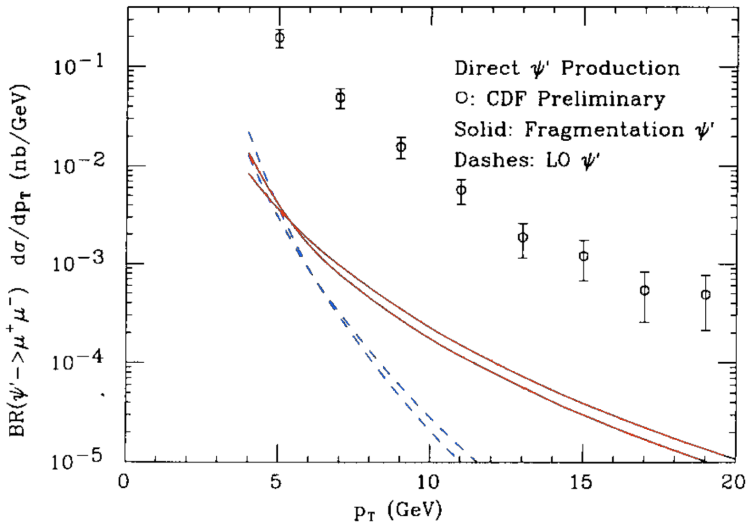

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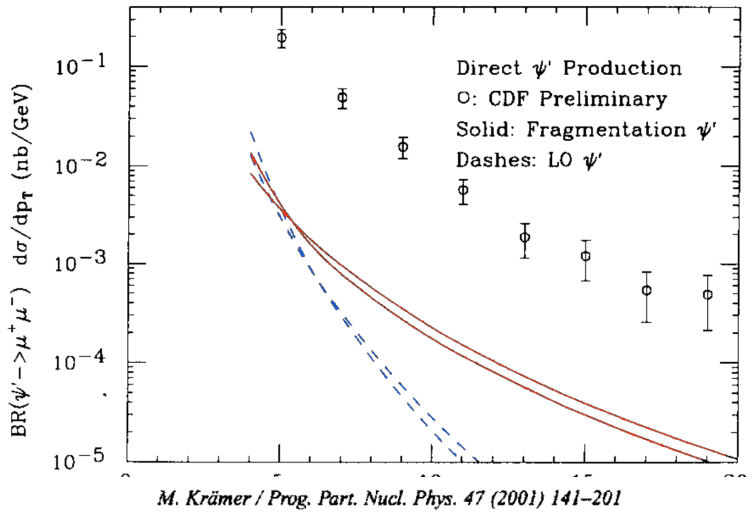

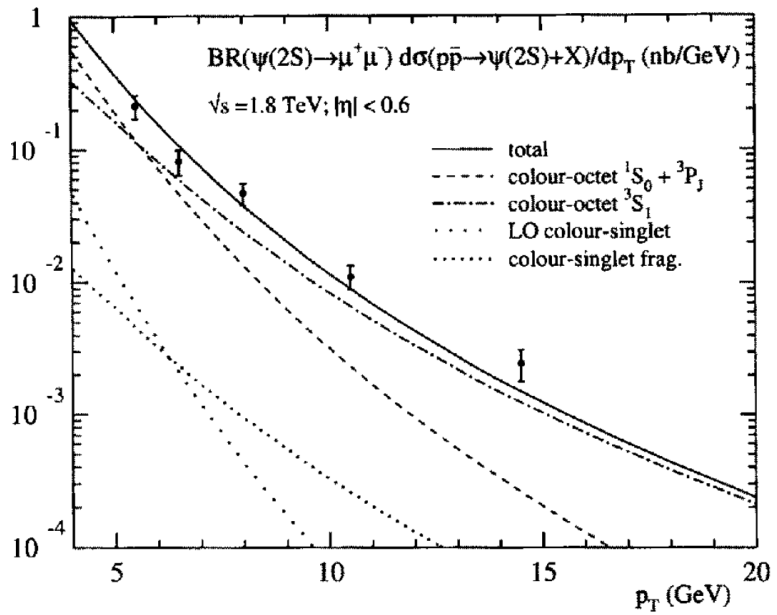

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G. Bodwin, E. Braaten, G. Lepage, PRD 51 (1995) 1125; P. Cho, A. Leibovich PRD 53 (1996) 6203

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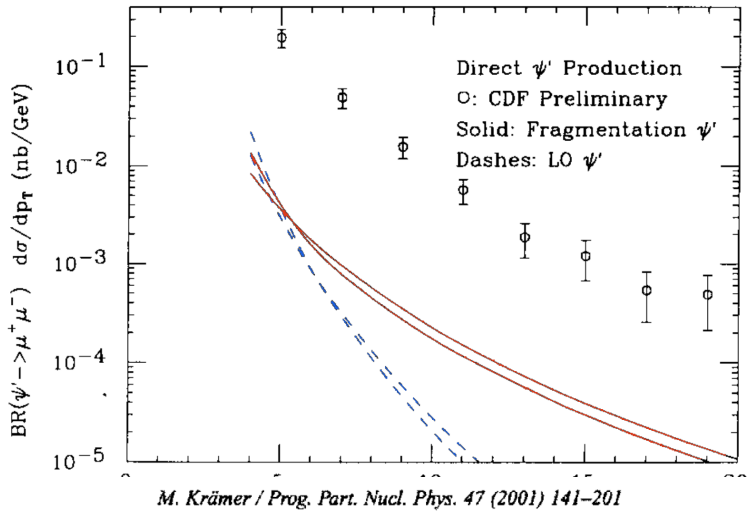

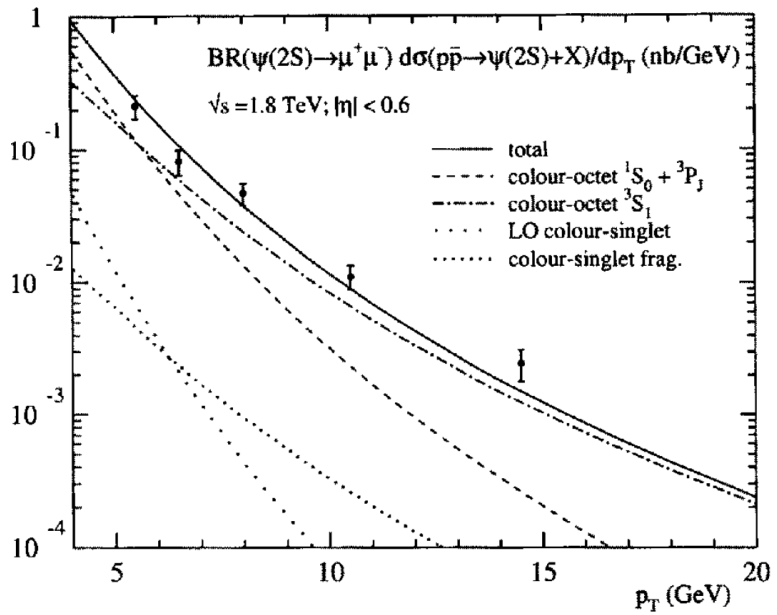

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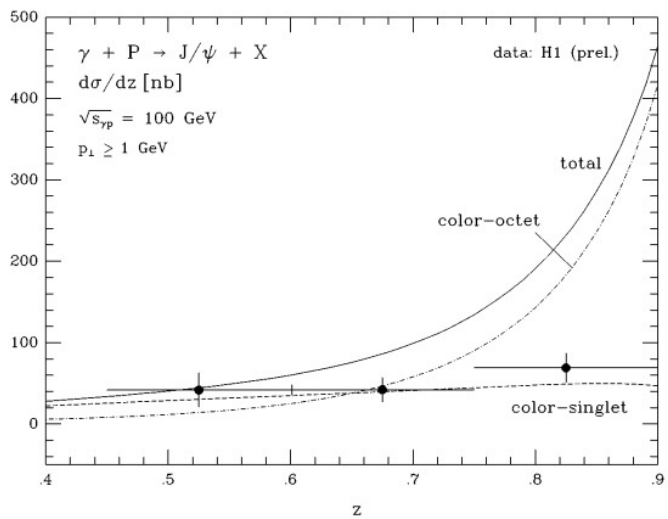

FIG. 4. The J/ψ energy distribution $d\sigma/dz$ at the photon-proton center of mass energy $\sqrt{s_{\gamma p}} = 100$ GeV integrated in the range $p_\perp \geq 1$ GeV. Experimental data from [26].

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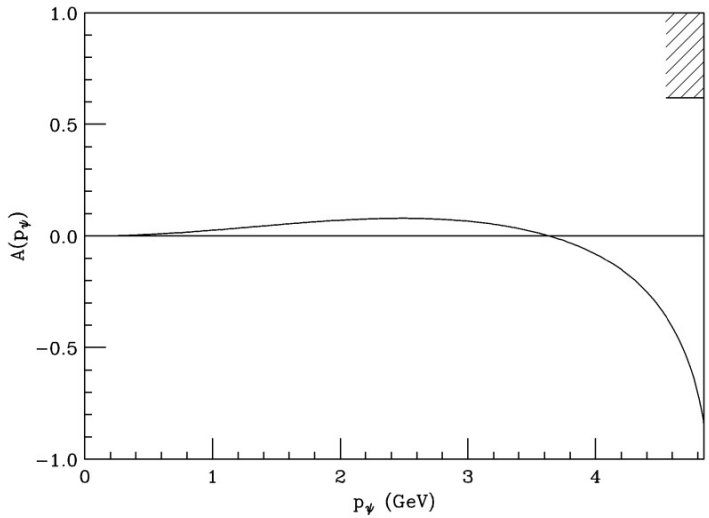

FIG. 1. Angular distribution parameter A as a function of the momentum p_ψ of the ψ for e^+e^- annihilation at $\sqrt{s} =$ 10.6 GeV. The solid line is the prediction of the color-singlet model. The hatched region is the band of predictions in the

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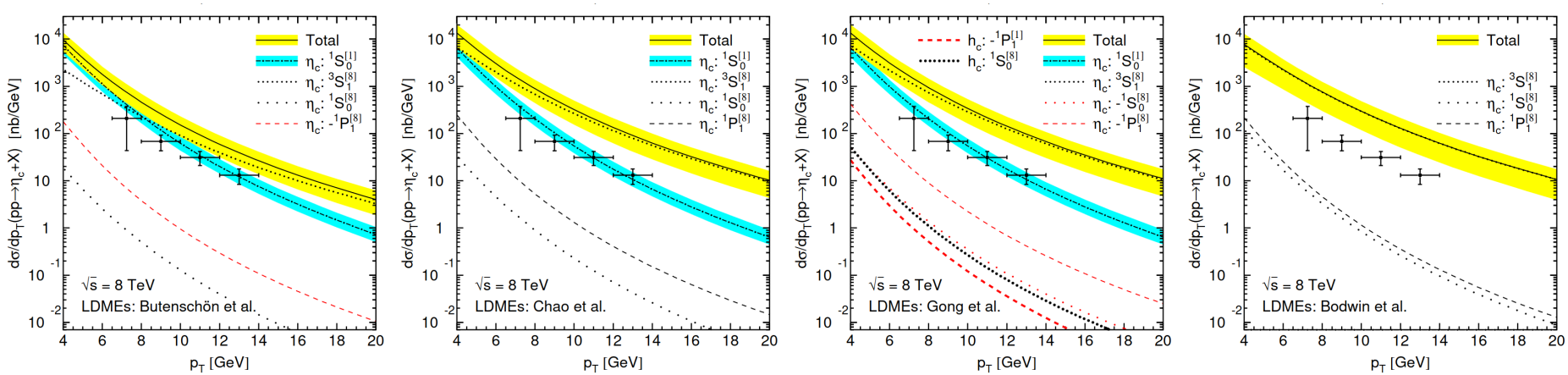

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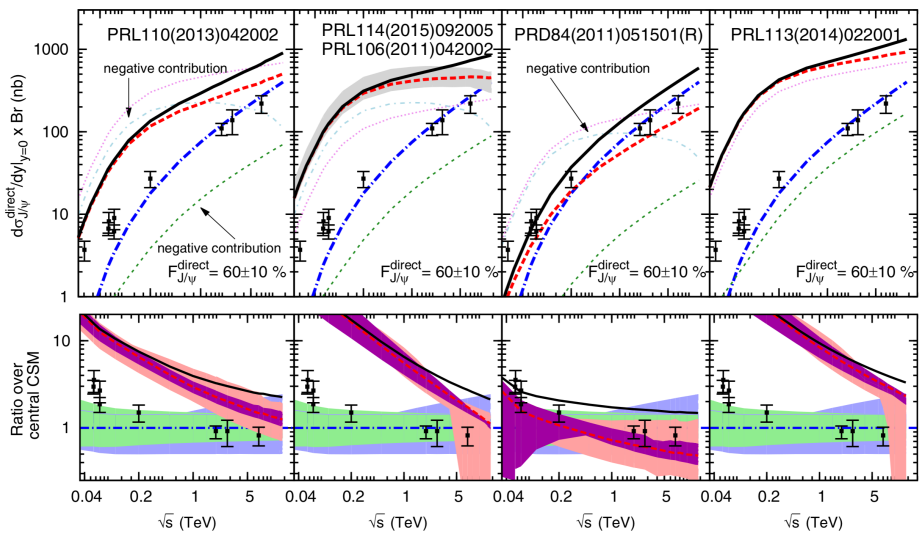

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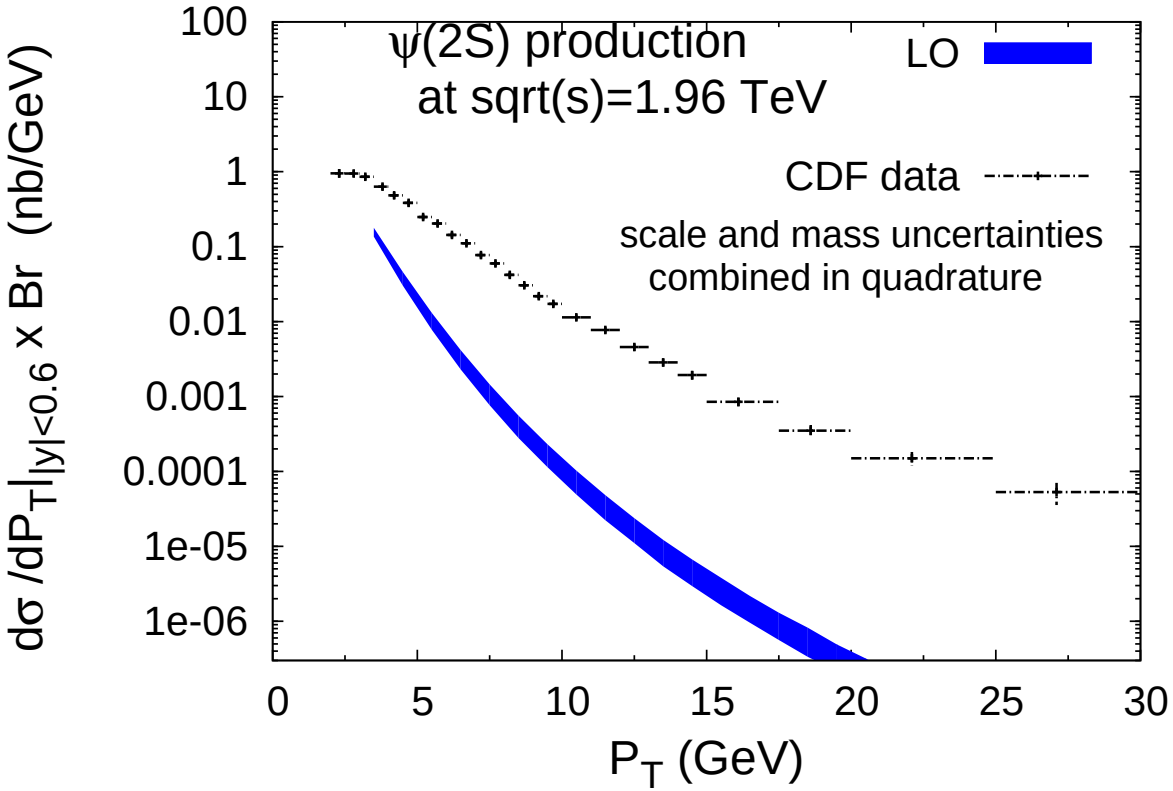

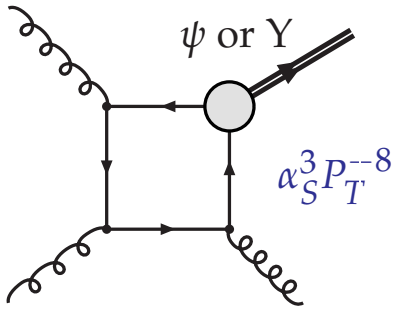

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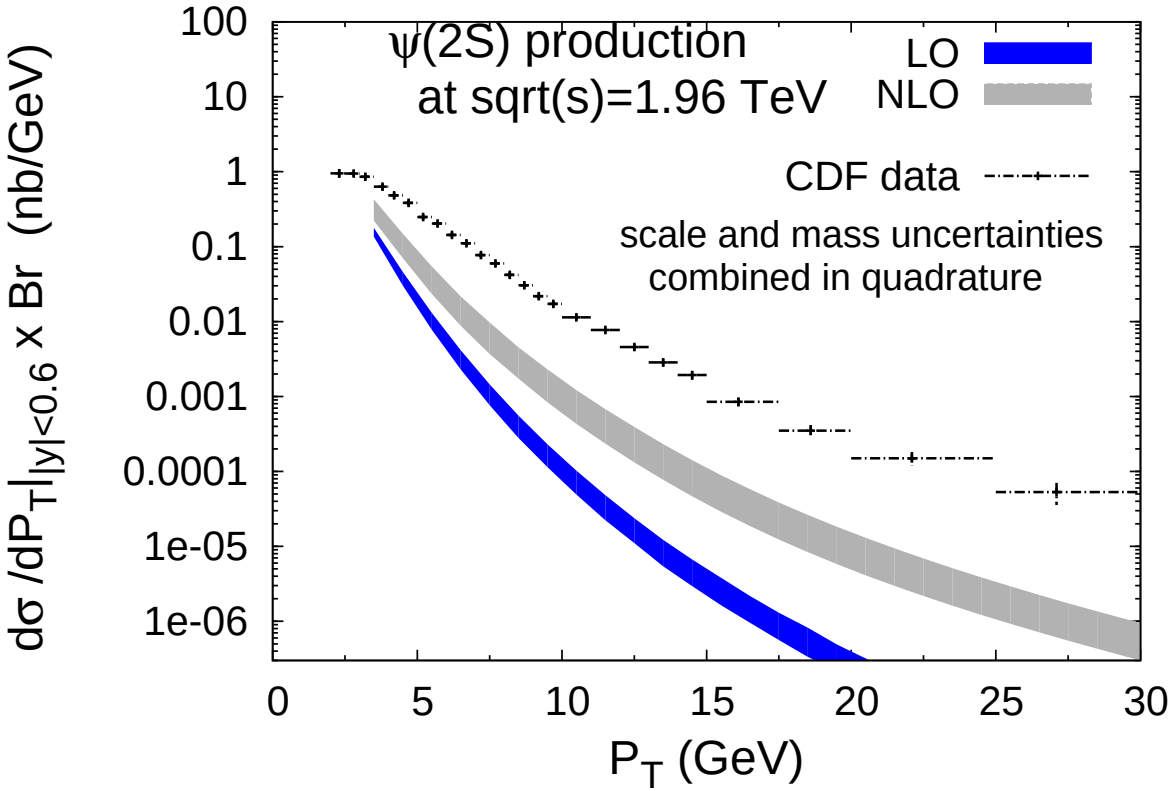

