SGR: ISR Resummation and pp cross-sections at 13 TeV

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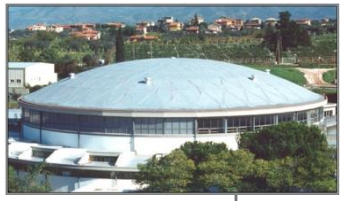
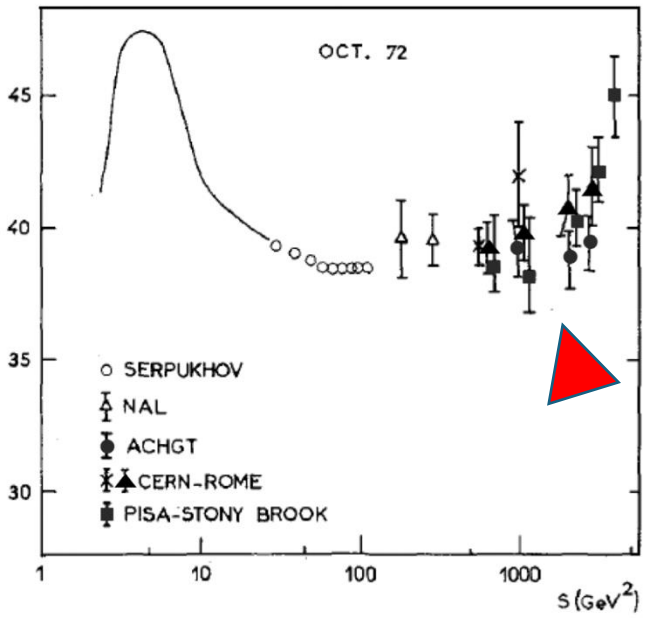
with
Y.N. Srivastava, INFN Perugia

Resummation, Evolution and Factorization REF 2026



1972 →LHC 2024 : do we understand the total cross section?

U. Amaldi (1973)



Turning point around 20-30 GeV

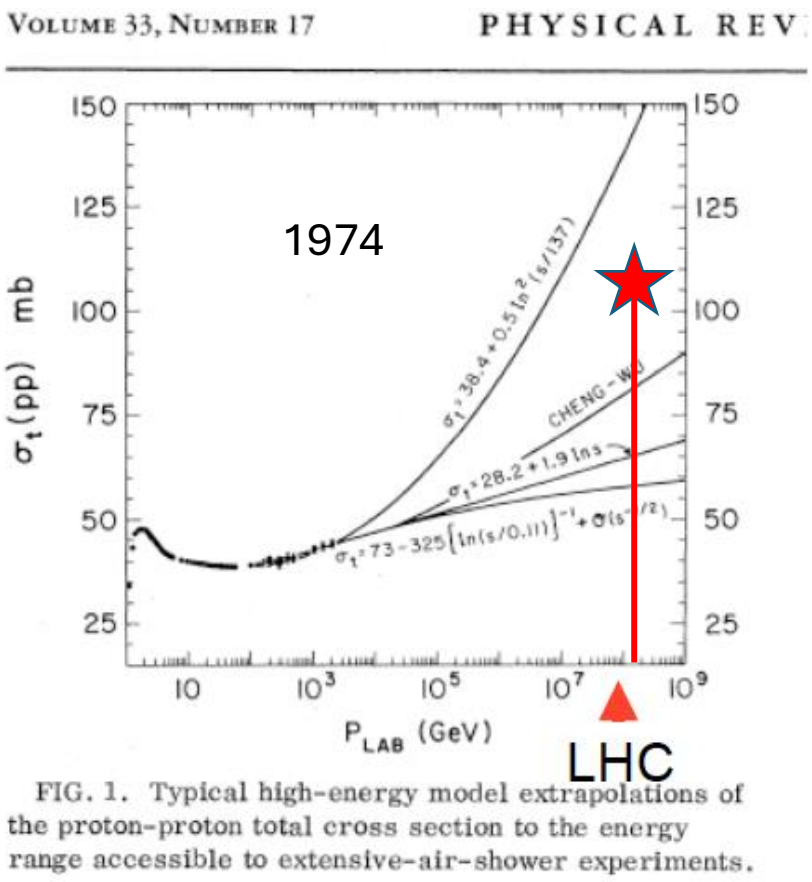
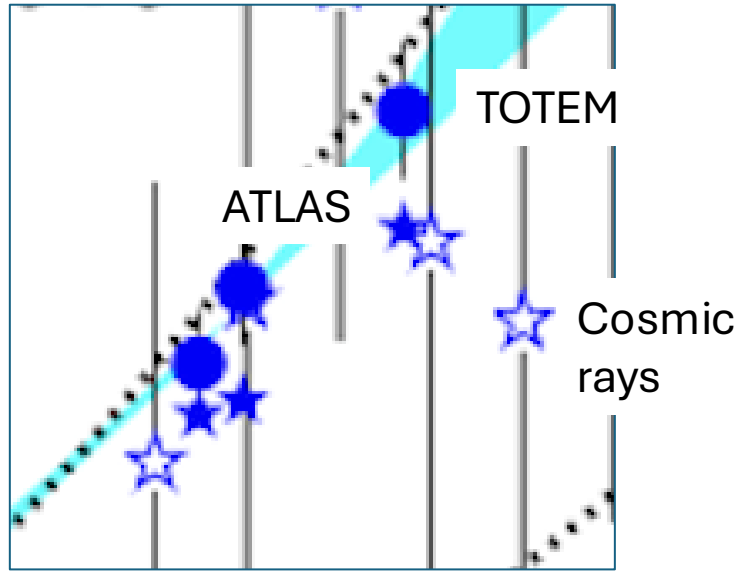


FIG. 1. Typical high-energy model extrapolations of the proton-proton total cross section to the energy range accessible to extensive-air-shower experiments.

Barger &Halzen, *Phys.Rev.Lett.* 33 (1974) 1051

σ_{pp}^{LHC}



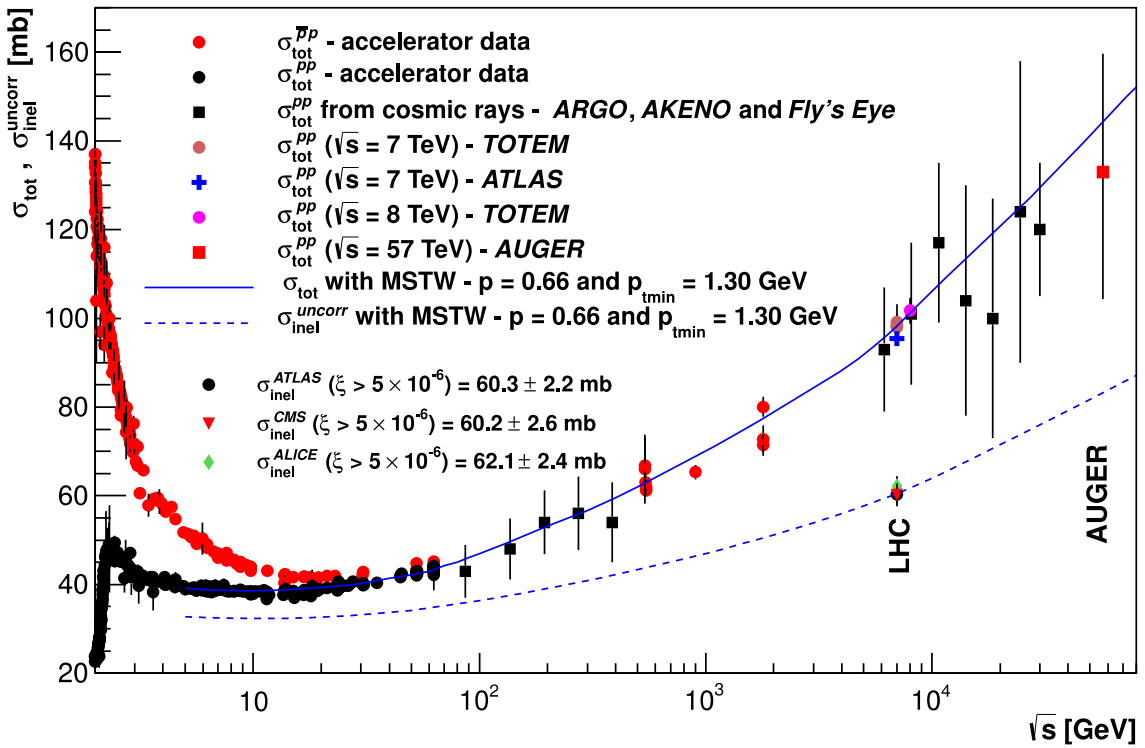
$\sqrt{s} = 7 \div 13$ TeV

Outline : revisiting previous work and updating with 13 TeV data

1. model for the **total** cross-section with Initial State Soft Gluon Resummation (SGR for initial state emission)
   - QCD **mini-jets**
   - **impact**-parameter distribution with SGR
   - Ansatz for $\alpha_s(k_t \to 0)$

2. The Differential elastic cross-section

3. SGR and QCD zero momentum mode



With R.M.Godbole et al, 2014 ISVECHRI

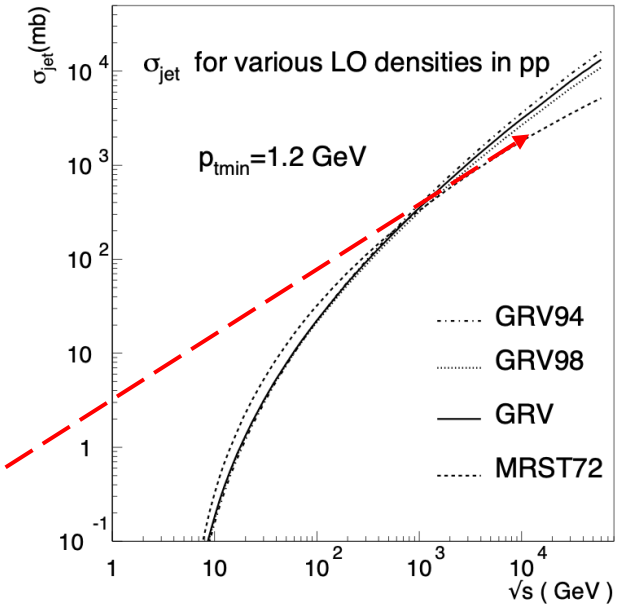
From the scattering amplitude → **total** and differential cross-section

1. **Optical Theorem**

$$\sigma_{total} = \sqrt{4\pi}\ \mathcal{I}m\mathcal{A}(s, t = 0)$$

2. **Eikonal** resummation in impact parameter **b**-space

$$\mathcal{A}(s,t) = i \int bdbJ_0(qb)[1 - e^{i\chi(b,s)}] \qquad t = -q^2$$



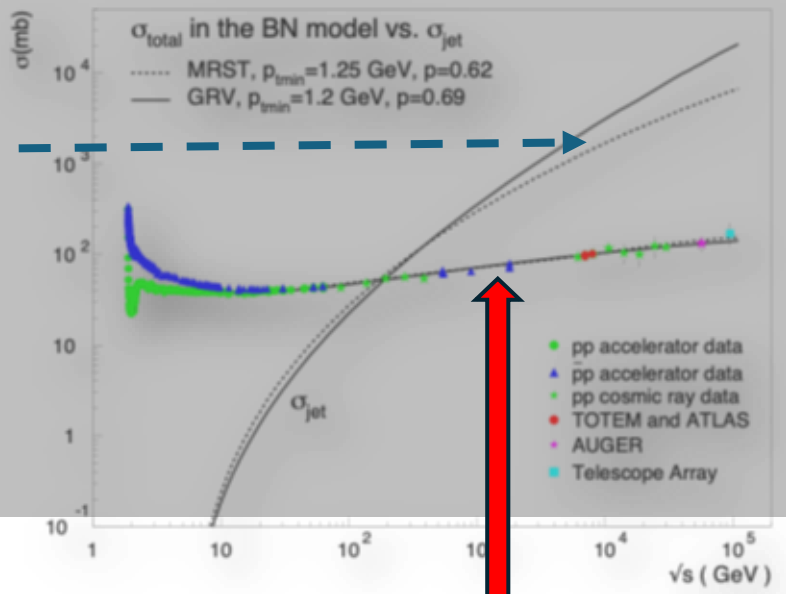
3. **The rise** → Energy dependence → QCD mini-jets?

$$\sigma_{\rm jet}^{AB}(s; p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \ \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}.$$

Initial State Soft Gluon Resummation (SGR) & minijets → **total** cross-sections

1. **Optical Theorem**
2. **The rise** → Energy dependence
   QCD mini-jet
3. **Eikonal** resummation
   in impact parameter **b**-space

With Grau and Godbole Phys. Rev 1996 → *Phys.Rev.D* 72 (2005) 076001



4. b-dependence

Initial State Radiation from Soft Gluon Resummation

(It is a universally recognized fact that) Initial State Radiation reduces the observed cross-section

Resumming Photons and … even gravitons, one by one

1937	F. Bloch& A. Nordsieck
1949	J. Schwinger
1952	L. Brown & R. Feyman
1954	M. Jauch & F. Rohrlich
1956	(V. V. Sudakov – no infrared)
1956-59	E. Lomon
1957-61	D.R. Yennie & F. Suura & with C. Frautschi
1965	S. Weinberg "Infrared Photons and Gravitons"
…..	

$$\Gamma_{\beta\alpha}^{QED,grav}(\leq E_{IR}) = b[A(QED,grav)](\frac{E_{IR}}{\Lambda_{UV}})^{A(QED,grav)}\Gamma_{\beta\alpha}^{0}$$

- A semiclassical derivation for QED (B. Touschek 1967)

$$d\sigma_{exp}(E) \propto (\frac{\Delta E_{exp}}{E})^{\beta(E)} d\sigma_{th}(E)$$

➔ was extended in 1979 to QCD transverse momentum distributions (Parisi&Petronzio , Curci, Greco&Srivastava)

➔ 1976 GP&YS had proposed to apply Touschek formalism for QED to transverse momentum distributions in strong interactions with a constant alphas strong

Bruno Touschek (1967) formalism for Soft photon distributions in QED

A statistical semi-classical approach + energy momentum conservation

$$d^4P(K) = \sum P(\{n_{\mathbf{k}}\})\delta_4(\sum_{\mathbf{k}} kn_{\mathbf{k}} - K)d^4K = \frac{d^4K}{(2\pi)^4}\int d^4x e^{iK\cdot x - h(x)}$$

$$P(\{n_{\mathbf{k}}\}) = \Pi_{\mathbf{k}} \frac{\bar{n}_{\mathbf{k}}^{n_{\mathbf{k}}}}{\bar{n}_{\mathbf{k}}!} e^{-\bar{n}_{\mathbf{k}}}$$

1. Integral representation of delta-function
2. Exchange sum with product
3. Sum over **k**

IR divergence cancelled

$$h(x) = \sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}[1 - e^{i(k\cdot x)}]$$

real emission

- Take the continuum limit

$$\bar{n}(\mathbf{k}) \quad \rightarrow continuum \rightarrow \quad d^3\bar{n}(\mathbf{k}) = (2\pi)^6 \frac{d^3\mathbf{k}}{\omega}|j_\perp(\omega, \mathbf{k})|^2$$

- In co-variant notation and first order in **α**

$$|\mathbf{j}_\perp|^2 = j_\mu j^\mu = \frac{\alpha}{(2\pi)^2}\sum_{i,j}\frac{(p_i\cdot p_j)\epsilon_i\epsilon_j}{(p_i\cdot k)(p_j\cdot k)}$$

F. Bloch and A. Nordsieck. Note on the Radiation Field of the electron. Phys. Rev. 52, 54 59, 1937
W. Thirring and B. Touschek, A covariant formulation of the Bloch-Nordsieck method, *Phil.Mag.Ser.7* 42 (1951) 244-249
E. G. Etim, G. P., and B. Touschek. The Infra-red Radiative Corrections to Colliding Beam (Electrons and Positrons) Experiments. Nuovo Cim. B, 51:276–302, 1967.