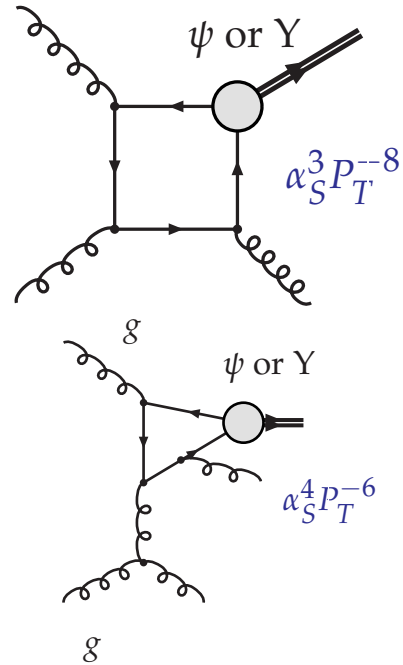

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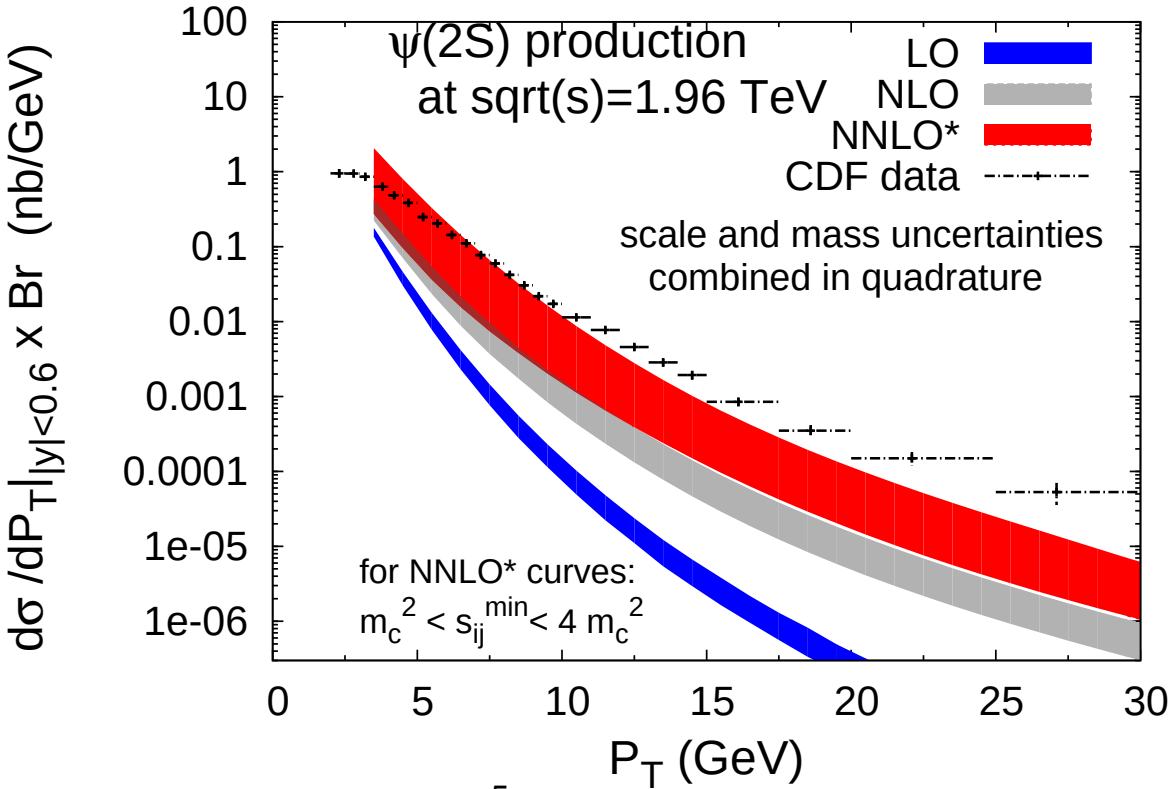

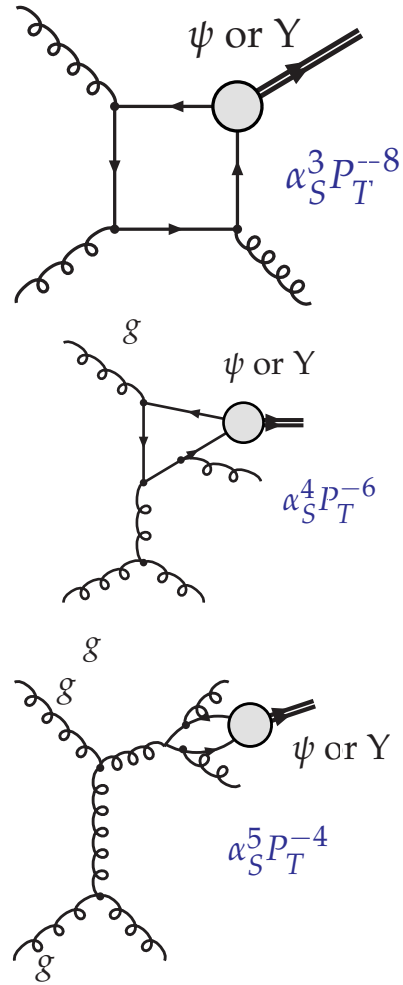

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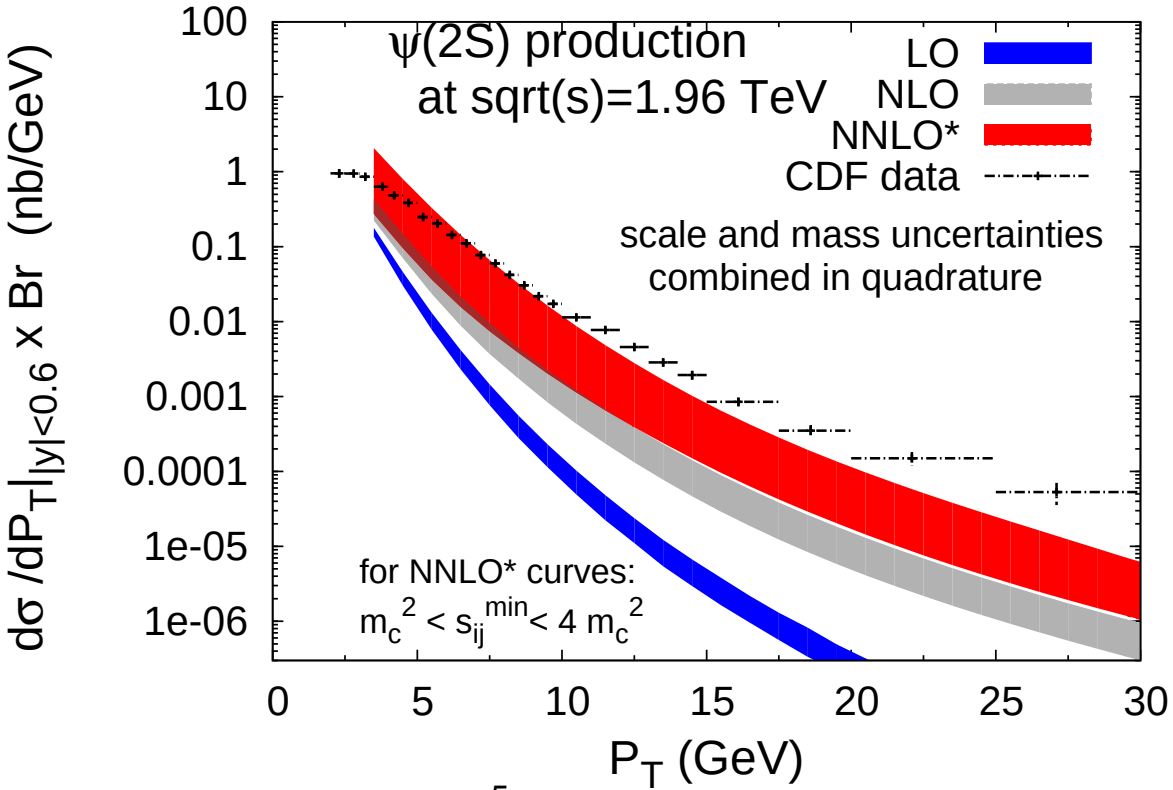

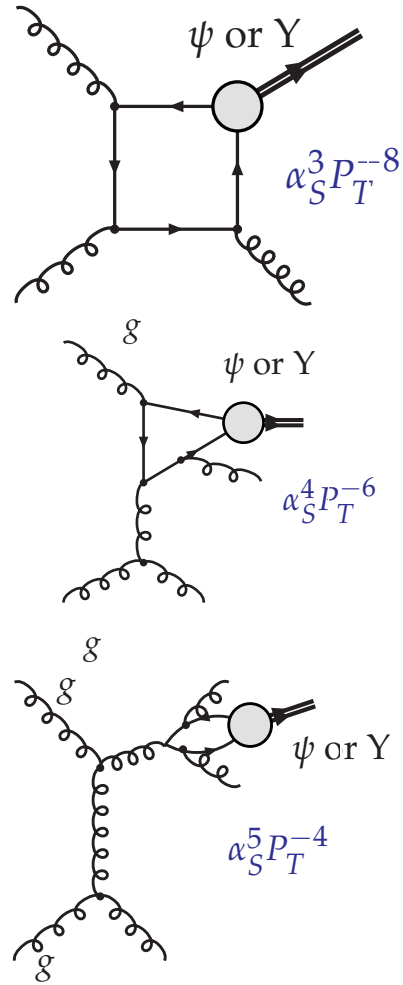

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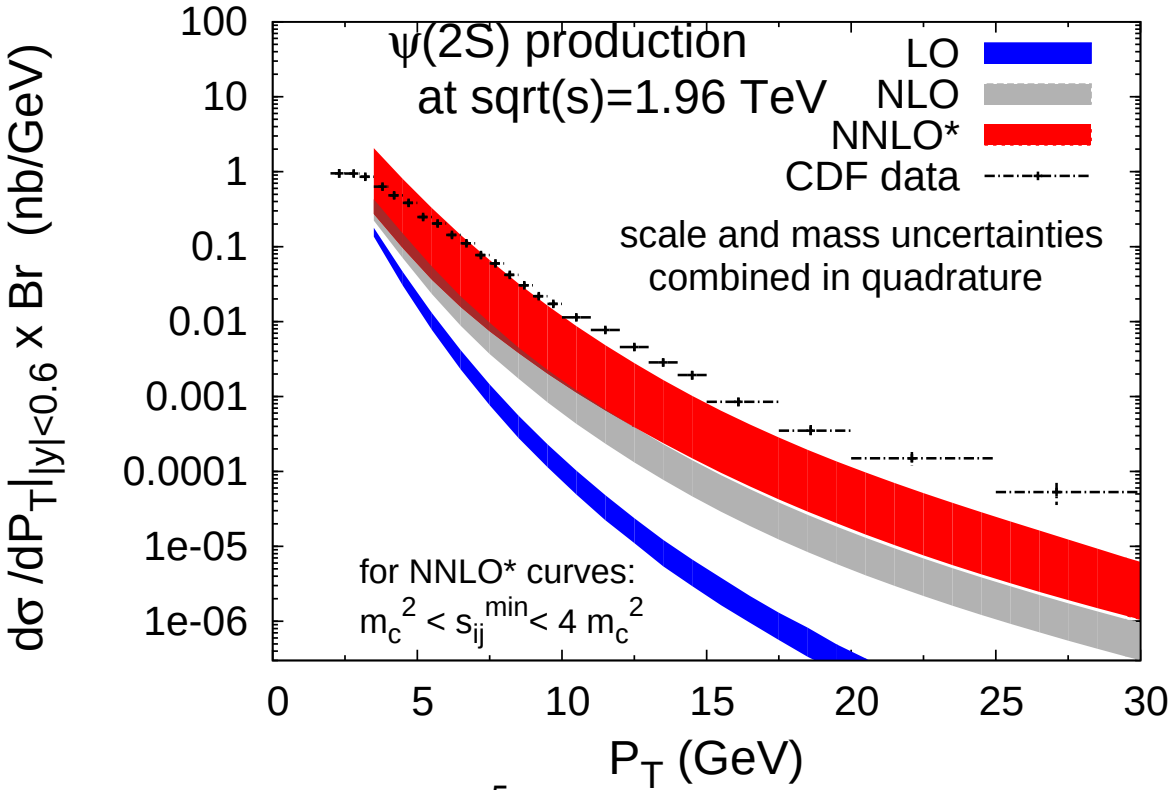

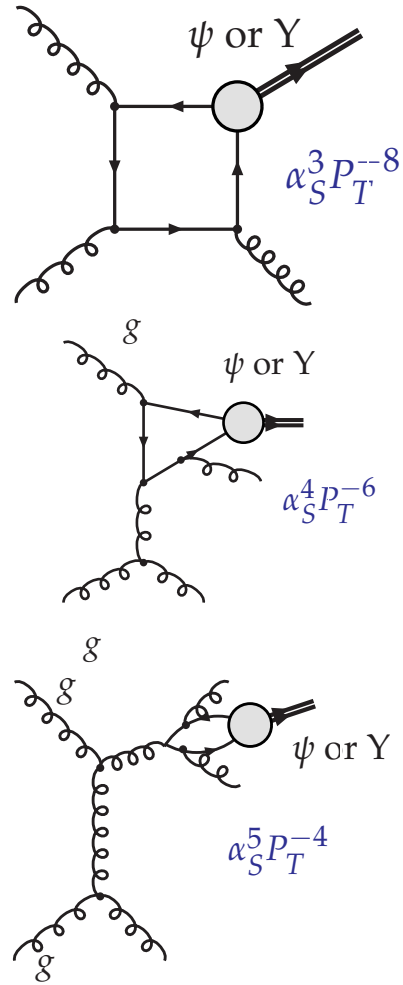

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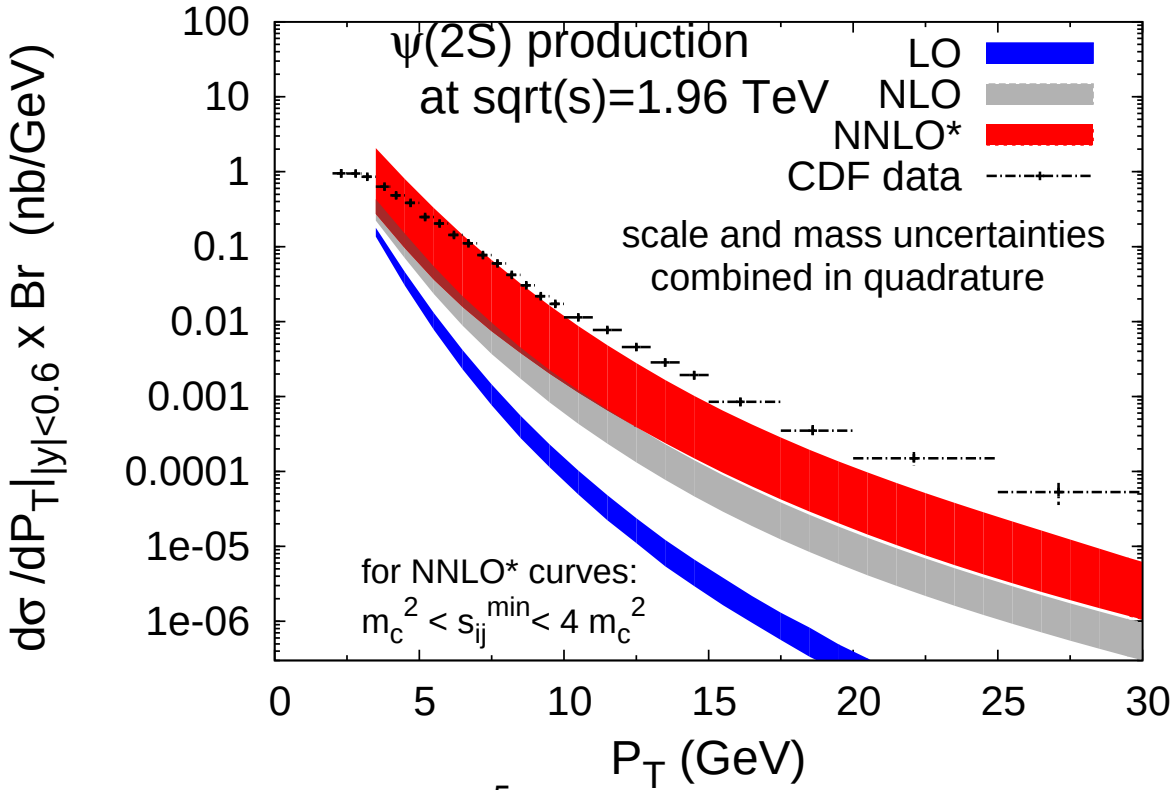

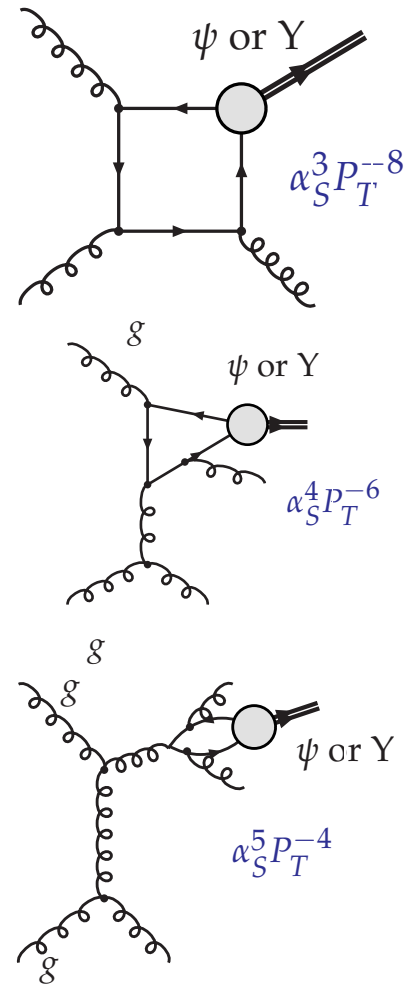

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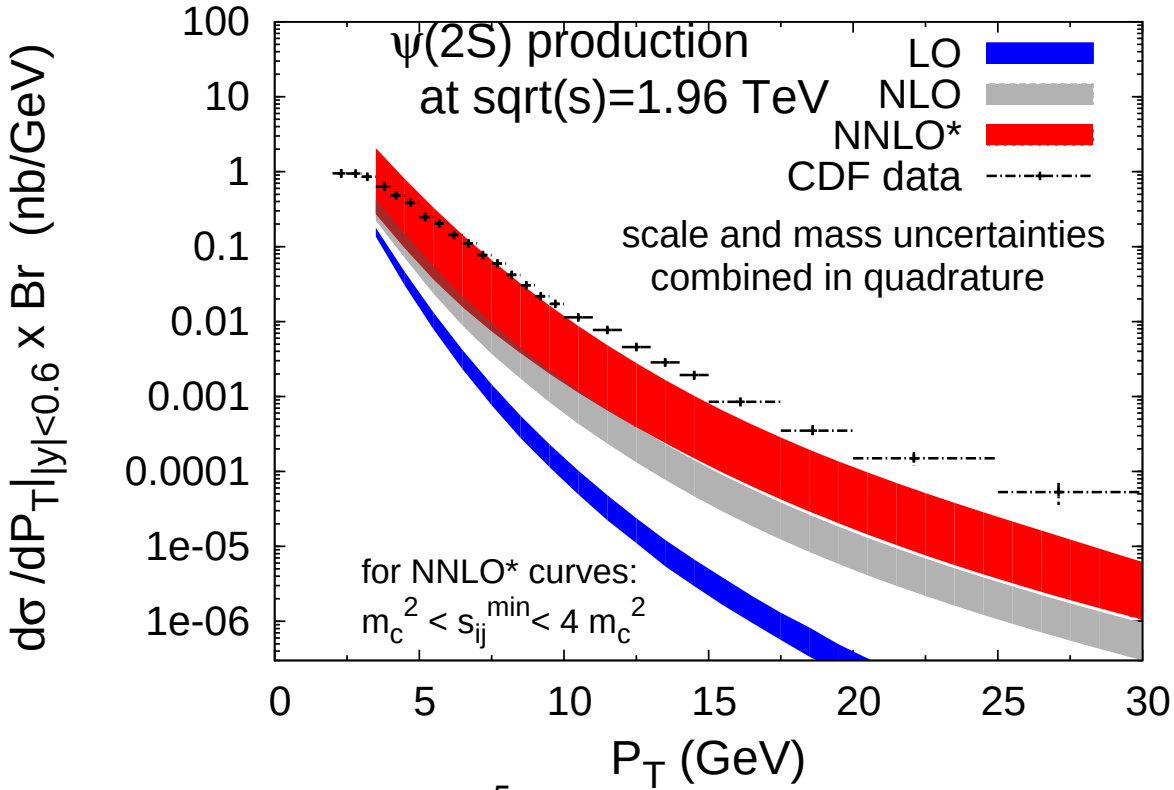

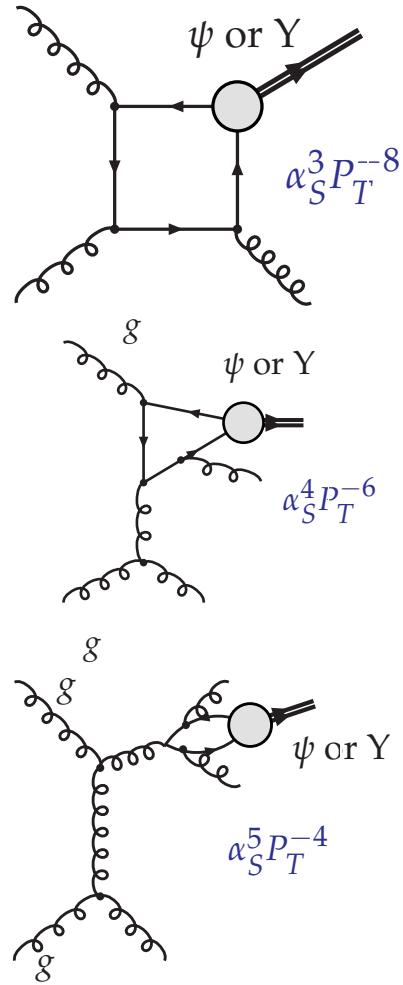

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- Let us analyse this with modern data and modern tools

Quarkonium production in NRQCD

1. Approach valid for $p_T \not\gg m_Q$ **usual fixed-order approach** (FFNS)

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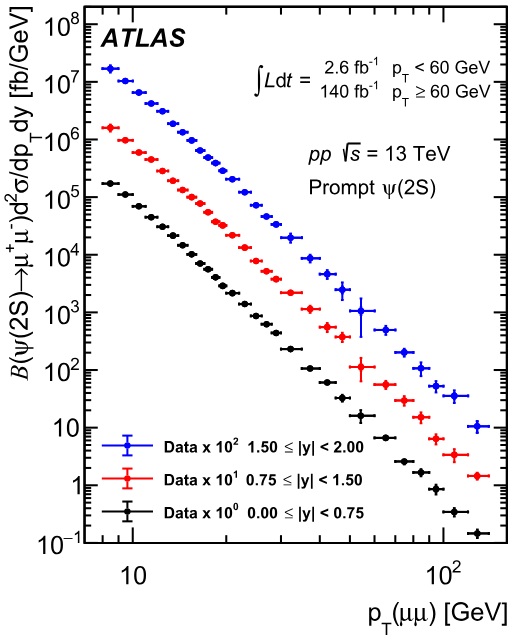
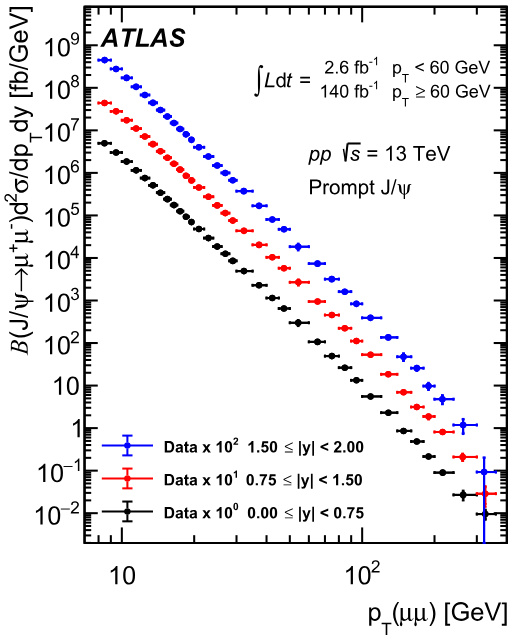
Defining $z = \frac{p_{T,\mathcal{Q}}}{p_{T,k}}$

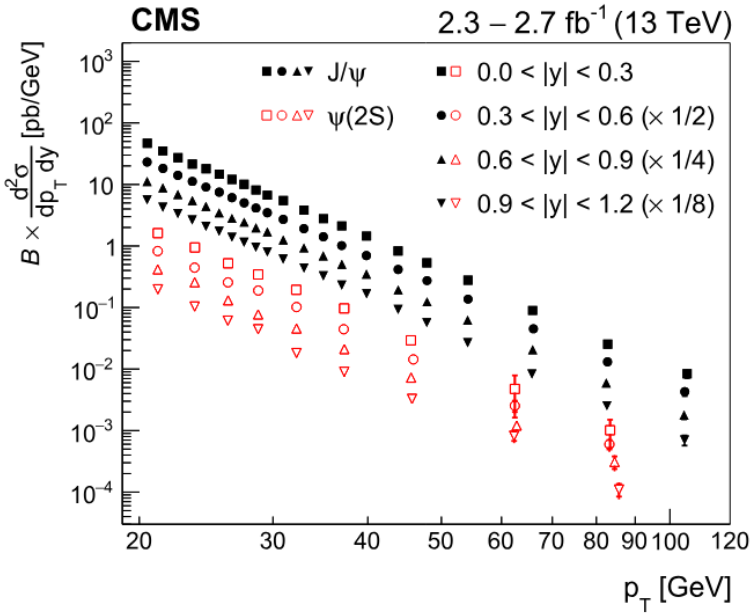
$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} \propto p_{T,\mathcal{Q}}^{-N} \sum_k \int dz\, z^{N-1}\, D_{k/\mathcal{Q}}(z, \mu_F(\simeq p_T)) = p_{T,\mathcal{Q}}^{-N} \sum_k \tilde{D}_{k/\mathcal{Q}}(N, \mu_F)$$

New ATLAS and CMS prompt ψ data

PLB 780 (2018) 251–272, EPJC 84 (2024) 2, 169, PLB 858 (2024) 139044







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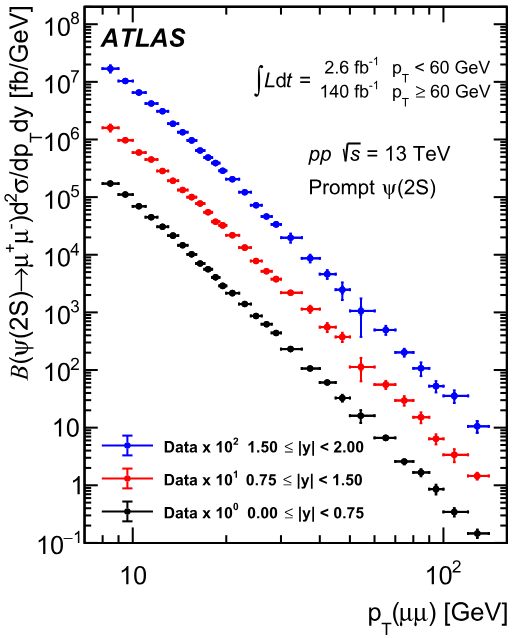

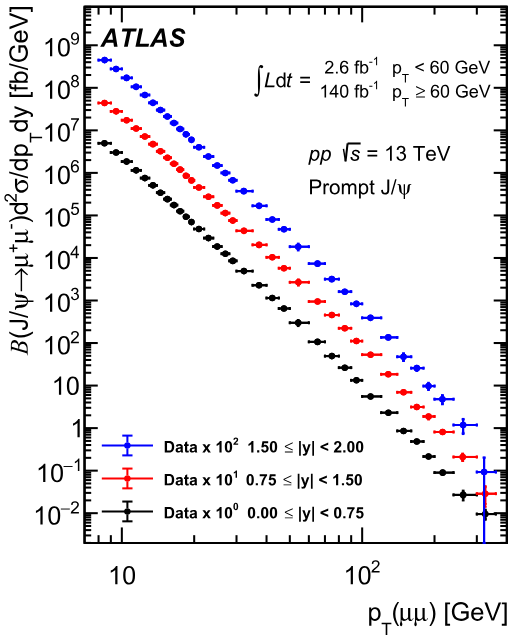

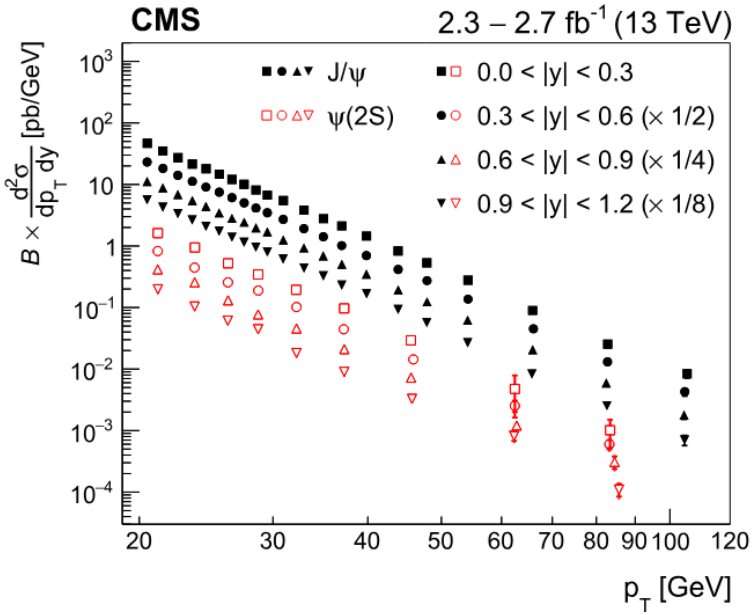

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- ATLAS P_T-differential: $\psi(2S) \rightarrow$ 140 GeV; $J/\psi \rightarrow$360 GeV

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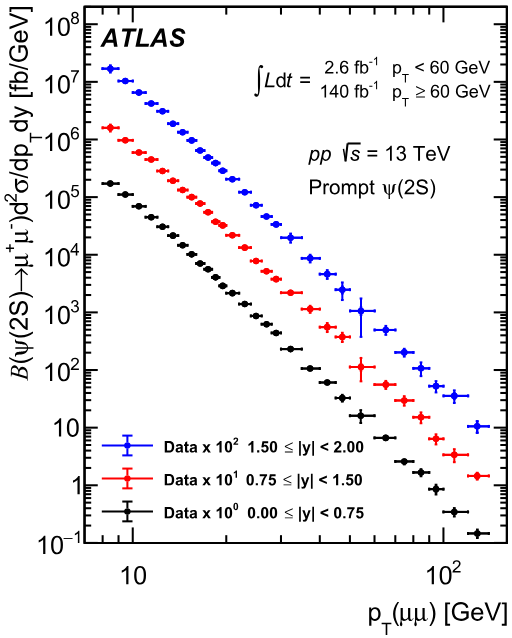

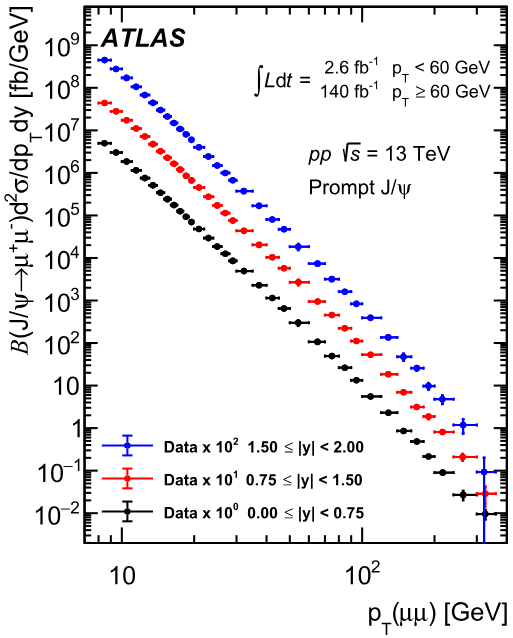

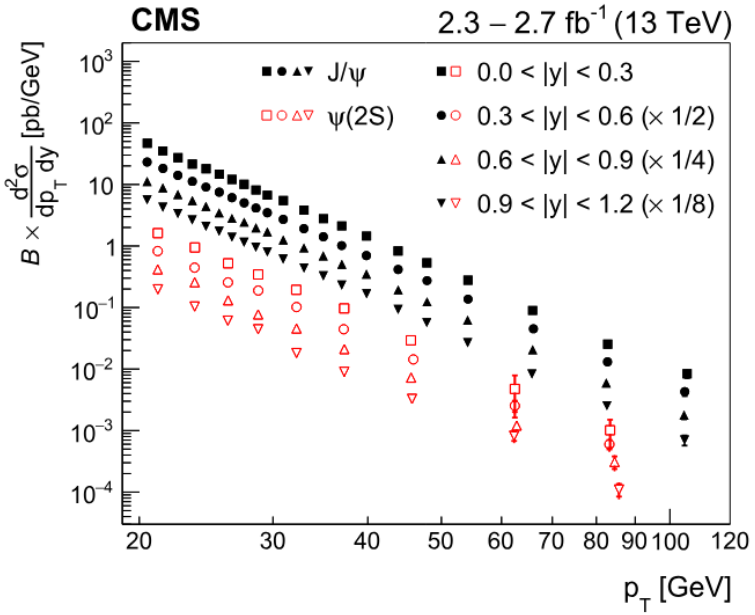

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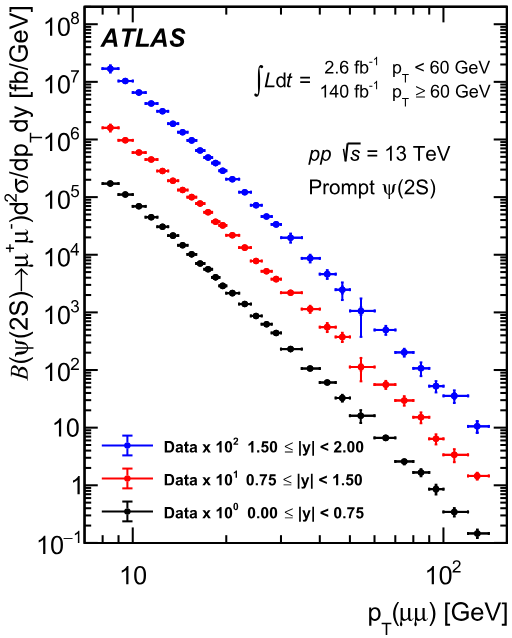