2 pigreco to be checked

How about resummation in QCD (I)?

Semi-classical &very generalGP&YS 1976

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b} e^{-i\mathbf{K}_\perp\cdot\mathbf{b} - \sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}[1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]}$$

Same statistical resummation and Poisson distribution as for QED

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b} e^{-i\mathbf{K}_\perp\cdot\mathbf{b} - h_{continuum}(\mathbf{b})}$$

$$h_{continuum}(b) = \frac{8}{3\pi^2}\int_?^{q_{max}} d^2\mathbf{k}_t \frac{\alpha_s(k_t^2)}{k_t^2}\ln[\frac{2q_{max}}{k_t}][1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]$$

Touschek inspired formalism in
Curci Greco&YS 1979, Parisi&Petronzio 1979

b-dependence : Poisson distributions + energy momentum conservation

$$A(\mathbf{b}) = \frac{\mathcal{F}\{d^2P(\mathbf{K}_\perp)\}}{\mathcal{N}} \qquad with \qquad \int d^2\mathbf{b}A(\mathbf{b}) = 1$$

- Touschek resummation with Poisson for Soft Initial State Radiation

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b}e^{-i\mathbf{K}_\perp\cdot\mathbf{b}-\frac{8}{3\pi^2}\int_0^{q_{max}} d^2\mathbf{k}_t \frac{\alpha_s(k_t^2)}{k_t^2}\ln[\frac{2q_{max}}{k_t}][1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]}$$

$$k_t \to 0 \qquad \alpha_s^{interpolation}(k_t) \propto \frac{p}{\ln[1+p(\frac{k_t}{\Lambda_{QCD}})^{2p}]} \qquad p<1$$

q_{max} from single collision kinematics (Chiappetta&Greco 1983)

$$q_{max}(s) \propto p_{tmin}^{mini-jets}$$

PRD with Grau et al., 1996→ GP, YS and Shekovstova 2024

$$large\ r: \quad V(r) \to r^{2p-1} \qquad 1/2 < p < 1$$

$$\lim_{b\to\infty} h(b,M,\Lambda) = \frac{2c_F\bar{b}}{\pi}(b^2\Lambda^2)^p\left[\frac{1}{8(1-p)}\left(2\ln(2Mb)+\frac{1}{1-p}\right)+\frac{1}{2p}\left(2\ln(Mb)-\frac{1}{p}\right)\right]$$

Recent ansatz

$$p = b_0$$

$$\alpha_s(k_t)^{interpolation} = \frac{1}{\ln[1+(\frac{k_t}{\Lambda})^{2b_0}]}$$

$$\to_{k_t\to 0} \quad (k_t/\Lambda)^{-2b_0}$$

$$\to_{k_t>>\Lambda} \quad \frac{1}{b_0\ln(\frac{k_t}{\Lambda})^2}$$

$$b_0 = \frac{11N_c - 3N_f}{12\pi}$$

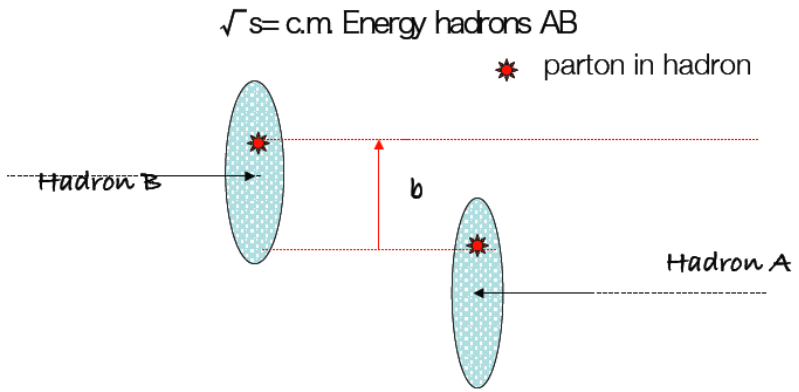
Phenomenology
N_f=2 for sigma elastic
N_f=6 sigma total

Add ref to UKrain

We model the impact parameter distribution as the Fourier-transform of ISR soft k_t distribution → a cut-off at large distances

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \, e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\, e^{-h(b,q_{max})}}$$

$$h(b,E) = \frac{16}{3\pi} \int_0^{qmax} \frac{dk_t}{k_t} \alpha_{eff}(k_t) \ln\left(\frac{2q_{max}}{k_t}\right)[1 - J_0(bk_t)]$$



$$\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}$$

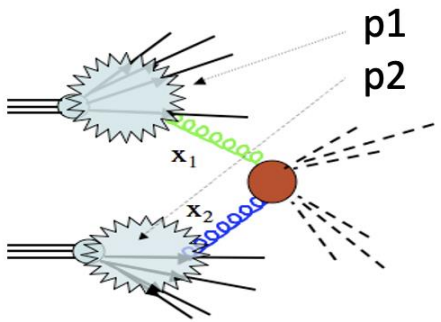
$$A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}$$

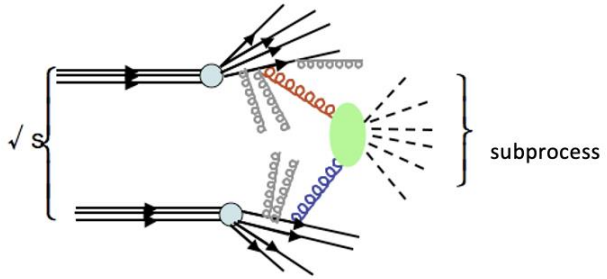
P<1 for convergence

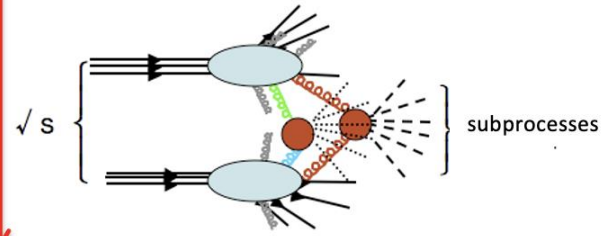
q_{tmax}

Fixed by single gluon emission kinematics

The model in images *Phys.Rev.D* 84 (2011) 094009



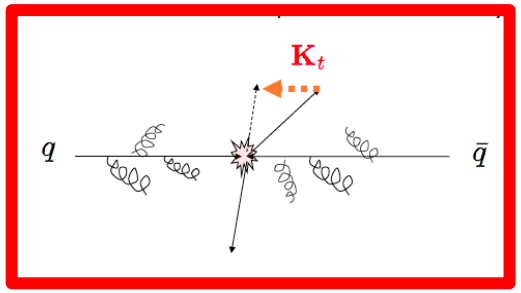


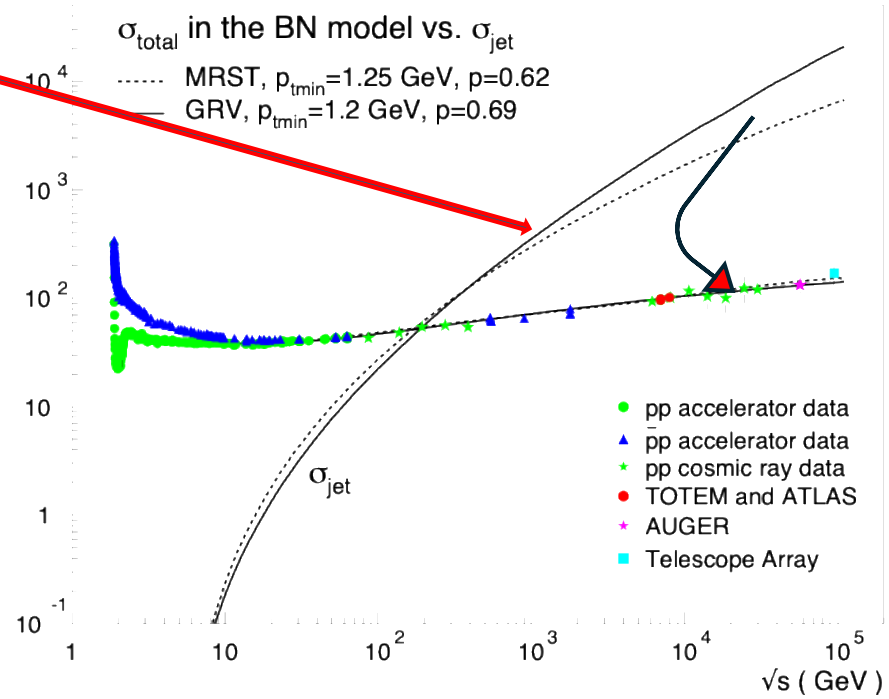


LO PDF

$$\sigma_{QCD-minijets(s)}$$

+





$$\sigma_{tot} = \int d^2\mathbf{b}[1 - e^{-\sigma_{QCD-minijets(s)} A_{SGR}(\mathbf{b},s)}]$$

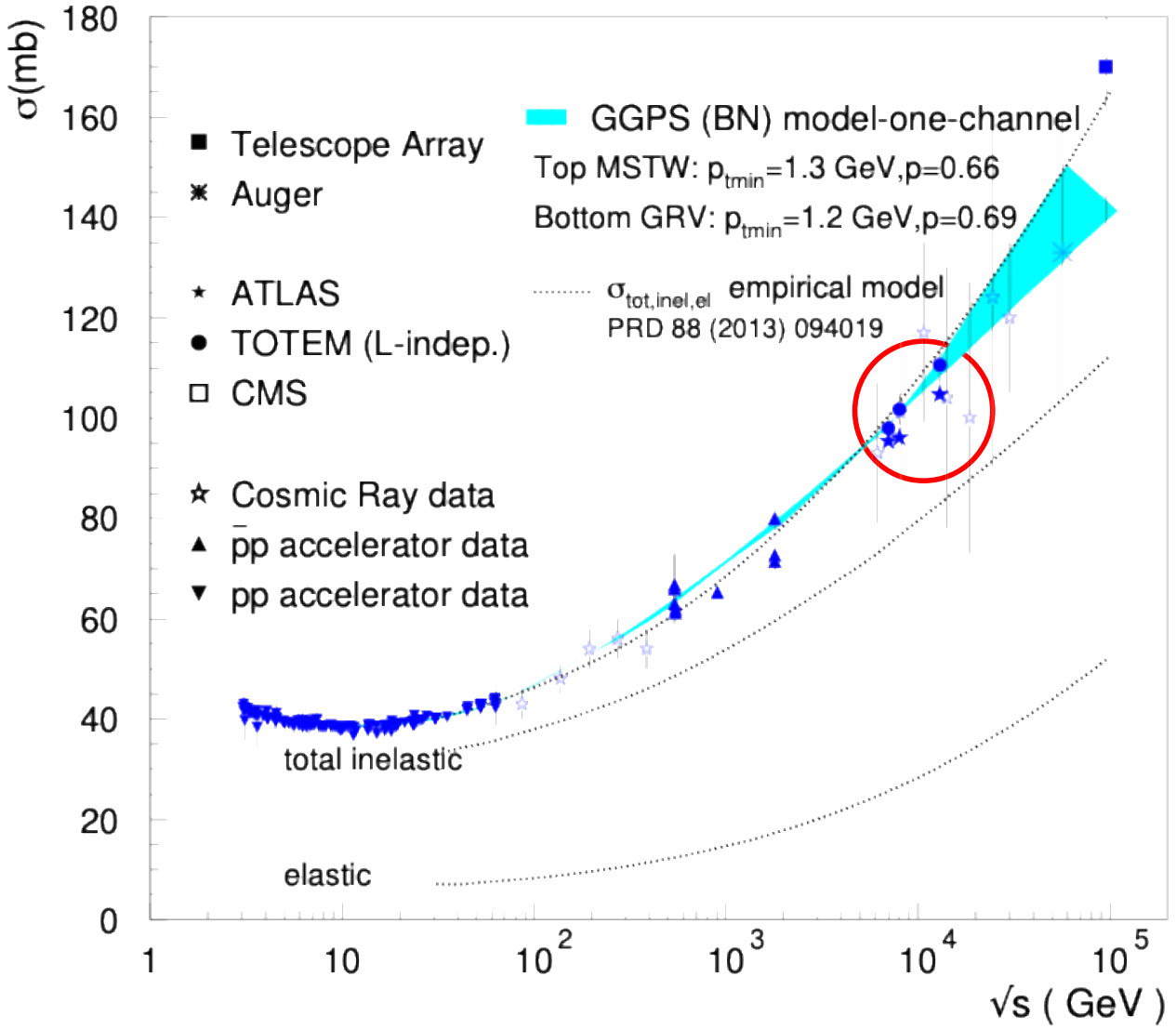
Updating previous work adding only 13 TeV data

- R. Godbole, A. Grau, G.P. & Y.S
- Phys. Rev. 2017→ 13 TeV data

- **impact**-parameter distribution with initial state SGR and Ansatz

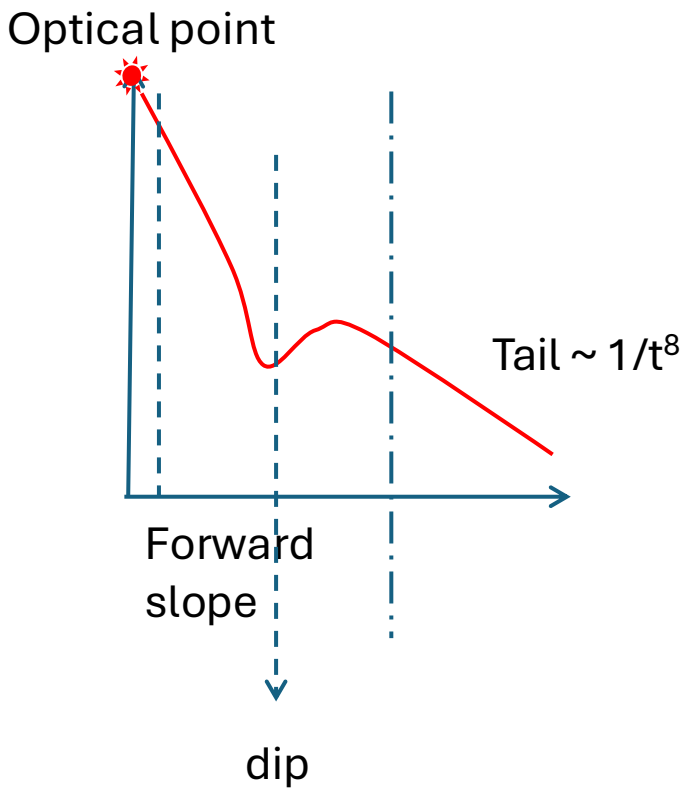
$$\alpha_s(k_t \to 0)$$

- + minijets



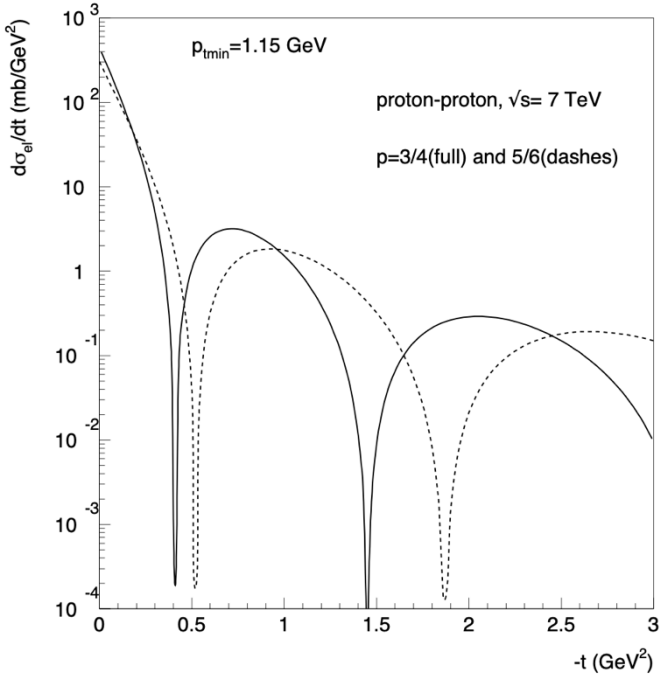
How well/bad does this model with the differential elastic cross-section?