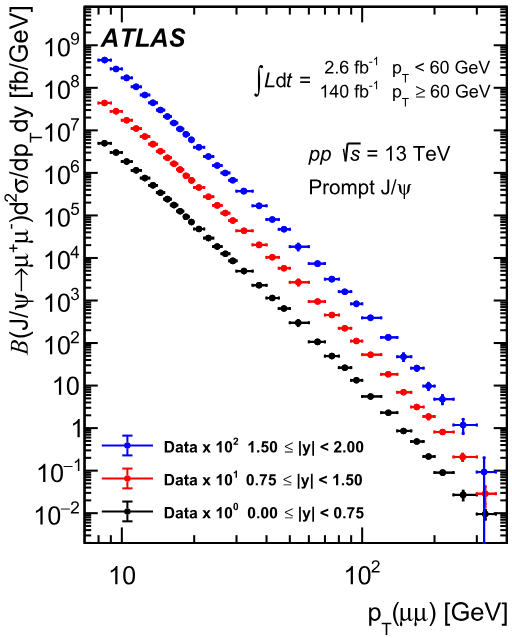

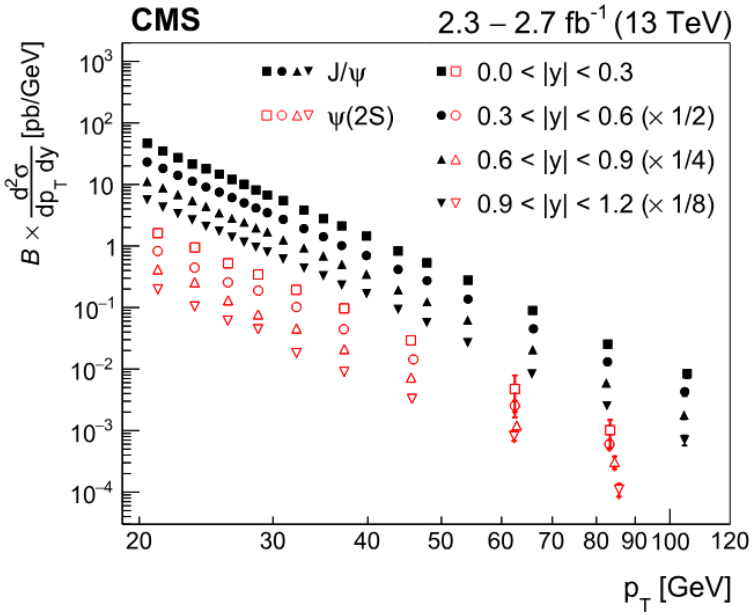

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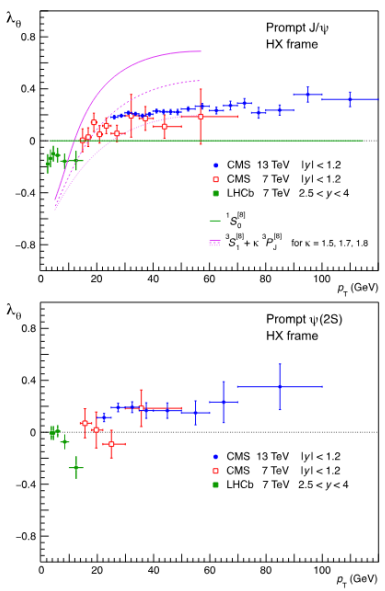

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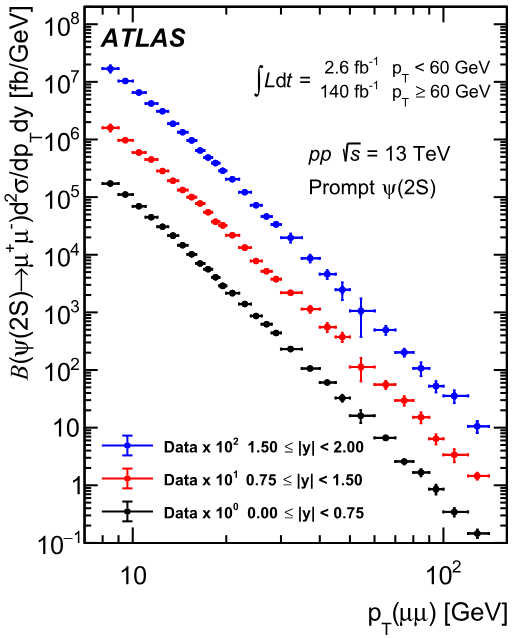

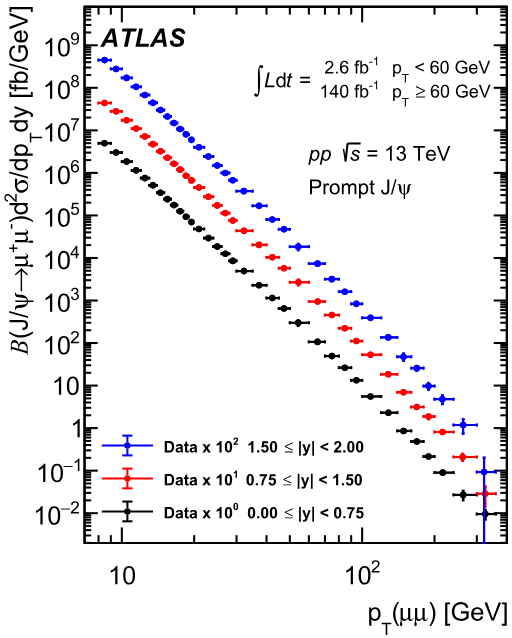

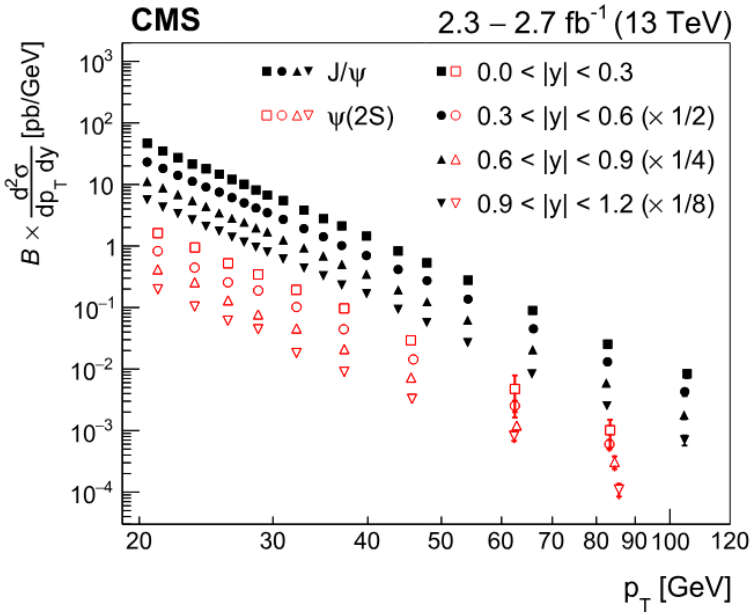

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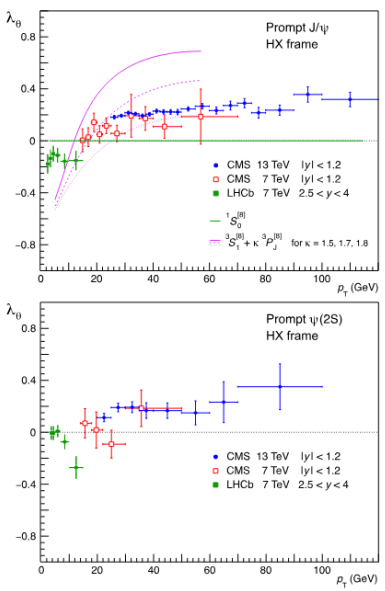

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PRD 52 (1995) 1726, PRD 24, 132 (1981), PRD 17, 3090 (1978), PLB 153 (1977), PLB 338 (1980)

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- **CO LDMEs unconstrained, but some related by HQSS**:

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Relativistic corrections to $g \to {}^3S_1^{[1]}$ FF

using notation of Bodwin, Kim, Lee, JHEP 11 (2012) 020

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		d_0	d_2	$m_Q^2\langle v^2\rangle d_2/d_0$
1st moment @μ_0:	$\tilde{D}(1,\mu_0)$	8.3	20.3 $/m_Q^2$	2.4 $\times\langle v^2\rangle$

JHEP 11 (2012) 020

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		d_0	d_2	$m_Q^2\langle v^2\rangle d_2/d_0$
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- Cross section ($N \simeq 6$) is very sensitive to relativistic corrections: 50% $\to$ 300%!
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G. Bodwin, J. Lee, PRD 69 054003 (2004)

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Impact of relativistic corrections on quarkonium phenomenology

Tools used in our study

- Fragmentation function initial conditions
  - Expressions detailed in supplementary materials of **2510.06456** and taken from listed refs.
    - $g \to \psi_{T,L} gg$ at LO α_s, v^0, v^2 **Zhang, Ma, Chen, Chao, PRD 96, 094016 (2017)**
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- Validations
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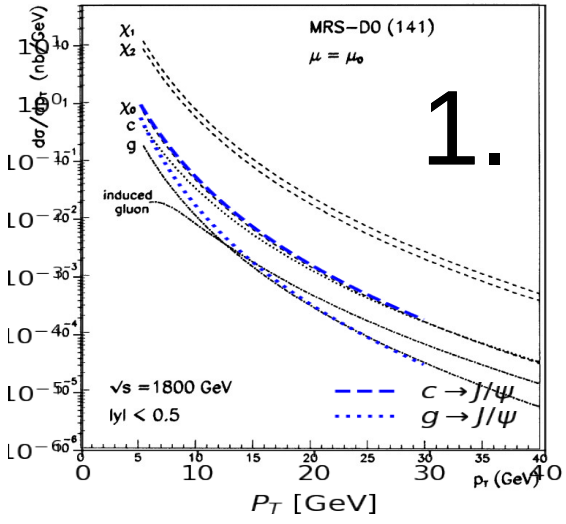
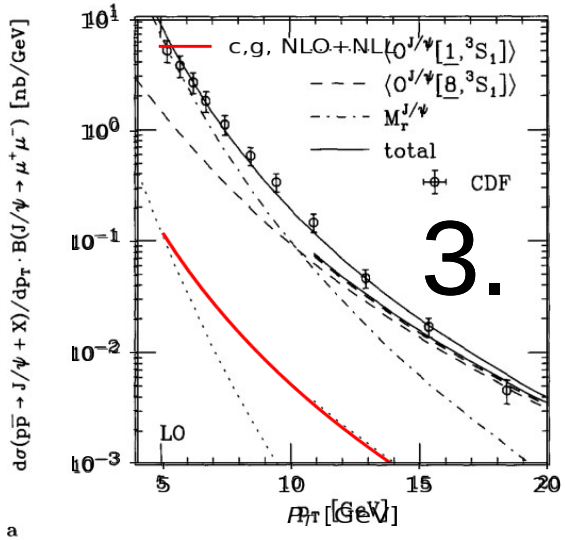
Validations





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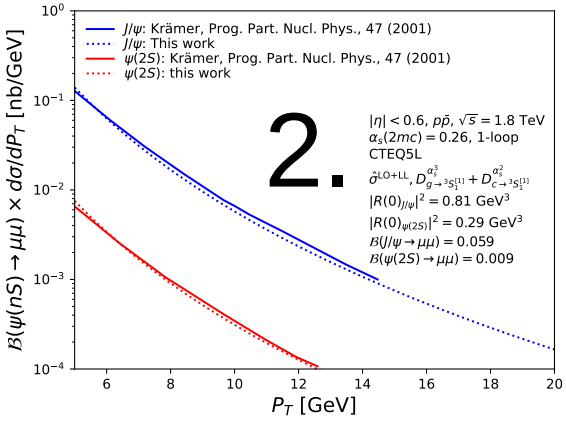
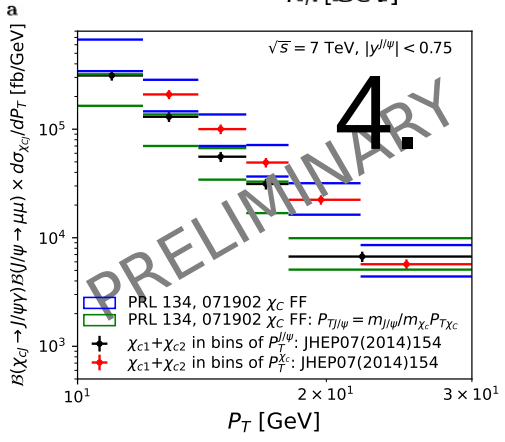
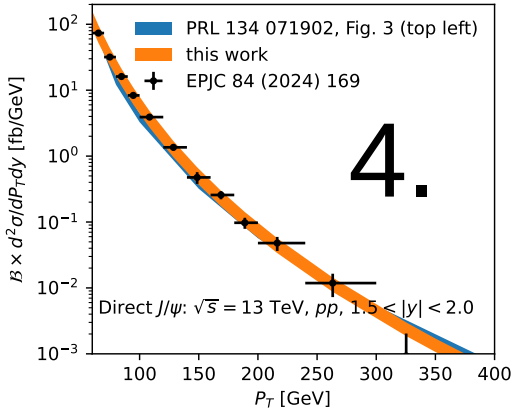
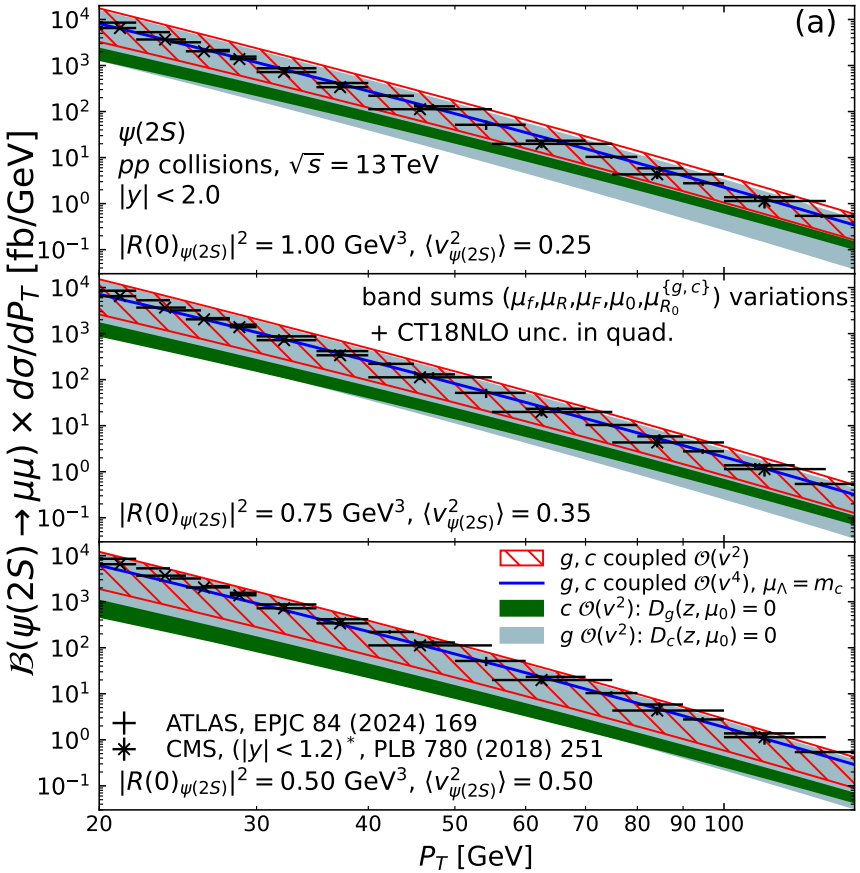






Prompt $\psi(2S)$ P_T-differential cross section results

arXiv:2510.06456



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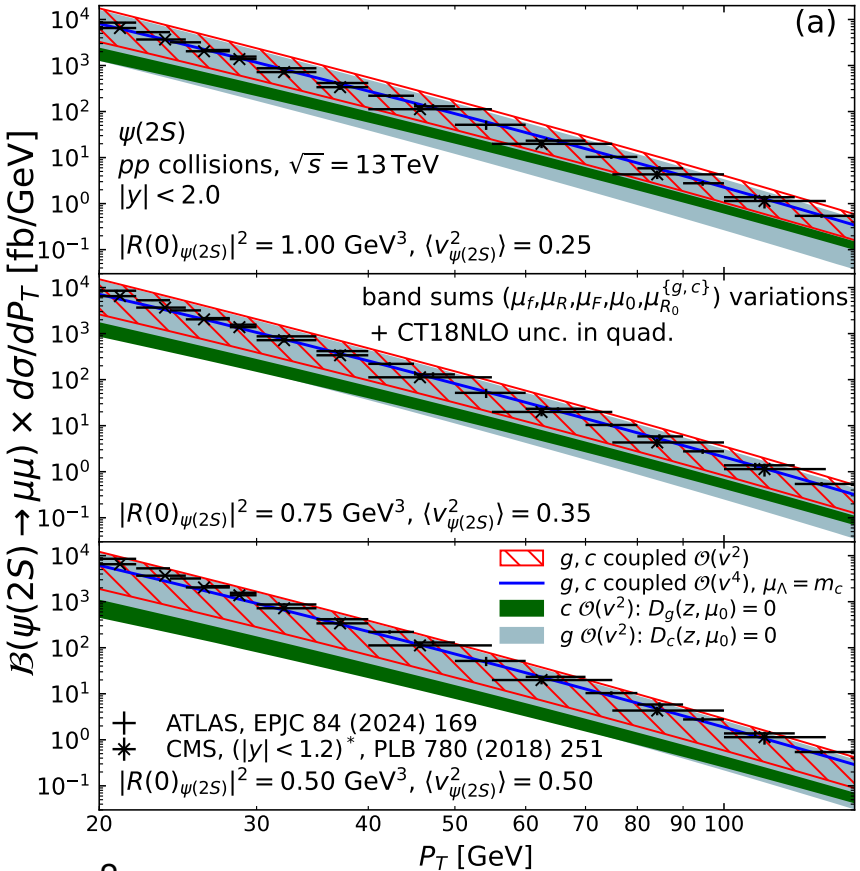

- v^2 corrections strongly enhance the gluon frag. contribution

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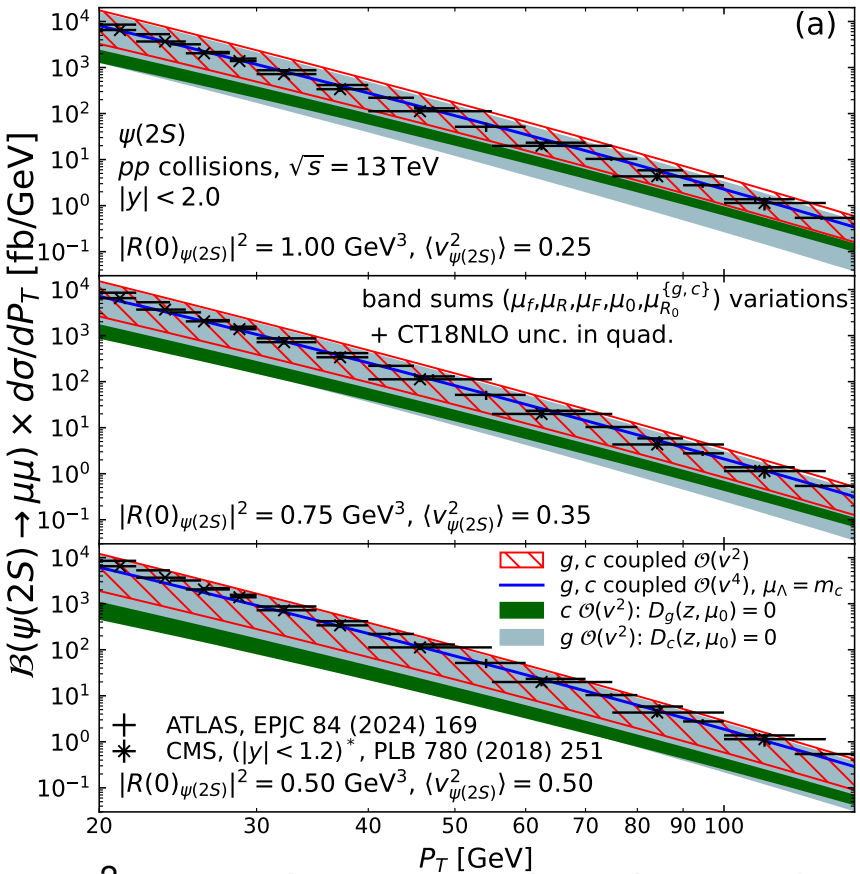