- Optical point : total cross-section
- Forward slope
- The dip and the bump
- The tail



Resort to an EMPIRICAL MODEL to try to fit the building blocks?

A diagnosis of the elastic differential cross-sections: 7 TeV data



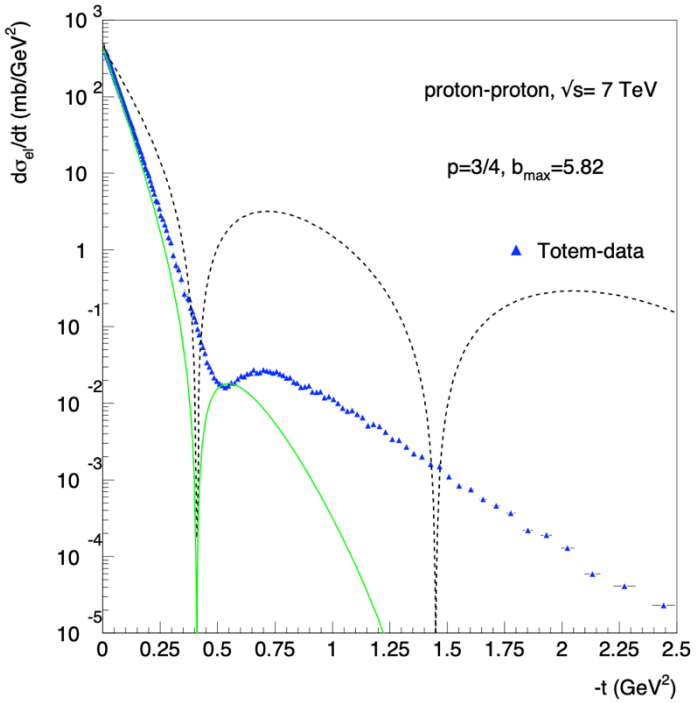
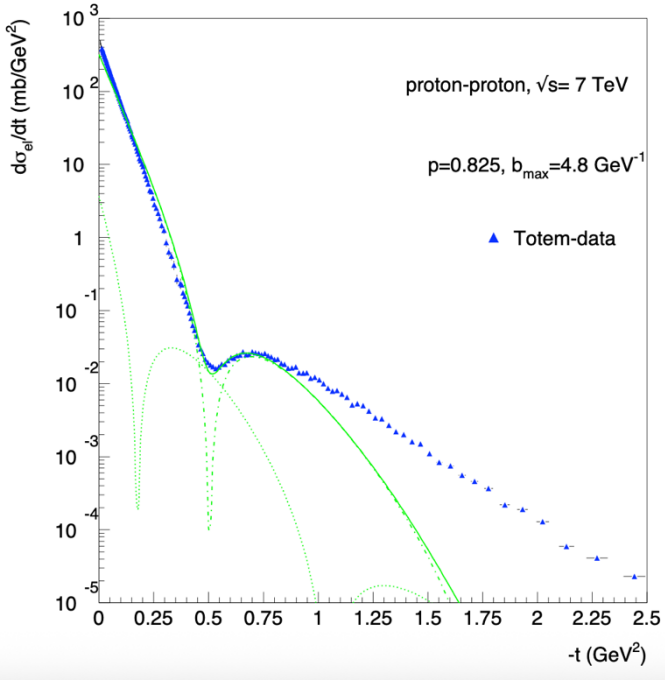


FIGURE 2. The bumps have been suppressed by $exp(tb^2_{max})$ but there is still no dip, only a zero.



Calculate **Imaginary** part of the eikonal
→bumps from the J0→ p dependence from b-distribution in eikonal function
→p as in sigma-total OK at zero t

Full green line → Adding *ad hoc* exponential cut-off
➔ no bumps
➔ misses position of the dip

Add real part from Martin's expression
→ dip and a single bump
→ misse

Things get better when

- Forward peak region : chosing larger p

→ only 2 flavours → ok with alpha_s model

- Adding a real part

→ eliminates the zeros of the J_0(qb)

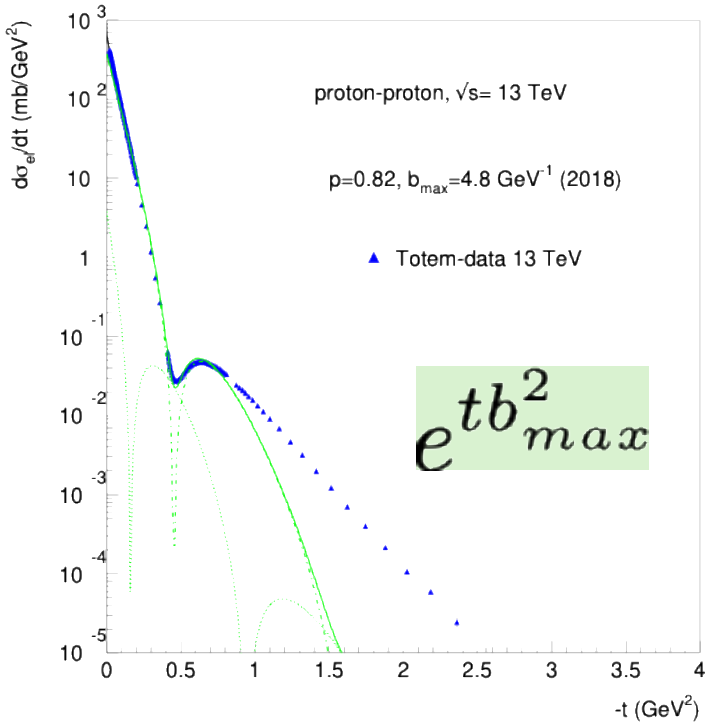
- $A(s,q) = i\int bdb(1-e^{-\chi(b,s)})[J_0(bq)+\rho(s)\int bdb(1-e^{-\chi(b,s)})[J_0(bq)-\frac{qb}{2}J_1(qb)]$

- Introducing an ad hoc exponential cut off in t

→ lowers the bump

b_{max} chosen in range 4.5 -5 GeV^2

13 TeV TOTEM DATA 2018



Where can such cutoff come in the SGR model ?