Results for three sets of parameters

$$|R(0)|^2 = 1 \text{ GeV}^3$$
$$\langle v^2 \rangle = 0.25$$

$$|R(0)|^2 = 0.75 \text{ GeV}^3$$
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- v^2 corrections strongly enhance the gluon frag. contribution
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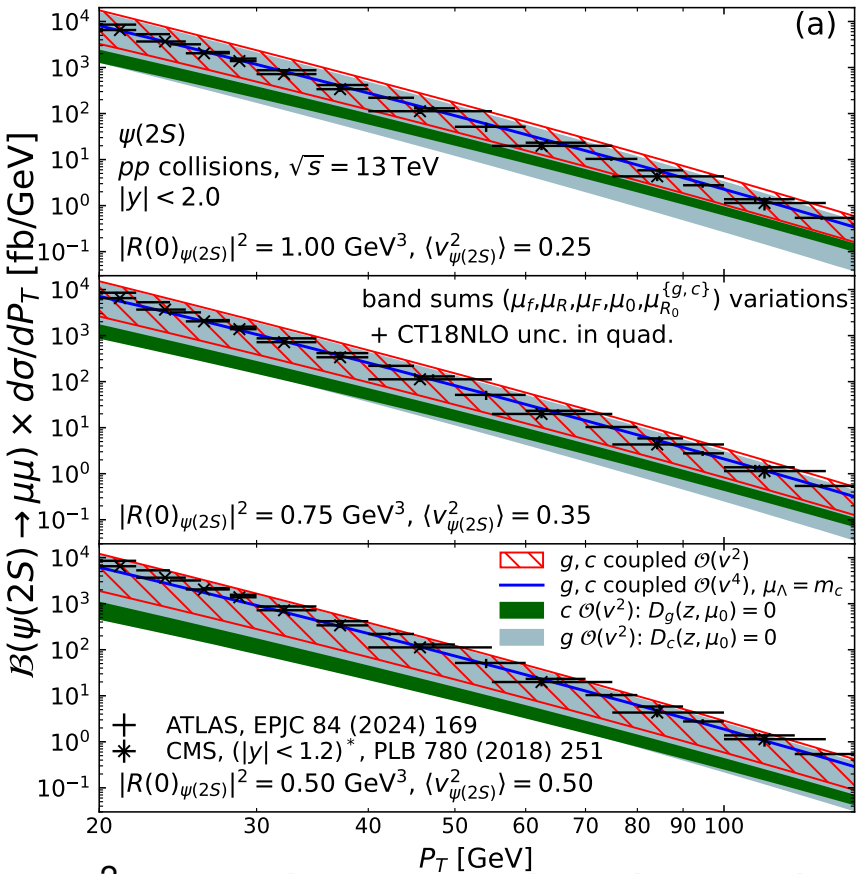

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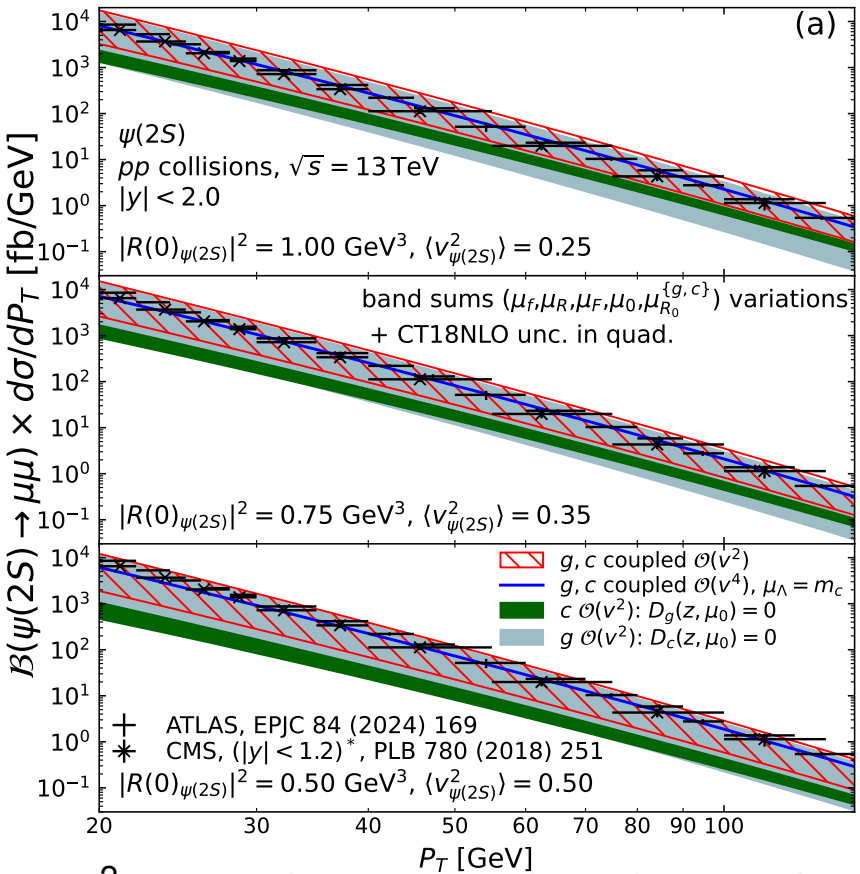

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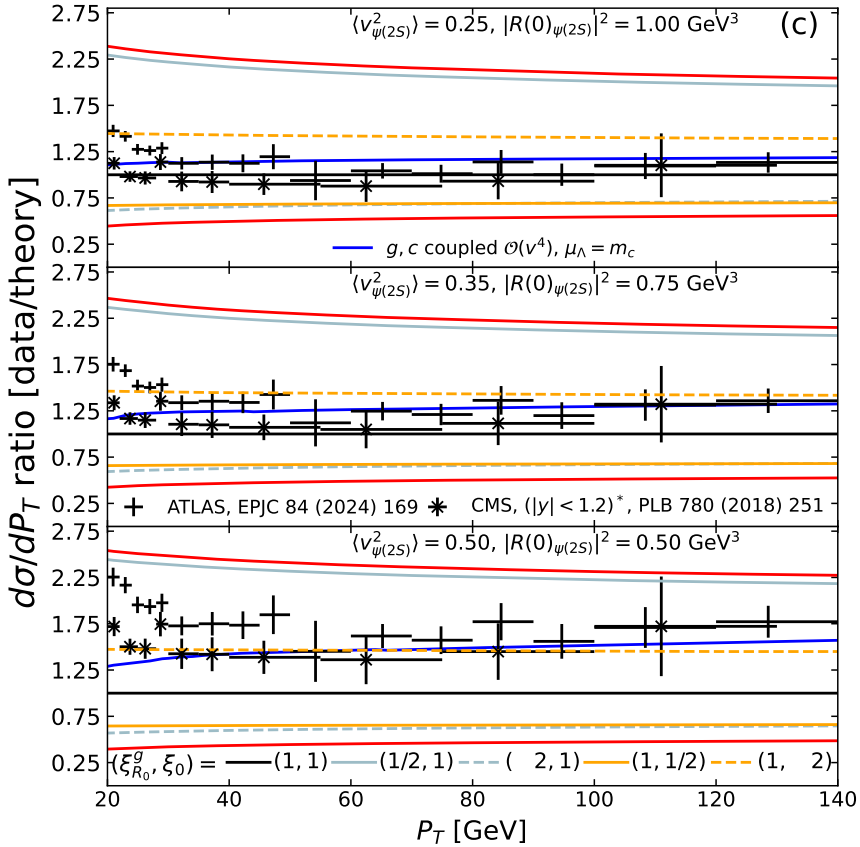
- The different parameter choices describes the data
- Differences mostly down to sharing between gluon and charm

A closer look at the uncertainties

arXiv:2510.06456



- Band includes PDF unc + full scale var: $\{\mu_{F_i}, \mu_R, \mu_{F_f}, \mu_0, \mu^g_{R_0}, \mu^c_{R_0}\}$
- Uncertainties dominated by $\mu^g_{R_0}$ in gluon frag. func. :
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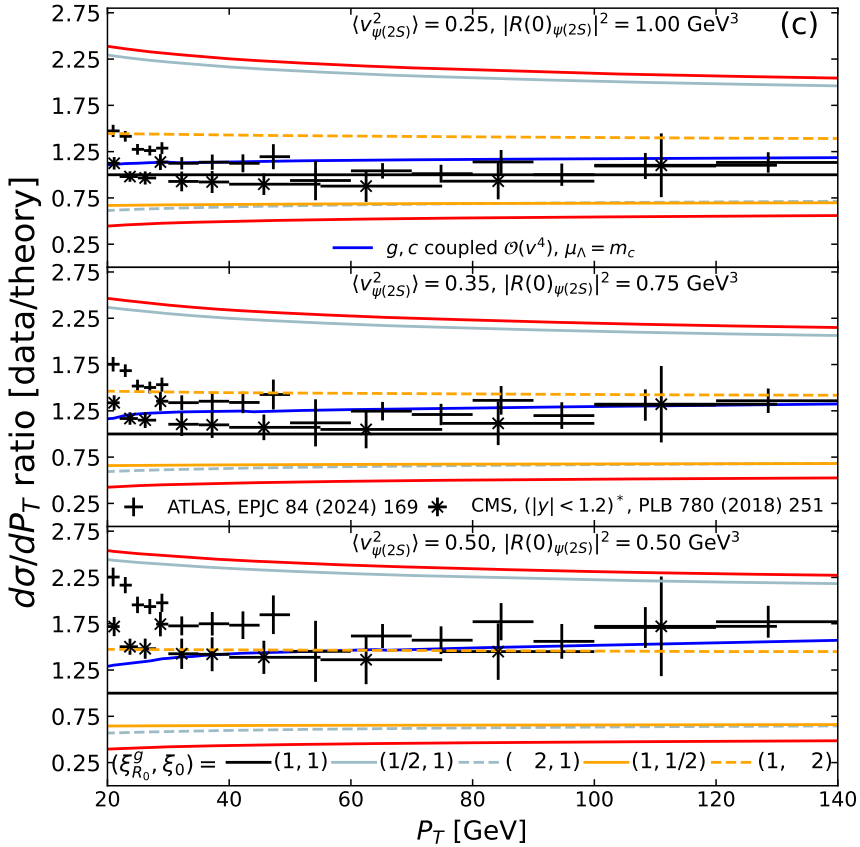

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- The rest of the scale variations + PDF uncertainty are smaller

$\psi(2S)$ polar anisotropy at $\mathcal{O}(v^2)$

arXiv:2510.06456

$$\frac{dN}{d\cos\theta} = \mathcal{N}\left(1 + \lambda_\theta \cos^2\theta\right), \qquad \lambda_\theta = \frac{1/2\sigma_T - \sigma_L}{1/2\sigma_T + \sigma_L} = \begin{cases} +1, & \text{transverse,} \\ -1, & \text{longitudinal,} \\ 0, & \text{unpolarised.} \end{cases}$$

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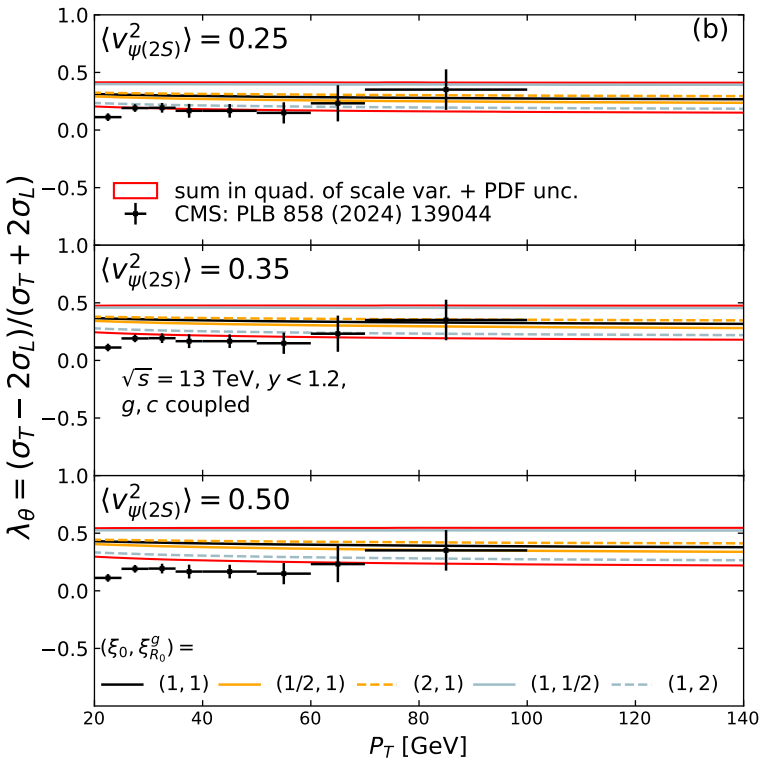

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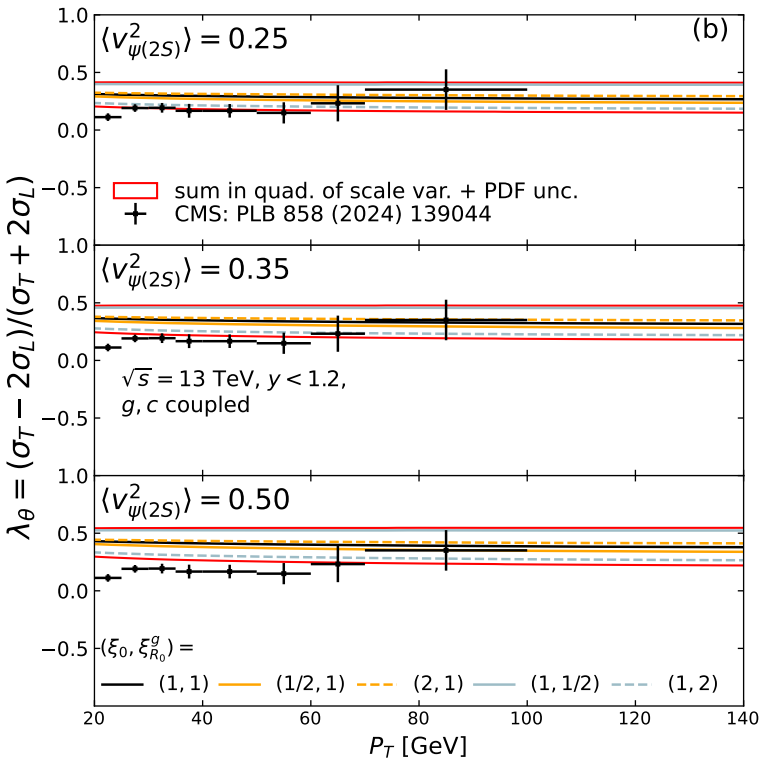

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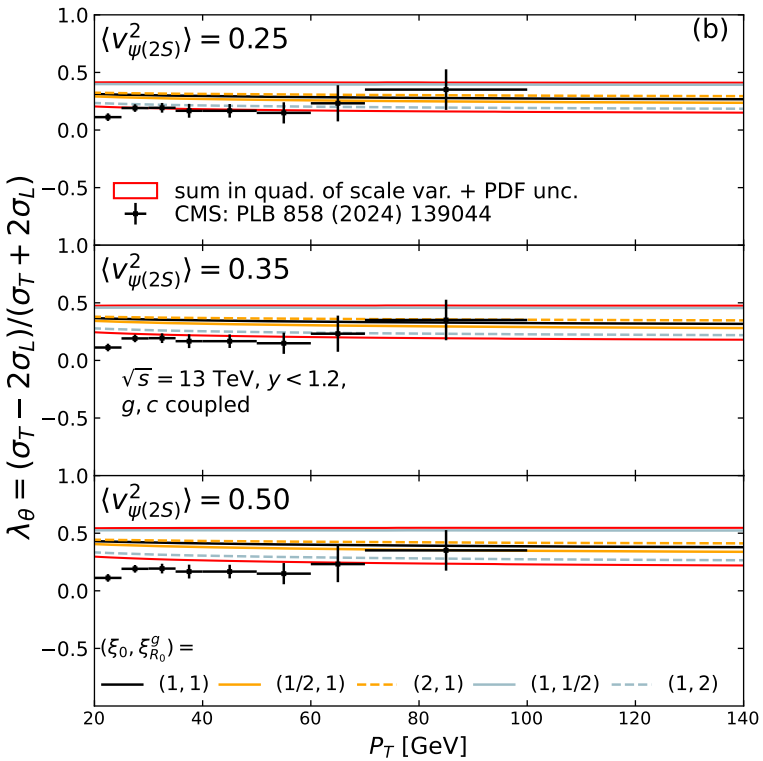

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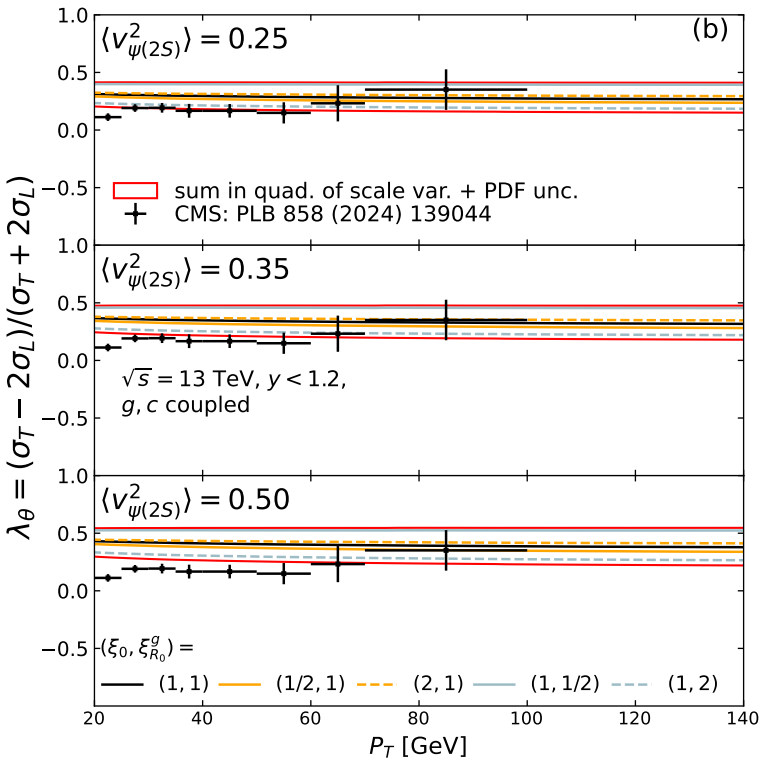

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- **NLO** corrections to gluon FF unknown, might be depolarising
- $pp \to J/\psi c\bar{c}$ at **NLO** hints no change of λ_θ **Feng, Qiao, PRD 112 (2025) 7, L071501**

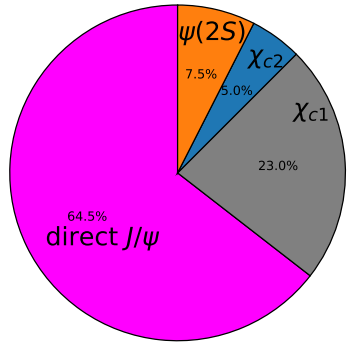
Moving to J/ψ

How to tackle J/ψ ?

JPL, Phys.Rept. 889 (2020)

- Unlike $\psi(2S)$, the J/ψ cross section has significant feedown

$$\sigma_{J/\psi} = \sigma^{\text{direct}}_{J/\psi} + \sigma^{\text{FD}}_{\psi(2S)} + \sigma^{\text{FD}}_{\chi_{c1}} + \sigma^{\text{FD}}_{\chi_{c2}}$$



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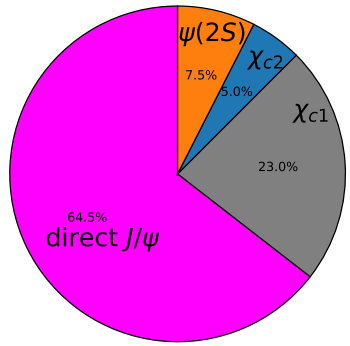

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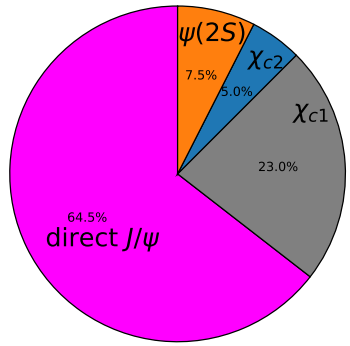

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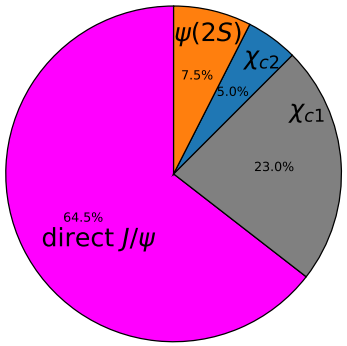

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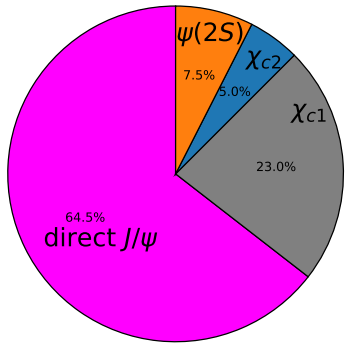
JPL, Phys.Rept. 889 (2020)



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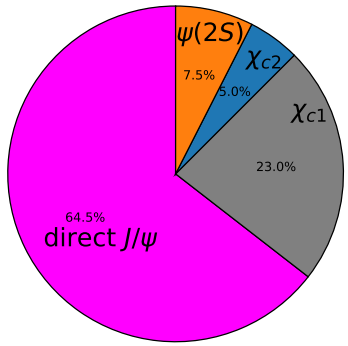

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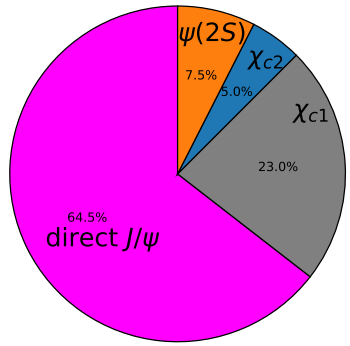

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- using two nuisance parameters, one for $\mu^g_{R_0}$ in the FF and one for $\alpha_s(\mu_R)$ in Γ
- valid if the nuisance parameters are close to their default value: 0

Our $\mathcal{O}(v^2)$ NRQCD fit

arXiv:2610.02096

- Our fit region is: [40:140] GeV
- Data included in the fit
  - $\sigma(\psi(2S))/\sigma(J/\psi)$ [ATLAS & CMS]
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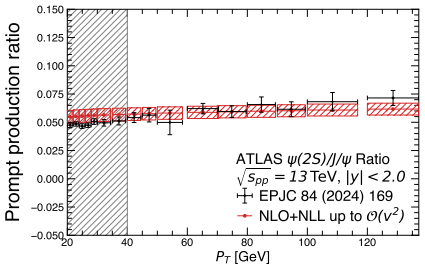

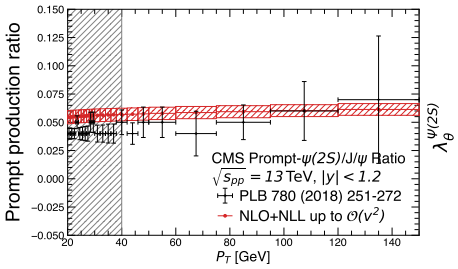

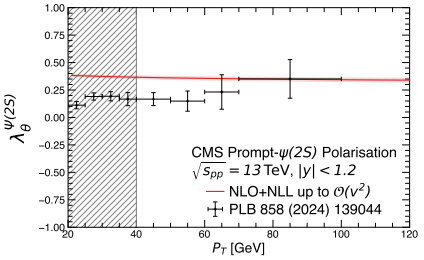

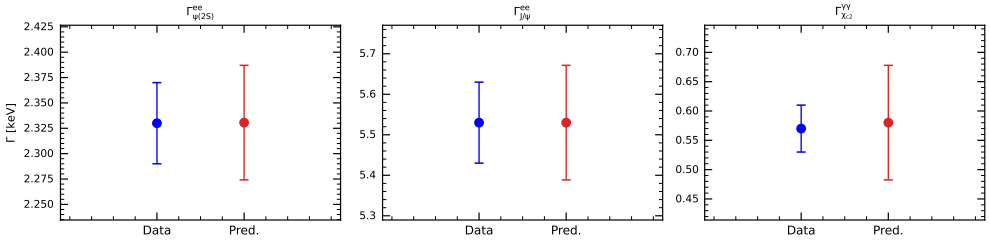

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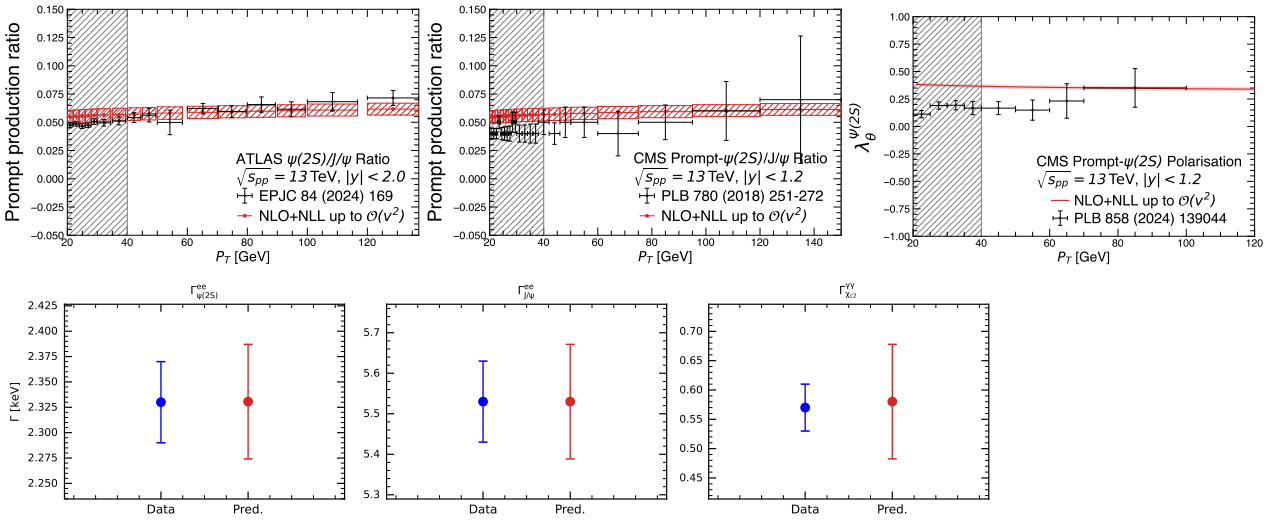
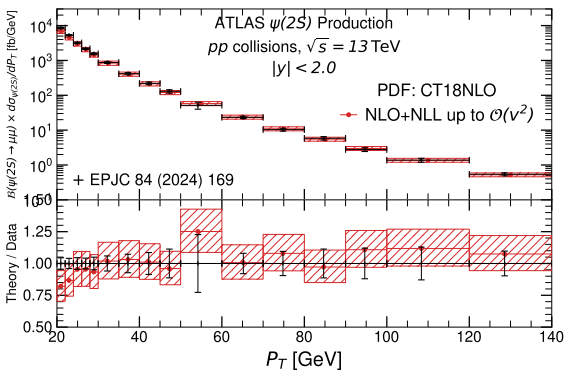
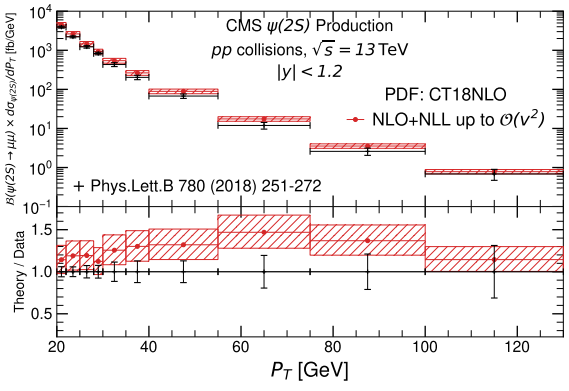


Outputs:

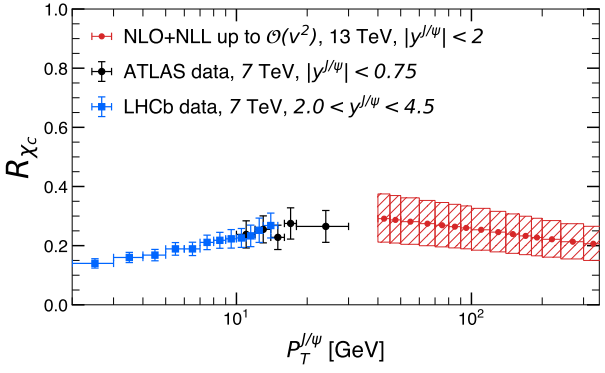
$\psi(2S)$ @ATLAS



$\psi(2S)$ @CMS



χ_c @ATLAS



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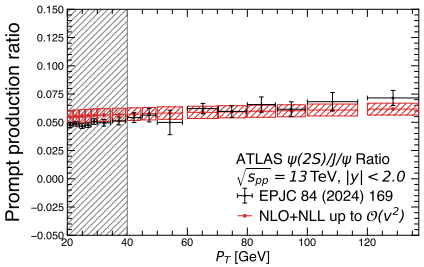

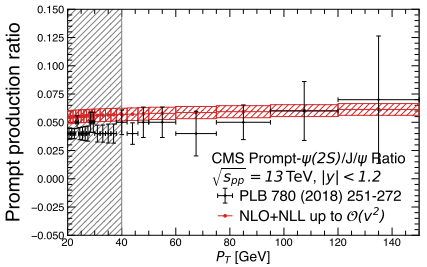

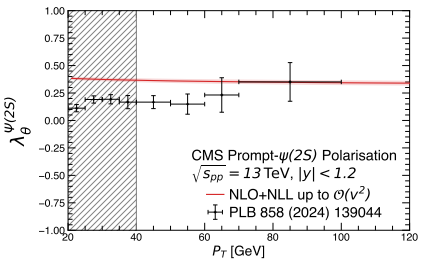

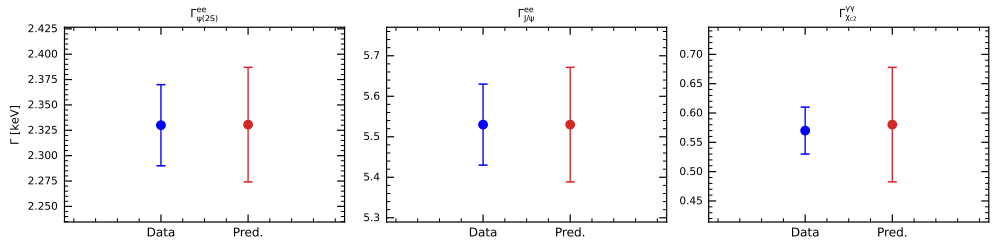

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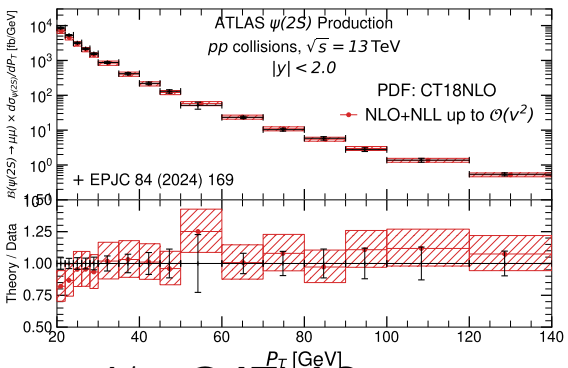

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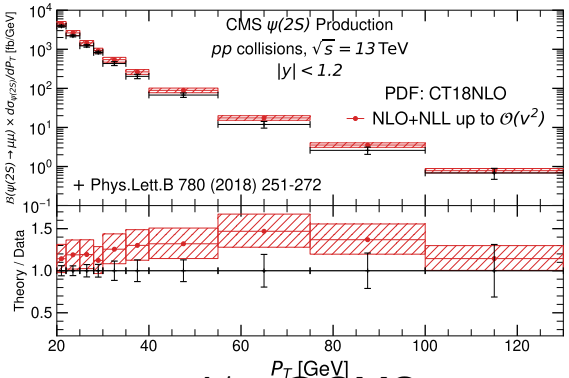
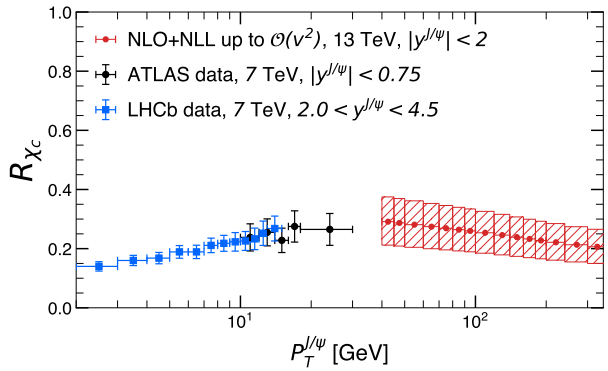
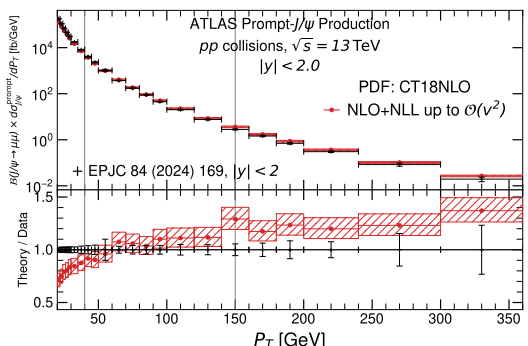


χ_c @ATLAS



J/ψ @ATLAS



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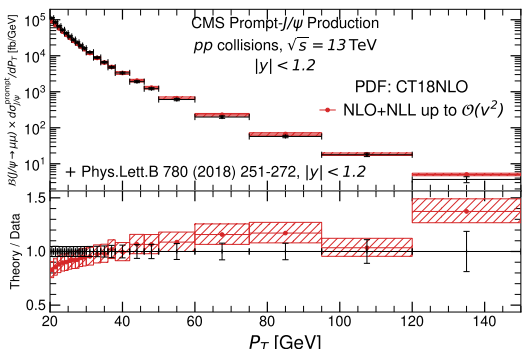

Our $\mathcal{O}(v^2)$ NRQCD fit II

arXiv:2610.02096

$\langle v^2_{\psi(2S)} \rangle$	$\langle v^2_{J/\psi} \rangle$	$\lvert R_{\psi(2S)}(0) \rvert^2$ [GeV^3]	$\lvert R_{J/\psi}(0) \rvert^2$ [GeV^3]	$\lvert R'_P(0) \rvert^2$ [GeV^5]	$\left\langle O^{\chi_{c0}}\left({}^3S_1^{[8]}\right) \right\rangle$ [$\times 10^{-2}$ GeV^3]
$0.38^{+0.06}_{-0.06}$	$0.22^{+0.07}_{-0.03}$	$0.93^{+0.17}_{-0.14}$	$1.36^{+0.21}_{-0.18}$	$0.08^{+0.01}_{-0.01}$	$0.36^{+0.05}_{-0.05}$

$\Gamma^{ee}_{\psi(2S)}$ [keV]	$\Gamma^{ee}_{J/\psi}$ [keV]	$\Gamma^{\gamma\gamma}_{\chi_{c2}}$ [keV]	δ_s	δ'_s	$\chi^2_{\text{data}}/n_{\text{data}}$
$2.33^{+0.06}_{-0.06}$	$5.53^{+0.14}_{-0.14}$	$0.57^{+0.09}_{-0.09}$	$0.07^{+0.16}_{-0.16}$	$0.05^{+0.06}_{-0.06}$	0.75

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- Thanks to the constraints from the vacuum-saturation approximation (up to v^4) applicable to CS LDMEs, our fit, contrary to CO LDME fits, does not inject any normalisation constraints from the cross section measurements.

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- Our resulting cross-section normalisation (and $\left\langle O^{\chi_{c0}}\left({}^3S_1^{[8]}\right)\right\rangle$) entirely comes from the computed NLO+NLL SDCs, the computed quarkonium FF and the normalisation constraints from the decay.

Our $\mathcal{O}(v^2)$ NRQCD fit II

arXiv:2610.02096

$\langle v^2_{\psi(2S)} \rangle$	$\langle v^2_{J/\psi} \rangle$	$\lvert R_{\psi(2S)}(0) \rvert^2$ [GeV^3]	$\lvert R_{J/\psi}(0) \rvert^2$ [GeV^3]	$\lvert R'_P(0) \rvert^2$ [GeV^5]	$\left\langle O^{\chi_{c0}}\left({}^3S_1^{[8]}\right)\right\rangle$ [$\times 10^{-2}$ GeV^3]
$0.38^{+0.06}_{-0.06}$	$0.22^{+0.07}_{-0.03}$	$0.93^{+0.17}_{-0.14}$	$1.36^{+0.21}_{-0.18}$	$0.08^{+0.01}_{-0.01}$	$0.36^{+0.05}_{-0.05}$

$\Gamma^{ee}_{\psi(2S)}$ [keV]	$\Gamma^{ee}_{J/\psi}$ [keV]	$\Gamma^{\gamma\gamma}_{\chi_{c2}}$ [keV]	δ_s	δ'_s	$\chi^2_{\text{data}}/n_{\text{data}}$
$2.33^{+0.06}_{-0.06}$	$5.53^{+0.14}_{-0.14}$	$0.57^{+0.09}_{-0.09}$	$0.07^{+0.16}_{-0.16}$	$0.05^{+0.06}_{-0.06}$	0.75

- Thanks to the constraints from the vacuum-saturation approximation (up to v^4) applicable to CS LDMEs, our fit, contrary to CO LDME fits, does not inject any normalisation constraints from the cross section measurements.
- Our resulting cross-section normalisation (and $\left\langle O^{\chi_{c0}}\left({}^3S_1^{[8]}\right)\right\rangle$) entirely comes from the computed NLO+NLL SDCs, the computed quarkonium FF and the normalisation constraints from the decay.
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- What about the J/ψ polarisation ?