What can give the exponential cutoff????
Discrete and continuum extension in Touschek's formalism : Zero momentum mode in QED

$$\sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}[1-e^{-i(k\cdot x)}] = \bar{n}_{\mathbf{k}=0}[1-e^{-i\omega_o t}] + \sum_{\mathbf{k}\neq 0} \bar{n}_{\mathbf{k}}[1-e^{-i(k\cdot x)}]$$

- Separate the zero mode and take **k**→0 with fictitious photon mass

$$h_0(t) = \bar{n}_0[1-e^{-i\omega_0 t}] = \frac{2\pi e^2}{L^3\mu^3}\left|\sum_i \epsilon_i \mathbf{v}_i\right|^2[1-e^{-i\mu t}] \approx i\tilde{\omega}\, t$$

$$\tilde{\omega} = \frac{2\pi e^2}{L^3\mu^3}\mu\left|\sum_i \epsilon_i \mathbf{v}_i\right|^2 \qquad n.r. \qquad \tilde{\omega} = 0$$

QED v.b.c

Limit depends on boundary conditions

F. Palumbo, Physical Effects of Boundary Conditions in Gauge Theories. Phys. Lett. B,132:165–168, 1983
F. Palumbo,G.P.,Infrared Radiative Corrections and the Zero Momentum Mode of Abelian Gauge Theories, Phys. Lett. B, 137:401– 403, 1984

How about resummation in QCD (II)?

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b}e^{-i\mathbf{K}_\perp\cdot\mathbf{b}-\sum_{\mathbf{k}}\bar{n}_{\mathbf{k}}[1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]}$$

Semi-classical &very general

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b}e^{-i\mathbf{K}_\perp\cdot\mathbf{b}-h_0(\mathbf{b})-\mathbf{h_{continuum}}(\mathbf{b})}$$

GP&YS &Shekhostova *Ukr.J.Phys.* 69 (2024) 11, 879 (2024)

$$h_0(\mathbf{b}) = \bar{n}_0[1-e^{i\mathbf{b}\cdot\mathbf{k_{t0}}}] \approx \bar{n}_0[-i\mathbf{b}\cdot\mathbf{k_{t0}} + b^2k_{t0}^2 + ...] \rightarrow -h_0 = -(\bar{n}_0\nu^2/4)b^2$$

Zero mode

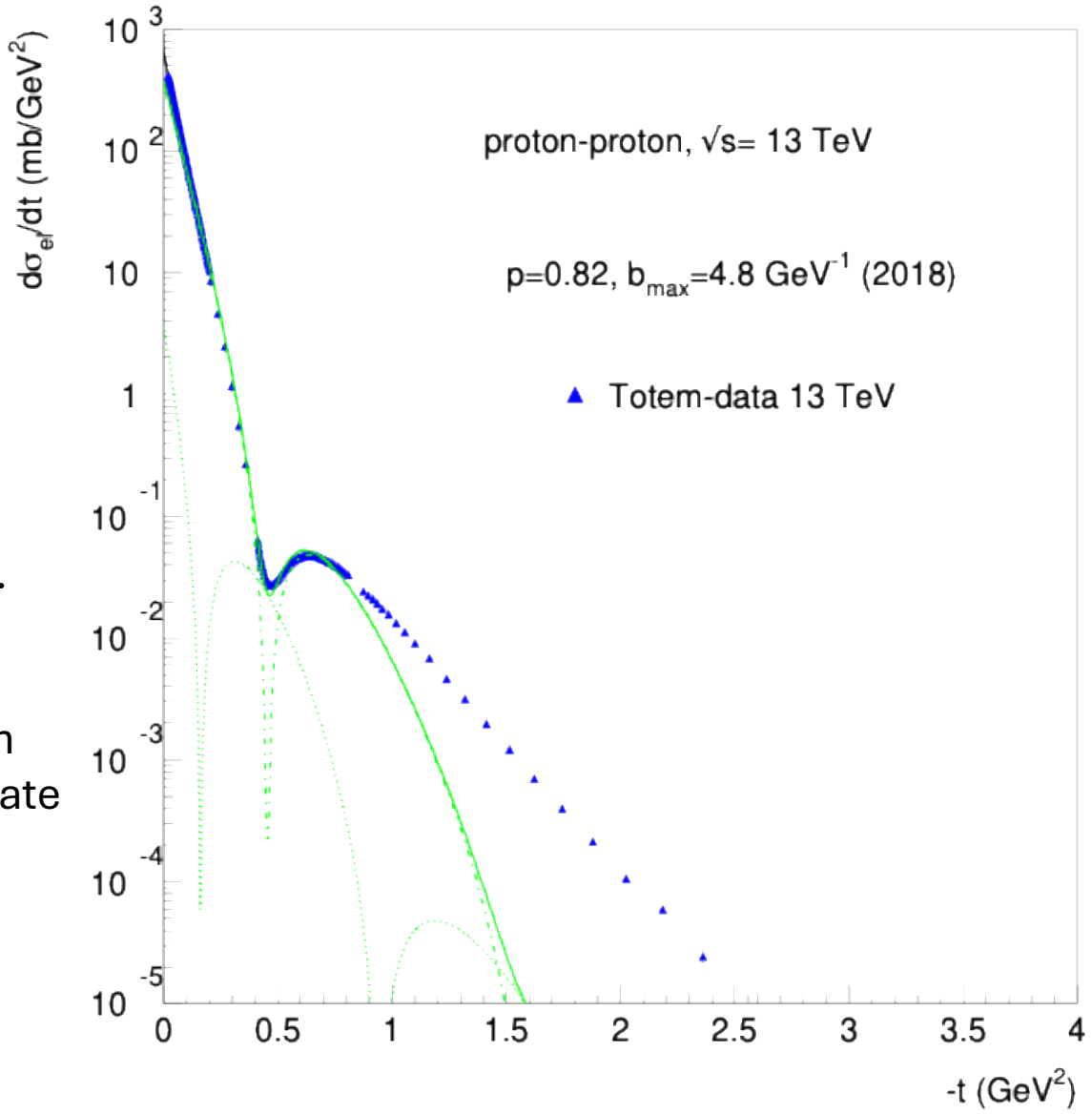
A gaussian cut off in b-space?

LHC@13 TeV data

- Good description 0.2 < -t <1 GeV
  - Follows exponential cut off → forward peaK
  - Agrees with position of the dip
    With p=0.82 but programs
    need to be rewritten

- Miss the total cross-section point for TOTEM data.
  Would it agree with ATLAS total?

- Miss the tail → probably 3 gluon contribution such
  q qbar +soft gluons + hard gluon in initial state
  or Odderon?



Conclusion (I) : Radiation of soft photons or gluons

- when the emitting charged particle is undergoing a collision and the emitted energy is much smaller than the available collision energy ➔emission can be factorized from the core process

- Different from fragmentation

- Impact parameter-distribution naturally arising in QCD

Conclusions (II)

- Our model for total cross-section: called Bloch Nordsieck (BN) model because of resummation of Poisson distribution and finite result for overall emission [PLB 1996, PRD 1999, PRD 2005]

- A laboratory to study alpha strong in continuum region

- Extending the model → elastic differential cross-section → A probe into the zero momentum mode in QCD

The END (for now)

Updating previous work adding only 13 TeV data

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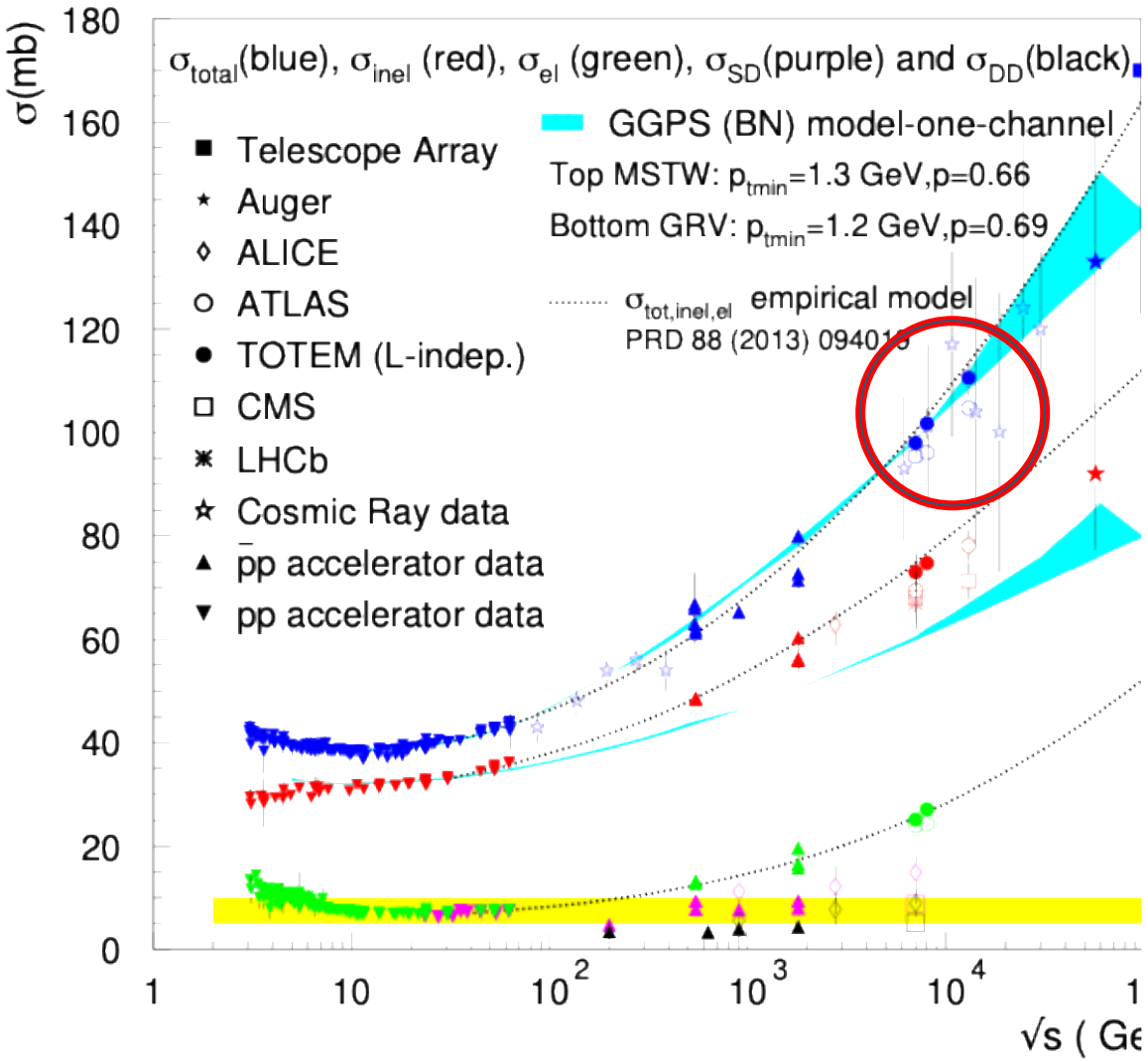
- Phys. Rev. 2017→ 13 TeV data

- **impact**-parameter distribution with initial state SGR and Ansatz

$$\alpha_s(k_t \to 0)$$

- + minijets



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Semi-classical &very general

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b} e^{-i\mathbf{K}_\perp\cdot\mathbf{b} - h_0(\mathbf{b}) - \mathbf{h_{continuum}}(\mathbf{b})}$$

GP&YS &shekhostova *Ukr.J.Phys.* 69 (2024) 11, 879 (2024)

$$h_{continuum}(b) = \frac{8}{3\pi^2}\int_0^{q_{max}} d^2\mathbf{k}_t \frac{\alpha_s(k_t^2)}{k_t^2}\ln[\frac{2q_{max}}{k_t}][1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]$$

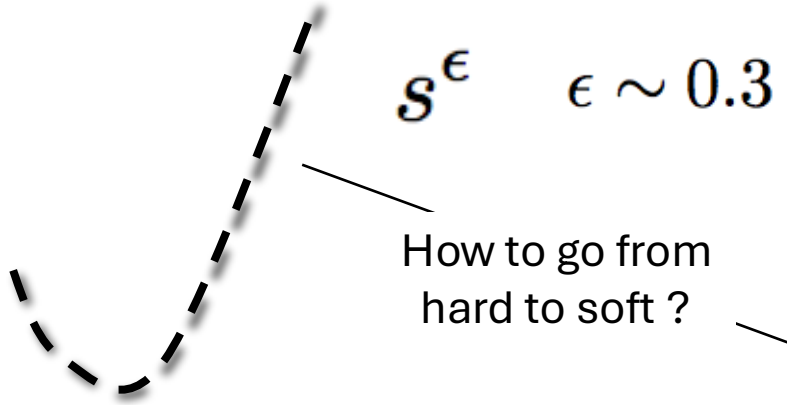
$$h_0(\mathbf{b}) = \bar{n}_0[1-e^{i\mathbf{b}\cdot\mathbf{k_{t0}}}] \approx \bar{n}_0[-i\mathbf{b}\cdot\mathbf{k_{t0}} + b^2k_{t0}^2 + ...] \rightarrow -h_0 = -(\bar{n}_0\nu^2/4)b^2$$

Zero mode

A gaussian cut off in b-space?

All total cross-sections rise... but not too much (Froissart dixit)

What generates the rise? Low-x parton collisions

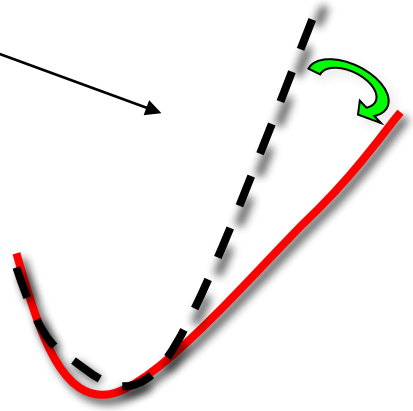


$$s^{\epsilon} \qquad \epsilon \sim 0.3$$

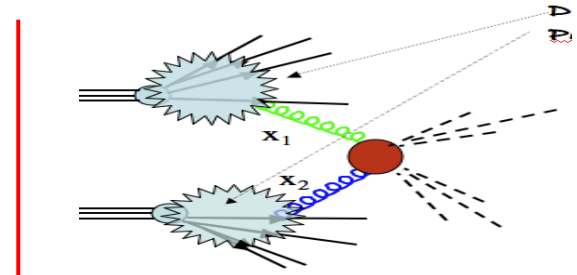
Cline,Halzen &Luthe 1973
Gaisser, Halzen,Stanev 1985
G.P., Y.N. Srivastava 1986
Durand,Pi 1987
Sjostrand, van Zijl 1987
...

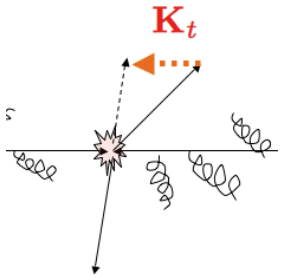
How to go from hard to soft ?

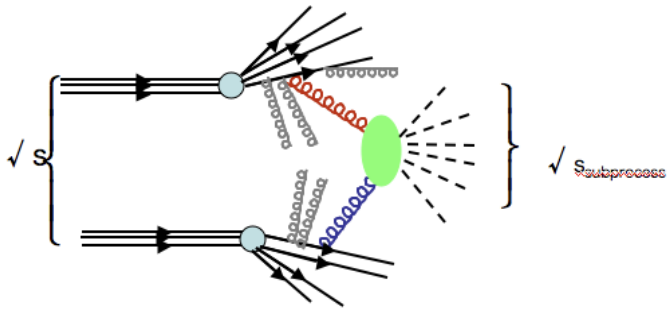
What tames the rise into to a Froissart-like behavior?