Towards J/ψ polar anisotropy results

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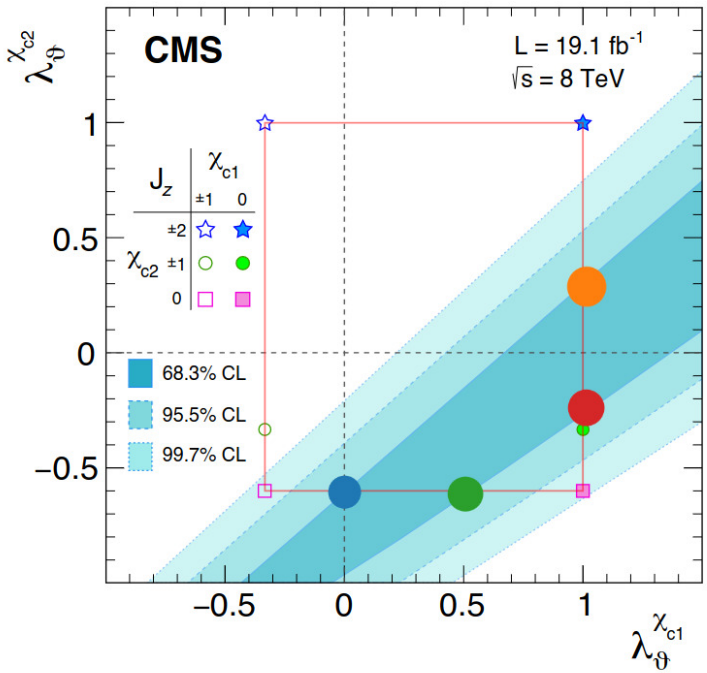
Phys.Rev.Lett. 124 (2020) 16, 162002



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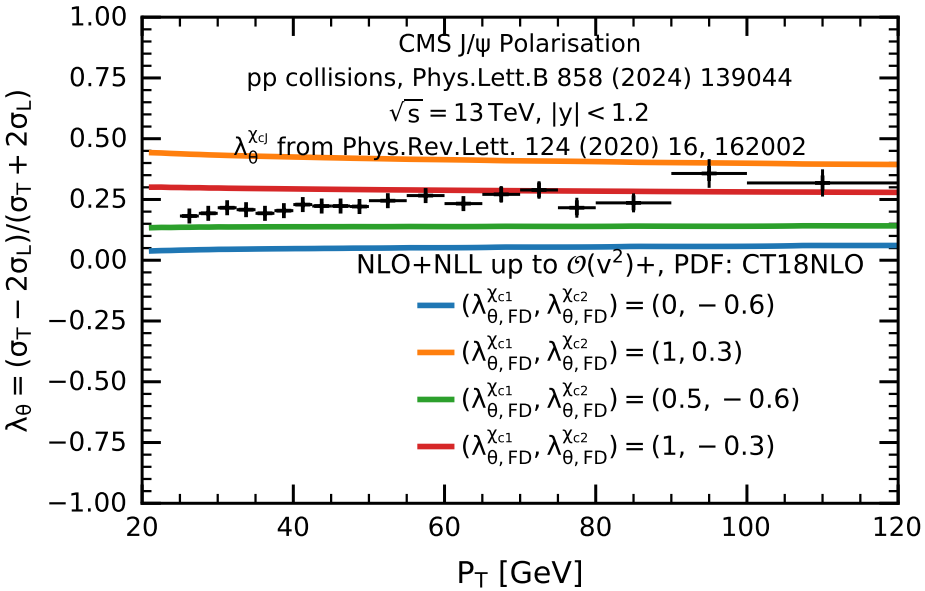

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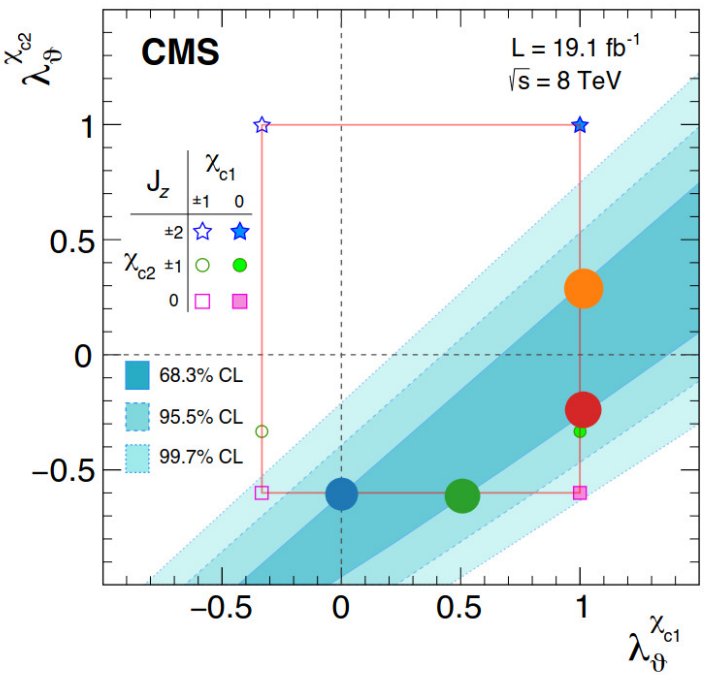

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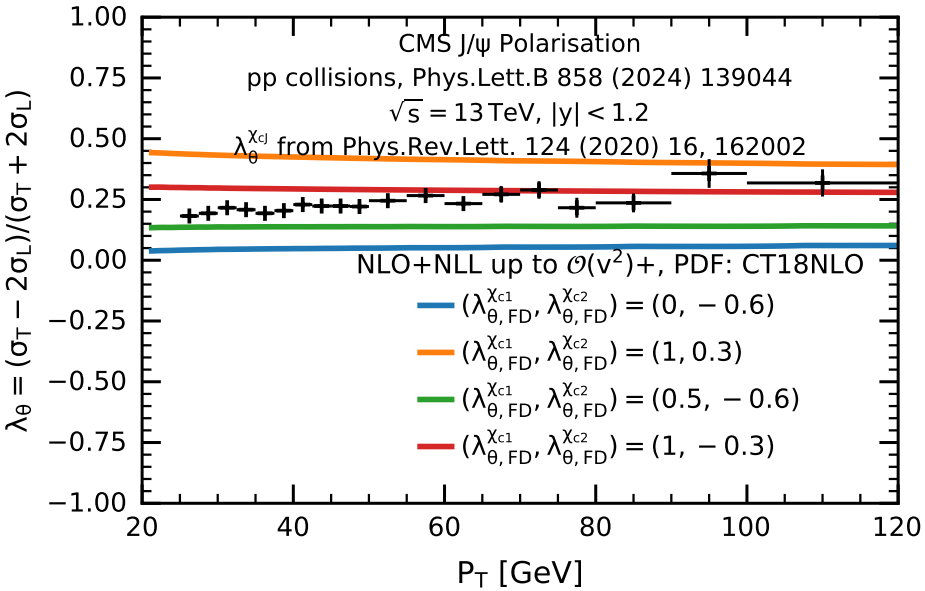

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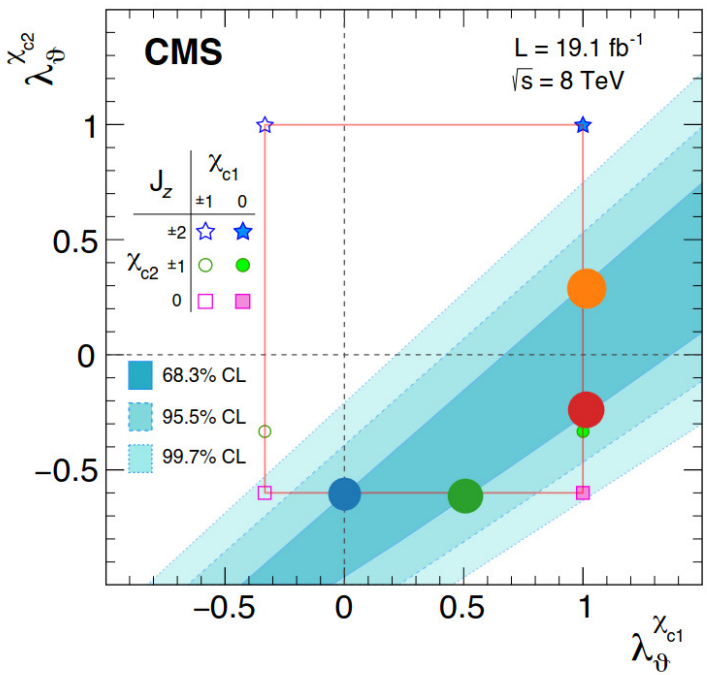

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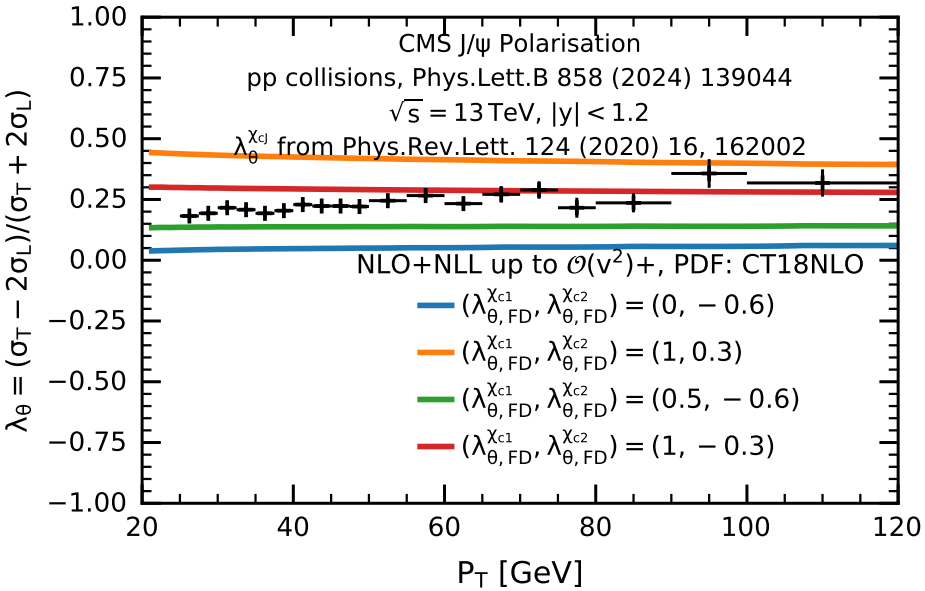

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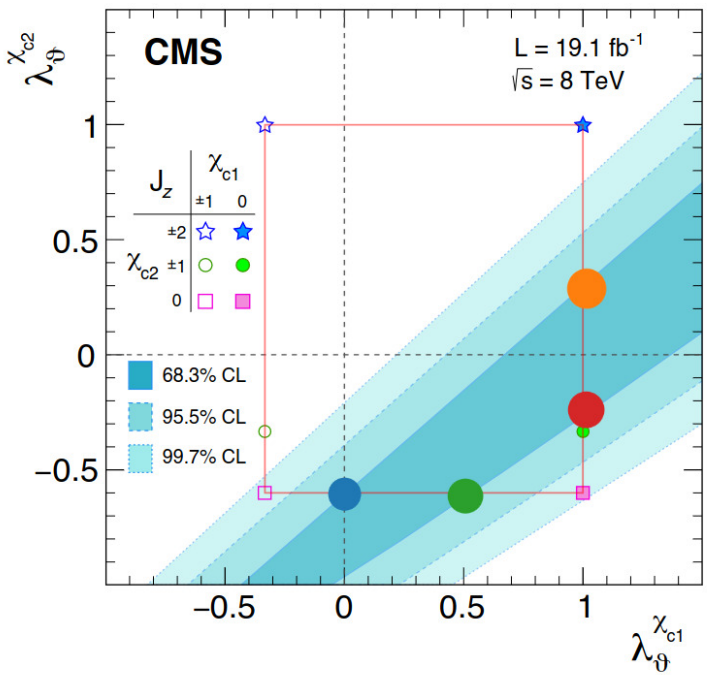

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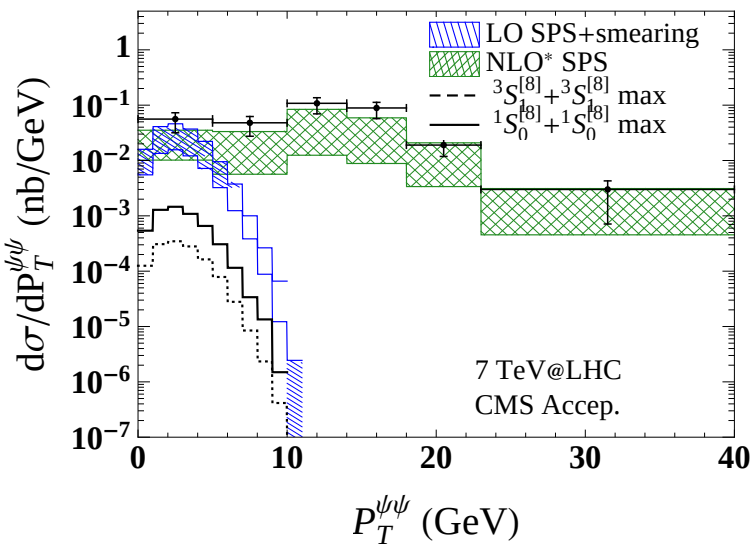

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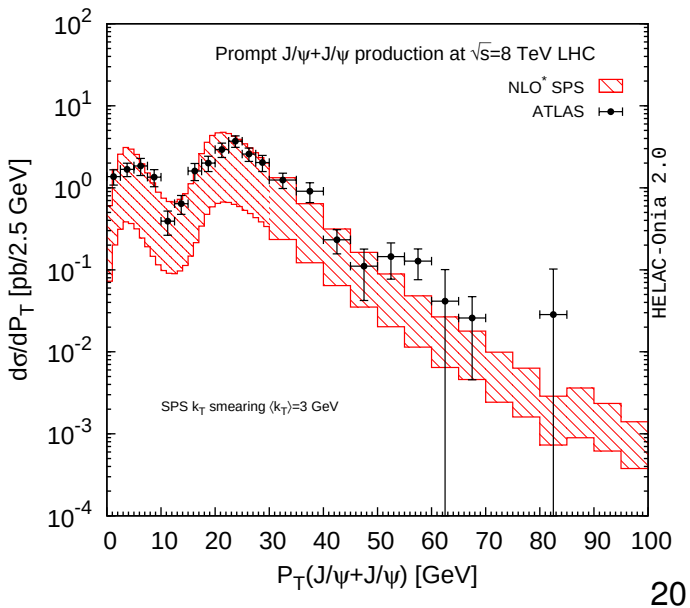

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  - evaluated to be much smaller than the data at Tevatron
  - uncertainties vastly underestimated
  - updating $|R(0)|^2$, α_s, and including newly computed contributions, gap with $\psi(2S)$ ATLAS data is much smaller than anticipated
- v^2-relativistic corrections to ${}^3S_1^{[1]}$
  - Large correction to FF at large N **G. Bodwin, U.R. Kim, J. Lee, JHEP 11 (2012) 020**
  - Impact on pheno overlooked No single impact study at the LHC
  - up to v^2, good agreement with high p_T $d\sigma/dP_T$ and λ_θ LHC $\psi(2S)$ data
  - Complete $\mathcal{O}(v^2)$ fit on J/ψ and $\psi(2S)$ data extremely succesful (yield, polarisation and decay)
  - ${}^3S_1^{[1]}$ $\psi(2S)$ also close to Tevatron data where NLP can be significant
- v^3 & v^4 COM: ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$
  - tuned to be dominant at large p_T
  - we predict a significant reduction of existing CO LDME values
- Theory to-do list
  - New CO LDME fit $\rightarrow$ better convergence with smaller $v^{3,4}$ contributions
  - NLO corrections to gluon FF
  - NLP corrections, to address $P_T < 40$ GeV

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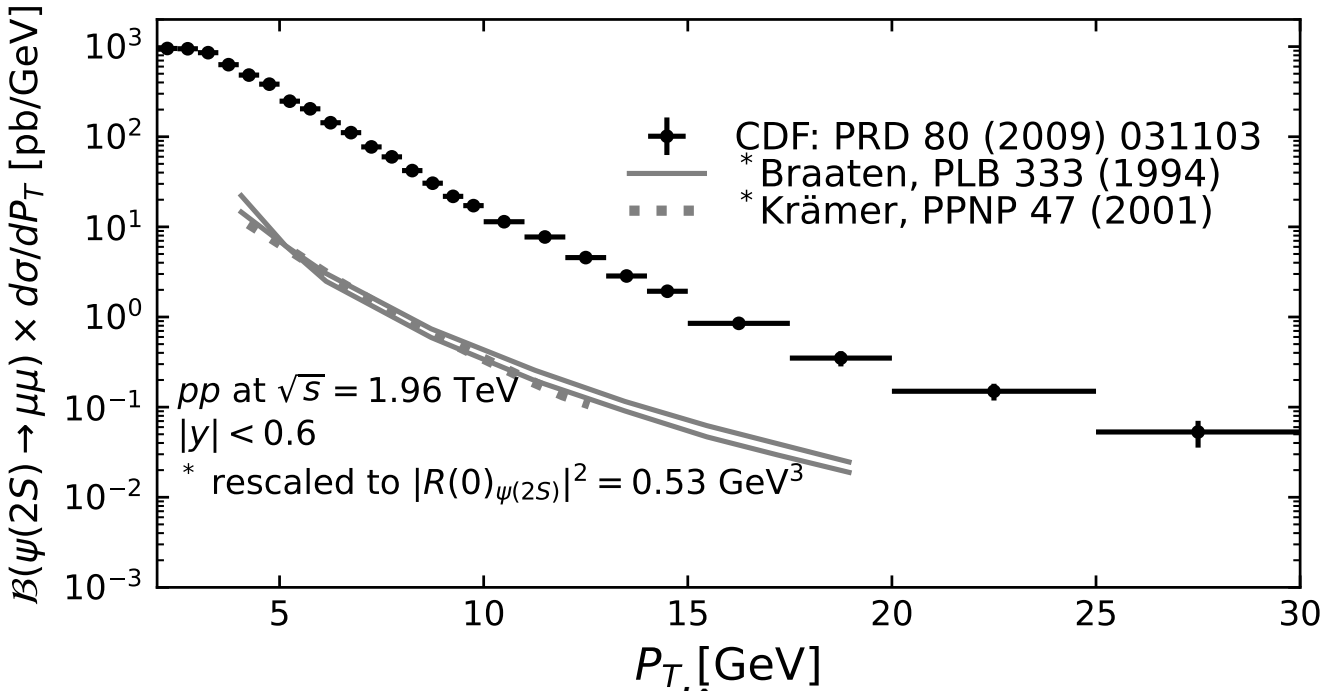
Backup