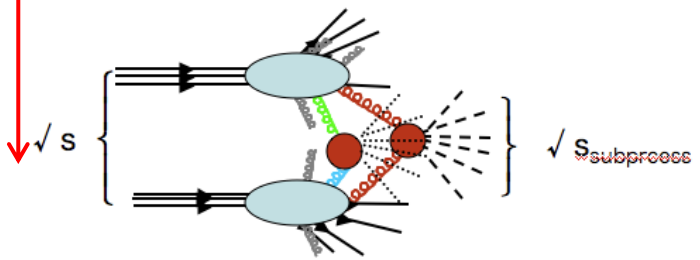


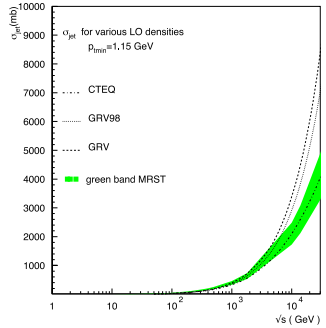
A cut off obtained by [embedding into the eikonal] the acollinearity induced by IR kt-emission
[our model, G.P. et al. **Phys.Lett.B382, 1996**]

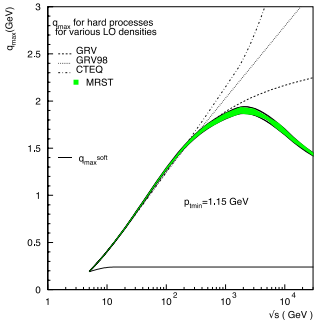


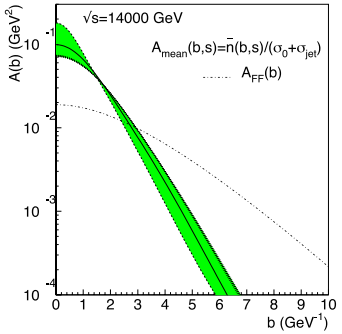


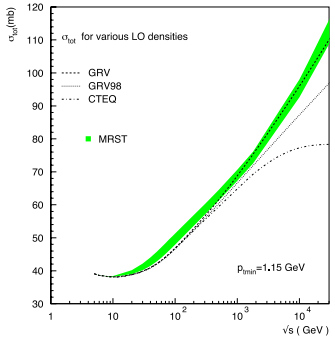












1. Calculate mini-jet cross-section
Choosing densities and ptmin

$$\sigma_{mini-jet} \simeq s^{\epsilon}$$
$$\epsilon \simeq 0.3 - 0.4$$

2. Calculate qmax: single soft gluon upper scale, for given PDF, ptmin

$$q_{max} \simeq p_{tmin}$$
$$\lesssim 2 - 3\ GeV$$

3. Calculate impact parameter distribution for given qmax and given infrared parameter p

$$\chi(b,s) = \chi_{low\ energy} +$$
$$+ A(b, qmax)\sigma_{jet}$$

4. Eikonalize

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi(b,s)}]$$

Why the mini-jet cross-section rises so much

Fagundes et al. , *Phys.Rev.D* 91 (2015) 11, 114011

further, the cross section for collisions resulting with a final state parton with $p_t \gtrsim 1$, will start rising because of $1/x$ behavior, entering into a *high gluon luminosity* region. We see that the contribution to the total cross section from these hard collisions can become sizable and rising with energy when

$$1/x = \sqrt{s}/2p_t >> 1 \; and \; p_t > 1 \; GeV. \tag{1}$$

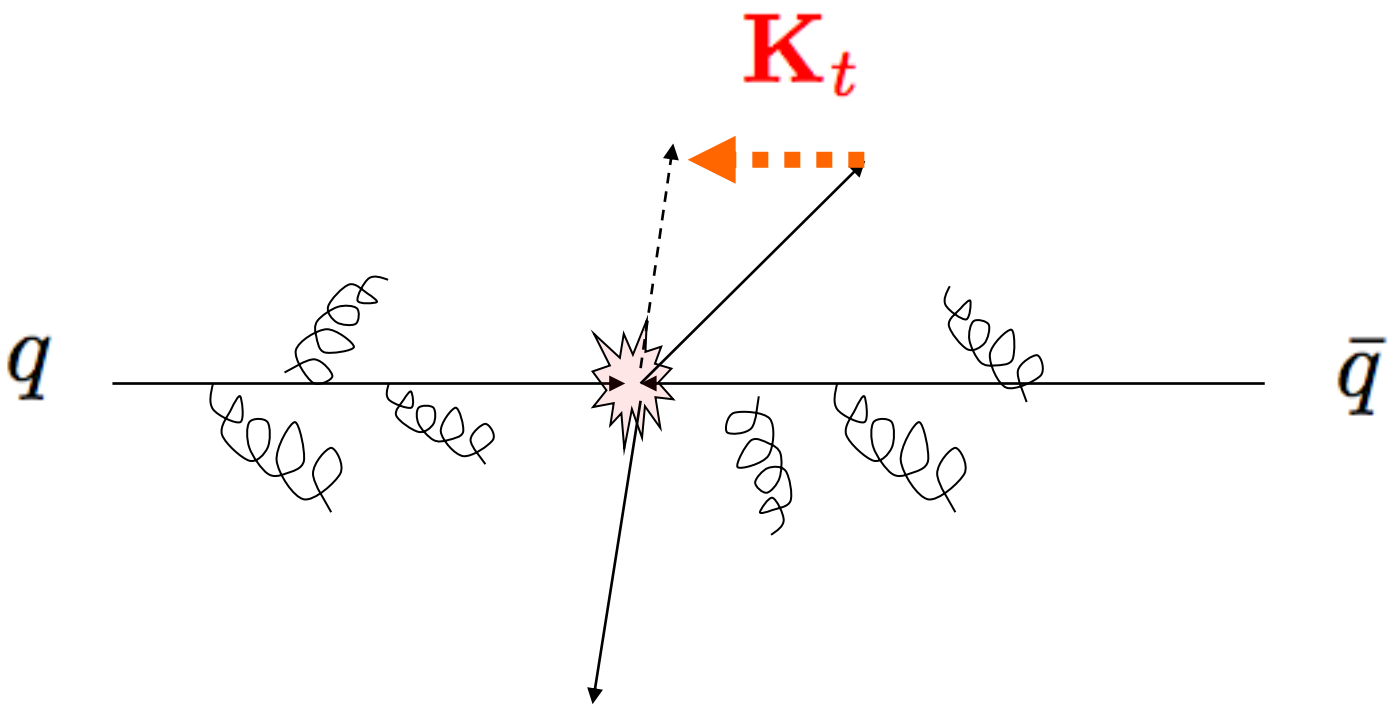
With the condition $p_t > 1 \; GeV$ for the *asymptotic freedom condition* to be satisfied, and the rise starting when $x << 1$, for instance $\simeq 0.1 - 0.2$, the turning point for the rise to start is for

$$\sqrt{s} \gtrsim (2/x) GeV \tag{2}$$

$$\sqrt{s} \sim (10 \div 20) \; GeV \tag{3}$$

as the data show. The transition from collisions which do not involve scattering of perturbative gluons, to a region dominated by mini-jets, is what is seen in the total cross section data, and is discussed, in a somewhat different language, in [1].

Soft gluon emission introduces acollinearity



Acollinearity reduces the collision cross-section as
partons do not scatter head-on any more, also explained as the gluon cloud
Becoming too thick for partons to see each other : gluon saturation

Major traits of our model

- Energy rise from mini-jets (to be obtained from DGLAP evoluted PDF and parton-parton x-sections)

$$\sigma_{jet}^{PDF} \sim s^{\epsilon_{PDF}} \sim s^{0.3-0.4}$$

- Saturation comes as a large distance effect: at large distances, soft gluon emission leads to a cut-off

$$A(b,s) \sim e^{-[b\bar{\Lambda}(s)]^{2p}}$$

- Embedded into eikonal formulation

In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering (mini-jets) and restores the Froissart bound

$$s^{\varepsilon}$$

$$\sigma_{tot}(s) \approx 2\pi \int_0^{\infty} db^2 [1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}]$$

$$\sigma_{tot}(s) \rightarrow [\varepsilon \ln(s)]^{(1/p)} \qquad \frac{1}{2} < p < 1$$

Grau, Godbole,GP,Srivastava,PLB682 2009