Revisiting the CDF ψ' anomaly

- Non-negligible NLP contribution expected: $P_T < 30$ GeV



$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} \simeq \sum_{i,j,k} \underbrace{f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i})}_{\text{cteq5l/D0}} \otimes \underbrace{\frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i}, \mu_{F_f}, \mu_R)}_{\text{LO}} \otimes \underbrace{D_k^{\mathcal{Q}}(\mu_{F_f}, \{\mu_0\})}_{\text{LL evolution}}$$

1. $g \to \psi(2S)gg$: *PRL 71 (1993) 1673; PRD 96 (2017) 094016*

2. $c \to \psi(2S)c$: *PRD 48 (1993) 4230–4235; JHEP 06 (2015) 021*

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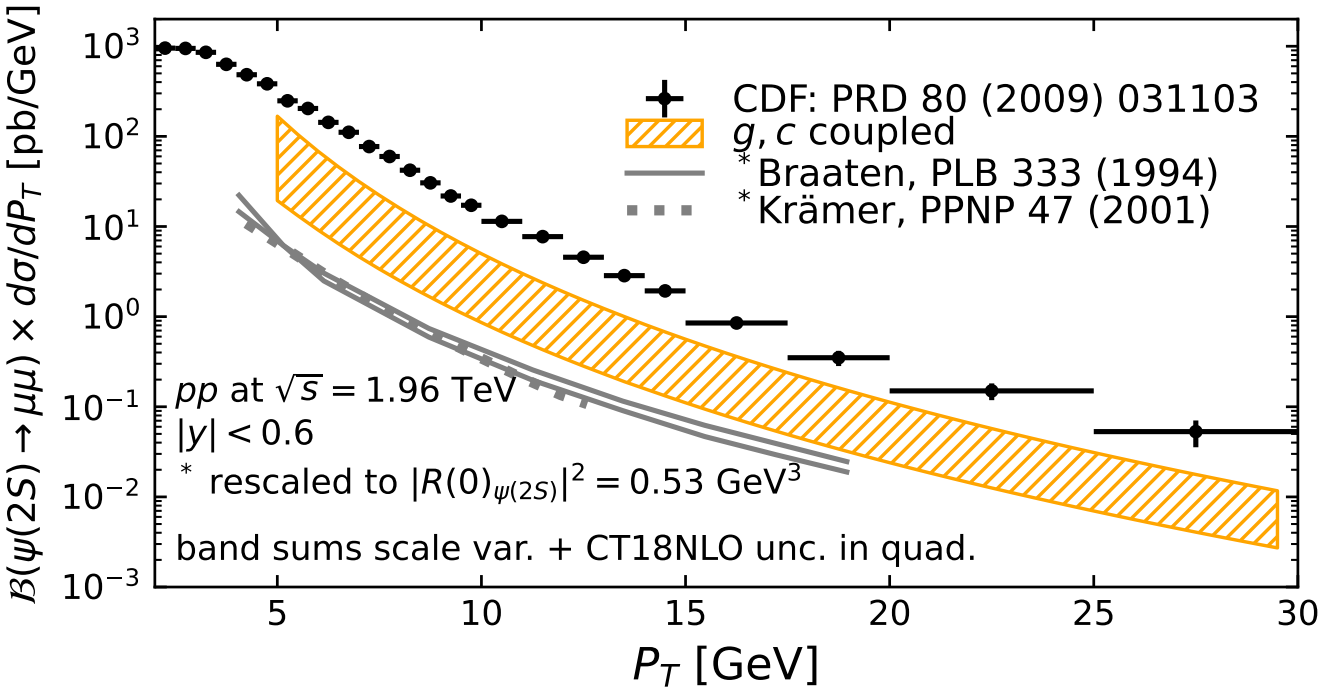

$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} \simeq \sum_{i,j,k} \underbrace{f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i})}_{\text{CT18NLO}} \otimes \underbrace{\frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i}, \mu_{F_f}, \mu_R)}_{\text{NLO}} \otimes \underbrace{D_k^{\mathcal{Q}}(\mu_{F_f}, \{\mu_0\})}_{\text{NLL evolution}}$$

Update α_s

1. $g \to \psi(2S)gg$: *PRL 71 (1993) 1673; PRD 96 (2017) 094016*

2. $c \to \psi(2S)c$: *PRD 48 (1993) 4230–4235; JHEP 06 (2015) 021*
   2.1 NLO corr: *PRD 100 (2019) 014005*

3. $g \to \psi(2S)c\bar{c}$: *PRD 106 (2022) 054030*

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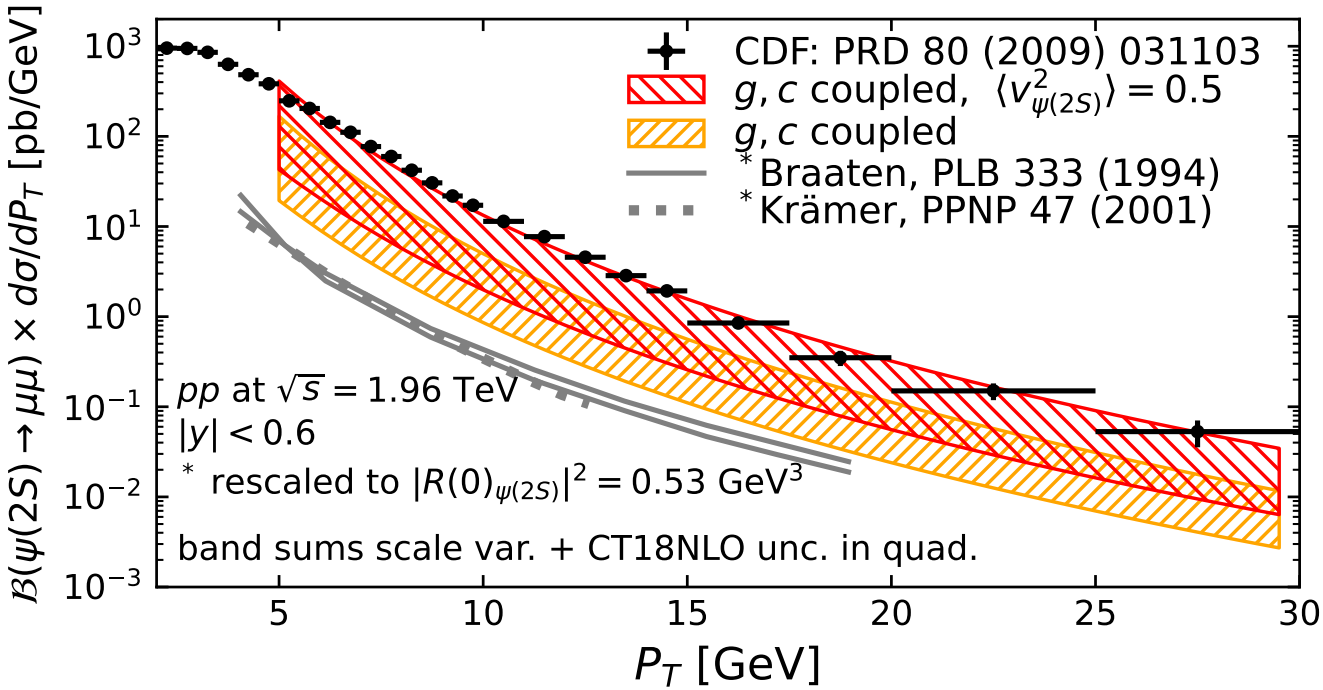

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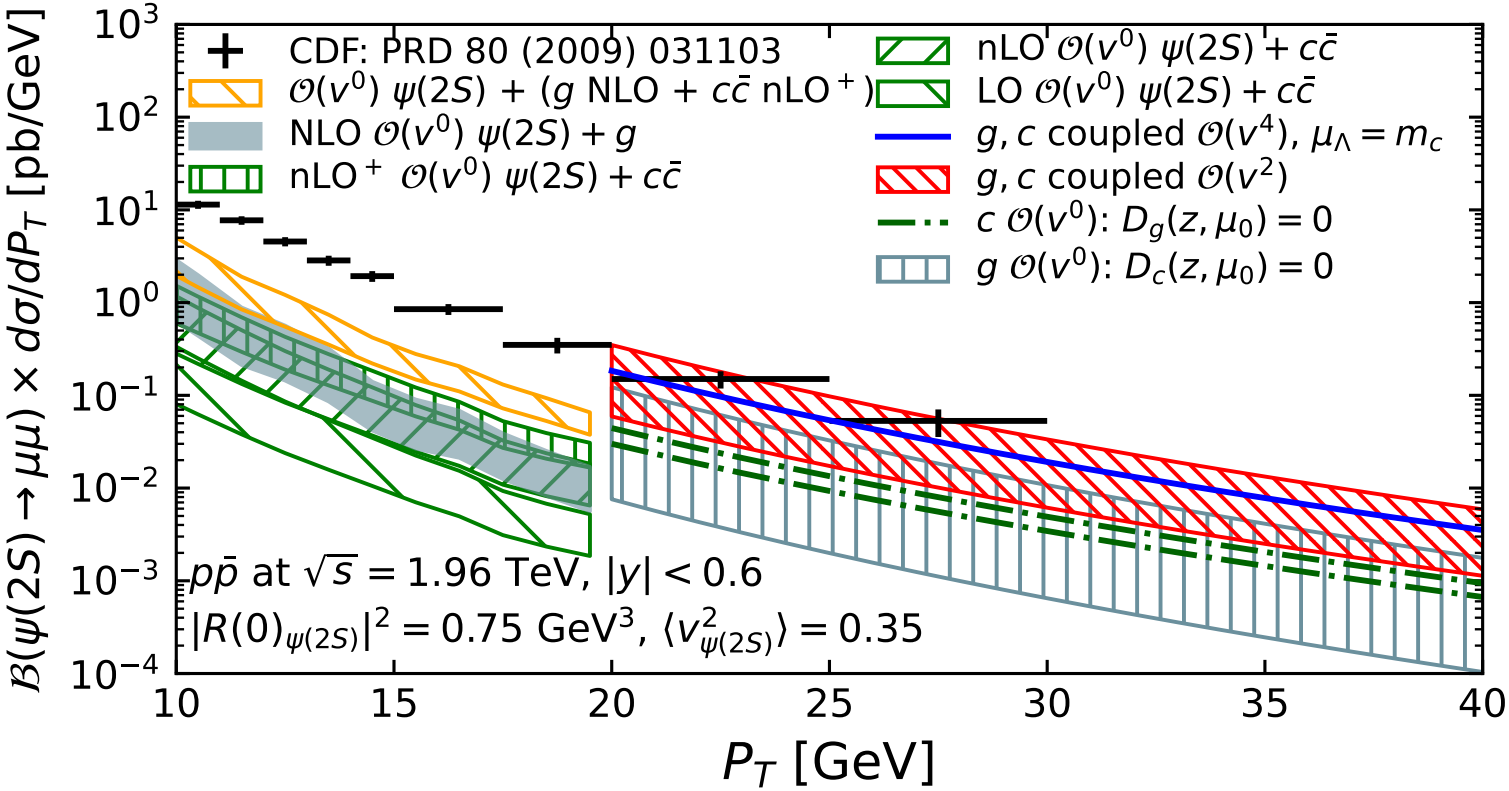
   - 1.1 v^2 corr: *JHEP 11 (2012) 020*
2. $c \to \psi(2S)c$: *PRD 48 (1993) 4230–4235; JHEP 06 (2015) 021*
   - 2.1 NLO corr: *PRD 100 (2019) 014005*
   - 2.2 v^2 corr *PRD 80 (2009) 014013*
3. $g \to \psi(2S)c\bar{c}$: *PRD 106 (2022) 054030*

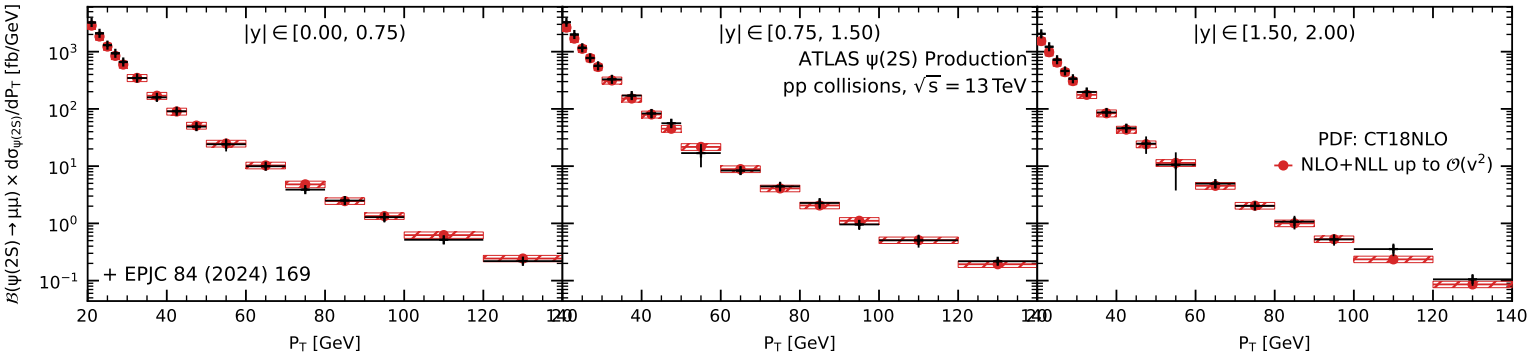
Revisiting the CDF ψ' anomaly II: FO vs LP

arXiv:2510.06456



- Similar–slightly better–results are expected with our ”fit” values: $\langle v^2_{\psi(2S)} \rangle = 0.38$ & $|R_{\psi(2S)}(0)|^2 = 0.93$ GeV^3
- Note: surprisingly CDF $\psi(2S)$ data up to higher P_T than J/ψ ones

More plots


|y| ∈ [0.00, 0.75)
|y| ∈ [0.75, 1.50)
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ATLAS ψ(2S) Production
pp collisions, √s = 13 TeV
PDF: CT18NLO
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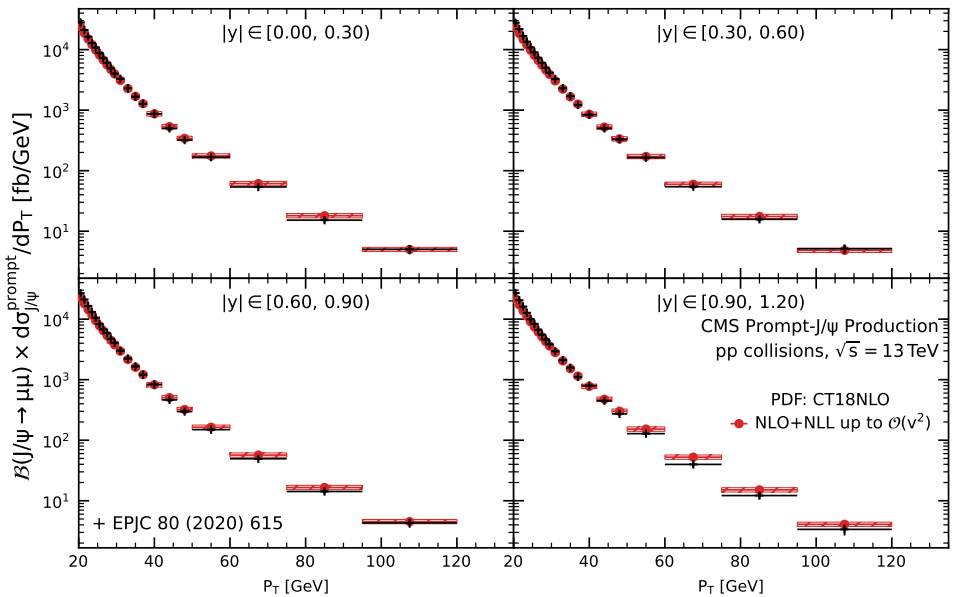
+ EPJC 84 (2024) 169
P_T [GeV]


|y| ∈ [0.00, 0.30)
|y| ∈ [0.30, 0.60)
|y| ∈ [0.60, 0.90)
|y| ∈ [0.90, 1.20)
CMS Prompt-J/ψ Production
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PDF: CT18NLO
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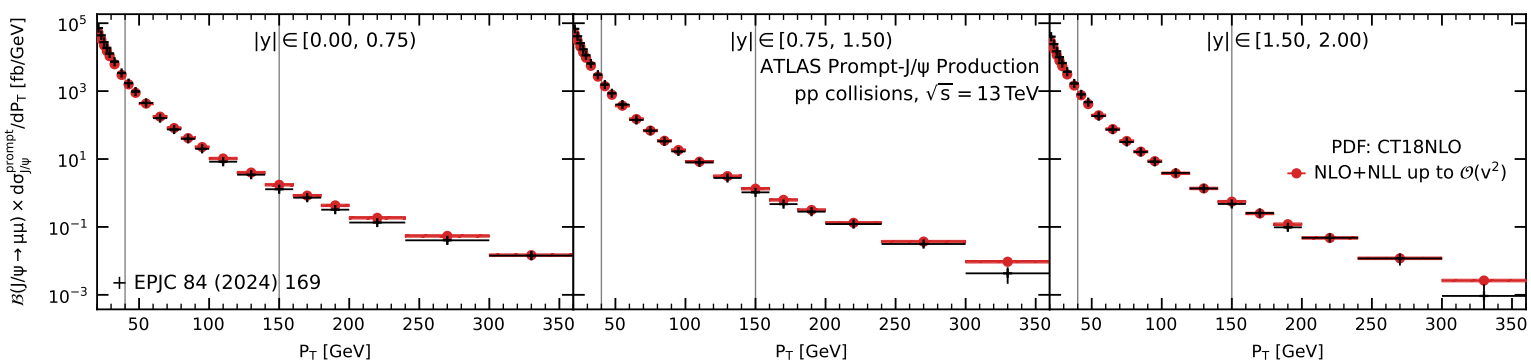
+ EPJC 80 (2020) 615
P_T [GeV]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